Untitled

Linear analysis

Additionally, we considered a standard *linear regression* model involving all predictors, as an alternative means to view the significance of each predictor. Note that in this context, performing *k-fold cross-validation* or *bootstrapping* isn't necessary as we're only interested in significant predictors, hence we performed an ordinary 80/20 splitting of the data into a training and a testing set as shown in the code below:

We then run a simple linear fit will all predictors, in order to analyse the significance levels of the parameters, provided that the linear test itself has a significant R^2 value.

```
##
## Call:
## lm(formula = crmrte ~ ., data = Crime)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.027793 -0.005201 -0.000603  0.004163  0.038831
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.115e-01  2.696e-02   4.136 4.03e-05 ***
## county       5.086e-06  6.285e-06   0.809 0.418738
## year        -1.463e-03  3.674e-04  -3.982 7.67e-05 ***
## prbarr      -3.334e-02  3.323e-03 -10.033  < 2e-16 ***
## prbconv     -2.062e-03  2.847e-04  -7.244 1.34e-12 ***
## prbpris      1.952e-03  4.264e-03   0.458 0.647334
## avgsen      -1.073e-04  1.355e-04  -0.792 0.428543
## polpc        1.725e+00  2.150e-01   8.021 5.51e-15 ***
## density      7.091e-03  5.542e-04  12.795  < 2e-16 ***
## taxpc        1.577e-04  3.932e-05   4.011 6.81e-05 ***
## regionother  4.816e-03  1.010e-03   4.770 2.32e-06 ***
## regionwest  -1.411e-03  1.235e-03  -1.143 0.253584
## smsayes     -3.429e-03  2.448e-03  -1.401 0.161817
## pctmin       1.326e-04  3.421e-05   3.876 0.000118 ***
## wcon        -6.811e-07  3.022e-06  -0.225 0.821741
## wtuc        -1.956e-07  1.330e-06  -0.147 0.883171
## wtrd         4.305e-06  4.261e-06   1.010 0.312703
## wfir        -1.014e-05  1.009e-05  -1.005 0.315466
## wser        -4.180e-06  3.529e-06  -1.184 0.236778
## wmfg        -1.401e-06  5.970e-06  -0.235 0.814550
## wfed         4.413e-05  9.551e-06   4.620 4.70e-06 ***
## wsta        -6.671e-06  9.998e-06  -0.667 0.504907
## wloc         4.167e-05  1.788e-05   2.331 0.020066 *
## mix          4.689e-03  2.218e-03   2.114 0.034926 *
## pctymle      8.609e-02  1.611e-02   5.344 1.29e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.008666 on 601 degrees of freedom
## Multiple R-squared:  0.7544, Adjusted R-squared:  0.7445
```

```
## F-statistic:  76.9 on 24 and 601 DF,  p-value: < 2.2e-16
```

as the R^2 value is sufficiently high (0.754353), we decided to perform `best subset selection` on the set of predictors. Although we are aware of the performance penalties of doing this for $p = 23$, the running times were considerably short and hence we decided to stick to this approach:

After getting all best subsets with size $k = 1...p$, we analysed both the *training* and *testing* by performing *k-fold cross validation* with $k = 10$ and then getting the minimum errors on all iterations:

The minimum obtained training error is

```
## [1] 9.415169e-05
```

The minimum obtained test error is

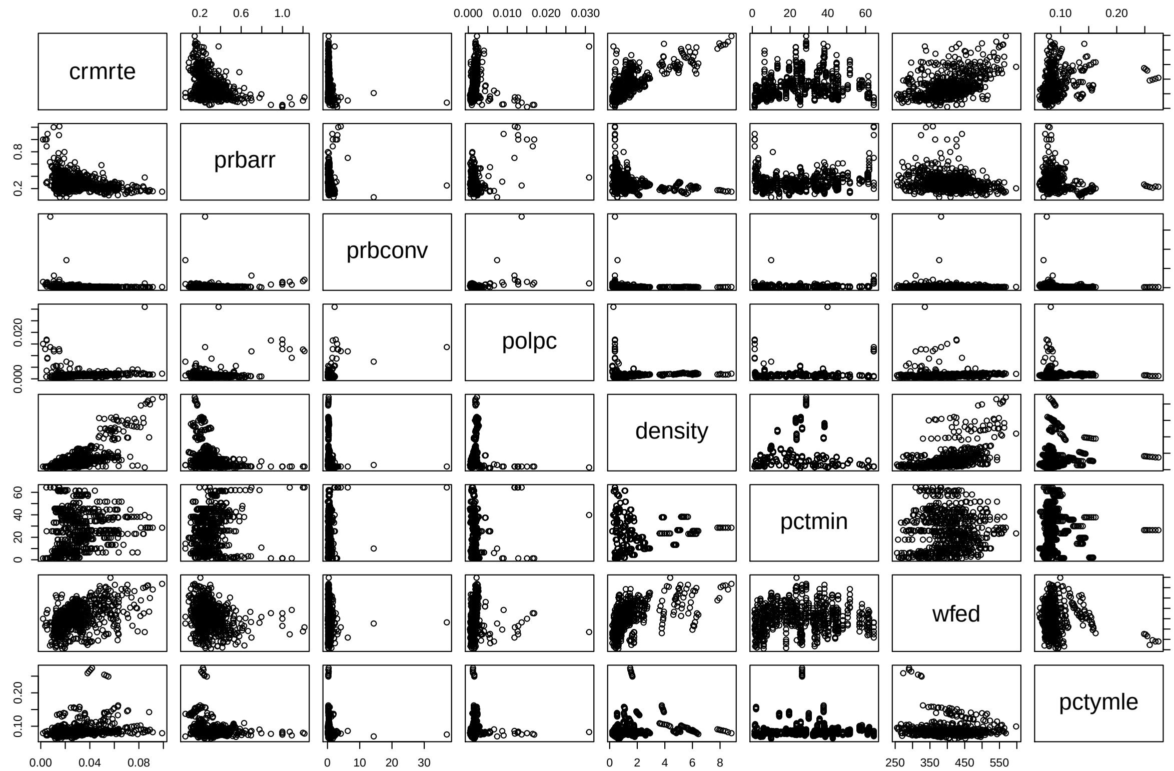
```
## [1] 6.455752e-05
```

both obtained when using the best subset with $k = 8$ predictors. The ratio between *testing* and *training* is (0.6856757). Consequently, we can conclude that the predictors yielded by the subset generated by *best subset selection* which minimizes both *s* belong to a consistent model and, hence, can be used as a basis for non linear models. Nevertheless, we decided to run a linear fit with these predictors in order to check our conclusions:

```
##
## Call:
## lm(formula = crmrte ~ prbarr + prbconv + polpc + density + as.factor(region) +
##     pctmin + wfed + pctymle, data = Crime)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.021247 -0.005769 -0.000725  0.004217  0.047783
##
## Coefficients:
##                         Estimate Std. Error t value Pr(>|t|)
## (Intercept)            7.205e-03  3.586e-03   2.009 0.044942 *
## prbarr                -3.202e-02  3.098e-03 -10.336  < 2e-16 ***
## prbconv               -1.807e-03  2.502e-04  -7.222 1.51e-12 ***
## polpc                  1.944e+00  2.112e-01   9.203  < 2e-16 ***
## density                7.193e-03  3.171e-04  22.684  < 2e-16 ***
## as.factor(region)other  4.681e-03  9.785e-04   4.784 2.15e-06 ***
## as.factor(region)west  -3.471e-03  1.161e-03  -2.990 0.002898 **
## pctmin                 1.243e-04  3.220e-05   3.861 0.000125 ***
## wfed                   2.641e-05  6.936e-06   3.808 0.000154 ***
## pctymle                7.466e-02  1.546e-02   4.828 1.74e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.00887 on 616 degrees of freedom
## Multiple R-squared:  0.7362, Adjusted R-squared:  0.7324
## F-statistic: 191.1 on 9 and 616 DF,  p-value: < 2.2e-16
```

We can note that all coefficients are significant and the R^2, as expected, was reduced but only marginally (0.7362415 versus 0.754353), which confirms that the model with this subset is indeed a good model.

Next, we proceeded to graphically analyse any nonlinearities between these predictors and the response, by looking at all pairwise plots:



It can be seen that `prbrarr`, `prbconv`and `polpc` have a peak structure that would benefit from applying a *log* to them in order to shrink those peaks. Additionally, `wfed` has a nonlinear relationship with `wfed`, which makes it suitable as a polynomial regression predictor. Consequently, we run a new, nonlinear model with these modified predictors:

```
##
## Call:
## lm(formula = crmrte ~ log(prbarr) + log(prbconv) + log(polpc) +
##     density + as.factor(region) + pctmin + poly(wfed, 3) + pctymle,
##     data = Crime)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.024202 -0.004384 -0.000246  0.003672  0.045724
##
## Coefficients:
##                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)             0.0564773  0.0046897  12.043  < 2e-16 ***
## log(prbarr)            -0.0144423  0.0009070 -15.923  < 2e-16 ***
## log(prbconv)           -0.0113128  0.0006276 -18.024  < 2e-16 ***
## log(polpc)              0.0099603  0.0006892  14.452  < 2e-16 ***
## density                 0.0052809  0.0003094  17.068  < 2e-16 ***
## as.factor(region)other  0.0039568  0.0008168   4.844 1.61e-06 ***
## as.factor(region)west  -0.0042023  0.0009729  -4.319 1.82e-05 ***
## pctmin                  0.0001813  0.0000271   6.691 5.01e-11 ***
```

```
## poly(wfed, 3)1          0.0155450  0.0094692   1.642    0.101
## poly(wfed, 3)2          0.0006990  0.0079938   0.087    0.930
## poly(wfed, 3)3         -0.0101414  0.0076411  -1.327    0.185
## pctymle                 0.0073072  0.0135778   0.538    0.591
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.007408 on 614 degrees of freedom
## Multiple R-squared:  0.8166, Adjusted R-squared:  0.8133
## F-statistic: 248.6 on 11 and 614 DF,  p-value: < 2.2e-16
```

The lack of significance of the polynomials for `wfed` and the increase in R^2 suggests that this model *overfits*. Hence, we decided to remove the polynomials related to `wfed`, but kept the *log* predictors as they have shown to be still very significant.

Next, we analysed all significant interactions between all interaction terms

```
## [1] 0.9706801
```

and plugged all interactions to out previous model:

```
## [1] 0.9725358
```

The number of interactions that we added to the model are:

```
## [1] 300   4
```

We can see that both models are seriously overfitting (R^2 values are 0.9706801 and 0.9725358 respectively, while the number of predictors has skyrocketed due to all interaction combinations). Even though it is tempting to keep only those interactions with a relevant significance value, since the removal of each of these predictors affects the overall model, we decided instead to choose a final model by using a *Stepwise Algorithm* applying *AIC* to decide. The stepwise procedure is too lengthy and cumbersome to show in text, but the code to generated is displayed below:

Once we got the fit, which has the following number of coefficients:

```
## [1] 190
```

which means a reduction on the number of interactions by 30%, we proceeded to calculate both *training MSE* and *testing MSE*:

```
## [1] 0.001248871
```

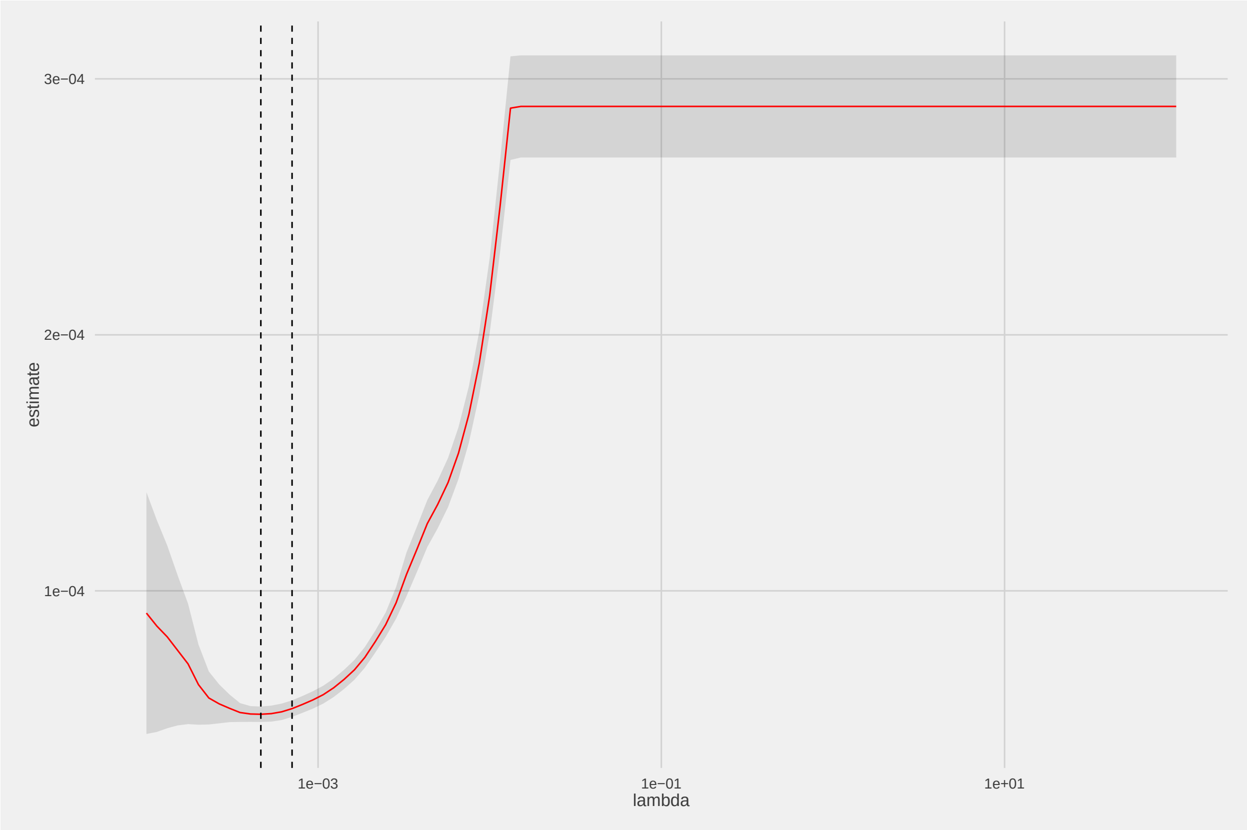
```
## [1] 0.001365305
```

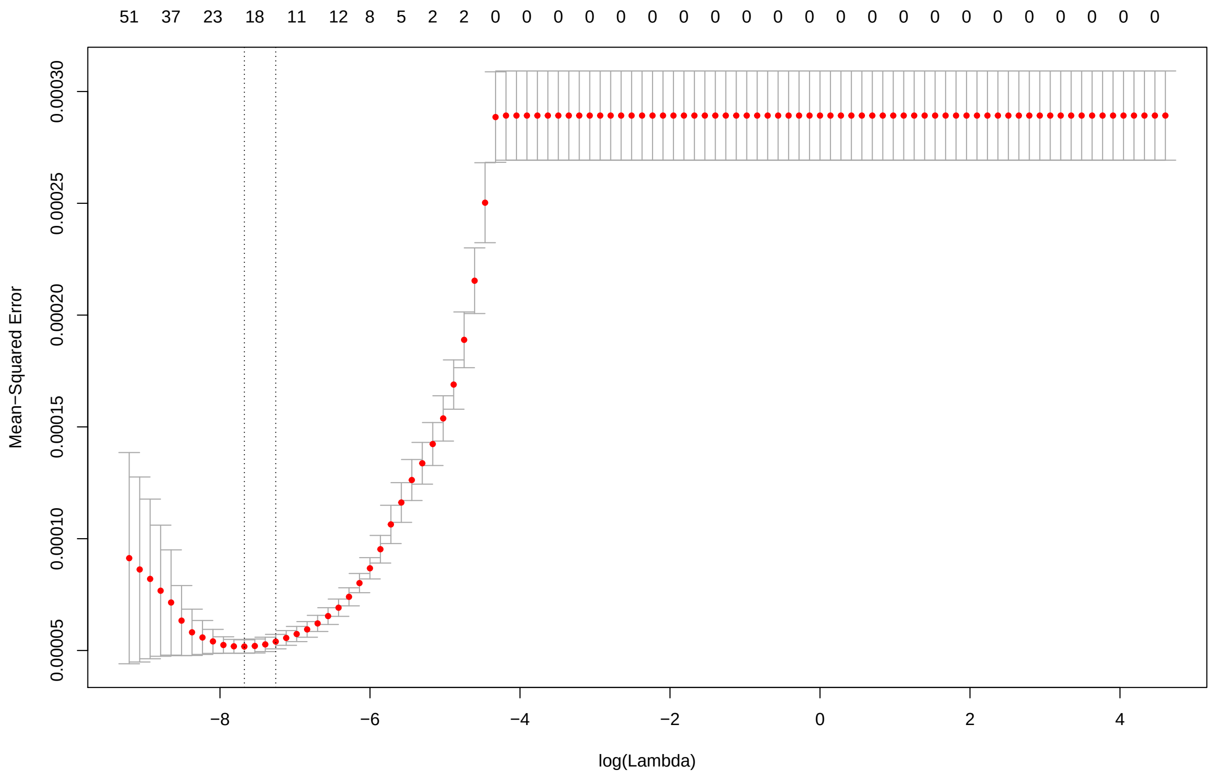
Here, we can see that the error rate remains close to one (1.0932317) which still proves that this model holds. Now that we obtained a complex model consisting of linear variables, *log* variables and *interaction* variables, we're gonna perform *Lasso* in order to remove all interaction terms that are not significant, so that we arrive to a model easy to understand.

Lasso Analysis

With the resulting model from all our previous steps, we performed *k-fold cross validation* using Lasso, in order to obtain the optimum value of λ for our model. The code generating the lasso is the following (note that the first line is a way to manually represent `interaction.fit` as the fit is not recognized by `cv.glmnet`, which requires a formula with a specific formatting):

Now that we have computed all possible values for *lambda*, we can create the plot showing the *training* and λ increases:





We then chose a value of λ within 1 standard deviation from the optimum value, as this is a commonly established good practice. The fit with that value is performed in the code below:

```
## [1] 16
```

Lasso actually yielded 16 variables with nonzero values, a reduction of 92% with respect to the *stepwise AIC*. The *testing* for this model is obtained using the same *k-folds* generated for the first model:

```
## [1] 5.141231e-05
```

and the error rate w.r.t. to our original model using *best subset selection* is:

```
## [1] 79.63799
```

Finally, we analyse the relative average $L1$ distance between our estimators and the true model:

```
## [1] 0.1969748
```

which yields a more than reasonable value, since it's below 30%. Consequently, the final set obtained in this section, after performing *linear*, *best subset selection*, *log*, *polynomial* , *stepwise AIC* and *Lasso* yielded a model that will be used in the following sections for more complex fits that will derive in our final model.