Report

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Contents

Introduction

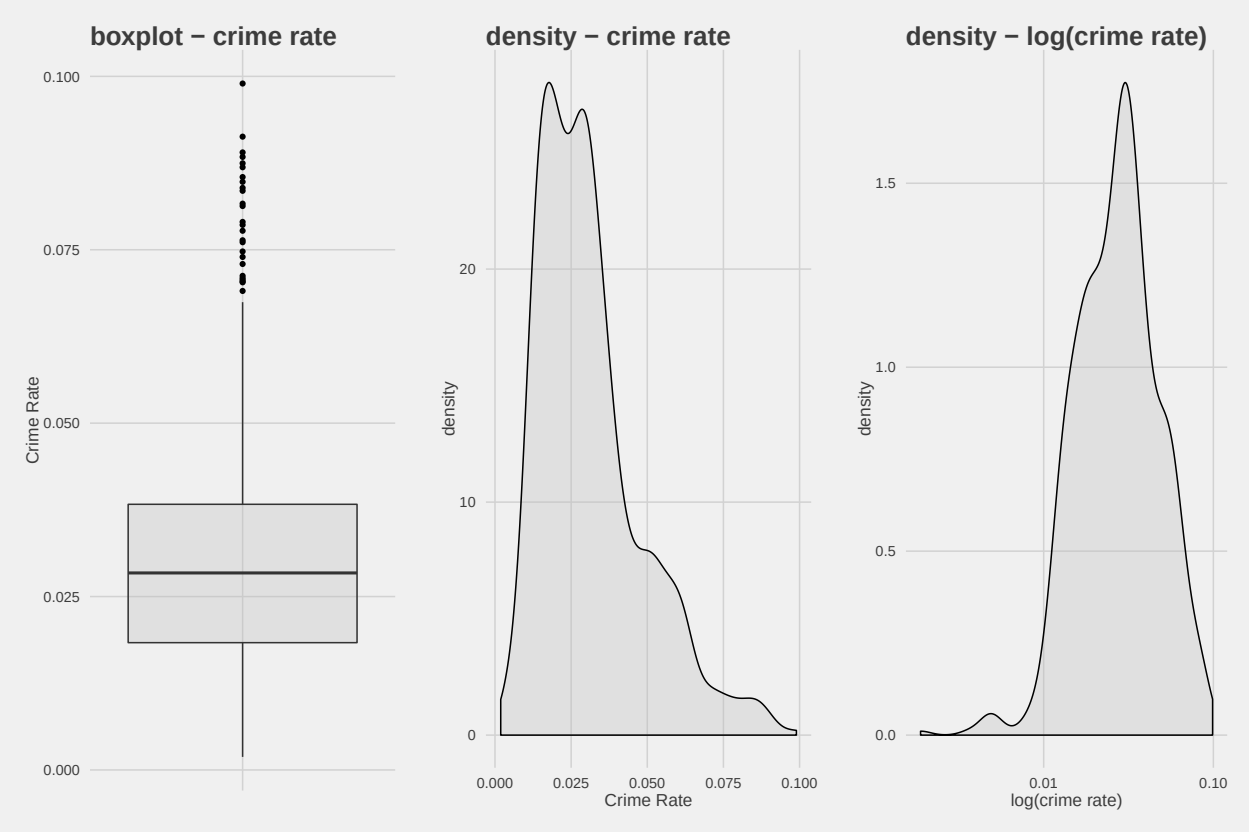
Understanding the factors behind criminal behaviour is one of the most crucial tasks for preventing and controlling future crime. In this report, we explore the potential factors affecting crime rates based on the demographics and econometrics data gathered from 197 counties in North Carolina from 1981 to 1987. Using various statistical methods and modeling techniques, we analyze and identify the most important factors and metrics tied to crime rates. We also present a predictive model capable of estimating the crime rate with under 25% error using the selected parameters.

Dataset

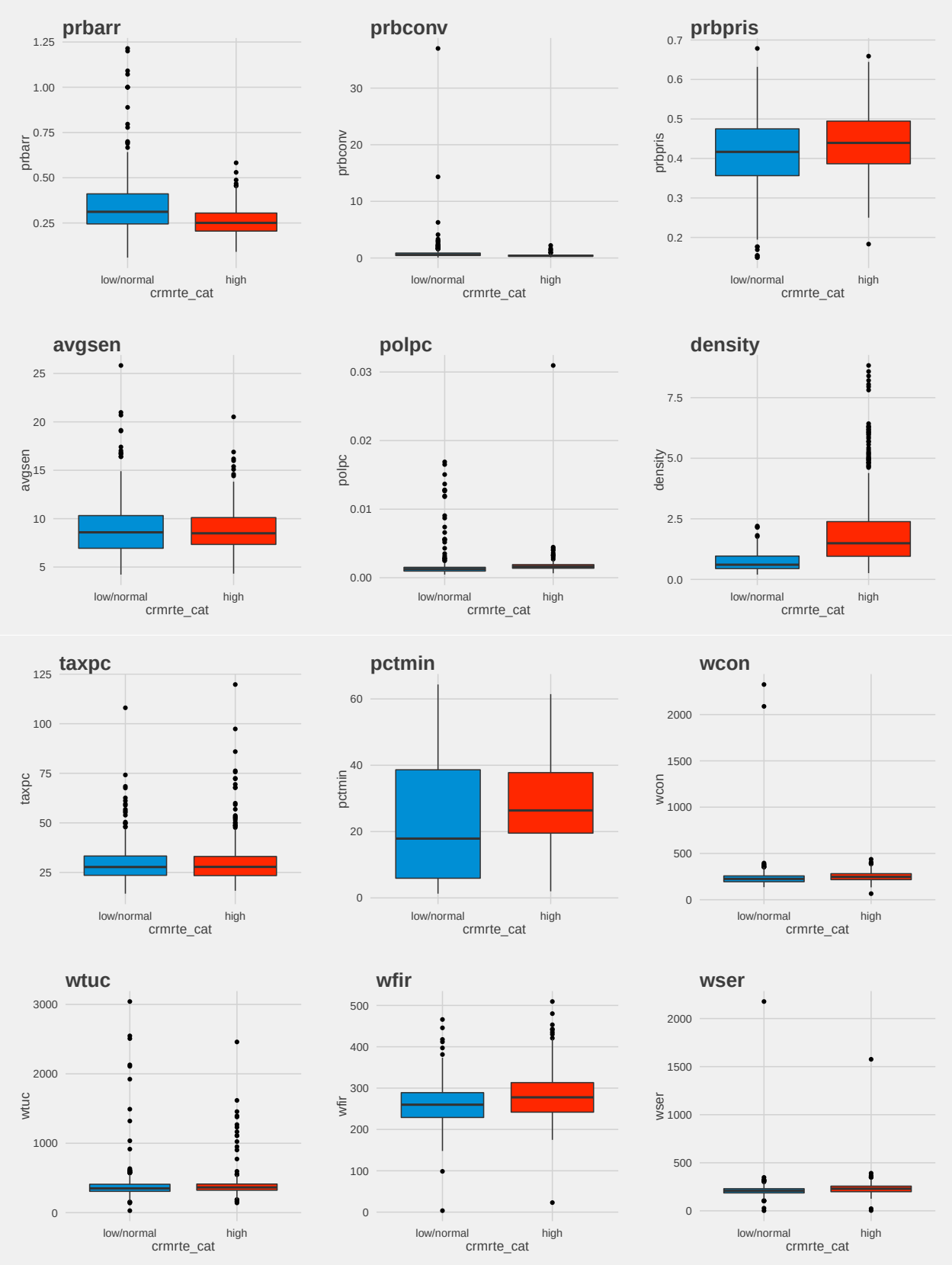
Predictor	Description
county	county identifier
year	year from 1981 to 1987
crmrte	crimes committed per person
prbarr	'probability' of arrest
prbconv	'probability' of conviction
prbpris	'probability' of prison sentence
avgsen	average sentence, days
polpc	police per capita
density	people per square mile
taxpc	tax revenue per capita
region	one of 'other', 'west' or 'central'
smsa	'yes' or 'no' if in SMSA
pctmin	percentage minority in 1980
wcon	weekly wage in construction
wtuc	weekly wage in trns, util, commun
wtrd	weekly wage in whole sales and retail trade
wfir	weekly wage in finance, insurance and real estate
wser	weekly wage in service industry
wmfg	weekly wage in manufacturing
wfed	weekly wage of federal emplyees
wsta	weekly wage of state employees
wloc	weekly wage of local governments employees mix offence mix: face-to-face/other
pctymle	percentage of young males

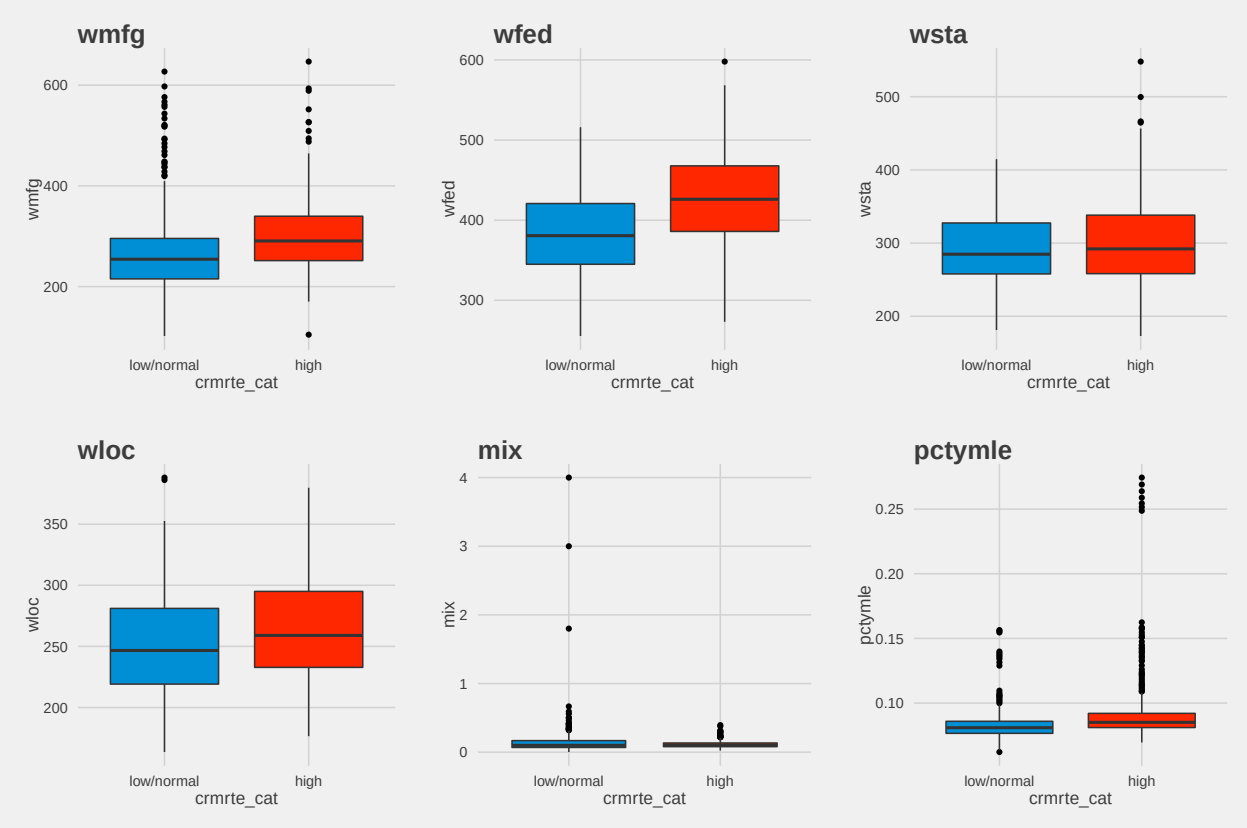
Table 1: Description of the predictors in the dataset

We analyzed the variables in the dataset starting with the target variable: `crmrte`, the crime rate. Along this study, we will use this variable in different forms. We define a categorical value equal to one representing high crime rate, when the value of the target variable is higher that its median value. We called this variable `crmrte_cat`. Also, we will use the natural logarithm of the variable to adequately transform it to apply different statisticals models to predict and describe the data. We assume that the target value depends on the other variables. The behaviour of the target is represented with a boxplot, a softened histogram of the variable, and a softened histogram of the logarithm of the variable.

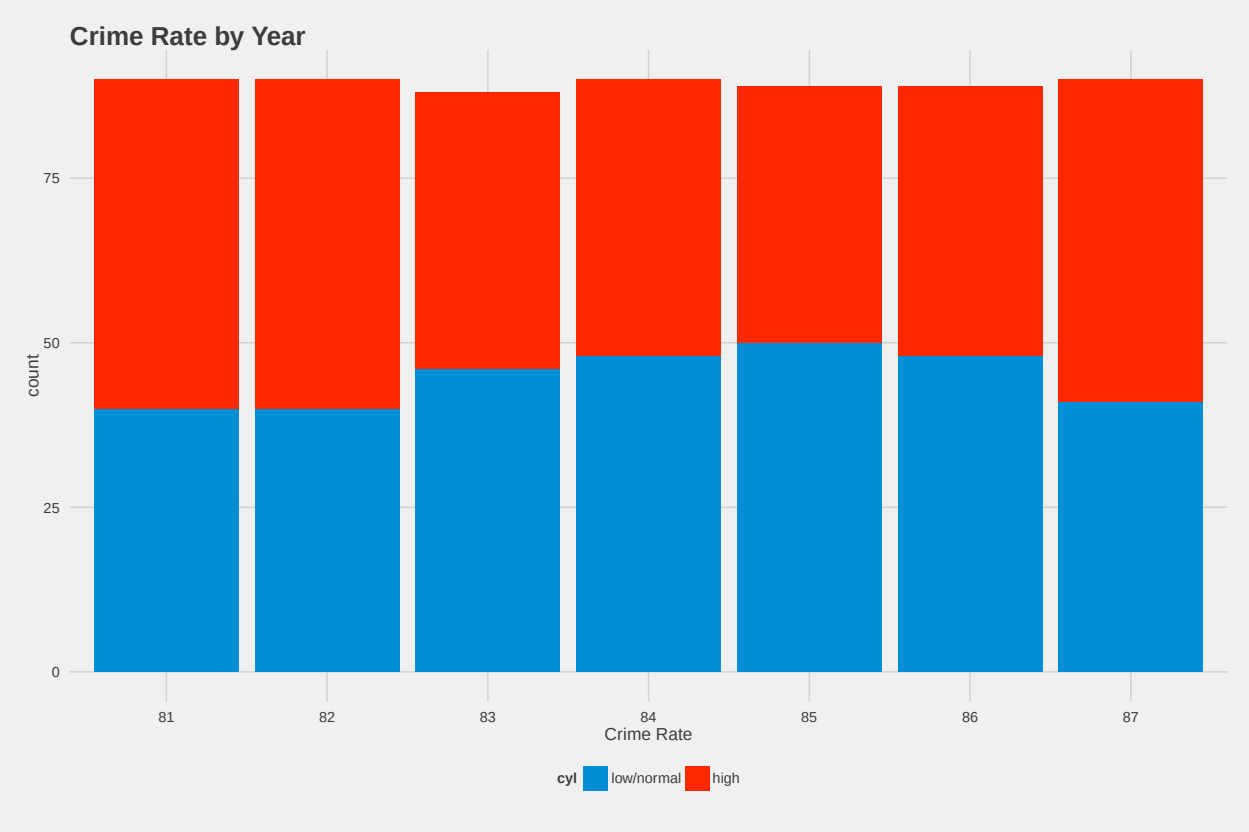


Besides the target variable, the dataset contains other 21 variables we used as predictors. Two of them have categorical values. The `region` variable can have 3 possible values: `other`, `west` or `central` and the `smsa` can have `yes` or `no`. The dataset also contains the `year` variable which can be considered as a time reference. A short description of each variable can be found in the table above. Next, we plot some charts to explore the behaviour of the variables and their relationships with the target.

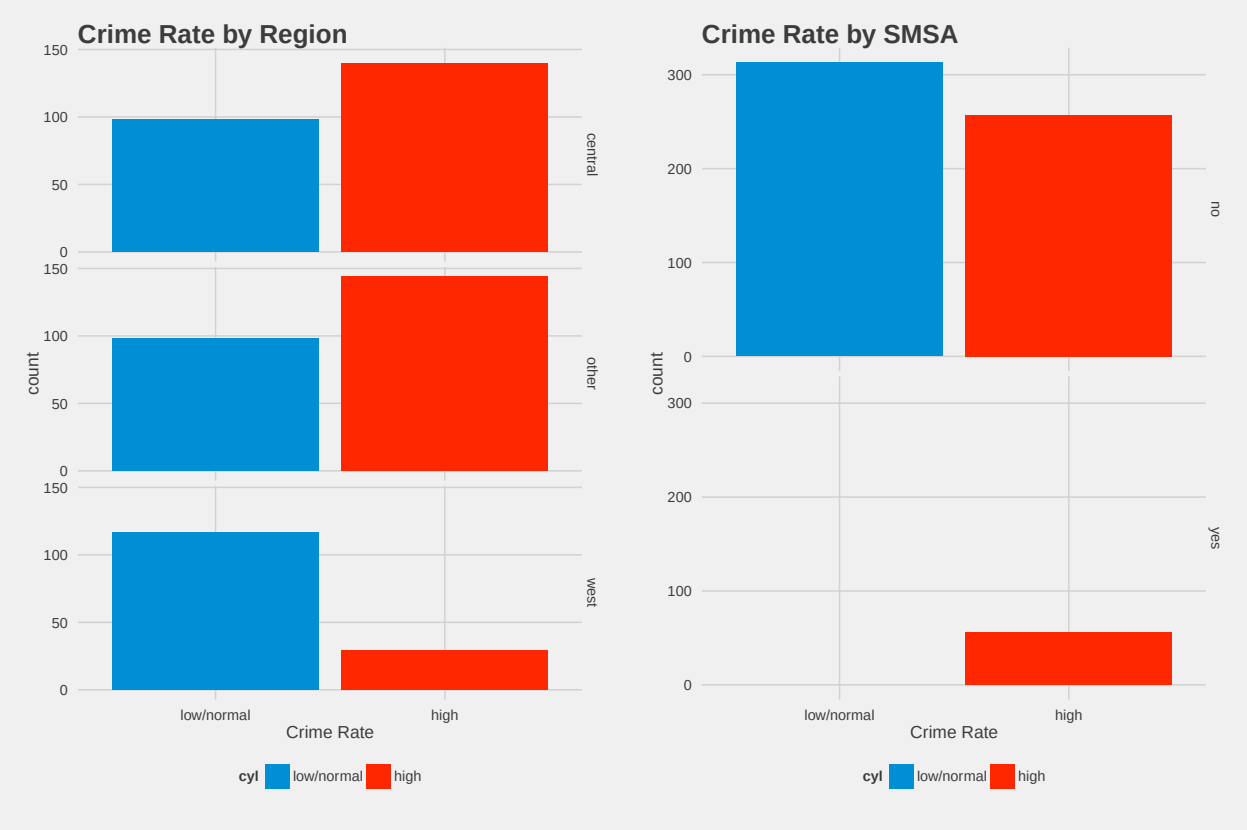

prbarr
prbarr
1.25
1.00
0.75
0.50
0.25
low/normal
high
crmrte_cat
prbconv
prbconv
30
20
10
0
low/normal
high
crmrte_cat
prbpris
prbpris
0.7
0.6
0.5
0.4
0.3
0.2
low/normal
high
crmrte_cat
avgsen
avgsen
25
20
15
10
5
low/normal
high
crmrte_cat
polpc
polpc
0.03
0.02
0.01
0.00
low/normal
high
crmrte_cat
density
density
7.5
5.0
2.5
0.0
low/normal
high
crmrte_cat
taxpc
taxpc
125
100
75
50
25
low/normal
high
crmrte_cat
pctmin
pctmin
60
40
20
0
low/normal
high
crmrte_cat
wcon
wcon
2000
1500
1000
500
0
low/normal
high
crmrte_cat
wtuc
wtuc
3000
2000
1000
0
low/normal
high
crmrte_cat
wfir
wfir
500
400
300
200
100
0
low/normal
high
crmrte_cat
wser
wser
2000
1500
1000
500
0
low/normal
high
crmrte_cat



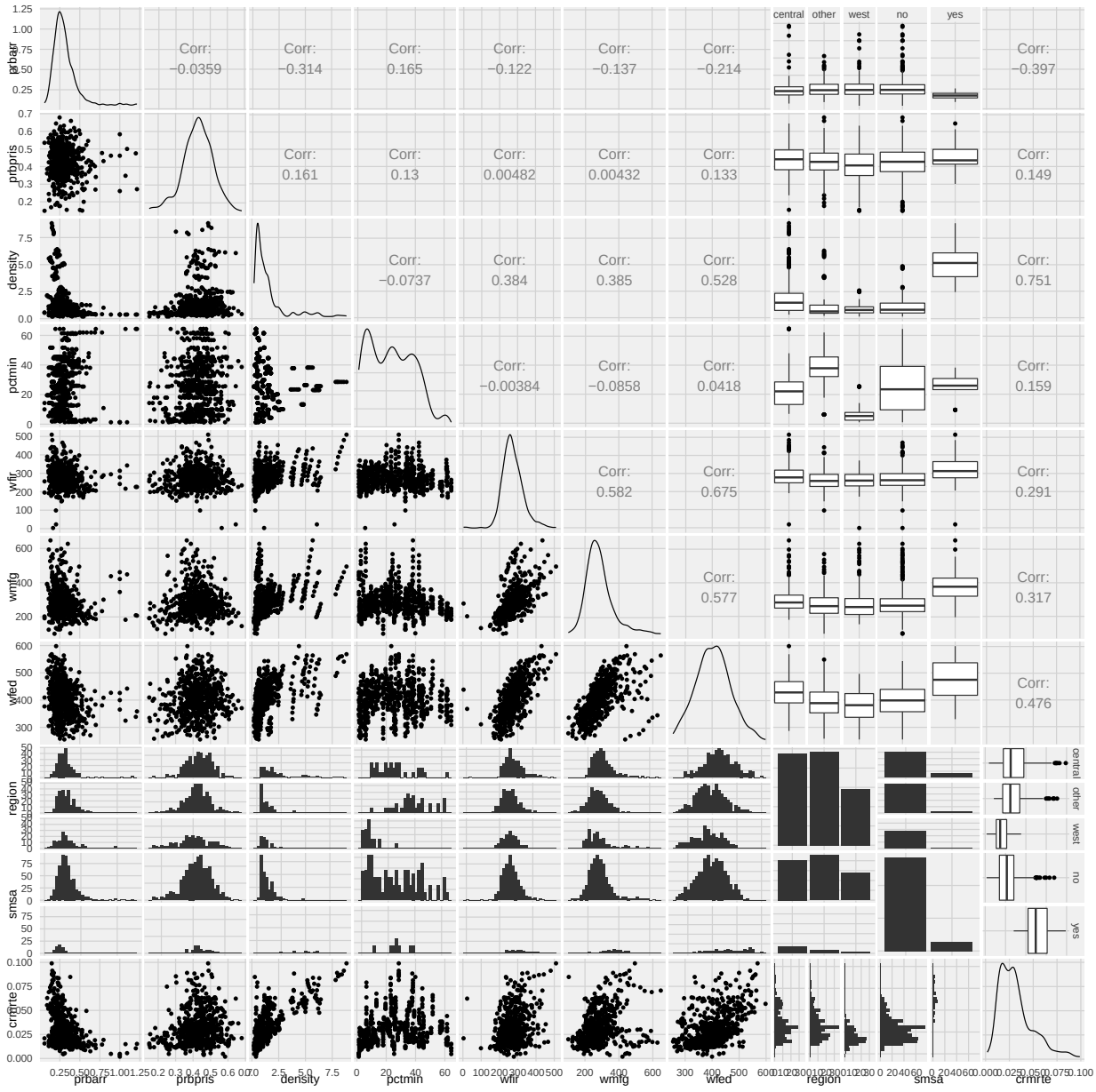
In the boxplots above, we can see that the variables that may have a predictive value with the target are variables `prbarr`, `density`, `pctmin`, `wfed`, `wmfg` and `pctymle` as they separate the population partially by the value of the defined target variable. We explore the rest of the predictors by tracing them on the following charts, starting with the variable `year`.



From the above plot, we notice that there is no significant trend on the crime rate along the timeline being considered. The other two variables with categorical values are `region` and `smsa`.



In these two charts above we see the crime rate decrease when the variable `region` takes the value `west` and when the `smsa` variable takes the value `yes`. Consequently, we continue to further explore the relationship between these two categorical variables and the target variable by implementing *ANOVA* in the next section, but first we analyze the variances and covariances between all predictors. We trace a paired graph with some selected variables in order to explore the correlation between the variables.



In the graphs above we show the correlation between the selected predictors and between the predictors and the target. The highest value of correlation is between the target variable and `density`. Other high values of correlation involve variables `wmfg`, `wfed` and `density`. We will later discuss whether these variables are significant for modeling.

ANOVA models

In the first analysis, we model the mean of the target variable using a two-level factor. We aggregate all values of `region` (`west`, `central` and `other`) into `w` and `nw`, whether they take value equal to `west` or not. Running *ANOVA*, we obtain the following output:

```
##              Df  Sum Sq  Mean Sq F value Pr(>F)
## region_w_nw   1 0.02476 0.024761   97.18 <2e-16 ***
```

```
## Residuals   624 0.15899 0.000255
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The results show a very low $p-$value for the variable, which means that the model is accurate. The null hypothesis (i.e., means are equal for both regions) is rejected. Then, we compare the means of the crime rate between the `west` and other regions.

```
##         nw          w
## 0.03475108 0.01987887
```

Considering the above analysis, we can assume that the model can correctly fit the value of the mean in each region: (`west`, `other`). The coefficients of the model can be extracted from the fit value retuned in the package.

```
##  (Intercept) region_w_nww
##   0.03475108  -0.01487221
```

The model is given by

$$\mu_{crmrte} = 0.0347 - 0.0148 I_{\{region='w'\}}$$

Now, we repeat the same analysis considering two factors. We alse include the other categorical variable: `smsa`. We fit an *ANOVA* model and we get the following output:

```
##              Df  Sum Sq Mean Sq F value Pr(>F)
## region_w_nw   1 0.02476 0.02476   155.8 <2e-16 ***
## smsa          1 0.05999 0.05999   377.5 <2e-16 ***
## Residuals   623 0.09900 0.00016
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Again, we obtain a good $p-$value for each of the two variables, which means that both factors have a strong relationship with the response variable. The null hypotesis (i.e., the means are equal) is rejected. The coefficients in this case are:

```
##  (Intercept) region_w_nww      smsayes
##   0.03123831  -0.01300927   0.03441082
```

Besides the two categorical variables, we include in the analysis of the variance the interaction effect between the two variables.

```
##                   Df  Sum Sq Mean Sq F value  Pr(>F)
## region_w_nw        1 0.02476 0.02476 157.927 < 2e-16 ***
## smsa               1 0.05999 0.05999 382.613 < 2e-16 ***
## region_w_nw:smsa   1 0.00147 0.00147   9.404 0.00226 **
## Residuals        622 0.09752 0.00016
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The $p-$values indicate that the two factors and the interaction between them are significant.

Confidence Interval for the Median

As discussed above, we defined a categorical target variable: `high`, if the value of the crime rate was higher that the median, and `low/normal` otherwise. This was done in order to be able to run models that require such variables, as we will see in the following sections. Therefore, it would be very useful to have confidence intervals about the median. Computing the median value, we get:

```
## [1] 0.02840405
```

We can simply obtain a confidence interval around that value. Considering the binomial distribution with $n = 626$ observations and a probability of 0.5, we want to obtain the $k - th$ observation that returns the 98%, the 95%, the 88% and the 81% of the probability by doing the following:

$$1 - 2 \times p_{binom}(k, n = 626, p = 0.5)$$

the $k - th$ values corresponding to those intervals are

```
## [1] 283 290 295 298
```

From the vector of sorted values for `crmrte`, we select the $k - th$ elements of the vector and the $n - k + 1$ elements corresponding to the four confidence intervals:

signif.%	low.level	up.level
0.99	0.0265029	0.0296409
0.95	0.0266806	0.0293452
0.88	0.0268714	0.0292154
0.81	0.0269836	0.0290823

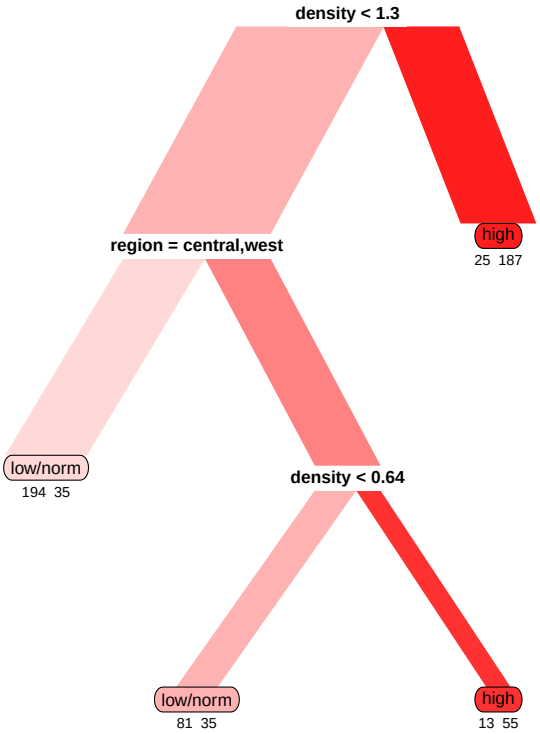
A more sofisticated method to obtain the confidence interval for the median is the *Wilcoxon Signed Rank Test*. As this test assumes symmetry of the variable's distribution, we applied it to the logarithm of the target variable, as discussed above. The results were then transformed to return the values to the original scale by applying the exponential function to the intervals obtained.

signif.%	low.level	up.level
0.99	0.0257032	0.0289377
0.95	0.0260857	0.0285302
0.88	0.0263252	0.0282818
0.81	0.0264716	0.0281065

The results are similar to the simpler sign test.

Dependency analysis with predictive models

Continuing with the analysis we fitted a decision tree model. To do so, we considered the target variable in the categorical format. The purpose of this model is to further explore the data and understand which variables are relevant to the response.




```
## Confusion Matrix and Statistics
##
##             Reference
## Prediction   low/normal high
##   low/normal        121   31
##   high               16  100
##
##                Accuracy : 0.8246
##                  95% CI : (0.7737, 0.8682)
##     No Information Rate : 0.5112
##     P-Value [Acc > NIR] : < 2e-16
##
##                   Kappa : 0.6482
##  Mcnemar's Test P-Value : 0.04114
##
##             Sensitivity : 0.8832
##             Specificity : 0.7634
##          Pos Pred Value : 0.7961
##          Neg Pred Value : 0.8621
##              Prevalence : 0.5112
##          Detection Rate : 0.4515
##    Detection Prevalence : 0.5672
##       Balanced Accuracy : 0.8233
##
##        'Positive' Class : low/normal
##
```

We get a testing accuracy of 83% and verify that the most relevant variables to the target are `region` and `density`.

Linear analysis

Additionally, we considered a standard *linear regression* model involving all predictors, as an alternative means to view the significance of each predictor. Note that in this context, performing *k-fold cross-validation* or *bootstrapping* isn't necessary as we are only interested in significant predictors, hence we performed an ordinary 80/20 splitting of the data into a training and a testing sets as shown in the code below:

We then run a simple linear fit will all predictors, in order to analyse the significance levels of the parameters, provided that the linear test itself has a significant R^2 value.

```
##
## Call:
## lm(formula = crmrte ~ ., data = Crime)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -0.0212397 -0.0043024  0.0000726  0.0037447  0.0309866
##
## Coefficients: (1 not defined because of singularities)
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)   7.111e-02  2.185e-02   3.254 0.001201 **
## county        1.288e-05  5.086e-06   2.532 0.011593 *
## year         -7.737e-04  2.987e-04  -2.591 0.009814 **
## prbarr       -2.182e-02  2.754e-03  -7.923 1.13e-14 ***
## prbconv      -1.339e-03  2.330e-04  -5.746 1.45e-08 ***
## prbpris      -3.512e-03  3.451e-03  -1.018 0.309238
## avgsen        6.224e-05  1.096e-04   0.568 0.570397
## polpc         1.092e+00  1.769e-01   6.172 1.25e-09 ***
## density       5.530e-03  4.551e-04  12.151  < 2e-16 ***
## taxpc         1.901e-04  3.175e-05   5.987 3.69e-09 ***
## regionother   2.501e-03  8.242e-04   3.034 0.002520 **
## regionwest    7.219e-05  9.990e-04   0.072 0.942419
## smsayes      -4.186e-04  1.981e-03  -0.211 0.832697
## pctmin        1.137e-04  2.760e-05   4.120 4.32e-05 ***
## wcon          2.487e-07  2.437e-06   0.102 0.918757
## wtuc         -5.245e-08  1.073e-06  -0.049 0.961010
## wtrd          5.217e-06  3.435e-06   1.519 0.129384
## wfir         -6.563e-06  8.139e-06  -0.806 0.420382
## wser         -4.666e-06  2.846e-06  -1.640 0.101613
## wmfg         -5.262e-06  4.818e-06  -1.092 0.275165
## wfed          2.252e-05  7.793e-06   2.889 0.004002 **
## wsta         -8.173e-06  8.061e-06  -1.014 0.311055
## wloc          2.040e-05  1.446e-05   1.411 0.158871
## mix           2.807e-03  1.791e-03   1.567 0.117698
## pctymle       4.517e-02  1.319e-02   3.426 0.000655 ***
## crmrte_cat.L  9.456e-03  5.249e-04  18.016  < 2e-16 ***
## region_w_nww         NA         NA      NA       NA
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Residual standard error: 0.006987 on 600 degrees of freedom
## Multiple R-squared:  0.8406, Adjusted R-squared:  0.8339
## F-statistic: 126.6 on 25 and 600 DF,  p-value: < 2.2e-16
```

as the R^2 value is sufficiently high (as the R^2 value is sufficiently high (0.8405914), we decided to perform '*bestsubsetselection*' on the set of predictors. Although we are aware of the performance penalties of doing this for $p = 23$, the running times were considerably short and hence we decided to stick to this approach:

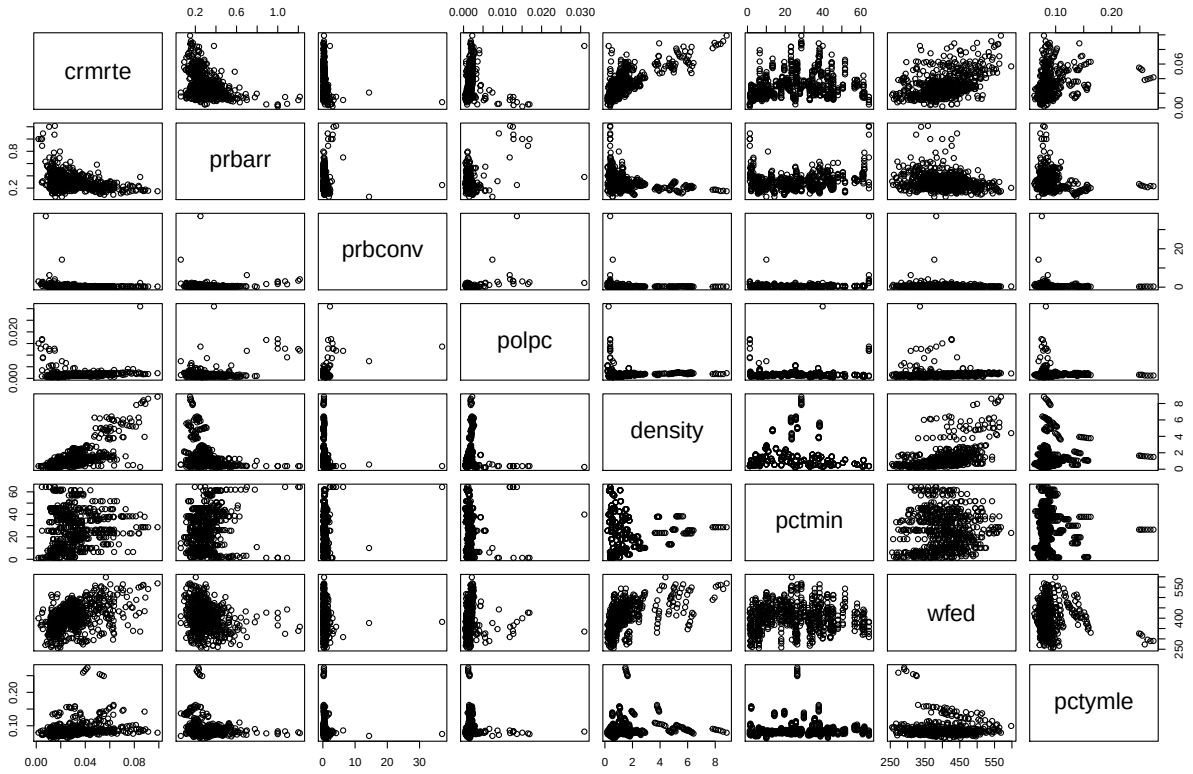
After getting all best subsets with size $k = 1...p$, we analysed both *training* and *testing* errors by performing *k-fold cross validation* with $k = 10$ and then getting the minimum errors on all iterations:

The cross-validation estimate of the training error is 4.7521724×10^{-4} and the cross-validation error is 5.4789369×10^{-4}. The actual training and test errors for this subset, on the original datasets, are 0.0012489 and 0.0013653, respectively. Both were obtained when using *best subset* with $k = 8$ predictors. The ratio between *testing* and *training* errors is (1.0932317). Consequently, we can conclude that the predictors yielded by the subset generated using *best subset selection* belong to a consistent model and, hence, can be used as a basis for non linear models. Nevertheless, we decided to run a linear fit with these predictors in order to check our conclusions:

```
##
## Call:
## lm(formula = crmrte ~ prbarr + prbconv + polpc + density + as.factor(region) +
##     pctmin + wfed + pctymle, data = Crime)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.021247 -0.005769 -0.000725  0.004217  0.047783
##
## Coefficients:
##                           Estimate Std. Error t value Pr(>|t|)
## (Intercept)             7.205e-03  3.586e-03   2.009 0.044942 *
## prbarr                 -3.202e-02  3.098e-03 -10.336  < 2e-16 ***
## prbconv                -1.807e-03  2.502e-04  -7.222 1.51e-12 ***
## polpc                   1.944e+00  2.112e-01   9.203  < 2e-16 ***
## density                 7.193e-03  3.171e-04  22.684  < 2e-16 ***
## as.factor(region)other  4.681e-03  9.785e-04   4.784 2.15e-06 ***
## as.factor(region)west  -3.471e-03  1.161e-03  -2.990 0.002898 **
## pctmin                  1.243e-04  3.220e-05   3.861 0.000125 ***
## wfed                    2.641e-05  6.936e-06   3.808 0.000154 ***
## pctymle                 7.466e-02  1.546e-02   4.828 1.74e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.00887 on 616 degrees of freedom
## Multiple R-squared:  0.7362, Adjusted R-squared:  0.7324
## F-statistic: 191.1 on 9 and 616 DF,  p-value: < 2.2e-16
```

We can note that all coefficients are significant and the R^2, as expected, was reduced but only marginally (0.7362415 versus 0.8405914), which confirms that the model with this subset is indeed a good model.

Next, we proceeded to graphically analyse any nonlinearities between these predictors and the response, by looking at all pairwise plots:



It can be seen that the relationship between `crmrte` and `prbrarr`, `prbconv` and `polpc`, respectively, could be better explained by applying a *log* to these predictors. Additionally, `wfed` and `pctmin` seem to have a nonlinear relationship with the response, which makes them suitable as polynomial regression predictors. Consequently, we run a new, nonlinear model with these modified predictors:

```
##
## Call:
## lm(formula = crmrte ~ log(prbarr) + log(prbconv) + log(polpc) +
##     density + as.factor(region) + poly(pctmin, 4) + poly(wfed,
##     3) + pctymle, data = Crime)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.023736 -0.004242 -0.000500  0.003928  0.043977
##
## Coefficients:
##                            Estimate Std. Error t value Pr(>|t|)
## (Intercept)                0.0629472  0.0047199  13.337  < 2e-16 ***
## log(prbarr)               -0.0143771  0.0009613 -14.955  < 2e-16 ***
## log(prbconv)              -0.0110102  0.0006437 -17.105  < 2e-16 ***
## log(polpc)                 0.0102260  0.0007065  14.473  < 2e-16 ***
## density                    0.0054673  0.0003141  17.408  < 2e-16 ***
## as.factor(region)other     0.0036592  0.0008355   4.380 1.40e-05 ***
## as.factor(region)west     -0.0056435  0.0011795  -4.785 2.15e-06 ***
## poly(pctmin, 4)1           0.0693350  0.0120000   5.778 1.21e-08 ***
## poly(pctmin, 4)2           0.0103876  0.0096010   1.082  0.27971
## poly(pctmin, 4)3          -0.0249664  0.0083587  -2.987  0.00293 **
```

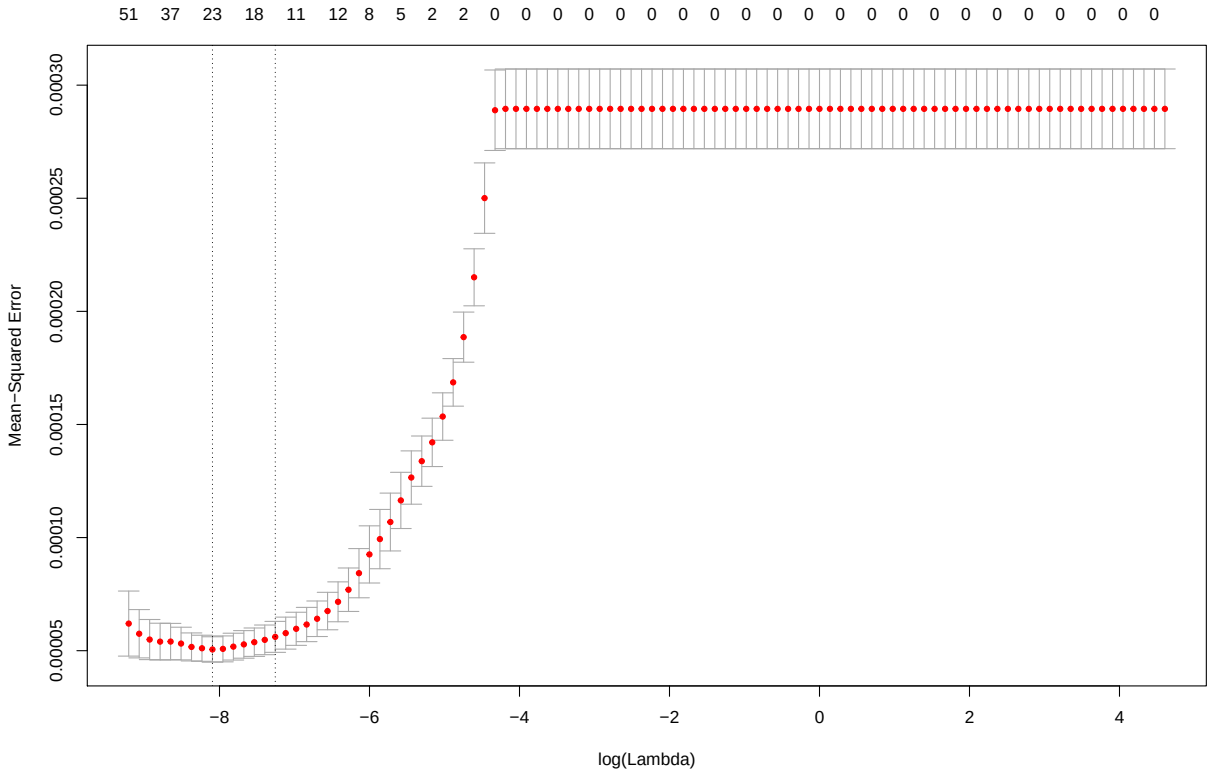
```
## poly(pctmin, 4)4       -0.0078389  0.0084322  -0.930  0.35292
## poly(wfed, 3)1          0.0136584  0.0095149   1.435  0.15166
## poly(wfed, 3)2         -0.0007147  0.0080329  -0.089  0.92913
## poly(wfed, 3)3         -0.0109946  0.0076187  -1.443  0.14950
## pctymle                 0.0118466  0.0136944   0.865  0.38734
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.007363 on 611 degrees of freedom
## Multiple R-squared:  0.8197, Adjusted R-squared:  0.8156
## F-statistic: 198.4 on 14 and 611 DF,  p-value: < 2.2e-16
```

The lack of significance of the polynomials for `wfed` and the increase in R^2 suggests that this model could possibly overfit. Hence, we removed the polynomials related to `wfed`, but kept the *log* predictors as they have shown to be very significant. `pctmin` has a special behaviour, where we see that only the $3rd$ degree polynomial is used and seems less significant that the linear approach. Consequently, we decided to remove both the polynomial and the linear coefficients and re attach `pctmin` as a spline in **??**. This updated model yielded a *testing MSE* of 4.7651687×10^{-5}, and an R^2 value of 0.8197077.

Next, we analysed all significant interactions between all original predictors and plugged them to our previous model. The number of interactions that we added to the model are 324 and the R^2 values for each fit are 0.9801254 and 0.9806373, respectively. Consequently, we can affirm that both models are seriously overfitting because the number of predictors has skyrocketed due to all interaction combinations. Even though it is tempting to keep only those interactions with a relevant significance value, since the removal of each of these predictors affects the overall model, we chose instead to refine it by using a *Stepwise Algorithm* applying *AIC* to decide. The resulting fit has 196 coefficients, which means a reduction on the number of interactions by 30%, we proceeded to calculate both *training MSE* (0.0012489) and *testing MSE* (0.0013653), yielding an error rate that remains close to one (1.0932317) which still proves that this model holds. Now that we obtained a complex model consisting of linear variables, *log* variables and *interaction* variables, we will perform *Lasso* in order to remove all interaction terms that are not significant, so that we arrive to a model easy to understand.

Lasso Analysis

With the resulting model from all our previous steps, we performed *k-fold cross validation* using Lasso, in order to obtain the optimum value of λ for our model. The plot showing the *cross-validation error* as λ increases is the following:



We then chose a value of λ within 1 standard deviation from the optimum value, as this is a commonly established good practice.

The predictors are

Lasso yielded the following 16 non-zero predictors:

log(prbconv)
log(polpc)
poly(wfed, 3)1
year
prbarr
wfed
pctmin:county
prbarr:wfir
polpc:regionother
density:regionother
density:pctmin
density:pctymle
pctmin:regionwest
regionwest:wsta
regionwest:mix
pctymle:regionwest

with nonzero values, a reduction of 92% with respect to the *stepwise AIC*. The *test error* for this model is obtained using the same *k-folds* generated for the first model:

```
## [1] 5.608461e-05
```

and this error is 4.1078442% of the training error of our original model using *best subset selection*. Finally, we analysed the $L1$-norm between our estimated responses and the true model, which yielded a value of 0.2026294, which is more than reasonable since it's below 30%. Consequently, the final set obtained in this section, after performing *linear*, *best subset selection*, *log*, *polynomial* , *stepwise AIC* and *Lasso* yielded a model that will be used in the following sections for more complex fits that will derive in our final model.

Non-linear modeling

From the pairwise plot we generated shown previously, we identified a predictor `pctmin`, corresponding to the proportion of minorities in the region, showing a clear non-linear relationship with the crime rate that can benefit from higher order polynomial regression/splines and help with predicting the overall crime rate.

Linear model

First we evaluated the regression model generated using only using a linear model:

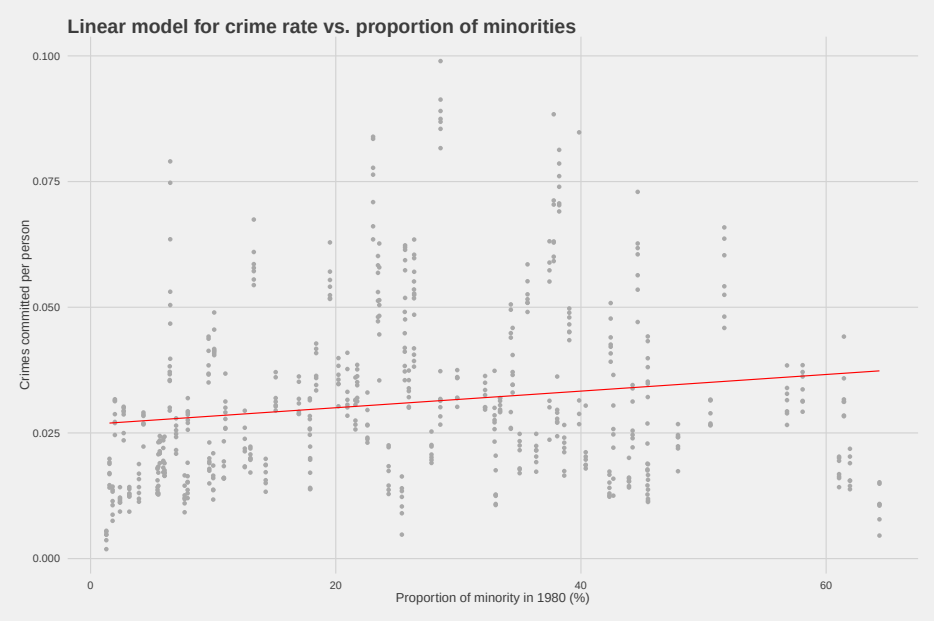


Figure 1: Linear model for crime rate vs. proportion of minorities

This naive model results in an error rate of 0.4808 for the testing set. From the plot, it is clear that the relationship between the crime rate and the proportion of minorities in the area is not linear.

Polynomial model

Next, we obtained a degree-4 polynomial function for a smooth fit over the `pctmin` data:

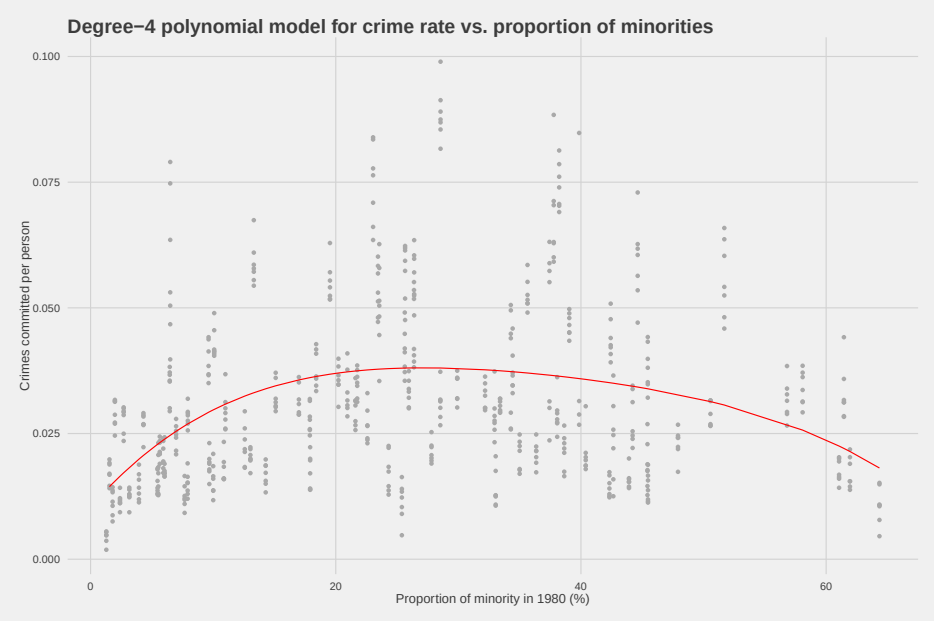


Figure 2: Degree-4 polynomial model for crime rate vs. proportion of minorities

The error rate was improved by reducing the bias of the model. This fit resulted in an error rate of 0.4755.

Splines

We further attempt to reduce the bias, introducing a more flexible piecewise polynomial by using knots. With a cubic spline, the fitted curves and their first and second derivatives are constrained to be continuous at the knots. As splines often lead to high variance at the outer ranges of the predictors, we fit a natural cubic spline, which forces the function to be linear at the boundary. `ns()` function was used to generate natural cubic knots with 6 degrees of freedom, with matrix of basis functions for splines and knots at 6.03%, 14.28%, 24.31%, 34.28%, and 42.64% of `pctmin`.

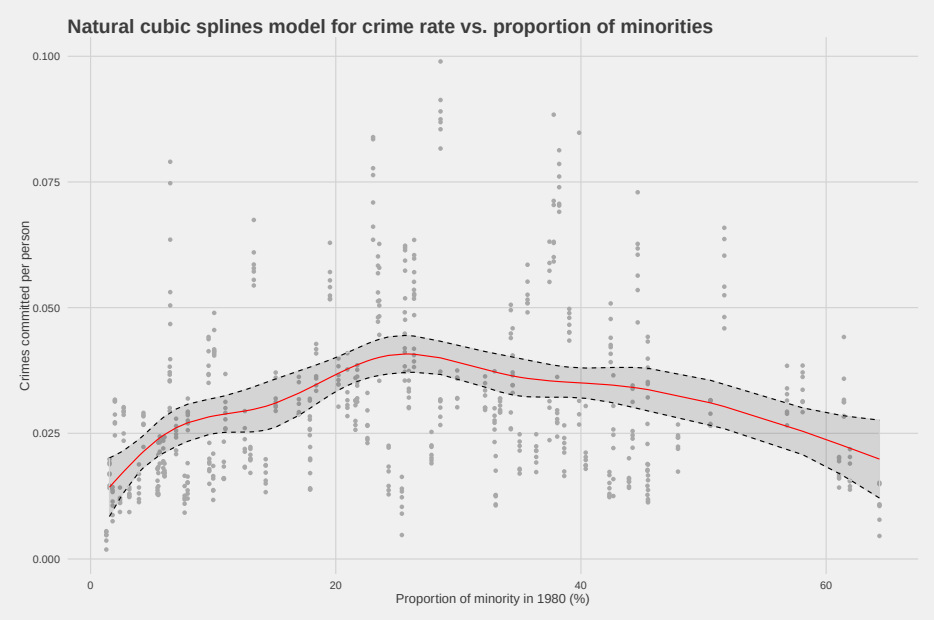


Figure 3: Natural cubic splines model for crime rate vs. proportion of minorities

Consequently, we see a modest improvement in the mean error: 0.468.

We also attempted to fit a smoothing spline with a value of λ chosen using cross-validation. This resulted in a model very similar to the polynomial fit and failed to improve the mean testing error.

Combined models

Combining the predictors from the previous section with just the linear `pctmin` results in a mean error of 0.2385.

Finally, we included the splined version of `pctmin` predictor alongside the selected predictors, which resulted in a slight improvement in mean error to 0.2355.