Instituto Politécnico Nacional

Neurofuzzy Systems

PRACTICE 2. BASIC FUZZY OPERATIONS

Carlos Josafath López Favila
Teacher Blanca Tovar Corona
Delivery Date: March 9th, 2023

I. Introduction

The membership function of a fuzzy set is a generalization of the indicator function in classical sets. In fuzzy logic, it represents the degree of truth as an extension of valuation. Degrees of truth are often confused with probabilities, although they are conceptually distinct, because fuzzy truth represents membership in vaguely defined sets, not likelihood of some event or condition.
A single fuzzy set by itself can model ordering information, but the main reason for employing fuzzy logic and developing fuzzy systems is to infer decisions and information from several pieces of expert knowledge.
The fuzzy set A can be alternatively denoted as follows:

- If X is discrete then $A = \frac{\sum \mu_A(x_i)}{x_i}$
- If X is continuous then $A = \int \frac{\mu_A(x)}{x}$

Here, $\mu_A(x)$ is the "membership function". Value of this function is between 0 and 1. This value represents the "degree of membership" (membership value) of element x in set A. The members of a fuzzy set are members to some degree, known as a membership grade or degree of membership.

A. Fuzzy Set Operations

1. Union

For union operation in fuzzy sets:

$$\mu_{A\cup B}(x) = \mu_A(x) \vee \mu_B(x) = max(\mu_A(x), \mu_B(x)) \tag{1}$$

2. Intersection

For intersection operation in fuzzy sets:

$$\mu_{A\cap B}(x) = \mu_A(x) \wedge \mu_B(x) = min(\mu_A(x), \mu_B(x)) \tag{2}$$

3. Complement

For complement operation in fuzzy sets:

$$\mu_{\overline{A}}(x)) = 1 - \mu_A(x) \tag{3}$$

B. Fuzzy Set Laws

Also, there exists some laws for fuzzy sets.

1. De Morgan Laws

$$\overline{\mu_A \cap \mu_B} = \overline{\mu_A} \cup \overline{\mu_B} \tag{4}$$

$$\overline{\mu_A \cup \mu_B} = \overline{\mu_A} \cap \overline{\mu_B} \tag{5}$$

II. Objective

In this practice I will recognize and use a the Fuzzy Set Operations and Laws in Python (which is my selected platform to work) for each membership functions to develop a subplot where each operation is described graphicly overlapping the result with the original fuzzy sets.

III. Develompent

Let X=[0,10], with a step of 0.1, be the universe for the following fuzzy sets:
Calculate the following operations

1. Complement

- Complement of A
- Complement of B
- Complement of C

2. Union

- A union B
- A union C
- B union C

3. Intersection

- A intersection B
- A intersection C
- B intersection C

4. De Morgan Laws

- $\overline{A \cap \overline{C}}$
- $\overline{\overline{B} \cap C}$
- $\overline{A \cup C}$

IV. Results

After a lot of code lines, the result is the following:

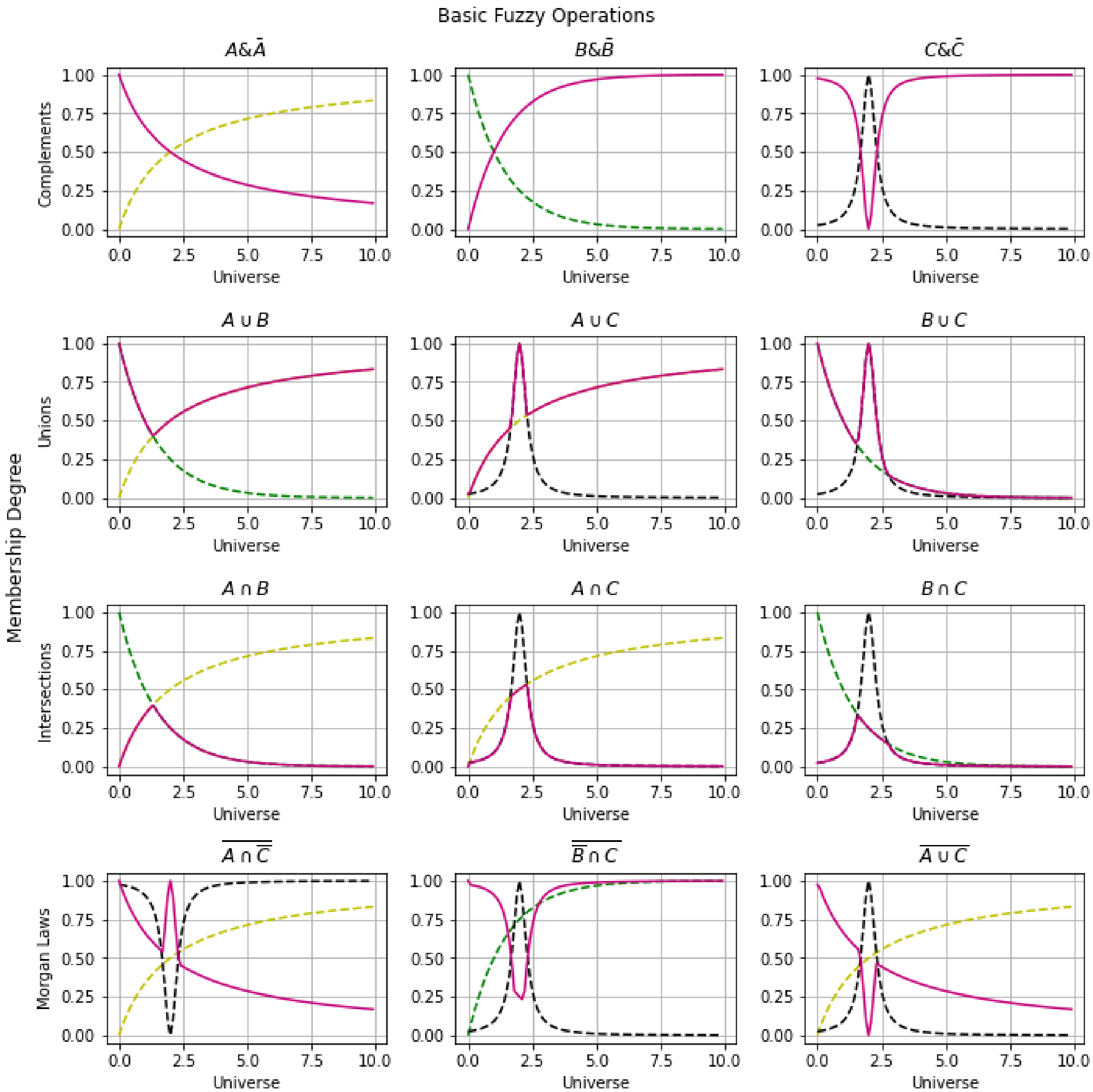

Figure 1: Fuzzy Operations and Laws

V. Conclusions

In this practice, I was able to learn and practice some of the basic fuzzy set operations and laws like DeMorgan's laws, and also graph them, which allowed me to practice my Python programming skills as well.

VI. References

- Brown, M.(1996). An Introduction to Fuzzy and Neurofuzzy Systems. http://libgen.is/book/index.php?md5=6EDA1FBA008F9F84B376003C79D0B25B

VII. Apenddix

A. Code

```python

import matplotlib.pyplot as plt
import numpy as np

A=[]
B=[]
C=[]
AuB=[]
AuC=[]
BuC=[]
AnB=[]
AnC=[]
BnC=[]
nA=[]
nB=[]
nC=[]

X=np.arange(0,10,0.1)

for k in X:
  A.append(k/(k+2))
  B.append(2**(-k))
  C.append(1/(1+10*(k-2)**(2)))

for k in range(len(X)):
  AuB.append(max(A[k],B[k]))
  AuC.append(max(A[k],C[k]))
  BuC.append(max(B[k],C[k]))
  AnB.append(min(A[k],B[k]))
  AnC.append(min(A[k],C[k]))
  BnC.append(min(B[k],C[k]))
  nA.append(1-A[k])
  nB.append(1-B[k])
  nC.append(1-C[k])


fig=plt.figure(figsize=(10,10),tight_layout=True)
fig.supylabel('Membership Degree')
fig.suptitle("Basic Fuzzy Operations")

plt.subplot(4,3,1)
plt.plot(X,A,'--y',X,[(1-A[k]) for k in range(len(X))],'#c4037a')
plt.title(r"$A & \bar{A} $")
plt.xlabel("Universe")
plt.ylabel("Complements")
plt.grid(True)

```

```python
plt.subplot(4,3,2)
plt.plot(X,B,'--g',X,[(1-B[k]) for k in range(len(X))],'#c4037a')
plt.title(r"$B & \bar{B} $")
plt.xlabel("Universe")
plt.grid(True)

plt.subplot(4,3,3)
plt.plot(X,C,'--k',X,[(1-C[k]) for k in range(len(X))],'#c4037a')
plt.title(r"$C & \bar{C} $")
plt.xlabel("Universe")
plt.grid(True)

plt.subplot(4,3,4)
plt.plot(X,A,'--y',X,B,'--g',X,AuB,'#c4037a')
plt.title(r"$A \cup B$")
plt.xlabel("Universe")
plt.ylabel("Unions")
plt.grid(True)

plt.subplot(4,3,5)
plt.plot(X,A,'--y',X,C,'--k',X,AuC,'#c4037a')
plt.title(r"$A \cup C$")
plt.xlabel("Universe")
plt.grid(True)

plt.subplot(4,3,6)
plt.plot(X,B,'--g',X,C,'--k',X,BuC,'#c4037a')
plt.title(r"$B \cup C$")
plt.xlabel("Universe")
plt.grid(True)

plt.subplot(4,3,7)
plt.plot(X,A,'--y',X,B,'--g',X,AnB,'#c4037a')
plt.title(r"$A \cap B$")
plt.ylabel("Intersections")
plt.xlabel("Universe")
plt.grid(True)

plt.subplot(4,3,8)
plt.plot(X,A,'--y',X,C,'--k',X,AnC,'#c4037a')
plt.title(r"$A \cap C$")
plt.xlabel("Universe")
plt.grid(True)

plt.subplot(4,3,9)
plt.plot(X,B,'--g',X,C,'--k',X,BnC,'#c4037a')
plt.title(r"$B \cap C$")
plt.xlabel("Universe")
plt.grid(True)

plt.subplot(4,3,10)
plt.plot(X,A,'--y',X,nC,'--k',X,[max(nA[k],C[k]) for k in range(len(X))],'#c4037a')
plt.title(r"$\overline{ A\cap \overline{C} }$")
plt.xlabel("Universe")
plt.ylabel("Morgan Laws")
plt.grid(True)

plt.subplot(4,3,11)
plt.plot(X,nB,'--g',X,C,'--k',X,[max(B[k],nC[k]) for k in range(len(X))],'#c4037a')
```

```
plt.title(r"$\overline{ \overline{B}\cap C }$")
plt.xlabel("Universe")
plt.grid(True)

plt.subplot(4,3,12)
plt.plot(X,A,'--y',X,C,'--k',X,[min(nA[k],nC[k]) for k in range(len(X))],'#c4037a')
plt.title("$\overline{A \cup C}$")
plt.xlabel("Universe")
plt.grid(True)
```