This problem set focuses in part on material covered in Week 5 (Lecture 7), so I recommend you to watch the lecture and attempt Assignment 7 before submitting your answers. The remainder of this problem set is revision material.

1. Let m, n denote any two natural numbers. Is the following a valid proof that mn is odd iff m and n are odd?

   If m, n are odd there are integers p, q such that $m = 2p+1, n = 2q+1$. Then $mn = (2p+1)(2q+1) = 2(2pq + p + q) + 1$, so mn is odd. That completes the proof.

2. Take the sentence:

   You can fool some of the people some of the time, but you cannot fool all of the people all the time.

   Let x be a variable for a person, t a variable for a period of time, and let $F(x, t)$ mean you can fool x at time t. Which of the following mathematical formulas is equivalent to the given statement?

   (a) $\exists x \exists t F(x, t) \wedge \exists x \exists t \neg F(x, t)$
   (b) $\exists x \exists t F(x, t) \wedge \neg \forall x \exists t F(x, t)$
   (c) $\exists x \exists t F(x, t) \wedge \neg \exists x \exists t F(x, t)$
   (d) None of the above.

3. True or false? For any two statements ϕ and ψ, either $\phi \Rightarrow \psi$ or its converse is true (or both).

4. Are the following two statements equivalent? $\neg(\phi \Rightarrow \psi)$ and $\phi \wedge (\neg\psi)$

5. Are the following two statements equivalent? $(\phi \vee \psi) \Rightarrow \theta$ and $(\phi \Rightarrow \theta) \wedge (\psi \Rightarrow \theta)$

6. True or false? There are infinitely many natural numbers n for which $\sqrt{n}$ is rational. (Before entering your answer, you should construct a proof of the statement or its negation, so you are sure.)

7. This argument is a proof that $1 = 2$. Obviously it is incorrect. Identify exactly what the error is, and grade the purported proof according to the course rubric. (Remember, this is not a regular mathematics course of the kind you are probably familiar with. We are working on various elements of mathematical thinking, mathematical exposition, and the communication of mathematics. The rubric is designed to focus attention on all of those factors.) Your "Overall valuation" figure is the grade you would assign a student if s/he submitted this proof in a first-year college *mathematics* course.

   Argument to show that $1 = 2$.

   We start with the identity
   $$1 - 3 = 4 - 6$$
   Adding $\frac{9}{4}$ to both sides to complete the squares, we get
   $$1 - 3 + \tfrac{9}{4} = 4 - 6 + \tfrac{9}{4}$$
   This factors as
   $$(1 - \tfrac{3}{2})^2 = (2 - \tfrac{3}{2})^2$$
   Taking the square root of both sides,
   $$1 - \tfrac{3}{2} = 2 - \tfrac{3}{2}$$
   Hence
   $$1 = 2$$