This problem set focuses on material covered in Lectures 3 and 4, so I recommend you to watch both lectures and attempt Assignments 3 and 4 before submitting your answers.

1. Which of the following conditions are *necessary* for the natural number n to be divisible by 6? Select all those you believe are necessary.
   (a) n is divisible by 3.
   (b) n is divisible by 9.
   (c) n is divisible by 12.
   (d) $n = 24$.
   (e) n^2 is divisible by 3.
   (f) n is even and divisible by 3.
2. Which of the following conditions are *sufficient* for the natural number n to be divisible by 6? Select all those you believe are sufficient.
   (a) n is divisible by 3.
   (b) n is divisible by 9.
   (c) n is divisible by 12.
   (d) $n = 24$.
   (e) n^2 is divisible by 3.
   (f) n is even and divisible by 3.
3. Which of the following conditions are *necessary and sufficient* for the natural number n to be divisible by 6? Select all those you believe are necessary and sufficient.
   (a) n is divisible by 3.
   (b) n is divisible by 9.
   (c) n is divisible by 12.
   (d) $n = 24$.
   (e) n^2 is divisible by 3.
   (f) n is even and divisible by 3.
4. Identify the antecedent in the conditional "If the apples are red, they are ready to eat."
   THE APPLES ARE RED **THE APPLES ARE READY TO EAT**
5. Identify the antecedent in the conditional "The differentiability of a function f is sufficient for f to be continuous."
   f **IS DIFFERENTIABLE** f **IS CONTINUOUS**
6. Identify the antecedent in the conditional "A function f is bounded if f is integrable."
   f **IS BOUNDED** f **IS INTEGRABLE**
7. Identify the antecedent in the conditional "A sequence S is bounded whenever S is convergent."
   S **IS BOUNDED** S **IS CONVERGENT**

8. Identify the antecedent in the conditional "It is necessary that n is prime in order for $2^n - 1$ to be prime."

   n **IS PRIME** $2^n - 1$ **IS PRIME**

9. Identify the antecedent in the conditional "The team wins only when Karl is playing."

   THE TEAM WINS **KARL IS PLAYING**

10. Identify the antecedent in the conditional "When Karl plays the team wins."

    THE TEAM WINS **KARL IS PLAYING**

11. Identify the antecedent in the conditional "The team wins when Karl plays."

    THE TEAM WINS **KARL IS PLAYING**

12. For natural numbers m, n, is it true that mn is even iff m and n are even?

13. Is it true that mn is odd iff m and n are odd?

14. Which of the following pairs of propositions are equivalent?

    (a) $\neg P \vee Q$, $P \Rightarrow Q$

    (b) $\neg(P \vee Q)$, $\neg P \wedge \neg Q$

    (c) $\neg P \vee \neg Q$, $\neg(P \vee \neg Q)$

    (d) $\neg(P \wedge Q)$, $\neg P \vee \neg Q$

    (e) $\neg(P \Rightarrow (Q \wedge R))$, $\neg(P \Rightarrow Q) \vee \neg(P \Rightarrow R)$

    (f) $P \Rightarrow (Q \Rightarrow R)$, $(P \wedge Q) \Rightarrow R$

15. A major focus of this course is learning how to assess mathematical reasoning. How good you are at doing that lies on a sliding scale. Your task is to evaluate the purported proof below, and evaluate it according to the course rubric. Enter your evaluation (which should be a whole number between 0 and 24, inclusive) in the box. A number within 4 points of the instructor's evaluation counts as correct. **You should read the article "Using the evaluation rubric" on the course website (it includes a short explanatory video) before attempting this question.** There will be many more proof evaluation questions like this as the course progresses.

    [The scoring system is somewhat arbitrary, due to limitations of the platform. But the goal is to provide opportunities for you to reflect on what makes an argument a good proof, and you are allowed to repeat the Problem Sets as many times as it takes to be able to progress. Your "score" is simply feedback information. Moreover, the "passing grade" for Problem Sets is a low 35%.]

    HERE IS WHAT THE INDIVIDUAL SUBMITTED:

    Claim: For any two propositions P, Q, $\neg P \wedge \neg Q$ is equivalent to $\neg[P \wedge Q]$.

    Proof: Suppose that $\neg P \wedge \neg Q$ is true. Then both $\neg P$ and $\neg Q$ are true.
    So P and Q are both false. Thus $P \wedge Q$ is false. Hence $\neg[P \wedge Q]$ is true.
    This argument clearly works the other way. So we have implication in both
    directions, which proves the claim.