1. Express as concisely and accurately as you can the relationship between $b|a$ and a/b.

2. Determine whether each of the following is true or false and prove your answer. (You saw these questions in the in-lecture quiz, so th first part is a repeat, except that now you should know the right answers.) The focus of this assignment is to *prove* each of your answers.

   (a) $0|7$ (b) $9|0$ (c) $0|0$ (d) $1|1$

   (e) $7|44$ (f) $7|(-42)$ (g) $(-7)|49$ (h) $(-7)|(-56)$

   (i) $(\forall n \in \mathcal{Z})[1|n]$ (j) $(\forall n \in \mathcal{N})[n|0]$ (k) $(\forall n \in \mathcal{Z})[n|0]$

3. Prove all the parts of the theorem in the lecture, giving the basic properties of divisibility. Namely, show that for any integers a, b, c, d, with $a \neq 0$:

   (a) $a|0$, $a|a$;
   (b) $a|1$ if and only if $a = \pm 1$;
   (c) if $a|b$ and $c|d$, then $ac|bd$ (for $c \neq 0$) ;
   (d) if $a|b$ and $b|c$, then $a|c$ (for $b \neq 0$) ;
   (e) $[a|b$ and $b|a]$ if and only if $a = \pm b$;
   (f) if $a|b$ and $b \neq 0$, then $|a| \leq |b|$;
   (g) if $a|b$ and $a|c$, then $a|(bx + cy)$ for any integers x, y.

OPTIONAL PROBLEMS

1. It is a standard result about primes (known as Euclid's Lemma) that if p is prime, then whenever p divides a product ab, p divides at least one of a, b. Prove the converse, that any natural number having this property (for any pair a, b) must be prime.

2. Try to prove Euclid's Lemma. If you do not succeed, you can find proofs in most textbooks on elementary number theory, and on the Web. Find a proof and make sure you understand it. If you find a proof on the Web, you will need to check that it is correct. There are false mathematical proofs all over the Internet. Though false proofs on Wikipedia usually get corrected fairly quickly, they also occasionally become corrupted when a well-intentioned contributor makes an attempted simplification that renders the proof incorrect. Learning how to make good use of Web resources is an important part of being a good mathematical thinker.