This problem set focuses on material covered in Lecture 6, so you should watch the lecture and attempt Assignment 6 before submitting your answers.

1. Which of the following is equivalent to $\neg\forall x[P(x) \Rightarrow (Q(x) \vee R(x))]$? (Only one is.)
   (a) $\exists x[P(x) \vee \neg Q(x) \vee \neg R(x)]$
   (b) $\exists x[\neg P(x) \wedge Q(x) \wedge R(x)]$
   (c) $\exists x[P(x) \wedge \neg Q(x) \wedge \neg R(x)]$
   (d) $\exists x[P(x) \wedge (\neg Q(x) \vee \neg R(x))]$
   (e) $\exists x[P(x) \vee (\neg Q(x) \wedge \neg R(x))]$
2. Let p, q be variables denoting tennis players, let t be a variable denoting games of tennis, and let $W(p, q, t)$ mean that p plays against q in game t and wins. Which of the following claims about tennis players mean the same as the symbolic formula $\forall p \exists q \exists t W(p, q, t)$? Select all that have that meaning.
   (a) Everyone wins a game
   (b) Everyone loses a game
   (c) For every player there is another player they beat all the time
   (d) There is a player who loses every game
   (e) There is a player who wins every game
3. Let p, q be variables denoting the tennis players in a club, let t be a variable denoting the club's games of tennis, and let $W(p, q, t)$ mean that p plays against q in game t and wins. Assuming that there are at least two tennis players and games between them do take place, which (if any) of the following symbolic formula cannot possibly be true? Select all you think cannot possibly be true.
   (a) $\forall p \exists q \exists t W(p, q, t)$
   (b) $\forall p \forall q \exists t W(p, q, t)$
   (c) $\forall q \exists p \exists t W(p, q, t)$
4. Which of the following means "Everybody loves a lover", where $L(x, y)$ means (person) x loves (person) y and a lover is defined to be someone in a mutual loving relationship? [If English is not your native language, you might want to discuss this sentence with a native English speaker before you answer. It's an idiomatic expression.]
   (a) $\forall x \forall y[\exists z(L(x, z) \wedge L(z, x)) \Rightarrow L(y, x)]$
   (b) $\forall x \forall y[\forall z(L(x, z) \vee L(z, x)) \Rightarrow L(y, x)]$
   (c) $\forall x[\exists z(L(x, z) \wedge L(z, x)) \wedge \forall y L(y, x)]$
5. Which of the following statements about the order relation on the real line is/are false?
   (a) $\forall x \forall y \forall z[(x \leq y) \wedge (y \leq z) \Rightarrow (x \leq z)]$
   (b) $\forall x \forall y[(x \leq y) \wedge (y \leq x) \Rightarrow (x = y)]$
   (c) $\forall x \exists y[(x \leq y) \wedge (y \leq x)]$
   (d) $\exists x \forall y[(y < x) \vee (x < y)]$
6. A student produced this purported proof while trying to understand Euclid's proof of the infinitude of the primes. Grade it according to the course rubric. SEE HANDWRITTEN PROOF ON NEXT PAGE.

IF AN INTEGER N IS DIVISIBLE BY A PRIME P, THEN N+1 IS NOT DIVISIBLE BY P.

PROOF: SUPPOSE N IS DIVISIBLE BY P. THEN THERE IS AN INTEGER Q SUCH THAT $N = PQ$. SO $N+1 = PQ+1$. THEN

$$\frac{N+1}{p} = q + \frac{1}{p}.$$

BUT $q + \frac{1}{p}$ IS NOT AN INTEGER. SO N+1 IS NOT DIVISIBLE BY P.