

This problem set focuses on material covered in Lecture 5, so I recommend you to watch the lecture and attempt Assignment 5 before submitting your answers.

1. Let x be a variable ranging over doubles tennis matches, and t be a variable ranging over doubles tennis matches when Rosario partners with Antonio. Let $W(x)$ mean that Rosario and her partner (whoever it is) win the doubles match x . Select the following English sentences that mean the same as the symbolic formula $\exists tW(t)$:
 - (a) Rosario and Antonio win every match where they are partners.
 - (b) Rosario and her partner sometimes win the match when she partners with Antonio.
 - (c) Whenever Rosario plays with Antonio, they win the match.
 - (d) Rosario and Antonio win exactly one match when they are partners.
 - (e) Rosario and Antonio win at least one match when they are partners.
 - (f) If Rosario and her partner win the match, she must be partnering with Antonio.
2. Let x be a variable ranging over doubles tennis matches, and t be a variable ranging over doubles tennis matches when Rosario partners with Antonio. Let $W(x)$ mean that Rosario and her partner (whoever it is) win the doubles match x . Select the following English sentences that mean the same as the symbolic formula $\forall tW(t)$:
 - (a) Rosario and Antonio win every match where they are partners.
 - (b) Rosario always partners with Antonio.
 - (c) Whenever Rosario partners with Antonio, they win the match.
 - (d) Sometimes, Rosario and her partner win the match.
 - (e) Rosario and her partner win the match whenever she partners with Antonio.
 - (f) If Rosario and her partner win the match, she must be partnering with Antonio.
3. Which of the following formal propositions says that there is no largest prime. (There may be more than one. You have to select all correct propositions.) The variables denote natural numbers.
 - (a) $\neg\exists x\exists y[\text{Prime}(x) \wedge \neg\text{Prime}(y) \wedge (x < y)]$
 - (b) $\forall x\exists y[\text{Prime}(x) \wedge \text{Prime}(y) \wedge (x < y)]$
 - (c) $\forall x\forall y[\text{Prime}(x) \wedge \text{Prime}(y) \wedge (x < y)]$
 - (d) $\forall x\exists y[\text{Prime}(y) \wedge (x < y)]$
 - (e) $\exists x\forall y[\text{Prime}(y) \wedge (x < y)]$
 - (f) $\forall x\exists y[\text{Prime}(x) \wedge (x < y)]$
4. The symbol $\exists!x$ means “There exists a unique x such that ...” Which of the following accurately defines the expression $\exists!x\phi(x)$?
 - (a) $\exists x\forall y[\phi(x) \wedge [\phi(y) \Rightarrow (x \neq y)]]$
 - (b) $\exists x[\phi(x) \wedge (\exists y)[\phi(y) \Rightarrow (x \neq y)]]$
 - (c) $\exists x\exists y[(\phi(x) \wedge \phi(y)) \Rightarrow (x = y)]$
 - (d) $[\exists x\phi(x)] \wedge (\forall y)[\phi(y) \Rightarrow (x = y)]$

(e) $\exists x[\phi(x) \wedge (\forall y)[\phi(y) \Rightarrow (x = y)]]$

5. Which of the following means “The arithmetic operation $x \uparrow y$ is not commutative.” (\uparrow is just some arbitrary binary operation.)

(a) $\forall x \forall y [x \uparrow y \neq y \uparrow x]$

(b) $\forall x \exists y [x \uparrow y \neq y \uparrow x]$

(c) $\exists x \exists y [x \uparrow y \neq y \uparrow x]$

(d) $\exists x \forall y [x \uparrow y \neq y \uparrow x]$

6. Evaluate this purported proof, and grade it according to the course rubric.

Claim: There does not exist a positive integer N such that $N^2 + 4N + 3$ is prime.

Proof: $N^2 + 4N + 3 = (N + 1)(N + 3)$. This is not prime.