Week 4 – Assignment 6

1. Show that ¬[∃xA(x)] is equivalent to ∀x[¬A(x)]
   1. 🡪 direction
      1. Assume ¬[∃xA(x)]
      2. (rewrite in English) It is not the case that there is an x such that A(x) is true
      3. (therefore) For all values of x, it is the case that A(x) is false
      4. (rewrite) For all values of x, it is not the case that A(x) is true
      5. (rewrite) ∀x[¬A(x)]
   2. 🡨 direction
      1. Assume ∀x[¬A(x)]
      2. (rewrite in English) For all values of x, it is not the case that A(x) is true
      3. (rewrite) For all values of x, it is the case that A(x) is false
      4. (same as) There are no values x such as A(x) is true
      5. (rewrite) It is not the case that there is a value x such that A(x) is true
      6. (rewrite) ¬[∃xA(x)]
2. Prove that “There is an even prime bigger than 2” is false
   1. (rewrite) There are no even prime numbers bigger than 2
   2. (Equivalent to) All prime numbers bigger than 2 are not even
   3. Let p be a prime bigger than 2; if p is even, then p can be written in the form p=2\*p’
   4. (therefore) p cannot be prime (since there is a number p’ greater than 1 and smaller than p that divides it)
3. .
   1. ∀p(IsStudent(p) => LikesPizza(p))
   2. ∃p(IsMyFriend(p) ∧ ¬HasCar(p))
   3. ∃a(IsElephant(a) ∧ ¬LikesMuffin(a))
   4. ∀x(IsTriangle(x) => IsIsoceles(x))
   5. ∃p(IsStudentInThisClass(p) ∧ IsNotHereToday(p))
   6. ∀p∃q(Loves(p, q))
   7. ∀p∃q(¬Loves(p, q))
   8. ∀p1((Man(p1) ∧ Comes(p1)) => ∀p2(Woman(p2) => Leaves(p2)))
   9. ∀p(Tall(p) OR Short(p))
   10. ∀p(Tall(p)) OR ∀p(Short(p))
   11. ∃s(Precious(s) AND ¬Beautiful(s))
   12. ∀p(¬Loves(p, me))
   13. ∃s(American(s) ∧ Poisonous(s))
   14. ∃s(Snake(s) ∧ American(s) ∧ Poisonous(s))
4. .
   1. ∃p(IsStudent(p) ∧ ¬LikesPizza(p))
      1. There is a student that does not like pizza.
   2. ∀p(IsMyFriend(p) => HasCar(p))
      1. All my friends have cars.
   3. ∀a(IsElephant(a) => LikesMuffin(a))
      1. All elephants like muffins
   4. ∃x(IsTriangle(x) ∧ ¬IsIsoceles(x))
      1. Some triangles are not isosceles.
   5. ∀p(IsStudentInThisClass(p) => IsHereToday(p))
      1. All students in this class are here today
   6. ∃p∀q(¬Loves(p, q))
      1. Someone doesn’t love anybody
   7. ∃p∀q(Loves(p, q))
      1. Someone loves everyone
   8. ∃p1(Man(p1) ∧ Comes(p1) ∧ ∃p2(Woman(p2) ∧ => ∀p2(Woman(p2) => ¬Leaves(p2)))
      1. For some men that come, not all women will leave
   9. ∃p(¬Tall(p) AND ¬Short(p))
      1. Someone is both not tall and not short
   10. ∃p(¬Tall(p)) AND ∃p(¬Short(p))
       1. Someone is not tall and someone is not short
   11. ∀s(Precious(s) => Beautiful(s))
       1. All precious stones are beautiful
   12. ∃p(Loves(p, me))
       1. Someone loves me
   13. ∀s(American(s) => ¬Poisonous(s))
       1. No American snakes are poisonous
   14. ∀s((Snake(s) ∧ American(s)) => ¬Poisonous(s))
       1. No American snakes are poisonous
5. .
   1. FALSE (x = 2/3, not natural)
   2. FALSE (x = SQRT(2), not rational)
   3. TRUE
   4. TRUE
   5. FALSE (y = 1, it’s not the case for all z)
   6. FALSE
   7. FALSE (e.g., x = -1)
   8. TRUE (by default)
6. .
   1. ∀x(2x + 3 ≠ 5x + 1)
   2. ∀x(x2 ≠ 2)
   3. ∃x∀y(y ≠ x2)
   4. ∃x∀y(y ≠ x2)
   5. ∃x∀y∃z(xy ≠ xz)
   6. ∃x∀y∃z(xy ≠ xz)
   7. ∃x(x < 0 ∧ ∀y(y2 ≠ x))
   8. ∃x(x < 0 ∧ ∀y(y2 ≠ x))
7. .
   1. (∃x ∊ ℕ)(∀y ∊ ℕ)(x + y ≠ 1)
   2. (∃x > 0)(∀y < 0)(x + y ≠ 0)
   3. ∀x(∃ε > 0)(x ≤ -ε OR x ≥ ε)
   4. (∃x ∊ ℕ)(∃y ∊ ℕ)(∀z ∊ ℕ) (x + y ≠ z2)
8. Negation of [((∃p)(∀t)(CanFoolAt(p,t))) AND  
    ((∃t)(∀p)(CanFoolAt(p,t))) AND ¬((∀t)(∀p)(CanFoolAt(p,t)))]
   1. [((∀p)(∃t)(¬CanFoolAt(p,t))) OR   
       ((∀t)(∃p)(¬CanFoolAt(p,t))) OR ((∀t)(∀p)(CanFoolAt(p,t)))]
   2. For every person, there’s a time you can’t fool them OR at any time there is someone you can’t fool OR you can always fool all the people all the time
9. Negation of (∀ε > 0)(∃δ > 0)(∀x)[|x-a| < δ => |f(x)-f(a)| < ε]
   1. (∃ε > 0)(∀δ > 0)(∃x)[|x-a| < δ AND |f(x)-f(a)| ≥ ε]

Week 4 – Problem Set 4

1. Which of the following is equivalent to ¬∀x[P(x) ⇒ (Q(x)∨R(x))]? (Only one is.)
   1. ∃x[P(x)∨¬Q(x)∨¬R(x)]
   2. ∃x[¬P(x)∧Q(x)∧R(x)]
   3. **∃x[P(x)∧¬Q(x)∧¬R(x)]**
   4. ∃x[P(x)∧(¬Q(x)∨¬R(x))]
   5. ∃x[P(x)∨(¬Q(x)∧¬R(x))]
      1. Demonstration: ¬∀x[P(x) ⇒ (Q(x)∨R(x))]
      2. ∃x[¬(P(x) ⇒ (Q(x)∨R(x)))]
      3. ∃x[¬(¬P(x) ∨ Q(x) ∨ R(x))]
      4. ∃x[(P(x) ∧ ¬Q(x) ∧ ¬R(x))]
2. Let p, q be variables denoting tennis players, let t be a variable denoting games of tennis, and let W(p,q,t) mean that p plays against q in game t and wins. Which of the following claims about tennis players mean the same as the symbolic formula ∀p∃q∃tW(p,q,t)? Select all that have that meaning.
   1. **Everyone wins a game**
   2. Everyone loses a game (e.g., AxB, AxC, BxC, CxB)
   3. For every player there is another player they beat all the time (e.g., AxB, BxA, AxC, CxA, BxC, CxB)
   4. There is a player who loses every game (same as c)
   5. There is a player who wins every game (same as c)
3. Let p,q be variables denoting the tennis players in a club, let t be a variable denoting the club’s games of tennis, and let W(p,q,t) mean that p plays against q in game t and wins. Assuming that there are at least two tennis players and games between them do take place, which (if any) of the following symbolic formula cannot possibly be true? Select all you think cannot possibly be true
   1. ∀p∃q∃tW(p,q,t)
   2. **∀p∀q∃tW(p,q,t)** (not conditioned that p != q)
   3. ∀q∃p∃tW(p,q,t)
4. Which of the following means “Everybody loves a lover”, where L(x,y) means (person) x loves (person) y and a lover is deﬁned to be someone in a mutual loving relationship? [If English is not your native language, you might want to discuss this sentence with a native English speaker before you answer. It’s an idiomatic expression.]
   1. **∀x∀y[∃z(L(x,z)∧L(z,x)) ⇒ L(y,x)]**
   2. ∀x∀y[∀z(L(x,z)∨L(z,x)) ⇒ L(y,x)]
   3. ∀x[∃z(L(x,z)∧L(z,x))∧∀yL(y,x)]
5. Which of the following statements about the order relation on the real line is/are false?TBC
   1. ∀x∀y∀z[(x ≤ y)∧(y ≤ z) ⇒ (x ≤ z)] TRUE (definition of total order)
   2. ∀x∀y[(x ≤ y)∧(y ≤ x) ⇒ (x = y)] TRUE
   3. ∀x∃y[(x ≤ y)∧(y ≤ x)] TRUE (x == y)
   4. **∃x∀y[(y < x)∨(x < y)]** FALSE (y == x)
6. A student produced this purported proof while trying to understand Euclid’s proof of the inﬁnitude of the primes. Grade it according to the course rubric.
   1. If an integer N is divisible by a prime P, then N+1 is not divisible by P.
   2. Proof: Suppose N is divisible by P. Then there is an integer Q such that N=PQ
   3. So N+1=PQ+1. Then
   4. (N+1)/P = Q + 1/P
   5. But Q+1/P is not an integer. So N+1 is not divisible by P.
   6. Logical correctness: 4 3 (cannot use division for integers)
   7. Clarity: 4 4
   8. Opening: 4 4
   9. Stating the conclusion: 4 4
   10. Reasons: 4 4
   11. Overall: 4 0
   12. => Total: 24 19

∀ ∃ φ ψ ¬ ∧ ∨ℕℚ∊∉ℝℤ≠⇐⇒⇔∃∄∀∴∧∨≤≥⊗