Week 4 – Assignment 6

1. Show that ¬[∃xA(x)] is equivalent to ∀x[¬A(x)]
   1. 🡪 direction
      1. Assume ¬[∃xA(x)]
      2. (rewrite in English) It is not the case that there is an x such that A(x) is true
      3. (therefore) For all values of x, it is the case that A(x) is false
      4. (rewrite) For all values of x, it is not the case that A(x) is true
      5. (rewrite) ∀x[¬A(x)]
   2. 🡨 direction
      1. Assume ∀x[¬A(x)]
      2. (rewrite in English) For all values of x, it is not the case that A(x) is true
      3. (rewrite) For all values of x, it is the case that A(x) is false
      4. (same as) There are no values x such as A(x) is true
      5. (rewrite) It is not the case that there is a value x such that A(x) is true
      6. (rewrite) ¬[∃xA(x)]
2. Prove that “There is an even prime bigger than 2” is false
   1. (rewrite) There are no even prime numbers bigger than 2
   2. (Equivalent to) All prime numbers bigger than 2 are not even
   3. Let p be a prime bigger than 2; if p is even, then p can be written in the form p=2\*p’
   4. (therefore) p cannot be prime (since there is a number p’ greater than 1 and smaller than p that divides it)
3. .
   1. ∀p(IsStudent(p) => LikesPizza(p))
   2. ∃p(IsMyFriend(p) ∧ ¬HasCar(p))
   3. ∃a(IsElephant(a) ∧ ¬LikesMuffin(a))
   4. ∀x(IsTriangle(x) => IsIsoceles(x))
   5. ∃p(IsStudentInThisClass(p) ∧ IsNotHereToday(p))
   6. ∀p∃q(Loves(p, q))
   7. ∀p∃q(¬Loves(p, q))
   8. ∀p1((Man(p1) ∧ Comes(p1)) => ∀p2(Woman(p2) => Leaves(p2)))
   9. ∀p(Tall(p) OR Short(p))
   10. ∀p(Tall(p)) OR ∀p(Short(p))
   11. ∃s(Precious(s) AND ¬Beautiful(s))
   12. ∀p(¬Loves(p, me))
   13. ∃s(American(s) ∧ Poisonous(s))
   14. ∃s(Snake(s) ∧ American(s) ∧ Poisonous(s))
4. .
   1. ∃p(IsStudent(p) ∧ ¬LikesPizza(p))
      1. There is a student that does not like pizza.
   2. ∀p(IsMyFriend(p) => HasCar(p))
      1. All my friends have cars.
   3. ∀a(IsElephant(a) => LikesMuffin(a))
      1. All elephants like muffins
   4. ∃x(IsTriangle(x) ∧ ¬IsIsoceles(x))
      1. Some triangles are not isosceles.
   5. ∀p(IsStudentInThisClass(p) => IsHereToday(p))
      1. All students in this class are here today
   6. ∃p∀q(¬Loves(p, q))
      1. Someone doesn’t love anybody
   7. ∃p∀q(Loves(p, q))
      1. Someone loves everyone
   8. ∃p1(Man(p1) ∧ Comes(p1) ∧ ∃p2(Woman(p2) ∧ => ∀p2(Woman(p2) => ¬Leaves(p2)))
      1. For some men that come, not all women will leave
   9. ∃p(¬Tall(p) AND ¬Short(p))
      1. Someone is both not tall and not short
   10. ∃p(¬Tall(p)) AND ∃p(¬Short(p))
       1. Someone is not tall and someone is not short
   11. ∀s(Precious(s) => Beautiful(s))
       1. All precious stones are beautiful
   12. ∃p(Loves(p, me))
       1. Someone loves me
   13. ∀s(American(s) => ¬Poisonous(s))
       1. No American snakes are poisonous
   14. ∀s((Snake(s) ∧ American(s)) => ¬Poisonous(s))
       1. No American snakes are poisonous
5. .
   1. FALSE (x = 2/3, not natural)
   2. FALSE (x = SQRT(2), not rational)
   3. TRUE
   4. TRUE
   5. FALSE (y = 1, it’s not the case for all z)
   6. FALSE
   7. FALSE (e.g., x = -1)
   8. TRUE (by default)
6. .
   1. ∀x(2x + 3 ≠ 5x + 1)
   2. ∀x(x2 ≠ 2)
   3. ∃x∀y(y ≠ x2)
   4. ∃x∀y(y ≠ x2)
   5. ∃x∀y∃z(xy ≠ xz)
   6. ∃x∀y∃z(xy ≠ xz)
   7. ∃x(x < 0 ∧ ∀y(y2 ≠ x))
   8. ∃x(x < 0 ∧ ∀y(y2 ≠ x))
7. .
   1. (∃x ∊ ℕ)(∀y ∊ ℕ)(x + y ≠ 1)
   2. (∃x > 0)(∀y < 0)(x + y ≠ 0)
   3. ∀x(∃ε > 0)(- ε ≥ x OR x ≤ ε)
   4. (∃x ∊ ℕ)(∃y ∊ ℕ)(∀z ∊ ℕ) (x + y ≠ z2)
8. Negation of [((∃p)(∀t)(CanFoolAt(p,t))) AND  
    ((∃t)(∀p)(CanFoolAt(p,t))) AND ¬((∀t)(∀p)(CanFoolAt(p,t)))]
   1. [((∀p)(∃t)(¬CanFoolAt(p,t))) OR   
       ((∀t)(∃p)(¬CanFoolAt(p,t))) OR ((∀t)(∀p)(CanFoolAt(p,t)))]
9. Negation of (∀ε > 0)(∃δ > 0)(∀x)[|x-a| < δ => |f(x)-f(a)| < ε]
   1. (∃ε > 0)(∀δ > 0)(∃x)[|x-a| < δ AND |f(x)-f(a)| ≥ ε]

Week 4 – Problem Set 4

1. TO CONTINUE FROM HERE
2. Let x be a variable ranging over doubles tennis matches, and t be a variable ranging over doubles tennis matches when Rosario partners with Antonio. Let W(x) mean that Rosario and her partner (whoever it is) win the doubles match x. Select the following English sentences that mean the same as the symbolic formula ∃tW(t)
   1. Rosario and Antonio win every match where they are partners.
   2. **Rosario and her partner sometimes win the match when she partners with Antonio.**
   3. Whenever Rosario plays with Antonio, they win the match.
   4. Rosario and Antonio win exactly one match when they are partners.
   5. **Rosario and Antonio win at least one match when they are partners.**
   6. If Rosario and her partner win the match, she must be partnering with Antonio.
3. Let x be a variable ranging over doubles tennis matches, and t be a variable ranging over doubles tennis matches when Rosario partners with Antonio. Let W(x) mean that Rosario and her partner (whoever it is) win the doubles match x. Select the following English sentences that mean the same as the symbolic formula ∀tW(t):
   1. **Rosario and Antonio win every match where they are partners.**
   2. Rosario always partners with Antonio.
   3. **Whenever Rosario partners with Antonio, they win the match.**
   4. Sometimes, Rosario and her partner win the match.
   5. **Rosario and her partner win the match whenever she partners with Antonio.**
   6. If Rosario and her partner win the match, she must be partnering with Antonio.
4. Which of the following formal propositions says that there is no largest prime? (There may be more than one. You have to select all correct propositions.) The variables denote natural numbers.
   1. ¬∃x∃y[Prime(x) ∧¬Prime(y)∧(x < y)]
   2. ∀x∃y[Prime(x)∧Prime(y)∧(x < y)] <<<=== False because Prime(x) is not always true
   3. ∀x∀y[Prime(x)∧Prime(y)∧(x < y)]
   4. **∀x∃y[Prime(y)∧(x < y)]**
   5. ∃x∀y[Prime(y)∧(x < y)]
   6. ∀x∃y[Prime(x)∧(x < y)]
5. The symbol ∃!x means “There exists a unique x such that ...” Which of the following accurately deﬁnes the expression ∃!xφ(x)?
   1. **∃x∀y[φ(x)∧[φ(y) ⇒ (x ≠ y)]] NO**
   2. ∃x[φ(x)∧(∃y)[φ(y) ⇒ (x ≠ y)]]
   3. ∃x∃y[(φ(x)∧φ(y)) ⇒ (x = y)]
   4. [∃xφ(x)]∧(∀y)[φ(y) ⇒ (x = y)]
   5. **∃x[φ(x)∧(∀y)[φ(y) ⇒ (x = y)]] <<<====**
6. Which of the following means “The arithmetic operation x↑y is not commutative.”? (↑ is just some arbitrary binary operation.)
   1. ∀x∀y[x↑y ≠ y↑x]
   2. ∀x∃y[x↑y ≠ y↑x]
   3. **∃x∃y[x↑y ≠ y↑x]**
   4. ∃x∀y[x↑y ≠ y↑x]
7. Evaluate this purported proof, and grade it according to the course rubric.  
   Claim: There does not exist a positive integer N such that N2 + 4N + 3 is prime.  
   Proof: N2 + 4N + 3 = (N + 1)(N + 3). This is not prime
   1. Logical correctness: 2 4
   2. Clarity: 2 0
   3. Opening: 2 0
   4. Stating the conclusion: 2 4
   5. Reasons: 0 0
   6. Overall: 0 2
   7. => Total: 8 10

∀ ∃ φ ψ ¬ ∧ ∨ℕℚ∊∉ℝℤ≠⇐⇒⇔∃∄∀∴∧∨≤≥⊗