Week 9 – Peer reviews

Grade the answers according to the course rubric. Enter your grade (which should be a whole number between 0 and 24, inclusive) in the box below.

Stephan, Feb 6th, 2017

1. Say whether the following is true or false and support your answer by a proof. (∃m∈N)(∃n∈N)(3m+5n=12)
   1. The statement is false.
   2. n can only be 1 or 2, given that 5 x 3 > 12. No m exists such that 3m = 7 for n = 1 or 3m = 2 for n = 2.True or false? (∃m∈N)(∃n∈N)(3m+5n=12)
   3. **Evaluation:**
      1. **Logical correctness: 4**
      2. **Clarity: 4**
      3. **Opening: 4**
      4. **Stating the conclusion: 4**
      5. **Reasons: 4**
      6. **Overall: 4**
      7. **=> Total: 24**
2. Say whether the following is true or false and support your answer by a proof: The sum of any five consecutive integers is divisible by 5 (without remainder).
   1. The statement is true.
   2. Given n + (n + 1) + (n + 2) + (n + 3) + (n + 4) = 5n + 10 = 5 (n + 1) + 5, division by 5 leaves no remainder.
   3. **Evaluation:**
      1. **Logical correctness: 4**
      2. **Clarity: 3**
      3. **Opening: 3**
      4. **Stating the conclusion: 4**
      5. **Reasons: 4**
      6. **Overall: 4**
      7. **=> Total: 22**
3. Say whether the following is true or false and support your answer by a proof: For any integer *n*, the number *n*2+*n*+1 is odd.
   1. n^2 + n + 1 = n(n + 1) + 1. n(n + 1) is even for all n. Therefore, n(n + 1) + 1 is odd for all n.
   2. **Evaluation:**
      1. **Logical correctness: 4**
      2. **Clarity: 4**
      3. **Opening: 4**
      4. **Stating the conclusion: 4**
      5. **Reasons: 3 (“why n(n+1) is even for all n”?)**
      6. **Overall: 4**
      7. **=> Total: 23**
4. Prove that every odd natural number is of one of the forms 4n+1 or 4n+3, where n is an integer.
   1. The statement is true.
   2. For the odd number m = 1: 4n + 1 = 1, gives n = 0. For 4n + 1 = (m + 2), 4n = m + 1 -- or 4n = m + 3 -- n covers all odd natural numbers.
   3. **Evaluation:**
      1. **Logical correctness: 0**
      2. **Clarity: 0**
      3. **Opening: 0**
      4. **Stating the conclusion: 2**
      5. **Reasons: 1**
      6. **Overall: 1**
      7. **=> Total: 4**
5. Prove that for any integer n, at least one of the integers n, n+2,n+4 is divisible by 3.
   1. n can be expressed in one of the forms 3q, 3q + 1, 3q + 2 for some q.
   2. In the first case, n is divisible by 3.
   3. In the second case n + 2 = 3q + 3 = 3(q + 1), so n + 2 is divisible by 3.
   4. In the third case n + 4 = 3q +6 = 3 (q + 2), so n + 4 is divisible by 3.
   5. **Evaluation:**
      1. **Logical correctness: 3 – has nothing to do with the problem**
      2. **Clarity: 4**
      3. **Opening: 4**
      4. **Stating the conclusion: 3 - Missing “since all cases are covered…”**
      5. **Reasons: 2 - Didn’t specify division theorem**
      6. **Overall: 2**
      7. **=> Total: 18**
6. A classic unsolved problem in number theory asks if there are infinitely many pairs of ‘twin primes’, pairs of primes separated by 2, such as 3 and 5, 11 and 13, or 71 and 73. Prove that the only prime triple (i.e. three primes, each 2 from the next) is 3, 5, 7.
   1. For the odd numbers n, n + 2, and n + 4 where n > 3, 3 divides one of those. If 3 does not divide n, then by the Div. Theorem, n = 3q + 1 or n = 3q + 2 for some q.
   2. If n + 2 = 3q + 3 than 3|(n+2) and if n + 4 = 3q + 6 than 3|(n+4). Therefore, 3 divides on of the three numbers.
   3. **Evaluation:**
      1. **Logical correctness: 4**
      2. **Clarity: 4**
      3. **Opening: 4**
      4. **Stating the conclusion: 4**
      5. **Reasons: 4**
      6. **Overall: 4**
      7. **=> Total: 24**
7. Prove that for any natural number n: 2 + 22 + 23 + . . . + 2n = 2n+1 – 2
   1. Via induction: For n = 1, 2 = 2^2 - 2, the identity is true.
   2. Assuming it holds for n, then adding 2^(n+1) gives 2^(n+2) - 2, the identity at n + 1.
   3. **Evaluation:**
      1. **Logical correctness: 2**
      2. **Clarity: 2**
      3. **Opening: 4**
      4. **Stating the conclusion: 2**
      5. **Reasons: 2**
      6. **Overall: 2**
      7. **=> Total: 14**
8. Prove (from the definition of a limit of a sequence) that if the sequence {an}[n=1..∞] tends to limit L as n→∞, then for any fixed number M>0, the sequence {Man}[n=1..∞] tends to the limit ML.
   1. --
   2. **Evaluation:**
      1. **Logical correctness: 0**
      2. **Clarity: 0**
      3. **Opening: 0**
      4. **Stating the conclusion: 0**
      5. **Reasons: 0**
      6. **Overall: 0**
      7. **=> Total: 0**
9. Given a collection An, n=1,2,… of intervals of the real line, their intersection is defined to be ⋂[n=1..∞]An={x|(∀n)(x∈An)}. Give an example of a family of intervals An, n=1,2,…, such that An+1⊂An for all n and ⋂[n=1..∞]An=∅. Prove that your example has the stated property.
   1. Given the sequence (0,1), (0,1/2), (0,1/4),... : A\_n = (0, 1/2^(n-1)).
   2. Because lim\_(n --> inf) {1/2n}\_(n=1) = 0, ⋂n=1∞An=∅.
   3. **Evaluation:**
      1. **Logical correctness: 3**
      2. **Clarity: 2**
      3. **Opening: 3**
      4. **Stating the conclusion: 2**
      5. **Reasons: 2**
      6. **Overall: 2**
      7. **=> Total: 14**
10. Given a collection An, n=1,2,… of intervals of the real line, their intersection is defined to be ⋂[n=1..∞]An={x|(∀n)(x∈An)}. Give an example of a family of intervals An, n=1,2,…, such that An+1⊂An for all n and ⋂[n=1..∞]An consists of a single real number. Prove that your example has the stated property.
    1. --
    2. **Evaluation:**
       1. **Logical correctness: 0**
       2. **Clarity: 0**
       3. **Opening: 0**
       4. **Stating the conclusion: 0**
       5. **Reasons: 0**
       6. **Overall: 0**
       7. **=> Total: 0**

Juras Šulcas, Feb 6th, 2017

1. Say whether the following is true or false and support your answer by a proof. (∃m∈N)(∃n∈N)(3m+5n=12)
   1. ∃m∈N means there exists at least one “m” which is an element of the set of natural numbers.
   2. ∃n∈N means there exists at least one “n” which is an element of the set of natural numbers.
   3. The answer depends on what definition of a natural number we use. There are 2 of them: traditional and alternative, the second one has zero included.
   4. So, it's true if we use alternative definition by using m=4, and n=0.
   5. Its false, if we use traditional definition, as no integers are suitable for our problem then.
   6. **Evaluation:**
      1. **Logical correctness: 3**
      2. **Clarity: 3**
      3. **Opening: 4**
      4. **Stating the conclusion: 4**
      5. **Reasons: 2 (why are there no integers suitable)**
      6. **Overall: 3**
      7. **=> Total: 19**
2. Say whether the following is true or false and support your answer by a proof: The sum of any five consecutive integers is divisible by 5 (without remainder).
   1. Lets take a as our smallest integer.
   2. Then the others are:
   3. a+1, a+2, a+3, a+4.
   4. Sum of these all is 5a + 10 = 5(a+2).
   5. So, it's true.
   6. **Evaluation:**
      1. **Logical correctness: 4**
      2. **Clarity: 4**
      3. **Opening: 4**
      4. **Stating the conclusion: 4**
      5. **Reasons: 4**
      6. **Overall: 4**
      7. **=> Total: 24**
3. Say whether the following is true or false and support your answer by a proof: For any integer *n*, the number *n*2+*n*+1 is odd.
   1. If we multiply any odd number by other odd number or also itself, we get another odd number.
   2. If we add an odd number to another odd number, we get an even number.
   3. If we add 1 to an even number, we get an odd number.
   4. So, if use n as any odd number, it's true.
   5. If we multiply any even number by other even number or also itself, we get another even number.
   6. If we add an even number to another even number, we get an even number.
   7. If we add 1 to an even number, we get an odd number.
   8. So, if use n as any even number, it's true.
   9. So, if use n as any number, it's true.
   10. **Evaluation:**
       1. **Logical correctness: 4**
       2. **Clarity: 4**
       3. **Opening: 4**
       4. **Stating the conclusion: 4**
       5. **Reasons: 4**
       6. **Overall: 4**
       7. **=> Total: 24**
4. Prove that every odd natural number is of one of the forms 4n+1 or 4n+3, where n is an integer.
   1. Any natural number multiplied by 2 gets us an even number. So, we take n, multiply by 2, then again, and by adding an odd number, we get a new odd.
   2. **Evaluation:**
      1. **Logical correctness: 0**
      2. **Clarity: 0**
      3. **Opening: 0**
      4. **Stating the conclusion: 0**
      5. **Reasons: 0**
      6. **Overall: 0**
      7. **=> Total: 0 – this is a proof that 4n+1,4n+3 are odd, not that it contains all odd numbers (per evaluation exercise 2)**
5. Prove that for any integer n, at least one of the integers n, n+2,n+4 is divisible by 3.
   1. Suppose that n is not divisible by 3.
   2. It means :
   3. n % 3 = {1,2} let call it x
   4. and
   5. n + 2 % 3 = (x + 2) % 3
   6. and n + 4 % 3 = (x + 4) % 3
   7. So if x = 1
   8. n + 2 % 3 = 3 % 3 = 0
   9. n + 4 % 3 = 5 % 3 = 2
   10. n + 2 is divisible by 3
   11. n + 4 is not divisible by 3
   12. else if x = 2
   13. n + 2 % 3 = 4 % 3 = 1
   14. n + 4 % 3 = 6 % 3 = 0
   15. n + 2 is not divisible by 3
   16. n + 4 is divisible by 3
   17. **Evaluation:**
       1. **Logical correctness: 4**
       2. **Clarity: 4**
       3. **Opening: 2**
       4. **Stating the conclusion: 2 (missing closing all cases)**
       5. **Reasons: 2 (why n%3 = {1,2}?)**
       6. **Overall: 3**
       7. **=> Total: 17**
6. A classic unsolved problem in number theory asks if there are infinitely many pairs of ‘twin primes’, pairs of primes separated by 2, such as 3 and 5, 11 and 13, or 71 and 73. Prove that the only prime triple (i.e. three primes, each 2 from the next) is 3, 5, 7.
   1. --
   2. **Evaluation:**
      1. **Logical correctness: 0**
      2. **Clarity: 0**
      3. **Opening: 0**
      4. **Stating the conclusion: 0**
      5. **Reasons: 0**
      6. **Overall: 0**
      7. **=> Total: 0**
7. Prove that for any natural number n: 2 + 22 + 23 + . . . + 2n = 2n+1 – 2
   1. --
   2. **Evaluation:**
      1. **Logical correctness: 0**
      2. **Clarity: 0**
      3. **Opening: 0**
      4. **Stating the conclusion: 0**
      5. **Reasons: 0**
      6. **Overall: 0**
      7. **=> Total: 0**
8. Prove (from the definition of a limit of a sequence) that if the sequence {an}[n=1..∞] tends to limit L as n→∞, then for any fixed number M>0, the sequence {Man}[n=1..∞] tends to the limit ML.
   1. Let ε>0 be given. By the assumption, we can find an N such that n⩾N⇒|an−L|<ε/M. then, n≥N⇒|Man−ML|=M|an−L|<M∗ϵ/M=ϵ which shows that if the sequence {an}[n=1..∞] tends to limit L as n→∞ then, for any fixed number M>0, the sequence {Man}[n=0..∞] tends to the limit ML.
   2. **Evaluation:**
      1. **Logical correctness: 4**
      2. **Clarity: 4**
      3. **Opening: 4**
      4. **Stating the conclusion: 4**
      5. **Reasons: 4**
      6. **Overall: 4**
      7. **=> Total: 24**
9. Given a collection An, n=1,2,… of intervals of the real line, their intersection is defined to be ⋂[n=1..∞]An={x|(∀n)(x∈An)}. Give an example of a family of intervals An, n=1,2,…, such that An+1⊂An for all n and ⋂[n=1..∞]An=∅. Prove that your example has the stated property.
   1. --
   2. **Evaluation:**
      1. **Logical correctness: 0**
      2. **Clarity: 0**
      3. **Opening: 0**
      4. **Stating the conclusion: 0**
      5. **Reasons: 0**
      6. **Overall: 0**
      7. **=> Total: 0**
10. Given a collection An, n=1,2,… of intervals of the real line, their intersection is defined to be ⋂[n=1..∞]An={x|(∀n)(x∈An)}. Give an example of a family of intervals An, n=1,2,…, such that An+1⊂An for all n and ⋂[n=1..∞]An consists of a single real number. Prove that your example has the stated property.
    1. Let An=(−1−(1/n), 1+(1/n)), and
    2. A=⋂[n=1..∞]An=[−1,1]
    3. **Evaluation:**
       1. **Logical correctness: 0**
       2. **Clarity: 0**
       3. **Opening: 0**
       4. **Stating the conclusion: 0**
       5. **Reasons: 0**
       6. **Overall: 0**
       7. **=> Total: 0**

Comments:

Overall: 18

- In question 1, you claimed that "false, if we use traditional definition, as no integers are suitable for our problem" - but didn't explain why there are no integers for that

- In question 4, you showed that any number of the form 4n+1 or 4n+3 is odd - that was fine. But the question asked to show that \*every\* natural number can be constructed of that form, which you did not do.

- In question 5, you stated that n % 3 = {1,2}; that is true, but why did you assume that; based on examples in this course, this truth comes from the division theorem.

- In the last question, the set of intervals that you provided does not have the required property (that their intersection is a set with a single value).

∀ ∃ φ ψ ¬ ∧ ∨ℕℚ∈∉ℝℤ≠⇐⇒⇔∃∄∀∴∧∨≤≥ε⊗∅ ∩∪⋂⋃⊂⊃⊅⊄⊆⊇⊈⊉