APMA 2120 Test 1 Study Guide Exercises

(Chapters 12 and 13)

To prepare for the test you should

- **Study the lecture notes posted on Collab.**
- **Review in-class worksheets, group work/quiz problems and homework problems**

Following are some additional practice problems in the review sections of the book:

- **Chapter 12 Review Section (Pages 884-886)**
  1, 5, 6, 10, 11, 16, 17, 18, 20, 21, 23, 24, 30, 31, 33, 36

- **Chapter 13 Review Section (Pages 928-929)**
  2, 3, 5, 6, 8, 9, 10, 12, 17, 18, 19, 22

Chapter 12 Review Section Exercises

1.

a. Find an equation of the sphere that passes through the point $(6, -2, 3)$ and has center $(-1, 2, 1)$.

b. Find the curve in which this sphere intersects the yz-plane.

c. Find the center and radius of the sphere

$$x^2 + y^2 + z^2 - 8x + 2y + 6z + 1 = 0$$

5. Find the values of x such that the vectors $\langle 3, 2, x \rangle$ and $\langle 2x, 4, x \rangle$ are orthogonal.

6. Find two unit vectors that are orthogonal to both $\mathbf{j} + 2\mathbf{k}$ and $\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$.

10. Given the points $A\,(1, 0, 1)$, $B\,(2, 3, 0)$, $C\,(-1, 1, 4)$, and $D\,(0, 3, 2)$, find the volume of the parallelepiped with adjacent edges AB, AC, and AD.

11.

a. Find a vector perpendicular to the plane through the points $A\,(1, 0, 0)$, $B\,(2, 0, -1)$, and $C\,(1, 4, 3)$.

b. Find the area of triangle ABC.

16. The line through $(1, 0, -1)$ and parallel to the line $\frac{1}{3}(x-4) = \frac{1}{2}y = z+2$

17. The line through $(-2, 2, 4)$ and perpendicular to the plane $2x - y + 5z = 12$

18. The plane through $(2, 1, 0)$ and parallel to $x + 4y - 3z = 1$

20. The plane through $(1, 2, -2)$ that contains the line $x = 2t, y = 3 - t, z = 1 + 3t$

21. Find the point in which the line with parametric equations $x = 2 - t$, $y = 1 + 3t$, $z = 4t$ intersects the plane $2x - y + z = 2$.

23. Determine whether the lines given by the symmetric equations

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

and

$$\frac{x+1}{6} = \frac{y-3}{-1} = \frac{z+5}{2}$$

are parallel, skew, or intersecting.

24.

a. Show that the planes $x + y - z = 1$ and $2x - 3y + 4z = 5$ are neither parallel nor perpendicular.

b. Find, correct to the nearest degree, the angle between these planes.

30. Identify and sketch the graph of $y = z^2$.

31. Identify and sketch the graph of $x^2 = y^2 + 4z^2$.

33. Identify and sketch the graph of $-4x^2 + y^2 - 4z^2 = 4$.

36. Identify and sketch the graph of $x = y^2 + z^2 - 2y - 4z + 5$.

Chapter 13 Review Section Exercises

2. Let $\mathbf{r}(t) = \langle \sqrt{2-t}, (e^t - 1)/t, \ln(t+1) \rangle$.

   a. Find the domain of $\mathbf{r}$.

   b. Find $\lim_{t\to 0} \mathbf{r}(t)$.

   c. Find $\mathbf{r}'(t)$.

3. Find a vector function that represents the curve of intersection of the cylinder $x^2 + y^2 = 16$ and the plane $x + z = 5$.

5. If $\mathbf{r}(t) = t^2\ \mathbf{i} + t\cos\pi t\ \mathbf{j} + \sin\pi t\ \mathbf{k}$, evaluate $\int_0^1 \mathbf{r}(t)\ dt$.

6. Let C be the curve with equations $x = 2 - t^3$, $y = 2t - 1$, $z = \ln t$. Find

a. the point where C intersects the xz-plane,

b. parametric equations of the tangent line at $(1, 1, 0)$, and

c. an equation of the normal plane to C at $(1, 1, 0)$.

8. Find the length of the curve $\mathbf{r}(t) = \langle 2t^{3/2},\ \cos 2t,\ \sin 2t\rangle$, $0 \leqslant t \leqslant 1$.

9. The helix $\mathbf{r}_1(t) = \cos t\ \mathbf{i} + \sin t\ \mathbf{j} + t\ \mathbf{k}$ intersects the curve $\mathbf{r}_2(t) = (1+t)\ \mathbf{i} + t^2\ \mathbf{j} + t^3\ \mathbf{k}$ at the point $(1, 0, 0)$. Find the angle of intersection of these curves.

10. Reparametrize the curve $\mathbf{r}(t) = e^t\ \mathbf{i} + e^t \sin t\ \mathbf{j} + e^t \cos t\ \mathbf{k}$ with respect to arc length measured from the point $(1, 0, 1)$ in the direction of increasing t.

12. Find the curvature of the ellipse $x = 3\cos t$, $y = 4\sin t$ at the points $(3, 0)$ and $(0, 4)$.

17. A particle moves with position function $\mathbf{r}(t) = t \ln t \, \mathbf{i} + t \, \mathbf{j} + e^{-t} \, \mathbf{k}$. Find the velocity, speed, and acceleration of the particle.

18. Find the velocity, speed, and acceleration of a particle moving with position function $\mathbf{r}(t) = (2t^2 - 3) \, \mathbf{i} + 2t \, \mathbf{j}$. Sketch the path of the particle and draw the position, velocity, and acceleration vectors for $t = 1$.

19. A particle starts at the origin with initial velocity $\mathbf{i} - \mathbf{j} + 3\mathbf{k}$. Its acceleration is $a(t) = 6t \, \mathbf{i} + 12t^2 \, \mathbf{j} - 6t \, \mathbf{k}$. Find its position function.

22. Find the tangential and normal components of the acceleration vector of a particle with position function

$$\mathbf{r}\left(t\right)=t\,\mathbf{i}+2t\,\mathbf{j}+t^{2}\,\mathbf{k}$$