

**Processamento Digital de Sinais**  
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**Lista de exercícios 4 – Transformada Discreta de Fourier (DFT)**

**P5.1** Compute the DFS coefficients of the following periodic sequences using the DFS definition, and then verify your answers using MATLAB.

1.  $\tilde{x}_1(n) = \{4, 1, -1, 1\}$ ,  $N = 4$
2.  $\tilde{x}_2(n) = \{2, 0, 0, 0, -1, 0, 0, 0\}$ ,  $N = 8$
3.  $\tilde{x}_3(n) = \{1, 0, -1, -1, 0\}$ ,  $N = 5$
4.  $\tilde{x}_4(n) = \{0, 0, 2j, 0, 2j, 0\}$ ,  $N = 6$
5.  $\tilde{x}_5(n) = \{3, 2, 1\}$ ,  $N = 3$

**P5.3** Let  $\tilde{x}_1(n)$  be periodic with fundamental period  $N = 40$  where one period is given by

$$\tilde{x}_1(n) = \begin{cases} 5 \sin(0.1\pi n), & 0 \leq n \leq 19 \\ 0, & 20 \leq n \leq 39 \end{cases}$$

and let  $\tilde{x}_2(n)$  be periodic with fundamental period  $N = 80$ , where one period is given by

$$\tilde{x}_2(n) = \begin{cases} 5 \sin(0.1\pi n), & 0 \leq n \leq 19 \\ 0, & 20 \leq n \leq 79 \end{cases}$$

These two periodic sequences differ in their periodicity but otherwise have the same nonzero samples.

1. Compute the DFS  $\tilde{X}_1(k)$  of  $\tilde{x}_1(n)$ , and plot samples (using the `stem` function) of its magnitude and angle versus  $k$ .
2. Compute the DFS  $\tilde{X}_2(k)$  of  $\tilde{x}_2(n)$ , and plot samples of its magnitude and angle versus  $k$ .
3. What is the difference between the two preceding DFS plots?

**P5.10** Plot the DTFT magnitude and angle of each of the following sequences using the DFT as a computation tool. Make an educated guess about the length  $N$  so that your plots are meaningful.

1.  $x(n) = (0.6)^{|n|} [u(n+10) - u(n-11)]$ .
2.  $x(n) = n(0.9)^n [u(n) - u(n-21)]$ .
3.  $x(n) = [\cos(0.5\pi n) + j \sin(0.5\pi n)][u(n) - u(n-51)]$ .
4.  $x(n) = \{1, 2, 3, 4, 3, 2, 1\}$ .
5.  $x(n) = \{-1, -2, -3, 0, 3, 2, 1\}$ .

**P5.33** Given the following sequences  $x_1(n)$  and  $x_2(n)$ :

$$x_1(n) = \{2, 1, 1, 2\}, \quad x_2(n) = \{1, -1, -1, 1\}$$

1. Compute the circular convolution  $x_1(n) \stackrel{(N)}{\circledast} x_2(n)$  for  $N = 4, 7$ , and  $8$ .
2. Compute the linear convolution  $x_1(n) * x_2(n)$ .
3. Using results of calculations, determine the minimum value of  $N$  necessary so that linear and circular convolutions are same on the  $N$ -point interval.
4. Without performing the actual convolutions, explain how you could have obtained the result of P5.33.3.

- P5.38** An analog signal  $x_a(t) = 2 \sin(4\pi t) + 5 \cos(8\pi t)$  is sampled at  $t = 0.01n$  for  $n = 0, 1, \dots, N-1$  to obtain an  $N$ -point sequence  $x(n)$ . An  $N$ -point DFT is used to obtain an estimate of the magnitude spectrum of  $x_a(t)$ .
1. From the following values of  $N$ , choose the one that will provide the accurate estimate of the spectrum of  $x_a(t)$ . Plot the real and imaginary parts of the DFT spectrum  $X(k)$ .  
(a)  $N = 40$ ,      (b)  $N = 50$ ,      (c)  $N = 60$ .
  2. From the following values of  $N$ , choose the one that will provide the least amount of leakage in the spectrum of  $x_a(t)$ . Plot the real and imaginary parts of the DFT spectrum  $X(k)$ . (a)  $N = 90$ ,      (b)  $N = 95$ ,      (c)  $N = 99$ .

Fonte: Ingle/Proakis, Digital Signal Processing Using Matlab - 3ª Edição.