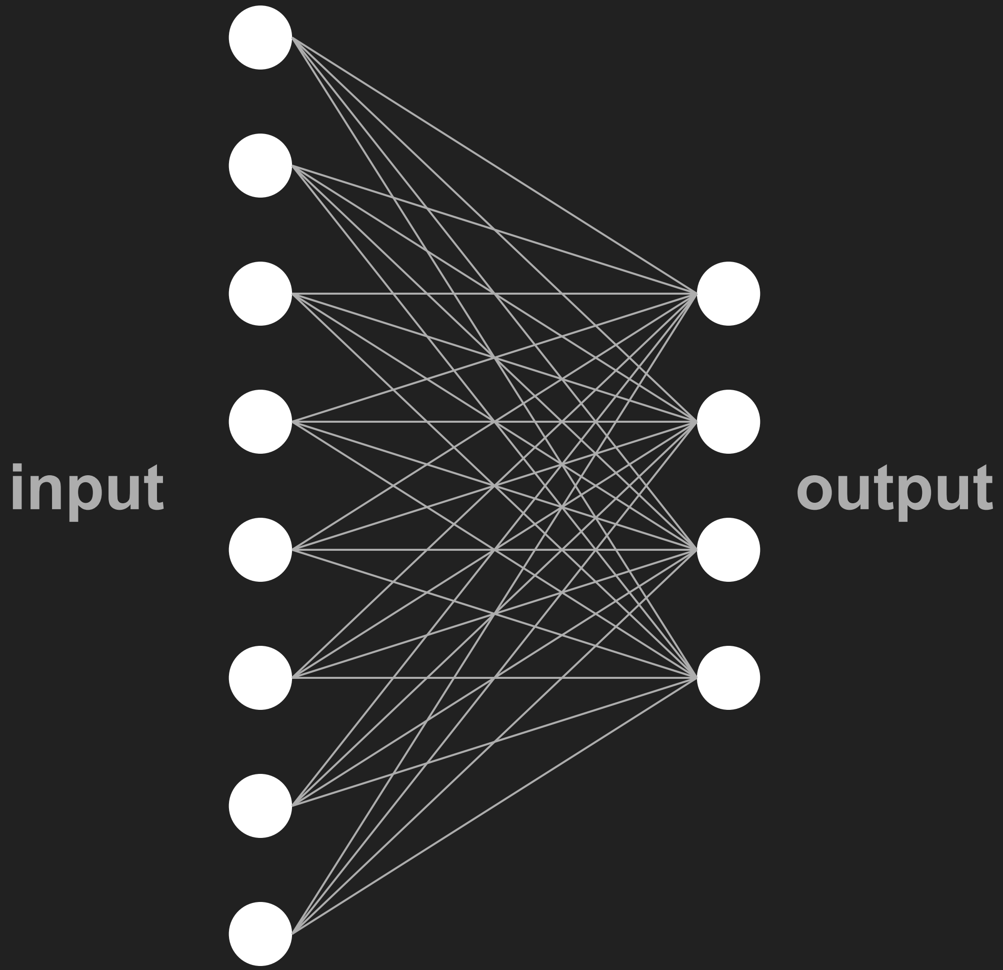
Neural networks

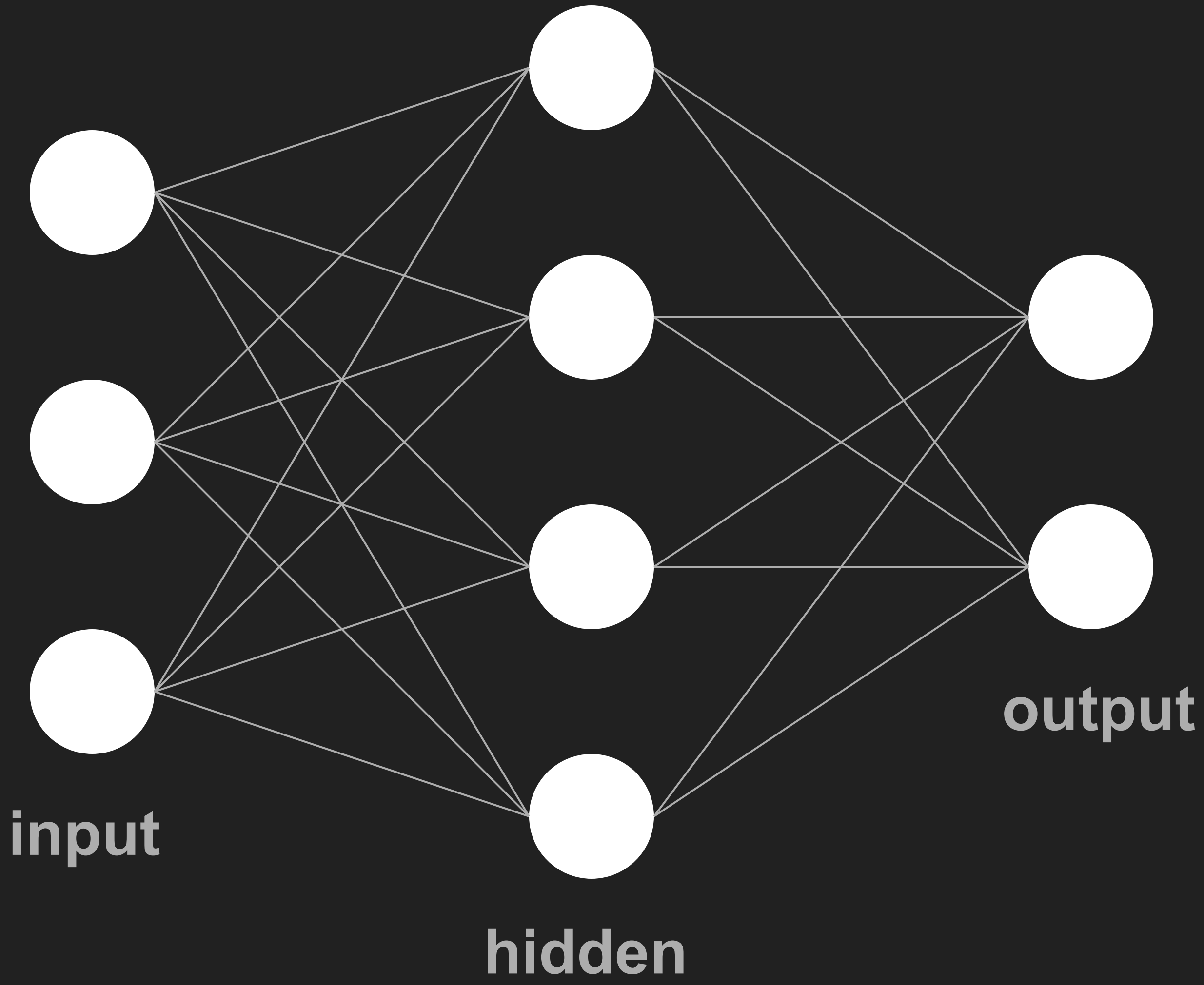
An introduction

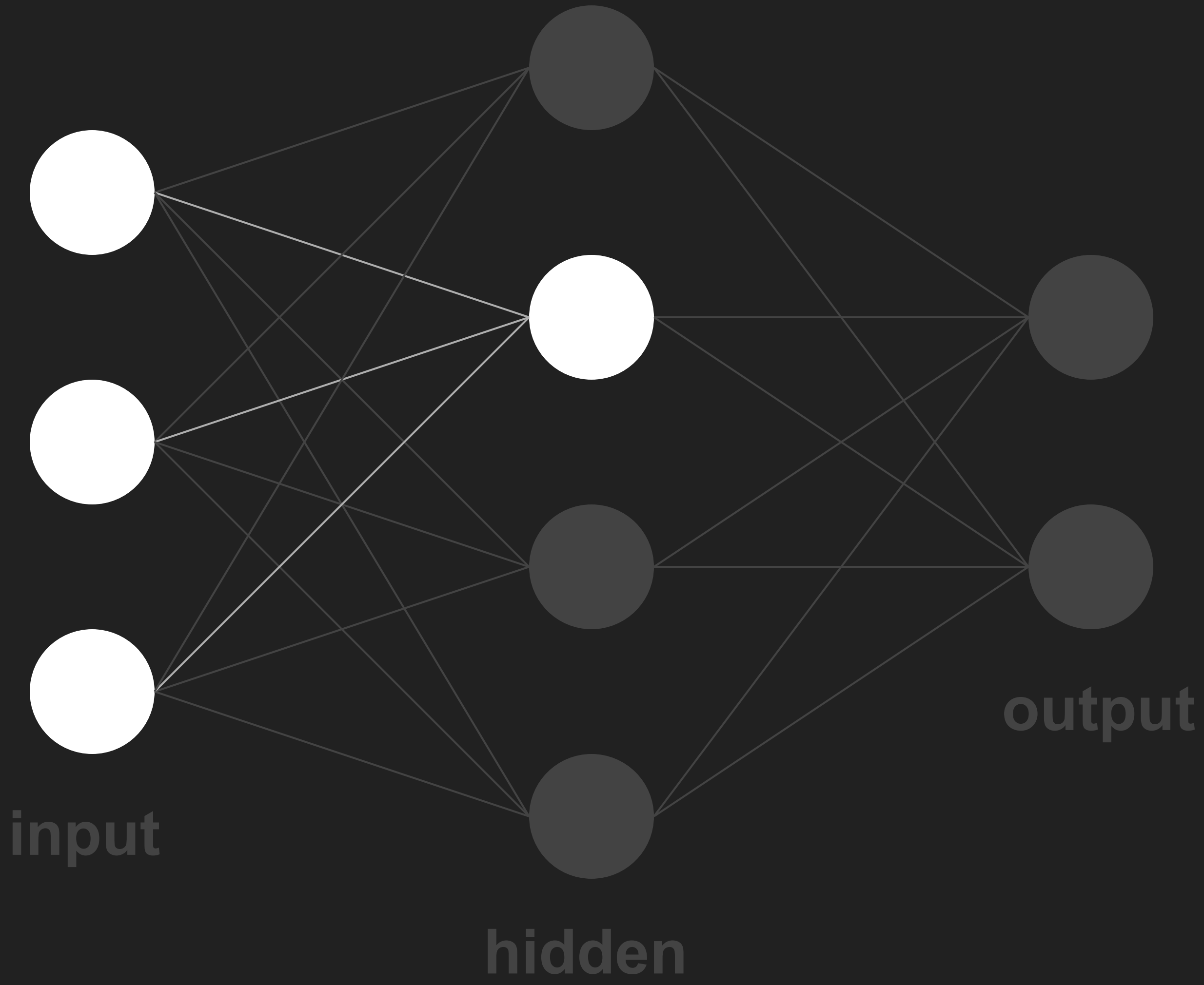
Single-layer perceptrons

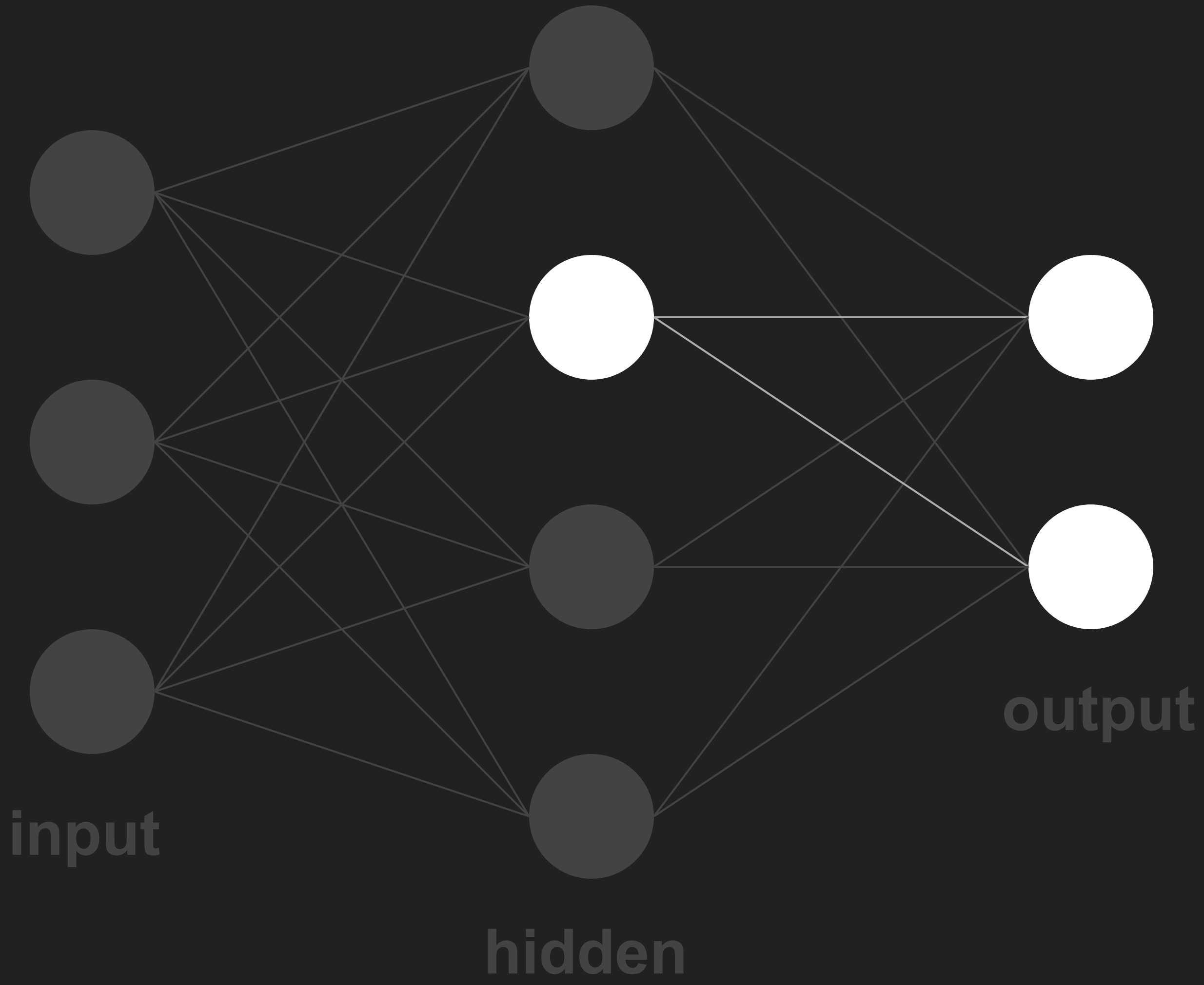

input
output

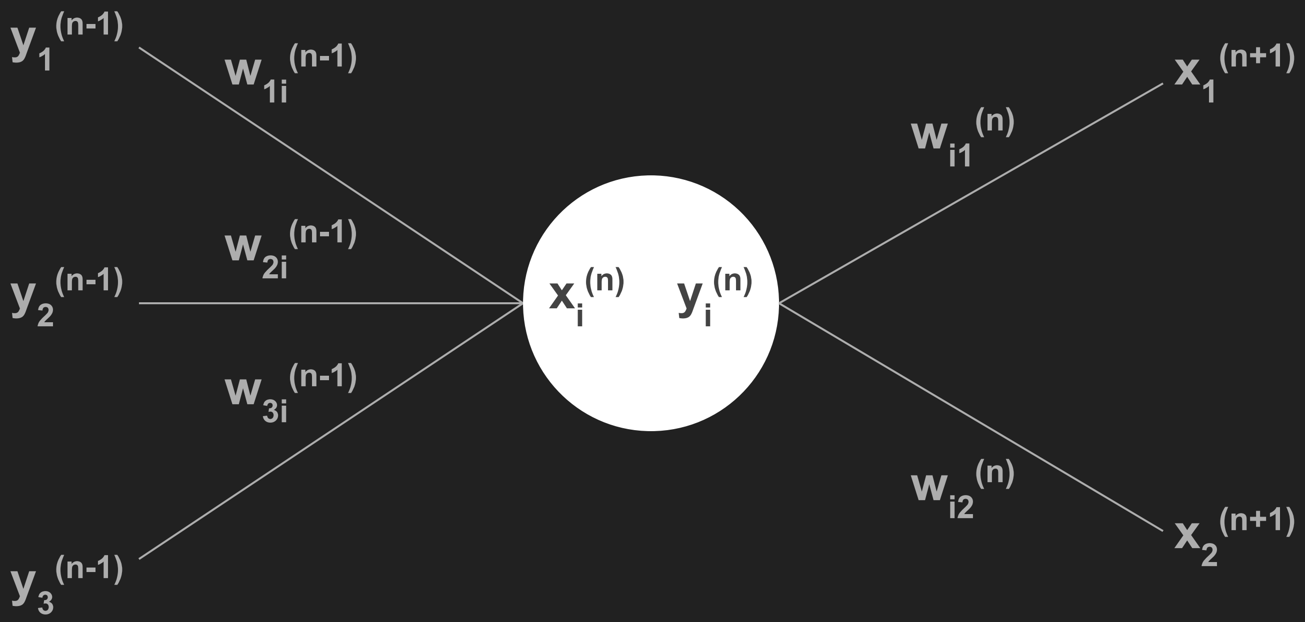
Example

Multiple layers


input
hidden
output


input
hidden
output

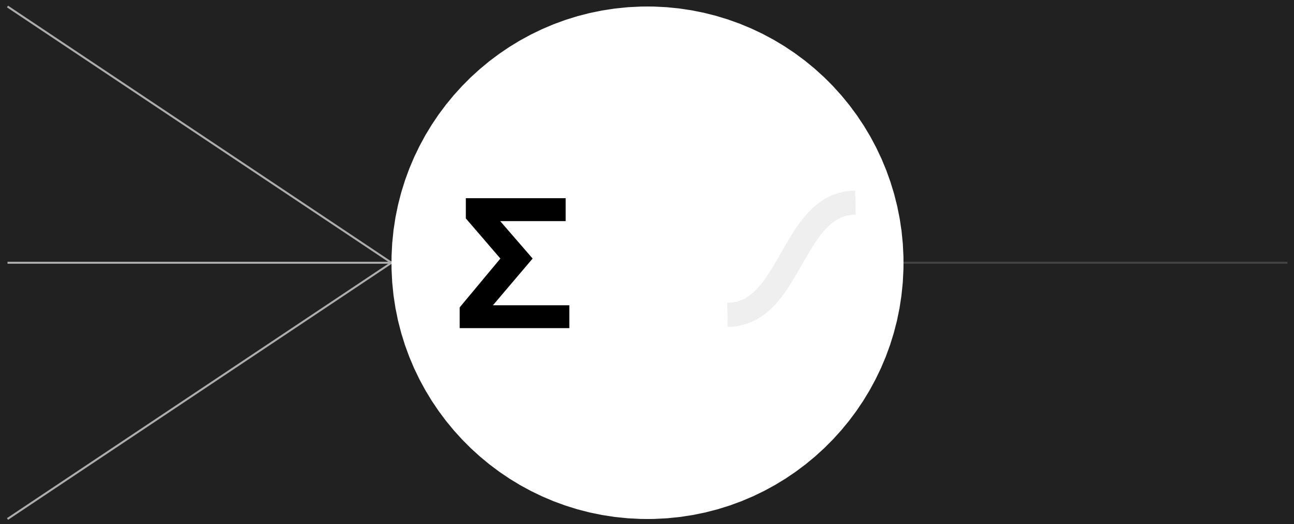

input
hidden
output



$y_1^{(n-1)}$
$w_{1i}^{(n-1)}$
$x_1^{(n+1)}$
$w_{i1}^{(n)}$
$w_{2i}^{(n-1)}$
$y_2^{(n-1)}$
$x_i^{(n)}$
$y_i^{(n)}$
$w_{3i}^{(n-1)}$
$w_{i2}^{(n)}$
$x_2^{(n+1)}$
$y_3^{(n-1)}$


Σ

$$x_i^{(n)} = \sum_j w_{ij}^{(n-1)} y_j^{(n-1)}$$

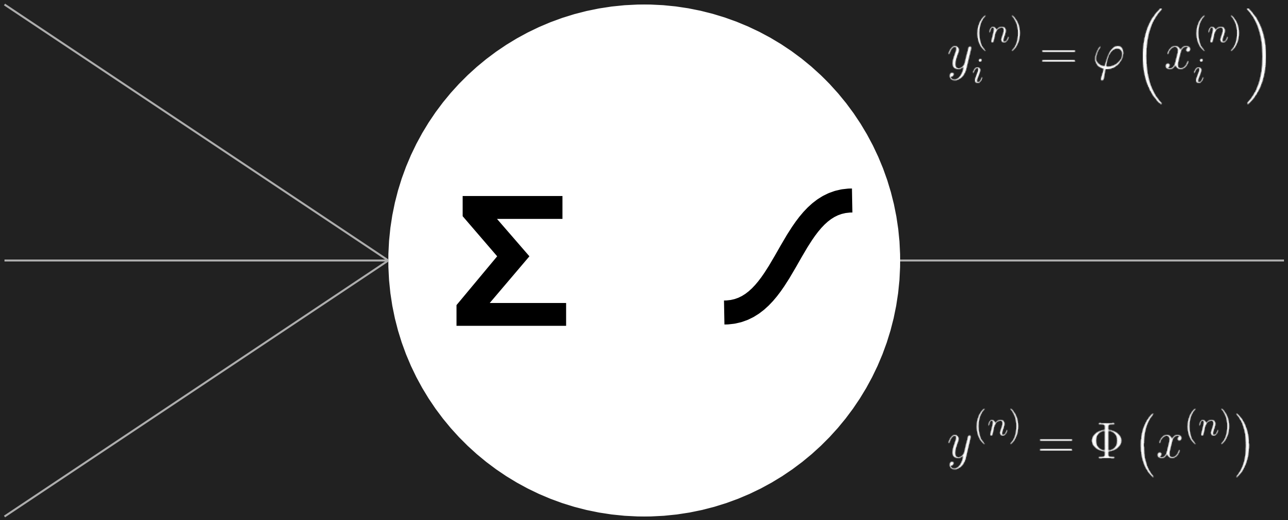


$$x^{(n)} = w^{(n-1)} \cdot y^{(n-1)}$$

$$y_i^{(n)} = \varphi\left(x_i^{(n)}\right)$$

$$y^{(n)} = \Phi\left(x^{(n)}\right)$$

$$x_i^{(n)} = \sum_j w_{ij}^{(n-1)} y_j^{(n-1)}$$

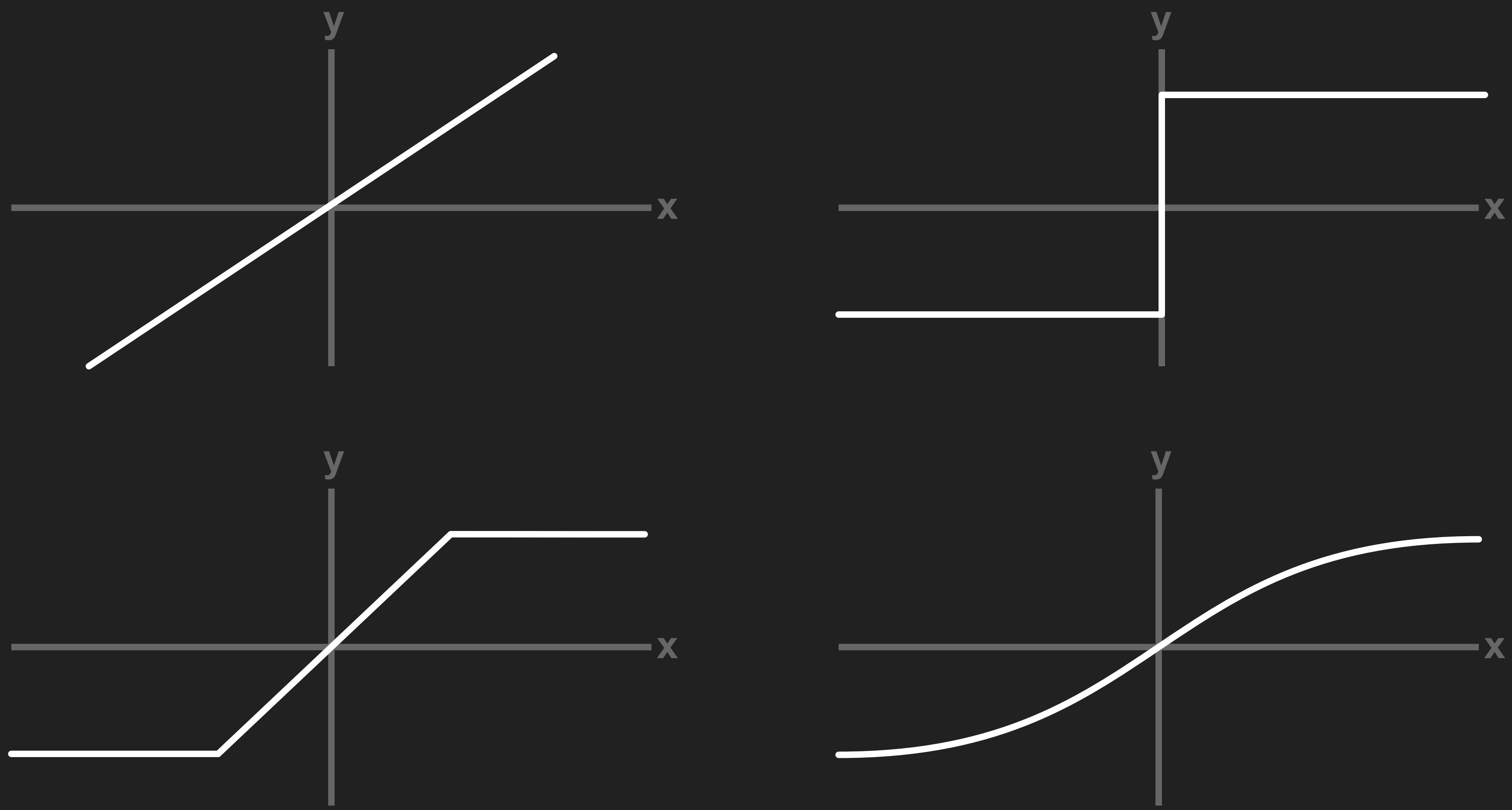


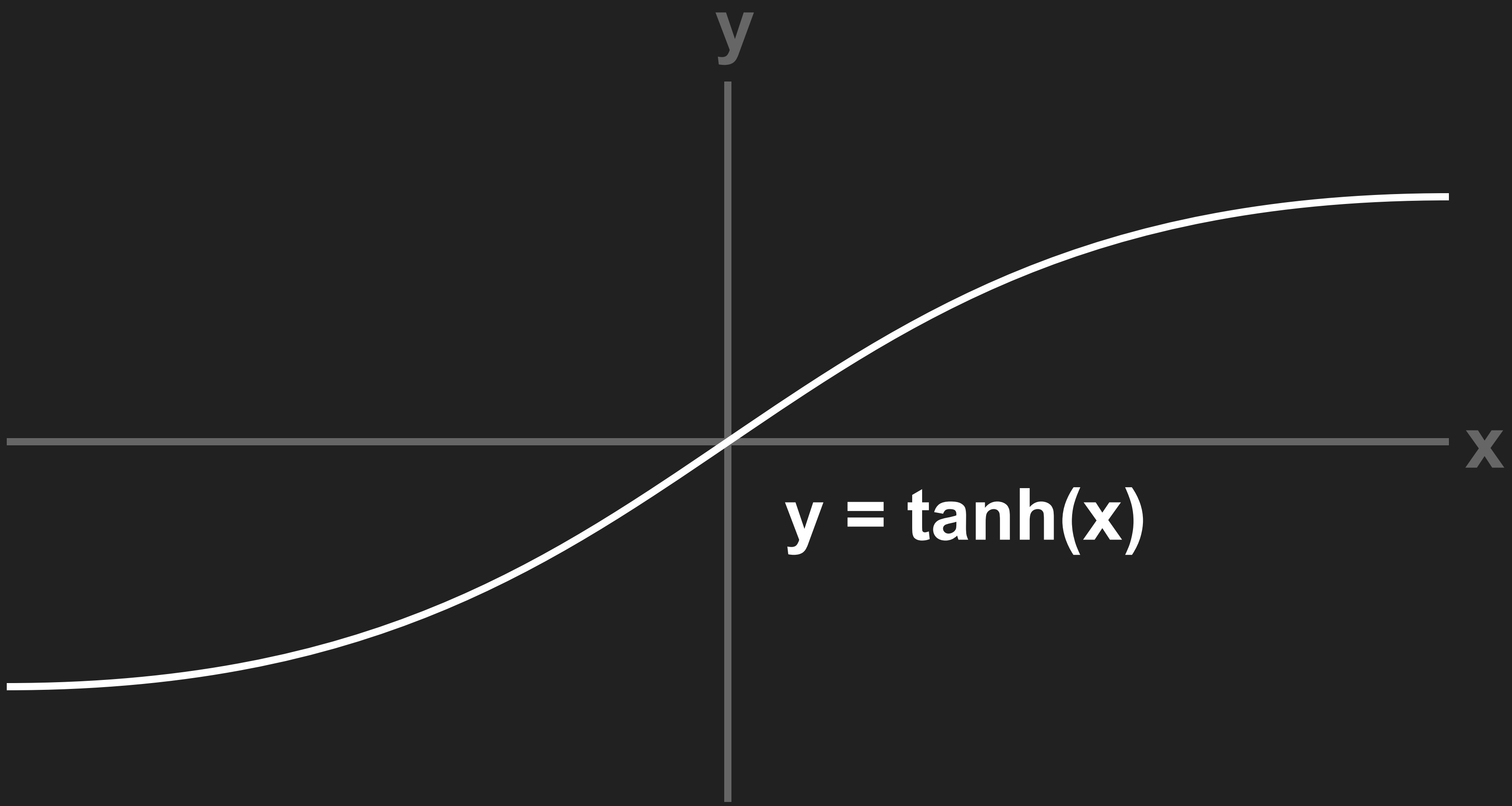
$$y_i^{(n)} = \varphi\left(x_i^{(n)}\right)$$

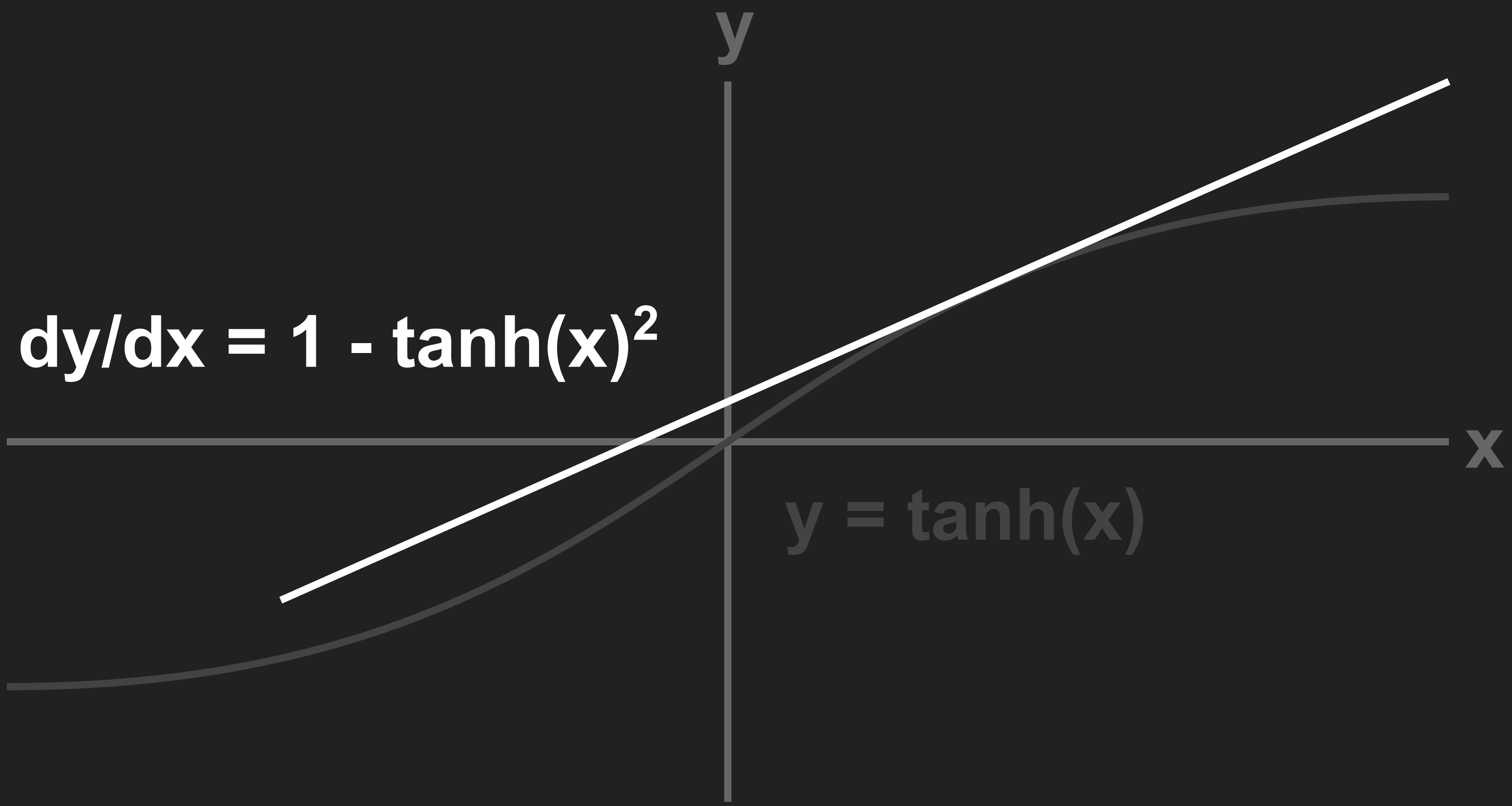
$$y^{(n)} = \Phi\left(x^{(n)}\right)$$

$$x^{(n)} = w^{(n-1)} \cdot y^{(n-1)}$$

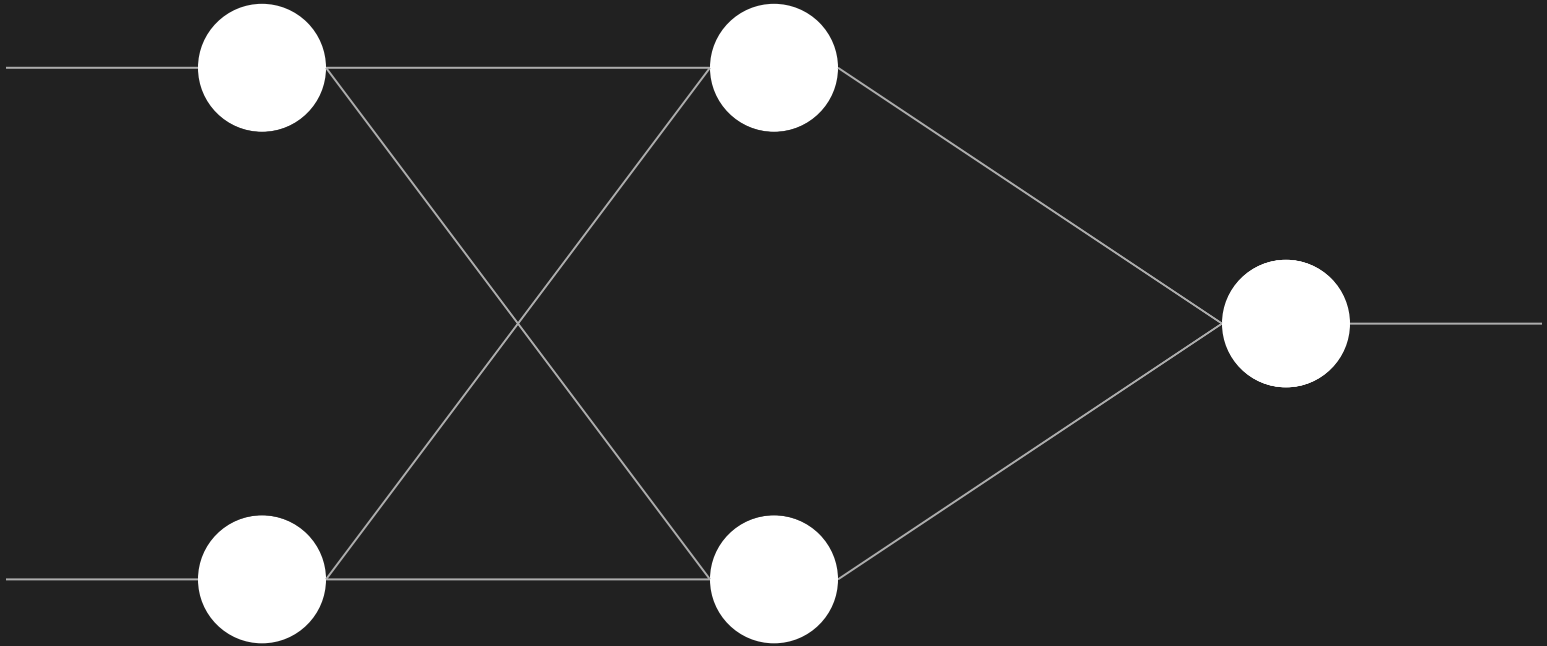
Activation functions


y
x
y
x
y
x
y
x

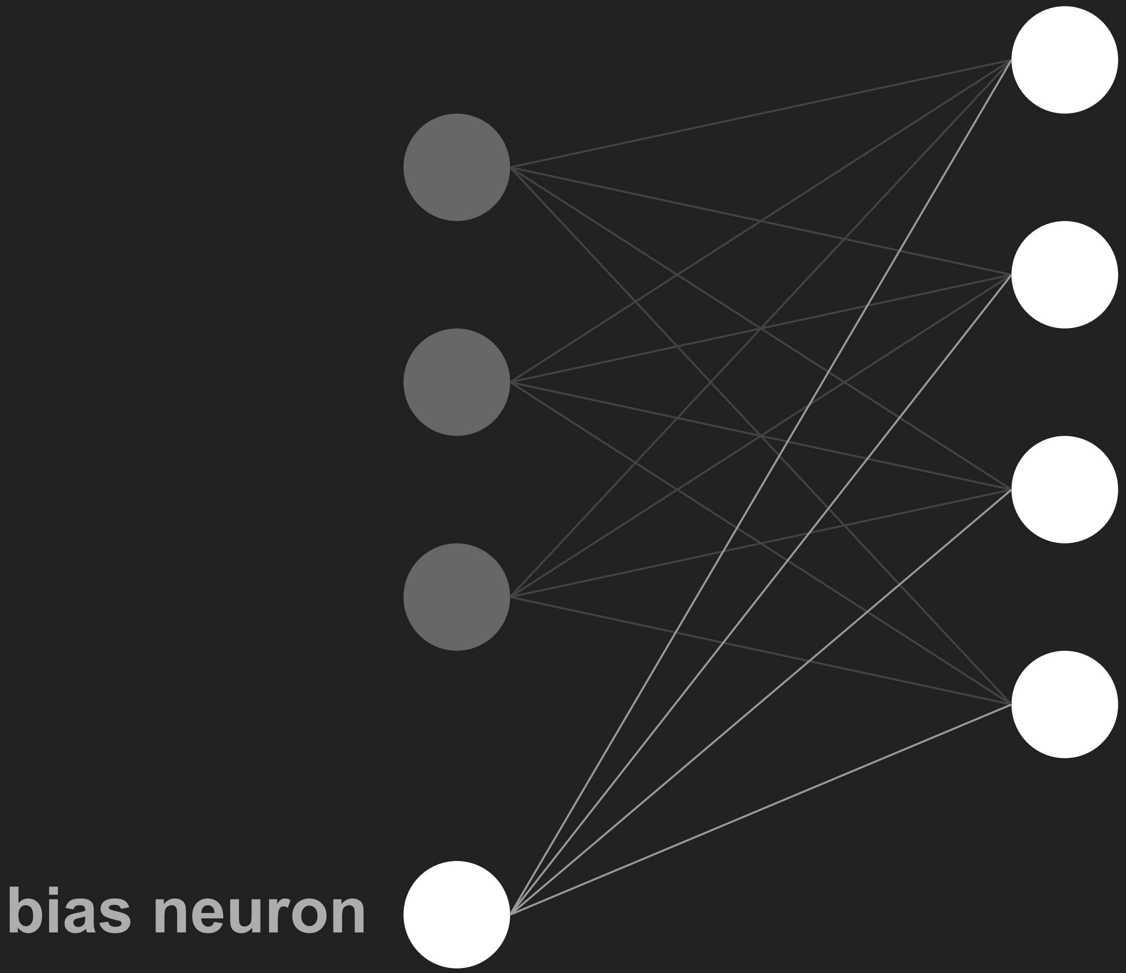

y
x
y = tanh(x)


y
x
dy/dx = 1 - tanh(x)^2
y = tanh(x)

XOR network



Bias neurons


bias neuron

Do we need more than 1 hidden layer?

Universal approximation theorem

Any continuous function on a compact interval can be approximated by a feed-forward neural network with a **single hidden layer**

For any *f* and ε there exist *a*, *b*, and *c* such that

$$g(x) = \sum_i c_i \varphi(a_i \cdot x + b_i)$$

$$|g(x) - f(x)| < \varepsilon$$

for all *x* in the interval

Training neural networks:

Backpropagation

Initializing weights

Draw from normal distribution

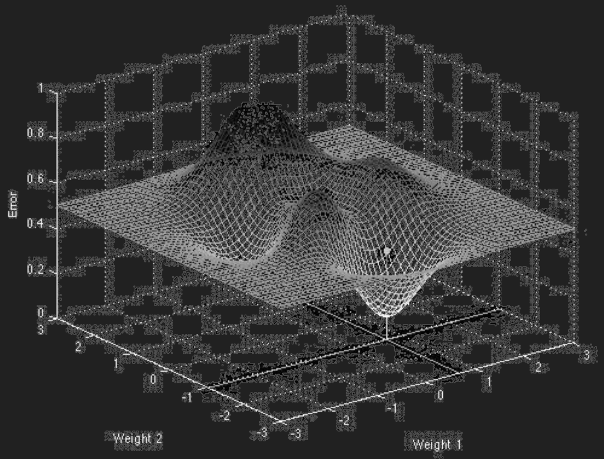
$$w_{ij}^{(n)} \sim N(0, \sigma)$$

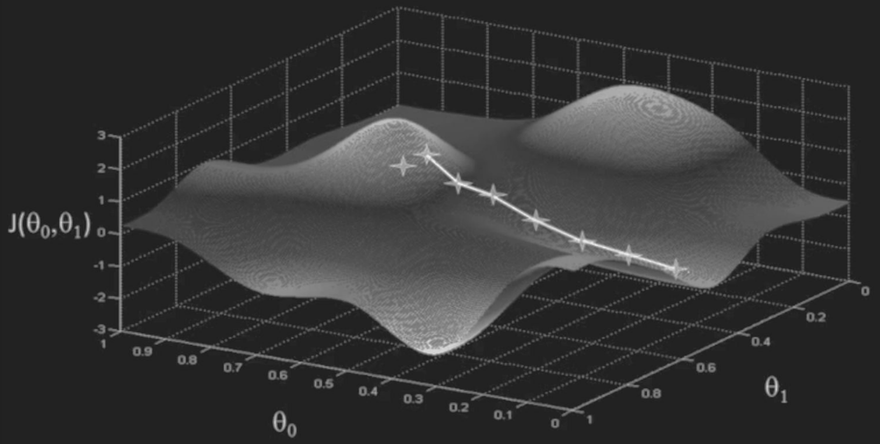
More inputs $\rightarrow$ Less variance

$$\sigma^2 = 1/N^{(n)}$$

This prevents saturation

Gradient descent

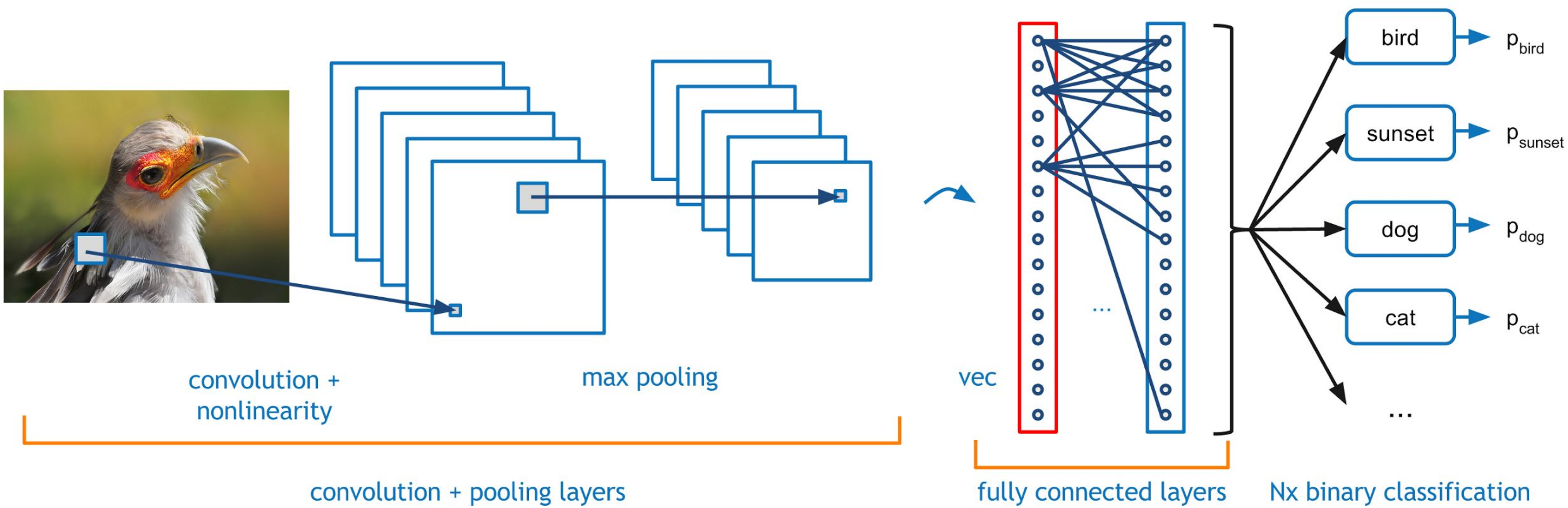




Automatic differentiation
(chain rule)

Data sets

Convolutional networks and deep learning


convolution +
nonlinearity
max pooling
vec
...
bird
sunset
dog
cat
...
p_{bird}
p_{sunset}
p_{dog}
p_{cat}
convolution + pooling layers
fully connected layers
Nx binary classification