Analysis of Matrix Completion Algorithms for Spectral Image Estimation from Compressive Coded Projections

High Dimensional Signal Processing Research Group

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Introduction

■ Hyperspectral Images

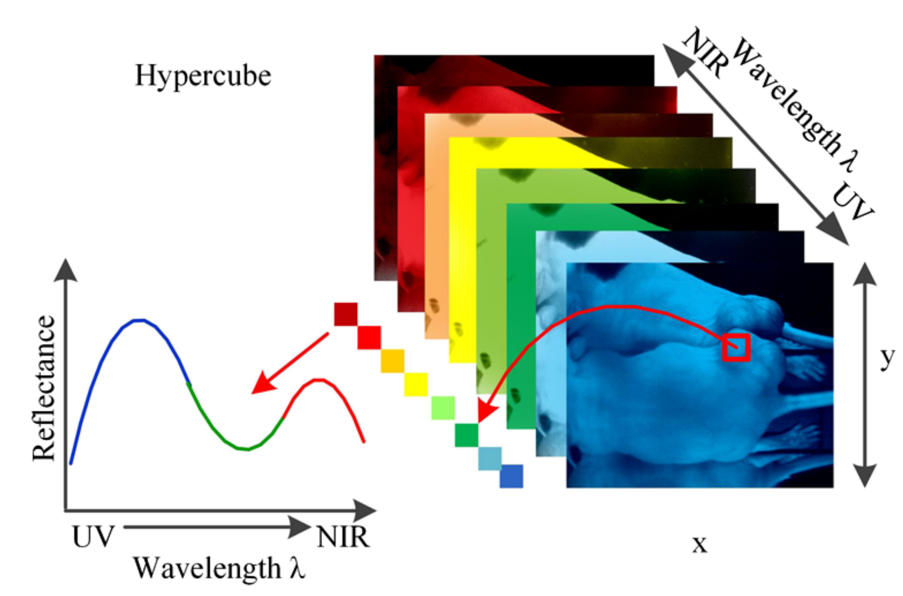

Figure 1. Hyperspectral Data Cube.

■ Coded Aperture Snapshot Spectral Imaging (CASSI) [1]

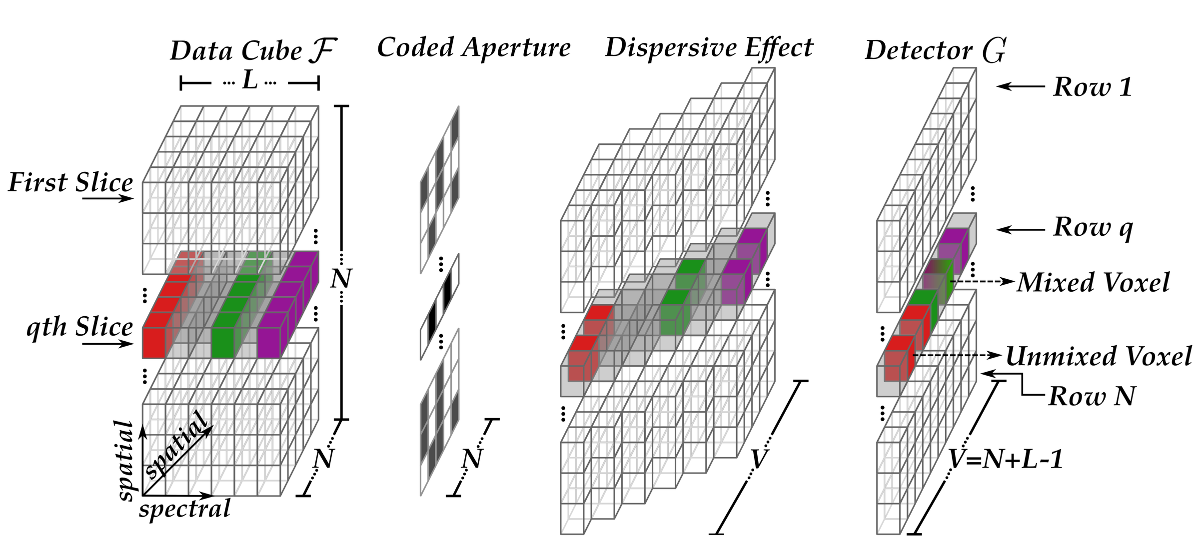

Figure 2. CASSI sensing mechanism.

- Based on compressive sensing (CS) theory.
- Takes a set of random coded projections from the underlying scene given by

$$\mathbf{g} = \mathbf{A}\boldsymbol{\theta} + \boldsymbol{\omega} = \mathbf{H}\boldsymbol{\Psi}\boldsymbol{\theta} + \boldsymbol{\omega} \tag{1}$$

- The underlying scene reconstruction process is accomplished by using a compressive sensing algorithm which recovers $\boldsymbol{\theta}$ by solving the $\ell_2 - \ell_1$ problem

$$\underset{\boldsymbol{\theta}}{\arg\min} \|\mathbf{g} - \mathbf{A}\boldsymbol{\theta}\|^2 + \tau\|\boldsymbol{\theta}\|_1 \tag{2}$$

The Model

■ The CASSI Spectral Sensing Model

- The discretized output at the CASSI detector is given by

$$\boldsymbol{G}_{mn} = \sum_{k=0}^{L-1} \boldsymbol{\mathcal{F}}_{m(n-k)(k)} \boldsymbol{T}_{m(n-k)} + \boldsymbol{\omega}_{mn} \tag{3}$$

- The proposed method consists of recovering the underlying scene using Matrix Completion. That is, given a subset of the source matrix entries, an MC algorithm recovers the missing entries by solving a convex optimization problem. In order to use MC, the input subset data cube $\boldsymbol{\mathcal{F}}^B$ of the MC algorithm is built by backprojecting the coded compressive measurement matrix $\boldsymbol{G}$. Due to the intensity multiplexing process occurring on the FPA, $\boldsymbol{\mathcal{F}}^B$ is composed of mixed and unmixed voxels. As depicted in Fig. 3, the mixed voxels are a combination of voxels from different spectral bands which "collide" during the integration process in the CASSI i.e, mixed voxels contain information from more than one spectral band. Furthermore, unmixed voxels are voxels from a specific spectral band which do not "collide" with others voxels.

$$\boldsymbol{\mathcal{F}^B}_{m(n-k)k} = \begin{cases} \boldsymbol{G}_{mn}, & \boldsymbol{T}_{m(n-k)} = 1 \Leftrightarrow k = k' \\ \boldsymbol{G}_{mn}/C_{mn}, & \text{otherwise} \end{cases} \tag{4}$$

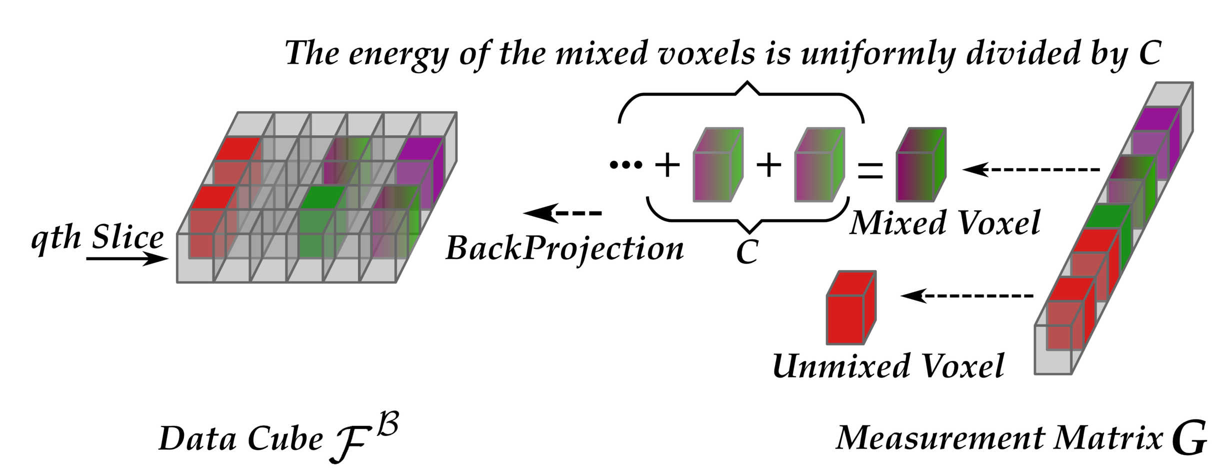

Figure 3. Backprojection Process.

■ The Matrix Completion Model

- Finding the missing entries of the $\boldsymbol{\mathcal{F}}^B$ data cube can be modeled as a MC problem.

$$\begin{aligned} &\text{minimize} && \|\boldsymbol{X}\|_* \\ &\text{subject to} && \mathcal{P}_\Omega(\boldsymbol{X}) = \mathcal{P}_\Omega(\boldsymbol{D}) \end{aligned} \tag{5}$$

- Extensive comparisons between MC solution approaches [2] revealed that the LMaFit [3] algorithm is overall the fastest and most reliable method for matrix completion.
- To complete a matrix with MC theory, this must satisfy certain constraints, such as, every row and column of the matrix must have an observed entry [4]. In order to apply the MC theory, we propose and evaluate the following six linear transformations to rearrange the $\boldsymbol{\mathcal{F}}^B \in \mathbb{R}^{N\times N\times L}$ data cube seeking to fulfill MC constraints and improve the reconstructions quality.

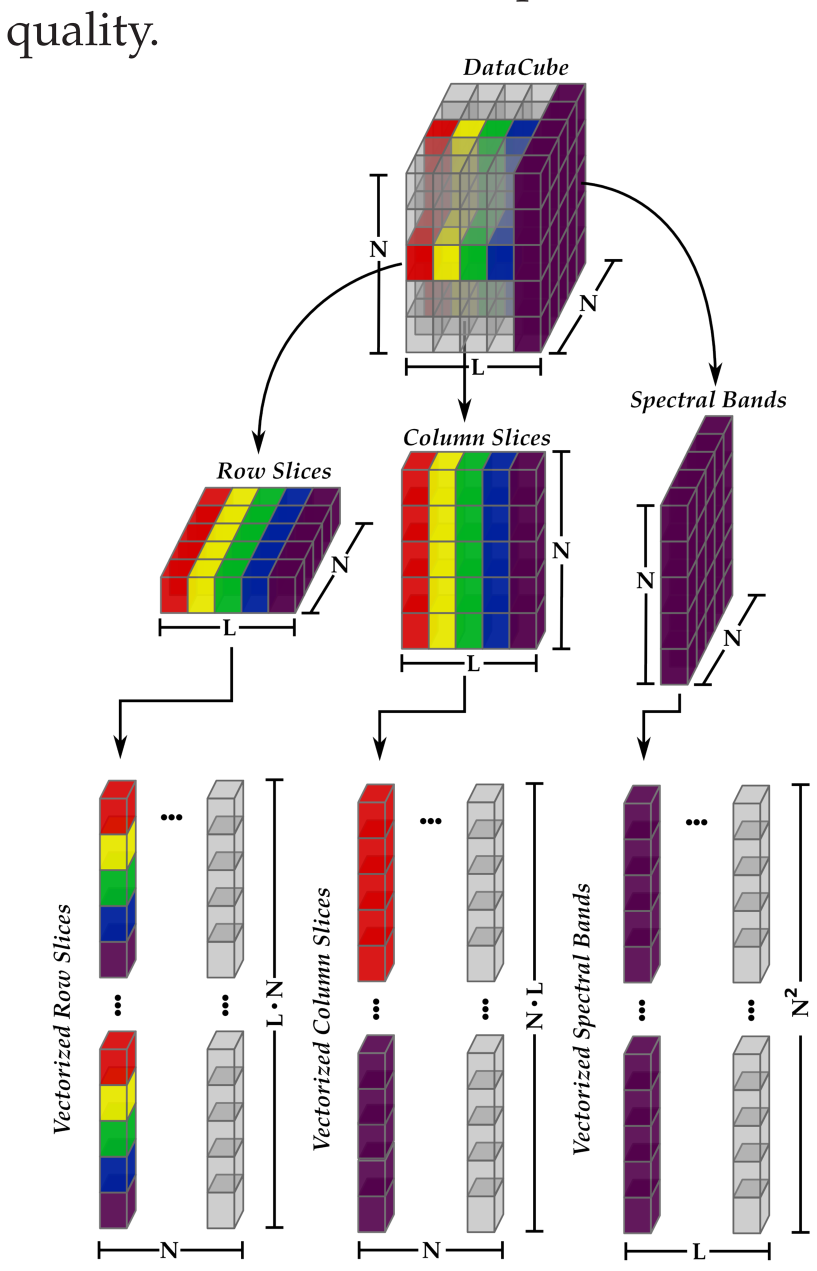

Figure 4. Linear Transformations.

Simulations Results

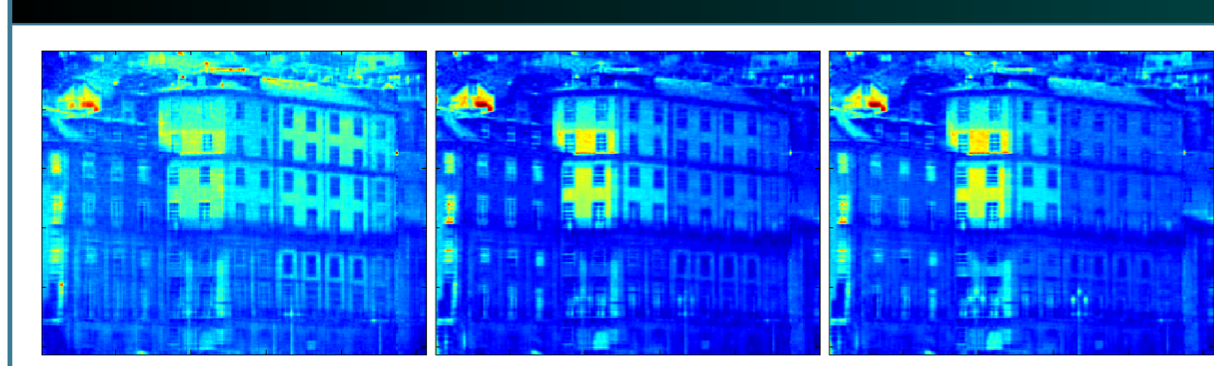

Figure 5. The 1st, 6th and 12th spectral bands of the original data cube.

Linear Transformation	20%	40%	60%	80%
1 Shot				
Spectral-Band-Splitting	15.2337	17.7609	21.8450	24.6646
Vectorized-Spectral-Band	✗	✗	✗	✗
Row-Slice-Splitting	✗	✗	✗	✗
Vectorized-Row-Slice	16.2814	22.3716	24.5309	25.0534
Column-Slice-Splitting	✗	✗	✗	✗
Vectorized-Column-Slice	15.8778	19.6311	24.5522	24.9127
3 Shots				
Spectral-Band-Splitting	17.3622	23.8337	26.2400	26.5590
Vectorized-Spectral-Band	✗	✗	26.7376	26.7791
Row-Slice-Splitting	✗	✗	26.7878	26.8321
Vectorized-Row-Slice	22.0313	26.1647	26.6790	26.8406
Column-Slice-Splitting	✗	✗	26.7395	26.7794
Vectorized-Column-Slice	18.9648	26.1157	26.6098	26.7665
8 Shots				
Spectral-Band-Splitting	24.7083	28.2627	29.3270	29.3622
Vectorized-Spectral-Band	✗	✗	28.7576	28.6711
Row-Slice-Splitting	✗	✗	28.9229	28.8333
Vectorized-Row-Slice	30.2322	31.0389	30.7210	30.4501
Column-Slice-Splitting	✗	✗	29.2410	29.1054
Vectorized-Column-Slice	31.2589	30.6264	30.7265	30.4180
11 Shots				
Spectral-Band-Splitting	30.0153	30.5562	30.6954	30.5326
Vectorized-Spectral-Band	✗	✗	29.5481	29.4037
Row-Slice-Splitting	✗	✗	29.7703	29.6113
Vectorized-Row-Slice	34.1704	33.1781	32.4893	32.0309
Column-Slice-Splitting	✗	✗	30.2408	30.0206
Vectorized-Column-Slice	34.2443	33.2419	32.5390	32.0389

Table 1. PSNR Reconstruction Results in decibels(dB).

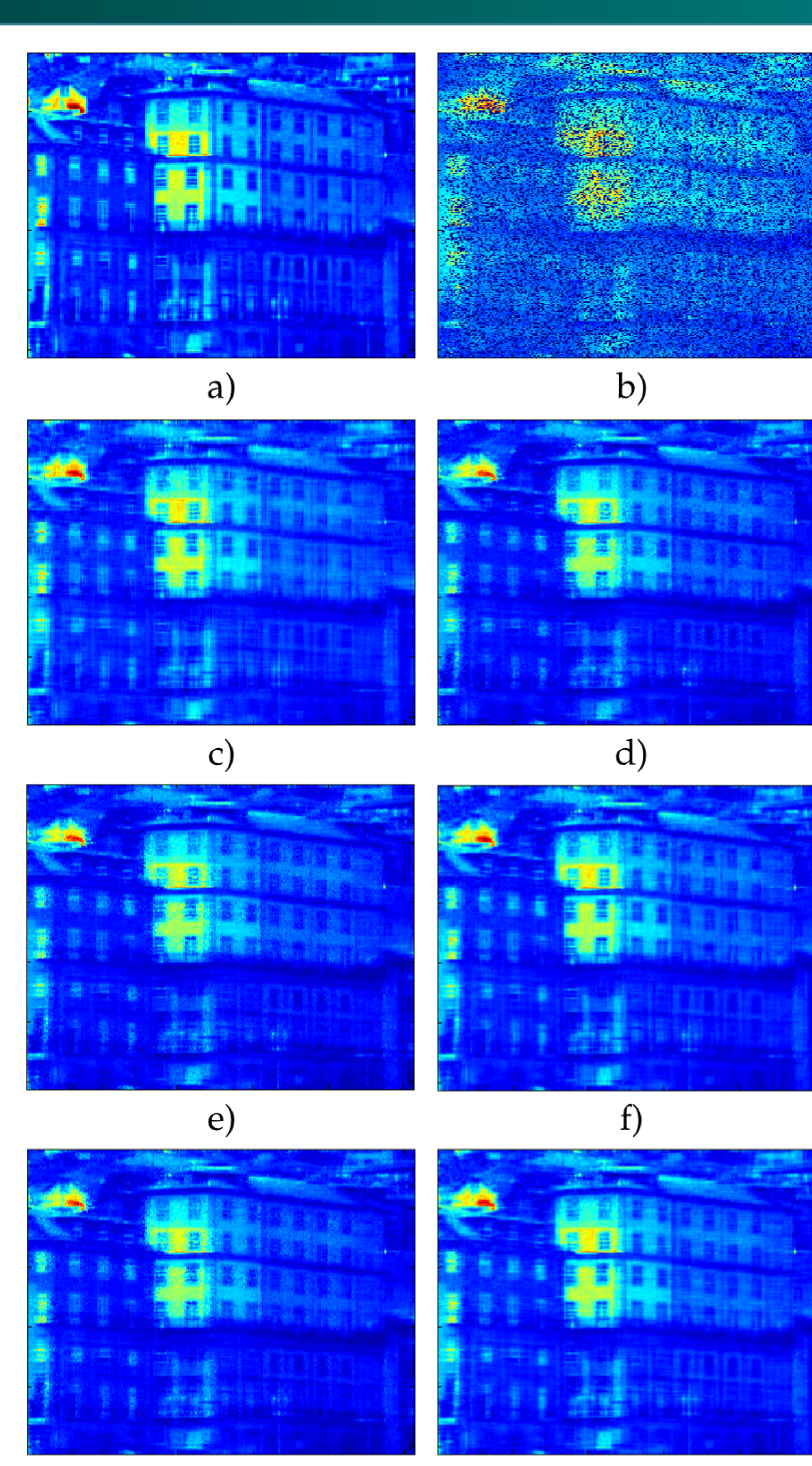

Figure 6. Reconstructions of the initial data cube using MC with the proposed linear transformations. The seventh spectral band of the original hyperspectral image is shown in a). Subfigure b) depicts the seventh spectral band of the backprojected data cube $\boldsymbol{\mathcal{F}^B}$ with sampling ratio of 0.6958. Subfigures $c) - h)$ presents the reconstruction results for the Spectral-Band-Splitting, Vectorized-Spectral-Band, Row-Slice-Splitting, Vectorized-Row-Slice, Column-Slice-Splitting and Vectorized-Column-Slice linear transformations, respectively.

Conclusions

In this paper it was proposed the use of the MC theory to achieve good spectral image estimations from compressive measurements using the CASSI system. Additionally, there were proposed six linear transformations to rearrange the data cube as a way to improve the estimations. Although the results are noisy, due to the presence of mixed voxels, the matrix completion algorithm provides good reconstructions in terms of PSNR. Also, the used matrix completion algorithm is faster than any compressive sensing algorithm, therefore this approach provides a fast and reliable estimation for hyperspectral data recovery.

References

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[2] M. Michenková, "Numerical algorithms for low-rank matrix completion problems," 2011.

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