Spectral Imaging Subspace Clustering with 3-D Spatial Regularizer

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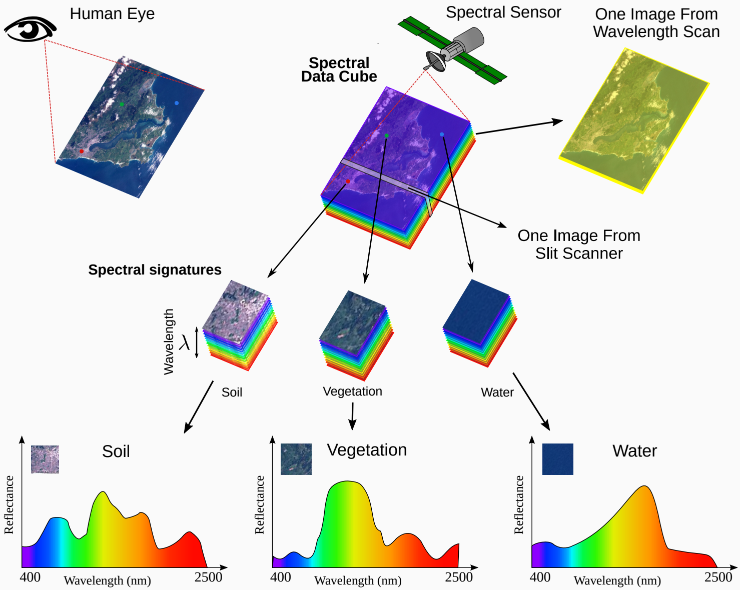
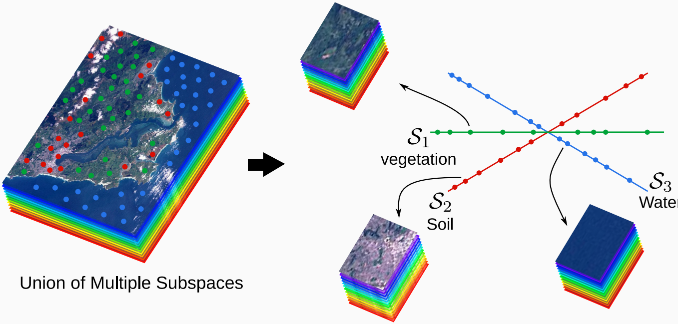
Introduction: Spectral Imaging



Introduction: Unsupervised Classification



Subspace Clustering Problem: Given a set of points lying in multiple subspaces, identify:

- The number of subspaces and their dimensions.
- A basis for each subspace.
- The segmentation of the data points.

Introduction: Sparse Subspace Clustering Algorithm (SSC)[1]

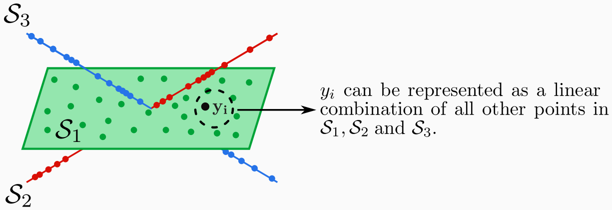
Consider a collection of N noise-free data points $\{\mathbf{y}_i\}_{i=1}^N$ that lie in the union of $\{\mathcal{S}_\ell\}_{\ell=1}^n$ subspaces.

$$\mathbf{Y} = [\mathbf{y}_1, \cdots \mathbf{y}_N] \tag{1}$$

- Data in a union of subspaces are **self-expressive**.

$$\mathbf{y}_i = \sum_{j=1}^{N} z_{ji}\mathbf{y}_j \quad \rightarrow \quad \mathbf{y}_i = \mathbf{Y}\mathbf{z}_i, \quad z_{ii} = 0. \tag{2}$$

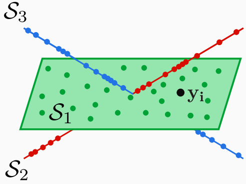
- Union of subspaces admits **subspace-sparse** representation.



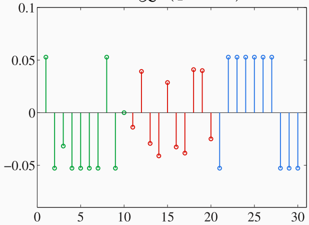
[1]E. Elhamifar, R. Vidal. Sparse Subspace Clustering. IEEE Conference on Computer Vision and Pattern Recognition (CVPR), 2009.

Optimization Problem: why impose a sparsity constraint on the y representation?

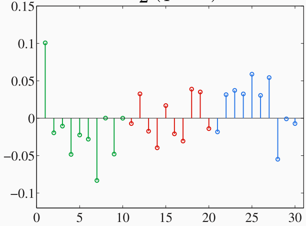
$$\min \|\mathbf{z}_i\|_q \quad \text{s.t.} \quad \mathbf{y}_i = \mathbf{Y}\mathbf{Z}_i, \quad z_{ii} = 0 \tag{3}$$



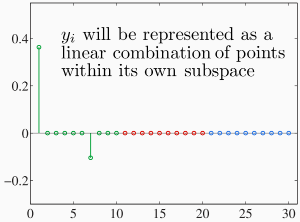
ℓ_∞ $(q = \infty)$



ℓ_2 $(q = 2)$



ℓ_1 $(q = 1)$



Introduction: Sparse Subspace Clustering Algorithm (SSC)

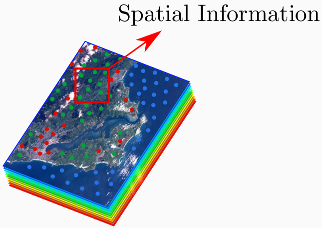
Original SSC Algorithm

1. Solve the optimization problem, where each column of $\mathbf{Y}$ corresponds to a spectral signature.

$$\begin{aligned} \min_{\mathbf{Z},\mathbf{R}} \quad & \|\mathbf{Z}\|_1 + \frac{\lambda}{2}\|\mathbf{R}\|_F^2 \\ \text{s.t.} \quad & \mathbf{Y} = \mathbf{Y}\mathbf{Z} + \mathbf{R},\ \text{diag}(\mathbf{Z}) = 0,\ \mathbf{Z}^T\mathbf{1} = \mathbf{1}, \end{aligned} \tag{4}$$

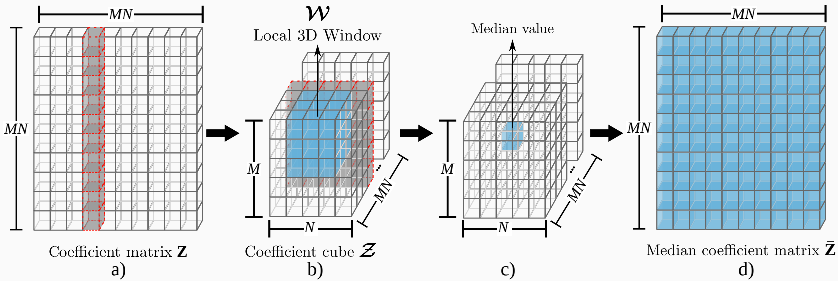
2. The segmentation of the spectral signatures is obtained by applying **spectral clustering** to the Laplacian matrix induced by the similarity matrix $\mathbf{W} = |\mathbf{Z}| + |\mathbf{Z}|^T$.

Limitation:

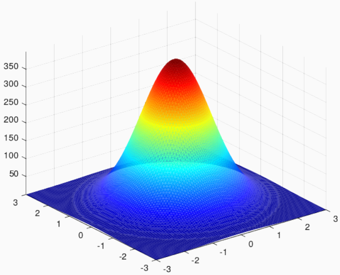


- This model does not take into account that neighboring pixels in a spectral image usually consist of similar materials belonging to the same class.

Proposed Method: 3D Filtering of the Coefficient matrix $\mathbf{Z}$



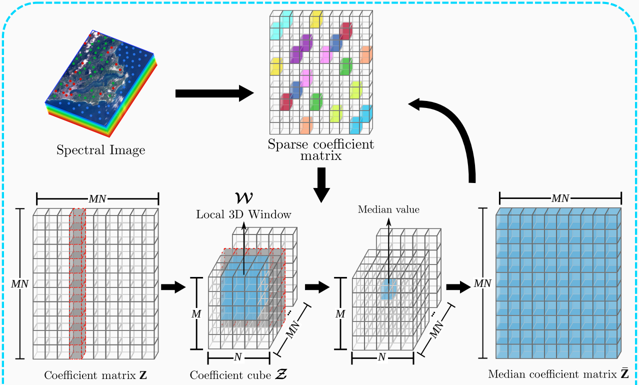
The filtering is performed using the isotropic 3D **Gaussian kernel** given by



$$G_{i,j,k} = -\frac{1}{\sqrt{2\pi\sigma}} \exp^{-\frac{i^2+j^2+k^2}{2\sigma^2}} \quad (5)$$

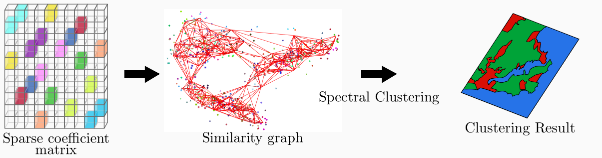
$$\frac{\alpha}{2}\|\mathbf{Z} - \bar{\mathbf{Z}}\|_F^2 \quad (6)$$

Proposed Method: SSC with 3D Spatial Regularizer

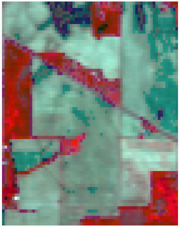


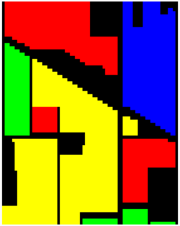
$$\min_{\mathbf{Z},\mathbf{R}} \quad \|\mathbf{Z}\|_1 + \frac{\lambda}{2}\|\mathbf{R}\|_F^2 + \frac{\alpha}{2}\|\mathbf{Z}-\bar{\mathbf{Z}}\|_F^2 \qquad (7)$$

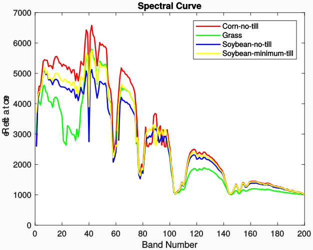
$$\text{s.t.} \quad \mathbf{Y} = \mathbf{Y}\mathbf{Z} + \mathbf{R},\ \operatorname{diag}(\mathbf{Z}) = 0,\ \mathbf{Z}^T\mathbf{1} = \mathbf{1},$$



Simulations and Results: Experimental Setup

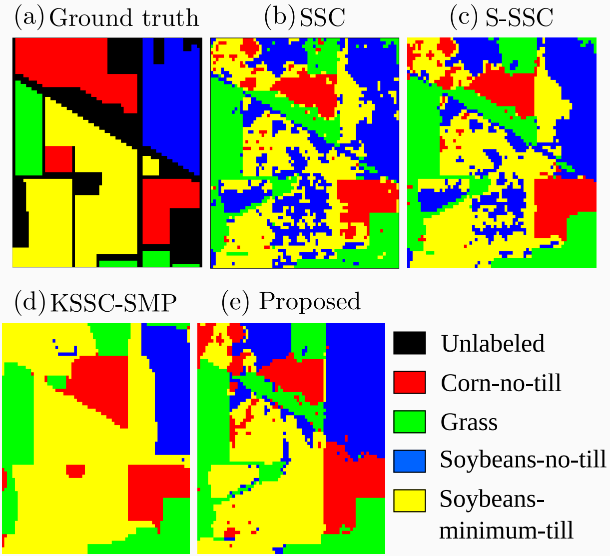






- Subimage of Aviris Indian Pines with 70×70 pixels and 200 spectral bands.
- Four main land-cover classes: corn-no-till, grass, soybeans-no-till, and soybeans-minimum-till.
- Comparison with three subspace clustering algorithms:
  - Original SSC.
  - SSC with spatial information for SI (S-SSC).
  - Total Variation kernel sparse subspace clustering algorithm with spatial max-pooling operation(TV-KSSC-SMP).
- 3D Gaussian kernel with window size $h = 3$ and $\sigma = 6$.

Simulations and Results: Experimental Results



Class	SSC	SSC-S[1]	TV-KSSC-SMP[2]	Proposed
Corn-no-till	48.96	56.12	45.17	**59.50**
Grass	98.60	**100**	**100**	**100**
Soybeans-no-till	70.63	70.77	63.52	**98.91**
Soybeans-minimun-till	59.23	67.44	**99.53**	84.74
Overall accuracy	62.62	68.20	76.88	**82.07**
Kappa	0.4758	0.5512	0.6525	**0.7467**

Table 1: Experiment result compared with other methods

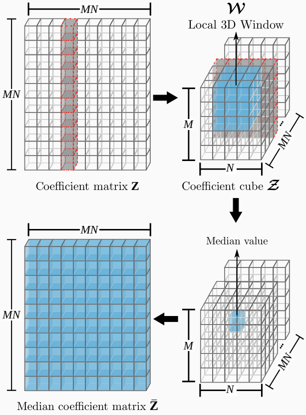
All the results, except the Kappa coefficients, are given in percentage.

[1] H. Zhang, H. Zhai, L. Zhang, P. Li. Spectral–spatial sparse subspace clustering for hyperspectral remote sensing images. IEEE Transactions on Geoscience and Remote Sensing, 54(6), 3672-3684, 2016

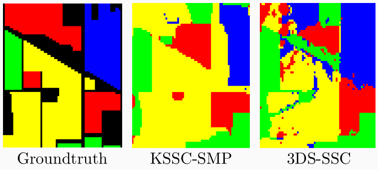
[2] J. Bacca, C. Hinojosa, and H. Arguello. Kernel Sparse Subspace Clustering with Total Variation Denoising for Hyperspectral Remote Sensing Images. Imaging and Applied Optics Conference, Optical Society of America (OSA), 2017.

Conclusion

- This work proposes to incorporate the spatial neighborhood information of the spectral scene in the SSC model by applying a 3-D Gaussian filter to the sparse representation coefficient matrix.



- The proposed method provides a significant improvement when compared with other state-of-the-art clustering methods.



Thank You !

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Questions?