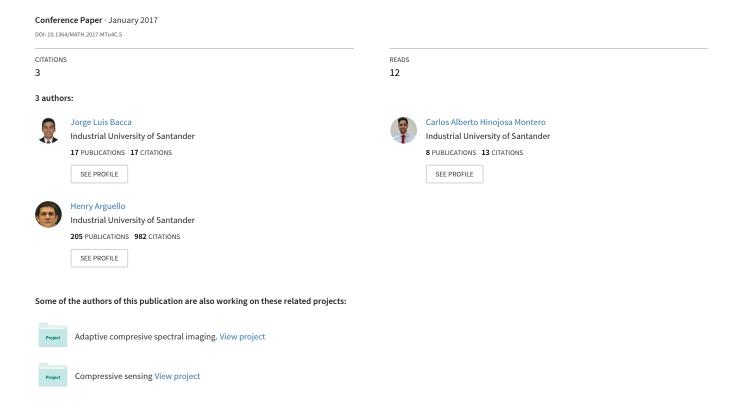
Kernel Sparse Subspace Clustering with Total Variation Denoising for Hyperspectral Remote Sensing Images



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Abstract: This paper proposes a new hyperspectral image subspace clustering framework which adds a total variation denoising constraint in order to improve the similarity between data points from the same subspace.

OCIS codes: 100.0100 Image processing, 030.4280 Noise in imaging systems, 280.0280 Remote sensing and sensors.

1. Introduction

Hyperspectral remote sensing image (HSI) classification is an important task for many practical applications, such as precision agriculture, monitoring and management of the environment and security and defense issues [1, 2]. Every pixel in a HSI is represented by a vector, also known as spectral signature of the pixel, whose values correspond to the intensity in different spectral bands. As different materials usually reflect electromagnetic energy differently at specific wavelengths, the spectral signatures can be used to distinguish different physical materials from the scene. Assuming that spectral signatures with similar spectral characteristics will lie in the same low-dimensional subspace, the sparse subspace clustering (SSC) algorithm can be used for clustering the hyperspectral remote sensing image. SSC capture the global geometric relationship among all data points by expressing each data point as a linear combination of all other points and then, the set of solutions is restricted to be sparse by minimizing the ℓ_1 norm of the representation coefficient matrix. Using the sparse representation matrix, a similarity graph is then built, from which the segmentation of the data is obtained [3]. The kernel sparse subspace clustering with spatial max pooling operation (KSSC-SMP) algorithm extends the SSC model to nonlinear manifolds by mapping the feature points from the original space into a higher dimensional space with a kernel strategy [4]. Then, the spatial max pooling operation is applied to the coefficient matrix in order to integrate the spatial contextual information such that a smoother clustering result is obtained.

In general, HSI clustering is a very challenging task due to the high complexity of the data and the large amount of noise. The noise in HSI is produced by numerous factors including thermal effects, sensor saturation, quantization errors and transmission errors. This work proposes and algorithm based on KSSC-SMP which adds a total variation (TV) denoising constraint to the optimization problem in order to remove noise from the spectral bands and to cluster spectral signatures in their corresponding low dimensional subspaces [5].

2. Proposed Approach

Every spectral signature in a HSI data cube can be denoted as an L-dimensional vector after lexicographically reordering the 3-D data cube into a 2-D matrix $\mathbf{Y} \in \mathbb{R}^{L \times M \cdot N}$, where M and N represents the spatial dimensions and L stands for the number of spectral bands. Assuming that the HSI image is corrupted by additive Gaussian noise $\mathbf{Y} = \mathbf{X} + \mathbf{Z}$, the TV denoising procedure is applied to the HSI data by solving the following optimization problem

$$\min_{\mathbf{X}} (1/2) \|\mathbf{Y} - \mathbf{X}\|_F^2 + \lambda_x \|\mathbf{D}\mathbf{X}\|_1, \tag{1}$$

where $\mathbf{D} = [\mathbf{D}_h; \mathbf{D}_v]$, with $\mathbf{D}_h, \mathbf{D}_v \in \mathbb{R}^{M \cdot N \times L}$, denotes the operator that computes the horizontal and vertical differences.

Once a denoised version of the HSI matrix is obtained, the KSSC-SMP algorithm can be used to perform the HSI clustering. The KSSC-SMP algorithm mainly consist of two steps. In the first step, the representation coefficient matrix **C** is acquired by solving the following optimization problem

$$\min_{\mathbf{C}} \|\mathbf{C}\|_{1} + \lambda_{c} \|\phi(\mathbf{X}) - \phi(\mathbf{X})\mathbf{C}\|_{F}^{2} \quad s.t \quad diag(\mathbf{C}) = 0, \ \mathbf{C}^{T}\mathbf{1} = \mathbf{1}.$$
(2)

where λ_c is a tradeoff between the data fidelity term and the sparse term, and $\phi(\cdot): \mathbb{R}^L \to H$ is a mapping function from the input space to the reproducing kernel Hilbert space H [4]. The kernel matrix is usually defined as $\mathbf{K}(\mathbf{x}_i, \mathbf{x}_j) = \{\phi(\mathbf{x}_i), \phi(\mathbf{x}_j)\} = \phi(\mathbf{x}_i)^T \phi(\mathbf{x}_j)$, where $\mathbf{K}(\cdot): \mathbb{R}^L \times \mathbb{R}^L \to \mathbb{R}$ represents the kernel function, which measures the similarity of two arguments denoting a pair of pixels. In this work, a radial basis function (RBF) kernel matrix, defined as $\mathbf{K}(\mathbf{x}_i, \mathbf{x}_j) = \exp(-\delta \|\mathbf{x}_i - \mathbf{x}_j\|^2)$, is used.

The second step of the KSSC-SMP algorithm consists of performing the spatial max pooling operation on the obtained coefficient matrix $\mathbf{C} = [\mathbf{c}_1, \mathbf{c}_2, \cdots, \mathbf{c}_{MN}]$. The spatial max pooling operation suppresses the low elements in the sparse representation vectors $\mathbf{c}_i \in \mathbb{R}^{MN}$ and preserve the large one. Therefore, the spatial contextual information is incorporated into the new coefficients to generate the spectral-spatial features.

This work proposes to integrate the HSI denosing and HSI clustering problems into a single optimization framework such a more accurate coefficient matrix \mathbf{C} can be achieved from the noise-free version of the data points. The proposed model is as follows:

$$\min_{\mathbf{C},\mathbf{X}} \mathbf{Q}(\mathbf{X}) + \mathbf{J}(\mathbf{C}, \hat{\mathbf{X}}) \quad s.t \quad diag(\mathbf{C}) = 0, \ \mathbf{C}^T \mathbf{1} = \mathbf{1}.$$
 (3)

with $\mathbf{Q}(\mathbf{X}) = (1/2) \|\mathbf{Y} - \mathbf{X}\|_F^2 + \lambda_x \|\mathbf{D}\mathbf{X}\|_1$ and $\mathbf{J}(\mathbf{C}, \hat{\mathbf{X}}) = \|\mathbf{C}\|_1 + \lambda_c \|\phi(\hat{\mathbf{X}}) - \phi(\hat{\mathbf{X}})\mathbf{C}\|_F^2$. In order to solve this optimization problem, the first step is to find an optimal value for \mathbf{X} while the variable \mathbf{C} is fixed. Then, the obtained solution in the previous step is assigned to $\hat{\mathbf{X}}$. The representation coefficient matrix \mathbf{C} is obtained by solving Eq. 3 while fixing the \mathbf{X} variable. This result is used to define the adjacency matrix as follows:

$$\mathbf{W} = |\mathbf{C}| + |\mathbf{C}|^T, \tag{4}$$

where each node *i* connects itself to a node *j* by an edge, and their weights are equal to $|\mathbf{C}_{i,j}| + |\mathbf{C}_{j,i}|$. Finally, the clustering result is obtained by applying spectral clustering to the Laplacian matrix induced by the adjacent matrix. The entire clustering process is shown in Fig 1.

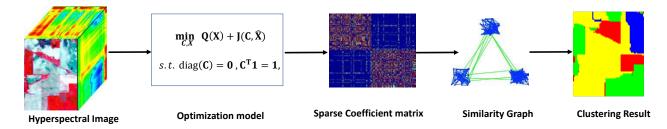


Fig. 1: Visual process step-by-step algorithm

Proposed Algorithm

Input: $\mathbf{Y} \in \mathbb{R}^{L \times M \cdot N}$. ℓ . λ

- 1: $\hat{\mathbf{X}} \leftarrow \text{Solve the equation (3)}$ with respect to \mathbf{X}
- 2: $\mathbf{C} \leftarrow \text{Solve the equation (3)}$ with respect to \mathbf{C}
- 3: $\mathbf{C} \leftarrow (\mathbf{C}/\|\mathbf{C}\|_{\infty})$ Normalize the columns
- 4: $\mathbf{C} \leftarrow$ apply max pooling operation to \mathbf{C}
- 5: $\mathbf{W} \leftarrow |\mathbf{C}| + |\mathbf{C}|^T$ Form a similarity graph with MN nodes representing the data points.
- 6: Apply spectral clustering to the similarity graph.

Output: A 2-D Matrix that records the labels of the clustering result.

3. Results

To evaluate the performance of the algorithm we use part of the Airborne Visible/Infrared Imaging Spectrometer (AVIRIS) data set from the Northwestern Indiana Indian Pines test [6], 20 bands were removed from the original 220 bands in the experiment, leaving 200. Spectral features and such dividing it in a with size of 85×75 , corn-no-till, grass, soybeans-no-till, and soybeans-minimum-till. The proposed algorithm is compared to three current algorithms

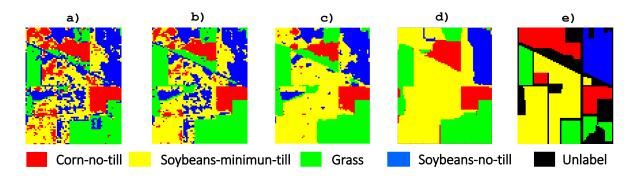


Fig. 2: Clustering result of different methods with the Indian Pines image: a) SSC b) S⁴C c) KSSC-SMP d)Proposed e) ground truth.

Class	SSC	S^4C	KSSC-SMP	Proposed
Corn-no-till	67.16	61.00	49.45	45.67
Soybeans-minimun-till	64.92	65.28	90.96	99.58
Grass	92.19	98.3562	100	100
Soybeans-no-till	66.12	65.30	75.41	68.72
overall accuracy	67.25	70.08	80.37	82.17

Table 1: Experiment result compared with other methods (units in percentage %).

for clustering as they are Sparse Subspace Clustering (SSC), SpectralSpatial Sparse Subspace Clustering (S⁴C) and the kernel sparse subspace clustering algorithm with spatial max pooling operation(KSSC-SMP).

Figure 2 depicts the visual clustering results for different subspace clustering methods. The ground-truth image is also provided for comparison in Fig 2(e). As observed, the proposed method provides a smoother result due to the noise removal. Table 1 shows the numerical results for each of the four land-cover classes (producer's accuracy) and the overall accuracy, given in percentage. From the numerical results, it can be observed that the proposed method outperforms the other methods.

The performace metric used

4. Conclusions

In this paper we propose an optimization framework for HSI clustering which removes noise of the scene via total variation denoising, obtaining a more accurate representation coefficient matrix. With the proposed method, a gain up to 2% in accuracy was obtained with respect to the state-of-the-art algorithm.

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