Kernel Sparse Subspace Clustering with Total Variation Denoising for Hyperspectral Remote Sensing Images

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Outline

Hyperspectral Remote Sensing Images

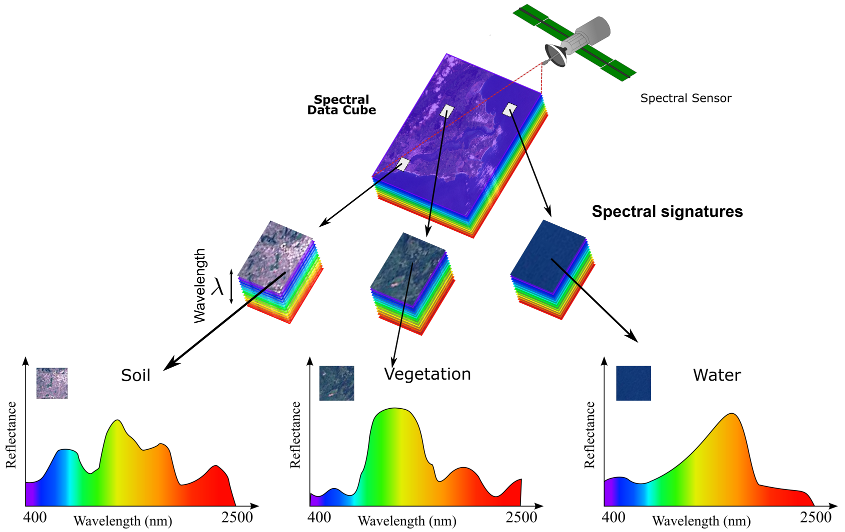
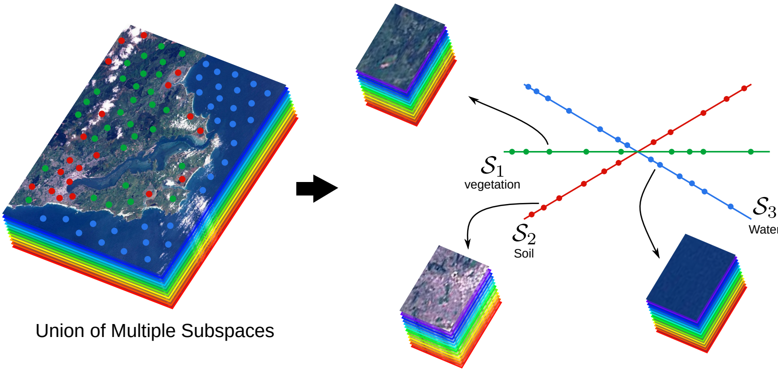


Figura: Representation of Hyperspectral Remote Sensing Images

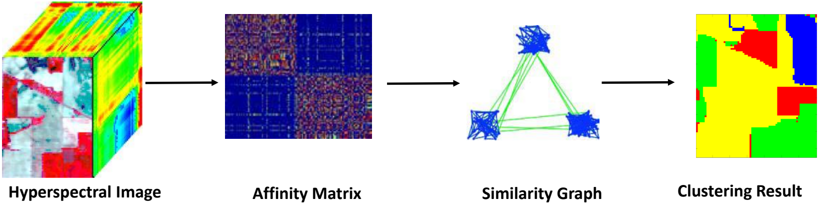
Subespace Clustering in Images



All spectral signatures with similar feature belong to the same Subspace.

Spectral Clustering Methods

- Represent data points as nodes in graph $\mathcal{G}$
- Use a symmetric nonnegative affinity matrix $\mathbf{C}$
- Connect nodes i and j with weights $w_{ij} = |c_{ij}| + |c_{ji}|$
- Infer clusters from Laplacian of $\mathcal{G}$



Sparse Subspace Clustering

Let $\mathbf{Y} = [\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_{MN}] \in \mathbb{R}^{L \times MN}$ data cube ordered.

- Data in a union of subspaces are **self-expressive**

$$\mathbf{y}_i = \sum_{j=1}^{MN} c_{ji}\mathbf{y}_j \quad \longrightarrow \quad \mathbf{y}_i = \mathbf{Y}\mathbf{c}_i, \quad c_{ii} = 0. \tag{1}$$

- Union of subspaces admits **subspace-sparse** representation

$$\min_{\mathbf{C}} \|\mathbf{C}\|_1 + \quad s.t \quad \mathbf{Y} = \mathbf{Y}\mathbf{C} + \mathbf{Z},\ diag(\mathbf{C}) = \mathbf{0},\ \mathbf{C}^T\mathbf{1} = \mathbf{1}. \tag{2}$$

where $\mathbf{Y} \in \mathbb{R}^{L \times MN}$ is data cube, $\mathbf{C} \in \mathbb{R}^{MN \times MN}$ is coefficient matrix, and $\mathbf{Z}$ is a noise term.

Kernel Sparse Subspace Clustering with Spatial Max Pooling Operation

Kernel-SSC with Spatial Max Pooling Operation [1]

$$\min_{\mathbf{C}} \|\mathbf{C}\|_1 + \lambda_c \|\phi(\mathbf{Y}) - \phi(\mathbf{Y})\mathbf{C}\|_F^2 \quad s.t \quad diag(\mathbf{C}) = \mathbf{0},\ \mathbf{C}^T\mathbf{1} = \mathbf{1}. \qquad (3)$$

- where λ_c is a tradeoff between the data fidelity term and the sparse term.
- $\mathbf{K}(\mathbf{y}_i, \mathbf{y}_j) = \{\phi(\mathbf{y}_i), \phi(\mathbf{y}_j)\} = \phi(\mathbf{y}_i)^T\phi(\mathbf{y}_j)$ where $\mathbf{K}(\cdot) : \mathbb{R}^L \times \mathbb{R}^L \longrightarrow \mathbb{R}$
- radial basis function is used $\mathbf{K}(\mathbf{y}_i, \mathbf{y}_j) = \exp(-\delta\|\mathbf{y}_i - \mathbf{y}_j\|^2)$

[1] Hyperspectral image kernel sparse subspace clustering with spatial max pooling operation [1]

Kernel-SSC with Spatial Max Pooling Operation [2]

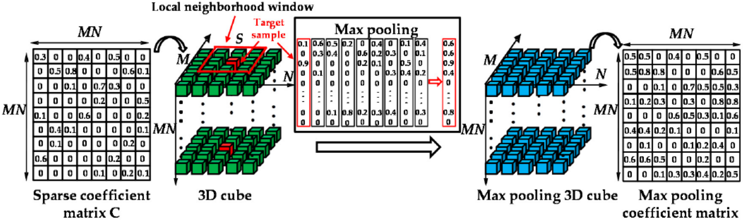
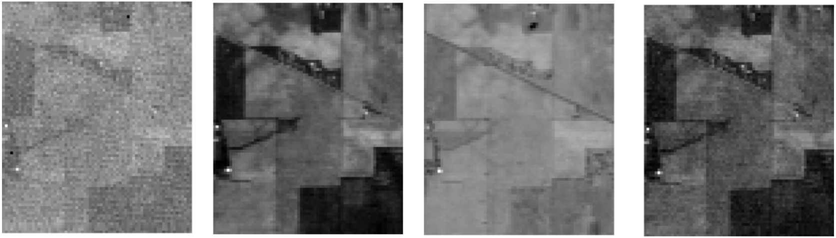


Figura: Max pooling operation process

Reordering coefficient matrix **C** in 3D cube, and then use the max pooling operation in each 2D matrix.

[2]Hyperspectral image kernel sparse subspace clustering with spatial max pooling operation [1]

Problem



Spectral Band 3 Spectral Band 29 Spectral Band 57 Spectral Band 100

Figura: Some spectral bands of real hyperspectral Images with noise

$$\mathbf{Y} = \mathbf{X} + \mathbf{Z} \tag{4}$$

Where **Y** is the data with Gaussian noise, **X** is a noiseless images and **Z** is a noise term.

Solution

$$\min_{\mathbf{X}} \, (1/2) \left\| \mathbf{Y} - \mathbf{X} \right\|_F^2 + \lambda_x \left\| \mathbf{D}\mathbf{X} \right\|_1 , \tag{5}$$

where $\mathbf{D} = [\mathbf{D}_h; \mathbf{D}_v]$, with $\mathbf{D}_h, \mathbf{D}_v \in \mathbb{R}^{M \cdot N \times L}$, denotes the operator that computes the horizontal and vertical differences.

Integrate

$$\min_{\mathbf{C},\mathbf{X}} \mathbf{Q}(\mathbf{X}) + \mathbf{J}(\mathbf{C}, \hat{\mathbf{X}}) \quad s.t \quad diag(\mathbf{C}) = \mathbf{0},\ \mathbf{C}^T\mathbf{1} = \mathbf{1}. \tag{6}$$

- with $\mathbf{Q}(\mathbf{X}) = (1/2)\,\|\mathbf{Y} - \mathbf{X}\|_F^2 + \lambda_x\,\|\mathbf{D}\mathbf{X}\|_1$
- with $\mathbf{J}(\mathbf{C}, \hat{\mathbf{X}}) = \|\mathbf{C}\|_1 + \lambda_c\|\phi(\hat{\mathbf{X}}) - \phi(\hat{\mathbf{X}})\mathbf{C}\|_F^2$
- where $\mathbf{D} = [\mathbf{D}_h; \mathbf{D}_v]$, with $\mathbf{D}_h, \mathbf{D}_v \in \mathbb{R}^{M \cdot N \times L}$,

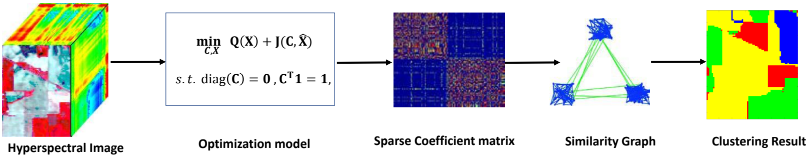


Figura: Visual process step-by-step algorithm

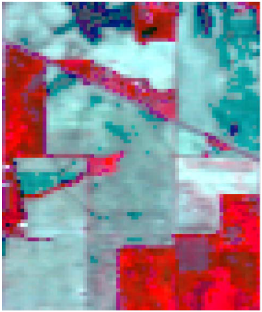
Proposed Algorithm

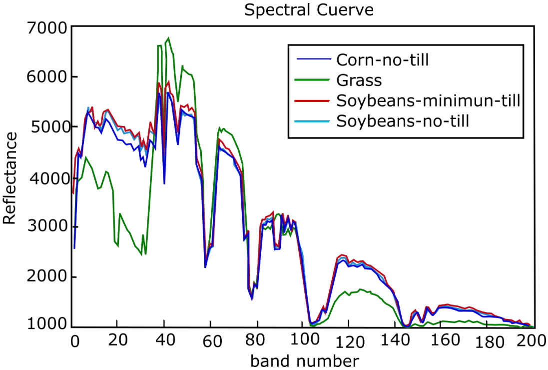
Input: $\mathbf{Y} \in \mathbb{R}^{L \times M \cdot N}, \ell, \lambda$

1: $\hat{\mathbf{X}} \leftarrow$ Solve the equation (6) with respect to $\mathbf{X}$
2: $\mathbf{C} \leftarrow$ Solve the equation (6) with respect to $\mathbf{C}$
3: $\mathbf{C} \leftarrow (\mathbf{C} / \|\mathbf{C}\|_\infty)$ Normalize the columns
4: $\mathbf{C} \leftarrow$ apply max pooling operation to $\mathbf{C}$
5: $\mathbf{W} \leftarrow |\mathbf{C}| + |\mathbf{C}|^T$ Form a similarity graph with MN nodes representing the data points.
6: Apply spectral clustering to the similarity graph.

Output: A 2-D Matrix that records the labels of the clustering result.

- part of the Airbone Visible/Infrared Imaging Spectrometer (AVIRIS) data set from the Northwestern **Indian Pines test**.





Results

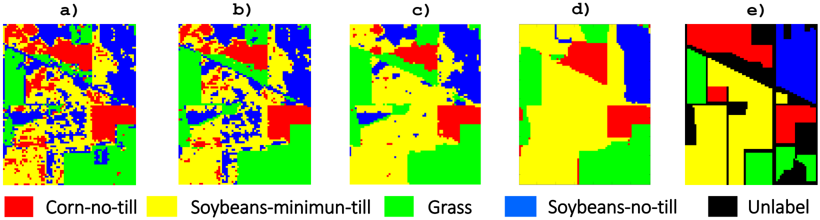


Figura: a), b), c) visual result of three current algorithms, d) proposed algorithm and e) ground truth. Where the colors represent land-cover classes in the hyperspectral image.

a) SSC (Sparse Subspace Clustering).

b) S^4C (Spatial-Spectral Subspace Clustering).

c) KSSC-SMP (Kernel Sparse Subspace Clustering with spatial mac pooling operation).

d) Proposed.

e) Ground truth.

The numerical results for each of the four land-cover classes (producer's accuracy) and the overall accuracy, given in percentage %.

Class	SSC	S^4C	KSSC-SMP	Proposed
Corn-no-till	67,16	61,00	**49,45**	45,67
Soybeans-minimun-till	64,92	65,28	90,96	**99,58**
Grass	92,19	98,3562	100	**100**
Soybeans-no-till	66,12	65,30	**75,41**	68,72
overall accuracy	67,25	70,08	80,37	**82, 17**

In this work we propose an optimization framework for Hyperspectral remote sensing imaging clustering which removes noise of the scene via total variation denoising, obtaining a more accurate representative coefficient matrix.

Thank You

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Questions?

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