Spectral Imaging Subspace Clustering with 3-D Spatial Regularizer

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Abstract: This paper proposes a spectral image clustering approach that uses a 3-D Gaussian filter to incorporate the spatial information of the scene in the Sparse Subspace Clustering model obtaining a more accurate representation coefficient matrix.

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1. Introduction

Spectral imaging senses spectral information at every spatial location of a scene. Spectral images (SI) are commonly regarded as three-dimensional (3-D) images where two of the coordinates correspond to the spatial domain and the third one represents the spectral wavelengths. Every pixel in a SI is represented by a vector, a.k.a spectral signature, whose values correspond to the intensity at different spectral bands. The substantial amount of information provided by the spectral signatures enables to better characterize and discriminate objects within the scene. In particular, assuming that spectral signatures with similar spectral characteristics lie in the same low-dimensional subspace, the subspace clustering theory can be used to model the spectral image classification problem. The sparse subspace clustering (SSC) model captures the global geometric relationship among all data points by expressing each point as a linear combination of all the others and then, the set of solutions is restricted to be sparse by minimizing the ℓ_1 norm of the representation coefficient matrix [1]. Using the sparse representation matrix, a similarity graph is then built, from which the segmentation of the data is obtained. Taking into account that neighboring pixels in a spectral image usually consist of similar materials, that have a high probability of belonging to the same land-cover class, different works have proposed to incorporate a spatial regularizer in the SSC optimization problem in order to obtain a more accurate representation coefficient matrix [3]. Commonly, the regularizer term proposed in such works tries to assign the same sparse basis to adjacent pixels by performing a two dimensional smoothing filtering, individually, over each column of the representation coefficient matrix. However, the information among columns is not taken into account. This work proposes to integrate the spatial-contextual information of the SI by performing a 3-D Gaussian filtering operation over the representation coefficient matrix such that the information among adjacent columns is used to improve the representation bias.

2. Proposed Approach

Rearrange the 3-D SI data cube into a 2-D matrix $\mathbf{X} \in \mathbb{R}^{L\times M\cdot N}$, where M and N represent the spatial dimensions and L is the number of spectral bands; each column of $\mathbf{X}$ corresponds to one spectral signature (data point). Using this representation, the SSC model with the spatial regularizer is written as follows

$$\begin{aligned} \min_{\mathbf{Z},\mathbf{R}} \quad & \|\mathbf{Z}\|_1 + \frac{\lambda}{2}\|\mathbf{R}\|_F^2 + \frac{\alpha}{2}\|\mathbf{Z}-\bar{\mathbf{Z}}\|_F^2 \\ \text{s.t.} \quad & \mathbf{X} = \mathbf{X}\mathbf{Z}+\mathbf{R},\ \text{diag}(\mathbf{Z}) = 0,\ \mathbf{Z}^T\mathbf{1} = \mathbf{1}, \end{aligned} \tag{1}$$

where $\mathbf{Z} \in \mathbb{R}^{MN\times MN}$ is the sparse representation coefficient matrix, $\mathbf{1}$ is a one-valued vector, $\mathbf{R}$ stands for the representation error, λ and α are regularization parameters. The constraint $\text{diag}(\mathbf{Z}) = 0$ is used to eliminate the trivial solution of writing a point as an affine combination of itself and the constraint $\mathbf{Z}^T\mathbf{1} = \mathbf{1}$ ensures that it is a case of an affine subspaces [1]. The spatial neighborhood information can be incorporated in the SSC procedure by using a smoothing filter which assigns similar sparse coefficients to adjacent pixels improving the representation bias. In Eq. 1, $\bar{\mathbf{Z}}$ is a filtered coefficient matrix which is used to regularize $\mathbf{Z}$. In previous works, $\bar{\mathbf{Z}}$ is constructed by rearranging the 2-D coefficient matrix $\mathbf{Z}$ to a 3-D cube $\hat{\mathbf{Z}} \in \mathbb{R}^{M\times N\times MN}$, such that each column of $\mathbf{Z}$ corresponds to a slice of $\hat{\mathbf{Z}}$, and then applying a 2-D smoothing filter, individually, to each slice [3]. However, these approaches do not take into account the information provided by adjacent pixels among the slices. By opening a 3-D window at each spatial location of $\hat{\mathbf{Z}}$, the proposed approach uses a 3-D Gaussian kernel to perform the smoothness operation taking into account the information among slices. Specifically, the filtering is performed using the isotropic 3-D Gaussian kernel given by

$$G_{i,j,k} = -\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{i^2+j^2+k^2}{2\sigma^2}}, \tag{2}$$

with a specific window size h and standard deviation σ. After the filtering process, the cube $\hat{\mathbf{Z}}$ is rearranged into the $\bar{\mathbf{Z}}$ filtered coefficient matrix to solve the optimization problem in Eq. 1. Then, the segmentation of the data points is obtained by applying spectral clustering to the Laplacian matrix induced by the similarity matrix $\mathbf{W} \in \mathbb{R}^{MN \times MN}$ which is defined as $\mathbf{W} = |\mathbf{Z}| + |\mathbf{Z}|^T$ [1,3].

3. Simulations and Results

The proposed SI subspace clustering approach was tested on a region of the AVIRIS Indian Pines data set with size 70×70 pixels and 200 spectral bands. This subimage includes four main land-cover classes: corn-no-till, grass, soybeans-no-till, and soybeans-minimum-till. The proposed approach was compared with three subspace clustering algorithms: the original SSC, the SSC with spatial information for SI (SSC-S) [3], and the Total Variation kernel sparse subspace clustering algorithm with spatial max pooling operation(TV-KSSC-SMP) [2]. In the simulations, the proposed method was tested using a 3-D Gaussian kernel with windows size $h = 3$ and standard deviation $\sigma = 6$. Figure 1 depicts the visual clustering results for different subspace clustering methods. The ground-truth image is also provided for comparison purposes in Fig 1(a). Furthermore, Table 1 shows the numerical results for each of the four land-cover classes (producer's accuracy), the overall accuracy and the kappa coefficients [4]. All the results, except the Kappa coefficients, are given in percentage. In the table, the optimal value of each row is shown in bold and the second-best results are underlined. From the visual and quantitative results, it can be clearly observed that the proposed method provides a significant improvement due to the coefficient matrix regularization.

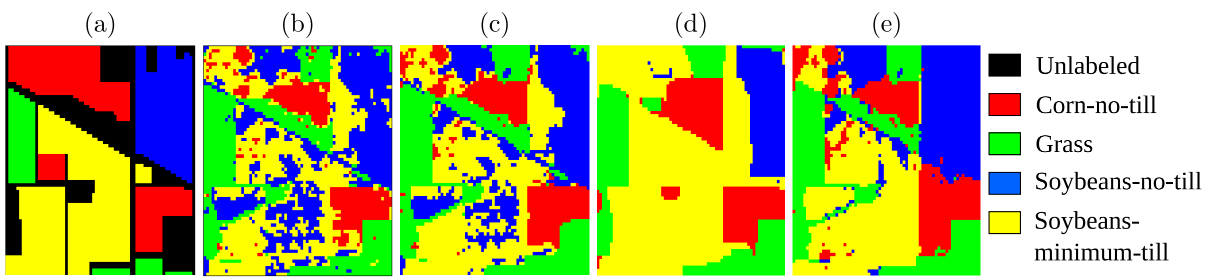

Fig. 1: Clustering result of different methods for the Indian Pines image: a) Ground truth b) SSC c) SSC-S d) TV-KSSC-SMP e) Proposed.

Class	SSC	SSC-S	TV-KSSC-SMP	Proposed
Corn-no-till	48.96	56.12	45.17	**59.50**
Grass	98.60	**100**	**100**	**100**
Soybeans-no-till	70.63	70.77	63.52	**98.91**
Soybeans-minimun-till	59.23	67.44	**99.53**	84.74
Overall accuracy	62.62	68.20	76.88	**82.07**
Kappa	0.4758	0.5512	0.6525	**0.7467**

Table 1: Quantitative evaluation of the different clustering algorithms for the Indian Pines image.

4. Conclusion

In this paper we proposed to incorporate the spatial neighborhood information of the spectral scene in the SSC model by applying a 3-D Gaussian filter to the sparse representation coefficient matrix. The proposed method provides a significant improvement when compared with other state-of-the-art clustering methods.

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