Escola d'Enginyeria de Telecomunicació i Aeroespacial de Castelldefels
UNIVERSITAT POLITÈCNICA DE CATALUNYA

Lab Report
RADIOLOCALIZATION

MATCHED FILTER

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The matched filter

The **matched filter** provides the instantaneous highest signal to noise ratio (SNR) at its output provided that the input signal is a replica of the transmitted pulse only affected by white noise.

Being $s(t)$ the (baseband) signal transmitted by the radar, the impulse response $h(t)$ of the matched filter is:

$$h(t) = K\, s^{*}(t_0 - t)$$

This response is the conjugate of a scaled (K), time reversed ($-t$), and shifted (t_0) version of the signal. t_0 corresponds to the time at which the output filter response to $s(t)$ will reach its peak although it can be considered as an arbitrary, but known, quantity.

The output $s_0(t)$ of the matched filter can be found as (assuming K=1 and t t_0=0):

$$s_0(t) = h(t) * s(t) = \int_{-\infty}^{\infty} s(\gamma)\, h(t-\gamma)\, d\gamma = \int_{-\infty}^{\infty} s(\gamma)\, s^{*}(t-\gamma)\, d\gamma$$

The interesting point of the matched filter is that the output SNR only depends on the signal energy and not on the waveform. Then the criteria to choose the pulse shape should consider the following trade-offs: maximum energy, high resolution, and Doppler frequency performance.

Waveform coding

A way to shape the pulse is using **waveform coding**, meaning that a phase modulation $\phi(t)$ is applied to the transmitted signal, for example a pulse having a pulsewidth τ_p. Constant amplitude of the transmitted signal is assumed because amplifiers in transmitters usually operate in saturation.

$$s(t) = e^{j\phi(t)}\, rect\left[\frac{t}{\tau_p}\right]$$

There are several ways of coding the waveform. Some ways consider continuous variations of the phase across the pulse, as for example the linear frequency modulation (chirp) or the non-linear frequency modulation. Another type of coding the waveform is changing the phase in the pulse using a discrete function of time, where the phase is constant over a time period τ_c called *chip* or subpulse.

$$\phi(t) = \sum_{k=0}^{K-1} \phi_k\, rect\left[\frac{t - k\tau_c - \tau_c/2}{\tau_c}\right]$$

Though many different codes have been adapted from other disciplines of technology in this lab we will work with two classical discrete phase codes: the **Barker codes** and the **Frank polyphase codes**. They will be enough to explain the advantages of using waveform coding.

Barker codes

A pulse coded using the Barker code consists of K sub pulses of width τ. Each sub pulse has a phase of 0º or 180º and then the signal amplitude changes between +1 and -1 according to that phase. Barker codes characterize by the fact that the maximum lobe level is $1/K$ which is convenient to avoid a false target detection. Only 8 Barker codes are known (see Table 1).

Code length	Code elements
1	+
2	+ −, or − −
3	+ + −, or + − +
4	+ + − +, or + + + −
5	+ + + − +
7	+ + + − − + −
11	+ + + − − − + − − + −
13	+ + + + + − − + + − + − +

Table 1. The 8 known Barker codes.

As an example, in Figure 1 (left) we have a 7-bits baseband Barker code and (right) a modulated version of the signal.

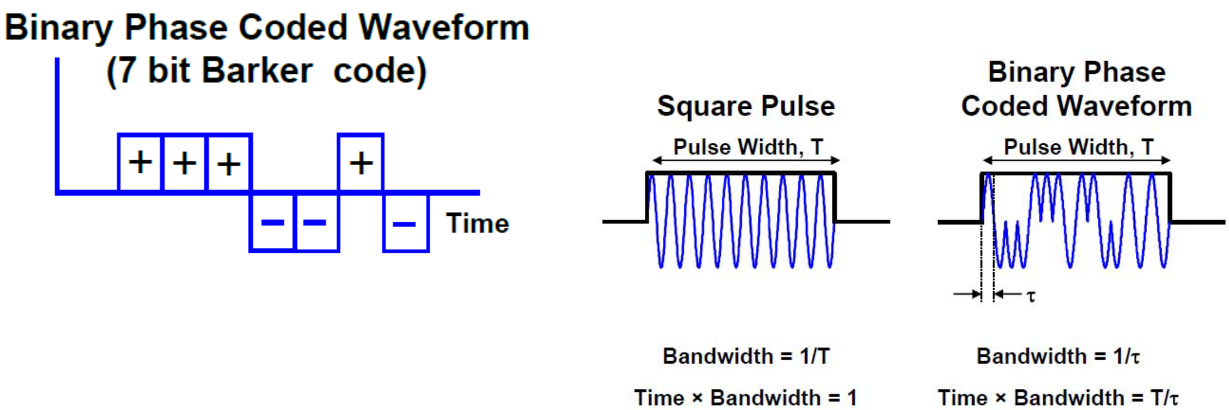

Figure 1. (*left*) 7-Bits baseband Barker code and (*right*) a modulated version of the signal.

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Lab Tasks

Part I

In this lab session you should plot the baseband complex signal and the output of the matched filter for a given code and a given signal to noise ratio.

4.1. Design a MATLAB function that generates a baseband signal corresponding to a given code at a given SNR level. The output of the function should be the vector of the baseband signal samples without the noise and the vector of the baseband signal samples with noise. To generate the noise, use the function ***randn*** of MATLAB.

Inputs	Pulse duration (s)
	Number of samples per chip [adim.]
	Vector of chip phases [degrees]
	Vector of chip amplitudes (1 by default) [adim.]
	Signal to noise ratio [dB]
Outputs	Samples of the baseband output signal vector without noise
	Samples of the generated noise vector
	Samples of the baseband output signal vector with noise
	Sampling Time (s)

4.2. Using the previously designed function make a plot of the pulse burst phases corresponding to Barker codes of length K=4, 7, 11, and 13. Consider that the SNR is 30 dB. Consider a pulse duration of 1 μs and 20 samples per chip. Any comment? (burst length, phases, ...)

4.3. Using ***conv*** (convolution), ***conj*** and ***fliplr*** functions of MATLAB find the output of the matched filter for the previously computed Barker codes (K=5, 7, 11, and 13). Make two plots, one considering the linear filter output and its corresponding response in dB. In the absence of noise (in fact SNR=30 dB), take note of the main to sidelobe level, number of lobes, and the length of the output pulse.

4.4. Make a linear plot of the matched filter output and the input signal for a Barker code of length K=13 when the SNR changes from 30 dB to -10 dB in steps of 10 dB. Make comments about the results (peak level, sidelobe level, peak width, ...).

Part II

If we remember from the previous lab session (False Alarm and Detection probabilities), a design parameter of a radar system is the threshold detection, which is directly selected depending on the amount of noise reaching the receiver and the targeted false alarm probability. During this part of the lab we are going to consider the voltage threshold for several scenarios with different SNRs and Pfa in order to find which Barker code meets the conditions of SNR to be detected in each case. Moreover, we are going to calculate the SNR with the help of the function generated at lab session 2 (Radar equation).

4.5. Using the functions of lab2 and the same input parameters (For the noise figure consider the first configuration Antenna-LNA-Wire-…), calculate the SNR of a radar situated at 30 Km from the target.

Peak power:	1.5 MW
Antenna gain:	45 dBi
Carrier frequency:	1.1 GHz
Bandwidth:	50 MHz
Noise figure:	Calculate for 1st config.
Additional losses:	0 dB
Radar cross section:	1 m^2

4.6. Execute the function of this code with this new SNR calculated, for a barker code K = 4, 7, 13. With the vector of output noise vector and considering a Pfa = 1e-3, 1e-5, 1e-7. Calculate the voltage threshold for each barker code. Plot the calculated voltage threshold over the output signal in dB. Analyze the results.

4.7. Reduce the SNR 3 dB and repeat the process of 3.6. Analyze the results.