A Triple Hazard Model for Price and Sales Crashes of New High-Technology Products

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ABSTRACT

The author propose a triple hazard model to analyze a phenomenon that occurs for information and high-technology new products that face short life cycles: the sales crash, the price crash, and the sales recovery. The model can untangle three important sources of variation in these interdependent events: i) lagged-event causality, ii) heterogeneity due to observed factors, and iii) heterogeneity due to unobserved (and possibly) correlated factors.

Results suggest that the price crash involves a 23% drop in the introductory price. The sales crash amounts to 60% drop in peak introductory unit sales. The occurrence of a price crash significantly decreases the hazard of a sales crash whereas the occurrence of a sales crash significantly increases the hazard of a price crash. The latter effect is significantly stronger than the former. The price crash significantly increases the hazard rate of a sales recovery whereas the sales recovery has a positive but insignificant effect on the occurrence of a price crash. The findings and model have important implications for managers of information and technology new products.

KEYWORDS: Duration Models, Hazard models, New Product Growth, Sales Saddles, Sales Crash, Price Crash, Information goods, High tech products,

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Introduction

The first iPhone was introduced on September 2007 at a price of $599. However, just 66 days after its launch, Apple decided to drop the iPhone's price by $200 and set its new price to $399. It was hypothesized that Apple's price cut was a response to sluggish sales (Business Week, 2007; Wired Magazine, 2007). This paper provides answers to questions like: How much more likely are market prices to fall after the sales of new products undergo sharp drops? Are such dramatic price reductions effective for halting sales slowdowns? Do price drops affect sales drop or vice versa?

Our key contributions are both methodological and substantive. First, we propose a hazard model that is new to the literature in marketing and duration models. This new model consists of three hazard functions that capture the timing of three interdependent events. We refer to it as the "triple-hazard model". The novelty of our triple-hazard model is that it allows us to simultaneously untangle three main sources of variation in the timing of events. These three sources of variation are: i), Lagged-event causality, ii), heterogeneity driven by observed factors, and iii), heterogeneity driven by unobserved (and possibly) correlated factors.

Lagged-event causality refers to the effect that the occurrence of an event has on the timing of a second future event. We apply our triple hazard model for analyzing a phenomenon, new to the marketing literature, the *sales-price crash*. This phenomenon occurs in new products in information and high-technology products with short life cycles. Information goods are those that provide consumers with knowledge or entertainment, such as video games, music, books, movies, etc. High-technology new products are those that require computing hardware and software, which have short life cycles. Both high-tech and information products often sell the highest number of units at their introduction. Their sales drop drastically after a certain point in time, after which they never recover initial sales levels. We define the *sales-price crash* as the inter-dependent (but not necessarily simultaneous) occurrence of two events: The sales crash and the price crash.

A sales crash is a substantial and permanent fall in the sales of a new product. The price crash is a deep and permanent reduction in the price of a new product that in some cases is followed by a sales recovery. The sales recovery is a modest increase in sales that may occur after the sales crash. This study addresses three important research questions:

1. How can one develop and estimate a triple hazard model for three potentially interdependent events? 2. What are the patterns of sales crashes, price crashes, and sales recoveries for information and high-tech new products? 3. What are the interdependent effect of sales crashes, price crashes, and sales recoveries, if any? We use hazard models to answer these questions. Hazard models have been applied in several marketing studies concerned with the sales of new products (e.g., Agarwal & Bayus 2002, Golder & Tellis 1997, Tellis, Stremersch, & Yin 2003). These studies all analyze the prediction of the sales takeoff. A sales takeoff is a dramatic increase in the sales of a new product that marks the transition from an introductory phase to the growth phase in new product's life cycle. These authors' rely on hazard models for a single event, which is the timing of the sales takeoff. Their models control for exogenous variation in the timing of the sales takeoff that is due to price and several other factors. Golder & Tellis (1997) find that price is the most important factor determining the timing of the sales takeoff and that a 1% decrease in the price of a new product is associated with a 4.5% increase in the probability of its takeoff. Similarly, Golder & Tellis (2004) find that a 1% decrease in price is

associated with a 5% decrease in the probability of a slowdown in the sales of a new product. More recently, Chandrasekaran & Tellis (2011) study the start timing and duration of the sales saddle. A sales saddle, as they define it, is a sudden, sustained, and deep drop in sales of a new product that is followed by a gradual recovery to a former sales peak.

The main limitation of this literature lies in the assumption that prices exogenously affect the timing of the sales takeoff or the saddle. However, the occurrence of a sales takeoff or saddle may also affect prices. Such bi-directionality is especially important in the case of price and sales crashes. . Our proposed model incorporates bi-directional effects between the timing of crashes in price and sales. Bi-directional effects between the timing of events may exist due to correlation introduced by unobserved factors or due to lagged-event causality. Recent advances in hazard models allow us to incorporate correlation, lagged-event causality, or both correlation and lagged dependence among the timing of multiple events.

Hazard models that introduce correlation dependence have been applied by Chintagunta & Haldar (1998), Park & Fader (2004), Schweidel, Fader, & Bradlow (2008), and Braun & Schweidel (2011). Chintagunta & Haldar (1998) propose a copula approach for modeling the joint density of the purchase timing of two related product categories; where the join density incorporates correlation between the timing of purchases. Other copula applications in marketing are described by Danaher & Smith (2011). Following a similar approach, Park & Fader (2004) develop a duration model that allows for correlation between the timing of user visits to multiple websites. In their model, correlation may be driven by both observed and unobserved factors. In a similar way, Schweidel, Fader, & Bradlow (2008) and Braun & Schweidel (2011) propose bivariate duration models that allow correlation between consumers' acquisition and retention times or consumer retention and churn.

Duration models simultaneously incorporating lagged-event causality and correlation, like ours, are new to the marketing literature but they have been previously applied in health and labor studies. For example, Van Ours (2003) and Van Ours & Williams (2009) test whether the timing of the first use of cannabis is a gateway to the later use of harder drugs or to school drop-out. Abbring, Van den Berg, & Van Ours (2005) analyze whether the timing of unemployment sanctions and unemployment are causally linked. Heckman & Borjas (1980) test whether current unemployment causes future unemployment. Duration models with lagged-event causality are part of a broader category of econometric models concerned with causal effects or what are commonly known as treatment effects (Heckman & Vytlacil, 2007). Such models are often applied for evaluating policy interventions like new regulations, job assistance programs, or new medical treatments.

We introduce a model that permits both statistical correlation and lagged-event causality between the timing of events. Our model is an extension of the bivariate duration model by Abbring & Van den Berg (2003). Briefly, Abbring & Van den Berg (2003) introduce a bivariate duration model for parallel duration variables, which proves that the lagged-event causality (or causal effect) can be rightly identified. We extend their model to the trivariate duration case, what we will refer to as a triple-hazard model. Further, we extend their method by adding dynamics to lagged-event causality by analyzing both sequential and parallel events. Sequential events always occur in a certain order (for example, when event B always follows event A at some point in time) whereas parallel events may occur without any order restriction.

Note that, lagged-event causality does not refer to the underlying causal and behavioral mechanism behind the marketing phenomena described in this paper. However, in some instances, lagged-event causality might be given a causal and behavioral interpretation if the lagged events represent a treatment or an intervention policy applied at the micro-level, like in Abbring, Van den Berg, & Van Ours (2005). Lagged-event causality, as we define in this study, refers to the effect of the occurrence of an event at time t_1 on the probability of occurrence of a second possibly forthcoming event occurring after time t_1. Our definition of lagged-event causality is close, but not identical, to the concept of Granger-causality. Our specification identifies the effect of one event on a second event after controlling for several factors. These control factors, together with the order in which events occur, is what allow us to statistically identify the effect of one event on another one. The formal statistical identification arguments of the model are available in the online appendix of the paper.

The plan of the paper is as follows. Section 1 describes the data and phenomena. Section 2 presents our definitions and measures. Section 3 presents the model, its parameterization, and the intuition behind its identification. Section 4 discusses the model identification. Section 5 describes six alternative model specifications. Section 6 presents our main results. Finally, Section 7 discusses the findings.

1. Market Context and Data

This study focuses on the market for video game software. Information goods like video games, books, music, movies, and apps, are purchased only once. Moreover, video games can be considered experience goods because their quality and entertainment value is typically assessed during or after consumption (Nelson, 1970). Finally, videogames are both information goods and high technology consumer products. Our data covers the period from 1995 to 2001. During this time video games were mainly sold through brick-and-mortar retailers. Nowadays, online channels are definitely important (Zhu & Zhang, 2010). The video game industry is receiving increasing media coverage, introducing very popular video games, and breaking records in sales. For example, the Black Ops II of Activision recently reached more than $1 billion dollars in revenues; this record is higher than the record for all-time gross sales set in the movie industry by Avatar in 2009 (USA Today, 2012). The penetration of video game consoles in US households is currently higher than the penetration of tablets, at 44%, representing close to 10% of all internet-connected consumer electronic devices (Forbes, 2013; Joystiq, 2013). In summary, the video game industry is an important segment of both the high-tech and the entertainment industries. Besides the contribution to modeling, our study derives new insights for marketing managers in this market. .

Our data consists of the monthly time series of sales and prices of 1562 video-game titles sold in the US from 1995 to 2001. From these data, we identify the timing of the sales crash, the timing of the price crash, and the timing of sales recovery (t_s, t_p and t_r, respectively). These data cover three major video-game platforms, nine product genres, eighty-five firms, and seven years. The data contain six observed characteristics for each video game: the video-game platform (or console), publisher, quality ratings, month and year of launch, and genre. These data come from the NPD Group from their point of sales system that covers 65% of the video game retailers in the US. The video games in our sample represent more than 400 million unit sales and a total of 14 billion in dollar gross revenues.

Several marketing forces are at play in the video game industry. First, the video game market is very likely composed of heterogeneous consumers segments such as hard-core gamers and casual players. In addition, consumers might be forward looking and might be able to strategically adjust their behavior to price expectations (Nair, 2007). Indirect network effects are important, whereby software may affect hardware (Clements & Ohashi, 2005) and hardware may affect software (Binken & Stremersch, 2009). Video game software companies face intense monopolistic competition due to which they may strategically price their products and practice inter-temporal price discrimination (Liu, 2010). Our model can account for several of these important factors at play in the data for the video game industry.

2. Sales Saddles vs. Sales and Price Crashes

This section defines and describes phenomena that apply to the market prices and sales of new information products. Based on our data, we observe the following regularities: 1. Prices stay relatively constant for an introductory period and then they fall substantially and fast at a point in time. 2. Sales levels are, for the great majority of products, highest at launch and decay deeply after an introductory period. That is, in this market we don’t find the usual pattern of the sales-take off or the sales saddle.

As stated before, a sales crash is a substantial and permanent fall in the sales of a new product. The price crash is a deep and permanent reduction in the price of a new product that in some cases is followed by a sales recovery. The sales recovery is a modest increase in sales that may occur after the sales crash. Note that the *price crash* refers to an overall market drop in prices and not to a temporal discount. Likewise, the *sales crash* refers to the market wide drop in sales for the product. This aggregation at the market (and not brand) level is similar to that generally used in most of the studies on new product diffusion and takeoff.

The sales crash is different from the concept of the *sales saddle*. The saddle is a sudden and deep drop in sales that starts after a rapid growth phase and that is followed by a gradual but strong sales recovery (Chandrasekaran & Tellis, 2011). The sales saddle normally occurs for durables. The sales saddle is a temporary drop in sales whereas the sales crash is a permanent reduction in sales. In addition, the sales saddle typically starts after the growth stage that follows takeoff, long after the product launch, whereas the sales crash starts within one year after launch.

Figure 1 illustrates the differences between the sales saddle and the sales crash. This figure shows that the sales saddle and the sales crash are embedded in two different diffusion patterns. The diffusion pattern on the left of Figure 1A begins with a period of slow sales, which then take off, marking the start of a rapid growth phase that ends in a sales saddle. In contrast, Figure 1B, products sell their highest number of units at market launch and their sales decay at a fast pace shortly after introduction. This latter diffusion pattern is typically accompanied by marketing activities adapted to it. It is well known that firms in the information goods industry focus their marketing activities, specially advertising and pricing, around the market launch date of their products (Eliashberg, Elberse, & Leenders, 2006). That is, several firms put considerable effort, in design and quality improvements, to sell the highest number of units at market launch. The recovery after a saddle tends to reach former peak levels (Chandrasekaran & Tellis, 2011) because it is driven by late adopters who represent a substantial share of the total market; whereas the recovery after a sales crash is typically weak, if it occurs at all. This means that late adopters represent a small share of the market. Such recoveries in the sales of new products might be due to the existence of multiple adopter segments or due to the fragmented communication between different groups of adopters (Goldenberg, Libai, & Muller, 2002).

2.1. Measures

Previous literature has made use of heuristics for identifying sales saddles and sales takeoffs. For example, Goldenberg et al. (2002) define the saddle as a decline of 10% or 20% in sales that follows an initial sales peak and they define the start of the saddle as the moment just before the sales drop. Moreover, Goldenberg et al. (2002) define the end of the saddle or recovery, as the time when sales recover their initial peak level and they focus their study on saddles that have a duration of more than two years. Likewise, Chandrasekaran & Tellis (2011) define the start of the saddle as the first drop in sales that is greater than 10% and that lasts more than two years. Finally, Golder & Tellis (2004) define the start of the slowdown as the moment when sales fall for two consecutive years after takeoff.

Most of the studies cited above first use visual inspection for identifying an event of interest and then develop heuristics to identify the events objectively. We use a similar strategy for identifying and formally defining the sales crash, the price crash, and the sales recovery. First, we identify these three events by visual inspection of the data for each of the 1,562 products. The goal of visual identification is to identify the timing of the first and steepest drop in the sales and price curves. The crash time, as illustrated in Figure 2, is defined as the month when the first steepest drop occurs. Likewise, the sales recovery timing is identified as the month of the first steepest increase in sales that occurs after the sales crash.

We start the visual identification task with the identification of the timing of the three events (timing of sales crash, price crash, and sales recovery) in a training set of 100 randomly selected videogames. Subsequently, we perform the visual identification of the events in the complete set of 1,562 products. Having a training identification task increases consistency. Second, we define and calibrate three heuristics to identify and predicting the timing of the three events as soon as they occur (and without the need of using data recorded after the events). The calibration goal is to maximize the match between the visually identified timing of events and the timing identified by the heuristics. The use of heuristics ensures objectivity, consistency, and replicability of our identification strategy as well as identification in the future that is contemporaneous with and not after the event.

We define the heuristics of the sales crash, price crash, and sales recovery respectively as follows:

1. The timing of the sales crash is the first month after introduction when the month-to-month sales decrease is greater than 25%.
2. The timing of the price crash is the first month after introduction when the month-to-month price cut is greater than 8%.
3. The timing of the sales recovery is the first month after the sales crash when month-to-month sales increase represents more than 20% of the peak sales before the sales crash. If such increase is not present, then the sales recovery is absent.

All the numeric thresholds above represent the exact numbers that maximize the fit between the timing of events identified by the heuristic and the timing of events identified visually. The first heuristic exactly matches the visually identified timing of the sales crash for

81% of the cases; the second heuristic identifies exactly the timing of the price crash as visually identified for 68% of the cases; the third heuristic exactly matches the visually identified timing of the sales recovery for 56% of the cases. Our success rate in identifying the events via heuristic rules indicates that: 1) Sales crashes (81% exactly identified) are more predictable than price crashes (68% exactly identified) and 2) that the sales recovery is the least predictable and probably the most elusive event. Note that the identifying the exact month of occurrence of these events is hard because we have an average time horizon of 39 months after the new product introduction. That is, there is a small chance, $(1/39)^3$, of naively identifying these three events for each product. As a robustness check, we built alternative heuristic and threshold rules similar to Golder & Tellis, (1997) but our alternative heuristics achieved very similar success rate in terms of event identification. Please note that Golder & Tellis, (1997) also built alternative heuristics to their threshold rule. Their alternative heuristics identified *exactly* the time of the sales takeoff 20% of the time by the logistic rule, and 74% of the time by the maximum growth rule. [1]

The timing of the sales crash, the timing of the price crash, and the timing of the sales recovery can be conceptually placed in two different timelines as shown in Figure 3. The sales recovery can only occur after the sales crash; hence these two events are *sequential* duration variables. In contrast, the timing of the price crash can occur on a timeline that extends from before to after the timing of the sales crash and recovery. Likewise, the sales crash may occur at any point on its timeline. Please note that the starting point of the timelines in Figure 3 represents the new product introduction. There are many possible locations of the sales crash timing (t_s), the price crash timing (t_p), and the timing of the sales recovery (t_r) on their respective timelines.

[1] Please see Golder & Tellis, (1997) for further details.

We may observe, for example, the occurrence of a sales crash at time t_s and the subsequent occurrence of a price crash at a time t_p (having $t_p > t_s$). Another plausible outcome consists of observing the realization of a price crash followed by the realization of a sales crash, hence observing $t_p < t_s$.

3. Triple Hazard Model

Our model consists of three conditional hazard functions for each product i, having i=1…1562. The three product-specific duration variables that we model are: 1) The timing of the sales crash, that is t_{si}, 2) The timing of the price crash, that is t_{pi}, and 3) The timing of the sales recovery, that is t_{ri}. We assume that these three duration variables are the realization of the random duration variables T_{si}, T_{pi}, and T_{ri}, respectively, and that their conditional hazard rates can be expressed as follows:

$$\begin{aligned} \lambda_{pi}\big(t_i \mid \mathcal{F}_{pi}\big)dt &= \mathrm{P}\big(t_i \leq T_{pi} < t_i + dt \mid T_{pi} \geq t_i, \mathcal{F}_{pi}\big), \\ \lambda_{si}(t_i \mid \mathcal{F}_{si})dt &= \mathrm{P}(t_i \leq T_{si} < t_i + dt \mid T_{si} \geq t_i, \mathcal{F}_{si}), \\ \lambda_{ri}(t_i \mid \mathcal{F}_{ri})dt &= \mathrm{P}(t_i \leq T_{ri} < t_i + dt \mid T_{ri} \geq t_i, \mathcal{F}_{ri}), \end{aligned} \quad (1)$$

for all $t > 0$ and where $\mathrm{P}(x|y)$ denotes a generic probability of x conditional on , and dt is an infinitesimal time increment. Thus, $\lambda_{pi}(t_i \mid \mathcal{F}_{pi})dt$ represents the probability of observing the price crash of product *i* occurring at time t_i given that is has not occurred before t_i, that is conditional on $T_{pi} \geq t_i$, and conditional on the information set $\mathcal{F}_{pi}$. Similarly, $\lambda_s(t_i \mid \mathcal{F}_{si})dt$ and $\lambda_r(t_i \mid \mathcal{F}_{ri})dt$ denotes the probability of observing a sales crash or a sales recovery at time t_i given they have not occurred before t_i and given the information sets $\mathcal{F}_{si}$ and $\mathcal{F}_{ri}$. A critical element in the conditional hazards is the information set $\mathcal{F}_{ji}$ for j $\in (s, p, r)$ because this element that will allow us to control for unobserved heterogeneity, lagged-event causality, indirect network effects, and observed heterogeneity.

The first element in the information sets $\mathcal{F}_{ji}$ consist of $\boldsymbol{x}_i$ that is a vector of observed product characteristics. The vector $\boldsymbol{x}_i$ includes 17 publisher dummies, 8 genre dummies, 17 monthly and yearly dummies, 4 hardware platform dummies, and 2 quality measures; that is a total of 48 control variables for 6 observed characteristics. The hardware platform (or video game consoles) dummies are our control for the possibility of indirect network effects; for example, products of certain platforms might have earlier or later crash timings and we permit for this possibility by including platforms in the $\boldsymbol{x}_i$ vector. The second element in the information sets $\mathcal{F}_{ji}$ are lagged events δ_i. For example, the occurrence of a price crash might influence the timing of forthcoming sales crashes, or vice versa. The influence of lagged-events on the hazard rate of subsequent events might be time varying and we will allow for this possibility. Finally, the third element in the information set $\mathcal{F}_{ji}$ is made by the unobserved factors v_{pi}, v_{si}, v_{ri}. These factors, like all others, are product-specific and we allow them to be correlated. As it is well known, observed prices and sales are the result of the interaction of firms and consumers and hence we aim at capturing such endogenous unobserved interactions by the unobserved correlated heterogeneity terms v_{pi}, v_{si}, v_{ri}.

We assume that the conditional hazard functions $\lambda_{ji}\left(t_i \mid \mathcal{F}_{ji} \right)$ for all j events follow a proportional hazard specification and that they are the product of: 1) A baseline hazard function $\lambda_j(t)$ that measures how the hazard rate of event j varies in time, 2) Effects of lagged-event effects δ_k^j that measure the effect of a lagged event j on the conditional hazard of an event k, 3) A regressor function $\phi_j(\boldsymbol{x}_i)$ that controls for observed heterogeneity in the timing of event j, and 4) An unobserved heterogeneity term v_{ji}. Hence, we write the conditional hazard rates as:

$$
\begin{aligned}
\lambda_p(t \mid \delta_{-pi}, \boldsymbol{x}_i, v_{pi}) &= \lambda_p(t)\phi_p(\boldsymbol{x}_i)\delta_p^s(t \mid t_{si})^{1\{t>t_{si}\}}\delta_p^r(t \mid t_{si}, t_{ri})^{1\{t>t_{si}+t_{ri}\}}v_{pi}, \\
\lambda_s(t \mid \delta_{-si}, \boldsymbol{x}_i, v_{si}) &= \lambda_s(t)\phi_s(\boldsymbol{x}_i)\delta_s^p(t \mid t_{pi})^{1\{t>t_{si}\}}v_{si}, \qquad (2) \\
\lambda_r(t \mid \delta_{-ri}, \boldsymbol{x}_i, v_{ri}) &= \lambda_r(t)\phi_r(\boldsymbol{x}_i)\delta_r^s(t_{si})\delta_r^p(t \mid t_{pi}, t_{si})^{1\{t_{si}<t_{pi}<t_{si}+t\}}v_{ri},
\end{aligned}
$$

where $\lambda_j(t \mid \delta_{-pi}, \boldsymbol{x}_i, v_{pi})$ is the conditional hazard rate of the event j, $1\{.\}$ is an indicator function that is equal to one if the event within the brackets occurs and zero otherwise, and δ_{-ji} represent the lagged events which occurred before event j. In the first equation of (2), δ_p^s represents the lagged-event effect of the sales crash on the price crash, δ_p^r represent the effect of the sales-recovery on the price crash; in the second equation of (2), δ_s^p represents the effect of the price crash on the sales crash; in the third equation of (2), δ_r^s represents the effect of the sales crash on the sales recovery, and δ_r^p represents the effect of the price crash on the sales recovery. Next, we present and discuss the exact parameterization for each of the elements affecting the conditional hazard functions.

3.1. Hazard Functions Specification

We assume that the baseline hazards follow an Expo-power specification defined as:

$$
\begin{aligned}
\lambda_p(t; \alpha_p, \gamma_p) &= \alpha_p t^{\alpha_p - 1} e^{\gamma_p t}, \\
\lambda_s(t; \alpha_s, \gamma_s) &= \alpha_s t^{\alpha_s - 1} e^{\gamma_s t}, \quad (3) \\
\lambda_r(t; \alpha_r, \gamma_r) &= \alpha_r t^{\alpha_r - 1} e^{\gamma_r t},
\end{aligned}
$$

for $t > 0$, $\alpha_j > 0$, and $\gamma_j \in \mathbf{R}$. The notation $x \in \mathbf{R}$ indicates that x may take any real value. We choose an Expo-power specification because it permits several shapes for the baseline hazards and because it is the most flexible function among several alternatives (Saha & Hilton, 1997; Seetharaman & Chintagunta, 2003). In particular, the Expo-power allows for U-shape, inverted U-shape, increasing, and decreasing hazard functions and it nests the Gompertz (when $\alpha_j = 1$) and the Weibull (when $\gamma_j = 0$), which are popular specifications in empirical analysis; see for example Franses & Paap (2004) or Seetharaman & Chintagunta (2003). The log-logistic is also a very flexible specification but we choose the Expo-power because the latter has a closed form expression for its integrated hazard and this greatly facilitates model estimation.

The regressor functions follow an exponential specification that is widely used in empirical analysis. That is,

$$\begin{aligned} \phi_p(\boldsymbol{x}_i; \boldsymbol{\beta}_p) &= exp\left(\boldsymbol{x}_i{}'\beta_p\right), \\ \phi_s(\boldsymbol{x}_i; \beta_s) &= exp(\boldsymbol{x}_i{}'\beta_s), \\ \phi_r(\boldsymbol{x}_i; \beta_r) &= exp(\boldsymbol{x}_i{}'\beta_r), \end{aligned} \quad (4)$$

where $\boldsymbol{\beta}_j \in \mathbf{R}^{48}$, $\boldsymbol{x}_i \in \mathbf{R}^{48}$, for $j \in \{p, s, r\}$, and where we use $\mathbf{R}^{48}$ to represent a vector of 48 real numbers. In view of the above definitions, the parameter vector $\boldsymbol{\beta}_j$ measures the effect of the observed covariates on the hazard rate of its corresponding event.

The lagged-event effects consist of four time-varying functions and one time-invariant function. The time-varying lagged-event effects measure: 1) The effect of the realization of the sales crash on the hazard rate of the price crash; 2) The effect of the price crash on the hazard rate of the sales crash; 3) The effect of the price crash realization on the hazard rate of the sales recovery; 4) The effect of sales recovery on the hazard rate of the price crash; and 5) The time-invariant effect function measures the effect of the realization of the sales crash on the sales recovery.

We assume that the hazard rate of the event of interest changes deterministically after the realization of one and/or the two other events. Specifically, we assume that the hazard rate of the sales crash increases/decreases by a factor of $\delta_s^p(t \mid t_p)$ as soon as the price crash occurs and hence this δ_s^p factor affects the hazard rate for all $t > t_p$. In a similar way, we assume that the hazard rate of the price crash increases/decreases by a factor of $\delta_p^s(t \mid t_s)$ as soon as the sales crash occurs and it affects the hazard for all $t > t_s$, and by a factor of $\delta_p^r(t \mid t_s, t_r)$ as soon as the sales recovery occurs and it affects the hazard for all $t > t_s + t_r$. In addition, we assume that the hazard rate of the sales recovery increases/decreases by a factor of $\delta_r^p(t \mid t_p, t_s)$ when the price crash occurs within the interval $(t_s, t_s + t_r)$, that is the interval between the sales crash and the sales recovery. Finally, we assume that the price crash affects the sales recovery indirectly through its effect on the sales crash but only when the price crash occurs before the sales crash. These conditions are explicit in the indicator functions that "turn on" or "turn off" the lagged event effects in Equation (1).

All the time-varying lagged-event effects are defined as piece-wise constant functions of K intervals with limits $[Y_l, Y_{l+1}]$ for $l = 0, \ldots, K-1$, $Y_0 = 0$,, and $Y_K = \infty$. We use a piece-wise specification to avoid any functional form in the effects of one event into another. Note that the larger the number of pieces the more non-parametric our specification becomes. For our analysis, we use seven intervals and these are $[0,2)$, $[2,4)$,..., and $[12, \infty)$. These are two-month intervals with the exception of the last lagged-event effect; this last effect takes place from the 12 month onwards and hence its upper limit is equal to infinity. A piece-wise specification is highly flexible and it is suitable for our study because we have no prior knowledge about the shape of the lagged-event effects. Notice that as the number of intervals increases, the more "nonparametric" this specification becomes. These piece-wise specifications were first introduced by Freund (1961) and they are commonly applied for measuring lagged dependence among parallel durations (Van den Berg, 2001). Below we provide further intuition about how these lagged-event functions work.

The lagged-event effect function δ_p^s measures the effect of the sales crash on the price crash and it is parameterized for $t > t_s > 0$ as

$$\delta_s^p(t \mid t_s; \theta_s^p) = \sum_{l=0}^{K-1} 1\,\{Y_l \leq t - t_s < Y_{l+1}\} exp\left(\theta_{sl}^p\right) \quad (5)$$

where we make use of an exponential function to ensure that the lagged-event effect is strictly positive.

In general, the role of the lagged-event effect functions is to select one of the "treatment" coefficients θ_{pl}^s for l=0,..,$K-1$ that affects the hazard rate at time t. Specifically, the function in Equation (5) selects the relevant $exp(\theta_{pl}^s)$ coefficient that affects the conditional hazard rate when the treatment duration is equal to $t - t_s$. Notice that there are K intervals $[Y_l, Y_{l+1}]$ for $l = 0, \dots, K-1$ and that the lagged-event coefficient $exp(\theta_{pl}^s)$ affects the hazard rate only when the treatment duration is within the coefficient's corresponding interval $[Y_l, Y_{l+1}]$. For example, the conditional hazard rate of the price crash is be equal to $\lambda_p(t)\phi_p(x)v_p$ before the realization of the sales crash and it is equal to $\lambda_p(t)\phi_p(x)\delta_p^s(t \mid t_s; \theta_p^s)v_p$ after the realization of the sales crash and for all subsequent periods (and before the recovery). And if we assume, for example, that the duration of the lagged-event effect δ_p^s is between zero and two months, that is $t - t_s < 2$ months, then $\delta_p^s(t \mid t_s; \theta_p^s)$ is equal to $exp(\theta_{p1}^s)$ and hence the conditional hazard rate would be equal to $\lambda_p(t)\phi_p(t)exp(\theta_{p1}^s)v_p$. If the duration of the lagged-event effect were in between two months and four, then the conditional hazard rate would be equal to $\lambda_p(t)\phi_p(t)exp(\theta^s_{p2})v_p$. In a similar way we could define the hazard rate for all subsequent periods after the lagged event takes place.

Next, the function δ^r_p measures the effect of the sales recovery on the price crash and it is modeled for $t > t_s + t_d > 0$ as

$$\delta^r_p(t \mid t_s, t_r; \theta^r_p) = \sum_{l=0}^{K-1} 1\{Y_l \leq t - t_s - t_d < Y_{l+1}\}exp\left(\theta^r_{pl}\right), \quad (6)$$

Similarly, the function δ^p_s measures the effect of a price crash on the hazard rate of a sales crash. We parameterize it for $t > t_p > 0$ as

$$\delta^p_s(t \mid t_p; \theta^p_s) = \sum_{l=0}^{K-1} 1\left\{Y_l \leq t - t_p < Y_{l+1}\right\}exp\left(\theta^p_{sl}\right). \quad (7)$$

Furthermore, δ^p_r measures the effect of a price crash on the hazard rate of a sales recovery. We adopt the next parameterization where

$$\delta^p_r(t \mid t_p, t_s; \theta^p_r) = \sum_{l=0}^{K-1} 1\left\{Y_l \leq t - t_p + t_s \leq Y_{l+1}\right\}exp\left(\theta^p_{rl}\right) \quad (8)$$

for $t > t_p - t_s > 0$.

Next, the effect of the sales crash on the sales recovery is defined as

$$\delta^s_r(t_s) \quad = \quad exp(t_s\beta^s_r). \quad (9)$$

The parameter β^s_r in the regressor function above measures the effect of the sales crash t_s on the hazard rate of the sales recovery. Specifications like (9) were first introduced by Heckman & Borjas (1980) as a way to measure the effect of a focal event on a second event that always proceeds the first focal event and it is also a form of lagged dependence (Heckman & Borjas, 1980; Van den Berg, 2001). In our context, the sales recovery is always preceded by a sales crash.

Finally, we assume that the random vector (v_p, v_s, v_r) follows a trivariate log-normal distribution. Hence,

$$\log\left(v_{pi}, v_{si}, v_{ri}\right) \sim \mathbf{N}(0, \mathbf{\Sigma}) \quad (10)$$

for all i and where $\mathbf{\Sigma}$ is the variance-covariance matrix and whose off-diagonal elements measure the covariance between the random effects.

3.2. Model Estimation

We first describe how to write the model likelihood contribution of each product (or SKU) and later on in the next subsection we discuss how do we estimate the triple hazards. The data consist of N realizations of random variables $(\tilde{T}_p, \tilde{T}_s, \tilde{T}_r, \gamma_p, \gamma_s, \gamma_r, X)$, where $\tilde{T}_j = min(C_j, T_j)$, $\gamma_j = 1\{T_j \leq C_j\}$ for $j \in \{p, s, r\}$. Throughout this section we denote the realization of $\tilde{T}_j$ and X by $\tilde{t}_j$ and x, respectively. C_j is a censoring variable that is equal to the number of months for which we observe data for each product. For example, if we have 24 months of data and we do not observe an event being realized within these 24 months, then $\tilde{t}_j$ will be equal to $C_j = 24$ for $j \in \{p, s\}$. In this section we suppress the sub-index i from all expressions and variables for clarity of exposition.

In addition, the data are such that when neither T_p nor T_s has been realized and one of them is censored the other duration variable is censored at the same point of time as well.

Similarly, when $T_s < T_p$ and none of T_p and T_r has been realized, censoring of one of the latter automatically results in censoring at the same point of time of the other variable as well. Let v_p, v_s and v_r denote realization of V_p, V_s, V_r, respectively. We first form the likelihood contribution of a single product and that is

$$l(\tilde{t}_p, \tilde{t}_s, \tilde{t}_r \mid x, v_p, v_s, v_r) = l_p(\tilde{t}_p \mid \tilde{t}_s, \tilde{t}_r, x, v_p) l_s(\tilde{t}_s \mid \tilde{t}_p, \tilde{t}_r, x, v_p) l_r(\tilde{t}_r \mid \tilde{t}_p, \tilde{t}_s, x, v_p), \quad (11)$$

where the likelihoods on the right hand side of the above display refers to $\tilde{t}_p, \tilde{t}_s$ and $\tilde{t}_r$, respectively. Next, we integrate out the unobserved terms v_p, v_s, v_r with respect to the probability measure G in order to obtain the likelihood contribution $\mathcal{L}(\tilde{t}_p, \tilde{t}_s, \tilde{t}_r \mid x)$, that is,

$$\mathcal{L}(\tilde{t}_p, \tilde{t}_s, \tilde{t}_r \mid x) := \int_{\mathbf{R}_+^3} l\,(\tilde{t}_p, \tilde{t}_s, \tilde{t}_{di} \mid x, v_p, v_s, v_r) dG(v_p, v_s, v_r). \quad (12)$$

The derivation of the form of the likelihoods is based on the following general idea. If we do not have uncensored observation, then the likelihood contribution equals the product between the respective instantaneous hazard rate and its survival function. On the other hand, in case an observation is censored, then its likelihood contribution equals the survival function. Let $\mathcal{T}$ denote a positive duration variable with conditional hazard rate $\ddot{\lambda}(.)$. Denote by $\tilde{\mathcal{T}} = min(\mathcal{T}, \mathcal{T}^c)$ and by $\tilde{\tau}$ the realization of $\tilde{\mathcal{T}}$. Then, the likelihood contribution for $\tilde{\tau}$ is given by

$$l(\tilde{\tau}) = 1\{\mathcal{T} \leq \mathcal{T}^c\} \ddot{\lambda}(\tilde{\tau}) exp\left(-\int_0^{\tilde{\tau}} \ddot{\lambda}\,(\omega) d\omega\right) + 1\{\mathcal{T} > \mathcal{T}^c\} exp\left(-\int_0^{\tilde{\tau}} \ddot{\lambda}\,(\omega) d\omega\right). \quad (13)$$

We need to introduce the following notation before writing the exact likelihood functions. First, we define the integrated baseline hazards as

$$
\begin{aligned}
\Lambda_p(t) &:= \int_0^t \lambda_p(\omega)d\omega, \\
\Lambda_s(t) &:= \int_0^t \lambda_s(\omega)d\omega, \qquad (14) \\
\Lambda_r(t) &:= \int_0^t \lambda_r(\omega)d\omega,
\end{aligned}
$$

for $t > 0$. Next, we define the integrated version of the product between the conditional hazard functions and the lagged-causal effects functions for $t_p, t_s, t_r > 0$ as (15):

$$
\begin{aligned}
\Upsilon_s^p(t_s \mid t_p) &= \int_{t_p}^{t_s} \lambda_j(\omega)\delta_s^p(\omega \mid t_p)d\omega, && \text{for } t_s > t_p, \\
\Upsilon_p^s(t_p \mid t_s) &= \int_{t_s}^{t_p} \lambda_j(\omega)\delta_p^s(\omega \mid t_s)d\omega, && \text{for } t_p > t_s, \\
\Upsilon_p^r(t_p \mid t_s, t_r) &= \int_{t_s+t_r}^{t_p} \lambda_p(\omega)\delta_p^s(\omega \mid t_s)\delta_p^r(\omega \mid t_s, t_r)d\omega, && \text{for } t_p > t_s + t_r, \\
\Upsilon_r^p(t_r \mid t_p, t_s) &= \int_{t_p-t_s}^{t_r} \lambda_r(\omega)\delta_r^p(\omega \mid t_p, t_s)d\omega, && \text{for } t_r > t_p - t_s > 0.
\end{aligned}
$$

Finally, the integrated causal effects are defined for $t_p, t_s, t_r > 0$ as (16):

$$
\begin{aligned}
\Delta_s^p(t_s \mid t_p) &= \int_{t_a}^{t_b} \delta_s^p(\omega \mid t_p)d\omega, && \text{for } t_s > t_p, \\
\Delta_p^s(t_p \mid t_s) &= \int_{t_a}^{t_b} \delta_p^s(\omega \mid t_s)d\omega, && \text{for } t_p > t_s, \\
\Delta_p^r(t_p \mid t_s, t_r) &= \int_{t_s+t_r}^{t_p} \delta_p^r(\omega \mid t_s, t_r)d\omega, && \text{for } t_p > t_s + t_r, \\
\Delta_r^p(t_r \mid t_p, t_s) &= \int_{t_p-t_s}^{t_r} \delta_r^p\left(\omega \mid t_p, t_s\right)d\omega, && \text{for } t_r > t_p - t_s > 0.
\end{aligned}
$$

The likelihood corresponding for $\tilde{t}_p$ is (17):

$$
\begin{aligned}
l_p(\tilde{t}_p \mid \tilde{t}_s, \tilde{t}_r, x, v_p) \quad = \quad & \gamma_p 1\{\tilde{t}_p < \tilde{t}_s\} v_p \phi_p(x) \lambda_p(\tilde{t}_p) exp(-\phi_p(x)\, \Lambda_p\,(\tilde{t}_p) v_p) \\
& + \gamma_s 1\{\tilde{t}_s < \tilde{t}_p \leq \tilde{t}_s + \tilde{t}_r\} \left(\phi_p(x) \lambda_p(\tilde{t}_p) \delta_p^s(\tilde{t}_p \mid \tilde{t}_s) v_p\right)^{\gamma_p} \times \\
& exp(-\phi_p(x)\left(\Lambda_p\,(\tilde{t}_s) + \Upsilon_p^s(\tilde{t}_p \mid \tilde{t}_s) v_p\right)) \\
& + \gamma_s \gamma_r 1\{\tilde{t}_s + \tilde{t}_r < \tilde{t}_p\} \left(\phi_p(x) \lambda_p(\tilde{t}_p) \delta_p^s(\tilde{t}_p \mid \tilde{t}_s) \delta_p^r(\tilde{t}_p \mid \tilde{t}_s, \tilde{t}_r) v_p\right)^{\gamma_p} \times \\
& exp(-\phi_p(x)\left(\Lambda_p\,(\tilde{t}_s) + \Upsilon_p^s(\tilde{t}_p \mid \tilde{t}_s) + \Upsilon_p^r(\tilde{t}_p \mid \tilde{t}_s, \tilde{t}_r)\right) v_p) \\
& + (1-\gamma_p)(1-\gamma_s) exp(-\phi_p(x)\, \Lambda_p \quad (\tilde{t}_p) v_p).
\end{aligned}
$$

We now provide some intuition for the expression above. The first term is the likelihood contribution in case the price crash occurs before the sales crash and consequently the hazard rate is affected by neither the sales crash nor the sales recovery. The second term describes the likelihood contribution in case the price crash happens after the sales crash but before the sales recovery. Note that it is at this latter case that the causal effect function δ_p^s enters the model and affects the instantaneous hazard rate. The third term equals the likelihood contribution in case the sales crash and sales recovery precede the price crash. In this latter case, two causal effects, δ_p^s and δ_p^r, affect the instantaneous hazard rate. Finally, the last term is equal to the likelihood contribution in case the timing of price crash and sales crash is censored. The term $\Upsilon_p^s(\tilde{t}_p \mid \tilde{t}_s, x)$ is defined as follows (18),

$$
\begin{aligned}
\Upsilon_p^s(\tilde{t}_p \mid \tilde{t}_s) \quad = \quad & \sum_{l=0}^{K-1} 1\left\{\Upsilon_l \leq \tilde{t}_p - \tilde{t}_s < \Upsilon_{l+1}\right\} \\
& \times \sum_{\eta=0}^{l} exp\left(\theta_{p\eta}^s\right) [\left(\Lambda_p\,(\tilde{t}_p)\right)^{1\{\eta=l\}} \left(\Lambda_p\,(\tilde{t}_s + \Upsilon_{\eta+1})\right)^{1\{0\leq\eta<l\}} \\
& - \left(\Lambda_p\,(\tilde{t}_s + \Upsilon_\eta)\right)^{1\{0<\eta\leq l\}} \left(\Lambda_p\,(\tilde{t}_s)\right)^{1\{\eta=0\}} \Big],
\end{aligned}
$$

where we use the definition of Υ_p^s and the fact that

$$\delta_p^s(\tilde{t}_p \mid \tilde{t}_s; \theta_p^s) = \textstyle\sum_{l=0}^{K-1} 1\left\{\Upsilon_l \leq \tilde{t}_p - \tilde{t}_s < \Upsilon_{l+1}\right\} exp(\theta_{pl}^s). \text{ (19)}$$

Concerning the term $\Upsilon_p^r(\tilde{t}_p \mid \tilde{t}_s, \tilde{t}_r)$, by using the corresponding definition and also that

$$\delta_p^r(\tilde{t}_p \mid \tilde{t}_s, \tilde{t}_r; \theta_p^r) = \sum_{l=0}^{K-1} 1\{Y_l \leq \tilde{t}_p - \tilde{t}_s - \tilde{t}_r < Y_{l+1}\} exp(\theta_{pl}^r), (20)$$

we get (21)

$$\begin{aligned} \Upsilon_p^r(\tilde{t}_p \mid \tilde{t}_s, \tilde{t}_r) &= \int_{\tilde{t}_s+\tilde{t}_r}^{\tilde{t}_p} \lambda_p(w) \delta_p^s(w \mid \tilde{t}_s) \delta_p^r(w \mid \tilde{t}_s, \tilde{t}_r) dw \\ &= \sum_{l=0}^{K-1} 1\{Y_l \leq \tilde{t}_p - \tilde{t}_s - \tilde{t}_r < Y_{l+1}\} \sum_{\eta=0}^{l} exp\left(\theta_{p\eta}^r\right) \\ &\quad \times \int_{\tilde{t}_s+\tilde{t}_r+Y_\eta}^{1\{\eta=l\}\tilde{t}_p + 1\{0 \leq \eta < l\}[Y_{\eta+1}+\tilde{t}_s+\tilde{t}_r]} \lambda_p(w) \delta_p^s(w \mid \tilde{t}_s) dw. \end{aligned}$$

Recall that

$$\delta_p^s(\tilde{t}_p \mid \tilde{t}_s; \theta_p^s) = \sum_{l=0}^{K-1} 1\{Y_l \leq \tilde{t}_p - \tilde{t}_s < Y_{l+1}\} exp(\theta_{pl}^s).$$

Then, the last line in the equation of $\Upsilon_p^r(\tilde{t}_p \mid \tilde{t}_s, \tilde{t}_r)$ becomes (22)

$$\begin{aligned} &\sum_{\rho=0}^{K-1} 1\{Y_\rho \leq \tilde{t}_r + Y_\eta < Y_{\rho+1}\} \sum_{\omega=\rho}^{K-1} 1\{Y_\omega \leq 1\{\eta = l\}\tilde{t}_p + 1\{0 \leq \eta < l\}[Y_{\eta+1} + \tilde{t}_s + \tilde{t}_r] - \tilde{t}_s < Y_{\omega+1}\} \\ &\sum_{\iota=\rho}^{\omega} exp\left(\theta_{p\iota}^s\right) [[\Lambda_p(1\{\eta = l\}\tilde{t}_p + 1\{0 \leq \eta < l\}[Y_{\eta+1} + \tilde{t}_s + \tilde{t}_r])]^{1\{\iota=\omega\}} [\Lambda_p(\tilde{t}_s + Y_{\rho+1})]^{1\{\rho \leq \iota < \omega\}} \\ &\qquad - [\Lambda_p(\tilde{t}_s + Y_\iota)]^{1\{\rho < \iota \leq \omega\}} [\Lambda_p(\tilde{t}_s + \tilde{t}_r + Y_\eta)]^{1\{\iota=\rho\}}]. \end{aligned}$$

Next we present the likelihood for $\tilde{t}_s$ and for $\tilde{t}_r$ but we outline less details as they are similar to the above derivations. The likelihood for $\tilde{t}_s$ is expressed as (23):

$$
\begin{aligned}
l_s(\tilde{t}_s \mid \tilde{t}_p, x, v_s) \quad = \quad & \gamma_s 1\{\tilde{t}_s < \tilde{t}_p\}\phi_s(x)\lambda_s(\tilde{t}_s)v_s exp(-\phi_s(x)\,\Lambda_s\,(\tilde{t}_s)v_s) \\
& +\gamma_p 1\{\tilde{t}_p < \tilde{t}_s\}\big(\phi_s(x)\lambda_s(\tilde{t}_s)\delta_s^p(\tilde{t}_s \mid \tilde{t}_p)v_s\big)^{\gamma_s} \\
& \times exp\big(-\phi_s(x)\,\Lambda_s\,(\tilde{t}_p) + \Upsilon_s^p(\tilde{t}_s \mid \tilde{t}_p)v_s\big) \\
& +(1-\gamma_s)(1-\gamma_p)exp(-\phi_s(x)\,\Lambda_s\,(\tilde{t}_s)v_s)
\end{aligned}
$$

Finally, the likelihood for $\tilde{t}_r$ is given by (24)

$$
\begin{aligned}
l_r(\tilde{t}_r \mid \tilde{t}_p, \tilde{t}_s, x, v_r) \quad = \quad & \gamma_s\gamma_p 1\{\tilde{t}_p < \tilde{t}_s\}\times exp(-\phi_r(x)\,\Lambda_r\,(\tilde{t}_r)\delta_r^s(\tilde{t}_s)v_r) \\
& \times(\phi_r(x)\lambda_r(\tilde{t}_r)\delta_r^s(\tilde{t}_s)v_r)^{\gamma_r} \\
& +\gamma_s\gamma_p 1\{\tilde{t}_s < \tilde{t}_p < \tilde{t}_s + \tilde{t}_r\} \\
& \times exp(-\phi_r(x)\delta_r^s(\tilde{t}_s)\big(\Lambda_r\,(\tilde{t}_p - \tilde{t}_s) + \Upsilon_r^p(\tilde{t}_r \mid \tilde{t}_p, \tilde{t}_s)\big)v_r) \\
& \times\big(\phi_r(x)\lambda_r(\tilde{t}_r)\delta_r^p(\tilde{t}_r \mid \tilde{t}_p, \tilde{t}_s)\delta_r^s(\tilde{t}_s)v_r\big)^{\gamma_r} \\
& +\gamma_s\gamma_p 1\{\tilde{t}_s + \tilde{t}_r < \tilde{t}_p\}exp(-\phi_r(x)\delta_r^s(\tilde{t}_s)\,\Lambda_r\,(\tilde{t}_r)v_r) \\
& +\gamma_s(1-\gamma_p)\times exp(-\phi_r(x)\,\Lambda_r\,(\tilde{t}_r)\delta_r^s(\tilde{t}_s)v_r) \\
& \times(\phi_r(x)\lambda_r(\tilde{t}_r)\delta_r^s(\tilde{t}_s)v_r)^{\gamma_r} + (1-\gamma_s).
\end{aligned}
$$

3.3. Prior Settings and Model Estimation

We use a Bayesian model specification and complete the model with the following priors. We assume that

$$
\begin{aligned}
p(\alpha_j) &\propto 1 && \text{for } j = p, s, r \\
p(\beta_j) &\propto \mid \Omega \mid^{-1/2} exp(-\frac{1}{2}\beta_j\Omega^{-1}\beta_j') && \text{for } j = p, s, r
\end{aligned}
$$

where Ω is equal to an identity matrix times 100. And we assume that

$$
\begin{aligned}
p(\theta_{j1}) &\propto (1/\tau_0^2)exp(-\frac{1}{2\tau_0^2}\theta_{j1}^2) && \text{for } j = p, s, r \\
p(\theta_{j\eta} - \theta_{j(\eta-1)}) &\propto (1/\tau_1^2)exp(-\frac{1}{2\tau_1^2}(\theta_{j\eta} - \theta_{j(\eta-1)})^2) && \text{for } j = p, s, r \text{ and } \eta = 2, \ldots, K.
\end{aligned}
$$

where τ_0^2 and τ_1^2 are set equal to 100. We set no prior for Σ and we define

$$p(\vartheta) = \prod_{j} \prod_{\eta} p\,(\beta_j) p(\alpha_j) p(\theta_{j1}) p(\theta_{j\eta} - \theta_{j(\eta-1)})$$

where ϑ is a vector containing α_j for $j = s, p, r$, $\theta_{j\eta}$ for $j = s, p, r$ and $\eta = 1, \ldots, K$, β_j for $j = s, p, r$, and Σ. The posterior distribution of ϑ is given by is then

$$p(\vartheta \mid \tilde{t}_{pi}, \tilde{t}_{si}, \tilde{t}_{ri}, x_i) = p(\vartheta) \prod_{i} l\,(\tilde{t}_{pi}, \tilde{t}_{si}, \tilde{t}_{ri} \mid x_i)$$

We sample each of the elements of ϑ using a Random Walk Metropolis step within a Gibbs sampler that updates each of the elements of ϑ at a time. We use a block update for β_j and a block update for $\theta_{j\eta}$. In addition, we use an auxiliary 3×3 matrix H with each r of its row equal to $[\varepsilon_{r1}, \varepsilon_{r2}, \varepsilon_{r3}]$ and we sample each ε_{rc} from a random walk and use $H'H$ as the proposal for Σ. This specification allows us to avoid an informative prior specification for Σ, like the Wishart, but we set a normal prior with variance 100 on the diagonal elements of $H'H$. The MCMC chain ran for 30,000 iterations and we discarded the first 10,000 draws. We ran a series of formal diagnostic tests (Plummer, Best, Cowles, & Vines, 2006) and visually inspected the MCMC output to confirm convergence.

4. Model Identification

The main purpose of our modeling effort is to untangle the main sources of variation in the timing of events being these unobserved correlated heterogeneity, observed heterogeneity, and lagged-events effects. Our identification results are an extension of the results of Abbring & Van den Berg, (2003) and Honoré, (1993). Abbring & Van den Berg, (2003) prove the identification of lagged-event effects and correlated random effects in a bivariate duration model. Honoré, (1993) prove the identification of lagged duration variables in multiple spell duration models. These two just cited results are the main basis of the identification of our triple hazard model.

In a typical economic model set-up, there is need for either instrumental variables (Angrist, Imbens, & Rubin, 1996), experimental data (Shadish, Cook, & Campbell, 2002), the incorporation of endogeneity (Berry, Levinsohn, & Pakes, 1995), or exclusion restrictions (Cameron & Trivedi, 2005) for identifying causal effects. In our setup, the occurrence sequence of events works as an exclusion restriction and in addition to this we incorporate endogeneity (pure statistical correlation) between the timing of events. Thus, we are able to identify the effect of the occurrence of a past event on the timing of occurrence of a subsequent one or what we call a lagged-event effect.

Our main modeling goal is to identify the lagged-event effects between the timing of three events: the sales crash, the price crash, and the sales recovery. We illustrate these lagged-effects in Figure 4 Lagged-Event Effects. The arrows connecting the events represent the lagged-event effects. As it can be seen immediately, the arrows in the diagram connect all events and at first hand the system seems completely endogenous and hence impossible to identify. However, the three events occur in different sequences and due to this sequential nature of events we can identify all events. For illustration purposes, we have marked with the letters A and B the lagged-event effects of the price crash on the sales crash and of the sales crash on the price crash, respectively. Whenever the sales crash occurs before the price crash, then the arrow marked with A cancels out of the system and hence we can identify how the variation in the sales crash affects the timing of occurrence of the price crash, effect represented via the arrow B. In a similar way, the effect A can be identified only when the price crash occurs before the sales crash, that is whenever the price crash is the lagged-event. Hence, one main source of identification in the data comes from the sequence in which events occur and this identification strategy is similar to the exclusion restrictions in models of simultaneous equations. The second source of identification is due to the incorporation of correlated unobserved heterogeneity. In other words, we control for statistical correlation between the timing of events and that allow us to identify the lagged-event effects.

Our identification results may be limited by the existence of price expectations or the anticipation of the timing of price cuts. However, our data shows that the timing of price crashes vary widely. Although this evidence is merely descriptive, it seems that on aggregate, the industry is randomizing the timing of its price discounts. Such randomization might prevent consumers and competitors from anticipating the timing of the price cuts (Baye, Morgan, & Scholten, 2006; Varian, 1980). Finally, it has been argued by (Abbring & Van den Berg, 2003, 2003) that the identification results hold as long as there is no perfect anticipation of policy changes.

4.1. The role of price expectations

In this section we discuss how expectations, particularly price expectations, could affect our results.

It has been argued that consumers in the video game industry are forward looking. Previous research, like Nair (2007), has taken into account such type of consumers for deriving the optimal pricing policies for video games. Moreover, Van Heerde, Dekimpe, & Putsis Jr. (2005) and Franses (2005) argue that ignoring the anticipation of policy changes (in our case price discounts) might lead to important biases in models' parameters. We agree with the common-sense view that most consumers are aware of the declining pricing patterns and of the deep price discounts offered by video game retailers. However, Moreover, the video game industry is launching close to two video games per day. Hence it is very costly to keep track of product launch dates and price cut timings for such a large number of products. That is, consumers would have a hard time developing such deep knowledge for anticipating the timing of price cuts unless consumers have rich data like ours. However, price anticipation is not strong, we acknowledge that this as a limitation of our model.

Nonetheless, randomization of price discounts in time cannot discourage consumers from expecting a price cut and from delaying their purchases until the time when their utility for a game is higher than its market price. That is, even when anticipation of price changes is hard, consumers could have heterogeneous preferences regarding price and timing of purchase. That is, a fraction of the total consumers may purchase a video game right after or only after a price crash. This kind of behavior can be captured by two elements of our model. The first element is the unobserved heterogeneity terms and the second is the lagged-event effects of the price crash on the timing of the sales crash and the sales recovery. If the price crash motivates further video game sales, then this price effect should decrease the hazard of a sales crash and it should increase the hazard rate of a sales recovery. This latter scenario can be fully captured by our model set up.

In summary, we believe that our model is suitable for the data and market being composed of enough elements to be robust to heterogeneous price preferences and expectations. We will compare our model to several other models, which are special cases of our specification, and report their fit statistics. The alternative models are described next.

5. Alternative Models

Our proportional triple hazard model consists of the following components: i). Expo-power baseline hazards functions, ii). Regressor functions, iii). Lagged-event functions, and iv). Correlated random effects. We compare our model against six different reduced versions of our general formulation. Each of these reduced models leave out some of the functions just listed.

The original formulation of the model does not allow for the unobserved heterogeneity to be zero so the second alternative specification (Model 2) leaves out the fourth component of the model; that is the correlated random effects are left out. The third alternative specification (Model 3) is composed of the same elements as the original formulation with the exception that all the lagged-event effects are left out. The fourth model specification (Model 4) assumes a Gompertz baseline hazard instead of an Expo-power specification. The fifth, six, and seventh alternative specifications (Model 5, 6, and 7) leave out the lagged-effects and all these latter models assume no correlation between their random effects. That is, these last three alternative models are simply single proportional hazard models with log-normal unobserved heterogeneity usually referred to in the literature as MPH (Mixed Proportional Hazard) models.

We compare our model to these seven alternative specifications using the Deviance Information Criterion (or DIC) proposed by Spiegelhalter et al. (2002). The DIC criterion is a common measure for comparing Bayesian model fit and it is defined as follows. The Bayesian deviance $D(\theta)$ is defined as

$$D(\vartheta) = -2\log\left(f(y|\vartheta)\right) + c$$

where $f(y|\vartheta)$ is the density of the data y given the model parameters θ and c is a normalizing constant. Spiegelhalter et al. (2002) define $DIC = D(\bar{\vartheta}) + 2p_D$ where $p_D = \overline{D(\vartheta)} - D(\bar{\vartheta})$ and $\bar{\vartheta}$ is the posterior mean of the parameters ϑ and the $\overline{D(\vartheta)}$ is the posterior mean of the Bayesan deviance $D(\vartheta)$.

6. Results

Our data reveals that the price crash occurs before the sales crash 73% of the time while it occurs after the sales crash 27% of the time (see Table 1). The price crash occurs before the sales recovery 90% of the time whereas it occurs after the recovery 10% of the time. The sales recovery is unobserved for 64% of the products in the sample and it is observed for the remaining 36%. In Table 2 we present the average depth of the price and sales crashes identified in our data. The overall depth of the sales crash is close to 60% while the depth of the price crash is close to 23%. Both the timing of the sales crash and the price crash seem to vary noticeably across firms, genres, and seasons.

In Table 3 we present the estimates of the baseline hazards. All coefficients are significantly different from zero with the exception of the γ_r coefficient in the sales recovery hazard. This implies that the baseline hazard of the sales recovery reduce to a Weibull specification whereas the baseline hazards of the sales crash and the price crash keep their Expo-power function. Recall that the Expo-power hazard reduces to a Gompertz when $\alpha = 1$ or to a

Weibull when $\gamma = 0$. These parameter estimates imply that the baseline hazard of the sales crash shows an inverted U-shape with a peak at the 9th month after introduction. The baseline hazard of the price crash shows a decreasing shape and it reaches zero by the second year after introduction. Finally, the baseline hazard of the sales recovery is initially high and that it decays at a mild rate while staying relatively constant after the tenth month.

Table 4 presents the four sets of lagged-event effects: i) The lagged-event effect (θ_s^p) of the price crash on the sales crash (upper panel), ii) the lagged-event effect (θ_r^p) of the price crash on the sales recovery (second panel from the top), iii) the lagged-event effects (θ_p^s) of the sales crash on the price crash (third panel from the top), and iv) the lagged-event effects (θ_p^r) of the sales recovery on the price crash (bottom panel). As we can see in the upper panel of Table 4, the signs of the lagged-event effects of the price crash on the sales crash are significantly negative. This result implies that the realization of a price crash significantly decreases the hazard rate of the sales crash at all subsequent periods following the price crash. In contrast, in the third panel, we notice that the realization of a sales crash increases the hazard rate of the price crash at all following periods after the sales crash occurrence. Notice how the effect between the sales crash and the price crash are asymmetric and that the stronger effect goes from the sales crash to the price crash. In Figure 5 we plot the four sets of lagged-event effects and their time variation. In the upper right panel, we notice how the realization of a price crash increases the hazard rate of a sales recovery after the third month that follows the price crash and it stays relatively constant (around 1.3) up to the twelfth month after the price crash. The reverse effect, that is the effect of a sales recovery on the price crash, is not significantly different from zero for most periods; notice how the confidence bounds for most of these latter coefficients contain zero.

In Table 5 we present the coefficients of the regressor functions. Our results indicate that there is substantial heterogeneity in the timing of the sales crash, the price crash, and the sales recovery associated to seasons, platforms, products' genres, publishers, and quality. Quality has a significant -1.208 coefficient in the sales crash regressor function and this implies that the hazard rate of a sales crash decreases as quality increases. In the same fashion, an increase in quality decreases the hazard rate of a price crash, see the -0.863 and -0.205 significant coefficients in the price crash regressor function. A higher quality also predicts a decrease in the hazard rate of the sales recovery given the significant -0.929 and -0.309 coefficients for quality in the regressor function of the sales recovery. A negative effect of quality on the sales recovery is surprising because it implies that high-quality products experience delayed sales recoveries. However, other researches (e.g., Garber, Jacob, Libai, & Muller, 2004), have found that the sales of successful products (which may be of high quality) may fall faster than less successful ones. That is, high quality not always results in positive sales trajectories.

Seasons are also significant. For example, the price crash is more likely during the shopping seasons of December but also in October, September, July and April than during all other months (see the significant coefficients of these months in the middle column of Table 5. Finally, in Table 6 we report the covariance matrix of the distribution of the random effects. Our results indicate that there is a positive dependence among the random effects and this implies that an unobserved shock would evenly delay or advance all events.

6.1. Model Fit

In Table 7 we present a comparison of model in-sample fit across the main models and all the alternative specifications described earlier. The main objective of this section is to evaluate our main model against several alternatives. The purpose of this comparison is to evaluate whether we need the main model and its elements or a reduced model that leaves out some of the main model's elements. In summary, our main triple hazard models fit the data better than any of the alternative specifications that we tried and better than the single-hazard models. As we can see in Table 7, our main specification achieved the lowest DIC criteria among all models. The model fits deteriorates when either the unobserved heterogeneity is left out (Model 2) or when we leave out all the lagged-event effects from our main specification (Model 3). The fit also deteriorates when we assume a Gompertz baseline hazard (Model 4) instead of the Expo-power that is assumed in our main specification. The triple hazard models fit the data substantially better than the single-hazard models. In brief, our main triple hazard model fits the data better and this evidence gives support to our main specification.

7. Discussion

This paper proposes a new causal triple hazard model useful for analyzing the inter-dependence among events. The model can statistically identify three sources of variation in the timing of events that may arise from i) observed covariates, ii) unobserved correlated covariates, and iii) lagged dependence between events. It also applies the model to the video game industry and obtains many novel findings. Hence, we contribute to the marketing literature by proposing a new triple hazard model and obtaining new findings in one industry. This section summarizes the main findings, presents managerial implications, and lists limitations and topics for further research.

7.1. Summary of Findings

- Results suggest that the price crash involves a 23% drop in the introductory price. The sales crash amounts to 60% drop in peak introductory unit sales. The price crash occurs before the sales crash 73% of the time while it occurs after the sales crash 27% of the time.
- The occurrence of price crash significantly lowers the hazard rate of a sales crash whereas the occurrence of a sales crash significantly increases the hazard of a price crash. The latter effect is stronger than the former.
- The sales crash is the main determining factor that significantly increases the hazard rate of price crashes given that the baseline hazard of the latter is decreasing in time.
- The price crash significantly increases the hazard rate of a sales recovery whereas the sales recovery has a positive but insignificant effect on the occurrence of a price crash. Products that face a late sales crash are less likely to face a sales recovery.
- The results are valid after controlling for observed and unobserved heterogeneity and endogeneity. The observed heterogeneity is significantly driven by seasons, quality, platforms, publishers and genres. Unobserved heterogeneity delays or advances evenly all events in time.

7.2. Managerial Implications

Our data and empirical analyzes seem to indicate that firms will inevitably face sales crashes and most of the time firms seem to wait for the crash before cutting prices. Surprisingly, we find that a sales crash is not completely inevitable and that managers may benefit from analyzing when to cut prices in order to delay such dramatic sales drop. Products like mobile phones, fiction novels, and designer's clothes usually sell high at introduction or during their relevant season and a sales crash will inevitably follow the high-season. Thus, managers in the information goods, fashion, and high-tech industries may benefit from analyzing whether such dramatic sales drop can be avoided or at least delayed by modifying their marketing strategies, especially pricing. Moreover, managers of new products could use our method to analyze whether the co-occurrence of sales and price drops are statistically correlated or lagged dependent. For example, the model can provide managers with new insights about whether the purchase timing of multiple durable goods, the click-stream across multiple websites, or the adoption timing of multiple technologies may trigger subsequent purchases or website visits.

7.3. Limitations and Further Research

Our empirical investigation should encourage further research due to its limitations. First, except for the time-varying causal effects among events, we do not control for any time-varying covariates of the marketing mix like advertising or promotion. Hence, we believe that a hazard model incorporating time-varying covariates would represent an important but challenging development. Second, we use aggregate data and we do not model individual level behavior. Therefore, we consider that the modeling of individual level behavior of consumers and firms facing price-sales crashes is an interesting avenue for further research. Finally, we do not model competitive expectations and strategies. . However, market participants may be dynamically updating their expectations and strategies every time when new information becomes available. Therefore, we consider that the incorporation of anticipation of the timing of policy changes into hazard models is a very interesting avenue for further research.

8. Tables and Figures

Sequence of Events	Frequency
Price Crash Before Sales Crash	26.7%
Price Crash After Sales Crash	73.3%
Sales Crash but Not Recovery	63.8%
Sales Crash and Sales Recovery	36.2%
Price Crash Before Sales Recovery	90.0%
Price Crash After Sales Recovery	10.0%

Table 1 Timing of Events

	Price Crash Timing	Sales Crash Timing	% Depth of Price Crash	% Depth of Sales Crash
Average by Firm				
ACCLAIM	9.45	3.61	-22.25%	-58.57%
ACTIVISION	7.35	3.77	-24.39%	-61.70%
CAPCOM	7.09	4.27	-25.08%	-59.64%
EIDOS INTERACTIVE	5.61	4.44	-22.24%	-63.77%
ELECTRONIC ARTS	5.82	4.63	-26.71%	-66.15%
HASBRO	7.47	4.04	-25.38%	-66.19%
INFOGRAMES	14.65	4.09	-21.27%	-60.55%
INTERPLAY	13.18	3.37	-20.04%	-65.36%
KONAMI	7.18	3.51	-28.02%	-61.96%
MIDWAY	5.33	4.62	-23.83%	-58.43%
NAMCO	6.91	4.17	-23.11%	-57.18%
NINTENDO	7.67	4.13	-22.56%	-63.70%
SEGA	5.18	4.38	-24.65%	-57.20%
SONY	6.82	3.95	-21.99%	-58.16%
THQ	7.22	3.93	-22.09%	-62.90%
Average by Genre				
ACTION	7.25	3.84	-24.28%	-62.69%
ADVENTURE	9.50	3.20	-22.84%	-55.98%
DRIVING	6.06	4.52	-23.79%	-57.56%
FAMILY	8.95	4.32	-26.29%	-55.11%
FIGHTING	7.84	4.60	-22.66%	-64.01%
SHOOTER	6.92	4.49	-23.02%	-63.83%
SIMULATIONS	7.78	3.69	-24.25%	-58.52%
SPORTS	6.93	4.10	-23.50%	-63.04%
STRATEGY	8.31	4.74	-24.08%	-61.06%
Average by Year				
1995	8.59	4.00	-24.93%	-60.00%
1996	6.86	4.47	-25.06%	-59.59%
1997	7.20	4.04	-23.15%	-61.42%
1998	7.69	3.80	-23.65%	-63.67%
1999	6.89	3.94	-23.81%	-64.01%
2000	7.84	4.12	-23.54%	-59.08%
2001	9.13	4.29	-12.89%	-40.38%

Table 2 Descriptive Statistics of the Timing of the Sales-Price Crash

	Posterior Mean	95% HPDR	
Sales Crash			
α_s	1.886 **	1.800	1.965
γ_s	-0.009 **	-0.012	-0.005
Price Crash			
α_p	0.908 **	0.841	0.968
γ_p	-0.242 **	-0.284	-0.211
Sales Recovery			
α_r	0.886 **	0.776	0.996
γ_r	-0.009	-0.028	0.026

Table 3 Coefficients of Baseline Hazard Functions

Price Crash -> Sales Crash	θ_s^p	95% HPDR
θ_{s1}^p Months 1-2	-0.409 **	(-0.563,-0.253)
θ_{s2}^p Months 3-4	-0.519 **	(-0.740,-0.319)
θ_{s3}^p Months 5-6	-0.947 **	(-1.275,-0.644)
θ_{s4}^p Months 7-8	-1.270 **	(-1.813,-0.852)
θ_{s5}^p Months 9-10	-0.728 **	(-1.224,-0.220)
θ_{s6}^p Months 11-12	-0.891 **	(-1.675,-0.172)
θ_{s7}^p Months 13 - ∞	-0.090	(-0.843,0.545)
Price Crash -> Sales Recovery	θ_r^p	**95% HPDR**
θ_{r1}^p Months 1-2	-0.330 **	(-0.607,-0.062)
θ_{r2}^p Months 3-4	0.556 **	(0.346,0.768)
θ_{r3}^p Months 5-6	0.764 **	(0.546,1.006)
θ_{r4}^p Months 7-8	0.744 **	(0.468,1.010)
θ_{r5}^p Months 9-10	0.670 **	(0.357,0.966)
θ_{r6}^p Months 11-12	0.921 **	(0.593,1.238)
θ_{r7}^p Months 13 - ∞	-0.756 **	(-1.119,-0.396)
Sales Crash -> Price Crash	θ_p^s	**95% HPDR**
θ_{p1}^s Month 1-2	1.112 **	(0.951,1.268)
θ_{p2}^s Month 3-4	1.810 **	(1.634,1.974)
θ_{p3}^s Month 5-6	2.249 **	(2.019,2.465)
θ_{p4}^s Month 7-8	2.806 **	(2.525,3.064)
θ_{p5}^s Month 9-10	2.840 **	(2.474,3.168)
θ_{p6}^s Month 11-12	3.088 **	(2.637,3.553)
θ_{p7}^s Months 13 - ∞	3.348 **	(2.802,3.900)
Sales Recovery -> Price Crash	θ_p^r	**95% HPDR**
θ_{p1}^r Month 1-2	-0.088	(-0.403,0.231)
θ_{p2}^r Month 3-4	-0.094	(-0.558,0.268)
θ_{p3}^r Month 5-6	0.526 **	(0.077,0.887)
θ_{p4}^r Month 7-8	0.838 **	(0.304,1.294)
θ_{p5}^r Month 9-10	0.358	(-0.482,1.022)
θ_{p6}^r Month 11-12	0.234	(-1.284,1.247)
θ_{p7}^r Months 13 - ∞	1.045 **	(0.050,1.864)

Table 4 Lagged-event Effects

Coefficient	β_s	β_p	β_r
Intercept	0.000	0.000	0.000
Platform			
N64	0.012	-0.059	-0.172 **
SATURN	-0.143 **	-0.158 **	0.064
SATURN SONYPS1	-0.198 **	-0.005	-0.085
SONYPS1 N64	-0.139 **	-0.005	-0.135 *
Genre			
ACTION	-0.187 **	-0.086	-0.118 **
ADVENTURE	-0.075	-0.165 **	-0.093
DRIVING	-0.299 **	-0.054	-0.017
FAMILY	-0.293 **	-0.095	-0.017
FIGHTING	-0.110 *	-0.210 **	-0.176 **
SHOOTER	-0.321 **	-0.051	-0.050
SIMULATIONS	-0.005	-0.299 **	-0.268 **
STRATEGY	-0.158 **	-0.119	-0.032
Publisher			
ACCLAIM	-0.356 **	-0.059	-0.086
ACTIVISION	-0.070	0.067	0.086
CAPCOM	0.036	-0.095	-0.010
EIDOS INTERACTIVE	-0.182 **	0.042	-0.006
ELECTRONIC ARTS	-0.078	-0.067	-0.045
HASBRO	-0.122	-0.012	-0.035
INFOGRAMES	-0.221 **	0.031	0.026
INTERPLAY	-0.103	0.001	0.019
KONAMI	-0.136	-0.117	-0.184 *
MIDWAY	-0.114 *	-0.147 **	-0.156 **
NAMCO	-0.038	-0.241 **	-0.068
NINTENDO	0.072	-0.256 **	0.017
SEGA	-0.035	-0.034	-0.024
Small Publishers 1	-0.325 **	-0.225 **	-0.169 **
Small Publishers 2	-0.170 **	-0.039	-0.103
Small Publishers 3	-0.233 **	-0.128	-0.158
THQ	-0.090	-0.037	-0.105
Launch Year			
1996	-0.486 **	-0.184 **	-0.090
1996	-0.368 **	-0.166 **	-0.179 **
1996	-0.263 **	-0.174 **	-0.324 **
1996	-0.325 **	-0.087	-0.272 **
1996	-0.472 **	-0.356 **	-0.320 **
1996	-0.222 **	-0.264 **	-0.064
Launch Month			
February	-0.098	-0.207 **	-0.413 **
March	-0.196 **	-0.169 *	0.038
April	-0.089	-0.024	0.012
May	-0.216 **	-0.160 **	0.059
June	-0.050	-0.165 **	-0.570 **
July	-0.299 **	-0.116	-0.040
August	-0.152 **	-0.188 **	-0.075
September	-0.263 **	-0.057	-0.203 **
October	-0.472 **	-0.107	-0.208 **
November	-0.319 **	-0.182 **	-0.047
December	-0.213 **	-0.060	0.006
Quality			
Quality	-1.208 **	-0.856 **	-0.832 **
Quality^2	0.071	-0.212 **	-0.244 **
Lagged Duration			
t_p	--	--	--
t_s	--	--	-0.215 **
t_r	--	--	--

Table 5 Coefficients of Regressor Functions

Covariance Matrix of Random Effects			
	v_s	v_p	v_r
v_s	0.0240 **	0.0265 *	0.0193 *
v_p		0.1569 **	0.1088 **
v_r			0.0794 **
Correlation Matrix of Random Effects			
	v_s	v_p	v_r
v_s	1	0.2631 *	0.3168
v_p		1	0.9792 **
v_r			1

Table 6 Random Effects Covariance Matrix

	Triple Hazard Models				Single Hazard Models		
Fit Criteria	**Model 1 (t_p,t_s,t_r)**	**Model 2 (t_p,t_s,t_r)**	**Model 3 (t_p,t_s,t_r)**	**Model 4 (t_p,t_s,t_r)**	**Model 5 (t_p)**	**Model 6 (t_s)**	**Model 7 (t_r)**
$D(\bar{\vartheta})$	23636.4	23814.8	24602.6	24231.2	8671.0	16938.5	10263.8
$\overline{D(\vartheta)}$	23736.9	24045.7	24672.1	24328.4	8708.8	18319.1	10208.9
$\overline{\log[f(y\|\vartheta)]}$	-11868.5	-12022.9	-12336.0	-12164.2	-4354.4	-9159.6	-5104.5
$\log[f(y\|\bar{\vartheta})]$	-11818.2	-11907.4	-12301.3	-12115.6	-4326.6	-8469.2	-5131.9
DIC	23837.5	24276.6	24741.5	24425.6	8746.7	19699.7	10154.1
DIC[a]	5.09	5.18	5.28	5.21	5.60	12.61	6.50

Notes: Model 1 is our main model with all its components. Model 2 is the same as the main model but it assumes that unobserved heterogeneity is absent. Model 3 is the same as the main model but it leaves out all lagged-event effects. Model 4 is equal to Model 1 except that it assumes a Gompertz baseline hazard. Model 5, 6, and 7 are single proportional hazard models with log-normal unobserved heterogeneity and Expo-power baseline hazards. DIC represents the Deviance Information Criteria. DIC[a] is the DIC divided by the number of observations so that comparison across triple hazard and single hazard models is facilitated. $f(y|\theta)$ represents the likelihood and the bar above a expression represents the posterior mean of the expression.

Table 7 Model Fit Comparison

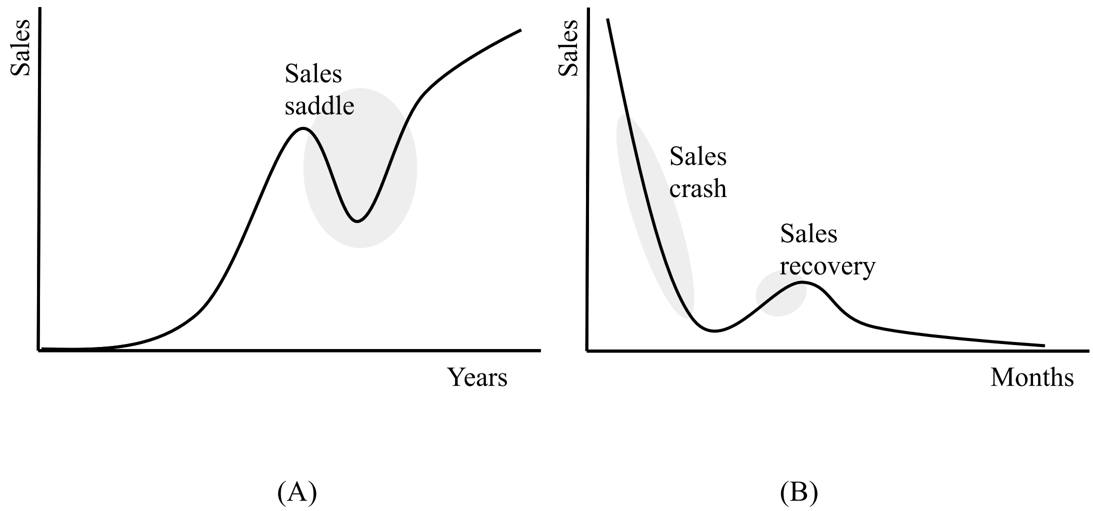

Figure 1 Sales Saddle vs. Sales Crash

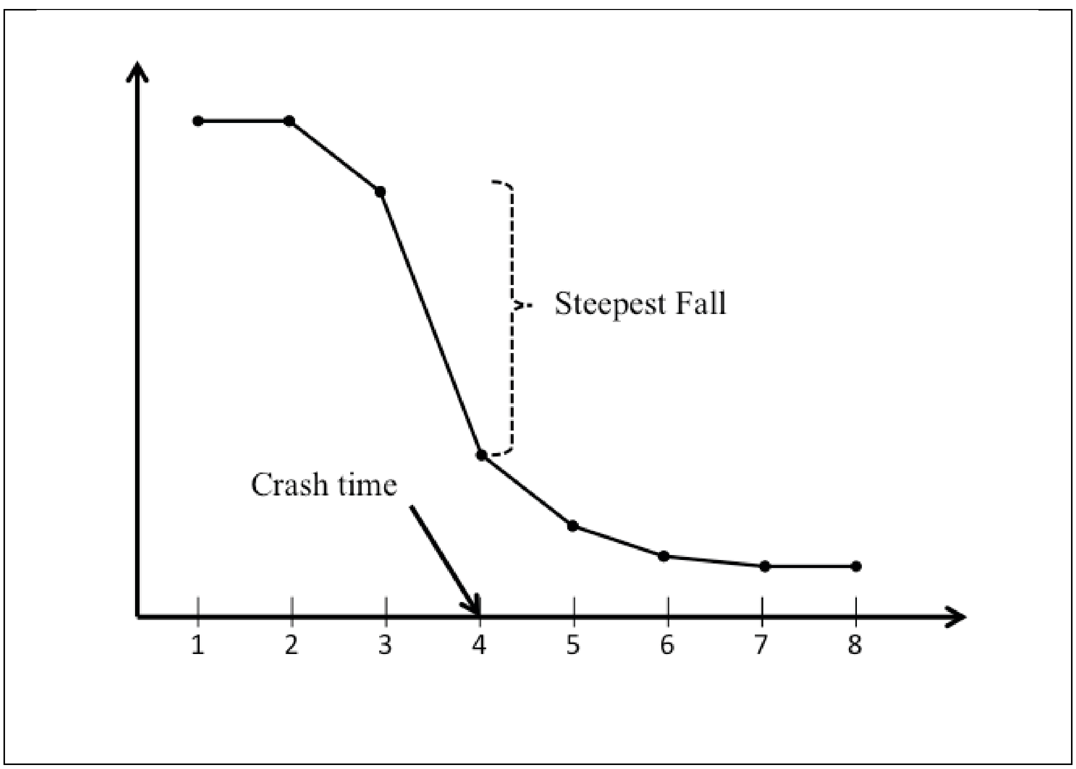

Figure 2 Defining Sales and Price Crashes

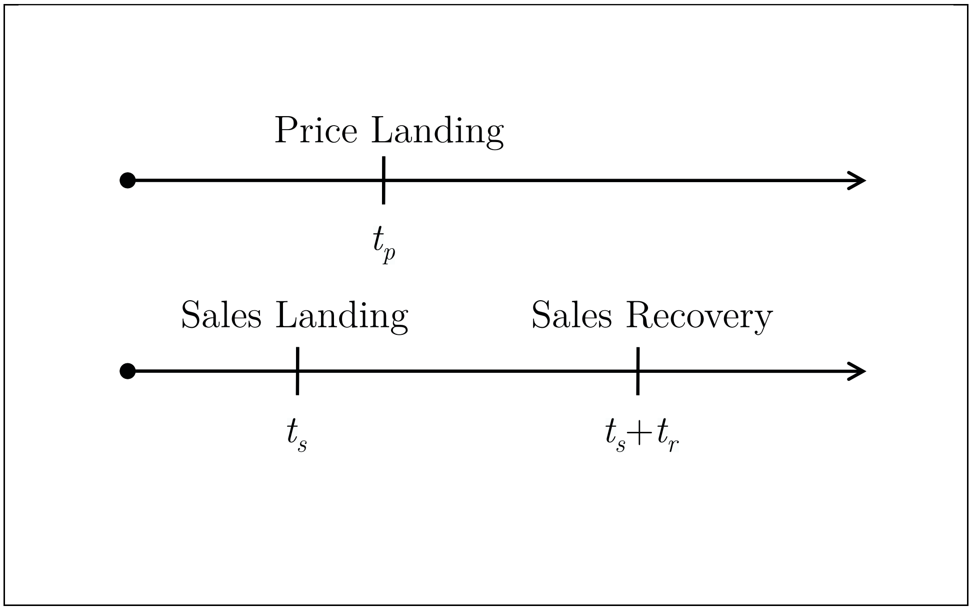

Figure 3 Timelines of Events

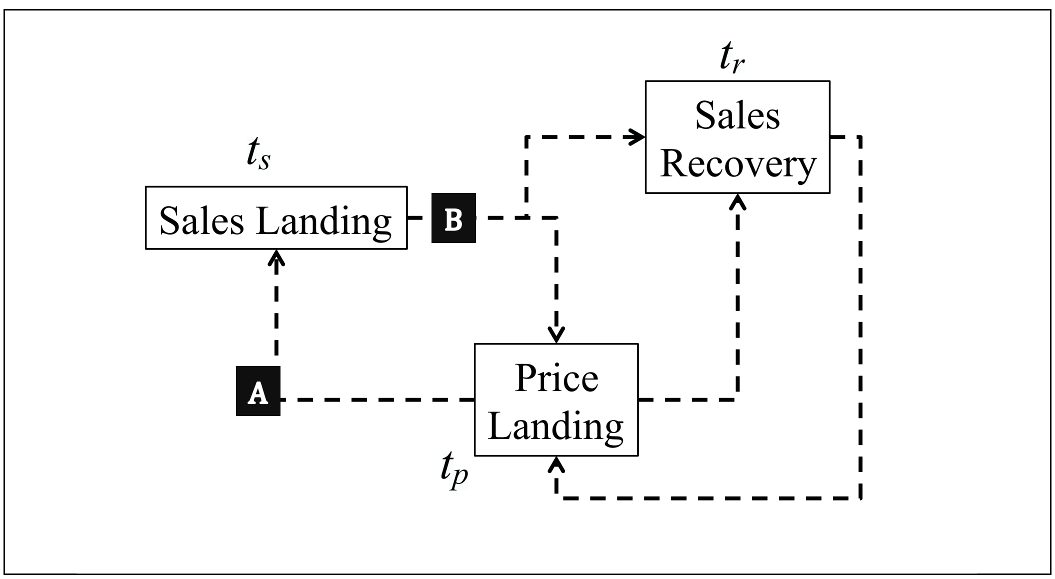

Figure 4 Lagged-Event Effects

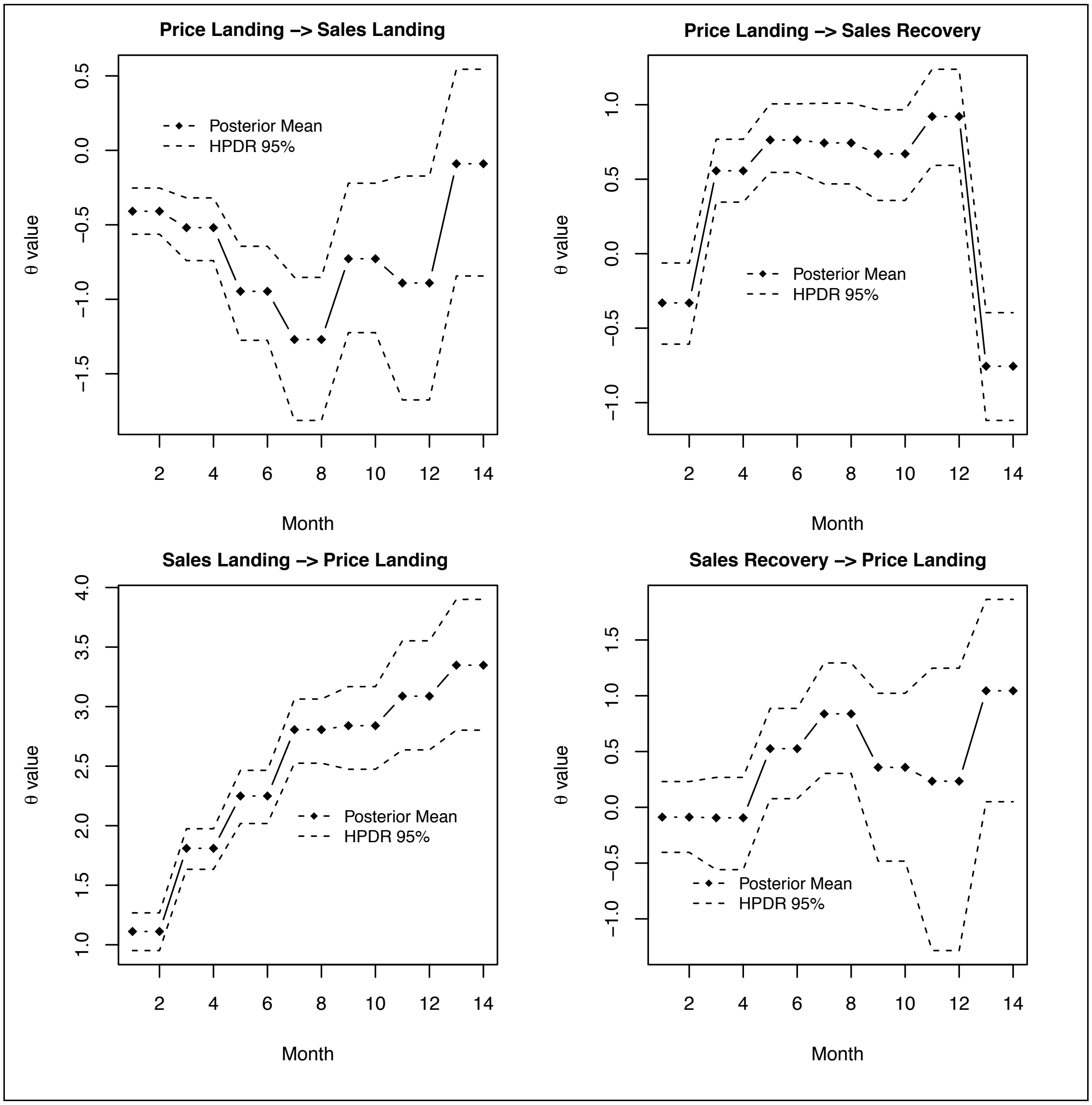

Figure 5 Causal Effects Coefficients

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