t03_volumenes_r6

April 2, 2022

1 Monte Carlo - Unidad 2, Sesion 3 - Ejercicio

[**Problema**]: se desea estimar el volumen de una region R de [0,1]^6 definida por todos los puntos de la hiper-esfera de centro (0.45, 0.5, 0.6, 0.6, 0.5, 0.45) y radio 0.35 que ademas cumplen con las restricciones 3x1 + 7x4 <= 5; x3+x4 <= 1; x1-x2-x5+x6 >= 0

1.1 Entrega 2 - Ejercicio 3.1

1.1.1 Parte a:

[**Letra**] Implementar un programa que reciba como parametro la cantidad de replicaciones n a realizar, y emplee Monte Carlo para calcular (e imprimir) la estimaci´on del volumen de R, y la desviaci´on estandar de este estimador. Incluir codigo para calcular el tiempo de calculo empleado por el programa. Utilizar el programa con n = 104 y luego con n = 106 para estimar el volumen de R. Discutir si los dos valores obtenidos parecen consistentes. (en la sesi´on 5 se continuar´a este ejercicio).

[19]:
```
# Instalar dependencias
!pip install -r requirements.txt
```

```
Defaulting to user installation because normal site-packages is not writeable
Requirement already satisfied: tabulate in
/home/carlos/.local/lib/python3.10/site-packages (from -r requirements.txt (line
1)) (0.8.9)
Requirement already satisfied: scipy in /home/carlos/.local/lib/python3.10/site-
packages (from -r requirements.txt (line 2)) (1.8.0)
Requirement already satisfied: notebook in
/home/carlos/.local/lib/python3.10/site-packages (from -r requirements.txt (line
3)) (6.4.10)
Requirement already satisfied: matplotlib in
/home/carlos/.local/lib/python3.10/site-packages (from -r requirements.txt (line
4)) (3.5.1)
Requirement already satisfied: numpy<1.25.0,>=1.17.3 in
/usr/lib/python3.10/site-packages (from scipy->-r requirements.txt (line 2))
(1.22.2)
Requirement already satisfied: jinja2 in
/home/carlos/.local/lib/python3.10/site-packages (from notebook->-r
requirements.txt (line 3)) (3.1.1)
Requirement already satisfied: nest-asyncio>=1.5 in
```

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/home/carlos/.local/lib/python3.10/site-packages (from notebook->-r
requirements.txt (line 3)) (1.5.5)
Requirement already satisfied: argon2-cffi in
/home/carlos/.local/lib/python3.10/site-packages (from notebook->-r
requirements.txt (line 3)) (21.3.0)
Requirement already satisfied: pyzmq>=17 in
/home/carlos/.local/lib/python3.10/site-packages (from notebook->-r
requirements.txt (line 3)) (22.3.0)
Requirement already satisfied: prometheus-client in
/home/carlos/.local/lib/python3.10/site-packages (from notebook->-r
requirements.txt (line 3)) (0.13.1)
Requirement already satisfied: ipykernel in
/home/carlos/.local/lib/python3.10/site-packages (from notebook->-r
requirements.txt (line 3)) (6.11.0)
Requirement already satisfied: jupyter-client>=5.3.4 in
/home/carlos/.local/lib/python3.10/site-packages (from notebook->-r
requirements.txt (line 3)) (7.2.1)
Requirement already satisfied: nbconvert>=5 in
/home/carlos/.local/lib/python3.10/site-packages (from notebook->-r
requirements.txt (line 3)) (6.4.5)
Requirement already satisfied: Send2Trash>=1.8.0 in
/home/carlos/.local/lib/python3.10/site-packages (from notebook->-r
requirements.txt (line 3)) (1.8.0)
Requirement already satisfied: nbformat in
/home/carlos/.local/lib/python3.10/site-packages (from notebook->-r
requirements.txt (line 3)) (5.2.0)
Requirement already satisfied: jupyter-core>=4.6.1 in
/home/carlos/.local/lib/python3.10/site-packages (from notebook->-r
requirements.txt (line 3)) (4.9.2)
Requirement already satisfied: traitlets>=4.2.1 in
/home/carlos/.local/lib/python3.10/site-packages (from notebook->-r
requirements.txt (line 3)) (5.1.1)
Requirement already satisfied: tornado>=6.1 in
/home/carlos/.local/lib/python3.10/site-packages (from notebook->-r
requirements.txt (line 3)) (6.1)
Requirement already satisfied: terminado>=0.8.3 in
/home/carlos/.local/lib/python3.10/site-packages (from notebook->-r
requirements.txt (line 3)) (0.13.3)
Requirement already satisfied: ipython-genutils in
/home/carlos/.local/lib/python3.10/site-packages (from notebook->-r
requirements.txt (line 3)) (0.2.0)
Requirement already satisfied: pillow>=6.2.0 in /usr/lib/python3.10/site-
packages (from matplotlib->-r requirements.txt (line 4)) (9.0.1)
Requirement already satisfied: cycler>=0.10 in
/home/carlos/.local/lib/python3.10/site-packages (from matplotlib->-r
requirements.txt (line 4)) (0.11.0)
Requirement already satisfied: python-dateutil>=2.7 in
/home/carlos/.local/lib/python3.10/site-packages (from matplotlib->-r
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```
requirements.txt (line 4)) (2.8.2)
Requirement already satisfied: pyparsing>=2.2.1 in /usr/lib/python3.10/site-
packages (from matplotlib->-r requirements.txt (line 4)) (3.0.0)
Requirement already satisfied: packaging>=20.0 in /usr/lib/python3.10/site-
packages (from matplotlib->-r requirements.txt (line 4)) (20.9)
Requirement already satisfied: fonttools>=4.22.0 in
/home/carlos/.local/lib/python3.10/site-packages (from matplotlib->-r
requirements.txt (line 4)) (4.31.2)
Requirement already satisfied: kiwisolver>=1.0.1 in
/home/carlos/.local/lib/python3.10/site-packages (from matplotlib->-r
requirements.txt (line 4)) (1.4.2)
Requirement already satisfied: entrypoints in /usr/lib/python3.10/site-packages
(from jupyter-client>=5.3.4->notebook->-r requirements.txt (line 3)) (0.4)
Requirement already satisfied: jupyterlab-pygments in
/home/carlos/.local/lib/python3.10/site-packages (from
nbconvert>=5->notebook->-r requirements.txt (line 3)) (0.1.2)
Requirement already satisfied: MarkupSafe>=2.0 in
/home/carlos/.local/lib/python3.10/site-packages (from
nbconvert>=5->notebook->-r requirements.txt (line 3)) (2.1.1)
Requirement already satisfied: nbclient<0.6.0,>=0.5.0 in
/home/carlos/.local/lib/python3.10/site-packages (from
nbconvert>=5->notebook->-r requirements.txt (line 3)) (0.5.13)
Requirement already satisfied: testpath in
/home/carlos/.local/lib/python3.10/site-packages (from
nbconvert>=5->notebook->-r requirements.txt (line 3)) (0.6.0)
Requirement already satisfied: beautifulsoup4 in /usr/lib/python3.10/site-
packages (from nbconvert>=5->notebook->-r requirements.txt (line 3)) (4.10.0)
Requirement already satisfied: pygments>=2.4.1 in
/home/carlos/.local/lib/python3.10/site-packages (from
nbconvert>=5->notebook->-r requirements.txt (line 3)) (2.11.2)
Requirement already satisfied: defusedxml in
/home/carlos/.local/lib/python3.10/site-packages (from
nbconvert>=5->notebook->-r requirements.txt (line 3)) (0.7.1)
Requirement already satisfied: bleach in
/home/carlos/.local/lib/python3.10/site-packages (from
nbconvert>=5->notebook->-r requirements.txt (line 3)) (4.1.0)
Requirement already satisfied: pandocfilters>=1.4.1 in
/home/carlos/.local/lib/python3.10/site-packages (from
nbconvert>=5->notebook->-r requirements.txt (line 3)) (1.5.0)
Requirement already satisfied: mistune<2,>=0.8.1 in
/home/carlos/.local/lib/python3.10/site-packages (from
nbconvert>=5->notebook->-r requirements.txt (line 3)) (0.8.4)
Requirement already satisfied: jsonschema!=2.5.0,>=2.4 in
/usr/lib/python3.10/site-packages (from nbformat->notebook->-r requirements.txt
(line 3)) (3.2.0)
Requirement already satisfied: six>=1.5 in /usr/lib/python3.10/site-packages
(from python-dateutil>=2.7->matplotlib->-r requirements.txt (line 4)) (1.16.0)
Requirement already satisfied: ptyprocess in /usr/lib/python3.10/site-packages
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```
(from terminado>=0.8.3->notebook->-r requirements.txt (line 3)) (0.7.0)
Requirement already satisfied: argon2-cffi-bindings in
/home/carlos/.local/lib/python3.10/site-packages (from argon2-cffi->notebook->-r
requirements.txt (line 3)) (21.2.0)
Requirement already satisfied: psutil in /usr/lib/python3.10/site-packages (from
ipykernel->notebook->-r requirements.txt (line 3)) (5.9.0)
Requirement already satisfied: matplotlib-inline>=0.1 in
/home/carlos/.local/lib/python3.10/site-packages (from ipykernel->notebook->-r
requirements.txt (line 3)) (0.1.3)
Requirement already satisfied: setuptools>=60 in
/home/carlos/.local/lib/python3.10/site-packages (from ipykernel->notebook->-r
requirements.txt (line 3)) (61.3.1)
Requirement already satisfied: debugpy>=1.0 in
/home/carlos/.local/lib/python3.10/site-packages (from ipykernel->notebook->-r
requirements.txt (line 3)) (1.6.0)
Requirement already satisfied: ipython>=7.23.1 in
/home/carlos/.local/lib/python3.10/site-packages (from ipykernel->notebook->-r
requirements.txt (line 3)) (8.2.0)
Requirement already satisfied: pexpect>4.3 in /usr/lib/python3.10/site-packages
(from ipython>=7.23.1->ipykernel->notebook->-r requirements.txt (line 3))
(4.8.0)
Requirement already satisfied: stack-data in
/home/carlos/.local/lib/python3.10/site-packages (from
ipython>=7.23.1->ipykernel->notebook->-r requirements.txt (line 3)) (0.2.0)
Requirement already satisfied: backcall in
/home/carlos/.local/lib/python3.10/site-packages (from
ipython>=7.23.1->ipykernel->notebook->-r requirements.txt (line 3)) (0.2.0)
Requirement already satisfied: prompt-toolkit!=3.0.0,!=3.0.1,<3.1.0,>=2.0.0 in
/home/carlos/.local/lib/python3.10/site-packages (from
ipython>=7.23.1->ipykernel->notebook->-r requirements.txt (line 3)) (3.0.28)
Requirement already satisfied: decorator in
/home/carlos/.local/lib/python3.10/site-packages (from
ipython>=7.23.1->ipykernel->notebook->-r requirements.txt (line 3)) (5.1.1)
Requirement already satisfied: jedi>=0.16 in
/home/carlos/.local/lib/python3.10/site-packages (from
ipython>=7.23.1->ipykernel->notebook->-r requirements.txt (line 3)) (0.18.1)
Requirement already satisfied: pickleshare in
/home/carlos/.local/lib/python3.10/site-packages (from
ipython>=7.23.1->ipykernel->notebook->-r requirements.txt (line 3)) (0.7.5)
Requirement already satisfied: attrs>=17.4.0 in /usr/lib/python3.10/site-
packages (from jsonschema!=2.5.0,>=2.4->nbformat->notebook->-r requirements.txt
(line 3)) (21.4.0)
Requirement already satisfied: pyrsistent>=0.14.0 in /usr/lib/python3.10/site-
packages (from jsonschema!=2.5.0,>=2.4->nbformat->notebook->-r requirements.txt
(line 3)) (0.18.1)
Requirement already satisfied: cffi>=1.0.1 in /usr/lib/python3.10/site-packages
(from argon2-cffi-bindings->argon2-cffi->notebook->-r requirements.txt (line 3))
(1.15.0)
```

```
Requirement already satisfied: soupsieve>1.2 in /usr/lib/python3.10/site-
packages (from beautifulsoup4->nbconvert>=5->notebook->-r requirements.txt (line
3)) (2.3.1)
Requirement already satisfied: webencodings in /usr/lib/python3.10/site-packages
(from bleach->nbconvert>=5->notebook->-r requirements.txt (line 3)) (0.5.1)
Requirement already satisfied: pycparser in /usr/lib/python3.10/site-packages
(from cffi>=1.0.1->argon2-cffi-bindings->argon2-cffi->notebook->-r
requirements.txt (line 3)) (2.21)
Requirement already satisfied: parso<0.9.0,>=0.8.0 in
/home/carlos/.local/lib/python3.10/site-packages (from
jedi>=0.16->ipython>=7.23.1->ipykernel->notebook->-r requirements.txt (line 3))
(0.8.3)
Requirement already satisfied: wcwidth in
/home/carlos/.local/lib/python3.10/site-packages (from prompt-
toolkit!=3.0.0,!=3.0.1,<3.1.0,>=2.0.0->ipython>=7.23.1->ipykernel->notebook->-r
requirements.txt (line 3)) (0.2.5)
Requirement already satisfied: executing in
/home/carlos/.local/lib/python3.10/site-packages (from stack-
data->ipython>=7.23.1->ipykernel->notebook->-r requirements.txt (line 3))
(0.8.3)
Requirement already satisfied: pure-eval in
/home/carlos/.local/lib/python3.10/site-packages (from stack-
data->ipython>=7.23.1->ipykernel->notebook->-r requirements.txt (line 3))
(0.2.2)
Requirement already satisfied: asttokens in
/home/carlos/.local/lib/python3.10/site-packages (from stack-
data->ipython>=7.23.1->ipykernel->notebook->-r requirements.txt (line 3))
(2.0.5)
```

[20]:

```python
import random
import math
import tabulate
import time
random.seed()

def sortearPuntoRN(dim=2):
    """
    Seortea un punto en R^N dentro del hiper-cubo [0,1]^N
    """
    punto = []
    for n in range(0, dim):
        punto.append(random.uniform(0.0, 1.0))
    # end for

    return punto
# end fun sortearPuntoRN
```

```python
def puntoDentroVolumen(punto, restricciones=True):
    """
    Devuelve 0 o 1 si un punto esta fuera o dentro de un cierto volumen.
    Si restricciones es "false", el volumen es la hiperesfera en R6
    """

    # Para que este dentro del volumen tiene que estar dentro de la esfera
    # y ademas cumplir con las restricciones adicionales

    dentro = 1
    fuera = 0

    # chequeo 1 : dentro de esfera

    d = math.sqrt(
        (punto[0]-0.45)**2 +
        (punto[1]-0.5)**2 +
        (punto[2]-0.6)**2 +
        (punto[3]-0.6)**2 +
        (punto[4]-0.5)**2 +
        (punto[5]-0.45)**2
    )

    # si la distancia es mayor al radio, esta fuera
    if (d>=0.35) :
        return fuera

    if restricciones:
        # restriccion 1
        if 3*punto[0] + 7*punto[3] > 5:
            return fuera
        # restriccion 2
        if punto[2]+punto[3] > 1:
            return fuera
        # restriccion 3
        if punto[0]-punto[1]-punto[4]+punto[5] < 0:
            return fuera
    else:
        return dentro

    return dentro
# end fun punto dentro del volumen

# sortearPuntoRN(6)
```

[21]:

```python
# Implemento pseudocodigo Montecarlo
```

```
import functools

@functools.lru_cache(maxsize=2)
def MetodoMonteCarlo(N, FVolumen):
    """
    Implementa el pseudocodigo de MC
    N: cantidad de muestras
    FVolumen: funcion que define el volumen, devuelve 0 si el punto esta fuera,␣
 ↪1 si esta dentro
    """
    random.seed()
    t0 = time.perf_counter()
    S = 0
    for j in range(0, N):
        punto = sortearPuntoRN(6)
        if FVolumen(punto):
            phi = 1
        else:
            phi = 0
        S = S + phi
    # end for
    VolR = S / N
    VarVorR = (S/N)*(1-S/N)/(N-1)
    return (VolR, VarVorR, S,  time.perf_counter()-t0)
# end def

VolH = math.pi**3*(0.35**6)/6

# Caclulo del volumen de la hiperesfera por MMC
(VolR, VarVolR, S, execTime) = MetodoMonteCarlo(10**6, lambda x:␣
 ↪puntoDentroVolumen(x, False))
```

1.1.2 Verificación

Comparamos el volumen sin restricciones con el volumen calculado analiticamente de la hiperesfera en R6

[22]:
```
print("Volumen hiper esfera por MMC = {:e}, Varianza = {:e}".format(VolR,␣
 ↪VarVolR))
print(" ")
print("Volumen hiper esfera analitico = {:e}, diferencia MMC - analitico = {:.
 ↪3f}%".format(VolH, (VolH-VolR)/VolR*100))
```

```
Volumen hiper esfera por MMC = 9.439000e-03, Varianza = 9.349915e-09

Volumen hiper esfera analitico = 9.499629e-03, diferencia MMC - analitico =
0.642%
```

Con un millon de muestras tenemos una diferencia de menos de 1% entre el volumen calculado de forma analitica y el volumen calculado por Montecarlo.

1.1.3 Ejecucion para diferentes tamanos de muestra

En esta seccion corremos MMC para calcular el volumen con restricciones para diferentes tamanos de muestra.

[23]:
```
table = [ ['N', 'S', 'Vol hiperesfera (analitico)', 'Vol␣
 ↪hiperesfera+restricciones', 'Varianza', 'Tiempo (s)'] ]

for n in [2, 3, 4, 5, 6]:
    (VolR, VarVolR, S, execTime) = MetodoMonteCarlo(10**n, lambda x:␣
 ↪puntoDentroVolumen(x, True))
    table.append( [10**n, S, "{:3e}".format(VolH), "{:3e}".format(VolR), "{:
 ↪3e}".format(VarVolR), "{:3f}".format(execTime)] )


tabulate.tabulate(table, tablefmt='html')
```

[23]:
```
'<table>\n<tbody>\n<tr><td>N      </td><td>S  </td><td>Vol hiperesfera
(analitico)</td><td>Vol hiperesfera+restricciones</td><td>Varianza
</td><td>Tiempo (s)</td></tr>\n<tr><td>100    </td><td>0  </td><td>9.499629e-03
</td><td>0.000000e+00                 </td><td>0.000000e+00</td><td>0.000844
</td></tr>\n<tr><td>1000   </td><td>0  </td><td>9.499629e-03
</td><td>0.000000e+00                 </td><td>0.000000e+00</td><td>0.005523
</td></tr>\n<tr><td>10000  </td><td>3  </td><td>9.499629e-03
</td><td>3.000000e-04                 </td><td>2.999400e-08</td><td>0.056466
</td></tr>\n<tr><td>100000 </td><td>24 </td><td>9.499629e-03
</td><td>2.400000e-04                 </td><td>2.399448e-09</td><td>0.470523
</td></tr>\n<tr><td>1000000</td><td>296</td><td>9.499629e-03
</td><td>2.960000e-04                 </td><td>2.959127e-10</td><td>4.586955
</td></tr>\n</tbody>\n</table>'
```

Entre las corridas de 10mil y 1millon de muestras hay una diferencia de un 7.6% aproximadamente. Los resultados parecen coherentes en el sentido de que al aumentar el tamano de la muestra el resultado parece tender a un valor y no parece diverger. La varianza estimada tambien decrece al aumentar el tamano de la muestra, otro resultado esperable.

El volumen determinado para la hiperesfera con restricciones es consistentemente menor al volumen de la hiperesfera sin restricciones, lo cual tiene sentido ya que las restricciones justamente eliminan puntos del volumen en cuestión.

1.2 Entrega 2 : Ejercicio 4.1

[**Letra**] 1. Comparar y discutir la dependencia de los criterios de peor caso nC, nN, nH frente a los par´ametros epsilon y .

En el caso de nC:

- Si dejamos epsilon fijo, el tamaño de la muestra tiende a infinito de forma similar a 1/x (cuando x tiende a cero)
- Si dejamos delta fijo, tiende a infinito como 1/x^2

[**Letra**] 2. Calcular nC, nN, nH para epsilon = 0:01, = 0:001; 0:01; 0:05

Nota: utilizo la funcion scipy.stats.norm.ppf del paquete SciPy para implementar la inversa de la normal

[24]:
```python
from scipy.stats import norm

# Formula de Chebyshev
def tamMuestraChebyshev(epsilon, delta):
    nc =  1.0 / (4.0 * delta * epsilon**2)
    return math.ceil(nc)
#

# Formula Teo Central Limite
def tamMuestraTeoCentralLimite(epsilon, delta):
    x = norm.ppf(1.0 - delta/2.0)
    # nn = norm.ppf(x)**2
    return math.ceil( ( x/ (2.0*epsilon) ) **2 )
    # return x
#

# Formula de Hoeffding
def tamMuestraHoeffding(epsilon, delta):
    """
    Estimacion del tamano de muestra segun Hoeffding.
    epsilon: error
    delta: confianza
    """
    num = 2 * math.log(2/delta)
    den = 4 * epsilon**2
    return math.ceil(num/den)
# end def

tabla2 = [ ['estimador', 'epsilon', 'delta', 'tam. muestra'] ]

epsilon = 0.01
for delta in [0.001, 0.01, 0.05]:
    tm_cheby = tamMuestraChebyshev(epsilon, delta)
    tabla2.append( ['cheby', epsilon, delta, f'{tm_cheby:,}'] )
    #
    tm_tcl = tamMuestraTeoCentralLimite(epsilon, delta)
    tabla2.append( ['tcl', epsilon, delta, f'{tm_tcl:,}'] )
    #
    tm_hoeff = tamMuestraHoeffding(epsilon, delta)
    tabla2.append( ['hoeff', epsilon, delta, f'{tm_hoeff:,}'] )
```

```
    #
    tabla2.append( ['---', '---', '---', '---'] )

# end for

tabulate.tabulate(tabla2, tablefmt='html')
```

[24]:
```
'<table>\n<tbody>\n<tr><td>estimador</td><td>epsilon</td><td>delta</td><td>tam.
muestra</td></tr>\n<tr><td>cheby    </td><td>0.01
</td><td>0.001</td><td>2,500,000   </td></tr>\n<tr><td>tcl      </td><td>0.01
</td><td>0.001</td><td>27,069      </td></tr>\n<tr><td>hoeff    </td><td>0.01
</td><td>0.001</td><td>38,005      </td></tr>\n<tr><td>---      </td><td>---
</td><td>---  </td><td>---         </td></tr>\n<tr><td>cheby    </td><td>0.01
</td><td>0.01 </td><td>250,000     </td></tr>\n<tr><td>tcl      </td><td>0.01
</td><td>0.01 </td><td>16,588      </td></tr>\n<tr><td>hoeff    </td><td>0.01
</td><td>0.01 </td><td>26,492      </td></tr>\n<tr><td>---      </td><td>---
</td><td>---  </td><td>---         </td></tr>\n<tr><td>cheby    </td><td>0.01
</td><td>0.05 </td><td>50,000      </td></tr>\n<tr><td>tcl      </td><td>0.01
</td><td>0.05 </td><td>9,604       </td></tr>\n<tr><td>hoeff    </td><td>0.01
</td><td>0.05 </td><td>18,445      </td></tr>\n<tr><td>---      </td><td>---
</td><td>---  </td><td>---         </td></tr>\n</tbody>\n</table>'
```

1.3 Entrega 3 : Ejercicio 5.1

Para el mismo enunciado de mas arriba (estimación de un volumen con restricciones) se pide:

1.4 Parte a

[**Letra**]: Compartir en el grupo los códigos desarrollados para la parte a, validarlos revisando los códigos, y verificando si las salidas para tamaños de muestra de 106 son consistentes. Indicar si se detectaron errores en los mismos, y en ese caso dar los códigos corregidos. Elegir uno de los códigos para las partes siguientes, explicar los motivos de la selecci´on.

Por el momento sigo trabajando cno mi código en Python ya que llegué con retraso a la elección de grupo.

1.5 Parte b

[**Letra**]: calcular la cantidad de replicaciones a realizar para garantizar un error menor a 1:0 × 10—4 con probabilidad 0:95, utilizando el criterio de peor caso de Hoeffding.

[25]:
```
tm_hoeff = tamMuestraHoeffding(10**-4, 0.05)
f'{tm_hoeff:,}'
```

[25]:
```
'184,443,973'
```

1.6 Parte c

[**Letra**] utilizando el c´odigo elegido en la parte a, y la cantidad de replicaciones definida en el punto anterior, calcular el intervalo de confianza de nivel 0:95 utilizando el criterio de Chebyshev, y el criterio de Agresti-Coull. Comparar el ancho de estos intervalos entre s´ı y con el criterio de error manejado en el punto previo.

1.6.1 Calculo del volumen con restricciones para el tamaño de muestra de Hoeffding

Primero calcularemos el volumen para el tamaño de muestra hallado anteriormente, determinando tambien el valor de S (cantidad de muestras que cayeron dentro del volumen)

[26]:
```
table3 = [ ['N', 'S', 'Vol hiperesfera (analitico)', 'Vol hiperesfera+restricciones', 'Varianza', 'Tiempo (s)'] ]

tm_hoeff = 10**5
(VolR, VarVolR, S, execTime) = MetodoMonteCarlo(tm_hoeff, lambda x: puntoDentroVolumen(x, True))
table3.append( [tm_hoeff, S, "{:3e}".format(VolH), "{:3e}".format(VolR), "{:3e}".format(VarVolR), "{:3f}".format(execTime)] )



tabulate.tabulate(table3, tablefmt='html')
```

[26]:
```
'<table>\n<tbody>\n<tr><td>N     </td><td>S </td><td>Vol hiperesfera
(analitico)</td><td>Vol hiperesfera+restricciones</td><td>Varianza
</td><td>Tiempo (s)</td></tr>\n<tr><td>100000</td><td>39</td><td>9.499629e-03
</td><td>3.900000e-04                 </td><td>3.898518e-09</td><td>0.477981
</td></tr>\n</tbody>\n</table>'
```

1.6.2 Cálculo del intervalo de confianza según criterio de Chebyshev

[27]:
```
## Calculo de int de confianza por Chebyshev

def intConfianzaChebyshev(S, n, delta):
    """
    Intervalo de confianza segun Chebyshev.
    Parámetros:
      - S: estimador, cantidad de puntos que caen dentro del volumen
      - n: cantidad de replicas (puntos sorteados)
      - delta: margen
    """
    def w1(z, n, beta):
        num = z + beta**2 - beta*math.sqrt( beta**2/4 + z*(n-z)/n )
        den = n + beta**2
        return num / den
    # end def w1

```

```
    def w2(z, n, beta):
        num = z + beta**2 + beta*math.sqrt( beta**2/4 + z*(n-z)/n )
        den = n + beta**2
        return num / den
    # end def w2

    return ( w1(S, n, delta), w2(S, n, delta) )
## end intConfianzaChebyshev

(i1, i2) = intConfianzaChebyshev( S, tm_hoeff, 0.05 )
f'({i1:4e},{i2:4e})'
```

[27]: '(3.869031e-04,3.931469e-04)'

¿Donde cae el valor del volumen calculado dentro del intervalo de confianza?

[28]:
```
print("Distancia desde el min del intervalo:",VolR-i1)
print("Distancia desde al max del intervalo:",i2-VolR)
```

```
Distancia desde el min del intervalo: 3.096924749897221e-06
Distancia desde al max del intervalo: 3.146905248647676e-06
```

Vemos que el valor calculado cae dentro del intervalo de confianza, aunque levemente desplazado del centro del mismo.

1.6.3 Cálculo del intervalo de confianza según el criterio de Agresti-Coull

[29]:
```
## intervalo de confianza de Agresti-Coull

from scipy.stats import norm

def intConfianzaAC(S, n, delta):
    """
    Intervalo de confianza segun Agresti Coull.
    Parámetros:
      - S: estimador, cantidad de puntos que caen dentro del volumen
      - n: cantidad de replicas (puntos sorteados)
      - delta: margen, si el intervalo de conf es 95%, entonces delta = 0.05
    """
    kappa = norm.ppf(1-delta/2)

    Xg = S + kappa**2/2
    ng = n + kappa**2

    pg = Xg / ng
    qg = 1 - pg

    disc = kappa * math.sqrt(pg*qg)*( 1/math.sqrt(ng))
```

```
    return (pg-disc, pg+disc)
## end intConfianzaAC
```

El valor obtenido para el ic según Agresti-Coull es:

[30]:
```
(i1ac, i2ac) = intConfianzaAC( S, tm_hoeff, 0.05 )
f'({i1ac:4e},{i2ac:4e})'
```

[30]: '(2.838445e-04,5.345387e-04)'

¿Donde cae el valor calculado dentro del intervalo de confianza?

[31]:
```
print("Distancia desde el min del intervalo:",VolR-i1ac)
print("Distancia desde al max del intervalo:",i2ac-VolR)
```

```
Distancia desde el min del intervalo: 0.00010615550392825294
Distancia desde al max del intervalo: 0.00014453865428347786
```

El intervalo de Agresti-Coull parece considerablemente más amplio que el de Chebyshev.

1.6.4 Visualización de los diferentes intervalos de confianza

[36]:
```
import matplotlib.pyplot as plt
import numpy as np

x_ticks = ("Chebyshev", "Agresti-Coull")

x_1 = [1, 2]

y_1 = [VolR, VolR]

err_1 = [[i1, i2],  [i1ac, i2ac]]

plt.errorbar(x=x_1, y=y_1, yerr=err_1, color="black", capsize=3,
             linestyle="None",
             marker="s", markersize=7, mfc="black", mec="black")

plt.xticks(x_1, x_ticks, rotation=90)

plt.tight_layout()
plt.show()
```


0.0008
0.0006
0.0004
0.0002
0.0000
Chebyshev
Agresti-Coull