Week 4: Decisions in Settings with High Uncertainty

- ◆ Session 1 – Decision Trees
  - ○ Example: Furniture maker IDEA Chooses a Supplier

- ◆ Session 2 – Using Simulation within Decision Trees
  - ○ Example: More Complex Demand Distributions for IDEA

- ◆ Session 3 – Using Optimization Together with Simulation
  - ○ Example: IDEA Chooses Order Quantities

- ◆ Session 4 – Wrap Up
  - ○ Example: Back to the Newsvendor Problem

# Original problem description for IDEA's Krusbär tent

◆ Order quantities and costs, by supplier

	Sweden (S)	**Poland (P)**
Order Quantity	5,000 units	10,000 units
Fixed Charge	0€	50,000€
Unit Cost	120€	100€

◆ Demand forecast given a unit price of 150€

○ 50% chance demand is strong: 10,000 units

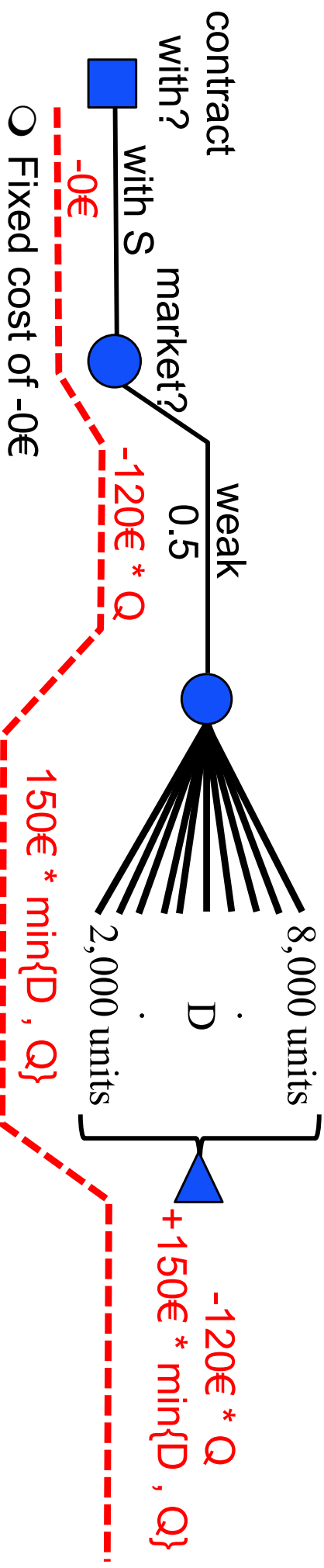○ 50% chance demand is weak: 5,000 units

But demand may be more complex

◆ Assume demand is <u>uniformly distributed</u>
  - ○ Defined by a minimum outcome and a maximum outcome
  - ○ Any outcome between a given minimum and maximum is equally likely
  - ○ For example, see J. R. Evans, *Business Analytics*, Pearson, 2013.

◆ More complex demand model
  - ○ 50%-50% chance demand is weak or strong
  - ○ Weak demand is <u>uniformly distributed</u>: 2,000-8,000 units
  - ○ Strong demand is <u>uniformly distributed</u>: 6,000-14,000 units

◆ How can we represent this in a tree?
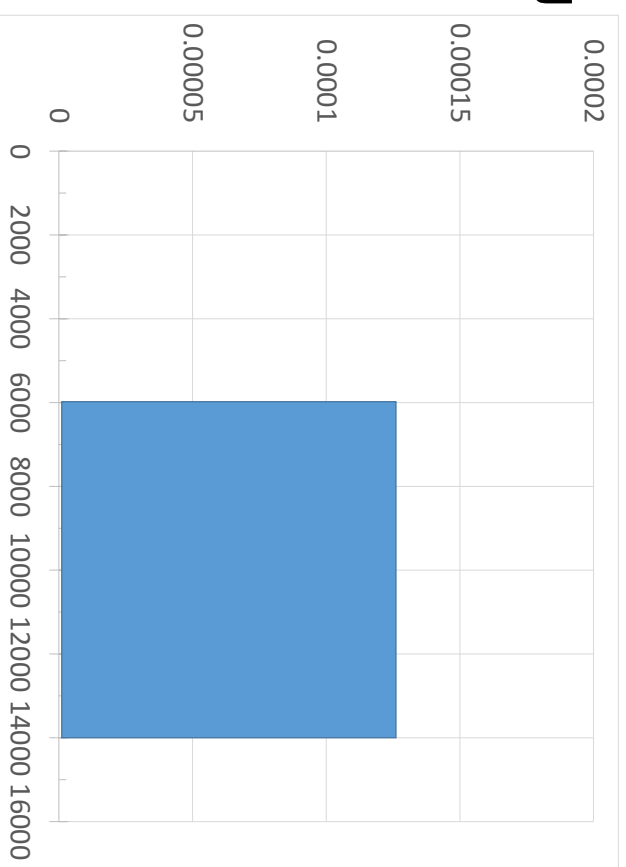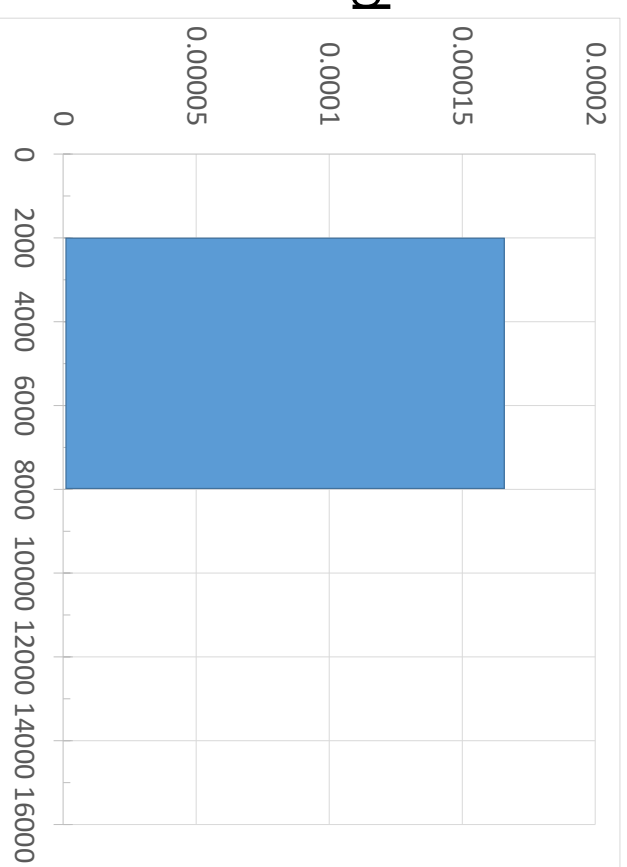
Consider the choice of Supplier S

◆ What happens if IDEA chooses supplier S and the market is weak?

contract with?

with S   market?

-0€

0.5   weak

-120€ * Q

150€ * min{D , Q}

-120€ * Q
+150€ * min{D , Q}

8,000 units

D

2,000 units

○ Fixed cost of -0€
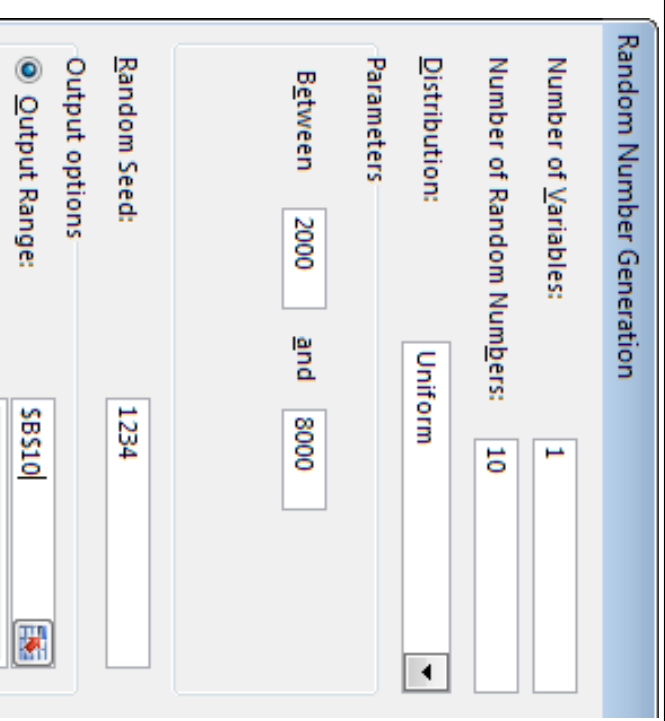
○ In a weak (or strong) market order Q = 5,000 units (100% of S's capacity)

  ☐ Order cost = -120€ * Q   (where Q=5,000)

○ Demand (D) is uniformly distributed between 2,000 and 8,000 units

  ☐ If D ≤ Q, then revenue = 150€ * D

  ☐ If D > Q, then revenue = 150€ * Q

  ☐ For any D IDEA earns revenue = 150€ * min{D , Q}

○ Add the cash flows to determine IDEA's profit

  ☐ Profit = - 120€ * Q + 150€ * min{D , Q}

◆ Set Q = 5,000 and simulate D to estimate IDEA's expected profit

Simulate Supplier S and a weak market (10 samples)

	A	B	C	D	E	F
1	IDEA.xlsx					
2						
3	Order Quantity (Q) =	5000	units			
4	Fixed Cost =	0	euros			
5	Price =	150	euros per unit			
6	Unit Cost =	120	euros per unit			
7						
8	Sample	Demand				
9	Number	Sample (D)	Revenue	Fixed Cost	Variable Cost	Profit
10	1	2,744.90	411,734.37	-	600,000.00	(188,265.63)
11	2	2,039.00	305,850.40	-	600,000.00	(294,149.60)
12	3	4,336.68	650,502.03	-	600,000.00	50,502.03
13	4	3,603.69	540,553.00	-	600,000.00	(59,447.00)
14	5	6,221.81	750,000.00	-	600,000.00	150,000.00
15	6	3,413.07	511,960.20	-	600,000.00	(88,039.80)
16	7	4,796.84	719,525.74	-	600,000.00	119,525.74
17	8	6,487.69	750,000.00	-	600,000.00	150,000.00
18	9	2,742.70	411,404.77	-	600,000.00	(188,595.23)
19	10	4,436.11	665,416.43	-	600,000.00	65,416.43
20						
21	average =	4,082.25				(28,305.31)
22	std. deviation =	1,471.92				158,788.93
23						

Callouts:

=$B$5*MIN($B$3,B10)

=$B$3*$B$6

=$B$4

=C10-D10-E10

=AVERAGE(B10:B19)

=STDEV(B10:B19)

Notes on the simulation

- ◆ We used Excel's Random Number Generator (RNG)
  - ○ 10 samples of 1 random variable (D)
  - ○ Uniformly distributed from 2,000 to 8,000
  - ○ Using random seed 1234

- ◆ Samples of the Uniformly distributed demand included fractional quantities
  - ○ That's by definition in a uniform distribution
  - ○ For simplicity, we'll use it in our example
  - ○ There are other distributions that ensure that samples are whole numbers
  - ○ See RJ Evans, *Business Analytics* Pearson, 2013

- ◆ The spreadsheet IDEA.xlsx has several worksheets, each with a different simulation

Random Number Generation

| Number of Variables: | 1 |
| Number of Random Numbers: | 10 |

Distribution: Uniform

Parameters

Between 2000 and 8000

Random Seed: 1234

Output options

◉ Output Range: $B$10|

Sample Number	Demand Sample (D)
1	2,744.90
2	2,039.00
3	4,336.68
4	3,603.69
5	6,221.81
6	3,413.07
7	4,796.84
8	6,487.69
9	2,742.70
10	4,436.11

Supplier S and a weak market, with 1000 samples

	A	B	C	D	E	F
1	IDEA.xlsx					
2						
3	Order Quantity (Q) =	5000	units			
4	Fixed Cost =	0	euros			
5	Price =	150	euros per unit			
6	Unit Cost =	120	euros per unit			
7						
8	Sample	Demand	Revenue	Fixed Cost	Variable Cost	Profit
9	Number	Sample (D)				
10	1	2,744.90	411,734.37	-	600,000.00	(188,265.63)
11	2	2,039.00	305,850.40	-	600,000.00	(294,149.60)
12	3	4,336.68	650,502.03	-	600,000.00	50,502.03
13	4	3,603.69	540,553.00	-	600,000.00	(59,447.00)
14	5	6,221.81	750,000.00	-	600,000.00	150,000.00
15	6	3,413.07	511,960.20	-	600,000.00	(88,039.80)
16	7	4,796.84	719,525.74	-	600,000.00	119,525.74
17	8	6,487.69	750,000.00	-	600,000.00	150,000.00
1008	999	2,642.54	396,380.50	-	600,000.00	(203,619.50)
1009	1000	3,713.92	557,087.92	-	600,000.00	(42,912.08)
1010						
1011	average =	5,044.06				42,404.97
1012	std. deviation =	1,712.70				145,039.38

We used the simulation to roll back a complex event

◆ Original event node with uniformly distributed demand

contract
with?

with S   market?

weak
0.5

8,000 units
·
D
·
2,000 units

-0€
+150€ * min{D , Q}
-120€ * Q
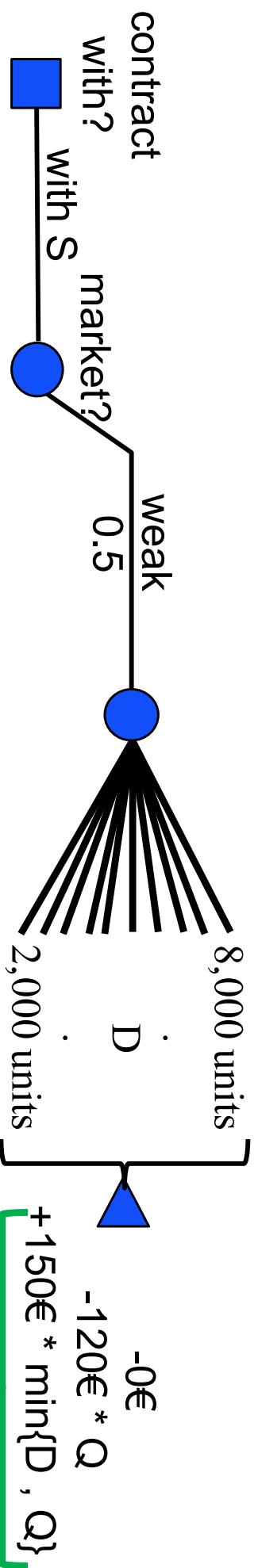
◆ Rolled back event node with expected value

contract
with?

with S   market?

weak
0.5

42,405€
simulation
results
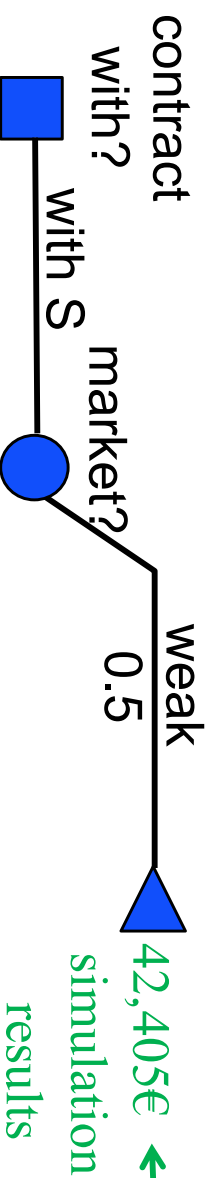
◆ We also can simulate to estimate the expected profit in the other 3 cases

Simulate Supplier S and a strong market (1000 samples)

	A	B	C	D	E	F
1	IDEA.xlsx					
2						
3	Order Quantity (Q) =	5000	units			
4	Fixed Cost =	0	euros			
5	Price =	150	euros per unit			
6	Unit Cost =	120	euros per unit			
7						
8	Sample	Demand				
9	Number	Sample (D)	Revenue	Fixed Cost	Variable Cost	Profit
10	1	6,993.19	750,000.00	-	600,000.00	150,000.00
11	2	6,052.00	750,000.00	-	600,000.00	150,000.00
12	3	9,115.57	750,000.00	-	600,000.00	150,000.00
13	4	8,138.25	750,000.00	-	600,000.00	150,000.00
14	5	11,629.08	750,000.00	-	600,000.00	150,000.00
15	6	7,884.09	750,000.00	-	600,000.00	150,000.00
16	7	9,729.12	750,000.00	-	600,000.00	150,000.00
17	8	11,983.58	750,000.00	-	600,000.00	150,000.00
1008	999	6,856.72	750,000.00	-	600,000.00	150,000.00
1009	1000	8,285.23	750,000.00	-	600,000.00	150,000.00
1010						
1011	average =	10,058.75				150,000.00
1012	std. deviation =	2,283.61				0.00

Simulate Supplier P and a weak market (1000 samples)

	A	B	C	D	E	F
1	IDEA.xlsx					
2						
3	Order Quantity (Q) =	10000	units			
4	Fixed Cost =	50000	euros			
5	Price =	150	euros per unit			
6	Unit Cost =	100	euros per unit			
7						
8		Sample	Demand			
9		Number	Sample (D)	Revenue	Fixed Cost	Variable Cost	Profit
10		1	2,744.90	411,734.37	50,000.00	1,000,000.00	(638,265.63)
11		2	2,039.00	305,850.40	50,000.00	1,000,000.00	(744,149.60)
12		3	4,336.68	650,502.03	50,000.00	1,000,000.00	(399,497.97)
13		4	3,603.69	540,553.00	50,000.00	1,000,000.00	(509,447.00)
14		5	6,221.81	933,271.28	50,000.00	1,000,000.00	(116,728.72)
15		6	3,413.07	511,960.20	50,000.00	1,000,000.00	(538,039.80)
16		7	4,796.84	719,525.74	50,000.00	1,000,000.00	(330,474.26)
17		8	6,487.69	973,152.87	50,000.00	1,000,000.00	(76,847.13)
1008		999	2,642.54	396,380.50	50,000.00	1,000,000.00	(653,619.50)
1009		1000	3,713.92	557,087.92	50,000.00	1,000,000.00	(492,912.08)
1010							
1011	average =	5,044.06					(293,391.16)
1012	std. deviation =	1,712.70					256,905.74

Simulate Supplier P and a strong market (1000 samples)

	A	B	C	D	E	F
1	IDEA.xlsx					
2						
3	Order Quantity (Q) =	10000	units			
4	Fixed Cost =	50000	euros			
5	Price =	150	euros per unit			
6	Unit Cost =	100	euros per unit			
7						
8	Sample	Demand				
9	Number	Sample (D)	Revenue	Fixed Cost	Variable Cost	Profit
10	1	6,993.19	1,048,979.16	50,000.00	1,000,000.00	(1,020.84)
11	2	6,052.00	907,800.53	50,000.00	1,000,000.00	(142,199.47)
12	3	9,115.57	1,367,336.04	50,000.00	1,000,000.00	317,336.04
13	4	8,138.25	1,220,737.33	50,000.00	1,000,000.00	170,737.33
14	5	11,629.08	1,500,000.00	50,000.00	1,000,000.00	450,000.00
15	6	7,884.09	1,182,613.61	50,000.00	1,000,000.00	132,613.61
16	7	9,729.12	1,459,367.66	50,000.00	1,000,000.00	409,367.66
17	8	11,983.58	1,500,000.00	50,000.00	1,000,000.00	450,000.00
1008	999	6,856.72	1,028,507.34	50,000.00	1,000,000.00	(21,492.66)
1009	1000	8,285.23	1,242,783.90	50,000.00	1,000,000.00	192,783.90
1010						
1011	average =	10,058.75				306,539.96
1012	std. deviation =	2,283.61				193,385.84

IDEA's tree with simulated estimates of expected profit

contract
with?

with no one — ▲ 0€

with S

market?

weak 0.5 — ▲ 42,405€

strong 0.5 — ▲ 150,000€

with P

market?

weak 0.5 — ▲ -293,391€

strong 0.5 — ▲ 306,540€
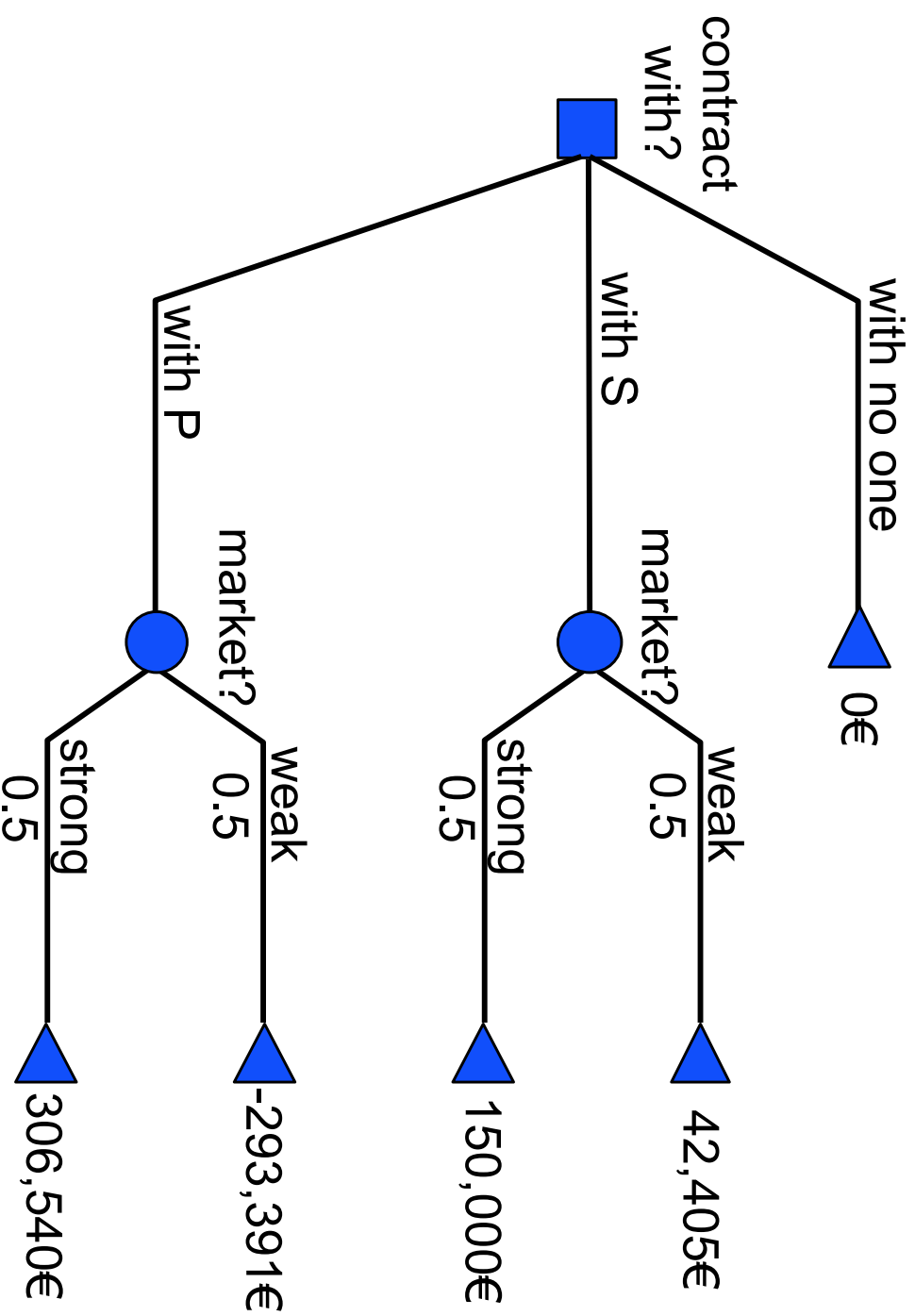
We then calculate the expected values for Supplier S, P

contract with?

with no one — 0€

with S

market?
- strong 0.5 → 150,000€
- weak 0.5 → 42,405€

0.5*42,405€
+0.5*150,000€
96,202.5€

with P

market?
- strong 0.5 → 306,540€
- weak 0.5 → -293,391€

-0.5*293,391€
+0.5*306,540€
6,574.5€
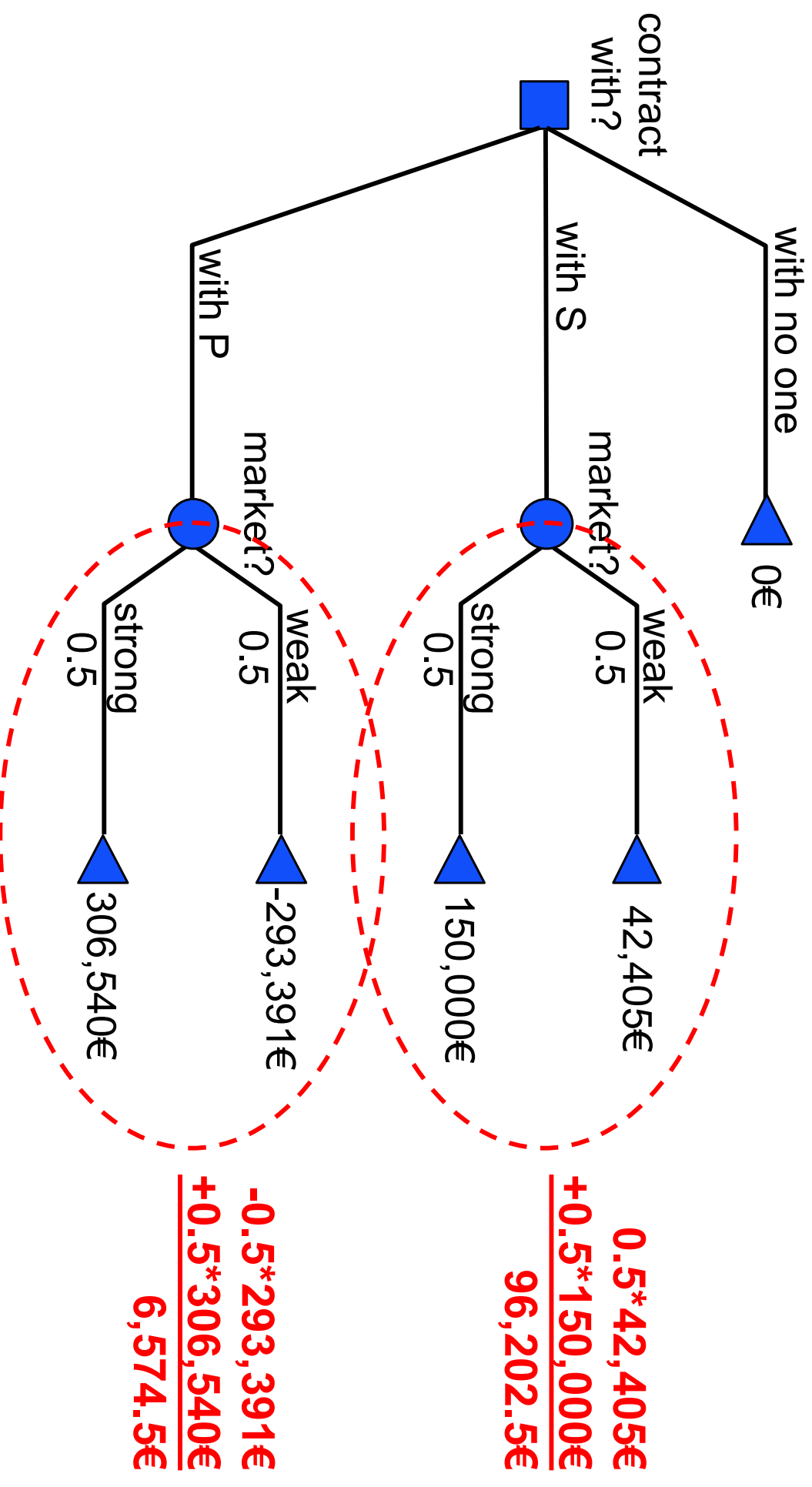
The exp. profit maximizing choice remains Supplier S

contract with?

with no one ——— 0€

with S ——— 96,202.50€

with P ——— 6,574.50€
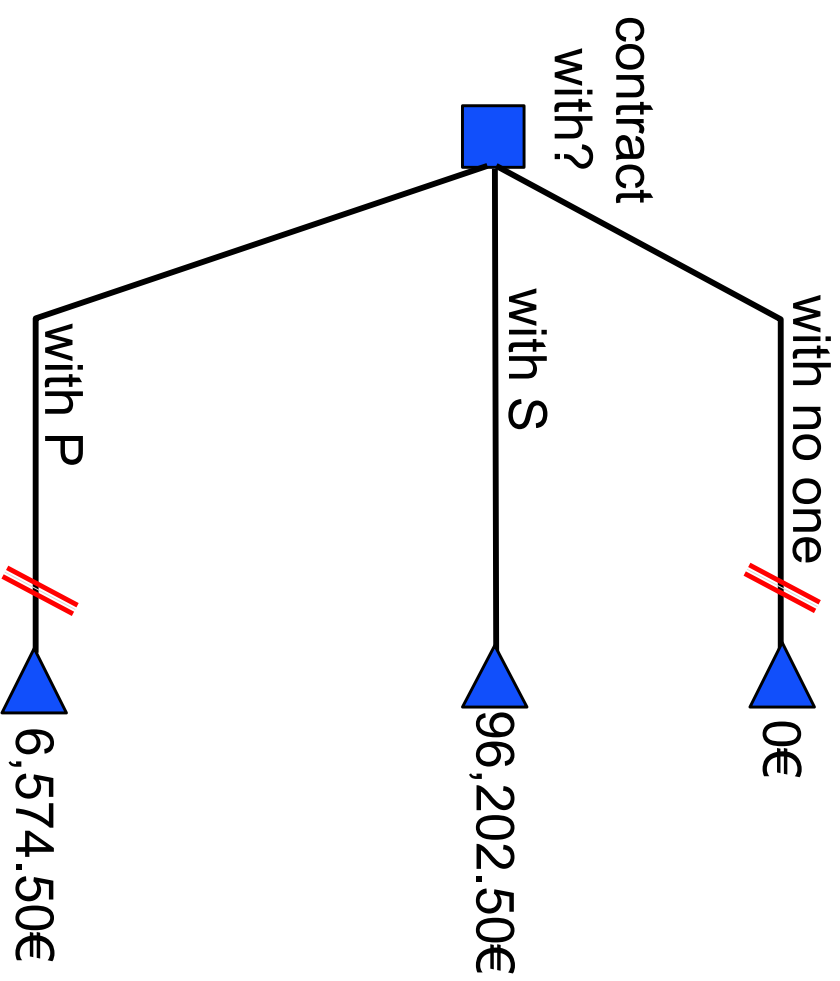
◆ Estimate for supplier S drops by >1/3
  ○ From 150,000€ to ~96,200€

◆ Estimate for supplier P drops by >90%
  ○ From 75,000€ to ~6,600€

◆ To maximize expected profit still contract with supplier S

Wrap-up for Session 2 of Week 4

◆ How did IDEA's problem change from last session to this one?

◆ The overall structure of the decision problem did not change

○ First decide on a supplier: S, P, or none

☐ Fixed costs and order quantities same as before

○ Then see if the market is weak or strong

☐ Same 50%/50% probabilities as before

◆ But the outcomes for weak and strong markets did become more complex

○ In the initial model they were fixed numbers, 5000 or 10,000

☐ In either case we could simply calculate IDEA's profits

○ In the updated model demand in weak and strong markets was still random

☐ We used simulation to estimate IDEA's expected profits

◆ Simulations let us evaluate the outcomes of a more complex event

○ Next session: we'll use optimization to help evaluate more complex decisions