Data Structures

Array Representation

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Outline

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Introduction

Observation

When a program manipulates many variables that contain "similar" forms of data, organizational problems quickly arise.

Example

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Making your life miserable

Because you can ask

Which is the highest score?

Look at the code

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```
double high_score = score0;
if ( score1 > high_score ) { high_score = score1; }
if ( score2 > high_score ) { high_score = score2; }
if ( score3 > high_score ) { high_score = score3; }
if ( score4 > high_score ) { high_score = score4; }
if ( score5 > high_score ) { high_score = score5; }
System.out.println(high_score);
```

How do we solve this?

Thus

Java, C and C++ use array variables to put together this collection of equal elements.

Memory Layout

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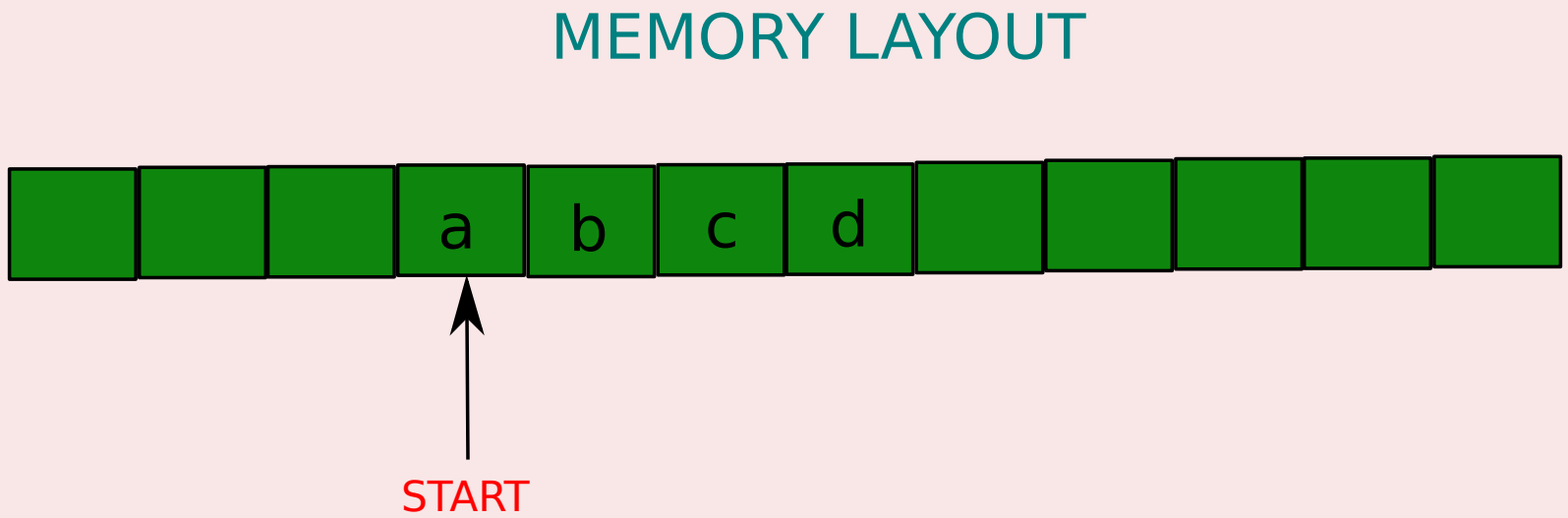

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1D Array Representation In Java, C, and C++

Notes

For 1-dimensional array:

- The elements are mapped into contiguous memory locations
- They are accessed through the use of $location\,(x\,[i]) = start + i$

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1. Space Overhead
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Total: $4 + 4 + 4n = 8 + 4n$ bytes

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We declare 2-dimensional arrays as follow (Java, C)

int[][] A = new int[3][4]

It can be shown as a table

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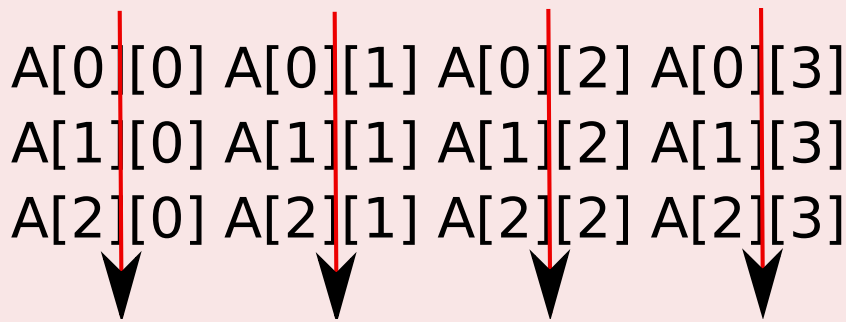

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This information is stored as 1D arrays with a reference to them

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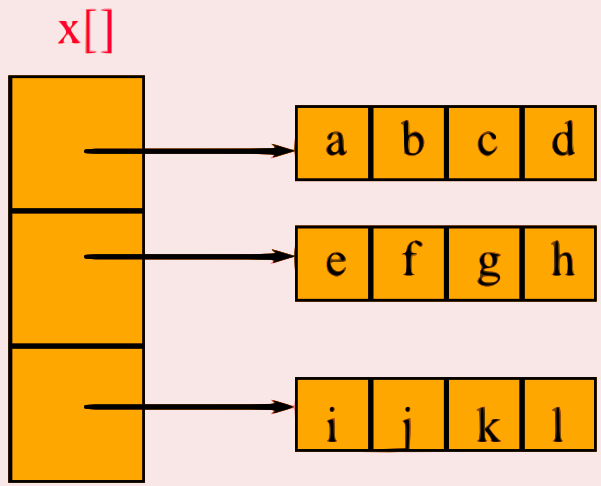

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This representation is called the **array-of-arrays** representation.

Second

It requires contiguous memory of size 3 bytes, 4 bytes, 4 bytes, and 4 bytes for the 4 1D-arrays.

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One memory block of size **number of rows** and **number of rows** blocks of size **number of columns**.

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For this

We will convert it into 1D-array y by collecting elements by rows.

- Rows are collected from top to bottom.

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An array

$\mathbf{y[] = \{a, b, c, d, e, f, g, h, i, j, k, l\}}$

Memory Map

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Memory Map

How do we locate an element?

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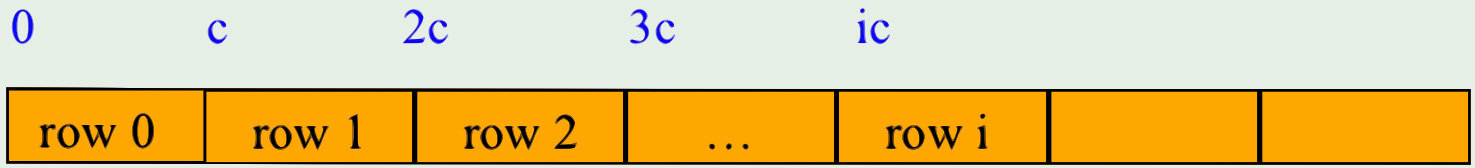

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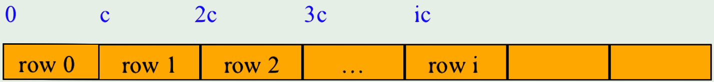

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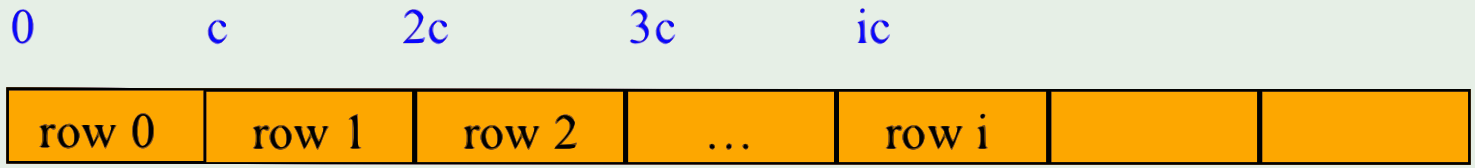

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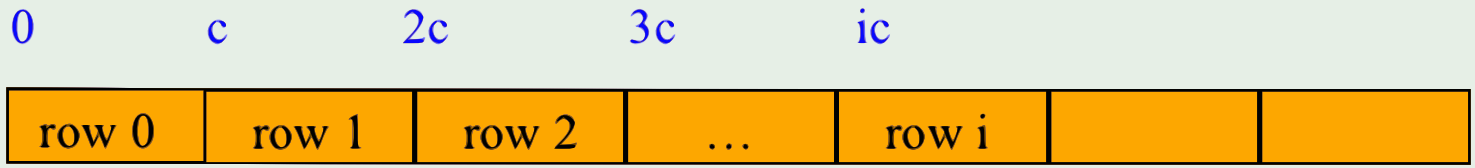

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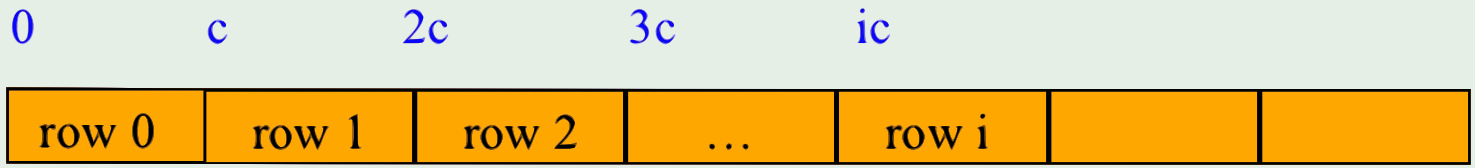

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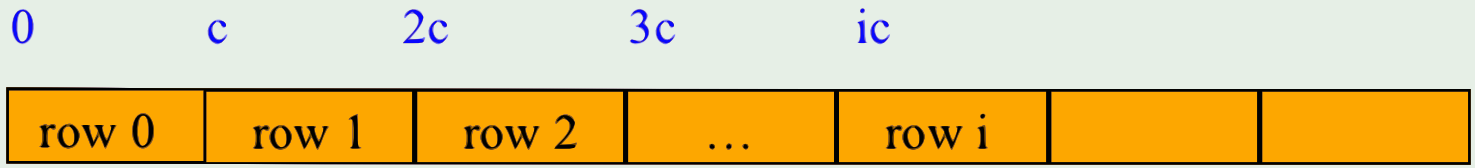

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Note: The number of rows = **length**/**c**

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Note

It may use a 2D array to represent a matrix.

Drawbacks of using a 2D Array

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Indexes are off by 1.

Second

Java arrays do not support matrix operations such as add, transpose, multiply, and so on.

However

You can develop a class Matrix for object-oriented support of all matrix operations.

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An $n \times n$ matrix in which all nonzero terms are on the diagonal.

Properties

- x(i,j) is on diagonal iff i = j

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For a $n \times n$ you are required to have:

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x(i,j) is part of lower triangle iff i >= j.

Number of elements in lower triangle is

$$1 + 2 + 3 + ... + n = \frac{n(n+1)}{2} \quad (1)$$

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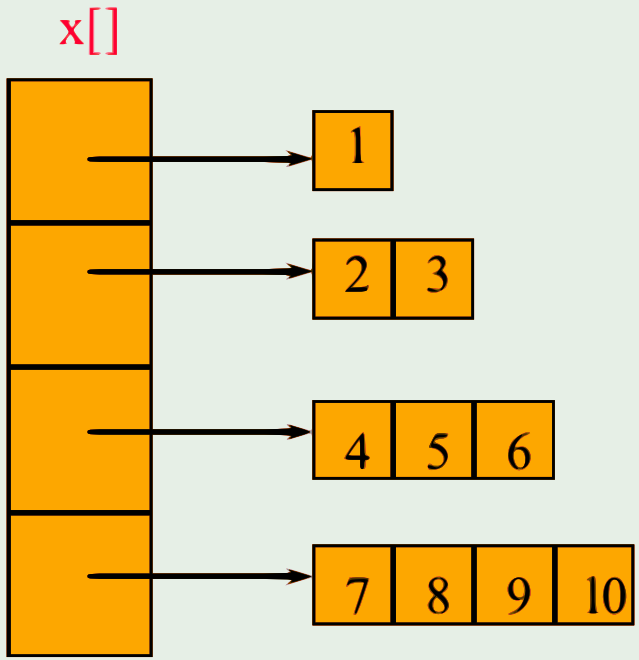

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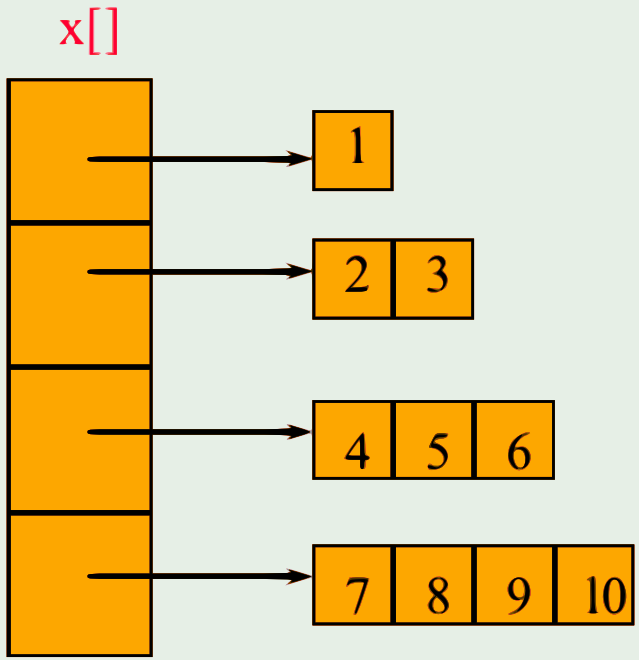

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Code

Code for irregular array

```
// declare a two-dimensional array variable
// and allocate the desired number of rows
int [][] irregularArray = new int [numberOfRows][];

// now allocate space for the elements in each row
for (int i = 0; i < numberOfRows; i++)
        irregularArray[i] = new int [size[i]];

// use the array like any regular array
irregularArray[2][3] = 5;
irregularArray[4][6] = irregularArray[2][3] + 2;
irregularArray[1][1] += 3;
```

However, You can do better

Map Lower Triangular Array Into A 1D Array

You can use a row-major order, but omit terms that are not part of the lower triangle.

For example

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The final mapping

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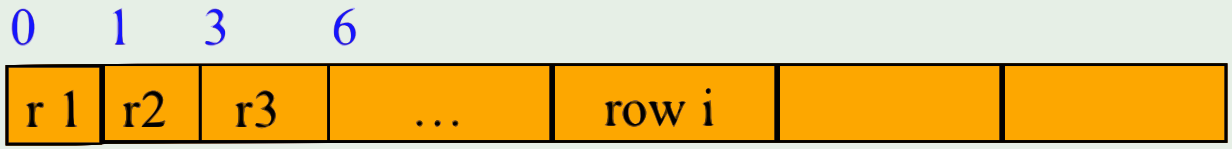

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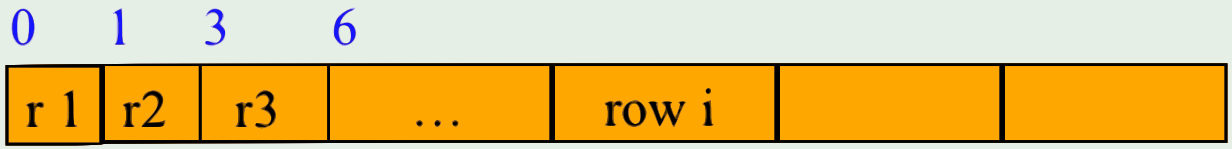

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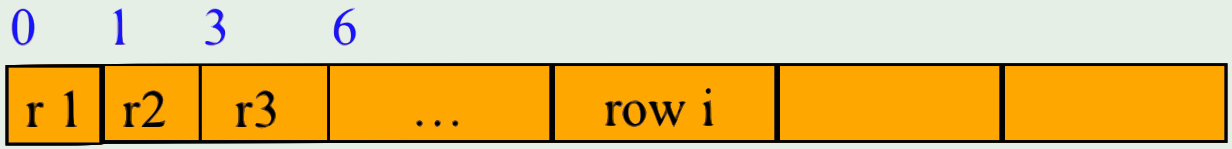

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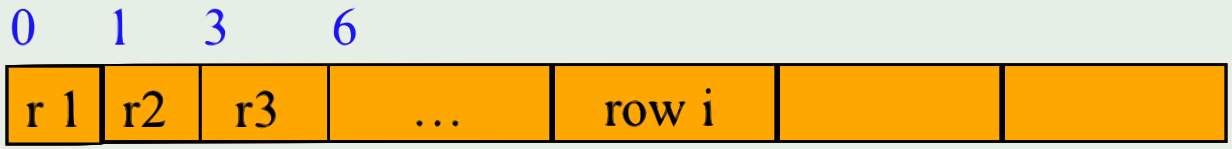

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Now the Interface Matrix

You will need this for your homework

```
public interface Matrix<Item>{
    public Item get(int row, int column);
    public void add(int row, int column,Item myobj);
    public Item remove(int row, int column);
    public void output();
}
```

Outline

Sparse Matrices

Definition

A sparse array is simply an array most of whose entries are zero (or null, or some other default value).

For Example

Suppose you wanted a 2-dimensional array of course grades, whose rows are students and whose columns are courses.

- There are about 5000 courses.
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Since most students take fewer than 5000 courses, there will be a lot of empty spaces in this array.

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What we need...

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There are ways to represent sparse arrays efficiently

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First

We will start with sparse one-dimensional arrays, which are simpler

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	0	1	2	3	4	5	6	7	8	9	10	11
Array	0	0	0	0	17	0	0	23	14	0	0	0

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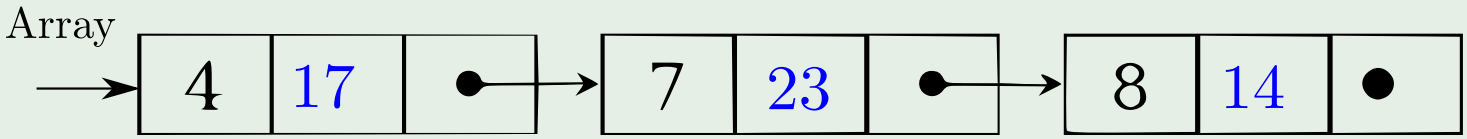

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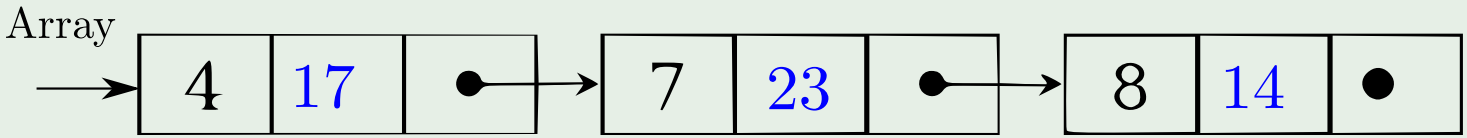

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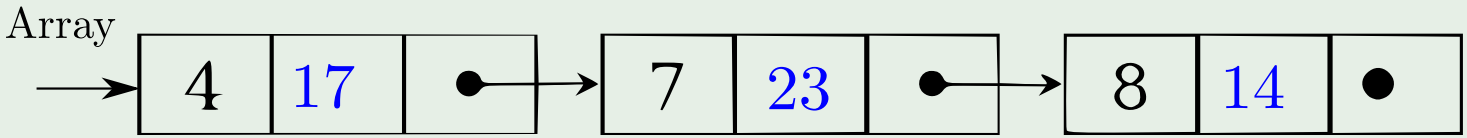

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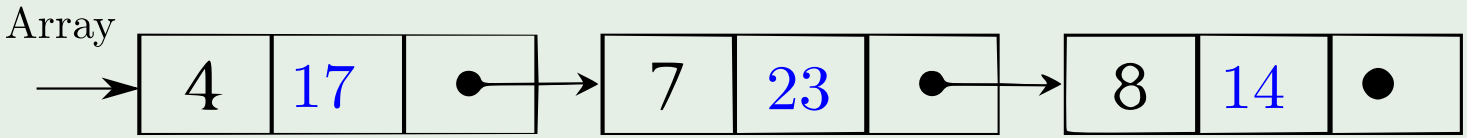

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Example of Get

Code

```
public Item get(int index) {
    ChainNode current = this.firstNode;
        do {
            if (index == current.index) {
                // found correct location
                return current.value;
            }
            current = current.next;
        } while (index < current.index && next != null);
    return null;
}
```

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length, elementCount $\Longrightarrow O(1)$

Problem with this analysis

True fact

In an ordinary array, indexing to find an element is the only operation we really need!!!

True fact

In a sparse array, we can do indexing reasonably quickly

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Look at this code

To find the maximum element in a normal array:

```
double max = array[0];
for (int i = 0; i < array.length; i++)
{
    if (array[i] > max) max = array[i];
}
```

Look at this code

To find the maximum element in a sparse array:

```
Double max = (Double) array.get(0);
for (int i = 0; i < array.length(); i++)
{
      Double temp = (Double) array.get(i);
      if (temp.compareTo(max) > 0) {
            max = temp;
      }
}
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What is the problem?

First

A lot of wrapping and casting because using generics.

Second

More importantly, in a normal array, every element is relevant

Third

- If a sparse array is 1% full, 99% of its elements will be zero.
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- Let the user step through the list.
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Code

```
public class SparseArrayIterator<Item> implements Iterator<Item> {
        // any data you need to keep track of goes here
        SparseArrayIterator() { ...You do need one... }

        public boolean hasNext ( ) { ...you write this code... }

        public Item next ( ) { ...you write this code... }

        public Item remove ( ) { ...you write this code... }
}
```

So, we have that

Code for Max

```
SparseArrayIterator iterator = new SparseArrayIterator(array);
Double max = (Double) array.get(0);

while (iterator.hasNext()) {
      temp = (Double) iterator.next();
      if (temp.compareTo(max) > 0) {
            max = temp;
      }
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- But it's worse than that, because next is defined to return an Item, so we would have to wrap the index.
- We could deal with this by overloading get to take an Item argument.

However

Problem

- Our SparseArrayIterator is fine for stepping through the elements of an array, but...
  - It does not tell us what index they were at.
  - For some problems, we may need this information

First Solution

Revise our iterator to tell us, not the value in each list cell, but the index in each list cell

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Instead of that use an IndexIterator Too

Code

```
public class IndexIterator
{
    private ChainNode current;
    IndexIterator() { // constructor
            current = firstNode;
      }
   public boolean hasNext()
       { // just like before }

    public int next()  {
        int index = current.index;
        current = current.next;
        return index;
    }
}
```

Now, Sparse Matrices

First

Something Simple!!

Using Array Linear List for Matrices

Example

$$\begin{bmatrix} 0 & 0 & 3 & 0 & 4 \\ 0 & 0 & 5 & 7 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 6 & 0 & 0 \end{bmatrix}$$

Thus

list =

row	1	1	2	2	4	4
column	3	5	3	4	2	3
value	3	4	5	6	2	6

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Now, How do we represent this list using arrays?

Why?

We want compact representations...

Use your friend element from arrays

Thus you declare an array element with the Node information

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Node element[] = new Node[1000];
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//Using Generic in Java
public class Node<Item>{
    //elements
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We can use chains too

We need to modify our node

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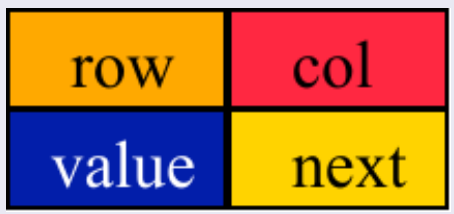
Graphically



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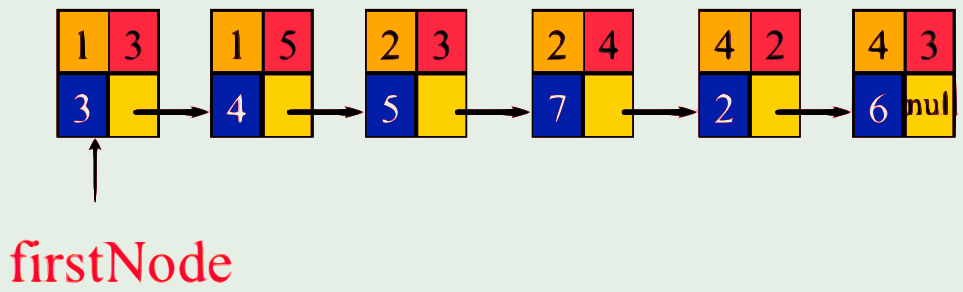
Example



One Linear List Per Row

Example

$$\begin{bmatrix} 0 & 0 & 3 & 0 & 4 \\ 0 & 0 & 5 & 7 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 6 & 0 & 0 \end{bmatrix}$$

Thus

row1 = [(3,3) (5,4)]
row2 = [(3,5) (4,7)]
row3 = []
row4 = [(2,2) (3,6)]

Node Structure

One Linear List Per Row

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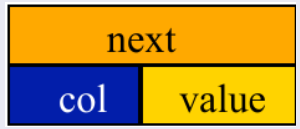
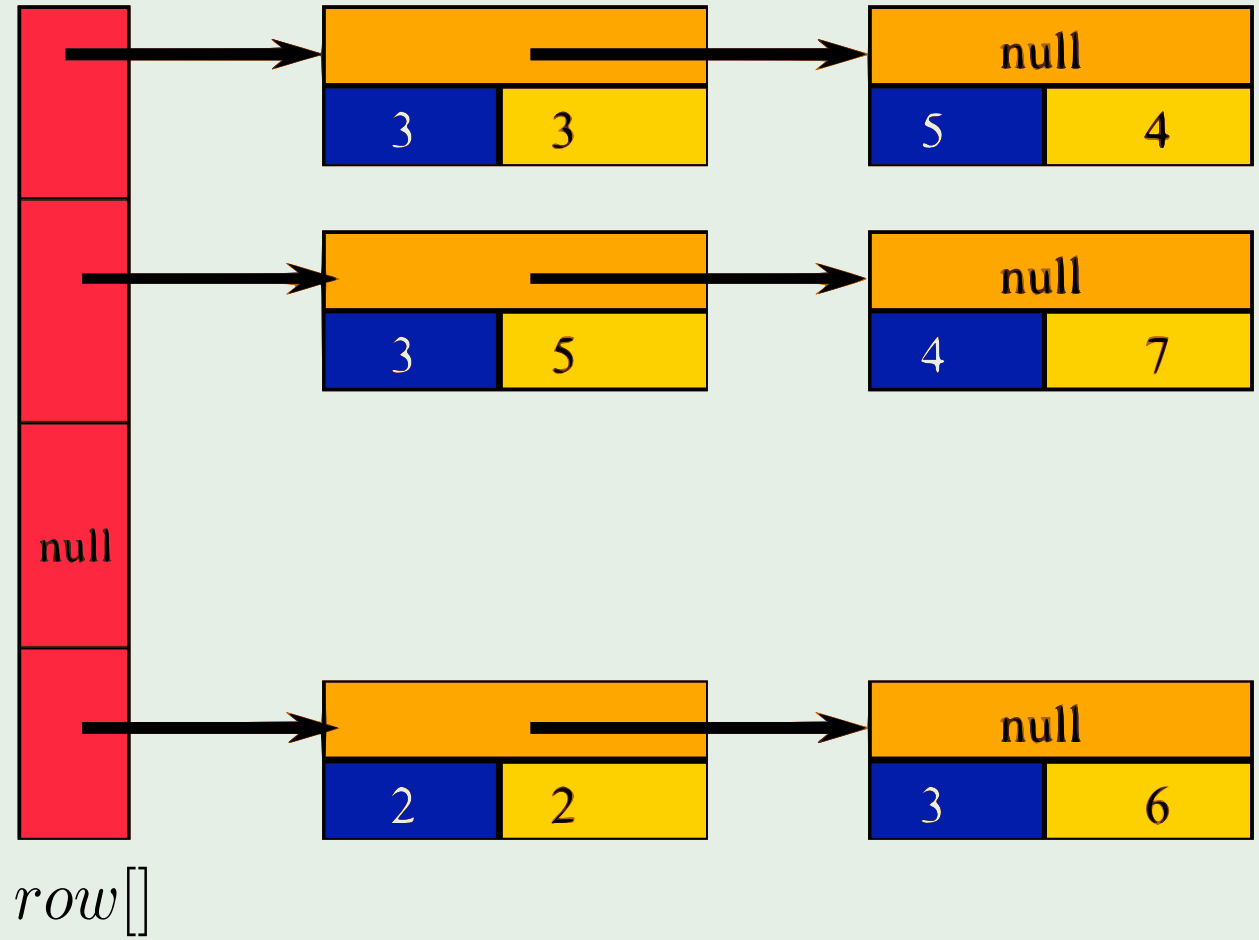


Array Of Row Chains

Example



We have a problem here

With 99% of the matrix as zeros or nulls

We have the problem that many row pointers are null...

What can you do?

Ideas

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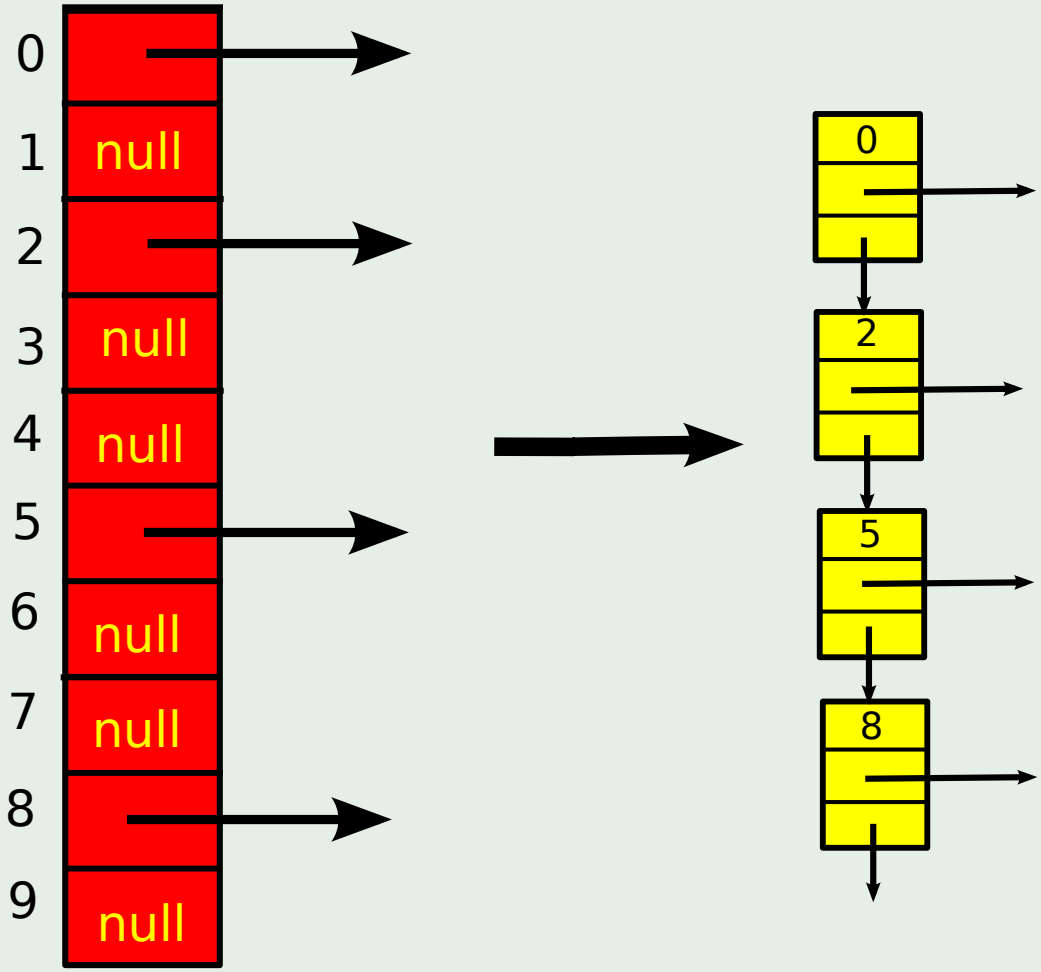
Compress the row

Use a sparse array!!!



However

Problems, problems Will Robinson!!!

Ideas?

Yes, access!! You still need to do an scanning!!!

We have a problem there!!

It is

Not so fast!!!! SOLUTIONS???

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Orthogonal List Representation

More complexity

Both row and column lists.

Node Structure

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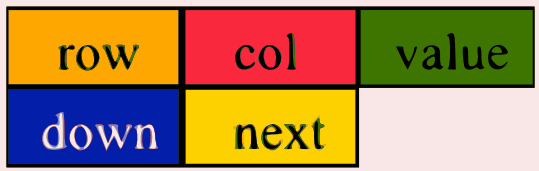

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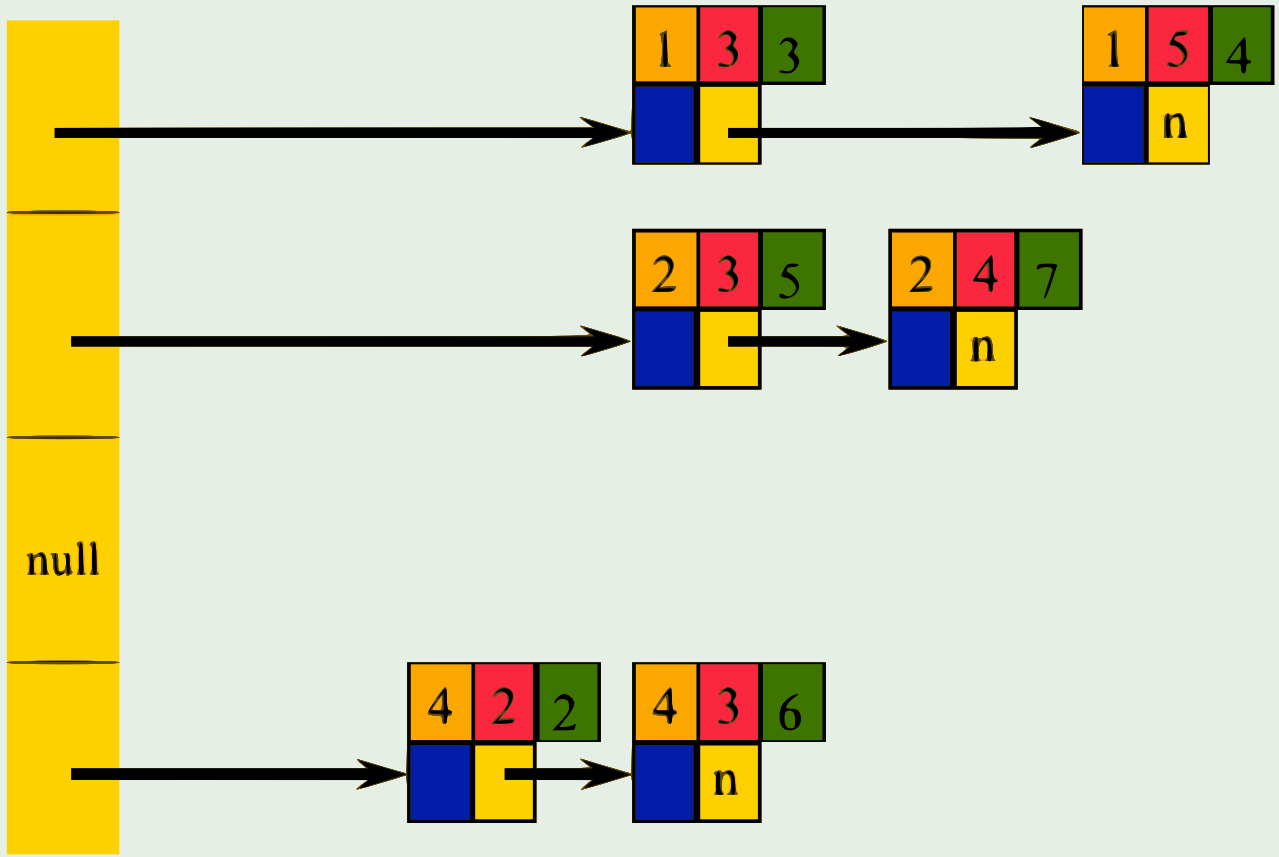
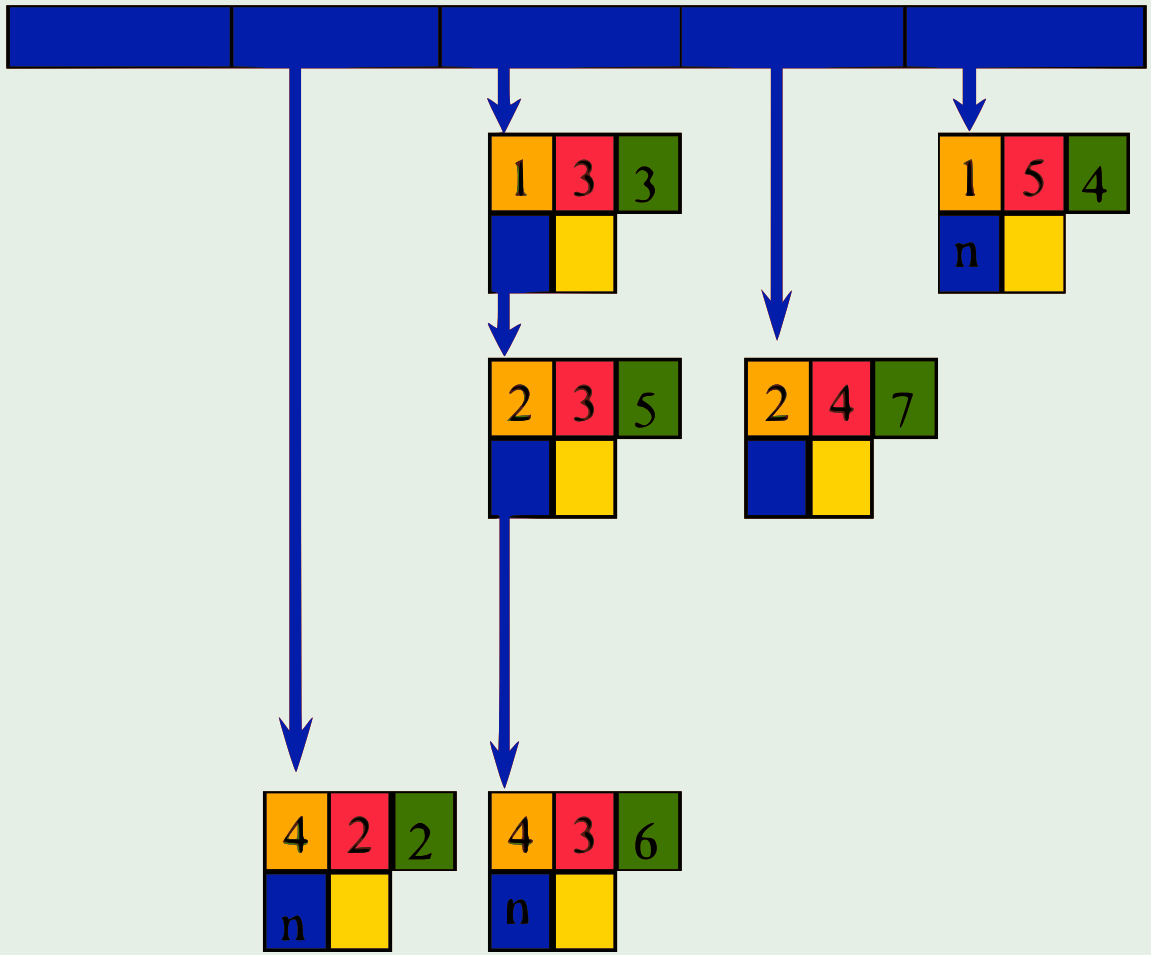
Row Lists

Row direction



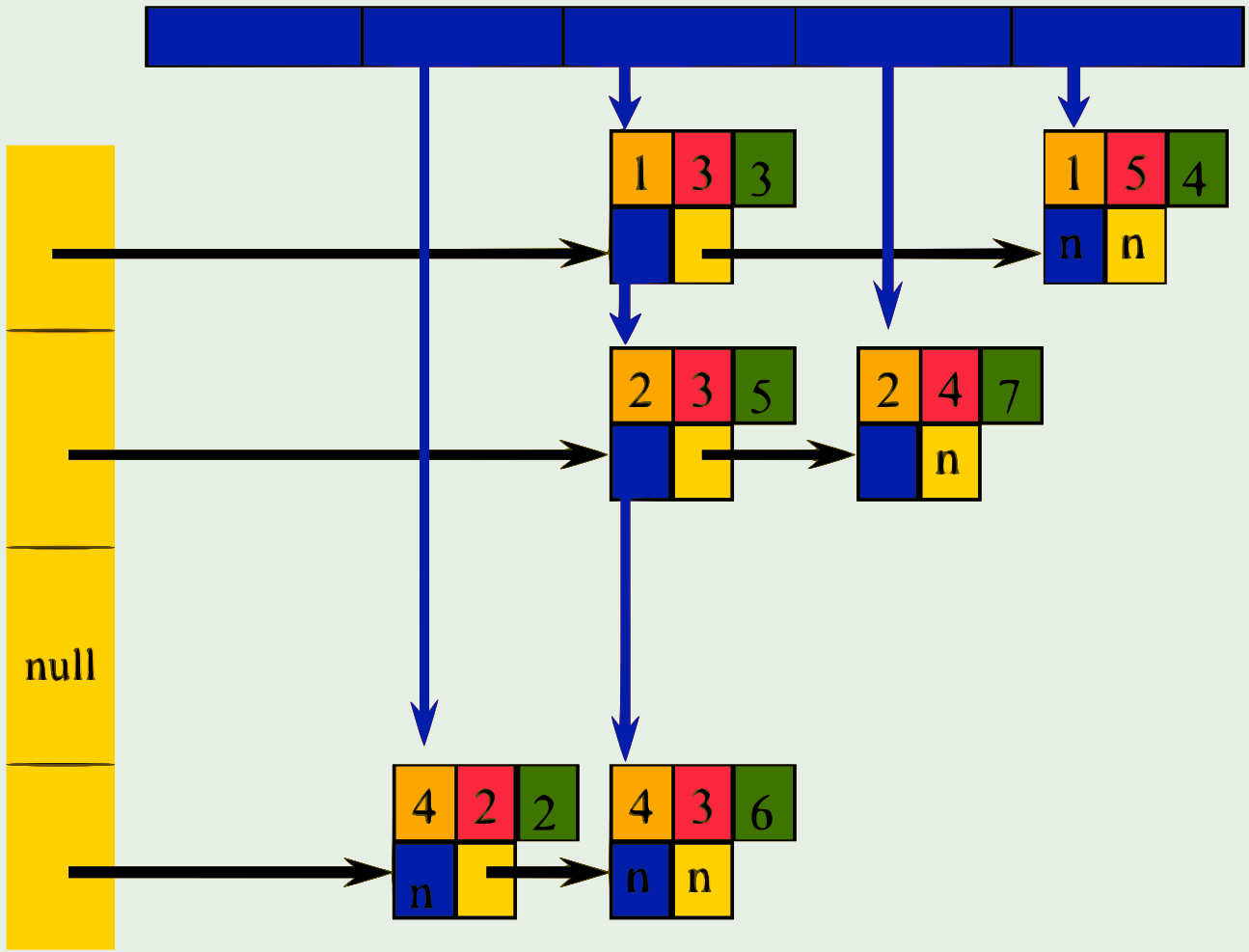
Column Lists

Column Direction



Orthogonal Lists

All Together



Go One Step Further

Question

We have another problem the sparse arrays in the column and row pointers!!!

Thus

How it will look like for this (You can try!!!)

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix} \tag{3}$$

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Do not worry!!!

You have MANY VARIATIONS!!!

Approximate Memory Requirements

Example

500 x 500 matrix with 1994 nonzero elements

- 2D array $500 \times 500 \times 4 = 1$ million bytes
- Single Array List $3 \times 1994 \times 4 = 23{,}928$ bytes
- One Chain Per Row $23928 + 500 \times 4 = 25{,}928$
- What about the sparse version even in row and column pointer?

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Run-time Performance

Example Matrix Transpose

500 x 500 matrix with 1994 nonzero elements:

Method	Time
2D Array	210 ms
Single Array List	6 ms
One Chain per Row	12 ms

Example Matrix Addition

500 x 500 matrix with 1994 nonzero elements:

Method	Time
2D Array	880 ms
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