

# Data Structures

## Stacks

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# Outline

## 1 Introduction

- Insertion and Deletion
- ADT

## 2 Examples

- Parentheses Matching
- Towers of Hanoi
- Chess
- Method Invocation And Return
- Method Invocation And Return
- Rat In A Maze

## 3 Implementation

- Derivation From ArrayLinearList
- Derivation From Chain

## 4 Code Snippets

# Introduction

## Definition of a stack

It is a linear list where:

- One end is called top
- Other end is called bottom

Additionally, adds and removes are at the top end only.

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Stores a set of elements in a particular order.

## Second

Stack principle: LAST IN FIRST OUT = LIFO

## Meaning

It means: the last element inserted is the first one to be removed

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# Example in Real Life

## Stack of Coins



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## Example

Insert the following items into a stack

List = {A, B, C, D, E}

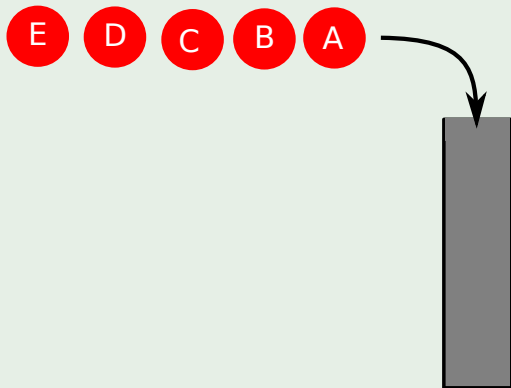
## Example

List = {A, B, C, D, E}



## Example

List = {A, B, C, D, E}, Push A



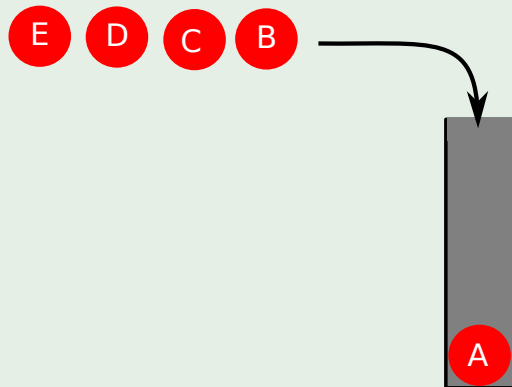
## Example

List = {B, C, D, E}



## Example

List = {B, C, D, E}, Push B





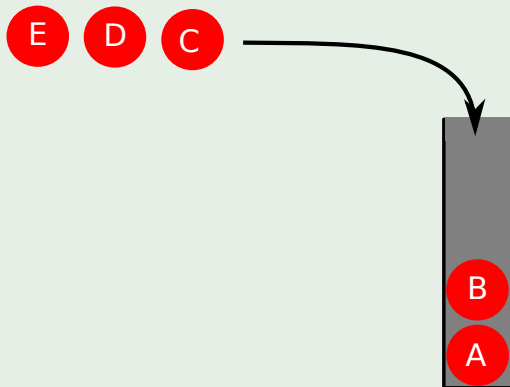
## Example

List = {C, D, E}



## Example

List = {C, D, E}, Push C



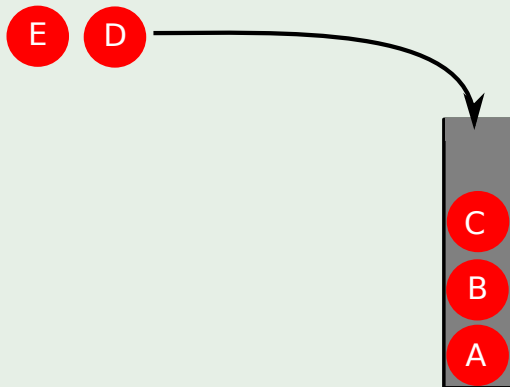
## Example

List = {D, E}



## Example

List = {D, E}, Push D



# Example

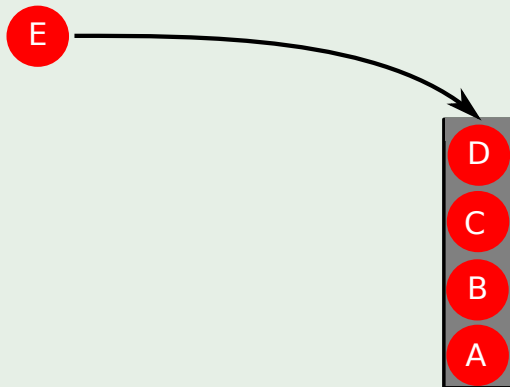
List = {E}

E

D  
C  
B  
A

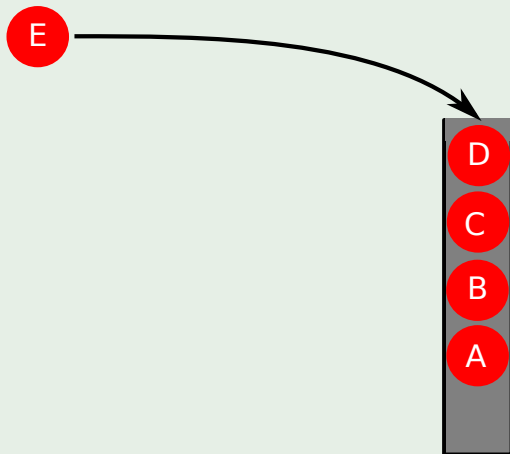
## Example

List = {E}, Push E



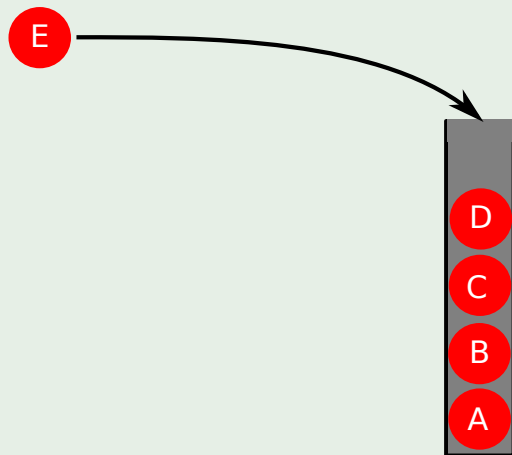
## Example

List = {E}, No space!!! Make Space



## Example

List = {E}, Push E





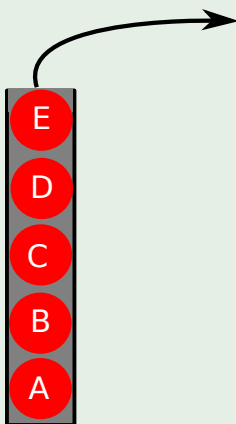
# Example

List = {}



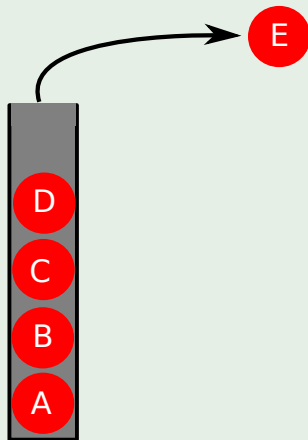
## Example

List = {}, Pop



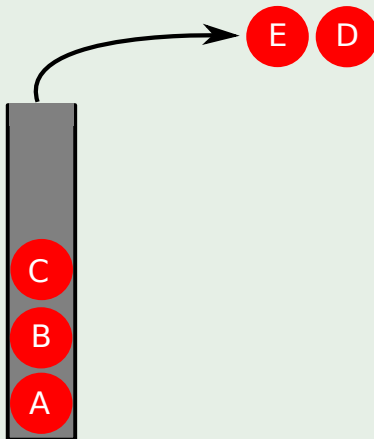
## Example

List = {E}, Pop



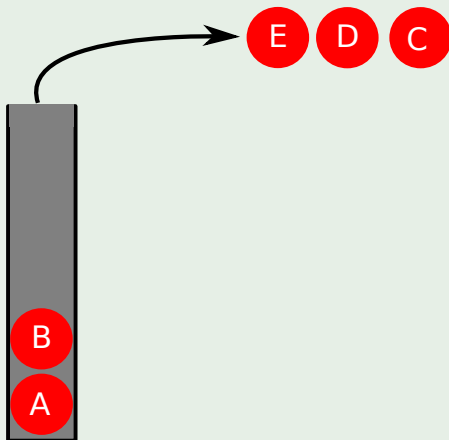
## Example

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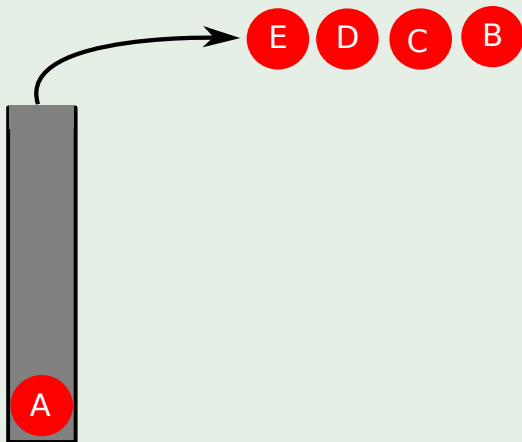
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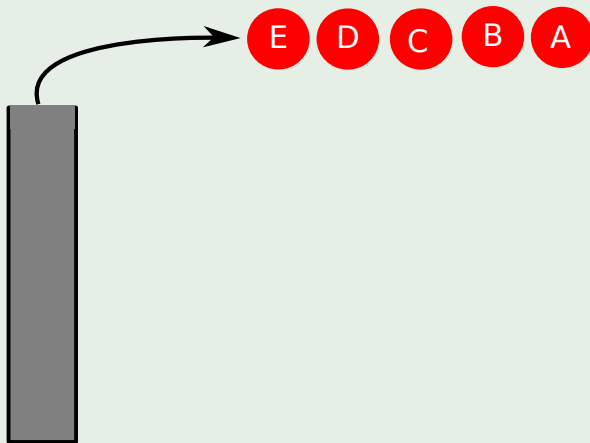
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List = {E,D,C,B,A}



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# Stacks ADT

## Interface

```
public interface Stack<Item>
{
    public boolean empty();
    public Item peek();
    public void push(Item TheObject);
    public Item pop();
}
```

# Explanation of the ADT I

`peek()`

This method allows to look at the top of the stack without removing it!!!

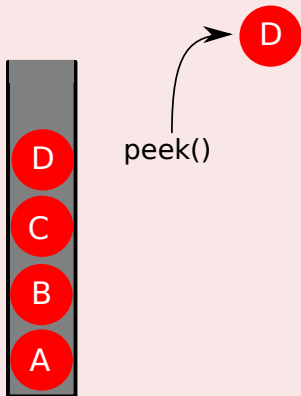
For Example

# Explanation of the ADT I

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This method allows to look at the top of the stack without removing it!!!

For Example



## Explanation of the ADT II

**pop()**

This method allows to pop stuff from the top of the stack!!!

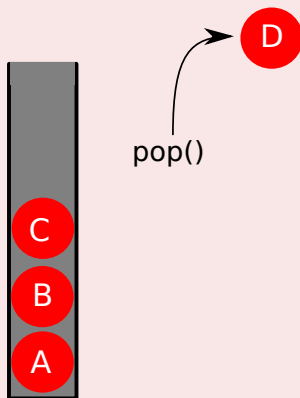
For Example

# Explanation of the ADT II

`pop()`

This method allows to pop stuff from the top of the stack!!!

For Example



# Explanation of the ADT III

**push()**

This method allows to push stuff to the top of the stack!!!

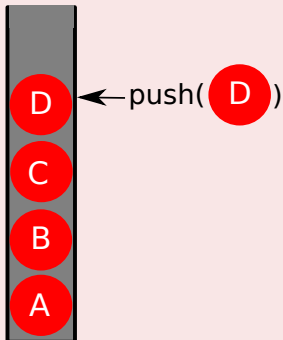
For Example

# Explanation of the ADT III

`push()`

This method allows to push stuff to the top of the stack!!!

For Example



## Explanation of the ADT III

`empty()`

This method allows to know if the stack is empty!!!



# Stack Applications

## Real life

- Pile of books
- Plate trays

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## More applications related to computer science

- Program execution stack (You will know about this in OS or CA)
- Evaluating expressions

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Instead of going toward the implementations!!!

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## Why not examples?

- ➊ Parentheses Matching
- ➋ Towers Of Hanoi/Brahma
- ➌ Switch Box Routing
- ➍ Try-Throw-Catch in Java
- ➎ Rat In A Maze

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# Parentheses Matching

## Example - Input

$((a+b)*c+d-e)/(f+g)-(h+j))$

(	(	(	<i>a</i>	+	<i>b</i>	)	*	<i>c</i>	+	<i>d</i>	-	<i>e</i>	)	/	...
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	...
(	<i>f</i>	+	<i>g</i>	)	-	(	<i>h</i>	+	<i>j</i>	)	)				
15	16	17	18	19	20	21	22	23	24	25	26				

## Output

Output pairs  $(u,v)$  such that the left parenthesis at position  $u$  is matched with the right parenthesis at  $v$ .

## Or

$(2,6)$   $(1,13)$   $(15,19)$   $(21,25)$   $(0,26)$

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$(a+b)*((c+d)$

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$(a+b))*((c+d)$

## Output

❶ (0,4)

❷ WRONG!!! Right parenthesis at 5 has no matching left parenthesis

❸ (8,12)

❹ WRONG!!! Left parenthesis at 7 has no matching right parenthesis



# Wrong Matching

## Input

$(a+b))*((c+d)$

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# Developing a Recursive Solution I

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What do we do? Ideas

Look at this

What if we have  $(a+b)$ ?

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# Initial Idea

```
boolean Rec-Paren(Chain List)
```

```
1 if (List.get(0)=='(')
```

```
2     List.remove(0)
```

```
3     return Rec-Paren(List)
```

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What else?



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Now

What else?

## Next Case

Cont...

2. else if (List.get(0)=='[0-9]|[+-]')

- List.remove(0)

- return Rec-Paren(List)

# Next Case

## Cont...

2. else if (List.get(0)=='[0-9][+-]')

① List.remove(0)

② return Rec-Paren(List)

## Last Step

3. else if (List.get(0)=='')

① List.remove(0)

② return true

3. else return false

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# Developing a Recursive Solution II

What if you have?

What if we have  $(a+b)$ ?

What about

What if we have  $a+b$ )?

## Developing a Recursive Solution II

What if you have?

What if we have  $(a+b)$ ?

What about

What if we have  $a+b$ )?

This solution fails!!!

So, we need to send something down the recursion

What do we do? Ideas

# What about...?

what if

We send down a flag!!

Thus

We need a flag to send down the recursion!!!

To tell the logic if we saw a left parenthesis

OK

For simple problems fine!!! However...

Thus

We need a flag to send down the recursion!!!

To tell the logic if we saw a left parenthesis

Ok

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Then

For problems like these ones

$((a+b)$

Nope

It does not work



Then

For problems like these ones

$((a+b)$

Nope

It does not work

We need something more complex

A counter!!!

To see how many “(” we have seen down the recursion!!!

# Recursive Solution - You assume a list of characters

```
public static boolean Balanced(ChainLinearList List ,
                                int Counter){
    if (List.isEmpty()){ // Check empty
        if (Counter == 0) // Check Counter
            return true;
        else
            return false;
    }
    if (List.get(0)=='(') // Case (
    {
        Counter++;
        List.remove(0);
        return Balanced(List , Counter);
    }
    else if (List.get(0)=='[0-9]|[+-]') // Case Number or +-
    {
        List.remove(0);
        return Balanced(List , Counter);
    }
    else if (List.get(0)=='') // Case )
    {
        if (Counter > 0){
            Counter--;
            List.remove(0);
            return Balanced(List , Counter);
        }
        else return false;
    }
    else return false
}
```

# Can we simplify our code?

Yes

Using this memory container the STACK!!!

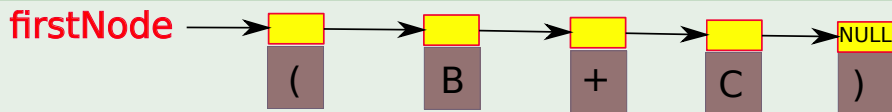
## Before Anything Else

Did you notice something about my Chain during the recursion?

What?

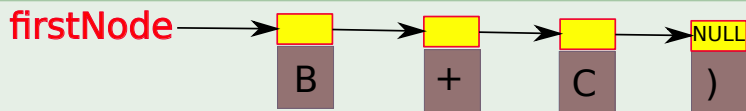
Yepi!!!

## Removing Elements



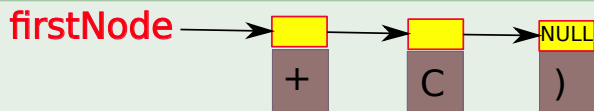
Yepi!!!

## Removing Elements



Yepi!!!

## Removing Elements





So, we can simulate the recursion using what?

First a way to...

ITERATE

A memory storage

A Stack

So somebody?

Can give a process for it?

So, we can simulate the recursion using what?

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So somebody?

Can give a process for it?

## Iterative Solution - You assume a list of characters

```
public static boolean Balanced(ChainLinearList List){
    Stack It = new Stack();
    if (List.isEmpty())
        return true
    while (!List.isEmpty()){ // Use a loop
        if (List.get(0)=='(')
        {
            It.push(List.get(0));
            List.remove(0);
        }
        else if (List.get(0)=='[0-9]|[+-]')
        { List.remove(0) }
        else if (List.get(0)=='')
        {
            if (!It.empty())
                It.pop();
            else
                return false;
            List.remove(0); // Yes remove the
                        // last ")"
        }
    }
    if (It.empty())
        return true;
    return false
}
```

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# The Puzzle

## The Legend

- There is a story about an Indian temple in Kashi Vishwanath which contains a large room with three time-worn posts in it surrounded by 64 golden disks.

## Something Notable

- Brahmin priests, acting out the command of an ancient prophecy, have been moving these disks, in accordance with the immutable rules of the Brahma, since that time.

## So

According to the legend, when the last move of the puzzle will be completed, the world will end.

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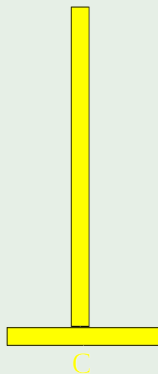
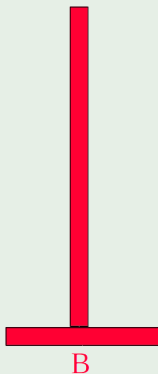
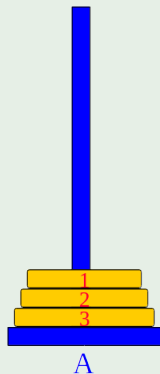
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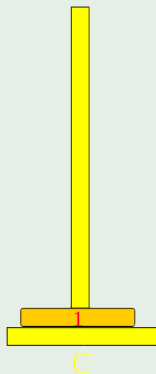
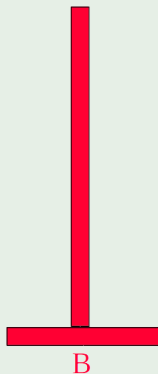
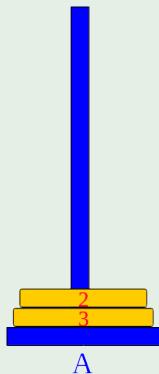
## A simple example with 3 disks

The objective of the puzzle is to move the entire stack to the end rod



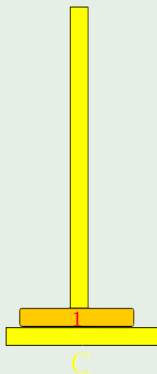
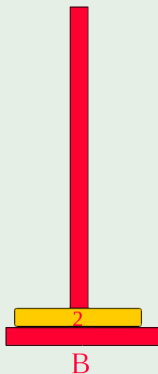
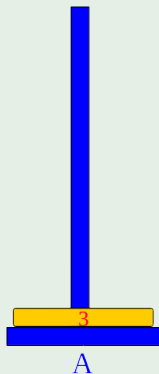
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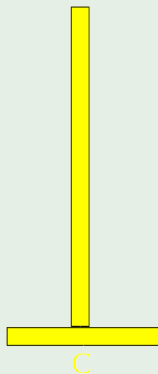
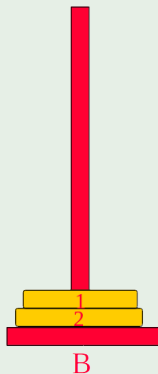
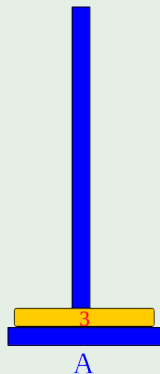
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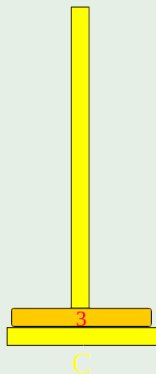
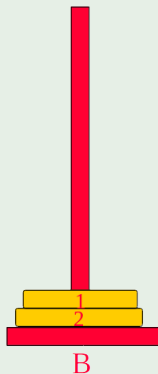
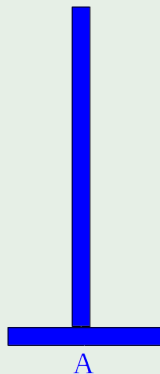
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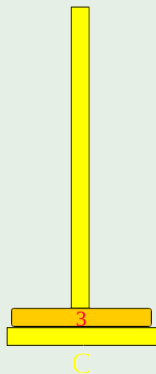
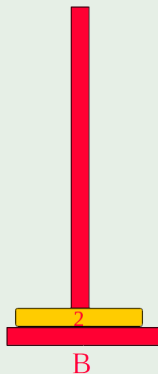
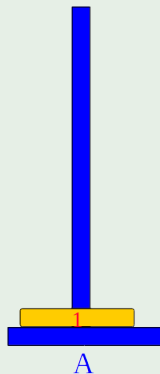
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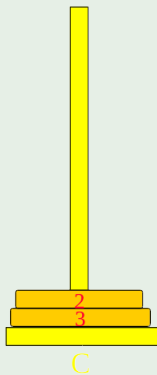
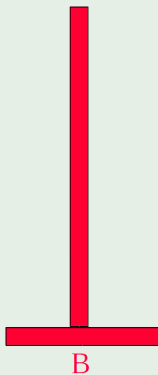
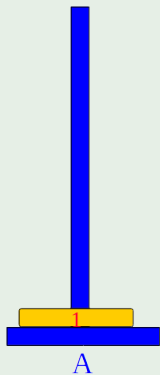
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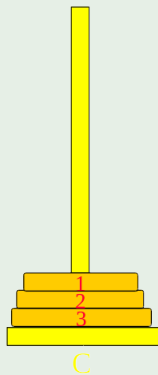
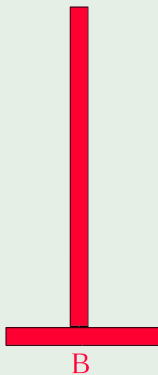
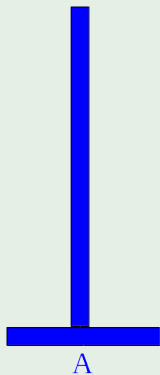
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# The Rules

## First One

Only one disk can be moved at a time.

## Second One

Each move consists of taking the upper disk from one of the stacks and placing it on top of another stack i.e. a disk can only be moved if it is the uppermost disk on a stack.

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With Three disk

You can solve it in seven moves

Thus, the minimum number of moves required for  $n$  disks

$$2^n - 1$$

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So how we solve the problem recursively?

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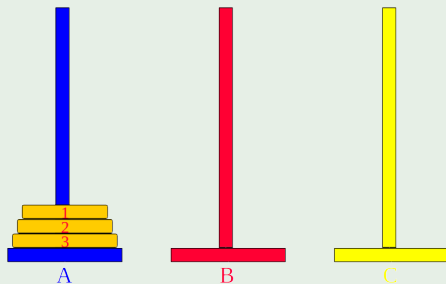
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# Recursive Idea

Label the posts



## The other things

- Let  $n$  be the total number of discs.
- Number the discs from 1 (smallest, topmost) to  $n$  (largest, bottommost)



# Next

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# Divide and Conquer

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It is an important algorithm design paradigm based on multi-branched recursion.

### Divide

- This phase of the algorithm works by recursively breaking down a problem into two or more sub-problems of the same (or related) type.

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# Divide and Conquer

## The Conquer II

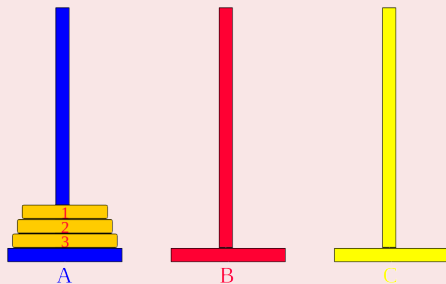
- The solutions to the sub-problems are then combined to give a solution to the original problem.

# Question!!

First

What is the clever thing to do?

Example



# A Simple Divide Phase

## Move everything

The things that are in front of disk  $n$ .

- In the previous case  $n = 3$

OK.

How the recursion looks as simple logic steps?



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# The Initial Recursion

## Logical Steps

To move  $n$  discs from tower A to tower C:

- Move  $n - 1$  discs from A to B using C as a temporary!!!
- This leaves disc  $n$  alone on tower A move disc  $n$  from A to C.
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# Recursive Solution

## Code

```
// Assume the list in A is in decreasing order
public static String TH(int n, Char Origin ,
                        Char Destination ,
                        Char Temp){

    String result;

    if (n == 1){
        return "Move_Disk_"+n+"_from_"+ Origin +
               "_to_" + Destination + "\n";
    }

    result = TH(n-1, Origin , Temp, Destination);
    result+= "Move_Disk_"+n+"_from_"+ Origin +
             "_to_" + Destination + "\n";
    result+= TH(n-1,Temp, Destination , Origin);

    return results;
}
```

# What do we need for the iterative solution?

Storage for the disks

Use one stack per tower!!!

A while loop to simulate the recursion

Doing What?

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# Simple Logic to the Iterative Version

## Logic

A simple solution for the toy puzzle:

- 1. Alternate moves between the smallest piece and a non-smallest piece.
- 2. When moving the smallest piece, always move it to the next position in the same direction.
- 3. If there is no tower position in the chosen direction, move the piece to the opposite end, but then continue to move in the correct direction.

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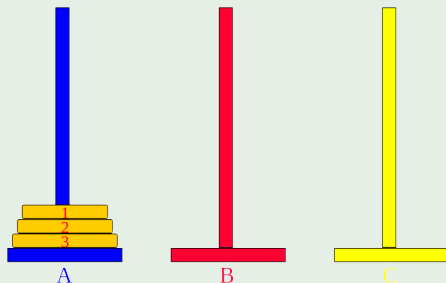
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## We have two cases

When there are  $n$  disks

$n$  is even

When there are  $n$  disk

$n$  is odd

Then a really simple solution involving  $2^{n-1} - 1$  moves

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We have for  $n$  even

Use Circular moves

$$A \implies B \implies C \implies A \quad (2)$$

We have for  $n$  odd

Use Circular Moves

$$A \implies C \implies B \implies A \quad (3)$$

## IterativeHT using three stacks : towerA, towerB, towerC

- ➊ for  $i = 1$  to  $2^{n-1}$ 
  - ➋ If  $(n \% 2 == 0)$ 
    - ➌ Select the smallest disk at the top of the stacks
    - ➍ Move the smallest disk one position in the direction of the cyclic order for even order.
    - ➎ Move the next largest disk to the only possible stack
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## You need a node state

### ① Number of Disks

● Origin Tower

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### Iterative-Hanoi(n)

- ❶ let S be an stack
- ❷ let Result be a empty string
- ❸ S.push(Node(n, A, C, B, Result))
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- ⑦         print "Move one from v.Origin to v.Destination \n"
- ⑧     else
- ⑨         S.push(Node(v.n-1, v.Temp, v.Destination, v.Origin))
- ⑩         S.push(Node(1, v.Origin, v.Destination, v.Temp))
- ⑪         S.push(Node(v.n-1, v.Origin, v.Temp, v.Destination))

We have then the following logic procedure

### Iterative-Hanoi(n)

- ① let S be an stack
- ② let Result be a empty string
- ③ S.push(Node(n, A, C, B, Result))
- ④ while S is not empty
- ⑤     v = S.pop()
- ⑥     if (v.n == 1)
- ⑦         print "Move one from v.Origin to v.Destination \n"
- ⑧     else
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- 8     else
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## What about the Legend?

To solve the towers of Hanoi for 64 disks

We need  $\approx 1.8 * 10^{19}$  moves

With a computer making  $10^6$  moves/second

A computer would take about 570 years to complete.



## What about the Legend?

To solve the towers of Hanoi for 64 disks

We need  $\approx 1.8 * 10^{19}$  moves

With a computer making  $10^9$  moves/second

A computer would take about 570 years to complete.

At 1 disk move/min

- The monks will take about  $3.4 \times 10^{13}$  years.
- The sun will destroy the life on earth in  $2.8 \times 10^9$ .

# Outline

- 1 Introduction
  - Insertion and Deletion
  - ADT
- 2 **Examples**
  - Parentheses Matching
  - Towers of Hanoi
  - **Chess**
  - Method Invocation And Return
  - Method Invocation And Return
  - Rat In A Maze
- 3 Implementation
  - Derivation From ArrayList
  - Derivation From Chain
- 4 Code Snippets

## As in the Tower of Hanoi

It is possible to devise a massive search solution based in the actual state of the chess board.

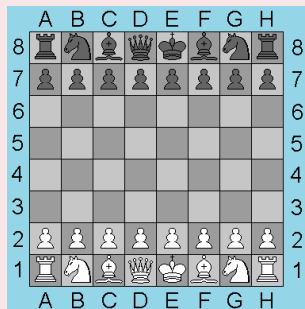
State 0

# Chess

## As in the Tower of Hanoi

It is possible to devise a massive search solution based in the actual state of the chess board.

## State 0



Then

### Something Notable

- If you put 1 penny for the first square, 2 for next, 4 for next, 8 for next, and so on.

You have

- $\$3.6 \times 10^{17}$  (federal budget  $\sim 2 \times 10^{12}$ ) .



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- Method Invocation And Return
- Rat In A Maze

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# Method Invocation And Return

We have stack in the memory system

```
public void a()  
{ ...; b(); ...}
```



return address in a()

# Method Invocation And Return

We have stack in the memory system

```
public void a()  
{ ...; b(); ...}  
public void b()  
{ ...; c(); ...}
```

return address in b()  
return address in a()

# Method Invocation And Return

We have stack in the memory system

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public void a()  
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public void c()  
{ ...; d(); ...}
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```
return address in c()  
return address in b()  
return address in a()
```

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# Try-Throw-Catch

## Thus

- ➊ When you enter a try block, push the address of this block on a stack.
- ➋ When an exception is thrown, pop the try block that is at the top of the stack (if the stack is empty, terminate).
- ➌ If the popped try block has no matching catch block, go back to the preceding step.
- ➍ If the popped try block has a matching catch block, execute the matching catch block.

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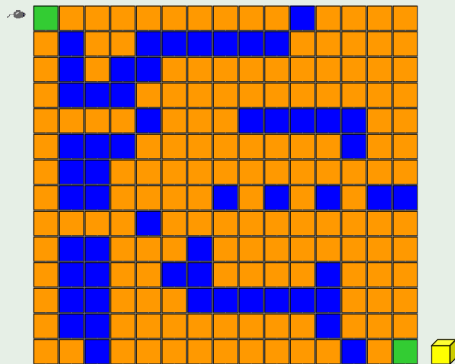
## 3 Implementation

- Derivation From ArrayList
- Derivation From Chain

## 4 Code Snippets

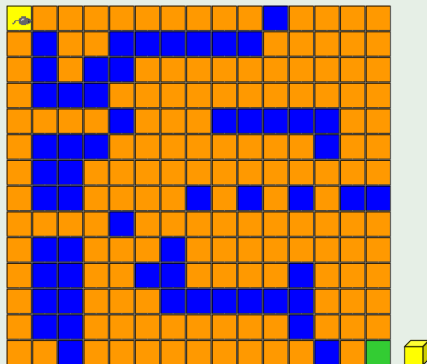
# Rat In A Maze

## Example



# Rat In A Maze

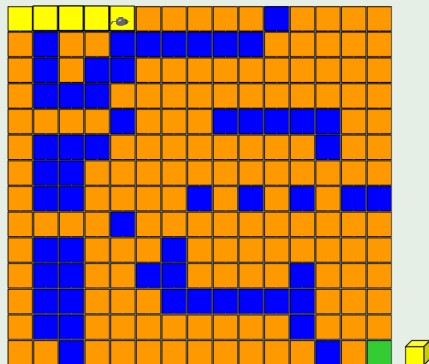
## Example



- Move order is: right, down, left, up
- Block positions to avoid revisit.

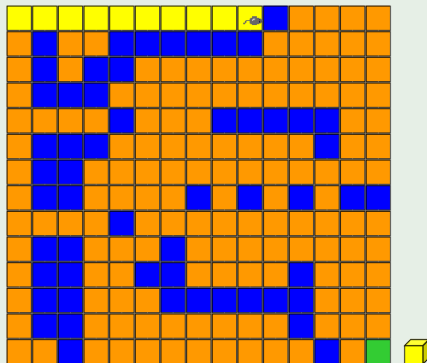
# Rat In A Maze

## Example



# Rat In A Maze

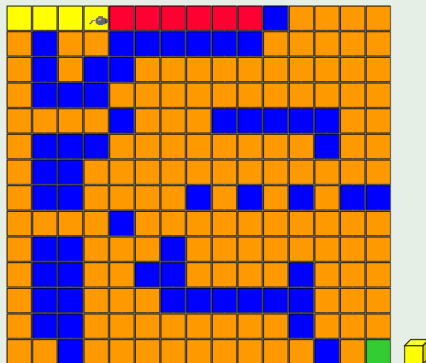
## Example



- Move backward until we reach a square from which a forward move is possible.

# Rat In A Maze

## Example

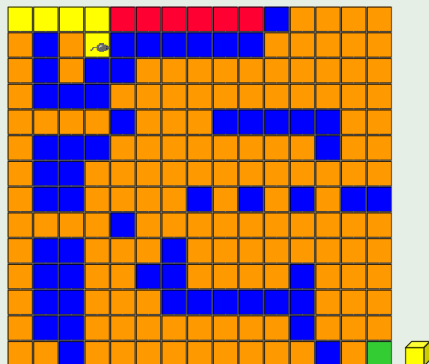


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# Rat In A Maze

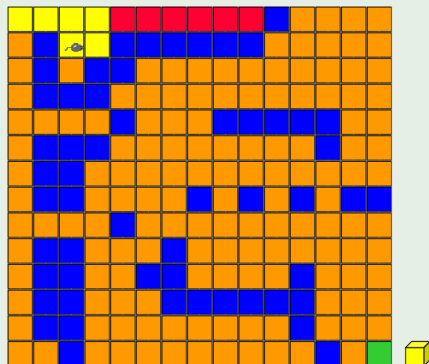
## Example



● MOVE DOWN

# Rat In A Maze

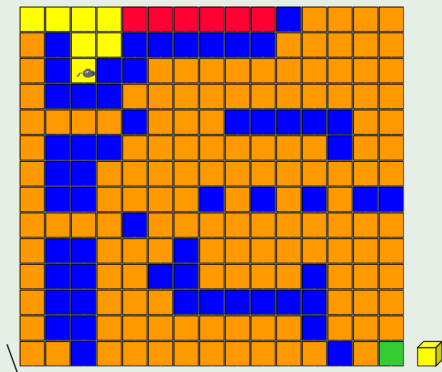
## Example



● MOVE LEFT

# Rat In A Maze

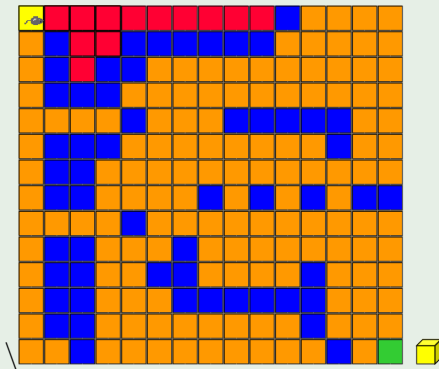
## Example



● MOVE DOWN

# Rat In A Maze

## Example



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- Derivation From Chain

## 4 Code Snippets

# Derivation From ArrayLinearList

We can do the following

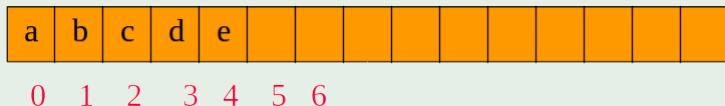
Stack top is either left end or right end of linear list



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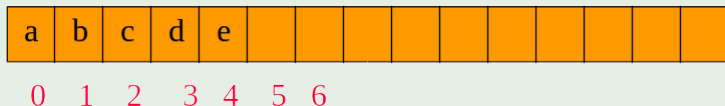
For any possible top

- `empty()`  $\implies$  `isEmpty()`
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- `peek()`  $\implies$  `get(0)` or `get(size() - 1)`
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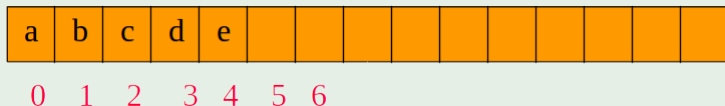
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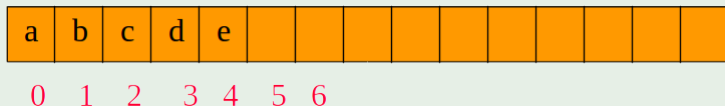
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# Derivation From ArrayLinearList

## When top is left end of linear list

- $\text{push}(\text{theObject}) \implies \text{add}(0, \text{theObject})$ 
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- $\text{pop}() \implies \text{remove}(0)$ 
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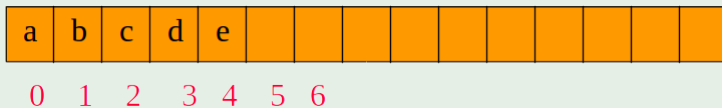
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When top is right end of linear list

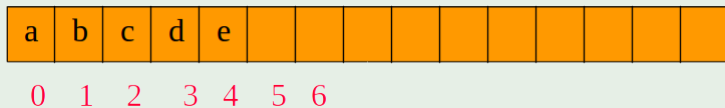


For any possible top

- `push(theObject) ==> add(size(), theObject)`
  - ▶  $O(1)$  time
- `pop() ==> remove(size()-1)`
  - ▶  $O(1)$  time

# Derivation From ArrayList

When top is right end of linear list



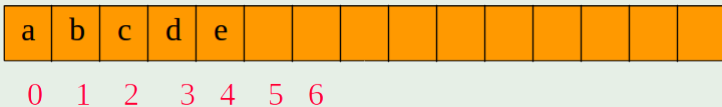
For any possible top

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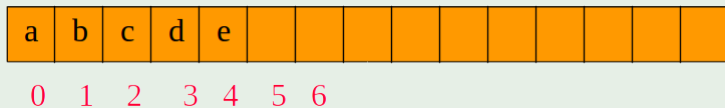


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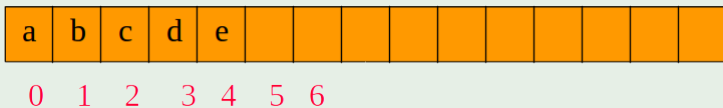


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Thus

## The Moral of the Story

You must use the right end of list as top of stack

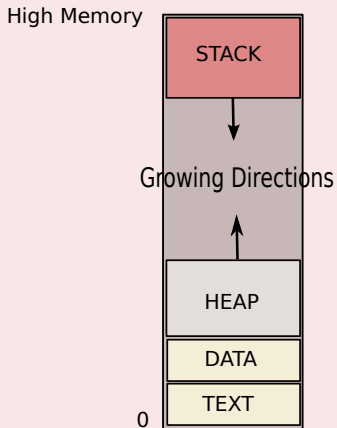
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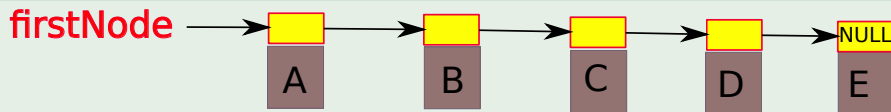
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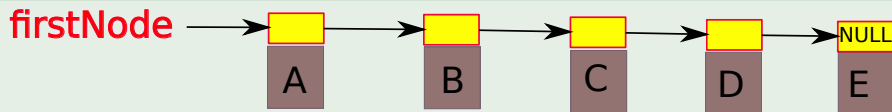
## Derivation from a Chain

Stack top is either left end or right end of linear list



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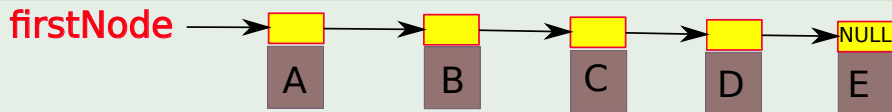
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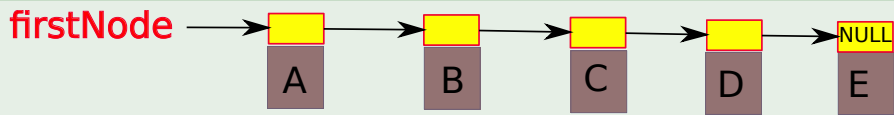


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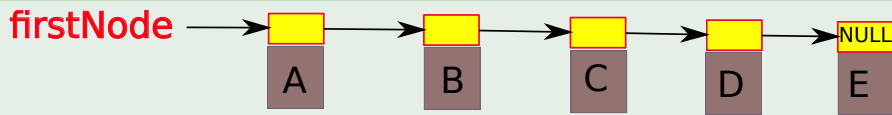
## Derivation from a Chain

When top is left end of linear list



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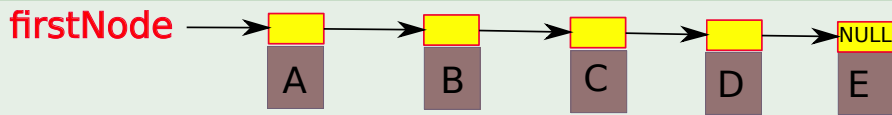


Thus

- `peek() ⇒ get(0)`
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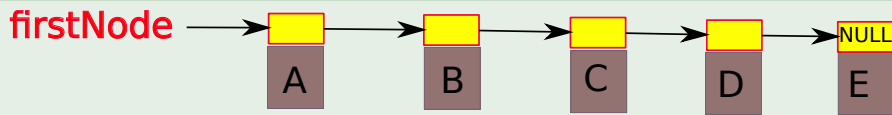


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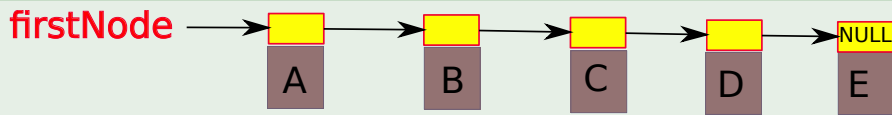


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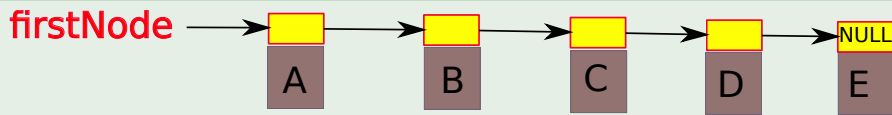


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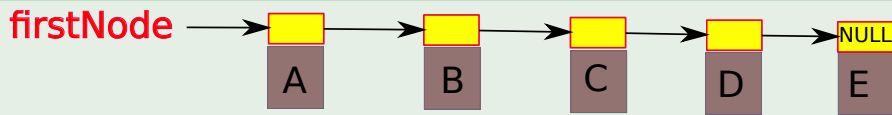


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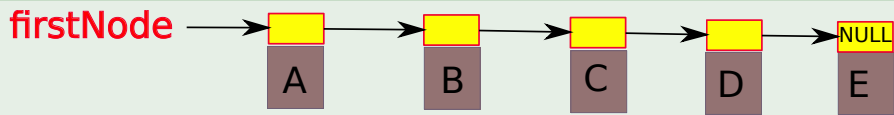
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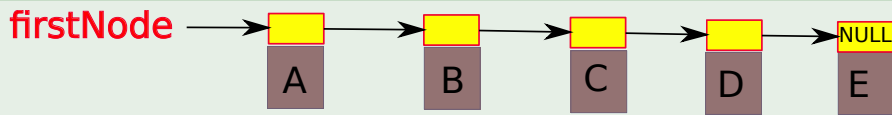
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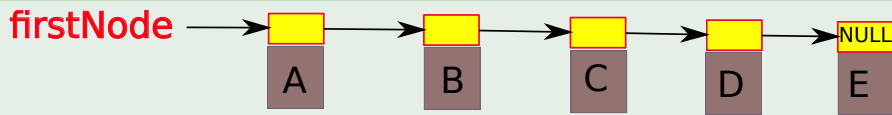


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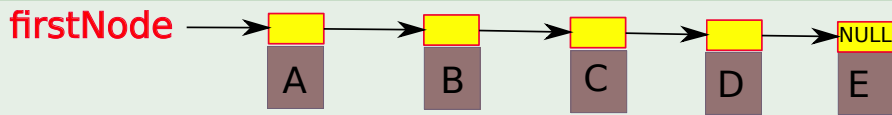


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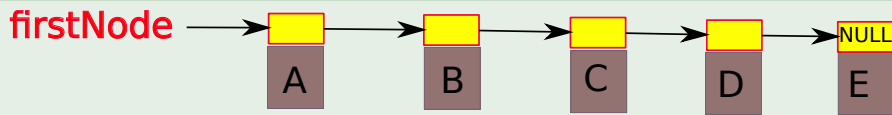


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# Derivation from a Chain

When top is left end of linear list

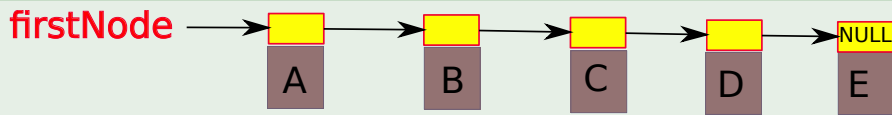


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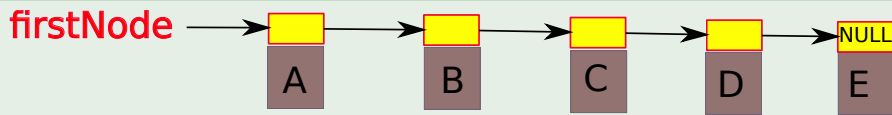


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## The Moral of the Story

You must use the left end of list as top of stack



# Derive From ArrayList

## Code

```
package MyStack;
import java.util.*; // It has stack exception
public class DerivedArrayStack extends
    ArrayList<Item>
{
    // constructors come here
    // Stack interface methods come here
}
```

# Derive From ArrayList

## Constructors

```
/** create a stack with the given initial  
 * capacity */  
public DerivedArrayStack(int initialCapacity)  
    {super(initialCapacity);}   
/** create a stack with initial capacity 10 */  
public DerivedArrayStack()  
    {this(10);}
```

## empty() And peek()

### Code

```
public boolean empty()
{
    return isEmpty();
}

public Object peek()
{
    if (empty())
        throw new EmptyStackException();
    return get(size() - 1);
}
```

## push(theObject) And pop()

### Code

```
public void push(Object theElement)
    {add(size(), theElement);}

public Object pop()
{
    if (empty())
        throw new EmptyStackException();
    return remove(size() - 1);
}
```

# The Pros

## The merits of deriving from ArrayLinearList

- Code for derived class is quite simple and easy to develop.
- Code is expected to require little debugging.
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- All public methods of `ArrayLinearList` are performed following the stack structure:
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So the derived code runs slower than necessary.

# Moral of the Story

## First

Code developed from scratch will run faster but will take more time (cost) to develop.

## Second

Tradeoff between software development cost and performance.

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It could be easy to develop the code first and later refine it to improve performance.

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# Code From Scratch

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- Use an int variable top.
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# Code From Scratch

## Data Member

```
package Stack;
import java.util.EmptyStackException;
import utilities.*;
public class ArrayStack<Item>
    implements Stack<Item>
{
    // data members
    int top;           // current top of stack
    Item [] stack;    // element array

    // Etc
}
```

# Constructors

## Code

```
public ArrayStack(int initialCapacity)
{
    if (initialCapacity < 1)
        throw new IllegalArgumentException
            ("initialCapacity must be ≥ 1");
    this.stack = (Item[]) new Object[initialCapacity];
    top = -1;
}
public ArrayStack()
{ this(10); }
```

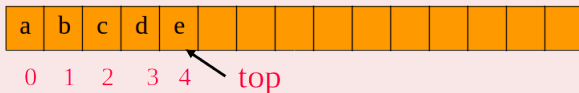


# Push(...)

## Code

```
public void push(Object theElement)
{
    // increase array size if necessary
    if (top == stack.length - 1)
        stack = ChangeArrayLength.changeLength1D
            (stack, 2 * stack.length);
    // put theElement at the top of the stack
    stack[++top] = theElement;
}
```

## Thus

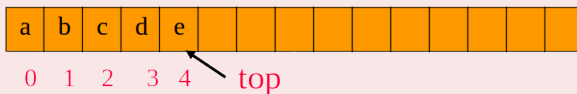


# Pop()

## Code

```
public Item pop()
{
    if (empty())
        throw new EmptyStackException();
    Object topElement = stack[top];
    stack[top--] = null; // enable garbage collection
    return topElement;
}
```

## Thus



# Actually

We have the following

`java.util.Stack`

- It derives from `java.util.Vector`.
- `java.util.Vector` is an array implementation of a linear list.

## Performance of 500,000 pop, push, and peek operations

We have

Class	500,000
DerivedArrayStack	0.38 s
Scratch Array Stack	0.22 s
DerivedArrayStackWithCatch	0.33 s
DerivedLinkedStack	3.20 s
Scratch LinkedStack	2.96 s