Data Structures

Introduction Algorithmic Thinking and Recursion

August 11, 2016

Outline

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Imagine the following

We have the following array of elements (Data Structure) $x_i \in \mathbb{R}$

1	2	3	4	5	6	7	8	9
x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9

- What if you are in one extreme or another?

- You need to scan from one extreme to the next... cost $O(n)$!!!

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How do we find the peak k?

- Ideas?
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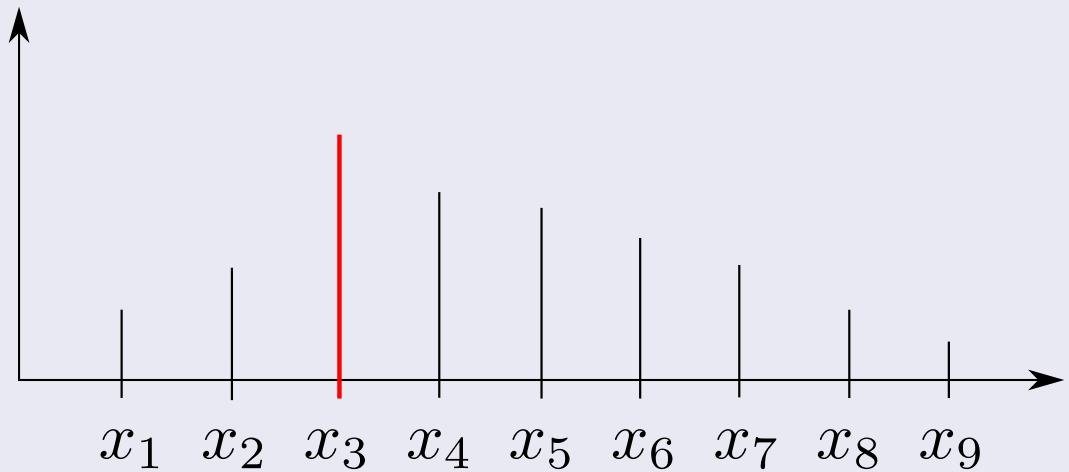

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What? if we have a new $index = \frac{n}{2}$

We can ask if:

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Go to the Right for the Peak!!!

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What is the complexity of this simple Strategy?

We have the following

$$T(n) = \underbrace{1+1+1+1+1+...+1}_{\text{Some Number of Times}}$$

Then what?

Imagine that you divide your input by 2 every time, then you stop when

$$\frac{n}{2^h} = 1$$

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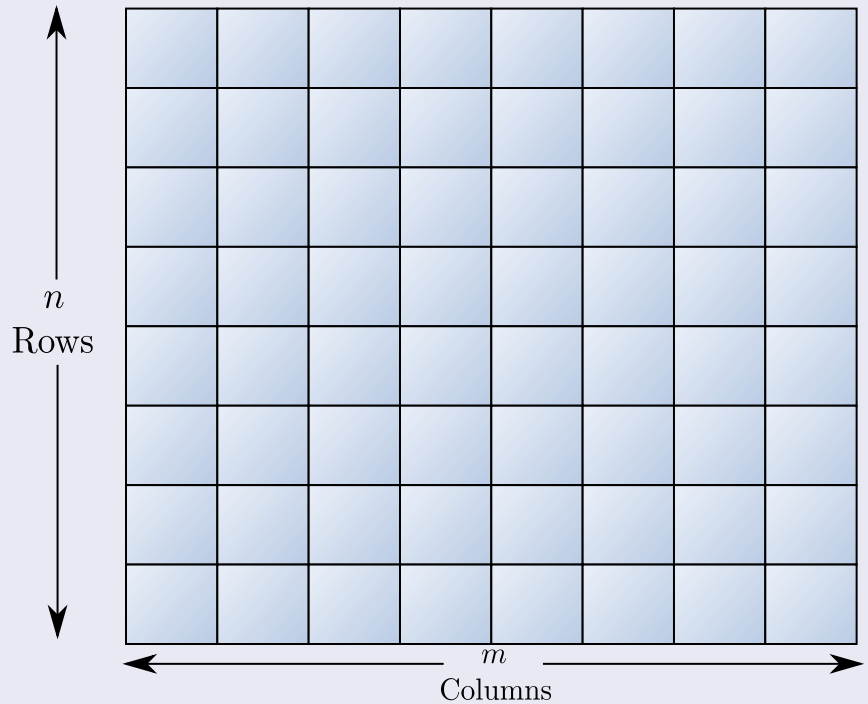

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A simple Greedy method

- Look for the larges one will take $O(nm) = O(n^2)$ if $n = m$

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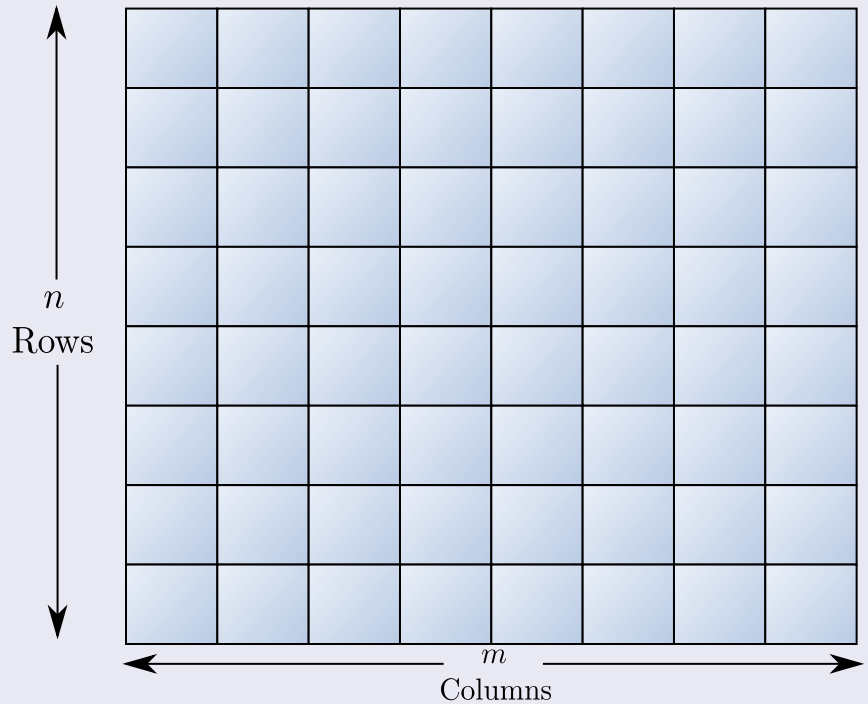

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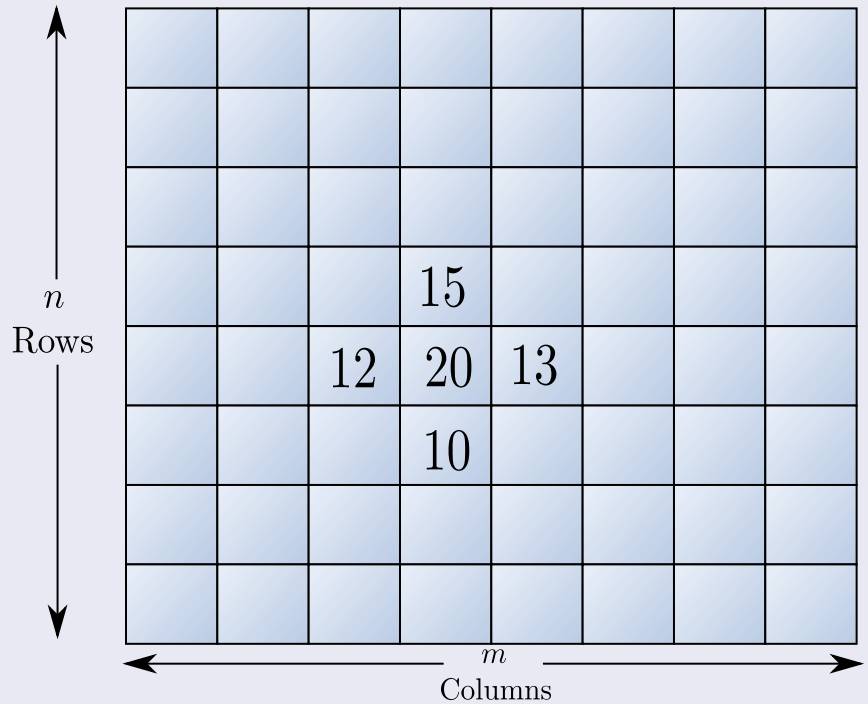

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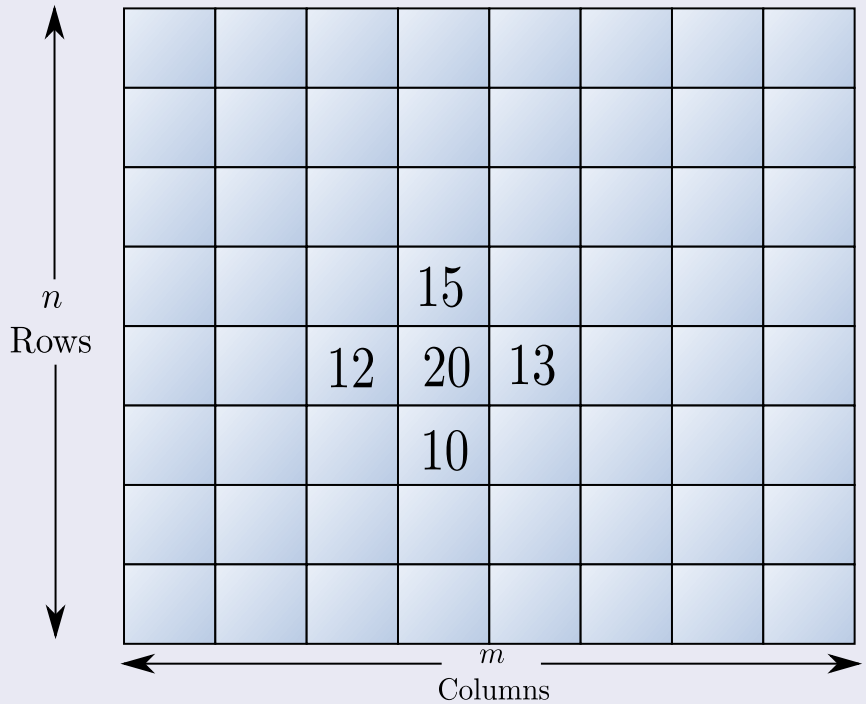

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Restrict Our Problem

We decide to have something like this

- Each column has a increasing-decreasing sequence!!!

Thus

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Second Attempt

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What Complexity?

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What about this?

We use the new constraint

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I propose the following

Find the peak of each column then?

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What do we need to find the peak between all the column-peaks?

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of steps for finding column peaks + # steps for finding the global peak

Then, we have

$$c_1 m \log n + c_2 m$$

In Big O notation

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Therefore... Algorithms are

We have that

Algorithms are procedures operating in data structures!!!

Therefore... given a data structure

We need to think about the algorithms operating on it!!!

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Another Example

Example: Principal Idea You do your part until your friend finishes and tells you

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A formal definition

Definition (Successor Function)

We define the successor operator:

- the function $S(x)$ that takes a number x to its successor.
- This gives one the nonnegative integers $\mathbb{N}_0 = \{0, 1, 2, \ldots\}$.

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$$add(x, z) = \begin{cases} x & \text{if } z = 0 \\ S(add(x, y)) & \text{if } z = S(y) \end{cases}$$

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We can prove that $2 + 2 = 4$

$$\begin{aligned} add\,(2,2) &= S\,(add\,(2,1)) \\ &= S\,(S\,(add\,(2,0))) \\ &= S\,(S\,(2)) \\ &= S\,(3) \\ &= 4 \end{aligned}$$

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$$P_1(x, y) \equiv x \text{ and } P_2(x, y) \equiv y \tag{1}$$

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There are two operations that make new functions from old:

- Composition and Primitive Recursion.

Definition (Composition)

Composition replaces the arguments of a function by another. For example,

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A function is primitive recursive if it can be built up using the base functions and the operations of composition and primitive recursion.

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Multiplication

$$mul\left(x,0\right)=0$$
$$mul\left(x,S\left(y\right)\right)=add\left(x,mul\left(x,y\right)\right)$$

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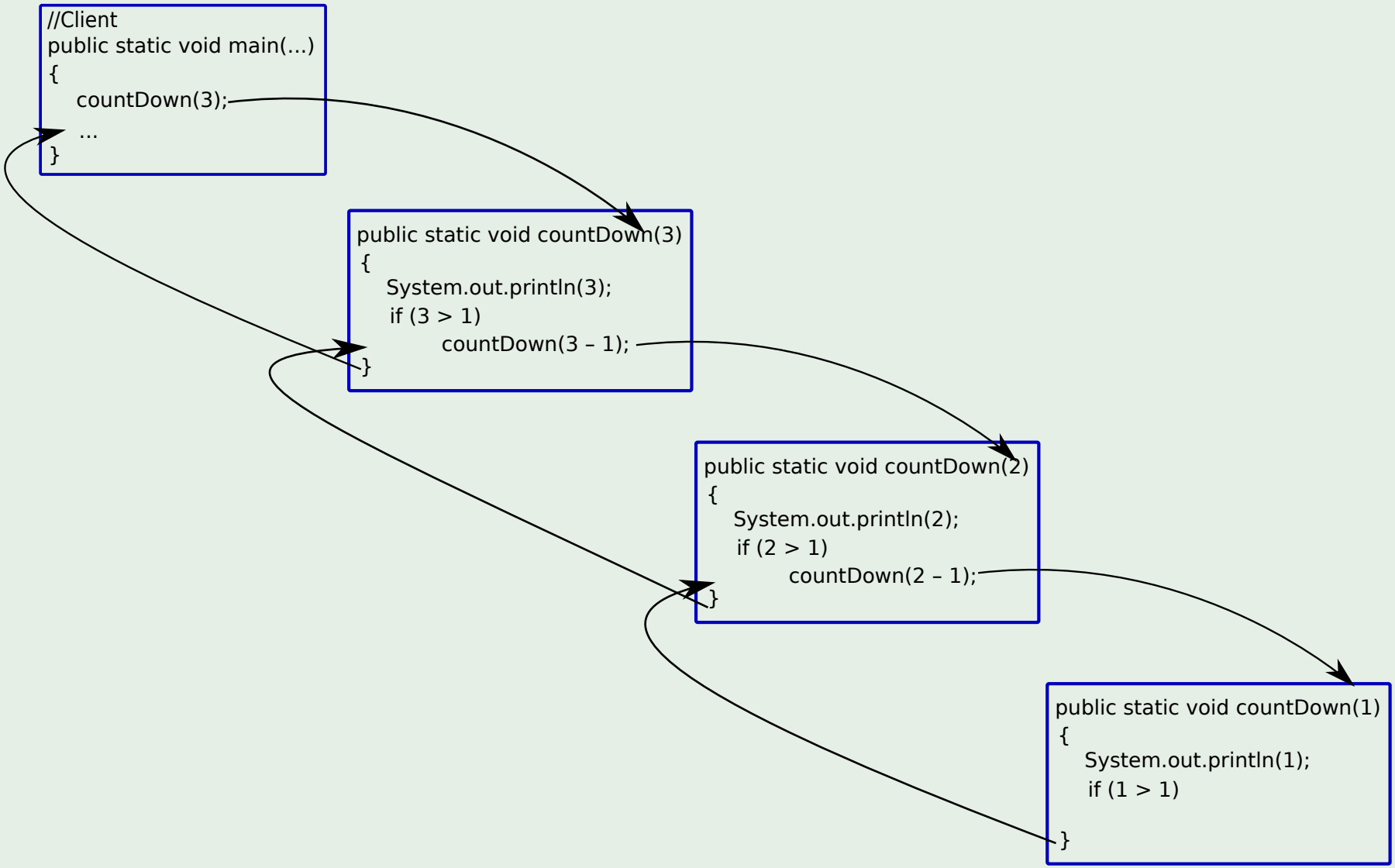
/** Counts down from a given positive integer.
@param integer an integer > 0 */
public static void countDown(int integer)
{
    System.out.println(integer);
    if (integer > 1)
        countDown(integer - 1);
} // end countDown
```

Example with 3

Tracing the recursive call countDown(3)



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Questions to answer when designing a recursive solution

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What part of the solution can you contribute directly?

Second

What smaller but identical problem has a solution that, when taken with your contribution, provides the solution to the original problem?

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When does the process end? That is, what smaller but identical problem has a known solution, and have you reached this problem, or base case?

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The method countDown displays the given integer first.

Answer 2

The smaller problem is counting down from **integer - 1**.

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3. RESTRICTION: There are no Boolean expressions over the alphabet other than those obtained from 1 and 2.

Remember Discrete Mathematics

During the definition of structural induction

A version of mathematical induction that is used to prove properties of recursively defined sets.

For example Boolean expressions

1. BASE: Each symbol of the alphabet is a Boolean expression.
2. RECURSION: If P and Q are Boolean expressions, then so are
   1. $P \wedge Q$
   2. $P \vee Q$
   3. $\neg P$
3. RESTRICTION: There are no Boolean expressions over the alphabet other than those obtained from 1 and 2.

Now, Recursive Functions

Recursive Functions

A function is said to be defined recursively or to be a recursive function if its rule of definition refers to itself.

We can use this functions to define the number of steps taken by recursive algorithms

For example:

$$T(n) = \begin{cases} 1 & \text{if } n \text{ is } 1 \\ T\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + 1 & \text{if } n > 1 \end{cases} \tag{2}$$

It describes

The total number of steps of the binary search in an ordered set of number using recursion.

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We can use our theory of solving recursive functions

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Outline

Design guidelines for successful recursion

First

The method must be given an input value.

Second

The method definition must contain logic that involves this input value and leads to different cases.

Third

One or more of these cases should provide a solution that does not require recursion. These are the base cases, or stopping cases.

Fourth

One or more cases must include a recursive invocation of the method using a "smaller" argument.

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Infinite recursion

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Outline

Example of code with infinite recursion

Code

```
/** Counts down from a given positive integer.
@param integer an integer > 0 */
public static void countDown(int integer)
{
    System.out.println(integer);
    countDown(integer - 1);
} // end countDown
```

Recursive Methods That Return a Value

The nice stuff about recursion

You can return values

For example

We can compute the sum $1 + 2 + 3 + \ldots + n$

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How the code will look

Partial Code

```
/** @param n an integer > 0
    @return the sum 1 + 2 + ... + n */
public static int sumOf(int n)
{
int sum;

// What do we put here?

return sum;
} // end sumOf
```

How To Debugging a Recursive Method

GENERAL RULES

1. Does the method have at least one input value?
2. Does the method contain a statement that tests an input value and leads to different cases?
3. Did you consider all possible cases? Does at least one of these cases cause at least one recursive call?
4. Do these recursive calls involve smaller arguments, smaller tasks, or tasks that get closer to the solution?
5. If these recursive calls produce or return correct results, will the method produce or return a correct result?
6. Is at least one of the cases a base case that has no recursive call?
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Outline

Now, what if we solve a recursive function

What we want to solve

n choose k (combinations)

We have

$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}, \text{ with } 1 < k < n \tag{3}$$

Base Case

$$\binom{n}{1} = n(k=1), \binom{n}{n} = 1(k=n) \tag{4}$$

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Base Code so you can start

Base Code

```
class Combinations{
        public static int Combinations(int n, int k)
        {

        }

        public static void main(String[] args)
        {

        }
}
```