Sieve of Eratosthenes

2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
32	33	34	35	36	37	38	39	40	41	42	43	44	45	46
47	48	49	50	51	52	53	54	55	56	57	58	59	60	61

Complexity: $\Theta(n \ln \ln n)$

Pseudo-code

1. Create list of unmarked natural numbers 2, 3, …, n
2. $k \leftarrow 2$
3. Repeat
   (a) Mark all multiples of k between k^2 and n
   (b) $k \leftarrow$ smallest unmarked number $> k$
   until $k^2 > n$
4. The unmarked numbers are primes

Parallelizing the Algorithm

3.a Mark all multiples of k between k^2 and n

$\Rightarrow$

for all j where $k^2 \leq j \leq n$ do
  if j mod $k = 0$ then
    mark j (it is not a prime)
  endif
endfor

3.b Find smallest unmarked number > k

$\Rightarrow$

Min-reduction (to find smallest unmarked number > k)
Broadcast (to get result to all tasks)

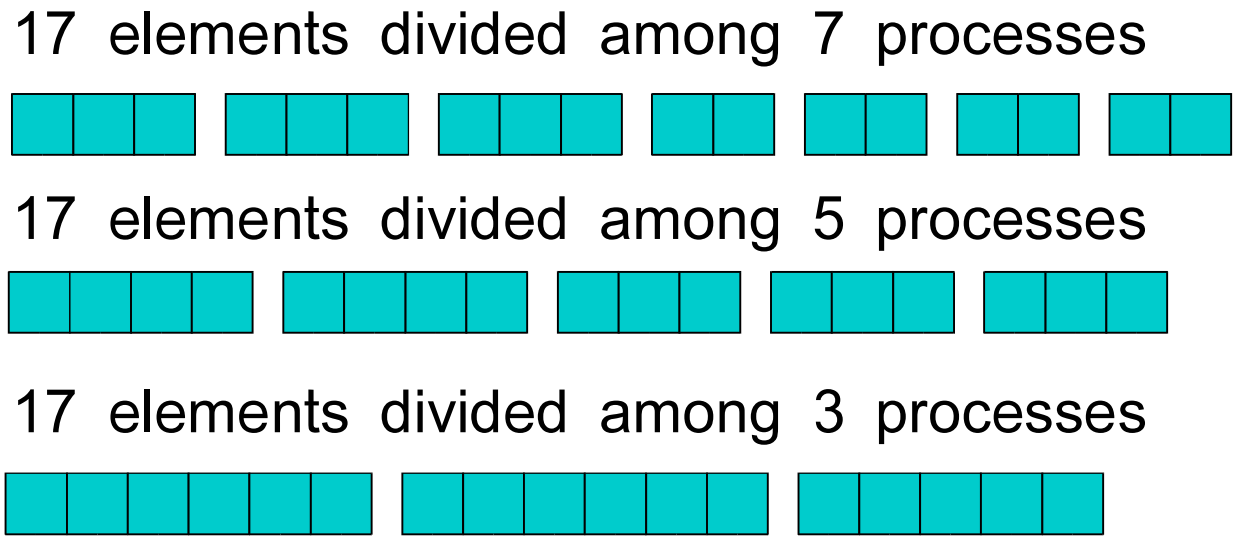
Block Decomposition Method #1

Let $r = n \bmod p$

If $r = 0$, all blocks have same size

Else

- First r blocks have size n/p
- Remaining $p-r$ blocks have size n/p



Block Decomposition Method #2

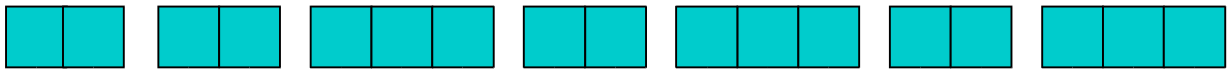
Scatters larger blocks among processes

First element controlled by process i $\lfloor in/p \rfloor$

Last element controlled by process i $\lfloor (i+1)n/p \rfloor - 1$

Process controlling element j $\lfloor p(j+1) - 1)/n \rfloor$

17 elements divided among 7 processes



17 elements divided among 5 processes

17 elements divided among 3 processes

Comparing Block Decompositions

Our choice

Operation	Method 1	Method 2
Low index	4	2
High index	6	4
Owner	7	4

Assuming no operations for “floor” function

Block Decomposition Macros

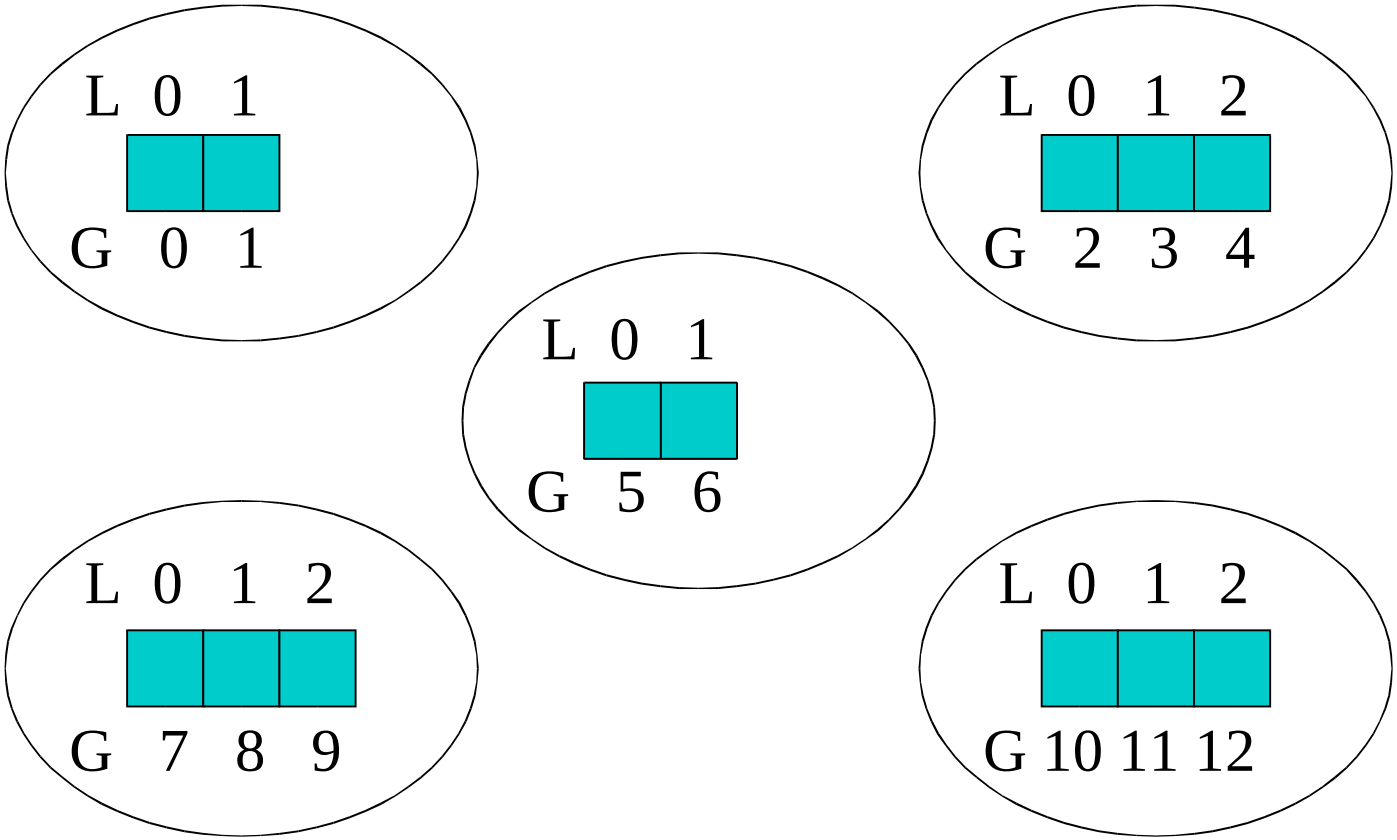
```
#define BLOCK_LOW(id,p,n)  ((i)*(n)/(p))

#define BLOCK_HIGH(id,p,n) \
        (BLOCK_LOW((id)+1,p,n)-1)

#define BLOCK_SIZE(id,p,n) \
        (BLOCK_LOW((id)+1)-BLOCK_LOW(id))

#define BLOCK_OWNER(index,p,n) \
        (((p)*(index)+1)-1)/(n))
```

Local and Global Indicies



Looping with Local/Global Index

- Sequential program

```
for (i = 0; i < n; i++) {
    …
}
```

- Parallel program

```
size = BLOCK_SIZE (id,p,n);
for (i = 0; i < size; i++) {
   gi = i + BLOCK_LOW(id,p,n);
}
```

Index *i* on this process…

…takes place of sequential program's index *gi*

Parallel Algorithm

1. Create list of unmarked natural numbers 2, 3, …, n

2. $k = 2$

Each process creates its share of list

Each process does this

3. Repeat

Each process marks its share of list

(a) Mark all multiples of k between k^2 and n

(b) k = smallest unmarked number > k Process 0 only

(c) Process 0 broadcasts k to rest of processes

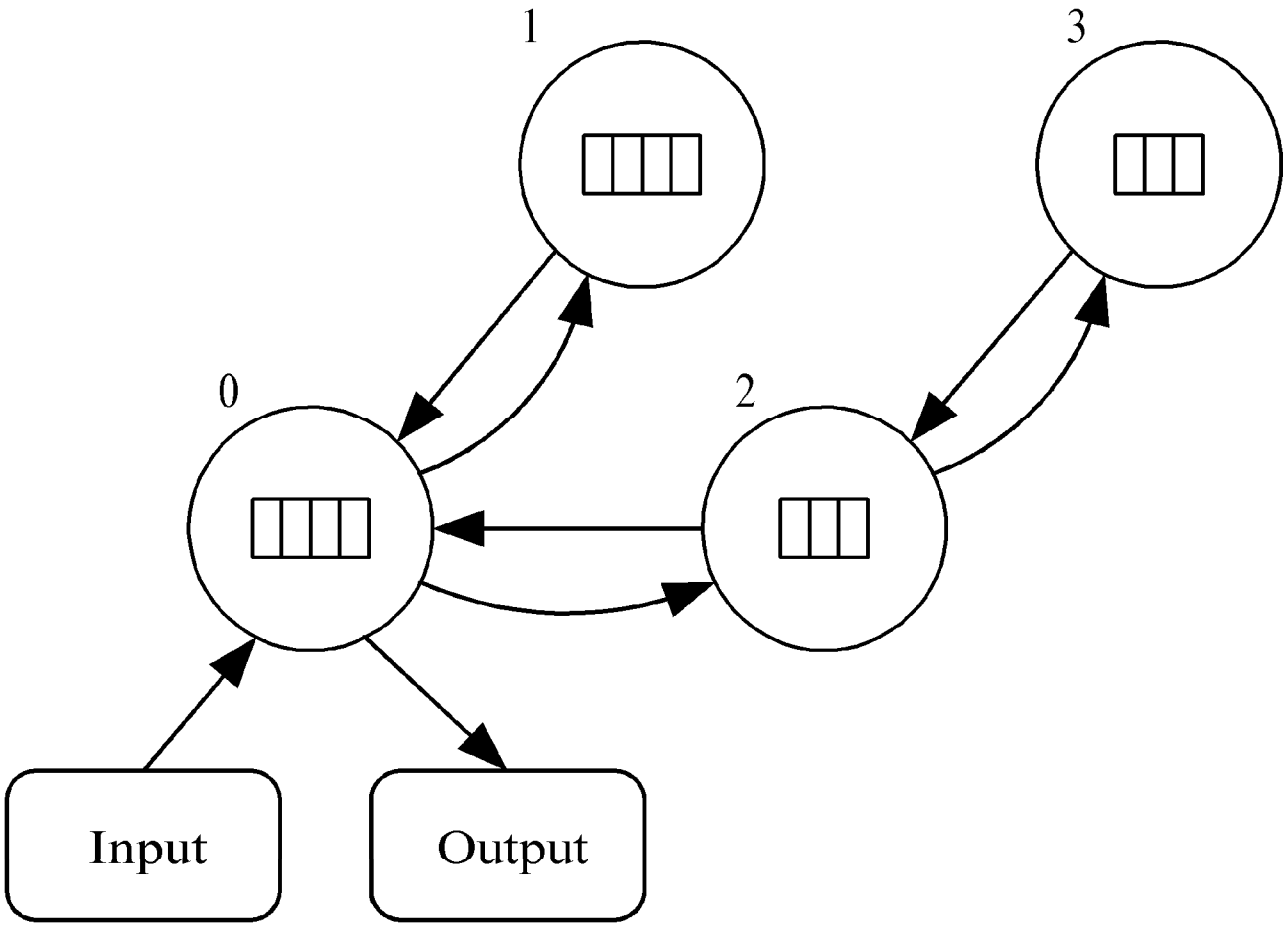
until $k^2 > m$

4. The unmarked numbers are primes

5. Reduction to determine number of primes

$$\chi\left(n \operatorname{lnln} n\right)/p + \left(\sqrt{n}/\ln\sqrt{n}\right)\lambda\lceil \log p \rceil$$

Task/Channel Graph



Code (part 1)

```
#include <mpi.h>
#include <math.h>
#include <stdio.h>
#include "MyMPI.h"
#define MIN(a,b)  ((a)<(b)?(a):(b))

int main (int argc, char *argv[])
{
   ...
   MPI_Init (&argc, &argv);
   MPI_Barrier(MPI_COMM_WORLD);
   elapsed_time = -MPI_Wtime();
   MPI_Comm_rank (MPI_COMM_WORLD, &id);
   MPI_Comm_size (MPI_COMM_WORLD, &p);
   if (argc != 2) {
      if (!id) printf ("Command line: %s <m>\n", argv[0]);
      MPI_Finalize(); exit (1);
}
```

Code (part 2)

```
n = atoi(argv[1]);
low_value = 2 + BLOCK_LOW(id,p,n-1);
high_value = 2 + BLOCK_HIGH(id,p,n-1);
size = BLOCK_SIZE(id,p,n-1);
proc0_size = (n-1)/p;
if ((2 + proc0_size) < (int) sqrt((double) n)) {
   if (!id) printf ("Too many processes\n");
   MPI_Finalize();
   exit (1);
}

marked = (char *) malloc (size);
if (marked == NULL) {
   printf ("Cannot allocate enough memory\n");
   MPI_Finalize();
   exit (1);
}
```

Code (part 4)

```
for (i = 0; i < size; i++) marked[i] = 0;
if (!id) index = 0;
prime = 2;
do {
   if (prime * prime > low_value)
      first = prime * prime - low_value;
   else {
      if (!(low_value % prime)) first = 0;
      else first = prime - (low_value % prime);
   }
   for (i = first; i < size; i += prime) marked[i] = 1;
   if (!id) {
      while (marked[++index]);
      prime = index + 2;
   }
   MPI_Bcast (&prime,  1, MPI_INT, 0, MPI_COMM_WORLD);
} while (prime * prime <= n);
```

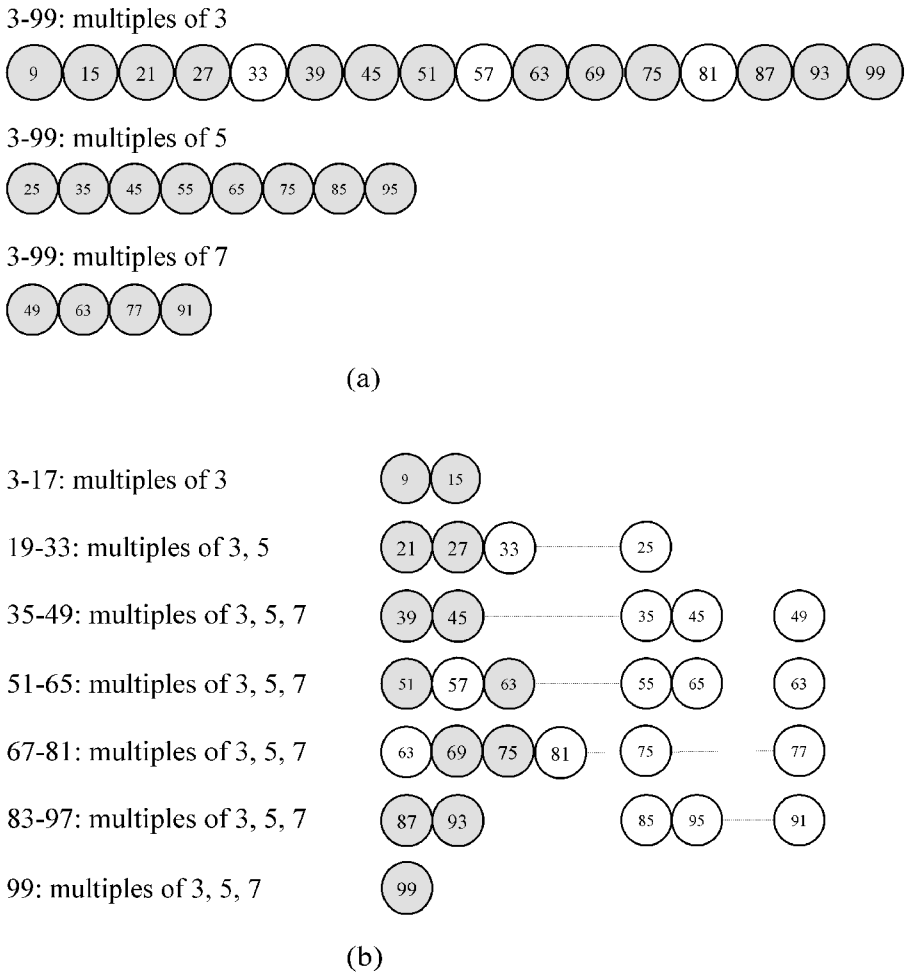
Code (part 4)

```
   count = 0;
   for (i = 0; i < size; i++)
      if (!marked[i]) count++;
   MPI_Reduce (&count, &global_count, 1, MPI_INT, MPI_SUM,
      0, MPI_COMM_WORLD);
   elapsed_time += MPI_Wtime();
   if (!id) {
      printf ("%d primes are less than or equal to %d\n",
         global_count, n);
      printf ("Total elapsed time: %10.6f\n", elapsed_time);
   }
   MPI_Finalize ();
   return 0;
}
```

Optimizations

- Delete even integers
  - Cuts number of computations in half
  - Frees storage for larger values of n
- Each process finds own sieving primes
  - Replicating computation of primes to $\sqrt{n}$
  - Eliminates broadcast step
- Reorganize loops
  - Increases cache hit rate

Loop Reorganization



User Defined Datatypes

- Methods for creating data types

```
MPI_Type_contiguous();
MPI_Type_vector();
MPI_Type_indexed();
MPI_Type_struct();
MPI_Pack();
MPI_Unpack();
```

- MPI datatypes defined similar to modern programming languages (C,C++,F90)
- Allows communication and I/O operations to use the same datatypes as rest of program
- Makes expressing the partitioning of datasets easier

Some datatype terminology

- Every MPI datatype has a few characteristics
  - type signature
    - list of the basic datatypes (in order) that make up the derived type
  - type map
    - basic dataypes
    - lower bound of each type
    - extent of the type (size + buffering)
- Some of this information is available about MPI datatypes through:

```
MPI_Get_extent
MPI_Get_size
```

Contiguous Array

- Creates an array of counts elements:

```
MPI_Type_contiguous(int count,
    MPI_Datatype oldtype,
    MPI_Datatype *newtype)
```

Strided Vector

- Constructs a cyclic set of elements

```
MPI_Type_vector(int count,
  int blocklength,
  int stride,
  MPI_Datatype oldtype,
  MPI_Datatype *newtype);
```

- Stride specified in number of elements
- Stride can be specified in bytes

  `MPI_Type_hvector();`
- Stride counts from start of block

Indexed Vector

- Allows an irregular pattern of elements

```
MPI_Type_indexed(int count,
    int *array_of_blocklengths,
    int *array_of_displacements,
    MPI_Datatype oldtype,
    MPI_Datatype *newtype);
```

- Displacements specified in number of elements
  - Displacements can be specified in bytes

```
MPI_Type_hindexed();
```

- A shortcut if all blocks are the same length:

```
MPI_Type_create_indexed_block()
```

Structured Records

- Allows different types to be combined

```
MPI_Type_struct(int count,
    int *array_of_blocklengths,
    MPI_Aint *array_of_displacements,
    MPI_Datatype *array_of_types,
    MPI_Datatype *newtype);
```

- Blocklengths specified in number of elements
- Displacements specified in bytes

Committing types

- In order for a user-defined derived datatype to be used as an argument to other MPI calls, the type must be “committed”.

```
MPI_Type_commit(type);
MPI_Type_free(type);
```

- Use commit after calling the type constructor, but before using the type anywhere else
- Call free after the type is no longer in use (no one actually does this, but it makes computer scientists happy...)

Pack and Unpack

- Packs sparse structures into contiguous memory

```
MPI_Pack(void* inbuf, int incount,
    MPI_Datatype datatype,
    void *outbuf, int outsize,
    int *position, MPI_Comm comm);
MPI_Unpack(void* inbuf, int insize,
    int *position, void *outbuf,
    int outcount,
    MPI_Datatype datatype,
    MPI_Comm comm);
```

Dealing with Groups

- A *group* is a set of tasks
- Groups are used to construct communicators
- Group accessors:

```
int MPI_Group_size(MPI_Group group, int
  *size)
int MPI_Group_rank(MPI_Group group, int
  *rank)
int MPI_Group_translate_ranks (MPI_Group
  group1, int n, int *ranks1, MPI_Group
  group2, int *ranks2)
int MPI_Group_compare(MPI_Group
  group1,MPI_Group group2, int *result)
```

Creating Groups

- Group constructors:

```
int MPI_Comm_group(MPI_Comm comm, MPI_Group *group)
int MPI_Group_union(MPI_Group group1, MPI_Group
  group2, MPI_Group *newgroup)
int MPI_Group_intersection(MPI_Group group1,
  MPI_Group group2, MPI_Group *newgroup)
int MPI_Group_difference(MPI_Group group1, MPI_Group
  group2, MPI_Group *newgroup)
int MPI_Group_incl(MPI_Group group, int n, int
  *ranks, MPI_Group *newgroup)
int MPI_Group_excl(MPI_Group group, int n, int
  *ranks, MPI_Group *newgroup)
int MPI_Group_range_incl(MPI_Group group, int n, int
  ranges[][3], MPI_Group *newgroup)
int MPI_Group_range_excl(MPI_Group group, int n, int
  ranges[][3], MPI_Group *newgroup)
```

Destroying Groups

- Group destructors:

```
int MPI_Group_free(MPI_Group *group)
```

Dealing with Communicators

- MPI collective operations deal with all the processes in a communicator
- MPI_COMM_WORLD by default contains every task in your MPI job
- Other communicators can be defined to allow collective operations on a subset of the tasks
- Communicator Accessors:

```
int MPI_Comm_size(MPI_Comm comm, int *size)
```

  - Returns the size of the group in comm

```
int MPI_Comm_rank(MPI_Comm comm, int *rank)
```

  - Returns the rank of the caller in that communicator

```
int MPI_Comm_compare(MPI_Comm comm1, MPI_Comm
  comm2,  int *result)
```

  - Returns if two communicators are the same, similar(same tasks but different ranks), or different

Creating Communicators

```
int MPI_Comm_dup(MPI_Comm comm, MPI_Comm
  *newcomm)
```

- Creates an exact copy of the communicator

```
int MPI_Comm_create(MPI_Comm comm, MPI_Group
  group, MPI_Comm *newcomm)
```

- Creates a new communicator with the contents of group
  - Group must be a subset of Comm

```
int MPI_Comm_split(MPI_Comm comm, int color,
  int key,MPI_Comm *newcomm)
```

- Creates a communicator for each distinct value of color, ranked by key

Destroying Communicators

```
int MPI_Comm_free(MPI_Comm comm)
```

- Destroys the named communicator

Topologies and Communicators

- MPI allows processes to be grouped in logical topologies
- Topologies can aid the programmer
  - Convenient naming methods for processes in a group
  - Naming can match communication patterns
  - a standard mechanism for representing common algorithmic concepts (i.e. 2D grids)
- Topologies can aid the runtime environment
  - Better mappings of MPI tasks to hardware nodes
  - Not really useful in a simple cluster environment....

Cartesian Topologies

```
int MPI_Cart_create(MPI_Comm comm_old, int ndims,
  int *dims, int *periods, int reorder, MPI_Comm
  *comm_cart)
```

- comm_old - input communicator
- ndims - # of dimensions in cartesian grid
- dims - integer array of size ndims specifying the number of processes in each dimension
- periods - true/false specifying whether each dimension is periodic (wraps around)
- reorder - ranks may be reordered or not
- comm_cart - new communicator containing new topology.

MPI_DIMS_CREATE

- A helper function for specifying a likely dimension decomposition.

```
int MPI_Dims_create(int nnodes, int
  ndims, int *dims)
```

  - nnodes - total nodes in grid
  - ndims - number of dimensions
  - dims - array returned with dimensions
- Example:

```
MPI_Dims_create(6,2,dims
```

    - will return (3,2) in dims

```
MPI_Dims_create(6,3,dims)
```

    - will return (3,2,1) in dims
- No rounding or ceiling function provided

Cartesian Inquiry Functions

- `MPI_Cartdim_get` will return the number of dimensions in a Cartesian structure

```
int MPI_Cartdim_get(MPI_Comm comm, int
  *ndims);
```

- `MPI_Cart_get` provides information on an existing topology
  - Arguments roughly mirror the create call

```
int MPI_Cart_get(MPI_Comm comm,int maxdims,
  int *dims, int *periods, int *coords);
```

  - Maxdims keeps a given communicator from overflowing your arguments

Cartesian Translator Functions

- Task IDs in a Cartesian coordinate system correspond to ranks in a "normal" communicator.
  - point-to-point communication routines (send/receive) rely on ranks

```
int MPI_Cart_rank(MPI_Comm comm, int
  *coords, int *rank)
int MPI_Cart_coords(MPI_Comm comm, int
  rank, int maxdims, int *coords)
```

- Coords - cartesian coordinates
- rank - ranks

Cartesian Shift function

```
int MPI_Cart_Shift(MPI_Comm comm, int
  direction, int disp, int *rank_source, int
  *rank_dest)
```

- direction - coordinate dimension of shift
- disp - displacement (can be positive or negative)
- rank_source and rank_dest are return values
  - Use that source and dest to call MPI_Sendrecv

Cartesian Shift Example

```
MPI_Comm ICOMM;
MPI_Status status;
int NY, srank, rrank;
int idims[2] = {4, 4};
int periods[2] = {1, 1};
void *plane1, *plane2;

MPI_Cart_create(MPI_COMM_WORLD, 2, idims, periods,
      0, &ICOMM);

MPI_Cart_shift(ICOMM, 0, 1, &rrank, &srank);
MPI_Sendrecv(plane1, NY, MPI_DOUBLE, srank,
      plane2, NY, MPI_DOUBLE, rrank,
      ICOMM, &status);

MPI_Cart_shift(ICOMM, 1, -1, &rrank, &srank);
MPI_Sendrecv(plane1, NY, MPI_DOUBLE, srank,
      plane2, NY, MPI_DOUBLE, rrank,
      ICOMM, &status);
```

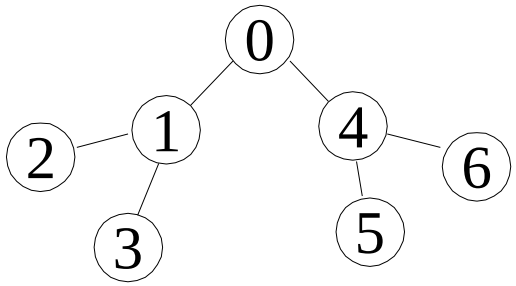
Graph Topologies

```
int MPI_Graph_create(MPI_Comm comm_old, int
  nnodes, int *index, int *edges, int reorder,
  MPI_Comm *comm_graph)
```

- nnodes - number of nodes
- index - the number of edges adjacent to all nodes 0 .. i
- edges - adjacent nodes for each node
- reorder - allow node ranks to be reordered

Thus, in C, index[0] is the degree of node zero, and index[i] - index[i-1] is the degree of node i, i=1, ..., nnodes-1; the list of neighbors of node zero is stored in edges[j], for 0 <= j <= index[0]-1 and the list of neighbors of node i, i>0, is stored in edges[j], index[i-1]+1 <= j <= index[i]-1.

Graph Example



node	adjacent nodes
0	1, 4
1	0, 2, 3
2	1
3	1
4	0, 5, 6
5	4
6	4

```
MPI_Comm newcomm;
int nnodes = 7;
int index[] = {2, 5, 6, 7, 10, 11, 12};
int edges[] = {1, 4, 0, 2, 3, 1, 1, 0, 5, 6, 4, 4};
MPI_Graph_create(MPI_COMM_WORLD, nnodes, index,
edges, 0, &newcomm);
```

Graph Inquiry Functions

```
int MPI_Graphdims_get(MPI_Comm comm, int
  *nnodes, int *nedges)
```

- Provides info needed to size index and edges

```
int MPI_Graph_get(MPI_Comm comm, int maxindex,
  int maxedges, int *index, int *edges)
```

- Get index and edges

```
int MPI_Graph_neighbors_count(MPI_Comm comm,
  int rank, int *nneighbors)
```

- Get number of neighbors

```
int MPI_Graph_neighbors(MPI_Comm comm, int
  rank, int maxneighbors, int *neighbors)
```

- Get neighbors