Benchmarking Linear Logic: sequents inspired from Kleene's intuitionistic theorems*

October 3, 2018

1 General Information

- Test run on a QEMU Virtual CPU, 2GHz, 64 bits, 2GiB of RAM running Ubuntu.
- Maude version: 2.7.1 built: Oct 2 2017.
- The search procedure considers proofs using up to 4 times the copy rule (focusing on one of the formulas of the classical context). Benchmarks 23 and 25 required the limit to be 5.

2 Translations

$\boxed{(1)\ \cdot \vdash A \to A}$

LJ (28ms)

$$\frac{}{\cdot \vdash A \to A}\ \star$$

MULTIPLICATIVE encoding (28ms)

$$\frac{\overline{\Gamma :\ A \vdash A}}{\Gamma :\ \cdot \vdash A \multimap A}\ \star$$

CALL-BY-NAME encoding (28ms)

$$\frac{\overline{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A) \multimap A}\ \star$$

CALL-BY-VALUE encoding (41ms)

$$\frac{\dfrac{\overline{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\ !}{\dfrac{\Gamma :\ \cdot \vdash !(A) \multimap !(A)}{\Gamma :\ \cdot \vdash !(!(A) \multimap !(A))}\ !}\ \star$$

01-ENC encoding (35ms)

$$\frac{\dfrac{\overline{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A) \multimap A}\ \star}{\Gamma :\ \cdot \vdash !(!(A) \multimap A)}\ !$$

$\boxed{(2)\ A \to B, B \to C \vdash A \to C}$

LJ (46ms)

$$\frac{\dfrac{}{A, A \to B, B \to C \vdash A}\ \star \quad \dfrac{\dfrac{\overline{A, B, B \to C \vdash B}\ \star \quad \overline{A, B, C \vdash C}\ \star}{A, B, B \to C \vdash C}\ \supset_L}{A, A \to B, B \to C \vdash C}\ \supset_L}{A \to B, B \to C \vdash A \to C}\ \star$$

MULTIPLICATIVE encoding (49ms)

$$\frac{\dfrac{\overline{\Gamma :\ A \vdash A} \quad \dfrac{\overline{\Gamma :\ B \vdash B} \quad \overline{\Gamma :\ C \vdash C}}{\Gamma :\ B, B \multimap C \vdash C}\ \multimap}{\Gamma :\ A, A \multimap B, B \multimap C \vdash C}\ \multimap}{\Gamma :\ A \multimap B, B \multimap C \vdash A \multimap C}\ \star$$

CALL-BY-NAME encoding (123ms)

$$\frac{\dfrac{\dfrac{\dfrac{\overline{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\ ! \quad \overline{\Gamma :\ B \vdash B}}{\Gamma :\ !(A) \multimap B \vdash B}\ \multimap}{\dfrac{\Gamma :\ \cdot \vdash B}{\Gamma :\ \cdot \vdash !(B)}\ !}\ D_C \quad \overline{\Gamma :\ C \vdash C}}{\Gamma :\ !(B) \multimap C \vdash C}\ \multimap}{\Gamma :\ \cdot \vdash C}\ D_C}{\Gamma :\ !(!(A) \multimap B), !(!(B) \multimap C) \vdash !(A) \multimap C}\ \star$$

CALL-BY-VALUE encoding (165ms)

$$\frac{\dfrac{\dfrac{\dfrac{\dfrac{\overline{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\ ! \quad \dfrac{\overline{\Gamma :\ \cdot \vdash B}}{\Gamma :\ !(B) \vdash B}\ \star}{\Gamma :\ !(A) \multimap !(B) \vdash B}\ \multimap}{\dfrac{\Gamma :\ \cdot \vdash B}{\Gamma :\ \cdot \vdash !(B)}\ !}\ D_C \quad \dfrac{\overline{\Gamma :\ \cdot \vdash C}}{\Gamma :\ \cdot \vdash !(C)}\ !}{\Gamma :\ !(B) \multimap !(C) \vdash !(C)}\ \star}{\Gamma :\ \cdot \vdash !(C)}\ \multimap}{\Gamma :\ \cdot \vdash !(A) \multimap !(C)}\ D_C}{\Gamma :\ \cdot \vdash !(!(A) \multimap !(C))}\ \star}{\Gamma :\ !(!(A) \multimap !(B)), !(!(B) \multimap !(C)) \vdash !(!(A) \multimap !(C))}\ !$$

01-ENC encoding (146ms)

$$\frac{\dfrac{\dfrac{\dfrac{\dfrac{\overline{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\ ! \quad \dfrac{\overline{\Gamma :\ \cdot \vdash B}}{\Gamma :\ !(B) \vdash B}\ \star}{\Gamma :\ !(A) \multimap !(B) \vdash B}\ \multimap}{\dfrac{\Gamma :\ \cdot \vdash B}{\Gamma :\ \cdot \vdash !(B)}\ !}\ D_C \quad \dfrac{\overline{\Gamma :\ \cdot \vdash C}}{\Gamma :\ !(C) \vdash C}\ \star}{\Gamma :\ !(B) \multimap !(C) \vdash C}\ \multimap}{\Gamma :\ \cdot \vdash C}\ D_C}{\Gamma :\ \cdot \vdash !(A) \multimap C}\ \star}{\Gamma :\ \cdot \vdash !(!(A) \multimap C)}\ !}{\Gamma :\ !(!(A) \multimap !(B)), !(!(B) \multimap !(C)) \vdash !(!(A) \multimap C)}\ \star$$

*See `https://github.com/carlosolarte/Benchmarking-Linear-Logic` for details on the encodings used.

(3) $A \to B \to C \vdash B \to A \to C$

LJ (46ms)

$$\dfrac{\dfrac{}{A, B, A \to B \to C \vdash A}\,\star \quad \dfrac{\dfrac{}{A, B, B \to C \vdash B}\,\star \quad \dfrac{}{A, B, C \vdash C}\,\star}{A, B, B \to C \vdash C}\,\supset_L}{\dfrac{A, B, A \to B \to C \vdash C}{A \to B \to C \vdash B \to A \to C}\,\star}\,\supset_L$$

MULTIPLICATIVE encoding (47ms)

$$\dfrac{\dfrac{\dfrac{}{\Gamma :\ A \vdash A} \quad \dfrac{\dfrac{}{\Gamma :\ B \vdash B} \quad \dfrac{}{\Gamma :\ C \vdash C}}{\Gamma :\ B, B \multimap C \vdash C}\,\multimap}{\Gamma :\ A, B, A \multimap B \multimap C \vdash C}\,\multimap}{\Gamma :\ A \multimap B \multimap C \vdash B \multimap A \multimap C}\,\star$$

CALL-BY-NAME encoding (71ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\,! \quad \dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash !(B)}\,! \quad \dfrac{}{\Gamma :\ C \vdash C}}{\Gamma :\ !(B) \multimap C \vdash C}\,\multimap}{\Gamma :\ !(A) \multimap !(B) \multimap C \vdash C}\,\multimap}{\Gamma :\ \cdot \vdash C}\,D_C}{\Gamma :\ !(!(A) \multimap !(B) \multimap C) \vdash !(B) \multimap !(A) \multimap C}\,\star$$

CALL-BY-VALUE encoding (162ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\,! \quad \dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash !(B)}\,! \quad \dfrac{\dfrac{}{\Gamma :\ \cdot \vdash C}}{\Gamma :\ \cdot \vdash !(C)}\,!}{\Gamma :\ !(B) \multimap !(C) \vdash !(C)}\,\star\,\multimap}{\Gamma :\ \cdot \vdash !(C)}\,D_C}{\Gamma :\ !(A) \multimap !(!(B) \multimap !(C)) \vdash !(C)}\,\star\,\multimap}{\Gamma :\ \cdot \vdash !(A) \multimap !(C)}\,D_C}{\Gamma :\ \cdot \vdash !(!(A) \multimap !(C))}\,\star}{\Gamma :\ \cdot \vdash !(B) \multimap !(!(A) \multimap !(C))}\,!}{\Gamma :\ \cdot \vdash !(!(B) \multimap !(!(A) \multimap !(C)))}\,!}{\Gamma :\ !(!(A) \multimap !(!(B) \multimap !(C))) \vdash !(!(B) \multimap !(!(A) \multimap !(C)))}\,\star$$

01-ENC encoding (158ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\,! \quad \dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash !(B)}\,! \quad \dfrac{\dfrac{}{\Gamma :\ \cdot \vdash C}}{\Gamma :\ !(C) \vdash C}\,\star}{\Gamma :\ !(B) \multimap !(C) \vdash C}\,\multimap}{\Gamma :\ \cdot \vdash C}\,D_C}{\Gamma :\ !(!(B) \multimap !(C)) \vdash C}\,\star\,\multimap}{\Gamma :\ !(A) \multimap !(!(B) \multimap !(C)) \vdash C}\,D_C}{\Gamma :\ \cdot \vdash C}}{\Gamma :\ \cdot \vdash !(A) \multimap C}\,\star}{\Gamma :\ \cdot \vdash !(!(A) \multimap C)}\,!}{\Gamma :\ \cdot \vdash !(B) \multimap !(!(A) \multimap C)}\,\star}{\Gamma :\ \cdot \vdash !(!(B) \multimap !(!(A) \multimap C))}\,!}{\Gamma :\ !(!(A) \multimap !(!(B) \multimap !(C))) \vdash !(!(B) \multimap !(!(A) \multimap C))}\,\star$$

(4) $A \to B \to C \vdash A \wedge B \to C$

LJ (47ms)

$$\dfrac{\dfrac{\dfrac{}{A, B, A \to B \to C \vdash A}\,\star \quad \dfrac{\dfrac{}{A, B, B \to C \vdash B}\,\star \quad \dfrac{}{A, B, C \vdash C}\,\star}{A, B, B \to C \vdash C}\,\supset_L}{A, B, A \to B \to C \vdash C}\,\supset_L}{A \to B \to C \vdash A \wedge B \to C}\,\star$$

MULTIPLICATIVE encoding (47ms)

$$\dfrac{\dfrac{\dfrac{}{\Gamma :\ A \vdash A} \quad \dfrac{\dfrac{}{\Gamma :\ B \vdash B} \quad \dfrac{}{\Gamma :\ C \vdash C}}{\Gamma :\ B, B \multimap C \vdash C}\,\multimap}{\Gamma :\ A, B, A \multimap B \multimap C \vdash C}\,\multimap}{\Gamma :\ A \multimap B \multimap C \vdash A \otimes B \multimap C}\,\star$$

CALL-BY-NAME encoding (71ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\,! \quad \dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash !(B)}\,! \quad \dfrac{}{\Gamma :\ C \vdash C}}{\Gamma :\ !(B) \multimap C \vdash C}\,\multimap}{\Gamma :\ !(A) \multimap !(B) \multimap C \vdash C}\,\multimap}{\Gamma :\ \cdot \vdash C}\,D_C}{\Gamma :\ !(!(A) \multimap !(B) \multimap C) \vdash !(A \,\&\, B) \multimap C}\,\star$$

CALL-BY-VALUE encoding (156ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\,! \quad \dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash !(B)}\,! \quad \dfrac{\dfrac{}{\Gamma :\ \cdot \vdash C}}{\Gamma :\ \cdot \vdash !(C)}\,!}{\Gamma :\ !(B) \multimap !(C) \vdash !(C)}\,\star\,\multimap}{\Gamma :\ \cdot \vdash !(C)}\,D_C}{\Gamma :\ !(A) \multimap !(!(B) \multimap !(C)) \vdash !(C)}\,\star\,\multimap}{\Gamma :\ \cdot \vdash !(C)}\,D_C}{\Gamma :\ \cdot \vdash !(A) \otimes !(B) \multimap !(C)}\,\star}{\Gamma :\ \cdot \vdash !(!(A) \otimes !(B) \multimap !(C))}\,!}{\Gamma :\ !(!(A) \multimap !(!(B) \multimap !(C))) \vdash !(!(A) \otimes !(B) \multimap !(C))}\,\star$$

01-ENC encoding (151ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\,! \quad \dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash !(B)}\,! \quad \dfrac{\dfrac{}{\Gamma :\ \cdot \vdash C}}{\Gamma :\ !(C) \vdash C}\,\star}{\Gamma :\ !(B) \multimap !(C) \vdash C}\,\multimap}{\Gamma :\ \cdot \vdash C}\,D_C}{\Gamma :\ !(!(B) \multimap !(C)) \vdash C}\,\star\,\multimap}{\Gamma :\ !(A) \multimap !(!(B) \multimap !(C)) \vdash C}\,D_C}{\Gamma :\ \cdot \vdash C}}{\Gamma :\ \cdot \vdash !(!(A) \,\&\, !(B)) \multimap C}\,\star}{\Gamma :\ \cdot \vdash !(!(!(A) \,\&\, !(B)) \multimap C)}\,!}{\Gamma :\ !(!(A) \multimap !(!(B) \multimap !(C))) \vdash !(!(!(A) \,\&\, !(B)) \multimap C)}\,\star$$

(5) $A \wedge B \to C \vdash A \to B \to C$

LJ (41ms)

$$\dfrac{\dfrac{}{A, B, A \wedge B \to C \vdash A \wedge B}\ \star \qquad \dfrac{}{A, B, C \vdash C}\ \star}{\dfrac{A, B, A \wedge B \to C \vdash C}{A \wedge B \to C \vdash A \to B \to C}\ \star}\ \supset_L$$

MULTIPLICATIVE encoding (54ms)

$$\dfrac{\dfrac{\dfrac{}{\Gamma :\ A \vdash A} \quad \dfrac{}{\Gamma :\ B \vdash B}}{\Gamma :\ A, B \vdash A \otimes B}\ \otimes \qquad \dfrac{}{\Gamma :\ C \vdash C}}{\dfrac{\Gamma :\ A, B, A \otimes B \multimap C \vdash C}{\Gamma :\ A \otimes B \multimap C \vdash A \multimap B \multimap C}\ \star}\ \multimap$$

CALL-BY-NAME encoding (73ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A} \quad \dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash A \,\&\, B}\ \star}{\Gamma :\ \cdot \vdash !(A \,\&\, B)}\ ! \qquad \dfrac{}{\Gamma :\ C \vdash C}}{\dfrac{\Gamma :\ !(A \,\&\, B) \multimap C \vdash C}{\Gamma :\ \cdot \vdash C}\ D_C}\ \multimap}{\Gamma :\ !(!(A \,\&\, B) \multimap C) \vdash !(A) \multimap !(B) \multimap C}\ \star$$

CALL-BY-VALUE encoding (205ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\ ! \quad \dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash !(B)}\ !}{\Gamma :\ \cdot \vdash !(A) \otimes !(B)}\ \otimes \quad \dfrac{\dfrac{}{\Gamma :\ \cdot \vdash C}}{\Gamma :\ \cdot \vdash !(C)}\ !}{\Gamma :\ !(A) \otimes !(B) \multimap !(C) \vdash !(C)}\ \star}{\Gamma :\ \cdot \vdash !(C)}\ D_C}{\Gamma :\ \cdot \vdash !(B) \multimap !(C)}\ \star}{\Gamma :\ \cdot \vdash !(!(B) \multimap !(C))}\ !}{\Gamma :\ \cdot \vdash !(A) \multimap !(!(B) \multimap !(C))}\ \star}{\Gamma :\ \cdot \vdash !(!(A) \multimap !(!(B) \multimap !(C)))}\ !}{\Gamma :\ !(!(A) \otimes !(B) \multimap !(C)) \vdash !(!(A) \multimap !(!(B) \multimap !(C)))}\ \star$$

01-ENC encoding (201ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A} \quad \dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash A \,\&\, B}\ \star}{\Gamma :\ \cdot \vdash !(A \,\&\, B)}\ !}{\Gamma :\ \cdot \vdash !(!(A \,\&\, B))}\ ! \qquad \dfrac{}{\Gamma :\ !(C) \vdash C}\ \star}{\Gamma :\ !(!(A \,\&\, B)) \multimap !(C) \vdash C}\ \multimap}{\Gamma :\ \cdot \vdash C}\ D_C}{\Gamma :\ \cdot \vdash !(B) \multimap C}\ \star}{\Gamma :\ \cdot \vdash !(!(B) \multimap C)}\ !}{\Gamma :\ \cdot \vdash !(A) \multimap !(!(B) \multimap C)}\ \star}{\Gamma :\ \cdot \vdash !(!(A) \multimap !(!(B) \multimap C))}\ !}{\Gamma :\ !(!(!(A \,\&\, B)) \multimap !(C)) \vdash !(!(A) \multimap !(!(B) \multimap C))}\ \star$$

(6) $A \to B \vdash B \to C \to A \to C$

LJ (47ms)

$$\dfrac{\dfrac{}{A, A \to B, B \to C \vdash A}\ \star \qquad \dfrac{\dfrac{}{A, B, B \to C \vdash B}\ \star \qquad \dfrac{}{A, B, C \vdash C}\ \star}{A, B, B \to C \vdash C}\ \supset_L}{A, A \to B, B \to C \vdash C}\ \supset_L}{A \to B \vdash B \to C \to A \to C}\ \star$$

MULTIPLICATIVE encoding (48ms)

$$\dfrac{\dfrac{}{\Gamma :\ A \vdash A} \qquad \dfrac{\dfrac{}{\Gamma :\ B \vdash B} \quad \dfrac{}{\Gamma :\ C \vdash C}}{\Gamma :\ B, B \multimap C \vdash C}\ \multimap}{\Gamma :\ A, A \multimap B, B \multimap C \vdash C}\ \multimap}{\Gamma :\ A \multimap B \vdash B \multimap C \multimap A \multimap C}\ \star$$

CALL-BY-NAME encoding (120ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{\Gamma :\ B \vdash B}}{\Gamma :\ !(A) \multimap B \vdash B}\ \multimap}{\Gamma :\ B \vdash B}\ D_C}{\Gamma :\ \cdot \vdash !(B)}\ ! \qquad \dfrac{}{\Gamma :\ C \vdash C}}{\dfrac{\Gamma :\ !(B) \multimap C \vdash C}{\Gamma :\ \cdot \vdash C}\ D_C}\ \multimap}{\Gamma :\ !(!(A) \multimap B) \vdash !(!(B) \multimap C) \multimap !(A) \multimap C}\ \star$$

CALL-BY-VALUE encoding (168ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\ ! \quad \dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ !(B) \vdash B}\ \star}{\Gamma :\ !(A) \multimap !(B) \vdash B}\ \multimap}{\Gamma :\ \cdot \vdash B}\ D_C}{\Gamma :\ \cdot \vdash !(B)}\ ! \qquad \dfrac{\dfrac{}{\Gamma :\ \cdot \vdash C}}{\Gamma :\ \cdot \vdash !(C)}\ !}{\Gamma :\ !(C) \vdash !(C)}\ \star}{\Gamma :\ !(B) \multimap !(C) \vdash !(C)}\ \multimap}{\Gamma :\ \cdot \vdash !(C)}\ D_C}{\Gamma :\ \cdot \vdash !(A) \multimap !(C)}\ \star}{\Gamma :\ \cdot \vdash !(!(A) \multimap !(C))}\ !}{\Gamma :\ \cdot \vdash !(!(B) \multimap !(C)) \multimap !(!(A) \multimap !(C))}\ \star}{\Gamma :\ \cdot \vdash !(!(!(B) \multimap !(C)) \multimap !(!(A) \multimap !(C)))}\ !}{\Gamma :\ !(!(A) \multimap !(B)) \vdash !(!(!(B) \multimap !(C)) \multimap !(!(A) \multimap !(C)))}\ \star$$

01-ENC encoding (159ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\ ! \quad \dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ !(B) \vdash B}\ \star}{\Gamma :\ !(A) \multimap !(B) \vdash B}\ \multimap}{\Gamma :\ \cdot \vdash B}\ D_C}{\Gamma :\ \cdot \vdash !(B)}\ ! \qquad \dfrac{\dfrac{}{\Gamma :\ \cdot \vdash C}}{\Gamma :\ !(C) \vdash C}\ \star}{\Gamma :\ !(B) \multimap !(C) \vdash C}\ \multimap}{\Gamma :\ \cdot \vdash C}\ D_C}{\Gamma :\ \cdot \vdash !(A) \multimap C}\ \star}{\Gamma :\ \cdot \vdash !(!(A) \multimap C)}\ !}{\Gamma :\ \cdot \vdash !(!(B) \multimap !(C)) \multimap !(!(A) \multimap C)}\ \star}{\Gamma :\ \cdot \vdash !(!(!(B) \multimap !(C)) \multimap !(!(A) \multimap C))}\ !}{\Gamma :\ !(!(A) \multimap !(B)) \vdash !(!(!(B) \multimap !(C)) \multimap !(!(A) \multimap C))}\ \star$$

(7) $A \to B \vdash C \to A \to C \to B$

LJ (47ms)

$$\dfrac{\dfrac{\overline{C, A \to B, C \to A \vdash C}^{\star} \quad \overline{A, C, A \to B \vdash A}^{\star}}{C, A \to B, C \to A \vdash A}\supset_L \quad \overline{B, C, C \to A \vdash B}^{\star}}{\dfrac{C, A \to B, C \to A \vdash B}{A \to B \vdash C \to A \to C \to B}^{\star}}\supset_L$$

MULTIPLICATIVE encoding (48ms)

$$\dfrac{\dfrac{\overline{\Gamma :\ C \vdash C} \quad \dfrac{\overline{\Gamma :\ A \vdash A} \quad \overline{\Gamma :\ B \vdash B}}{\Gamma :\ A, A \multimap B \vdash B}\multimap}{\Gamma :\ C, A \multimap B, C \multimap A \vdash B}\multimap}{\Gamma :\ A \multimap B \vdash C \multimap A \multimap C \multimap B}^{\star}$$

CALL-BY-NAME encoding (120ms)

$$\dfrac{\dfrac{\dfrac{\overline{\Gamma :\ \cdot \vdash C}}{\Gamma :\ \cdot \vdash !(C)}! \quad \overline{\Gamma :\ A \vdash A}}{\Gamma :\ !(C) \multimap A \vdash A}\multimap}{\dfrac{\Gamma :\ \cdot \vdash A}{\Gamma :\ \cdot \vdash !(A)}! \quad \dfrac{\overline{\Gamma :\ B \vdash B}}{\Gamma :\ !(A) \multimap B \vdash B}\multimap}{\Gamma :\ \cdot \vdash B}D_C}{\Gamma :\ !(!(A) \multimap B) \vdash !(!(C) \multimap A) \multimap !(C) \multimap B}^{\star}$$

CALL-BY-VALUE encoding (167ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\overline{\Gamma :\ \cdot \vdash C}}{\Gamma :\ \cdot \vdash !(C)}! \quad \dfrac{\overline{\Gamma :\ \cdot \vdash A}}{\Gamma :\ !(A) \vdash A}^{\star}}{\Gamma :\ !(C) \multimap !(A) \vdash A}\multimap}{\dfrac{\Gamma :\ \cdot \vdash A}{\Gamma :\ \cdot \vdash !(A)}!}D_C \quad \dfrac{\overline{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash !(B)}!}{\Gamma :\ !(A) \multimap !(B) \vdash !(B)}^{\star}\multimap}{\Gamma :\ \cdot \vdash !(B)}D_C}{\Gamma :\ \cdot \vdash !(C) \multimap !(B)}^{\star}}{\Gamma :\ \cdot \vdash !(!(C) \multimap !(B))}!}{\Gamma :\ \cdot \vdash !(!(C) \multimap !(A)) \multimap !(!(C) \multimap !(B))}^{\star}}{\Gamma :\ \cdot \vdash !(!(!(C) \multimap !(A)) \multimap !(!(C) \multimap !(B)))}!}{\Gamma :\ !(!(A) \multimap !(B)) \vdash !(!(!(C) \multimap !(A)) \multimap !(!(C) \multimap !(B)))}^{\star}$$

01-ENC encoding (156ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\overline{\Gamma :\ \cdot \vdash C}}{\Gamma :\ \cdot \vdash !(C)}! \quad \dfrac{\overline{\Gamma :\ \cdot \vdash A}}{\Gamma :\ !(A) \vdash A}^{\star}}{\Gamma :\ !(C) \multimap !(A) \vdash A}\multimap}{\dfrac{\Gamma :\ \cdot \vdash A}{\Gamma :\ \cdot \vdash !(A)}!}D_C \quad \dfrac{\overline{\Gamma :\ \cdot \vdash B}}{\Gamma :\ !(B) \vdash B}^{\star}}{\Gamma :\ !(A) \multimap !(B) \vdash B}\multimap}{\Gamma :\ \cdot \vdash B}D_C}{\Gamma :\ \cdot \vdash !(C) \multimap B}^{\star}}{\Gamma :\ \cdot \vdash !(!(C) \multimap B)}!}{\Gamma :\ \cdot \vdash !(!(C) \multimap !(A)) \multimap !(!(C) \multimap B)}^{\star}}{\Gamma :\ \cdot \vdash !(!(!(C) \multimap !(A)) \multimap !(!(C) \multimap B))}!}{\Gamma :\ !(!(A) \multimap !(B)) \vdash !(!(!(C) \multimap !(A)) \multimap !(!(C) \multimap B))}^{\star}$$

(8) $A \to B \vdash A \wedge C \to B \wedge C$

LJ (37ms)

$$\dfrac{\dfrac{\overline{A, C, A \to B \vdash A}^{\star} \quad \overline{A, B, C \vdash B}^{\star}}{A, C, A \to B \vdash B}\supset_L}{A \to B \vdash A \wedge C \to B \wedge C}^{\star}$$

MULTIPLICATIVE encoding (48ms)

$$\dfrac{\dfrac{\overline{\Gamma :\ A \vdash A} \quad \dfrac{\overline{\Gamma :\ B \vdash B} \quad \overline{\Gamma :\ C \vdash C}}{\Gamma :\ B, C \vdash B \otimes C}\otimes}{\Gamma :\ A, C, A \multimap B \vdash B \otimes C}\multimap}{\Gamma :\ A \multimap B \vdash A \otimes C \multimap B \otimes C}^{\star}$$

CALL-BY-NAME encoding (73ms)

$$\dfrac{\dfrac{\dfrac{\overline{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}! \quad \overline{\Gamma :\ B \vdash B}}{\Gamma :\ !(A) \multimap B \vdash B}\multimap}{\Gamma :\ \cdot \vdash B}D_C \quad \overline{\Gamma :\ \cdot \vdash C}}{\Gamma :\ !(!(A) \multimap B) \vdash !(A \,\&\, C) \multimap B \,\&\, C}^{\star}$$

CALL-BY-VALUE encoding (135ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\overline{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}! \quad \dfrac{\overline{\Gamma :\ \cdot \vdash B}}{\Gamma :\ !(B) \vdash B}^{\star}}{\Gamma :\ !(A) \multimap !(B) \vdash B}\multimap}{\dfrac{\Gamma :\ \cdot \vdash B}{\Gamma :\ \cdot \vdash !(B)}!}D_C \quad \dfrac{\overline{\Gamma :\ \cdot \vdash C}}{\Gamma :\ \cdot \vdash !(C)}!}{\Gamma :\ \cdot \vdash !(B) \otimes !(C)}\otimes}{\Gamma :\ \cdot \vdash !(A) \otimes !(C) \multimap !(B) \otimes !(C)}^{\star}}{\Gamma :\ \cdot \vdash !(!(A) \otimes !(C) \multimap !(B) \otimes !(C))}!}{\Gamma :\ !(!(A) \multimap !(B)) \vdash !(!(A) \otimes !(C) \multimap !(B) \otimes !(C))}^{\star}$$

01-ENC encoding (130ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\overline{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}! \quad \dfrac{\overline{\Gamma :\ \cdot \vdash B}}{\Gamma :\ !(B) \vdash B}^{\star}}{\Gamma :\ !(A) \multimap !(B) \vdash B}\multimap}{\Gamma :\ \cdot \vdash B}D_C \quad \overline{\Gamma :\ \cdot \vdash C}}{\Gamma :\ \cdot \vdash B \,\&\, C}^{\star}}{\Gamma :\ \cdot \vdash !(B \,\&\, C)}!}{\Gamma :\ \cdot \vdash !(!(A) \,\&\, !(C)) \multimap !(B \,\&\, C)}^{\star}}{\Gamma :\ \cdot \vdash !(!(!(A) \,\&\, !(C)) \multimap !(B \,\&\, C))}!}{\Gamma :\ !(!(A) \multimap !(B)) \vdash !(!(!(A) \,\&\, !(C)) \multimap !(B \,\&\, C))}^{\star}$$

(9) $A \to B \vdash C \wedge A \to C \wedge B$

LJ (37ms)

$$\dfrac{\dfrac{\dfrac{}{A, C, A \to B \vdash A}\,\star \quad \dfrac{}{A, B, C \vdash B}\,\star}{A, C, A \to B \vdash B}\,\supset_L}{A \to B \vdash C \wedge A \to C \wedge B}\,\star$$

MULTIPLICATIVE encoding (48ms)

$$\dfrac{\dfrac{}{\Gamma :\ A \vdash A} \quad \dfrac{\dfrac{}{\Gamma :\ C \vdash C} \quad \dfrac{}{\Gamma :\ B \vdash B}}{\Gamma :\ B, C \vdash C \otimes B}\,\otimes}{\Gamma :\ A, C, A \multimap B \vdash C \otimes B}\,\multimap}{\Gamma :\ A \multimap B \vdash C \otimes A \multimap C \otimes B}\,\star$$

CALL-BY-NAME encoding (60ms)

$$\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash C} \quad \dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\,! \quad \dfrac{}{\Gamma :\ B \vdash B}}{\Gamma :\ !(A) \multimap B \vdash B} \quad \dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ !(!(A) \multimap B) \vdash !(C \,\&\, A) \multimap C \,\&\, B}\,\star\,D_C\,\multimap$$

CALL-BY-VALUE encoding (137ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\,! \quad \dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash C}}{\Gamma :\ \cdot \vdash !(C)}\,! \quad \dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash !(B)}\,!}{\Gamma :\ \cdot \vdash !(C) \otimes !(B)}\,\otimes}{\Gamma :\ !(B) \vdash !(C) \otimes !(B)}\,\star}{\Gamma :\ !(A) \multimap !(B) \vdash !(C) \otimes !(B)}\,\multimap}{\Gamma :\ \cdot \vdash !(C) \otimes !(A) \multimap !(C) \otimes !(B)}\,D_C}{\Gamma :\ \cdot \vdash !(!(C) \otimes !(A) \multimap !(C) \otimes !(B))}\,\star}{\Gamma :\ !(!(A) \multimap !(B)) \vdash !(!(C) \otimes !(A) \multimap !(C) \otimes !(B))}\,!\,\star$$

01-ENC encoding (131ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\,! \quad \dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash C} \quad \dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash C \,\&\, B}\,\star}{\Gamma :\ \cdot \vdash !(C \,\&\, B)}\,!}{\Gamma :\ !(B) \vdash !(C \,\&\, B)}\,\star}{\Gamma :\ !(A) \multimap !(B) \vdash !(C \,\&\, B)}\,\multimap}{\Gamma :\ \cdot \vdash !(C \,\&\, B)}\,D_C}{\Gamma :\ \cdot \vdash !(!(C) \,\&\, !(A)) \multimap !(C \,\&\, B)}\,\star}{\Gamma :\ \cdot \vdash !(!(!(C) \,\&\, !(A)) \multimap !(C \,\&\, B))}\,!\,\star$$

(10) $\neg A \vdash A \to B$

LJ (37ms)

$$\dfrac{\dfrac{\dfrac{}{A, A \to \bot \vdash A}\,\star \quad \dfrac{}{A, \bot \vdash B}\,\star}{A, A \to \bot \vdash B}\,\supset_L}{A \to \bot \vdash A \to B}\,\star$$

MULTIPLICATIVE encoding (41ms)

Not provable

CALL-BY-NAME encoding (66ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\,! \quad \dfrac{}{\Gamma :\ \mathbf{0} \vdash B}\,\star}{\Gamma :\ !(A) \multimap \mathbf{0} \vdash B}\,\multimap}{\Gamma :\ \cdot \vdash B}\,D_C}{\Gamma :\ !(!(A) \multimap \mathbf{0}) \vdash !(A) \multimap B}\,\star$$

CALL-BY-VALUE encoding (117ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\,! \quad \dfrac{}{\Gamma :\ \mathbf{0} \vdash !(B)}\,\star}{\Gamma :\ !(A) \multimap \mathbf{0} \vdash !(B)}\,\multimap}{\Gamma :\ \cdot \vdash !(B)}\,D_C}{\Gamma :\ \cdot \vdash !(A) \multimap !(B)}\,\star}{\Gamma :\ \cdot \vdash !(!(A) \multimap !(B))}\,!}{\Gamma :\ !(!(A) \multimap \mathbf{0}) \vdash !(!(A) \multimap !(B))}\,\star$$

01-ENC encoding (118ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\,! \quad \dfrac{}{\Gamma :\ !(\mathbf{0}) \vdash B}\,\star}{\Gamma :\ !(A) \multimap !(\mathbf{0}) \vdash B}\,\multimap}{\Gamma :\ \cdot \vdash B}\,D_C}{\Gamma :\ \cdot \vdash !(A) \multimap B}\,\star}{\Gamma :\ \cdot \vdash !(!(A) \multimap B)}\,!}{\Gamma :\ !(!(A) \multimap !(\mathbf{0})) \vdash !(!(A) \multimap B)}\,\star$$

(11) $A \vdash \neg A \to B$

LJ (37ms)

$$\dfrac{\dfrac{}{A, A \to \bot \vdash A}\ \star \quad \dfrac{}{A, \bot \vdash B}\ \star}{\dfrac{A, A \to \bot \vdash B}{A \vdash A \to \bot \to B}\ \star}\ \supset_L$$

MULTIPLICATIVE encoding (28ms)

Not provable

CALL-BY-NAME encoding (65ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{\Gamma :\ \mathbf{0} \vdash B}\ \star}{\Gamma :\ !(A) \multimap \mathbf{0} \vdash B}\ \multimap}{\Gamma :\ \cdot \vdash B}\ D_C}{\Gamma :\ !(A) \vdash !(!(A) \multimap \mathbf{0}) \multimap B}\ \star$$

CALL-BY-VALUE encoding (78ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{\Gamma :\ \mathbf{0} \vdash !(B)}\ \star}{\Gamma :\ !(A) \multimap \mathbf{0} \vdash !(B)}\ \multimap}{\Gamma :\ \cdot \vdash !(B)}\ D_C}{\Gamma :\ \cdot \vdash !(!(A) \multimap \mathbf{0}) \multimap !(B)}\ \star}{\Gamma :\ \cdot \vdash !(!(!(A) \multimap \mathbf{0}) \multimap !(B))}\ !}{\Gamma :\ !(A) \vdash !(!(!(A) \multimap \mathbf{0}) \multimap !(B))}\ \star$$

01-ENC encoding (76ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{\Gamma :\ !(\mathbf{0}) \vdash B}\ \star}{\Gamma :\ !(A) \multimap !(\mathbf{0}) \vdash B}\ \multimap}{\Gamma :\ \cdot \vdash B}\ D_C}{\Gamma :\ \cdot \vdash !(!(A) \multimap !(\mathbf{0})) \multimap B}\ \star}{\Gamma :\ \cdot \vdash !(!(!(A) \multimap !(\mathbf{0})) \multimap B)}\ !}{\Gamma :\ !(A) \vdash !(!(!(A) \multimap !(\mathbf{0})) \multimap B)}\ \star$$

(12) $B \vdash A \to B$

LJ (21ms)

$$\dfrac{}{B \vdash A \to B}\ \star$$

MULTIPLICATIVE encoding (21ms)

Not provable

CALL-BY-NAME encoding (27ms)

$$\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ !(B) \vdash !(A) \multimap B}\ \star$$

CALL-BY-VALUE encoding (47ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash !(B)}\ !}{\Gamma :\ \cdot \vdash !(A) \multimap !(B)}\ \star}{\Gamma :\ \cdot \vdash !(!(A) \multimap !(B))}\ !}{\Gamma :\ !(B) \vdash !(!(A) \multimap !(B))}\ \star$$

01-ENC encoding (39ms)

$$\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash !(A) \multimap B}\ \star}{\Gamma :\ \cdot \vdash !(!(A) \multimap B)}\ !}{\Gamma :\ !(B) \vdash !(!(A) \multimap B)}\ \star$$

$\boxed{(13)\ A \to B \vdash \neg B \to \neg A}$

LJ (47ms)

$$\dfrac{\dfrac{}{A, A \to B, B \to \bot \vdash A}\ \star \qquad \dfrac{\dfrac{}{A, B, B \to \bot \vdash B}\ \star \qquad \dfrac{}{A, B, \bot \vdash \bot}\ \star}{A, B, B \to \bot \vdash \bot}\ \supset_L}{\dfrac{A, A \to B, B \to \bot \vdash \bot}{A \to B \vdash B \to \bot \to A \to \bot}\ \star}\ \supset_L$$

MULTIPLICATIVE encoding (47ms)

$$\dfrac{\dfrac{}{\Gamma :\ A \vdash A} \qquad \dfrac{\dfrac{}{\Gamma :\ B \vdash B} \qquad \dfrac{}{\Gamma :\ \ \bot \vdash \bot}}{\Gamma :\ B, B \multimap \bot \vdash \bot}\ \multimap}{\dfrac{\Gamma :\ A, A \multimap B, B \multimap \bot \vdash \bot}{\Gamma :\ A \multimap B \vdash B \multimap \bot \multimap A \multimap \bot}\ \star}\ \multimap$$

CALL-BY-NAME encoding (143ms)

$$\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}\ !}{\Gamma :\ \cdot \vdash !(A)} \quad \dfrac{}{\Gamma :\ B \vdash B}\ \multimap}{\dfrac{\Gamma :\ !(A) \multimap B \vdash B}{\dfrac{\Gamma :\ \cdot \vdash B}{\Gamma :\ \cdot \vdash !(B)}\ !}\ D_C} \quad \dfrac{}{\Gamma :\ \mathbf{0} \vdash B}\ \star}\ \multimap}{\dfrac{\Gamma :\ !(B) \multimap \mathbf{0} \vdash B}{\dfrac{\Gamma :\ \cdot \vdash B}{\Gamma :\ \cdot \vdash !(B)}\ !}\ D_C} \quad \dfrac{}{\Gamma :\ \mathbf{0} \vdash B}\ \star}\ \multimap}{\dfrac{\Gamma :\ !(B) \multimap \mathbf{0} \vdash B}{\dfrac{\Gamma :\ \cdot \vdash B}{\Gamma :\ \cdot \vdash !(B)}\ !}\ D_C} \quad \dfrac{}{\Gamma :\ \mathbf{0} \vdash \mathbf{0}}\ \star}\ \multimap}{\dfrac{\Gamma :\ !(B) \multimap \mathbf{0} \vdash \mathbf{0}}{\Gamma :\ \cdot \vdash \mathbf{0}}\ D_C}}{\Gamma :\ !(!(A) \multimap B) \vdash !(!(B) \multimap \mathbf{0}) \multimap !(A) \multimap \mathbf{0}}\ \star}$$

CALL-BY-VALUE encoding (174ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}\ !}{\Gamma :\ \cdot \vdash !(A)} \quad \dfrac{}{\Gamma :\ \cdot \vdash B}\ !}{\Gamma :\ \cdot \vdash !(B)}\ \star}\ \multimap}{\Gamma :\ !(A) \multimap !(B) \vdash B}\ D_C} \quad \dfrac{}{\Gamma :\ \mathbf{0} \vdash B}\ \star}\ \multimap}{\Gamma :\ !(B) \multimap \mathbf{0} \vdash B}\ D_C} \quad \dfrac{}{\Gamma :\ \mathbf{0} \vdash B}\ \star}\ \multimap}{\Gamma :\ !(B) \multimap \mathbf{0} \vdash B}\ D_C} \quad \dfrac{}{\Gamma :\ \mathbf{0} \vdash B}\ \star}\ \multimap}{\Gamma :\ !(B) \multimap \mathbf{0} \vdash \mathbf{0}}\ D_C} \quad \dfrac{}{\Gamma :\ \mathbf{0} \vdash \mathbf{0}}\ \star}\ \multimap}{\dfrac{\Gamma :\ \cdot \vdash \mathbf{0}}{\Gamma :\ \cdot \vdash !(A) \multimap \mathbf{0}}\ \star}\ !}{\Gamma :\ \cdot \vdash !(!(A) \multimap \mathbf{0})}\ \star}\ !}{\Gamma :\ \cdot \vdash !(!(!(B) \multimap \mathbf{0}) \multimap !(!(A) \multimap \mathbf{0}))}\ \star}\ !}{\Gamma :\ !(!(A) \multimap !(B)) \vdash !(!(!(B) \multimap \mathbf{0}) \multimap !(!(A) \multimap \mathbf{0}))}\ \star}$$

01-ENC encoding (178ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}\ !}{\Gamma :\ \cdot \vdash !(A)} \quad \dfrac{}{\Gamma :\ \cdot \vdash B}\ !}{\Gamma :\ \cdot \vdash !(B)}\ \star}\ \multimap}{\Gamma :\ !(A) \multimap !(B) \vdash B}\ D_C} \quad \dfrac{}{\Gamma :\ !(\mathbf{0}) \vdash B}\ \star}\ \multimap}{\Gamma :\ !(B) \multimap !(\mathbf{0}) \vdash B}\ D_C} \quad \dfrac{}{\Gamma :\ !(\mathbf{0}) \vdash B}\ \star}\ \multimap}{\Gamma :\ !(B) \multimap !(\mathbf{0}) \vdash B}\ D_C} \quad \dfrac{}{\Gamma :\ !(\mathbf{0}) \vdash \mathbf{0}}\ \star}\ \multimap}{\dfrac{\Gamma :\ !(B) \multimap !(\mathbf{0}) \vdash \mathbf{0}}{\Gamma :\ \cdot \vdash \mathbf{0}}\ D_C}}{\dfrac{\Gamma :\ \cdot \vdash !(A) \multimap \mathbf{0}}{\Gamma :\ \cdot \vdash !(!(A) \multimap \mathbf{0})}\ \star}\ !}{\Gamma :\ \cdot \vdash !(!(!(B) \multimap \mathbf{0}) \multimap !(!(A) \multimap \mathbf{0}))}\ \star}\ !}{\Gamma :\ !(!(A) \multimap !(B)) \vdash !(!(!(B) \multimap !(\mathbf{0})) \multimap !(!(A) \multimap \mathbf{0}))}\ \star}$$

$\boxed{(14)\ A \to \neg B \vdash \neg\neg B \to \neg A}$

LJ (64ms)

$$\dfrac{\dfrac{}{A, A \to B \to \bot, B \to \bot \to \bot \vdash A}\ \star \qquad \dfrac{\dfrac{\dfrac{}{A, B, B \to \bot, B \to \bot \to \bot \vdash B \to \bot}\ \star \qquad \dfrac{}{A, B, \bot, B \to \bot \to \bot \vdash \bot}\ \star}{A, B, B \to \bot, B \to \bot \to \bot \vdash \bot}\ \supset_L \qquad \dfrac{}{A, \bot, B \to \bot \vdash \bot}\ \star}{A, B \to \bot, B \to \bot \to \bot \vdash \bot}\ \supset_L}{A, A \to B \to \bot, B \to \bot \to \bot \vdash \bot}\ \supset_L}{A \to B \vdash B \to \bot \to \bot \to A \to \bot}\ \star$$

MULTIPLICATIVE encoding (75ms)

$$\dfrac{\dfrac{\dfrac{}{\Gamma :\ A \vdash A}}{\Gamma :\ A, B, A \multimap B \multimap \bot \vdash \bot}\ \multimap \qquad \dfrac{\dfrac{}{\Gamma :\ B \vdash B} \qquad \dfrac{}{\Gamma :\ \ \bot \vdash \bot}}{\Gamma :\ B, B \multimap \bot \vdash \bot}\ \multimap}{\dfrac{\Gamma :\ A, A \multimap B \multimap \bot \vdash B \multimap \bot}{\Gamma :\ A \multimap B \multimap \bot \vdash B \multimap \bot \multimap \bot \vdash \bot}\ \star}\ \multimap \qquad \dfrac{}{\Gamma :\ \ \bot \vdash \bot}\ \multimap}{\Gamma :\ A \multimap B \multimap \bot \vdash B \multimap \bot \multimap \bot \multimap \bot \vdash \bot}\ \star$$

CALL-BY-NAME encoding (180ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}\ !}{\Gamma :\ \cdot \vdash !(A)} \quad \dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}\ !}{\Gamma :\ \cdot \vdash !(B)} \quad \dfrac{}{\Gamma :\ \mathbf{0} \vdash \mathbf{0}}\ \star}\ \multimap}{\Gamma :\ !(A) \multimap !(B) \multimap \mathbf{0} \vdash \mathbf{0}}\ \multimap}{\Gamma :\ \cdot \vdash \mathbf{0}}\ D_C}{\Gamma :\ \cdot \vdash !(B) \multimap \mathbf{0}}\ \star}{\Gamma :\ \cdot \vdash !(!(B) \multimap \mathbf{0})}\ !} \quad \dfrac{}{\Gamma :\ \mathbf{0} \vdash \mathbf{0}}\ \star}\ \multimap}{\Gamma :\ !(!(B) \multimap \mathbf{0}) \multimap \mathbf{0} \vdash \mathbf{0}}\ D_C}{\Gamma :\ \cdot \vdash \mathbf{0}}\ \star}{\Gamma :\ \cdot \vdash !(B) \multimap \mathbf{0}}\ !} \quad \dfrac{}{\Gamma :\ \mathbf{0} \vdash \mathbf{0}}\ \star}\ \multimap}{\Gamma :\ !(!(B) \multimap \mathbf{0}) \multimap \mathbf{0} \vdash \mathbf{0}}\ D_C}{\Gamma :\ \cdot \vdash \mathbf{0}}\ \star}{\Gamma :\ \cdot \vdash !(B) \multimap \mathbf{0}}\ !} \quad \dfrac{}{\Gamma :\ \mathbf{0} \vdash \mathbf{0}}\ \star}\ \multimap}{\Gamma :\ !(!(B) \multimap \mathbf{0}) \multimap \mathbf{0} \vdash \mathbf{0}}\ D_C}{\Gamma :\ \cdot \vdash \mathbf{0}}\ \star}{\Gamma :\ !(!(A) \multimap !(B) \multimap \mathbf{0}) \vdash !(!(!(B) \multimap \mathbf{0}) \multimap \mathbf{0}) \multimap !(A) \multimap \mathbf{0}}\ \star}$$

CALL-BY-VALUE encoding (217ms)

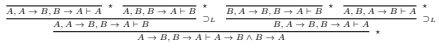
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}\ !}{\Gamma :\ \cdot \vdash !(A)} \quad \dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}\ !}{\Gamma :\ \cdot \vdash !(B)} \quad \dfrac{}{\Gamma :\ \mathbf{0} \vdash \mathbf{0}}\ \star}\ \multimap}{\Gamma :\ !(B) \multimap \mathbf{0} \vdash \mathbf{0}}\ D_C}{\Gamma :\ \cdot \vdash \mathbf{0}}\ \star}{\Gamma :\ \cdot \vdash !(B) \multimap \mathbf{0}}\ !} \quad \dfrac{}{\Gamma :\ \mathbf{0} \vdash \mathbf{0}}\ \star}\ \multimap}{\Gamma :\ !(!(B) \multimap \mathbf{0}) \multimap \mathbf{0} \vdash \mathbf{0}}\ D_C}{\Gamma :\ \cdot \vdash \mathbf{0}}\ \star}{\Gamma :\ \cdot \vdash !(!(B) \multimap \mathbf{0})}\ !} \quad \dfrac{}{\Gamma :\ \mathbf{0} \vdash \mathbf{0}}\ \star}\ \multimap}{\Gamma :\ !(!(!(B) \multimap \mathbf{0}) \multimap \mathbf{0}) \multimap \mathbf{0} \vdash \mathbf{0}}\ D_C}{\Gamma :\ \cdot \vdash \mathbf{0}}\ \star}{\Gamma :\ \cdot \vdash !(A) \multimap \mathbf{0}}\ !}{\Gamma :\ \cdot \vdash !(!(!(B) \multimap \mathbf{0}) \multimap \mathbf{0}) \multimap !(!(A) \multimap \mathbf{0}))}\ \star}\ !}{\Gamma :\ !(!(A) \multimap !(B) \multimap \mathbf{0})) \vdash !(!(!(!(B) \multimap \mathbf{0}) \multimap \mathbf{0}) \multimap !(!(A) \multimap \mathbf{0}))}\ \star}$$

01-ENC encoding (226ms)

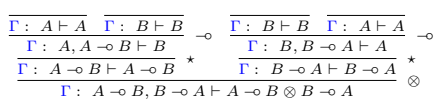
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}\ !}{\Gamma :\ \cdot \vdash !(A)} \quad \dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}\ !}{\Gamma :\ \cdot \vdash !(B)} \quad \dfrac{}{\Gamma :\ !(\mathbf{0}) \vdash \mathbf{0}}\ \star}\ \multimap}{\Gamma :\ !(B) \multimap !(\mathbf{0}) \vdash \mathbf{0}}\ D_C}{\Gamma :\ \cdot \vdash \mathbf{0}}\ \star}{\Gamma :\ \cdot \vdash !(B) \multimap \mathbf{0}}\ !} \quad \dfrac{}{\Gamma :\ !(\mathbf{0}) \vdash !(!(B) \multimap \mathbf{0})}\ \star}\ \multimap}{\Gamma :\ !(A) \multimap !(!(B) \multimap !(\mathbf{0})) \vdash !(!(B) \multimap \mathbf{0})}\ D_C}{\Gamma :\ \cdot \vdash !(!(B) \multimap \mathbf{0})}\ !} \quad \dfrac{}{\Gamma :\ !(\mathbf{0}) \vdash !(!(B) \multimap \mathbf{0})}\ \star}\ \multimap}{\Gamma :\ !(!(!(B) \multimap \mathbf{0}) \multimap !(\mathbf{0})) \vdash !(!(!(B) \multimap \mathbf{0}) \multimap \mathbf{0})}\ D_C}{\Gamma :\ \cdot \vdash !(!(!(B) \multimap \mathbf{0}) \multimap \mathbf{0})}\ !} \quad \dfrac{}{\Gamma :\ !(\mathbf{0}) \vdash \mathbf{0}}\ \star}\ \multimap}{\Gamma :\ !(!(!(B) \multimap \mathbf{0}) \multimap !(\mathbf{0})) \vdash !(!(!(!(B) \multimap \mathbf{0}) \multimap \mathbf{0}) \multimap !(\mathbf{0}))}\ D_C}{\Gamma :\ \cdot \vdash !(\mathbf{0})}\ !}{\Gamma :\ \cdot \vdash !(A) \multimap \mathbf{0}}\ \star}{\Gamma :\ \cdot \vdash !(!(A) \multimap \mathbf{0})}\ !}{\Gamma :\ \cdot \vdash !(!(!(!(B) \multimap \mathbf{0}) \multimap !(\mathbf{0})) \multimap !(!(A) \multimap \mathbf{0}))}\ !}{\Gamma :\ !(!(A) \multimap !(!(B) \multimap !(\mathbf{0}))) \vdash !(!(!(!(!(B) \multimap \mathbf{0}) \multimap \mathbf{0}) \multimap !(\mathbf{0})) \multimap !(!(A) \multimap \mathbf{0}))}\ \star}$$

(15) $A \to B, B \to A \vdash (A) \leftrightarrow (B)$

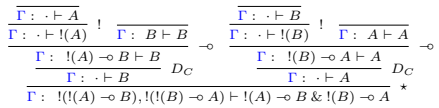
LJ (51ms)



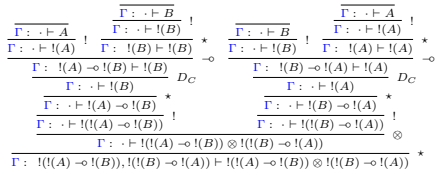
MULTIPLICATIVE encoding (75ms)



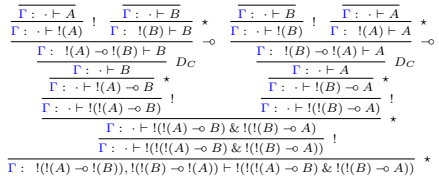
CALL-BY-NAME encoding (138ms)



CALL-BY-VALUE encoding (205ms)



01-ENC encoding (191ms)



(16) $(A) \leftrightarrow (B) \vdash A \to B$

LJ (37ms)

MULTIPLICATIVE encoding (29ms)

Not provable

CALL-BY-NAME encoding (102ms)

CALL-BY-VALUE encoding (132ms)

01-ENC encoding (126ms)

(17) $(A) \leftrightarrow (B) \vdash B \to A$

LJ (37ms)

$$\dfrac{\dfrac{}{B, A \to B, B \to A \vdash B}\ \star \quad \dfrac{}{A, B, A \to B \vdash A}\ \star}{\dfrac{B, A \to B, B \to A \vdash A}{A \to B \wedge B \to A \vdash B \to A}\ \star}\ \supset_L$$

MULTIPLICATIVE encoding (29ms)

Not provable

CALL-BY-NAME encoding (105ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash !(B)}\ ! \quad \dfrac{}{\Gamma :\ A \vdash A}}{\dfrac{\Gamma :\ !(B) \multimap A \vdash A}{\Gamma :\ \cdot \vdash A}\ D_C}\ \multimap}{\Gamma :\ !(!(A) \multimap B \,\&\, !(B) \multimap A) \vdash !(B) \multimap A}\ \star$$

CALL-BY-VALUE encoding (140ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash !(B)}\ ! \quad \dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\ !}{\Gamma :\ !(B) \multimap !(A) \vdash !(A)}\ \star}{\Gamma :\ \cdot \vdash !(A)}\ D_C \quad \dfrac{\dfrac{}{\Gamma :\ \cdot \vdash !(B) \multimap !(A)}\ \star}{\Gamma :\ \cdot \vdash !(!(B) \multimap !(A))}\ !}{\Gamma :\ !(!(A) \multimap !(B)) \otimes !(!(B) \multimap !(A)) \vdash !(!(B) \multimap !(A))}\ \star$$

01-ENC encoding (122ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash !(B)}\ ! \quad \dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ !(A) \vdash A}\ \star}{\Gamma :\ !(B) \multimap !(A) \vdash A}\ \multimap}{\Gamma :\ \cdot \vdash A}\ D_C}{\Gamma :\ \cdot \vdash !(B) \multimap A}\ \star}{\Gamma :\ \cdot \vdash !(!(B) \multimap A)}\ !}{\Gamma :\ !(!(!(A) \multimap !(B)) \,\&\, !(!(B) \multimap !(A))) \vdash !(!(B) \multimap A)}\ \star$$

(18) $(A) \leftrightarrow (B), A \vdash B$

LJ (38ms)

$$\dfrac{\dfrac{}{A, A \to B, B \to A \vdash A}\ \star \quad \dfrac{}{A, B, B \to A \vdash B}\ \star}{\dfrac{A, A \to B, B \to A \vdash B}{A, A \to B \wedge B \to A \vdash B}\ \star}\ \supset_L$$

MULTIPLICATIVE encoding (29ms)

Not provable

CALL-BY-NAME encoding (102ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{\Gamma :\ B \vdash B}}{\dfrac{\Gamma :\ !(A) \multimap B \vdash B}{\Gamma :\ \cdot \vdash B}\ D_C}\ \multimap}{\Gamma :\ !(A), !(!(A) \multimap B \,\&\, !(B) \multimap A) \vdash B}\ \star$$

CALL-BY-VALUE encoding (119ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash !(B)}\ ! \quad \dfrac{}{\Gamma :\ !(B) \vdash !(B)}\ \star}{\Gamma :\ !(A) \multimap !(B) \vdash !(B)}\ D_C}{\Gamma :\ \cdot \vdash !(B)}\ \multimap}{\Gamma :\ !(A), !(!(A) \multimap !(B)) \otimes !(!(B) \multimap !(A)) \vdash !(B)}\ \star$$

01-ENC encoding (111ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{\Gamma :\ !(B) \vdash B}\ \star}{\Gamma :\ !(A) \multimap !(B) \vdash B}\ \multimap}{\Gamma :\ \cdot \vdash B}\ D_C}{\Gamma :\ !(A), !(!(!(A) \multimap !(B)) \,\&\, !(!(B) \multimap !(A))) \vdash B}\ \star$$

(19) $(A) \leftrightarrow (B), B \vdash A$

LJ (38ms)

$$\dfrac{\dfrac{}{B, A \to B, B \to A \vdash B}\ \star \quad \dfrac{}{A, B, A \to B \vdash A}\ \star}{\dfrac{B, A \to B, B \to A \vdash A}{B, A \to B \wedge B \to A \vdash A}\ \star}\ \supset_L$$

MULTIPLICATIVE encoding (32ms)

Not provable

CALL-BY-NAME encoding (102ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash !(B)}\ ! \quad \dfrac{}{\Gamma :\ A \vdash A}}{\Gamma :\ !(B) \multimap A \vdash A}\ \multimap}{\Gamma :\ \cdot \vdash A}\ D_C}{\Gamma :\ !(B), !(!(A) \multimap B\ \&\ !(B) \multimap A) \vdash A}\ \star$$

CALL-BY-VALUE encoding (118ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash !(B)}\ ! \quad \dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\ !}{\Gamma :\ !(B) \multimap !(A) \vdash !(A)}\ \star\ \multimap}{\Gamma :\ \cdot \vdash !(A)}\ D_C}{\Gamma :\ !(B), !(!(A) \multimap !(B)) \otimes !(!(B) \multimap !(A)) \vdash !(A)}\ \star$$

01-ENC encoding (108ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash !(B)}\ ! \quad \dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ !(A) \vdash A}\ \star}{\Gamma :\ !(B) \multimap !(A) \vdash A}\ \multimap}{\Gamma :\ \cdot \vdash A}\ D_C}{\Gamma :\ !(B), !(!(!(A) \multimap !(B)) \ \&\ !(!(B) \multimap !(A))) \vdash A}\ \star$$

(20) $\cdot \vdash (A) \leftrightarrow (A)$

LJ (21ms)

$$\dfrac{}{\cdot \vdash A \to A \wedge A \to A}\ \star$$

MULTIPLICATIVE encoding (34ms)

$$\dfrac{\dfrac{\dfrac{}{\Gamma :\ A \vdash A}}{\Gamma :\ \cdot \vdash A \multimap A}\ \star \quad \dfrac{\dfrac{}{\Gamma :\ A \vdash A}}{\Gamma :\ \cdot \vdash A \multimap A}\ \star}{\Gamma :\ \cdot \vdash A \multimap A \otimes A \multimap A}\ \otimes$$

CALL-BY-NAME encoding (27ms)

$$\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A} \quad \dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A) \multimap A\ \&\ !(A) \multimap A}\ \star$$

CALL-BY-VALUE encoding (47ms)

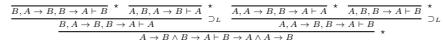
$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\ ! \quad \dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\ !}{\Gamma :\ \cdot \vdash !(A) \multimap !(A)}\ \star \quad \dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\ ! \quad \dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\ !}{\Gamma :\ \cdot \vdash !(A) \multimap !(A)}\ \star}{\Gamma :\ \cdot \vdash !(!(A) \multimap !(A)) \otimes !(!(A) \multimap !(A))}\ \otimes$$

01-ENC encoding (47ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A) \multimap A}\ \star \quad \dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A) \multimap A}\ \star}{\Gamma :\ \cdot \vdash !(!(A) \multimap A)}\ ! \quad \dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A) \multimap A}\ \star}{\Gamma :\ \cdot \vdash !(!(A) \multimap A)\ \&\ !(!(A) \multimap A)}\ !}{\Gamma :\ \cdot \vdash !(!(!(A) \multimap A)\ \&\ !(!(A) \multimap A))}\ !$$

$\boxed{(21)\ (A) \leftrightarrow (B) \vdash (B) \leftrightarrow (A)}$

LJ (50ms)



MULTIPLICATIVE encoding (81ms)

$$\dfrac{\dfrac{\dfrac{\overline{\Gamma :\ B \vdash B}\quad \overline{\Gamma :\ A \vdash A}}{\Gamma :\ B, B \multimap A \vdash A}\multimap}{\Gamma :\ B \multimap A \vdash B \multimap A}\star \qquad \dfrac{\dfrac{\overline{\Gamma :\ A \vdash A}\quad \overline{\Gamma :\ B \vdash B}}{\Gamma :\ A, A \multimap B \vdash B}\multimap}{\Gamma :\ A \multimap B \vdash A \multimap B}\star}{\dfrac{\Gamma :\ A \multimap B, B \multimap A \vdash B \multimap A \otimes A \multimap B}{\Gamma :\ A \multimap B \otimes B \multimap A \vdash B \multimap A \otimes A \multimap B}\star}\otimes$$

CALL-BY-NAME encoding (138ms)

$$\dfrac{\dfrac{\dfrac{\overline{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash !(B)}! \quad \overline{\Gamma :\ A \vdash A}}{\Gamma :\ !(B) \multimap A \vdash A}\multimap}{\Gamma :\ \cdot \vdash A}D_C \qquad \dfrac{\dfrac{\dfrac{\overline{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}! \quad \overline{\Gamma :\ B \vdash B}}{\Gamma :\ !(A) \multimap B \vdash B}\multimap}{\Gamma :\ \cdot \vdash B}D_C}{\Gamma :\ !(!(A) \multimap B \,\&\, !(B) \multimap A) \vdash !(B) \multimap A \,\&\, !(A) \multimap B}\star$$

CALL-BY-VALUE encoding (203ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\overline{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash !(B)}! \quad \dfrac{\overline{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}!}{\Gamma :\ !(B) \multimap !(A) \vdash !(A)}\star}{\Gamma :\ \cdot \vdash !(A)}\multimap}{\Gamma :\ \cdot \vdash !(B) \multimap !(A)}D_C}{\Gamma :\ \cdot \vdash !(!(B) \multimap !(A))}\star}{\Gamma :\ \cdot \vdash !(!(B) \multimap !(A)) \otimes !(!(A) \multimap !(B))}! \qquad \dfrac{\dfrac{\dfrac{\dfrac{\overline{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}! \quad \dfrac{\overline{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash !(B)}!}{\Gamma :\ !(A) \multimap !(B) \vdash !(B)}\star}{\Gamma :\ \cdot \vdash !(B)}\multimap}{\Gamma :\ !(A) \multimap !(B) \vdash B}D_C}{\Gamma :\ \cdot \vdash !(!(A) \multimap !(B))}\star}{\Gamma :\ \cdot \vdash !(!(A) \multimap !(B))}! \Bigg}{\Gamma :\ !(!(A) \multimap !(B)) \otimes !(!(B) \multimap !(A)) \vdash !(!(B) \multimap !(A)) \otimes !(!(A) \multimap !(B))}\star$$

01-ENC encoding (188ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\overline{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash !(B)}! \quad \dfrac{\overline{\Gamma :\ \cdot \vdash A}}{\Gamma :\ !(A) \vdash A}\star}{\Gamma :\ !(B) \multimap !(A) \vdash A}\multimap}{\Gamma :\ \cdot \vdash A}D_C}{\Gamma :\ \cdot \vdash !(B) \multimap A}\star}{\Gamma :\ \cdot \vdash !(!(B) \multimap A)}! \qquad \dfrac{\dfrac{\dfrac{\dfrac{\overline{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}! \quad \dfrac{\overline{\Gamma :\ \cdot \vdash B}}{\Gamma :\ !(A) \vdash B}\star}{\Gamma :\ !(A) \multimap !(B) \vdash B}\multimap}{\Gamma :\ \cdot \vdash B}D_C}{\Gamma :\ \cdot \vdash !(A) \multimap B}\star}{\Gamma :\ \cdot \vdash !(!(A) \multimap B)}! \Bigg}{\Gamma :\ \cdot \vdash !(!(B) \multimap A) \,\&\, !(!(A) \multimap B)}! \Bigg}{\Gamma :\ !(!(!(A) \multimap !(B)) \,\&\, !(!(B) \multimap !(A))) \vdash !(!(!(B) \multimap A) \,\&\, !(!(A) \multimap B))}\star$$

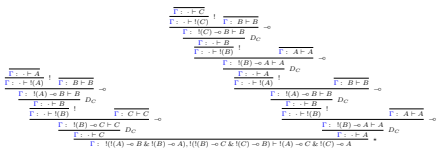
$\boxed{(22)\ (A) \leftrightarrow (B), (B) \leftrightarrow (C) \vdash (A) \leftrightarrow (C)}$

LJ (75ms)

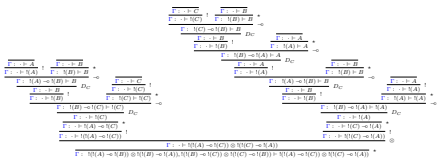
MULTIPLICATIVE encoding (115ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\overline{\Gamma :\ A \vdash A}}{\Gamma :\ A, A \multimap B, B \multimap C \vdash C}\multimap}{\Gamma :\ A, A \multimap B, B \multimap C \vdash A \multimap C}\star \qquad \dfrac{\dfrac{\overline{\Gamma :\ C \vdash C}}{\Gamma :\ C, B \multimap A, C \multimap B \vdash A}\multimap}{\Gamma :\ B \multimap A, C \multimap B \vdash C \multimap A}\star}{\Gamma :\ A \multimap B, B \multimap A, B \multimap C, C \multimap B \vdash A \multimap C \otimes C \multimap A}\otimes}{\Gamma :\ A \multimap B \otimes B \multimap A, B \multimap C \otimes C \multimap B \vdash A \multimap C \otimes C \multimap A}\star$$

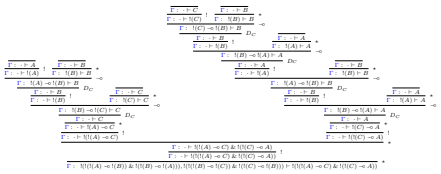
CALL-BY-NAME encoding (218ms)



CALL-BY-VALUE encoding (331ms)



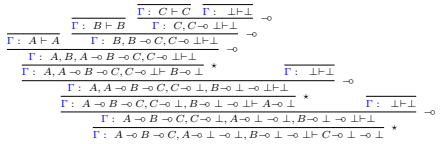
01-ENC encoding (291ms)



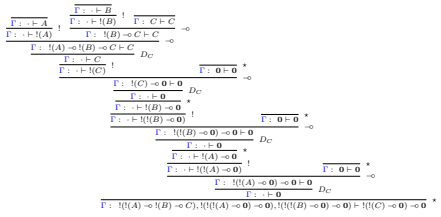
(23) $A \to B \to C, \neg\neg A, \neg\neg B \vdash \neg\neg C$

LJ (206ms)

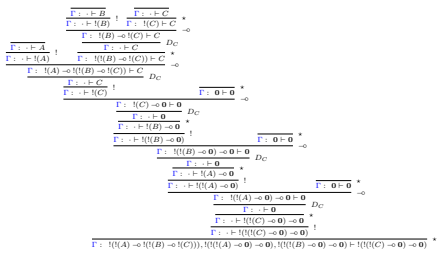
MULTIPLICATIVE encoding (124ms)



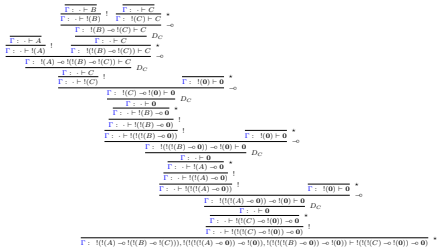
CALL-BY-NAME encoding (221ms)



CALL-BY-VALUE encoding (324ms)



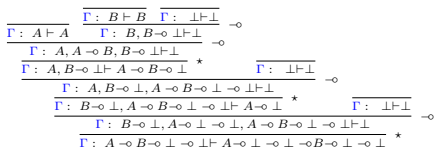
01-ENC encoding (380ms)



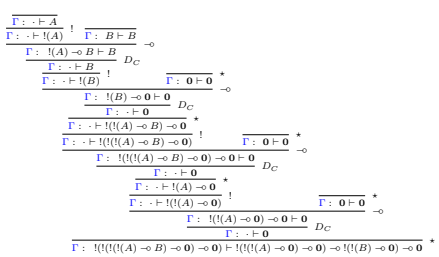
(24) $\neg\neg A \to B \vdash \neg\neg A \to \neg\neg B$

LJ (94ms)

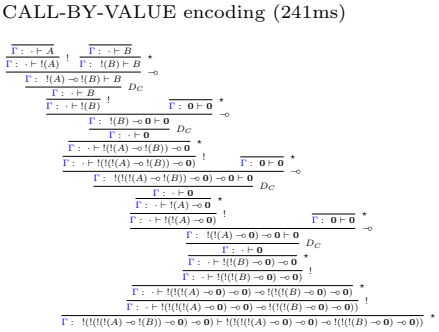
MULTIPLICATIVE encoding (106ms)



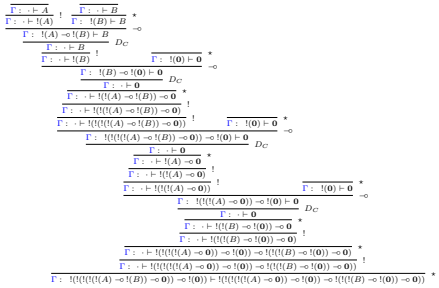
CALL-BY-NAME encoding (199ms)



CALL-BY-VALUE encoding (241ms)



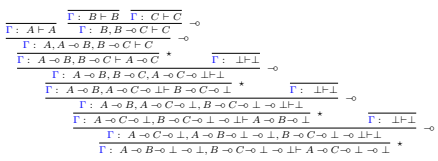
01-ENC encoding (269ms)



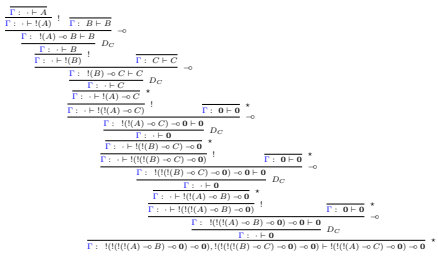
(25) $\neg\neg A \to B, \neg\neg B \to C \vdash \neg\neg A \to C$

LJ (324ms)

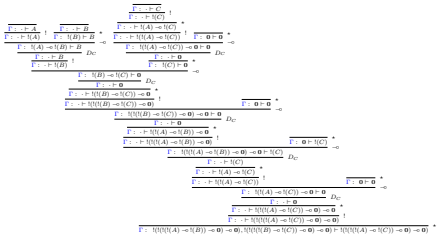
MULTIPLICATIVE encoding (145ms)



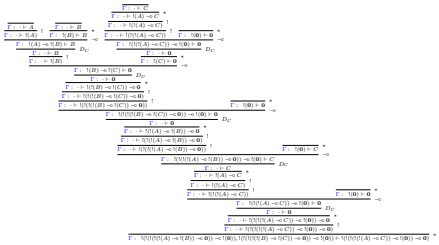
CALL-BY-NAME encoding (283ms)



CALL-BY-VALUE encoding (375ms)

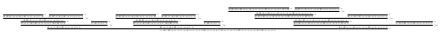


01-ENC encoding (421ms)



(26) $\cdot \vdash (\neg\neg A \wedge B) \leftrightarrow (\neg\neg A \wedge \neg\neg B)$

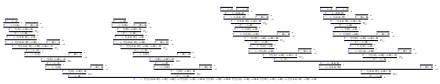
LJ (137ms)



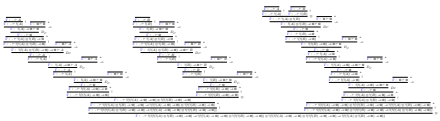
MULTIPLICATIVE encoding (66ms)

Not provable

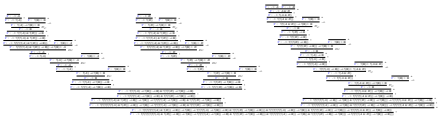
CALL-BY-NAME encoding (601ms)



CALL-BY-VALUE encoding (584ms)



01-ENC encoding (646ms)



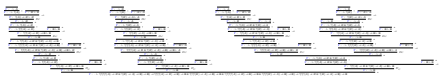
(27) $\cdot \vdash (\neg\neg(A) \leftrightarrow (B)) \leftrightarrow (\neg\neg A \to B \wedge \neg\neg B \to A)$

LJ (2703ms)

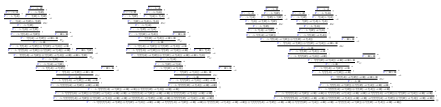
MULTIPLICATIVE encoding (84ms)

Not provable

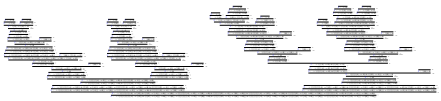
CALL-BY-NAME encoding (661ms)



CALL-BY-VALUE encoding (841ms)



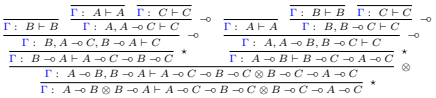
01-ENC encoding (1132ms)



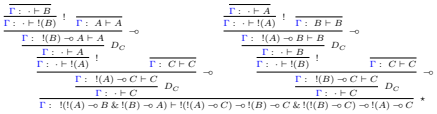
(28) $(A) \leftrightarrow (B) \vdash (A \to C) \leftrightarrow (B \to C)$

LJ (75ms)

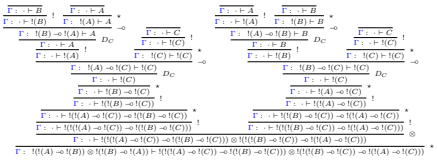
MULTIPLICATIVE encoding (95ms)



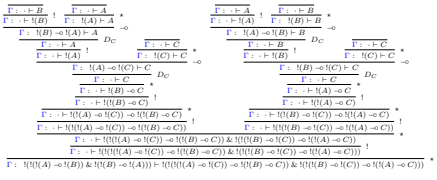
CALL-BY-NAME encoding (173ms)



CALL-BY-VALUE encoding (288ms)



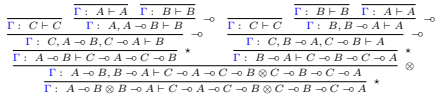
01-ENC encoding (268ms)



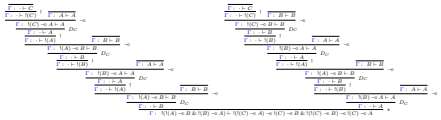
(29) $(A) \leftrightarrow (B) \vdash (C \to A) \leftrightarrow (C \to B)$

LJ (72ms)

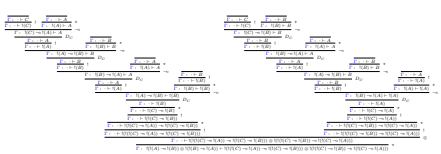
MULTIPLICATIVE encoding (96ms)



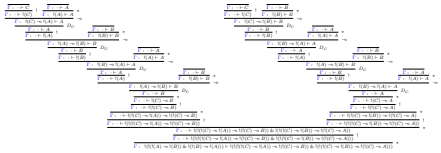
CALL-BY-NAME encoding (250ms)



CALL-BY-VALUE encoding (387ms)

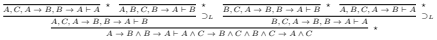


01-ENC encoding (356ms)

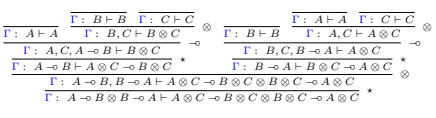


(30) $(A) \leftrightarrow (B) \vdash (A \wedge C) \leftrightarrow (B \wedge C)$

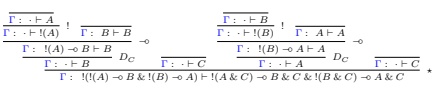
LJ (51ms)



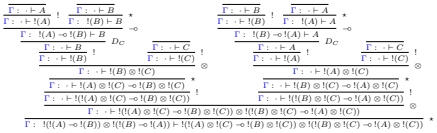
MULTIPLICATIVE encoding (99ms)



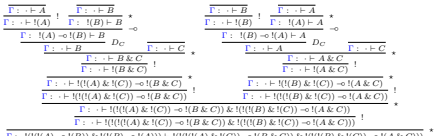
CALL-BY-NAME encoding (138ms)



CALL-BY-VALUE encoding (238ms)

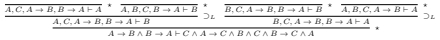


01-ENC encoding (234ms)

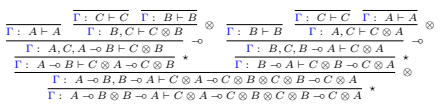


(31) $(A) \leftrightarrow (B) \vdash (C \wedge A) \leftrightarrow (C \wedge B)$

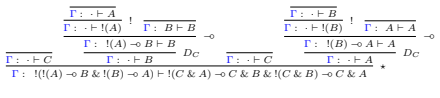
LJ (51ms)



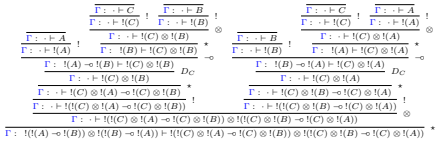
MULTIPLICATIVE encoding (98ms)



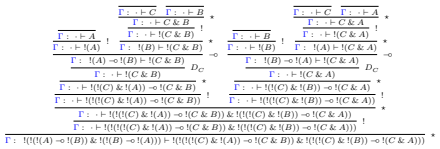
CALL-BY-NAME encoding (142ms)



CALL-BY-VALUE encoding (227ms)

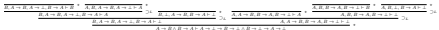


01-ENC encoding (229ms)

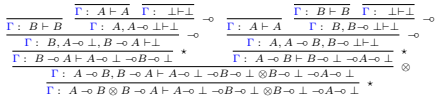


(32) $(A) \leftrightarrow (B) \vdash (\neg A) \leftrightarrow (\neg B)$

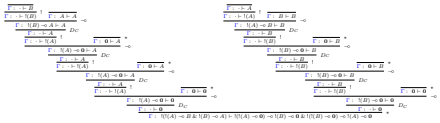
LJ (74ms)



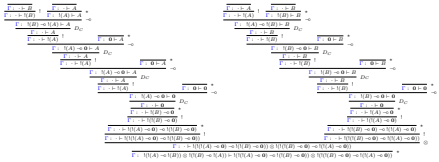
MULTIPLICATIVE encoding (97ms)



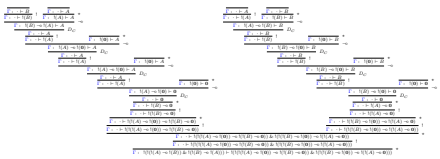
CALL-BY-NAME encoding (223ms)



CALL-BY-VALUE encoding (304ms)



01-ENC encoding (320ms)

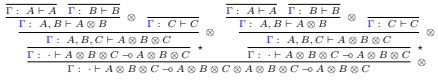


(33) $\cdot \vdash (A \wedge B \wedge C) \leftrightarrow (A \wedge B \wedge C)$

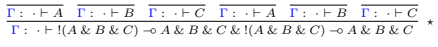
LJ (21ms)

$$\frac{}{\cdot \vdash A \wedge B \wedge C \to A \wedge B \wedge C \wedge A \wedge B \wedge C \to A \wedge B \wedge C}\ \star$$

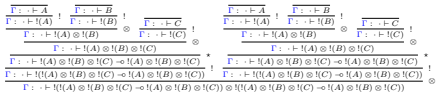
MULTIPLICATIVE encoding (66ms)



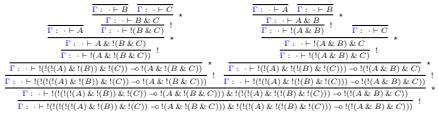
CALL-BY-NAME encoding (34ms)



CALL-BY-VALUE encoding (99ms)



01-ENC encoding (124ms)

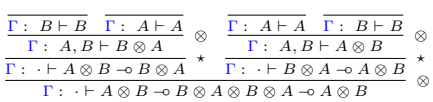


(34) $\cdot \vdash (A \wedge B) \leftrightarrow (B \wedge A)$

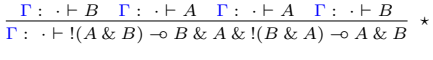
LJ (21ms)

$$\frac{}{\cdot \vdash A \wedge B \to B \wedge A \wedge B \wedge A \to A \wedge B}\ \star$$

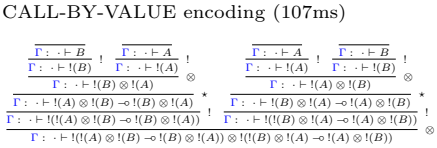
MULTIPLICATIVE encoding (73ms)



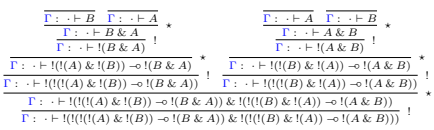
CALL-BY-NAME encoding (35ms)



CALL-BY-VALUE encoding (107ms)



01-ENC encoding (102ms)



$\boxed{(35)\ \cdot \vdash (A \wedge A) \leftrightarrow (A)}$

LJ (21ms)

$$\dfrac{}{\cdot \vdash A \wedge A \to A \wedge A \to A \wedge A}\ \star$$

MULTIPLICATIVE encoding (34ms)

Not provable

CALL-BY-NAME encoding (28ms)

$$\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}\quad \dfrac{}{\Gamma :\ \cdot \vdash A}\quad \dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash\ !(A\ \&\ A) \multimap A\ \&\ !(A) \multimap A\ \&\ A}\ \star$$

CALL-BY-VALUE encoding (76ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash\ !(A)}\ !}{\Gamma :\ \cdot \vdash\ !(A) \otimes\ !(A) \multimap\ !(A)}\ \star}{\Gamma :\ \cdot \vdash\ !(!(A) \otimes\ !(A) \multimap\ !(A))}\ !\quad \dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash\ !(A)}\ !\quad \dfrac{}{\Gamma :\ \cdot \vdash\ !(A)}}{\Gamma :\ \cdot \vdash\ !(A) \otimes\ !(A)}\ !\otimes}{\Gamma :\ \cdot \vdash\ !(!(A) \otimes\ !(A) \multimap\ !(A)) \otimes\ !(!(A) \multimap\ !(A) \otimes\ !(A))}\ \star}{\Gamma :\ \cdot \vdash\ !(!(A) \otimes\ !(A) \multimap\ !(A)) \otimes\ !(!(A) \multimap\ !(A) \otimes\ !(A))}\ !\otimes$$

01-ENC encoding (81ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash\ !(!(A)\ \&\ !(A)) \multimap A}\ \star}{\Gamma :\ \cdot \vdash\ !(!(!(A)\ \&\ !(A)) \multimap A)}\ !\quad \dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}\quad \dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash A\ \&\ A}\ !}{\Gamma :\ \cdot \vdash\ !(A \multimap A\ \&\ A)}\ \star}{\Gamma :\ \cdot \vdash\ !(!(!(A)\ \&\ !(A)) \multimap A)\ \&\ !(!(A) \multimap\ !(A\ \&\ A))}\ !}{\Gamma :\ \cdot \vdash\ !(!(!(!(A)\ \&\ !(A)) \multimap A)\ \&\ !(!(A) \multimap\ !(A\ \&\ A)))}\ !$$

$\boxed{(36)\ A \vdash (A \to B) \leftrightarrow (B)}$

LJ (37ms)

$$\dfrac{\dfrac{\dfrac{}{A, A \to B \vdash A}\ \star\quad \dfrac{}{A, B \vdash B}\ \star}{A, A \to B \vdash B}\ \supset_L}{A \vdash A \to B \to B \wedge B \to A \to B}\ \star$$

MULTIPLICATIVE encoding (42ms)

Not provable

CALL-BY-NAME encoding (73ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash\ !(A)}\ !\quad \dfrac{}{\Gamma :\ B \vdash B}}{\Gamma :\ \ !(A) \multimap B \vdash B}\ \multimap}{\Gamma :\ \cdot \vdash B}\ D_C\quad \dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \ !(A) \vdash\ !(!(A) \multimap B) \multimap B\ \&\ !(B) \multimap\ !(A) \multimap B}\ \star$$

CALL-BY-VALUE encoding (129ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash\ !(A)}\ !\quad \dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash\ !(B)}\ !}{\Gamma :\ \ !(A) \multimap\ !(B) \vdash\ !(B)}\ \star}{\Gamma :\ \cdot \vdash\ !(!(A) \multimap\ !(B))}\ D_C}{\Gamma :\ \cdot \vdash\ !(!(!(A) \multimap\ !(B)) \multimap\ !(B))}\ \star\quad \dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash\ !(B)}\ !}{\Gamma :\ \cdot \vdash\ !(!(A) \multimap\ !(B))}\ \star}{\Gamma :\ \cdot \vdash\ !(!(!(A) \multimap\ !(B)) \multimap\ !(B)) \otimes\ !(!(B) \multimap\ !(!(A) \multimap\ !(B)))}\ !\otimes}{\Gamma :\ \ !(A) \vdash\ !(!(!(A) \multimap\ !(B)) \multimap\ !(B)) \otimes\ !(!(B) \multimap\ !(!(A) \multimap\ !(B)))}\ \star$$

01-ENC encoding (123ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash\ !(A)}\ !\quad \dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \ !(B) \vdash B}\ \star}{\Gamma :\ \ !(A) \multimap\ !(B) \vdash B}\ \multimap}{\Gamma :\ \cdot \vdash B}\ D_C}{\Gamma :\ \cdot \vdash\ !(!(A) \multimap\ !(B)) \multimap B}\ \star\quad \dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash\ !(A) \multimap B}\ \star}{\Gamma :\ \cdot \vdash\ !(!(A) \multimap B)}\ !}{\Gamma :\ \cdot \vdash\ !(!(!(A) \multimap\ !(B)) \multimap B)}\ !\quad \dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash\ !(B) \multimap B}\ \star}{\Gamma :\ \cdot \vdash\ !(!(B) \multimap\ !(!(A) \multimap B))}\ !}{\Gamma :\ \cdot \vdash\ !(!(!(A) \multimap\ !(B)) \multimap B)\ \&\ !(!(B) \multimap\ !(!(A) \multimap B))}\ \star}{\Gamma :\ \cdot \vdash\ !(!(!(!(A) \multimap\ !(B)) \multimap B)\ \&\ !(!(B) \multimap\ !(!(A) \multimap B)))}\ !}{\Gamma :\ \ !(A) \vdash\ !(!(!(!(A) \multimap\ !(B)) \multimap B)\ \&\ !(!(B) \multimap\ !(!(A) \multimap B)))}\ \star$$

(37) $B \vdash (A \to B) \leftrightarrow (B)$

LJ (21ms)

$$\frac{}{B \vdash A \to B \to B \wedge B \to A \to B}\ \star$$

MULTIPLICATIVE encoding (40ms)

Not provable

CALL-BY-NAME encoding (39ms)

$$\frac{\dfrac{}{\Gamma :\ \cdot \vdash B} \quad \dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \ !(B) \vdash !(!(A) \multimap B) \multimap B \,\&\, !(B) \multimap !(A) \multimap B}\ \star$$

CALL-BY-VALUE encoding (152ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}\,!}{\Gamma :\ \cdot \vdash !(B)}\,\star}{\Gamma :\ \cdot \vdash !(!(A) \multimap !(B)) \multimap !(B)}\,! \quad \dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}\,!}{\Gamma :\ \cdot \vdash !(B)}\,\star}{\Gamma :\ \cdot \vdash !(B) \multimap !(!(A) \multimap !(B))}\,!}{\Gamma :\ \cdot \vdash !(!(!(A) \multimap !(B)) \multimap !(B)) \otimes !(!(B) \multimap !(!(A) \multimap !(B)))}\,\otimes}{\Gamma :\ \ !(B) \vdash !(!(!(A) \multimap !(B)) \multimap !(B)) \otimes !(!(B) \multimap !(!(A) \multimap !(B)))}\ \star$$

01-ENC encoding (88ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}\,\star}{\Gamma :\ \cdot \vdash !(!(A) \multimap !(B)) \multimap B}\,! \quad \dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}\,\star}{\Gamma :\ \cdot \vdash !(A) \multimap B}\,!}{\Gamma :\ \cdot \vdash !(B) \multimap !(!(A) \multimap B)}\,\star}{\Gamma :\ \cdot \vdash !(!(!(A) \multimap !(B)) \multimap B) \,\&\, !(!(B) \multimap !(!(A) \multimap B))}\,!}{\Gamma :\ \ !(B) \vdash !(!(!(!(A) \multimap !(B)) \multimap B) \,\&\, !(!(B) \multimap !(!(A) \multimap B)))}\ \star$$

(38) $\neg A \vdash (A \to B) \leftrightarrow (\neg A)$

LJ (52ms)

$$\dfrac{\dfrac{\dfrac{}{A, A \to B, A \to \bot \vdash A}\,\star \quad \dfrac{}{A, \bot, A \to B \vdash \bot}\,\star}{A, A \to B, A \to \bot \vdash \bot}\,\supset_L \quad \dfrac{\dfrac{}{A, A \to \bot \vdash A}\,\star \quad \dfrac{}{A, \bot \vdash B}\,\star}{A, A \to \bot \vdash B}\,\supset_L}{A \to \bot \vdash A \to B \to A \to \bot \wedge A \to \bot \to A \to B}\ \star$$

MULTIPLICATIVE encoding (42ms)

Not provable

CALL-BY-NAME encoding (101ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\,! \quad \dfrac{}{\Gamma :\ \mathbf{0} \vdash \mathbf{0}}\,\star}{\Gamma :\ \ !(A) \multimap \mathbf{0} \vdash \mathbf{0}}\,\multimap \quad \dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\,! \quad \dfrac{}{\Gamma :\ \mathbf{0} \vdash B}\,\star}{\Gamma :\ \ !(A) \multimap \mathbf{0} \vdash B}\,\multimap}{\Gamma :\ \cdot \vdash \mathbf{0}}\,D_C \quad \dfrac{\Gamma :\ \cdot \vdash B}{\Gamma :\ \cdot \vdash B}\,D_C}{\Gamma :\ \ !(!(A) \multimap \mathbf{0}) \vdash !(!(A) \multimap B) \multimap !(A) \multimap \mathbf{0} \,\&\, !(!(A) \multimap \mathbf{0}) \multimap !(A) \multimap B}\ \star$$

CALL-BY-VALUE encoding (201ms)

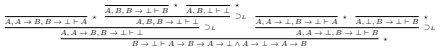
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\,! \quad \dfrac{}{\Gamma :\ \mathbf{0} \vdash \mathbf{0}}\,\star}{\Gamma :\ \ !(A) \multimap \mathbf{0} \vdash \mathbf{0}}\,\multimap}{\Gamma :\ \cdot \vdash \mathbf{0}}\,D_C}{\Gamma :\ \cdot \vdash !(A) \multimap \mathbf{0}}\,\star}{\Gamma :\ \cdot \vdash !(!(A) \multimap \mathbf{0})}\,! \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\,! \quad \dfrac{}{\Gamma :\ \mathbf{0} \vdash !(B)}\,\star}{\Gamma :\ \ !(A) \multimap \mathbf{0} \vdash !(B)}\,\multimap}{\Gamma :\ \cdot \vdash !(B)}\,D_C}{\Gamma :\ \cdot \vdash !(A) \multimap !(B)}\,\star}{\Gamma :\ \cdot \vdash !(!(A) \multimap \mathbf{0}) \multimap !(!(A) \multimap !(B))}\,!}{\Gamma :\ \cdot \vdash !(!(!(A) \multimap \mathbf{0}) \multimap !(!(A) \multimap !(B))) \otimes !(!(!(A) \multimap \mathbf{0}) \multimap !(!(A) \multimap !(B)))}\,\otimes}{\Gamma :\ \ !(!(A) \multimap \mathbf{0}) \vdash !(!(!(A) \multimap !(B)) \multimap !(!(A) \multimap \mathbf{0})) \otimes !(!(!(A) \multimap \mathbf{0}) \multimap !(!(A) \multimap !(B)))}\ \star$$

01-ENC encoding (203ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\,! \quad \dfrac{}{\Gamma :\ !(\mathbf{0}) \vdash \mathbf{0}}\,\star}{\Gamma :\ \ !(A) \multimap !(\mathbf{0}) \vdash \mathbf{0}}\,\multimap}{\Gamma :\ \cdot \vdash \mathbf{0}}\,D_C}{\Gamma :\ \cdot \vdash !(A) \multimap \mathbf{0}}\,\star}{\Gamma :\ \cdot \vdash !(!(A) \multimap \mathbf{0})}\,! \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\,! \quad \dfrac{}{\Gamma :\ !(\mathbf{0}) \vdash B}\,\star}{\Gamma :\ \ !(A) \multimap !(\mathbf{0}) \vdash B}\,\multimap}{\Gamma :\ \cdot \vdash B}\,D_C}{\Gamma :\ \cdot \vdash !(A) \multimap B}\,\star}{\Gamma :\ \cdot \vdash !(!(A) \multimap !(\mathbf{0})) \multimap !(!(A) \multimap B)}\,!}{\Gamma :\ \cdot \vdash !(!(!(A) \multimap !(B)) \multimap !(!(A) \multimap \mathbf{0})) \,\&\, !(!(!(A) \multimap !(\mathbf{0})) \multimap !(!(A) \multimap B))}\,\star}{\Gamma :\ \cdot \vdash !(!(!(!(A) \multimap !(B)) \multimap !(!(A) \multimap \mathbf{0})) \,\&\, !(!(!(A) \multimap !(\mathbf{0})) \multimap !(!(A) \multimap B)))}\,!}{\Gamma :\ \ !(!(A) \multimap !(\mathbf{0})) \vdash !(!(!(!(A) \multimap !(B)) \multimap !(!(A) \multimap \mathbf{0})) \,\&\, !(!(!(A) \multimap !(\mathbf{0})) \multimap !(!(A) \multimap B)))}\ \star$$

(39) $\neg B \vdash (A \to B) \leftrightarrow (\neg A)$

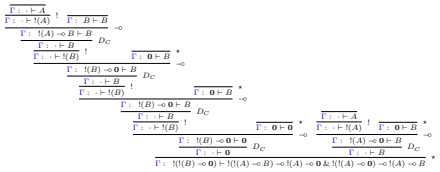
LJ (61ms)



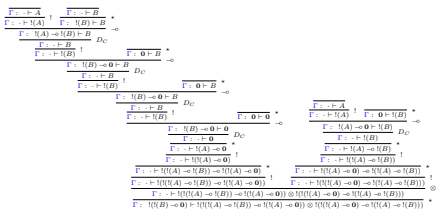
MULTIPLICATIVE encoding (43ms)

Not provable

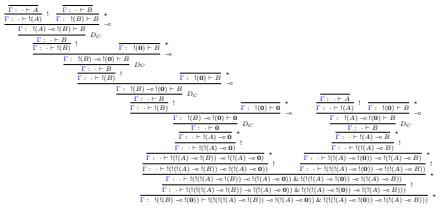
CALL-BY-NAME encoding (185ms)



CALL-BY-VALUE encoding (256ms)



01-ENC encoding (259ms)



(40) $B \vdash (A \wedge B) \leftrightarrow (A)$

LJ (21ms)

$$\frac{}{B \vdash A \wedge B \to A \wedge A \to A \wedge B}\ \star$$

MULTIPLICATIVE encoding (34ms)

Not provable

CALL-BY-NAME encoding (34ms)

$$\frac{\dfrac{}{\Gamma :\ \cdot \vdash A}\quad \dfrac{}{\Gamma :\ \cdot \vdash A}\quad \dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \ !(B) \vdash\ !(A \,\&\, B) \multimap A \,\&\, !(A) \multimap A \,\&\, B}\ \star$$

CALL-BY-VALUE encoding (88ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}\ !}{\Gamma :\ \cdot \vdash\ !(A)}\ \star}{\Gamma :\ \cdot \vdash\ !(A) \otimes\ !(B) \multimap\ !(A)}\ !\quad \dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}\ !}{\Gamma :\ \cdot \vdash\ !(A)}\quad \dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}\ !}{\Gamma :\ \cdot \vdash\ !(B)}}{\Gamma :\ \cdot \vdash\ !(A) \otimes\ !(B)}\ \otimes}{\Gamma :\ \cdot \vdash\ !(A) \otimes\ !(B) \multimap\ !(A) \otimes\ !(B)}\ \star}{\Gamma :\ \ !(B) \vdash\ !(A) \otimes\ !(B) \multimap\ !(A) \otimes\ !(B)}\ \star$$

01-ENC encoding (88ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash\ !(!(A) \,\&\, !(B)) \multimap A}\ \star}{\Gamma :\ \cdot \vdash\ !(!(!(A) \,\&\, !(B)) \multimap A)}\ !\quad \dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}\quad \dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash A \,\&\, B}\ !}{\Gamma :\ \cdot \vdash\ !(A) \multimap\ !(A \,\&\, B)}\ \star}{\Gamma :\ \cdot \vdash\ !(!(!(A) \,\&\, !(B)) \multimap A) \,\&\, !(!(A) \multimap\ !(A \,\&\, B))}\ !}{\Gamma :\ \cdot \vdash\ !(!(!(!(A) \,\&\, !(B)) \multimap A) \,\&\, !(!(A) \multimap\ !(A \,\&\, B)))}\ \star$$

(41) $\neg B \vdash (A \wedge B) \leftrightarrow (B)$

LJ (37ms)

$$\dfrac{\dfrac{\dfrac{}{B, B \to \bot \vdash B}\ \star \quad \dfrac{}{B, \bot \vdash A}\ \star}{B, B \to \bot \vdash A}\ \supset_L}{B \to \bot \vdash A \wedge B \to B \wedge B \to A \wedge B}\ \star$$

MULTIPLICATIVE encoding (41ms)

Not provable

CALL-BY-NAME encoding (78ms)

$$\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B} \quad \dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash !(B)}\ ! \quad \dfrac{}{\Gamma :\ \mathbf{0} \vdash A}\ \star}{\Gamma :\ !(B) \multimap \mathbf{0} \vdash A}\ \multimap}{\Gamma :\ \cdot \vdash A}\ D_C \quad \dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ !(!(B) \multimap \mathbf{0}) \vdash !(A \,\&\, B) \multimap B \,\&\, !(B) \multimap A \,\&\, B}\ \star$$

CALL-BY-VALUE encoding (172ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash !(B)}\ ! \quad \dfrac{}{\Gamma :\ \mathbf{0} \vdash !(A) \otimes !(B)}\ \star}{\Gamma :\ !(B) \multimap \mathbf{0} \vdash !(A) \otimes !(B)}\ \multimap}{\Gamma :\ \cdot \vdash !(A) \otimes !(B)}\ D_C \quad \dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash !(B)}\ !}{\Gamma :\ \cdot \vdash !(A) \otimes !(B) \multimap !(B)}\ \star}{\Gamma :\ \cdot \vdash !(!(A) \otimes !(B) \multimap !(B)) \otimes !(!(B) \multimap !(A) \otimes !(B))}\ \otimes$$
$$\dfrac{\dfrac{\Gamma :\ \cdot \vdash !(A) \otimes !(B) \multimap !(B)}{\Gamma :\ \cdot \vdash !(!(A) \otimes !(B) \multimap !(B))}\ ! \quad \dfrac{\Gamma :\ \cdot \vdash !(B) \multimap !(A) \otimes !(B)}{\Gamma :\ \cdot \vdash !(!(B) \multimap !(A) \otimes !(B))}\ !}{\Gamma :\ !(!(B) \multimap \mathbf{0}) \vdash !(!(A) \otimes !(B) \multimap !(B)) \otimes !(!(B) \multimap !(A) \otimes !(B))}\ \star$$

01-ENC encoding (194ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash !(B)}\ ! \quad \dfrac{}{\Gamma :\ !(\mathbf{0}) \vdash !(A \,\&\, B)}\ \star}{\Gamma :\ !(B) \multimap !(\mathbf{0}) \vdash !(A \,\&\, B)}\ \multimap}{\Gamma :\ \cdot \vdash A \,\&\, B}\ D_C \quad \dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash !(!(!(A) \,\&\, !(B)) \multimap B)}\ \star}{\Gamma :\ \cdot \vdash !(!(!(A) \,\&\, !(B)) \multimap B) \,\&\, !(!(B) \multimap !(A \,\&\, B))}\ !}{\Gamma :\ \cdot \vdash !(!(!(A) \,\&\, !(B)) \multimap B) \,\&\, !(!(B) \multimap !(A \,\&\, B)))}\ \star$$
$$\dfrac{\dfrac{\Gamma :\ \cdot \vdash !(A \,\&\, B)}{\Gamma :\ \cdot \vdash !(!(A \,\&\, B))}\ ! \quad \dfrac{\Gamma :\ \cdot \vdash !(B) \multimap !(A \,\&\, B)}{\Gamma :\ \cdot \vdash !(!(B) \multimap !(A \,\&\, B))}\ !}{\Gamma :\ !(!(B) \multimap !(\mathbf{0})) \vdash !(!(!(A) \,\&\, !(B)) \multimap B) \,\&\, !(!(B) \multimap !(A \,\&\, B)))}\ \star$$

(42) $\cdot \vdash A \to \neg\neg A$

LJ (38ms)

$$\dfrac{\dfrac{\dfrac{}{A, A \to \bot \vdash A}\ \star \quad \dfrac{}{A, \bot \vdash \bot}\ \star}{A, A \to \bot \vdash \bot}\ \supset_L}{\cdot \vdash A \to A \to \bot \to \bot}\ \star$$

MULTIPLICATIVE encoding (41ms)

$$\dfrac{\dfrac{\dfrac{}{\Gamma :\ A \vdash A} \quad \dfrac{}{\Gamma :\ \bot \vdash \bot}}{\Gamma :\ A, A \multimap \bot \vdash \bot}\ \multimap}{\Gamma :\ \cdot \vdash A \multimap A \multimap \bot \multimap \bot}\ \star$$

CALL-BY-NAME encoding (66ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{\Gamma :\ \mathbf{0} \vdash \mathbf{0}}\ \star}{\Gamma :\ !(A) \multimap \mathbf{0} \vdash \mathbf{0}}\ \multimap}{\Gamma :\ \cdot \vdash \mathbf{0}}\ D_C}{\Gamma :\ \cdot \vdash !(A) \multimap !(!(A) \multimap \mathbf{0}) \multimap \mathbf{0}}\ \star$$

CALL-BY-VALUE encoding (84ms)

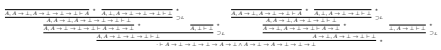
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{\Gamma :\ \mathbf{0} \vdash \mathbf{0}}\ \star}{\Gamma :\ !(A) \multimap \mathbf{0} \vdash \mathbf{0}}\ \multimap}{\Gamma :\ \cdot \vdash \mathbf{0}}\ D_C}{\Gamma :\ \cdot \vdash !(!(A) \multimap \mathbf{0}) \multimap \mathbf{0}}\ \star}{\Gamma :\ \cdot \vdash !(!(!(A) \multimap \mathbf{0}) \multimap \mathbf{0})}\ !}{\Gamma :\ \cdot \vdash !(A) \multimap !(!(!(A) \multimap \mathbf{0}) \multimap \mathbf{0})}\ \star}{\Gamma :\ \cdot \vdash !(!(A) \multimap !(!(!(A) \multimap \mathbf{0}) \multimap \mathbf{0}))}\ !$$

01-ENC encoding (81ms)

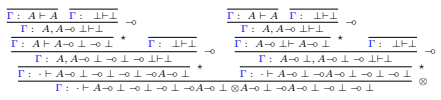
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{\Gamma :\ !(\mathbf{0}) \vdash \mathbf{0}}\ \star}{\Gamma :\ !(A) \multimap !(\mathbf{0}) \vdash \mathbf{0}}\ \multimap}{\Gamma :\ \cdot \vdash \mathbf{0}}\ D_C}{\Gamma :\ \cdot \vdash !(!(A) \multimap !(\mathbf{0})) \multimap \mathbf{0}}\ \star}{\Gamma :\ \cdot \vdash !(!(!(A) \multimap !(\mathbf{0})) \multimap \mathbf{0})}\ !}{\Gamma :\ \cdot \vdash !(A) \multimap !(!(!(A) \multimap !(\mathbf{0})) \multimap \mathbf{0})}\ \star}{\Gamma :\ \cdot \vdash !(!(A) \multimap !(!(!(A) \multimap !(\mathbf{0})) \multimap \mathbf{0}))}\ !$$

$(43)\ \cdot \vdash (\neg\neg\neg A) \leftrightarrow (\neg A)$

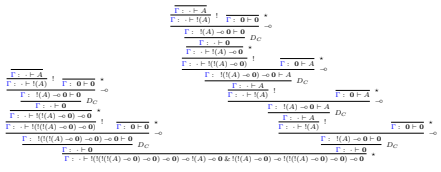
LJ (81ms)



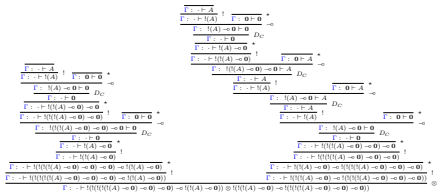
MULTIPLICATIVE encoding (105ms)



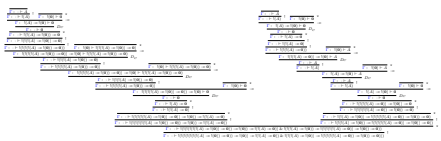
CALL-BY-NAME encoding (244ms)



CALL-BY-VALUE encoding (295ms)



01-ENC encoding (343ms)



$(44)\ \cdot \vdash \neg A \wedge \neg A$

LJ (38ms)

$$\dfrac{\dfrac{}{A, A \to \bot \vdash A}\ \star \quad \dfrac{}{A, \bot \vdash \bot}\ \star}{\dfrac{A, A \to \bot \vdash \bot}{\cdot \vdash A \wedge A \to \bot \to \bot}\ \star}\ \supset_L$$

MULTIPLICATIVE encoding (41ms)

$$\dfrac{\dfrac{\Gamma :\ A \vdash A \quad \Gamma :\ \ \bot \vdash \bot}{\Gamma :\ A, A \multimap \bot \vdash \bot}\ \multimap}{\Gamma :\ \cdot \vdash A \otimes A \multimap \bot \multimap \bot}\ \star$$

CALL-BY-NAME encoding (65ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\Gamma :\ \cdot \vdash A}{\Gamma :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{\Gamma :\ \mathbf{0} \vdash \mathbf{0}}\ \star}{\Gamma :\ !(A) \multimap \mathbf{0} \vdash \mathbf{0}}\ \multimap}{\Gamma :\ \cdot \vdash \mathbf{0}}\ D_C}{\Gamma :\ \cdot \vdash !(A \,\&\, !(A) \multimap \mathbf{0}) \multimap \mathbf{0}}\ \star$$

CALL-BY-VALUE encoding (70ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\Gamma :\ \cdot \vdash A}{\Gamma :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{\Gamma :\ \mathbf{0} \vdash \mathbf{0}}\ \star}{\Gamma :\ !(A) \multimap \mathbf{0} \vdash \mathbf{0}}\ \multimap}{\Gamma :\ \cdot \vdash \mathbf{0}}\ D_C}{\Gamma :\ \cdot \vdash !(A) \otimes !(!(A) \multimap \mathbf{0}) \multimap \mathbf{0}}\ \star}{\Gamma :\ \cdot \vdash !(!(A) \otimes !(!(A) \multimap \mathbf{0}) \multimap \mathbf{0})}\ !$$

01-ENC encoding (70ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\Gamma :\ \cdot \vdash A}{\Gamma :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{\Gamma :\ !(\mathbf{0}) \vdash \mathbf{0}}\ \star}{\Gamma :\ !(A) \multimap !(\mathbf{0}) \vdash \mathbf{0}}\ \multimap}{\Gamma :\ \cdot \vdash \mathbf{0}}\ D_C}{\Gamma :\ \cdot \vdash !(!(A) \,\&\, !(!(A) \multimap !(\mathbf{0}))) \multimap \mathbf{0}}\ \star}{\Gamma :\ \cdot \vdash !(!(!(A) \,\&\, !(!(A) \multimap !(\mathbf{0}))) \multimap \mathbf{0})}\ !$$

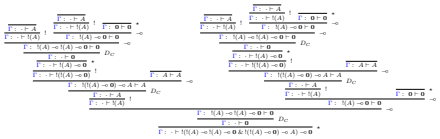
$(45)\ \cdot \vdash \neg(A) \leftrightarrow (\neg A)$

LJ (80ms)

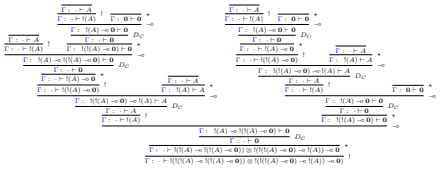
MULTIPLICATIVE encoding (41ms)

Not provable

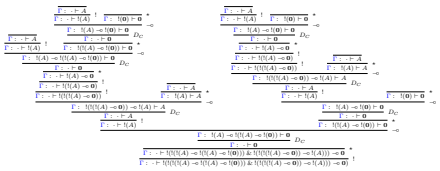
CALL-BY-NAME encoding (231ms)



CALL-BY-VALUE encoding (282ms)

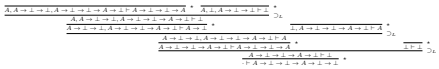


01-ENC encoding (300ms)



$(46)\ \cdot \vdash \neg\neg\neg\neg A \to A$

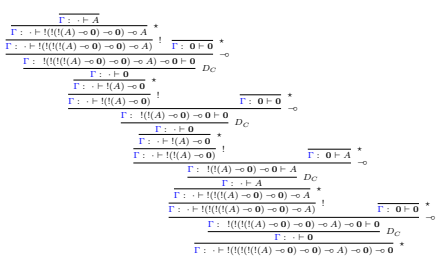
LJ (66ms)



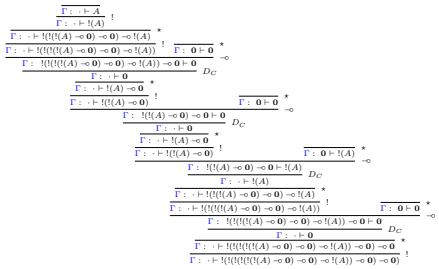
MULTIPLICATIVE encoding (41ms)

Not provable

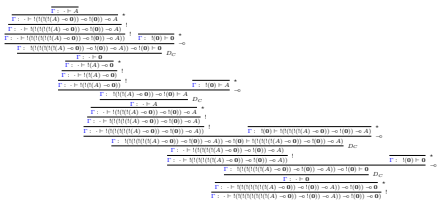
CALL-BY-NAME encoding (198ms)



CALL-BY-VALUE encoding (210ms)

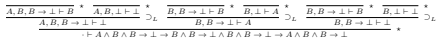


01-ENC encoding (217ms)



(47) $\cdot \vdash (A \wedge B \wedge \neg B) \leftrightarrow (B \wedge \neg B)$

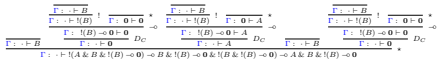
LJ (61ms)



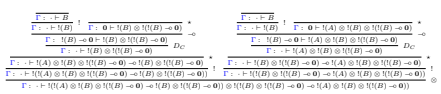
MULTIPLICATIVE encoding (42ms)

Not provable

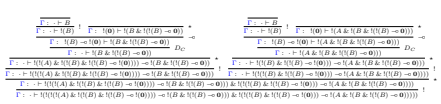
CALL-BY-NAME encoding (121ms)



CALL-BY-VALUE encoding (140ms)

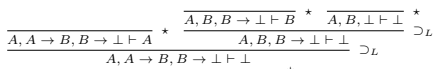


01-ENC encoding (132ms)

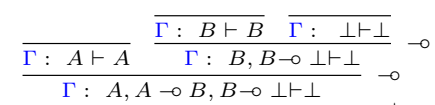


(48) $\cdot \vdash A \to B \to \neg A \wedge \neg B$

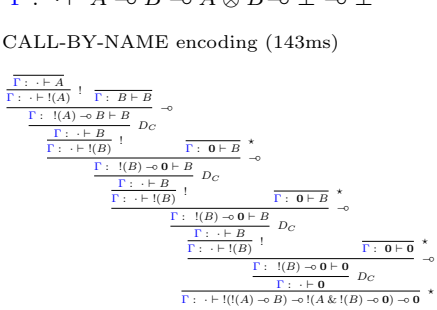
LJ (49ms)



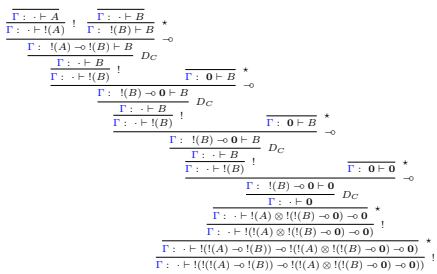
MULTIPLICATIVE encoding (48ms)



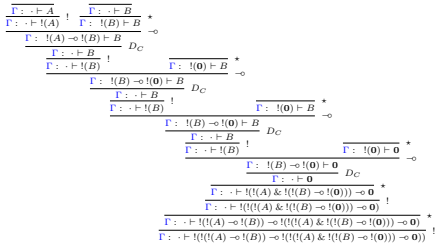
CALL-BY-NAME encoding (143ms)



CALL-BY-VALUE encoding (167ms)

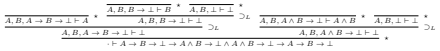


01-ENC encoding (169ms)

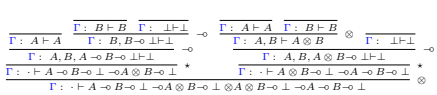


$(49)\ \cdot \vdash (A \to \neg B) \leftrightarrow (\neg A \wedge B)$

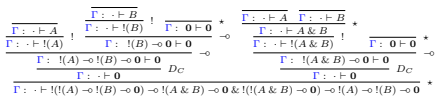
LJ (58ms)



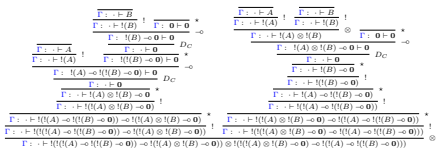
MULTIPLICATIVE encoding (88ms)



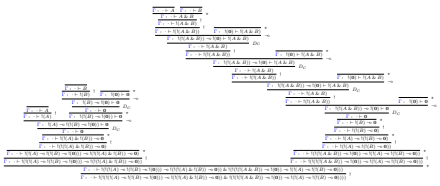
CALL-BY-NAME encoding (118ms)



CALL-BY-VALUE encoding (281ms)

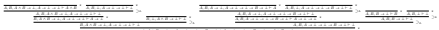


01-ENC encoding (318ms)



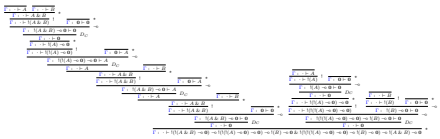
$(50)\ \cdot \vdash (\neg A \wedge B) \leftrightarrow (\neg\neg A \to \neg B)$

LJ (93ms)

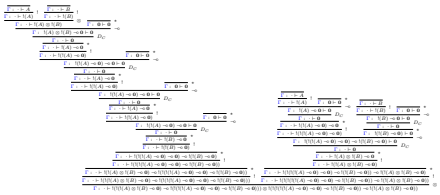


MULTIPLICATIVE encoding (134ms)

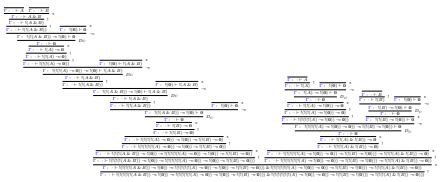
CALL-BY-NAME encoding (333ms)



CALL-BY-VALUE encoding (421ms)

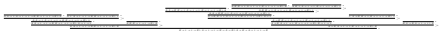


01-ENC encoding (470ms)



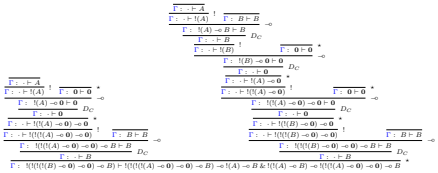
(51) $\neg\neg B \to B \vdash (\neg\neg A \to B) \leftrightarrow (A \to B)$

LJ (135ms)

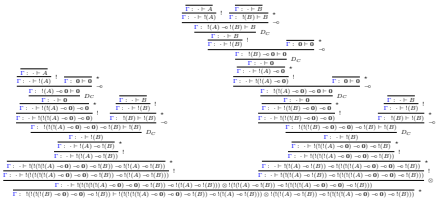


MULTIPLICATIVE encoding (156ms)

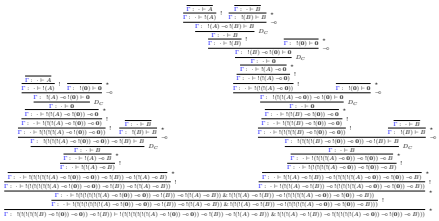
CALL-BY-NAME encoding (278ms)



CALL-BY-VALUE encoding (414ms)

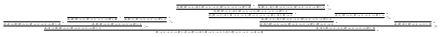


01-ENC encoding (449ms)



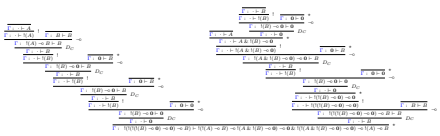
(52) $\neg\neg B \to B \vdash (A \to B) \leftrightarrow (\neg A \wedge \neg B)$

LJ (94ms)

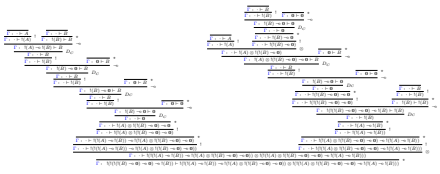


MULTIPLICATIVE encoding (134ms)

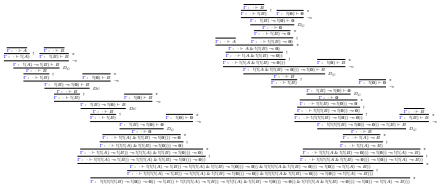
CALL-BY-NAME encoding (296ms)



CALL-BY-VALUE encoding (457ms)



01-ENC encoding (489ms)



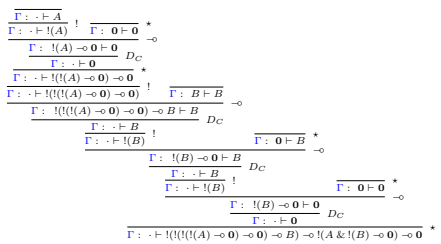
$\boxed{(53)\ \cdot \vdash \neg\neg A \to B \to \neg A \wedge \neg B}$

LJ (65ms)

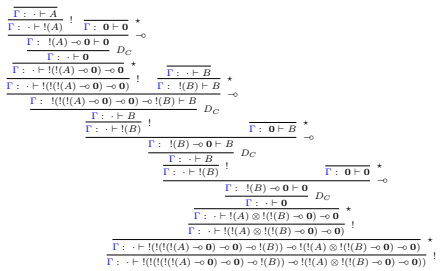
MULTIPLICATIVE encoding (75ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ A \vdash A}\quad\dfrac{}{\Gamma :\ \ \perp\vdash\perp}}{\Gamma :\ A, A\multimap \perp\vdash\perp}\multimap}{\Gamma :\ A \vdash A\multimap \perp \multimap \perp}\star\qquad\dfrac{\dfrac{\dfrac{}{\Gamma :\ B \vdash B}\quad\dfrac{}{\Gamma :\ \ \perp\vdash\perp}}{\Gamma :\ B, B\multimap \perp\vdash\perp}\multimap}{\Gamma :\ A, B\multimap \perp, A\multimap \perp \multimap \perp \multimap B\vdash\perp}\multimap}{\Gamma :\ \cdot \vdash A\multimap \perp \multimap \perp \multimap B \multimap A \otimes B\multimap \perp \multimap \perp}\star$$

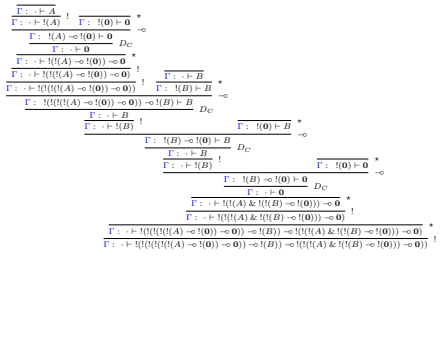
CALL-BY-NAME encoding (178ms)



CALL-BY-VALUE encoding (204ms)



01-ENC encoding (215ms)



$\boxed{(54)\ \cdot \vdash A \wedge B \to \neg A \to \neg B}$

LJ (47ms)

$$\dfrac{\dfrac{}{A, B, A \to B \to \perp \vdash A}\star\qquad\dfrac{\dfrac{}{A, B, B \to \perp \vdash B}\star\qquad\dfrac{}{A, B, \perp \vdash \perp}\star}{A, B, B \to \perp \vdash \perp}\supset_L}{A, B, A \to B \to \perp \vdash \perp}\supset_L$$
$$\cdot \vdash A \wedge B \to A \to B \to \perp \to \perp\ \star$$

MULTIPLICATIVE encoding (48ms)

$$\dfrac{\dfrac{\dfrac{}{\Gamma :\ A \vdash A}\quad\dfrac{\dfrac{}{\Gamma :\ B \vdash B}\quad\dfrac{}{\Gamma :\ \ \perp\vdash\perp}}{\Gamma :\ B, B\multimap \perp\vdash\perp}\multimap}{\Gamma :\ A, B, A\multimap B\multimap \perp\vdash\perp}\multimap}{\Gamma :\ \cdot \vdash A \otimes B \multimap A \multimap B\multimap \perp \multimap \perp}\star$$

CALL-BY-NAME encoding (77ms)

$$\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\,!\quad\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash !(B)}\,!\quad\dfrac{}{\Gamma :\ \mathbf{0} \vdash \mathbf{0}}\star}{\Gamma :\ !(B) \multimap \mathbf{0} \vdash \mathbf{0}}\multimap}{\Gamma :\ !(A) \multimap !(B) \multimap \mathbf{0} \vdash \mathbf{0}}\,D_C}{\Gamma :\ \cdot \vdash !(A \,\&\, B) \multimap !(!(A) \multimap !(B) \multimap \mathbf{0}) \multimap \mathbf{0}}\star$$

CALL-BY-VALUE encoding (105ms)

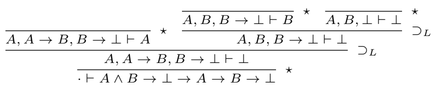
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\,!\quad\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash !(B)}\,!\quad\dfrac{}{\Gamma :\ \mathbf{0} \vdash \mathbf{0}}\star}{\Gamma :\ !(B) \multimap \mathbf{0} \vdash \mathbf{0}}\multimap}{\Gamma :\ \cdot \vdash \mathbf{0}}\,D_C}{\Gamma :\ !(A) \multimap !(!(B) \multimap \mathbf{0}) \vdash \mathbf{0}}\star}{\Gamma :\ \cdot \vdash !(!(A) \multimap !(!(B) \multimap \mathbf{0})) \multimap \mathbf{0}}\multimap}{\Gamma :\ !(A) \multimap !(!(B) \multimap \mathbf{0})) \multimap \mathbf{0})}\,D_C}{\Gamma :\ \cdot \vdash !(!(!(A) \multimap !(!(B) \multimap \mathbf{0})) \multimap \mathbf{0})}\star}{\Gamma :\ \cdot \vdash !(!(A) \otimes !(B) \multimap !(!(!(A) \multimap !(!(B) \multimap \mathbf{0})) \multimap \mathbf{0})}\,!}{\Gamma :\ \cdot \vdash !(!(A) \otimes !(B) \multimap !(!(!(A) \multimap !(!(B) \multimap \mathbf{0})) \multimap \mathbf{0}))}\star$$

01-ENC encoding (106ms)

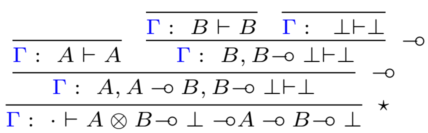
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}\,!\quad\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash B}}{\Gamma :\ \cdot \vdash !(B)}\,!\quad\dfrac{}{\Gamma :\ !(\mathbf{0}) \vdash \mathbf{0}}\star}{\Gamma :\ !(B) \multimap !(\mathbf{0}) \vdash \mathbf{0}}\multimap}{\Gamma :\ \cdot \vdash \mathbf{0}}\,D_C}{\Gamma :\ !(A) \multimap !(!(B) \multimap !(\mathbf{0})) \vdash \mathbf{0}}\star}{\Gamma :\ \cdot \vdash !(!(A) \multimap !(!(B) \multimap !(\mathbf{0}))) \multimap \mathbf{0}}\multimap}{\Gamma :\ !(A) \multimap !(!(B) \multimap !(\mathbf{0}))) \multimap \mathbf{0})}\,D_C}{\Gamma :\ \cdot \vdash !(!(!(A) \multimap !(!(B) \multimap !(\mathbf{0}))) \multimap \mathbf{0})}\star}{\Gamma :\ \cdot \vdash !(!(A) \,\&\, !(B)) \multimap !(!(!(A) \multimap !(!(B) \multimap !(\mathbf{0}))) \multimap \mathbf{0})}\,!}{\Gamma :\ \cdot \vdash !(!(A) \,\&\, !(B)) \multimap !(!(!(A) \multimap !(!(B) \multimap !(\mathbf{0}))) \multimap \mathbf{0}))}\star$$

(55) $\cdot \vdash A \wedge \neg B \to \neg A \to B$

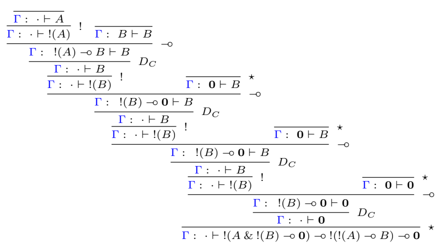
LJ (48ms)



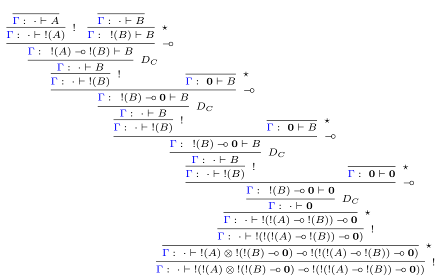
MULTIPLICATIVE encoding (47ms)



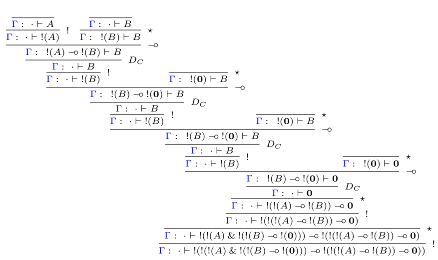
CALL-BY-NAME encoding (144ms)



CALL-BY-VALUE encoding (168ms)

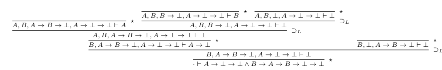


01-ENC encoding (168ms)

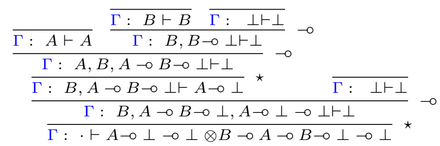


(56) $\cdot \vdash \neg\neg A \wedge B \to \neg A \to \neg B$

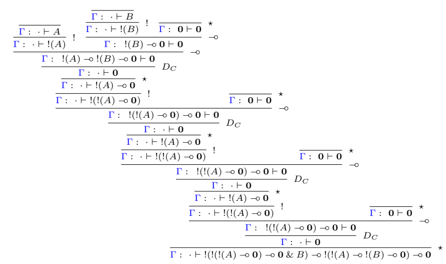
LJ (62ms)



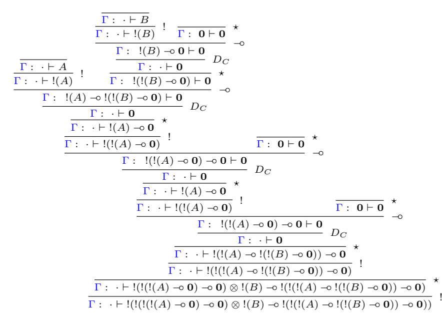
MULTIPLICATIVE encoding (77ms)



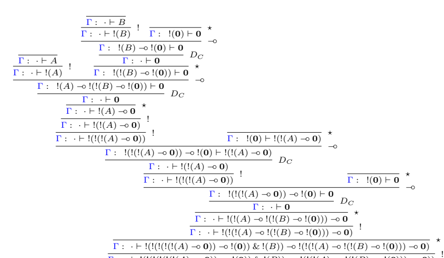
CALL-BY-NAME encoding (182ms)



CALL-BY-VALUE encoding (212ms)

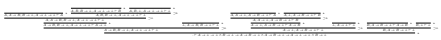


01-ENC encoding (210ms)



(57) $\cdot \vdash (\neg\neg A \wedge \neg B) \leftrightarrow (\neg A \to B)$

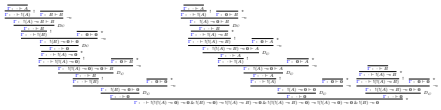
LJ (114ms)



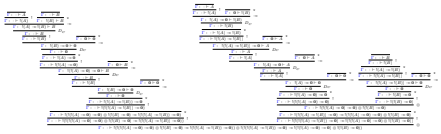
MULTIPLICATIVE encoding (58ms)

Not provable

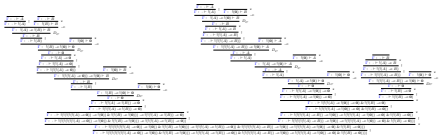
CALL-BY-NAME encoding (326ms)



CALL-BY-VALUE encoding (421ms)



01-ENC encoding (454ms)



(58) $\cdot \vdash (\neg A \to B) \leftrightarrow (\neg\neg A \wedge \neg B)$

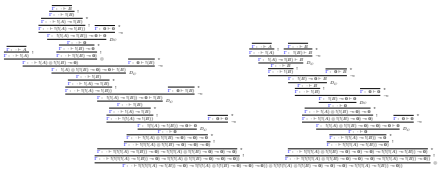
LJ (109ms)

MULTIPLICATIVE encoding (41ms)

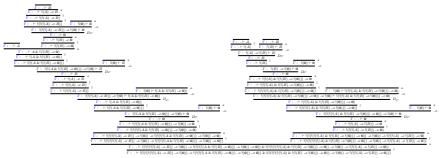
Not provable

CALL-BY-NAME encoding (597ms)

CALL-BY-VALUE encoding (481ms)

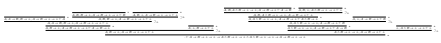


01-ENC encoding (501ms)



(59) $\cdot \vdash (\neg\neg A \to B) \leftrightarrow (\neg A \wedge \neg B)$

LJ (112ms)

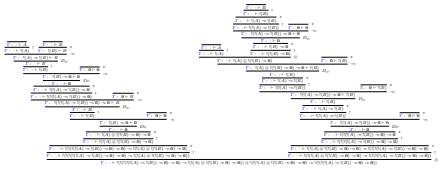


MULTIPLICATIVE encoding (57ms)

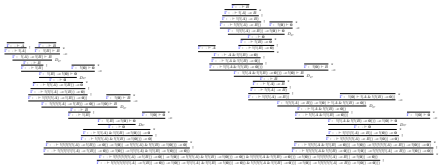
Not provable

CALL-BY-NAME encoding (594ms)

CALL-BY-VALUE encoding (468ms)

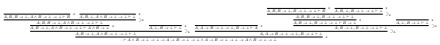


01-ENC encoding (498ms)

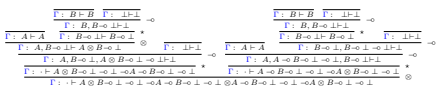


(60) $\cdot \vdash (\neg A \wedge \neg B) \leftrightarrow (A \to \neg\neg B)$

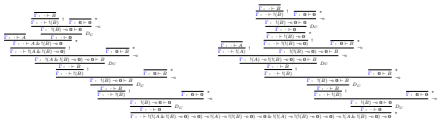
LJ (92ms)



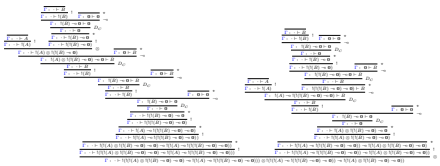
MULTIPLICATIVE encoding (124ms)



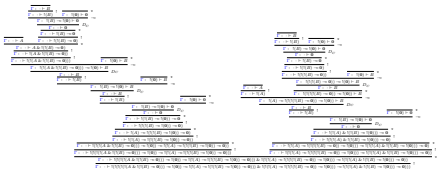
CALL-BY-NAME encoding (317ms)



CALL-BY-VALUE encoding (436ms)

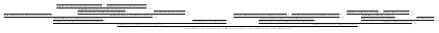


01-ENC encoding (468ms)

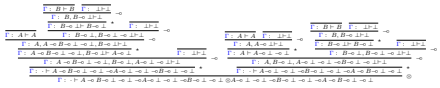


$$\boxed{(61)\ \cdot \vdash (A \to \neg\neg B) \leftrightarrow (\neg\neg A \to \neg\neg B)}$$

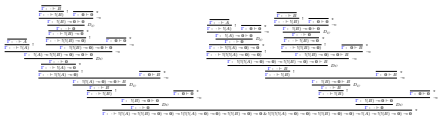
LJ (169ms)



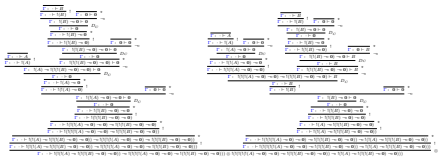
MULTIPLICATIVE encoding (163ms)



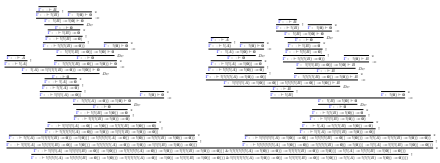
CALL-BY-NAME encoding (412ms)



CALL-BY-VALUE encoding (479ms)



01-ENC encoding (552ms)



3 Alternative Translations

(10) $\Gamma :\ A \multimap \mathbf{0} \vdash A \multimap B$

encoding (55ms)

$$\dfrac{\dfrac{\overline{\Gamma :\ A \vdash A} \quad \overline{\Gamma :\ \mathbf{0} \vdash B}}{\Gamma :\ A, A \multimap \mathbf{0} \vdash B}\multimap}{\Gamma :\ A \multimap \mathbf{0} \vdash A \multimap B}\star$$

(11) $\Gamma :\ A \vdash A \multimap \mathbf{0} \multimap B$

encoding (41ms)

$$\dfrac{\dfrac{\overline{\Gamma :\ A \vdash A} \quad \overline{\Gamma :\ \mathbf{0} \vdash B}}{\Gamma :\ A, A \multimap \mathbf{0} \vdash B}\multimap}{\Gamma :\ A \vdash A \multimap \mathbf{0} \multimap B}\star$$

(12) $\Gamma :\ B \vdash !(A) \multimap B$

encoding (22ms)

$$\dfrac{\overline{\Gamma :\ B \vdash B}}{\Gamma :\ B \vdash !(A) \multimap B}\star$$

(16) $\Gamma :\ A \multimap B \otimes !(B \multimap A) \vdash A \multimap B$

encoding (35ms)

$$\dfrac{\dfrac{\overline{\Gamma :\ A \vdash A} \quad \overline{\Gamma :\ B \vdash B}}{\Gamma :\ A, A \multimap B \vdash B}\multimap}{\Gamma :\ A \multimap B \otimes !(B \multimap A) \vdash A \multimap B}\star$$

(17) $\Gamma :\ !(A \multimap B) \otimes B \multimap A \vdash B \multimap A$

encoding (38ms)

$$\dfrac{\dfrac{\overline{\Gamma :\ B \vdash B} \quad \overline{\Gamma :\ A \vdash A}}{\Gamma :\ B, B \multimap A \vdash A}\multimap}{\Gamma :\ !(A \multimap B) \otimes B \multimap A \vdash B \multimap A}\star$$

(18) $\Gamma :\ A, A \multimap B \otimes B \multimap A \vdash B \otimes B \multimap A$

encoding (56ms)

$$\dfrac{\dfrac{\overline{\Gamma :\ A \vdash A} \quad \dfrac{\overline{\Gamma :\ B \vdash B} \quad \overline{\Gamma :\ A \vdash A}}{\Gamma :\ B, B \multimap A \vdash A}\multimap \quad \dfrac{\overline{\Gamma :\ B \vdash B} \quad \overline{\Gamma :\ B \multimap A \vdash B \multimap A}}{\Gamma :\ B, B \multimap A \vdash B \otimes B \multimap A}\star\otimes}{\Gamma :\ A, A \multimap B, B \multimap A \vdash B \otimes B \multimap A}\multimap}{\Gamma :\ A, A \multimap B \otimes B \multimap A \vdash B \otimes B \multimap A}\star$$

(19) $\Gamma :\ B, A \multimap B \otimes B \multimap A \vdash A \otimes A \multimap B$

encoding (55ms)

$$\dfrac{\dfrac{\overline{\Gamma :\ B \vdash B} \quad \dfrac{\dfrac{\overline{\Gamma :\ A \vdash A} \quad \overline{\Gamma :\ B \vdash B}}{\Gamma :\ A, A \multimap B \vdash B}\multimap \quad \dfrac{\overline{\Gamma :\ A \vdash A} \quad \overline{\Gamma :\ A \multimap B \vdash A \multimap B}}{\Gamma :\ A, A \multimap B \vdash A \otimes A \multimap B}\star\otimes}{\Gamma :\ B, A \multimap B, B \multimap A \vdash A \otimes A \multimap B}\multimap}{\Gamma :\ B, A \multimap B \otimes B \multimap A \vdash A \otimes A \multimap B}\star$$

(26a) $\Gamma :\ \cdot \vdash A \,\&\, B \multimap \bot \multimap \bot \multimap A \multimap \bot \multimap \bot \,\&\, B \multimap \bot \multimap \bot$

encoding (107ms)

$$\dfrac{\dfrac{\dfrac{\overline{\Gamma :\ A \vdash A} \quad \overline{\Gamma :\ \bot \vdash \bot}}{\Gamma :\ A, A \multimap \bot \vdash \bot}\multimap}{\Gamma :\ A \,\&\, B, A \multimap \bot \vdash \bot}\&_1}{\Gamma :\ A \multimap \bot, A \,\&\, B \multimap \bot \multimap \bot \vdash \bot}\star \quad \overline{\Gamma :\ \bot \vdash \bot}\multimap \qquad \dfrac{\dfrac{\dfrac{\overline{\Gamma :\ B \vdash B} \quad \overline{\Gamma :\ \bot \vdash \bot}}{\Gamma :\ B, B \multimap \bot \vdash \bot}\multimap}{\Gamma :\ A \,\&\, B, B \multimap \bot \vdash \bot}\&_2}{\Gamma :\ B \multimap \bot \vdash A \,\&\, B \multimap \bot}\star \quad \overline{\Gamma :\ \bot \vdash \bot}\multimap}{\Gamma :\ B \multimap \bot, A \,\&\, B \multimap \bot \multimap \bot \vdash \bot}\multimap}{\Gamma :\ \cdot \vdash A \,\&\, B \multimap \bot \multimap \bot \multimap A \multimap \bot \multimap \bot \,\&\, B \multimap \bot \multimap \bot}\star$$

(26b) $\Gamma :\ \cdot \vdash A \multimap \bot \multimap \bot \otimes B \multimap \bot \multimap \bot \multimap A \otimes B \multimap \bot \multimap \bot$

encoding (109ms)

$$\dfrac{\dfrac{\dfrac{\overline{\Gamma :\ A \vdash A} \quad \overline{\Gamma :\ B \vdash B}}{\Gamma :\ A, B \vdash A \otimes B}\otimes \quad \overline{\Gamma :\ \bot \vdash \bot}}{\Gamma :\ A, B, A \otimes B \multimap \bot \vdash \bot}\multimap}{\Gamma :\ A, A \otimes B \multimap \bot, B \multimap \bot \vdash \bot}\star \quad \overline{\Gamma :\ \bot \vdash \bot}\multimap}{\Gamma :\ A \otimes B \multimap \bot, B \multimap \bot \multimap \bot \vdash A \multimap \bot}\star \quad \overline{\Gamma :\ \bot \vdash \bot}\multimap}{\Gamma :\ A \otimes B \multimap \bot, A \multimap \bot \multimap \bot, B \multimap \bot \multimap \bot \vdash \bot}\multimap}{\Gamma :\ \cdot \vdash A \multimap \bot \multimap \bot \otimes B \multimap \bot \multimap \bot \multimap A \otimes B \multimap \bot \multimap \bot}\star$$

(27a) $\Gamma :\ \cdot \vdash !(A \multimap B) \otimes !(B \multimap A) \multimap \bot \multimap \bot \multimap A \multimap B \multimap \bot \multimap \bot \,\&\, B \multimap A \multimap \bot \multimap \bot$

encoding (155ms)

$$\dfrac{\dfrac{\dfrac{\overline{\Gamma :\ A \vdash A} \quad \overline{\Gamma :\ B \vdash B}}{\Gamma :\ A, A \multimap B \vdash B}\multimap}{\Gamma :\ A \vdash A \multimap B}D_C \quad \overline{\Gamma :\ \bot \vdash \bot}\multimap}{\Gamma :\ A \multimap B \vdash A \multimap B}\star \quad \overline{\Gamma :\ \bot \vdash \bot}\multimap}{\Gamma :\ A \multimap B \multimap \bot \vdash A \multimap B \multimap \bot}\multimap \qquad \dfrac{\dfrac{\dfrac{\overline{\Gamma :\ B \vdash B} \quad \overline{\Gamma :\ A \vdash A}}{\Gamma :\ B, B \multimap A \vdash A}\multimap}{\Gamma :\ B \vdash A}D_C \quad \overline{\Gamma :\ \bot \vdash \bot}\multimap}{\Gamma :\ B \multimap A \vdash B \multimap A}\star \quad \overline{\Gamma :\ \bot \vdash \bot}\multimap}{\Gamma :\ B \multimap A \multimap \bot \vdash B \multimap A \multimap \bot}\multimap}{\Gamma :\ \cdot \vdash !(A \multimap B) \otimes !(B \multimap A) \multimap \bot \multimap \bot \multimap A \multimap B \otimes B \multimap A \multimap \bot \multimap \bot}\multimap$$

(27b) $\Gamma :\ \cdot \vdash A \multimap B \multimap \bot \multimap \bot \otimes B \multimap A \multimap \bot \multimap \bot \multimap A \multimap B \otimes B \multimap A \multimap \bot \multimap \bot$

encoding (168ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\overline{\Gamma :\ A \vdash A} \quad \overline{\Gamma :\ B \vdash B}}{\Gamma :\ A, A \multimap B \vdash B}\multimap \quad \dfrac{\overline{\Gamma :\ B \vdash B} \quad \overline{\Gamma :\ A \vdash A}}{\Gamma :\ B, B \multimap A \vdash A}\multimap}{\Gamma :\ A \multimap B \vdash A \multimap B}\star \quad \dfrac{\overline{\Gamma :\ B \vdash B} \quad \overline{\Gamma :\ B \multimap A \vdash B \multimap A}}{\Gamma :\ B \multimap A \vdash B \multimap A}\star\otimes}{\Gamma :\ A \multimap B, B \multimap A \vdash A \multimap B \otimes B \multimap A}\multimap \quad \overline{\Gamma :\ \bot \vdash \bot}\multimap}{\Gamma :\ A \multimap B, A \multimap B \otimes B \multimap A \multimap \bot \vdash B \multimap A \multimap \bot}\multimap}{\Gamma :\ A \multimap B, A \multimap B \otimes B \multimap A \multimap \bot, B \multimap A \multimap \bot \vdash \bot}\star \quad \overline{\Gamma :\ \bot \vdash \bot}\multimap}{\Gamma :\ A \multimap B \otimes B \multimap A \multimap \bot, A \multimap B \multimap \bot \multimap \bot, B \multimap A \multimap \bot \vdash \bot}\star \quad \overline{\Gamma :\ \bot \vdash \bot}\multimap}{\Gamma :\ \cdot \vdash A \multimap B \multimap \bot \multimap \bot \otimes B \multimap A \multimap \bot \multimap \bot \multimap A \multimap B \otimes B \multimap A \multimap \bot \multimap \bot}\star$$

(35) $\Gamma :\ \cdot \vdash !(A) \otimes !(A) \multimap !(A) \otimes !(A) \multimap !(A) \otimes !(A)$

encoding (59ms)

$$\dfrac{\dfrac{\overline{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}! \quad \dfrac{\dfrac{\overline{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}! \quad \dfrac{\overline{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}!}{\Gamma :\ \cdot \vdash !(A) \otimes !(A)}\otimes}{\Gamma :\ \cdot \vdash !(A) \otimes !(A) \multimap !(A)}\star \quad \dfrac{\dfrac{\overline{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}! \quad \dfrac{\overline{\Gamma :\ \cdot \vdash A}}{\Gamma :\ \cdot \vdash !(A)}!}{\Gamma :\ \cdot \vdash !(A) \otimes !(A)}\otimes}{\Gamma :\ \cdot \vdash !(A) \otimes !(A) \multimap !(A) \otimes !(A)}\star\otimes$$

(36) $\Gamma :\ A \vdash A \multimap B \multimap B \otimes B \multimap !(A) \multimap B$

encoding (62ms)

$$\dfrac{\dfrac{\dfrac{}{\Gamma :\ A \vdash A}\quad\dfrac{}{\Gamma :\ B \vdash B}}{\Gamma :\ A, A \multimap B \vdash B}\multimap}{\Gamma :\ A \vdash A \multimap B \multimap B}\star \qquad \dfrac{\dfrac{}{\Gamma :\ B \vdash B}}{\Gamma :\ \cdot \vdash B \multimap !(A) \multimap B}\star}{\Gamma :\ A \vdash A \multimap B \multimap B \otimes B \multimap !(A) \multimap B}\otimes$$

(37) $\Gamma :\ B \vdash !(A \multimap B) \multimap B \otimes B \multimap !(A) \multimap B$

encoding (54ms)

$$\dfrac{\dfrac{}{\Gamma :\ B \vdash B}}{\Gamma :\ B \vdash !(A \multimap B) \multimap B}\star \qquad \dfrac{\dfrac{}{\Gamma :\ B \vdash B}}{\Gamma :\ \cdot \vdash B \multimap !(A) \multimap B}\star}{\Gamma :\ B \vdash !(A \multimap B) \multimap B \otimes B \multimap !(A) \multimap B}\otimes$$

(38) $\Gamma :\ A \multimap \bot \vdash !(A \multimap B) \multimap A \multimap \bot \otimes A \multimap \mathbf{0} \multimap A \multimap B$

encoding (74ms)

$$\dfrac{\dfrac{\dfrac{}{\Gamma :\ A \vdash A}\quad\dfrac{}{\Gamma :\ \bot \vdash \bot}}{\Gamma :\ A, A \multimap \bot \vdash \bot}\multimap}{\Gamma :\ A \multimap \bot \vdash !(A \multimap B) \multimap A \multimap \bot}\star \qquad \dfrac{\dfrac{\dfrac{}{\Gamma :\ A \vdash A}\quad\dfrac{}{\Gamma :\ \mathbf{0} \vdash B}}{\Gamma :\ A, A \multimap \mathbf{0} \vdash B}\multimap}{\Gamma :\ \cdot \vdash A \multimap \mathbf{0} \multimap A \multimap B}\star}{\Gamma :\ A \multimap \bot \vdash !(A \multimap B) \multimap A \multimap \bot \otimes A \multimap \mathbf{0} \multimap A \multimap B}\otimes$$

(39) $\Gamma :\ B \multimap \mathbf{0} \vdash A \multimap B \multimap A \multimap \mathbf{0} \otimes A \multimap \mathbf{0} \multimap A \multimap B$

encoding (86ms)

$$\dfrac{\dfrac{\dfrac{}{\Gamma :\ A \vdash A}\quad\dfrac{\dfrac{}{\Gamma :\ B \vdash B}\quad\dfrac{}{\Gamma :\ \mathbf{0} \vdash \mathbf{0}}}{\Gamma :\ B, B \multimap \mathbf{0} \vdash \mathbf{0}}\multimap}{\Gamma :\ A, A \multimap B, B \multimap \mathbf{0} \vdash \mathbf{0}}\multimap}{\Gamma :\ B \multimap \mathbf{0} \vdash A \multimap B \multimap A \multimap \mathbf{0}}\star \qquad \dfrac{\dfrac{\dfrac{}{\Gamma :\ A \vdash A}\quad\dfrac{}{\Gamma :\ \mathbf{0} \vdash B}}{\Gamma :\ A, A \multimap \mathbf{0} \vdash B}\multimap}{\Gamma :\ \cdot \vdash A \multimap \mathbf{0} \multimap A \multimap B}\star}{\Gamma :\ B \multimap \mathbf{0} \vdash A \multimap B \multimap A \multimap \mathbf{0} \otimes A \multimap \mathbf{0} \multimap A \multimap B}\otimes$$

(40) $\Gamma :\ B \vdash A \otimes !(B) \multimap A \otimes A \multimap A \otimes B$

encoding (55ms)

$$\dfrac{\dfrac{}{\Gamma :\ A \vdash A}}{\Gamma :\ \cdot \vdash A \otimes !(B) \multimap A}\star \qquad \dfrac{\dfrac{}{\Gamma :\ A \vdash A}\quad\dfrac{}{\Gamma :\ B \vdash B}}{\Gamma :\ A, B \vdash A \otimes B}\otimes}{\Gamma :\ B \vdash A \otimes !(B) \multimap A \otimes A \multimap A \otimes B}\star\otimes$$

(41) $\Gamma :\ B \multimap \mathbf{0} \vdash !(A) \otimes B \multimap B \otimes B \multimap A \otimes B$

encoding (69ms)

$$\dfrac{\dfrac{}{\Gamma :\ B \vdash B}}{\Gamma :\ \cdot \vdash !(A) \otimes B \multimap B}\star \qquad \dfrac{\dfrac{}{\Gamma :\ B \vdash B}\quad\dfrac{}{\Gamma :\ \mathbf{0} \vdash A \otimes B}\otimes}{\Gamma :\ B, B \multimap \mathbf{0} \vdash A \otimes B}\multimap}{\Gamma :\ B \multimap \mathbf{0} \vdash !(A) \otimes B \multimap B \otimes B \multimap A \otimes B}\star\otimes$$

(45) $\Gamma :\ \cdot \vdash !(A \multimap A \multimap \bot) \otimes !(A) \multimap \bot \multimap !(A) \multimap \bot$

encoding (65ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \bot \vdash \bot}}{\Gamma :\ \cdot \vdash \bot}D_C}{\Gamma :\ \cdot \vdash !(A) \multimap \bot}\star \qquad \dfrac{\dfrac{\dfrac{}{\Gamma :\ \bot \vdash \bot}}{\Gamma :\ \cdot \vdash \bot}D_C}{\Gamma :\ !(A) \vdash \bot}\star}{\Gamma :\ !(A) \multimap \bot \multimap !(A) \vdash \bot}\multimap}{\Gamma :\ \cdot \vdash !(A \multimap A \multimap \bot) \otimes !(A) \multimap \bot \multimap !(A) \multimap \bot}\star$$

(46) $\Gamma :\ \cdot \vdash !(!(A \multimap \bot \multimap \mathbf{0}) \multimap A \multimap \bot) \multimap \bot$

encoding (138ms)

$$\dfrac{\dfrac{\dfrac{}{\Gamma :\ A \vdash A}}{\Gamma :\ A \vdash !(A \multimap \bot \multimap \mathbf{0}) \multimap A}\star \qquad \dfrac{}{\Gamma :\ \bot \vdash \bot}}{\Gamma :\ A, !(A \multimap \bot \multimap \mathbf{0}) \multimap A \multimap \bot \vdash \bot}\multimap}{\Gamma :\ \cdot \vdash !(!(A \multimap \bot \multimap \mathbf{0}) \multimap A \multimap \bot) \multimap \bot}\star$$

(47) $\Gamma :\ \cdot \vdash A \otimes B \otimes B \multimap \mathbf{0} \multimap B \otimes B \multimap \mathbf{0} \otimes B \otimes B \multimap \mathbf{0} \multimap A \otimes B \otimes B \multimap \mathbf{0}$

encoding (83ms)

$$\dfrac{\dfrac{\dfrac{}{\Gamma :\ B \vdash B}\quad\dfrac{}{\Gamma :\ A, \mathbf{0} \vdash B \otimes B \multimap \mathbf{0}}\star}{\Gamma :\ A, B, B \multimap \mathbf{0} \vdash B \otimes B \multimap \mathbf{0}}\multimap}{\Gamma :\ \cdot \vdash A \otimes B \otimes B \multimap \mathbf{0} \multimap B \otimes B \multimap \mathbf{0}}\star \qquad \dfrac{\dfrac{\dfrac{}{\Gamma :\ B \vdash B}\quad\dfrac{}{\Gamma :\ \mathbf{0} \vdash A \otimes B \otimes B \multimap \mathbf{0}}\star}{\Gamma :\ B, B \multimap \mathbf{0} \vdash A \otimes B \otimes B \multimap \mathbf{0}}\multimap}{\Gamma :\ \cdot \vdash B \otimes B \multimap \mathbf{0} \multimap A \otimes B \otimes B \multimap \mathbf{0}}\star}{\Gamma :\ \cdot \vdash A \otimes B \otimes B \multimap \mathbf{0} \multimap B \otimes B \multimap \mathbf{0} \otimes B \otimes B \multimap \mathbf{0} \multimap A \otimes B \otimes B \multimap \mathbf{0}}\otimes$$

(57a) $\Gamma :\ \cdot \vdash A \multimap \bot \multimap \bot \otimes B \multimap \bot \multimap A \multimap B \multimap \bot$

encoding (71ms)

$$\dfrac{\dfrac{\dfrac{}{\Gamma :\ A \vdash A}\qquad\dfrac{\dfrac{}{\Gamma :\ B \vdash B}\quad\dfrac{}{\Gamma :\ \bot \vdash \bot}}{\Gamma :\ B, B \multimap \bot \vdash \bot}\multimap}{\Gamma :\ A, A \multimap B, B \multimap \bot \vdash \bot}\multimap}{\Gamma :\ A \multimap B, B \multimap \bot \vdash A \multimap \bot}\star \qquad \dfrac{}{\Gamma :\ \bot \vdash \bot}}{\Gamma :\ A \multimap B, B \multimap \bot, A \multimap \bot \multimap \bot \vdash \bot}\multimap}{\Gamma :\ \cdot \vdash A \multimap \bot \multimap \bot \otimes B \multimap \bot \multimap A \multimap B \multimap \bot}\star$$

(57b) $\Gamma :\ \cdot \vdash !(A) \multimap B \multimap \bot \multimap A \multimap \mathbf{0} \multimap \bot \,\&\, B \multimap \bot$

encoding (92ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}\quad\dfrac{}{\Gamma :\ \mathbf{0} \vdash B}\star}{\Gamma :\ A \multimap \mathbf{0} \vdash B}\multimap}{\Gamma :\ A \multimap \mathbf{0} \vdash !(A) \multimap B}\star \qquad \dfrac{}{\Gamma :\ \bot \vdash \bot}}{\Gamma :\ A \multimap \mathbf{0}, !(A) \multimap B \multimap \bot \vdash \bot}\multimap \qquad \dfrac{\dfrac{}{\Gamma :\ B \vdash B}}{\Gamma :\ B \vdash !(A) \multimap B}\star \qquad \dfrac{}{\Gamma :\ \bot \vdash \bot}}{\Gamma :\ B, !(A) \multimap B \multimap \bot \vdash \bot}\multimap}{\Gamma :\ \cdot \vdash !(A) \multimap B \multimap \bot \multimap A \multimap \mathbf{0} \multimap \bot \,\&\, B \multimap \bot}\star$$

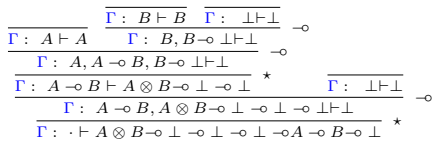
(58a) $\Gamma :\ \cdot \vdash !(!(A) \multimap B \multimap \bot) \multimap A \otimes B \multimap \bot \multimap \mathbf{0} \multimap \bot$

encoding (162ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma :\ B \vdash B}}{\Gamma :\ B \vdash !(A) \multimap B}\star \qquad \dfrac{}{\Gamma :\ \bot \vdash \bot}}{\Gamma :\ B, !(A) \multimap B \multimap \bot \vdash \bot}\multimap}{\Gamma :\ \cdot \vdash !(A) \multimap B \multimap \bot \multimap A \multimap \mathbf{0} \multimap \bot \,\&\, B \multimap \bot}D_C \qquad \dfrac{\dfrac{\dfrac{}{\Gamma :\ \cdot \vdash A}\qquad\dfrac{\dfrac{}{\Gamma :\ B \vdash B}}{\Gamma :\ \cdot \vdash A \otimes B \multimap \bot}\star}{\Gamma :\ \cdot \vdash A \otimes B \multimap \bot}\otimes \qquad \dfrac{}{\Gamma :\ \mathbf{0} \vdash B}\star}{\Gamma :\ A \otimes B \multimap \bot \multimap \mathbf{0} \vdash B}\multimap}{\Gamma :\ A \otimes B \multimap \bot \multimap \mathbf{0} \vdash !(A) \multimap B}\star \qquad \dfrac{}{\Gamma :\ \bot \vdash \bot}}{\Gamma :\ A \otimes B \multimap \bot \multimap \mathbf{0}, !(A) \multimap B \multimap \bot \vdash \bot}\multimap}{\Gamma :\ A \otimes B \multimap \bot \multimap \mathbf{0} \vdash \bot}D_C}{\Gamma :\ \cdot \vdash !(!(A) \multimap B \multimap \bot) \multimap A \otimes B \multimap \bot \multimap \mathbf{0} \multimap \bot}\star$$

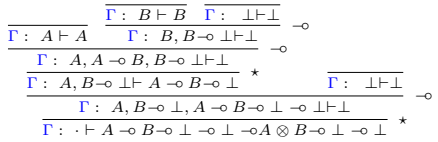
(58b) $\Gamma :\ \cdot \vdash A \otimes B \multimap \bot \multimap \bot \multimap \bot \multimap A \multimap B \multimap \bot$

encoding (69ms)



(59a) $\Gamma :\ \cdot \vdash A \multimap B \multimap \bot \multimap \bot \multimap A \otimes B \multimap \bot \multimap \bot$

encoding (71ms)



(59b) $\Gamma :\ \cdot \vdash A \otimes B \multimap \bot \multimap \mathbf{0} \multimap !(!(A) \multimap B \multimap \bot) \multimap \bot$

encoding (163ms)