Benchmarking Linear Logic: sequents inspired from Kleene's intuitionistic theorems*

October 10, 2018

1 General Information

- Test run on a QEMU Virtual CPU, 2GHz, 64 bits, 2GB of RAM running Ubuntu.
- Timeout in all the cases was 2 minutes.
- Maude version: 2.7.1 built: Oct 2 2017.
- The search procedure considers proofs using up to 4 times the copy rule (focusing on one of the formulas of the classical context). Benchmarks 23 and 25 required the limit to be 5.

2 Translations

$\boxed{(1)\ \cdot \vdash A \to A}$

LJ (28ms)

$$\frac{}{\cdot \vdash A \to A}\ \star$$

MULTIPLICATIVE encoding (28ms)

$$\frac{\dfrac{}{\cdot :\ A \vdash A}}{\cdot :\ \cdot \vdash A \multimap A}\ \star$$

CALL-BY-NAME encoding (28ms)

$$\frac{\dfrac{}{A :\ \cdot \vdash A}}{\cdot :\ \cdot \vdash\ !(A) \multimap A}\ \star$$

CALL-BY-VALUE encoding (41ms)

$$\frac{\dfrac{\dfrac{\dfrac{}{A :\ \cdot \vdash A}}{A :\ \cdot \vdash\ !(A)}\ !}{\cdot :\ \cdot \vdash\ !(A) \multimap\ !(A)}\ \star}{\cdot :\ \cdot \vdash\ !(!(A) \multimap\ !(A))}\ !$$

01-ENC encoding (35ms)

$$\frac{\dfrac{\dfrac{}{A :\ \cdot \vdash A}}{\cdot :\ \cdot \vdash\ !(A) \multimap A}\ \star}{\cdot :\ \cdot \vdash\ !(!(A) \multimap A)}\ !$$

*See `https://github.com/carlosolarte/Benchmarking-Linear-Logic` for details on the encodings used.

(2) $A \to B, B \to C \vdash A \to C$

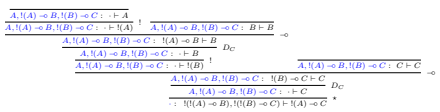
LJ (46ms)

$$\dfrac{\dfrac{}{A, A \to B, B \to C \vdash A}\ \star \quad \dfrac{\dfrac{}{A, B, B \to C \vdash B}\ \star \quad \dfrac{}{A, B, C \vdash C}\ \star}{A, B, B \to C \vdash C}\ \supset_L}{\dfrac{A, A \to B, B \to C \vdash C}{A \to B, B \to C \vdash A \to C}\ \star}\ \supset_L$$

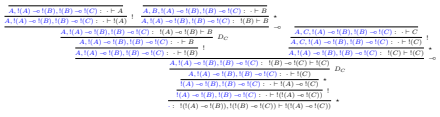
MULTIPLICATIVE encoding (49ms)

$$\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{\dfrac{}{\cdot :\ B \vdash B} \quad \dfrac{}{\cdot :\ C \vdash C}}{\cdot :\ B, B \multimap C \vdash C}\ \multimap}{\cdot :\ A, A \multimap B, B \multimap C \vdash C}\ \multimap}{\cdot :\ A \multimap B, B \multimap C \vdash A \multimap C}\ \star$$

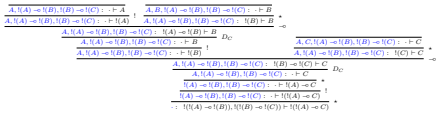
CALL-BY-NAME encoding (123ms)



CALL-BY-VALUE encoding (165ms)



01-ENC encoding (146ms)



(3) $A \to B \to C \vdash B \to A \to C$

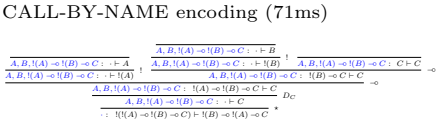
LJ (46ms)

$$\dfrac{\dfrac{}{A, B, A \to B \to C \vdash A}\ \star \quad \dfrac{\dfrac{}{A, B, B \to C \vdash B}\ \star \quad \dfrac{}{A, B, C \vdash C}\ \star}{A, B, B \to C \vdash C}\ \supset_L}{\dfrac{A, B, A \to B \to C \vdash C}{A \to B \to C \vdash B \to A \to C}\ \star}\ \supset_L$$

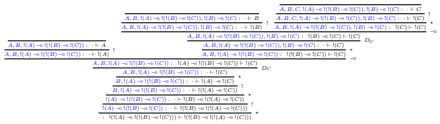
MULTIPLICATIVE encoding (47ms)

$$\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{\dfrac{}{\cdot :\ B \vdash B} \quad \dfrac{}{\cdot :\ C \vdash C}}{\cdot :\ B, B \multimap C \vdash C}\ \multimap}{\cdot :\ A, B, A \multimap B \multimap C \vdash C}\ \multimap}{\cdot :\ A \multimap B \multimap C \vdash B \multimap A \multimap C}\ \star$$

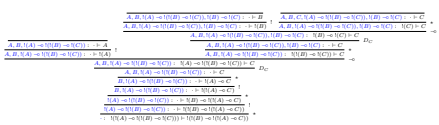
CALL-BY-NAME encoding (71ms)



CALL-BY-VALUE encoding (162ms)

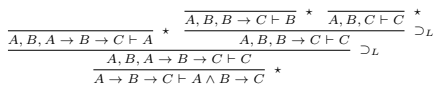


01-ENC encoding (158ms)

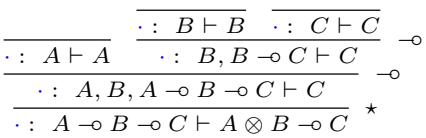


(4) $A \to B \to C \vdash A \wedge B \to C$

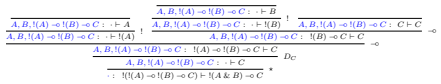
LJ (47ms)



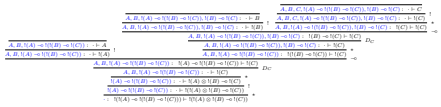
MULTIPLICATIVE encoding (47ms)



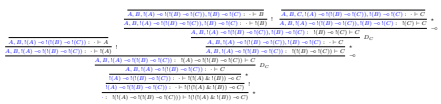
CALL-BY-NAME encoding (71ms)



CALL-BY-VALUE encoding (156ms)

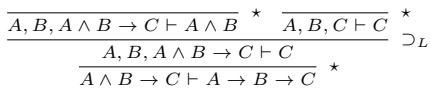


01-ENC encoding (151ms)

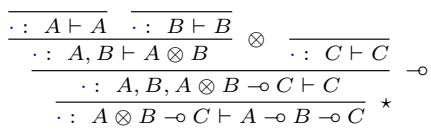


(5) $A \wedge B \to C \vdash A \to B \to C$

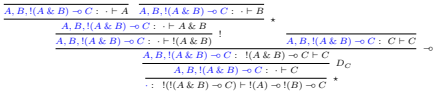
LJ (41ms)



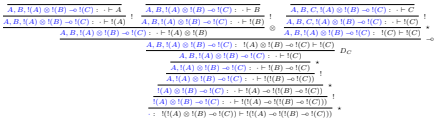
MULTIPLICATIVE encoding (54ms)



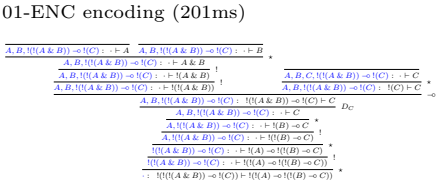
CALL-BY-NAME encoding (73ms)



CALL-BY-VALUE encoding (205ms)

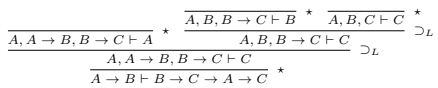


01-ENC encoding (201ms)

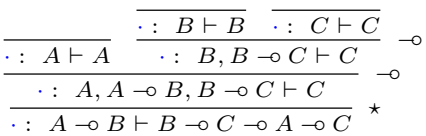


(6) $A \to B \vdash B \to C \to A \to C$

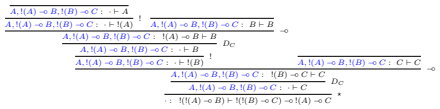
LJ (47ms)



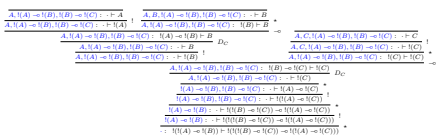
MULTIPLICATIVE encoding (48ms)



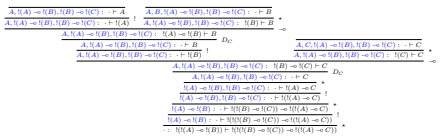
CALL-BY-NAME encoding (120ms)



CALL-BY-VALUE encoding (168ms)

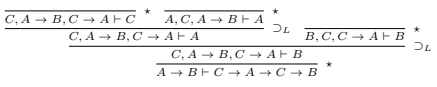


01-ENC encoding (159ms)

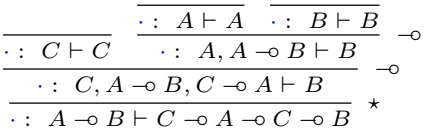


(7) $A \to B \vdash C \to A \to C \to B$

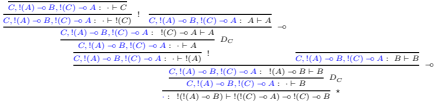
LJ (47ms)



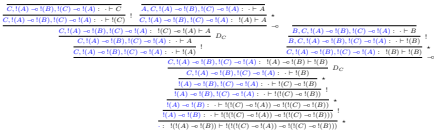
MULTIPLICATIVE encoding (48ms)



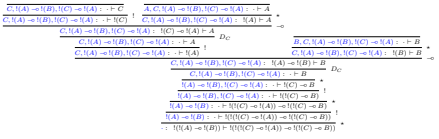
CALL-BY-NAME encoding (120ms)



CALL-BY-VALUE encoding (167ms)



01-ENC encoding (156ms)



(8) $A \to B \vdash A \wedge C \to B \wedge C$

LJ (37ms)

$$\dfrac{\dfrac{}{A, C, A \to B \vdash A}\ \star \quad \dfrac{}{A, B, C \vdash B}\ \star}{\dfrac{A, C, A \to B \vdash B}{A \to B \vdash A \wedge C \to B \wedge C}\ \star}\ \supset_L$$

MULTIPLICATIVE encoding (48ms)

$$\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{\dfrac{}{\cdot :\ B \vdash B} \quad \dfrac{}{\cdot :\ C \vdash C}\ \otimes}{\cdot :\ B, C \vdash B \otimes C}}{\dfrac{\cdot :\ A, C, A \multimap B \vdash B \otimes C}{\cdot :\ A \multimap B \vdash A \otimes C \multimap B \otimes C}\ \star}\ \multimap$$

CALL-BY-NAME encoding (73ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A, C, !(A) \multimap B :\ \cdot \vdash A}}{A, C, !(A) \multimap B :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, C, !(A) \multimap B :\ B \vdash B}}{A, C, !(A) \multimap B :\ !(A) \multimap B \vdash B}\ \multimap}{A, C, !(A) \multimap B :\ \cdot \vdash B}\ D_C \qquad \dfrac{}{A, C, !(A) \multimap B :\ \cdot \vdash C}}{\cdot :\ !(!(A) \multimap B) \vdash !(A \,\&\, C) \multimap B \,\&\, C}\ \star$$

CALL-BY-VALUE encoding (135ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, C, !(A) \multimap !(B) :\ \cdot \vdash A}}{A, C, !(A) \multimap !(B) :\ \cdot \vdash !(A)}\ ! \quad \dfrac{\dfrac{}{A, B, C, !(A) \multimap !(B) :\ \cdot \vdash B}}{A, C, !(A) \multimap !(B) :\ !(B) \vdash B}\ \star}{A, C, !(A) \multimap !(B) :\ !(A) \multimap !(B) \vdash B}\ \multimap}{A, C, !(A) \multimap !(B) :\ \cdot \vdash B}\ D_C \qquad \dfrac{}{A, C, !(A) \multimap !(B) :\ \cdot \vdash C}}{A, C, !(A) \multimap !(B) :\ \cdot \vdash !(B)}\ ! \qquad \dfrac{}{A, C, !(A) \multimap !(B) :\ \cdot \vdash !(C)}\ !}{A, C, !(A) \multimap !(B) :\ \cdot \vdash !(B) \otimes !(C)}\ \otimes$$
$$\dfrac{\dfrac{!(A) \multimap !(B) :\ \cdot \vdash !(A) \otimes !(C) \multimap !(B) \otimes !(C)}{!(A) \multimap !(B) :\ \cdot \vdash !(!(A) \otimes !(C) \multimap !(B) \otimes !(C))}\ !}{\cdot :\ !(!(A) \multimap !(B)) \vdash !(!(A) \otimes !(C) \multimap !(B) \otimes !(C))}\ \star$$

01-ENC encoding (130ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, C, !(A) \multimap !(B) :\ \cdot \vdash A}}{A, C, !(A) \multimap !(B) :\ \cdot \vdash !(A)}\ ! \quad \dfrac{\dfrac{}{A, B, C, !(A) \multimap !(B) :\ \cdot \vdash B}}{A, C, !(A) \multimap !(B) :\ !(B) \vdash B}\ \star}{A, C, !(A) \multimap !(B) :\ !(A) \multimap !(B) \vdash B}\ \multimap}{A, C, !(A) \multimap !(B) :\ \cdot \vdash B}\ D_C \qquad \dfrac{}{A, C, !(A) \multimap !(B) :\ \cdot \vdash C}}{A, C, !(A) \multimap !(B) :\ \cdot \vdash B \,\&\, C}\ !}{!(A) \multimap !(B) :\ \cdot \vdash !(!(A) \,\&\, !(C)) \multimap !(B \,\&\, C)}\ \star}{!(A) \multimap !(B) :\ \cdot \vdash !(!(!(A) \,\&\, !(C)) \multimap !(B \,\&\, C))}\ !}{\cdot :\ !(!(A) \multimap !(B)) \vdash !(!(!(A) \,\&\, !(C)) \multimap !(B \,\&\, C))}\ \star$$

(9) $A \to B \vdash C \wedge A \to C \wedge B$

LJ (37ms)

$$\dfrac{\dfrac{}{A, C, A \to B \vdash A}\ \star \quad \dfrac{}{A, B, C \vdash B}\ \star}{\dfrac{A, C, A \to B \vdash B}{A \to B \vdash C \wedge A \to C \wedge B}\ \star}\ \supset_L$$

MULTIPLICATIVE encoding (48ms)

$$\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{\dfrac{}{\cdot :\ C \vdash C} \quad \dfrac{}{\cdot :\ B \vdash B}\ \otimes}{\cdot :\ B, C \vdash C \otimes B}}{\dfrac{\cdot :\ A, C, A \multimap B \vdash C \otimes B}{\cdot :\ A \multimap B \vdash C \otimes A \multimap C \otimes B}\ \star}\ \multimap$$

CALL-BY-NAME encoding (60ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, C, !(A) \multimap B :\ \cdot \vdash A}}{A, C, !(A) \multimap B :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, C, !(A) \multimap B :\ B \vdash B}}{A, C, !(A) \multimap B :\ !(A) \multimap B \vdash B}\ \multimap}{A, C, !(A) \multimap B :\ \cdot \vdash B}\ D_C}{A, C, !(A) \multimap B :\ \cdot \vdash C}}{\cdot :\ !(!(A) \multimap B) \vdash !(C \,\&\, A) \multimap C \,\&\, B}\ \star$$

CALL-BY-VALUE encoding (137ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, C, !(A) \multimap !(B) :\ \cdot \vdash A}}{A, C, !(A) \multimap !(B) :\ \cdot \vdash !(A)}\ ! \quad \dfrac{\dfrac{}{A, B, C, !(A) \multimap !(B) :\ \cdot \vdash C}}{A, B, C, !(A) \multimap !(B) :\ !(B) \vdash !(C) \otimes !(B)}\ \star}{A, C, !(A) \multimap !(B) :\ !(A) \multimap !(B) \vdash !(C) \otimes !(B)}\ \multimap}{A, C, !(A) \multimap !(B) :\ \cdot \vdash !(C) \otimes !(B)}\ D_C \qquad \dfrac{}{A, B, C, !(A) \multimap !(B) :\ \cdot \vdash !(C)}\ !}{!(A) \multimap !(B) :\ \cdot \vdash !(C) \otimes !(A) \multimap !(C) \otimes !(B)}\ \star}{!(A) \multimap !(B) :\ \cdot \vdash !(!(C) \otimes !(A) \multimap !(C) \otimes !(B))}\ !}{\cdot :\ !(!(A) \multimap !(B)) \vdash !(!(C) \otimes !(A) \multimap !(C) \otimes !(B))}\ \star$$

01-ENC encoding (131ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, C, !(A) \multimap !(B) :\ \cdot \vdash A}}{A, C, !(A) \multimap !(B) :\ \cdot \vdash !(A)}\ ! \quad \dfrac{\dfrac{\dfrac{}{A, B, C, !(A) \multimap !(B) :\ \cdot \vdash C} \quad \dfrac{}{A, B, C, !(A) \multimap !(B) :\ \cdot \vdash B}\ \star}{A, B, C, !(A) \multimap !(B) :\ \cdot \vdash C \,\&\, B}}{A, C, !(A) \multimap !(B) :\ !(B) \vdash !(C \,\&\, B)}\ \star}{A, C, !(A) \multimap !(B) :\ !(A) \multimap !(B) \vdash !(C \,\&\, B)}\ \multimap}{A, C, !(A) \multimap !(B) :\ \cdot \vdash !(C \,\&\, B)}\ D_C}{!(A) \multimap !(B) :\ \cdot \vdash !(!(C) \,\&\, !(A)) \multimap !(C \,\&\, B)}\ \star}{!(A) \multimap !(B) :\ \cdot \vdash !(!(!(C) \,\&\, !(A)) \multimap !(C \,\&\, B))}\ !}{\cdot :\ !(!(A) \multimap !(B)) \vdash !(!(!(C) \,\&\, !(A)) \multimap !(C \,\&\, B))}\ \star$$

(10) $\neg A \vdash A \to B$

LJ (37ms)

$$\dfrac{\dfrac{}{A, A \to \bot \vdash A}\ \star \quad \dfrac{}{A, \bot \vdash B}\ \star}{\dfrac{A, A \to \bot \vdash B}{A \to \bot \vdash A \to B}\ \star}\ \supset_L$$

MULTIPLICATIVE encoding (41ms)

Not provable

CALL-BY-NAME encoding (66ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap \mathbf{0} :\ \mathbf{0} \vdash B}\ \star}{\dfrac{A, !(A) \multimap \mathbf{0} :\ !(A) \multimap \mathbf{0} \vdash B}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash B}\ D_C}\ \multimap}{\cdot :\ !(!(A) \multimap \mathbf{0}) \vdash !(A) \multimap B}\ \star$$

CALL-BY-VALUE encoding (117ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap \mathbf{0} :\ \mathbf{0} \vdash !(B)}\ \star}{\dfrac{A, !(A) \multimap \mathbf{0} :\ !(A) \multimap \mathbf{0} \vdash !(B)}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(B)}\ D_C}\ \multimap}{\dfrac{!(A) \multimap \mathbf{0} :\ \cdot \vdash !(A) \multimap !(B)}{!(A) \multimap \mathbf{0} :\ \cdot \vdash !(!(A) \multimap !(B))}\ !}\ \star}{\cdot :\ !(!(A) \multimap \mathbf{0}) \vdash !(!(A) \multimap !(B))}\ \star$$

01-ENC encoding (118ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash A}}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap !(\mathbf{0}) :\ !(\mathbf{0}) \vdash B}\ \star}{\dfrac{A, !(A) \multimap !(\mathbf{0}) :\ !(A) \multimap !(\mathbf{0}) \vdash B}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash B}\ D_C}\ \multimap}{!(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(A) \multimap B}\ \star}{!(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(!(A) \multimap B)}\ !}{\cdot :\ !(!(A) \multimap !(\mathbf{0})) \vdash !(!(A) \multimap B)}\ \star$$

(11) $A \vdash \neg A \to B$

LJ (37ms)

$$\dfrac{\dfrac{}{A, A \to \bot \vdash A}\ \star \quad \dfrac{}{A, \bot \vdash B}\ \star}{\dfrac{A, A \to \bot \vdash B}{A \vdash A \to \bot \to B}\ \star}\ \supset_L$$

MULTIPLICATIVE encoding (28ms)

Not provable

CALL-BY-NAME encoding (65ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap \mathbf{0} :\ \mathbf{0} \vdash B}\ \star}{\dfrac{A, !(A) \multimap \mathbf{0} :\ !(A) \multimap \mathbf{0} \vdash B}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash B}\ D_C}\ \multimap}{\cdot :\ !(A) \vdash !(!(A) \multimap \mathbf{0}) \multimap B}\ \star$$

CALL-BY-VALUE encoding (78ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap \mathbf{0} :\ \mathbf{0} \vdash !(B)}\ \star}{\dfrac{A, !(A) \multimap \mathbf{0} :\ !(A) \multimap \mathbf{0} \vdash !(B)}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(B)}\ D_C}\ \multimap}{\dfrac{A :\ \cdot \vdash !(!(A) \multimap \mathbf{0}) \multimap !(B)}{A :\ \cdot \vdash !(!(!(A) \multimap \mathbf{0}) \multimap !(B))}\ !}\ \star}{\cdot :\ !(A) \vdash !(!(!(A) \multimap \mathbf{0}) \multimap !(B))}\ \star$$

01-ENC encoding (76ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash A}}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap !(\mathbf{0}) :\ !(\mathbf{0}) \vdash B}\ \star}{\dfrac{A, !(A) \multimap !(\mathbf{0}) :\ !(A) \multimap !(\mathbf{0}) \vdash B}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash B}\ D_C}\ \multimap}{A :\ \cdot \vdash !(!(A) \multimap !(\mathbf{0})) \multimap B}\ \star}{A :\ \cdot \vdash !(!(!(A) \multimap !(\mathbf{0})) \multimap B)}\ !}{\cdot :\ !(A) \vdash !(!(!(A) \multimap !(\mathbf{0})) \multimap B)}\ \star$$

(12) $B \vdash A \to B$

LJ (21ms)

$$\frac{}{B \vdash A \to B}\ \star$$

MULTIPLICATIVE encoding (21ms)

Not provable

CALL-BY-NAME encoding (27ms)

$$\frac{\dfrac{}{A, B :\ \cdot \vdash B}}{\cdot :\ \ !(B) \vdash\ !(A) \multimap B}\ \star$$

CALL-BY-VALUE encoding (47ms)

$$\frac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \vdash B}}{A, B :\ \cdot \vdash\ !(B)}\ !}{B :\ \cdot \vdash\ !(A) \multimap\ !(B)}\ \star}{B :\ \cdot \vdash\ !(!(A) \multimap\ !(B))}\ !}{\cdot :\ \ !(B) \vdash\ !(!(A) \multimap\ !(B))}\ \star$$

01-ENC encoding (39ms)

$$\frac{\dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \vdash B}}{B :\ \cdot \vdash\ !(A) \multimap B}\ \star}{B :\ \cdot \vdash\ !(!(A) \multimap B)}\ !}{\cdot :\ \ !(B) \vdash\ !(!(A) \multimap B)}\ \star$$

(13) $A \to B \vdash \neg B \to \neg A$

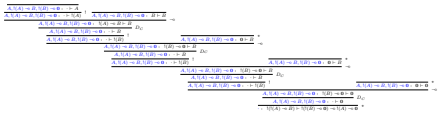
LJ (47ms)

$$\dfrac{\dfrac{\dfrac{}{A, A \to B, B \to \bot \vdash A}\ \star \quad \dfrac{\dfrac{}{A, B, B \to \bot \vdash B}\ \star \quad \dfrac{}{A, B, \bot \vdash \bot}\ \star}{A, B, B \to \bot \vdash \bot}\ \supset_L}{A, A \to B, B \to \bot \vdash \bot}\ \supset_L}{A \to B \vdash B \to \bot \to A \to \bot}\ \star$$

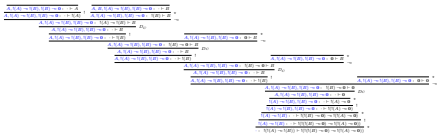
MULTIPLICATIVE encoding (47ms)

$$\dfrac{\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{\dfrac{}{\cdot :\ B \vdash B} \quad \dfrac{}{\cdot :\ \ \bot \vdash \bot}}{\cdot :\ B, B \multimap \bot \vdash \bot}\ \multimap}{\cdot :\ A, A \multimap B, B \multimap \bot \vdash \bot}\ \multimap}{\cdot :\ A \multimap B \vdash B \multimap \bot \multimap A \multimap \bot}\ \star$$

CALL-BY-NAME encoding (143ms)



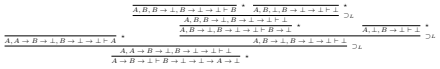
CALL-BY-VALUE encoding (174ms)



01-ENC encoding (178ms)

(14) $A \to \neg B \vdash \neg\neg B \to \neg A$

LJ (64ms)



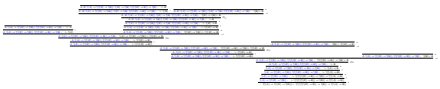
MULTIPLICATIVE encoding (75ms)

$$\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{\dfrac{}{\cdot :\ B \vdash B} \quad \dfrac{}{\cdot :\ \bot \vdash \bot}}{\cdot :\ B, B \multimap \bot \vdash \bot}\ \multimap}{\cdot :\ A, B, A \multimap B \multimap \bot \vdash \bot}\ \multimap \qquad \dfrac{\dfrac{}{\cdot :\ A, A \multimap B \multimap \bot \vdash B \multimap \bot}\ \star \qquad \dfrac{}{\cdot :\ \bot \vdash \bot}}{\cdot :\ A, A \multimap B \multimap \bot, B \multimap \bot \multimap \bot \vdash \bot}\ \multimap}{\cdot :\ A \multimap B \multimap \bot \vdash B \multimap \bot \multimap \bot \multimap A \multimap \bot}\ \star$$

CALL-BY-NAME encoding (180ms)

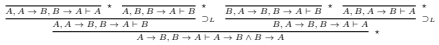
CALL-BY-VALUE encoding (217ms)

01-ENC encoding (226ms)



(15) $A \to B, B \to A \vdash (A) \leftrightarrow (B)$

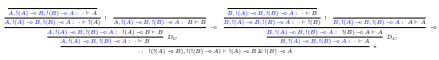
LJ (51ms)



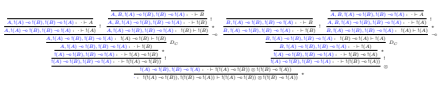
MULTIPLICATIVE encoding (75ms)

$$\dfrac{\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{}{\cdot :\ B \vdash B}\ \multimap}{\cdot :\ A, A \multimap B \vdash B}\ \multimap \quad \star}{\cdot :\ A \multimap B \vdash A \multimap B} \qquad \dfrac{\dfrac{\dfrac{}{\cdot :\ B \vdash B} \quad \dfrac{}{\cdot :\ A \vdash A}\ \multimap}{\cdot :\ B, B \multimap A \vdash A}\ \multimap \quad \star}{\cdot :\ B \multimap A \vdash B \multimap A} \otimes$$
$$\cdot :\ A \multimap B, B \multimap A \vdash A \multimap B \otimes B \multimap A$$

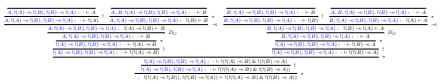
CALL-BY-NAME encoding (138ms)



CALL-BY-VALUE encoding (205ms)



01-ENC encoding (191ms)



$\boxed{(16)\ (A) \leftrightarrow (B) \vdash A \to B}$

LJ (37ms)

$$\dfrac{\dfrac{}{A, A \to B, B \to A \vdash A}\ \star \quad \dfrac{}{A, B, B \to A \vdash B}\ \star}{\dfrac{A, A \to B, B \to A \vdash B}{A \to B \wedge B \to A \vdash A \to B}\ \star}\ \supset_L$$

MULTIPLICATIVE encoding (29ms)

Not provable

CALL-BY-NAME encoding (102ms)

$$\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash A}}{A, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash\ !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap B, !(B) \multimap A :\ B \vdash B}}{\dfrac{A, !(A) \multimap B, !(B) \multimap A :\ !(A) \multimap B \vdash B}{A, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash B}\ D_C}\ \multimap$$
$$\dfrac{A, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash B}{\cdot :\ !(!(A) \multimap B\ \&\ !(B) \multimap A) \vdash !(A) \multimap B}\ \star$$

CALL-BY-VALUE encoding (132ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash A}}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash\ !(A)}\ ! \quad \dfrac{\dfrac{}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash B}}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash\ !(B)}\ !}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ !(A) \multimap !(B) \vdash !(B)}\ \star\ \multimap$$
$$\dfrac{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash\ !(B)}{!(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(A) \multimap !(B)}\ D_C$$
$$\dfrac{!(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(A) \multimap !(B)}{!(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(!(A) \multimap !(B))}\ \star$$
$$\dfrac{}{\cdot :\ !(!(A) \multimap !(B)) \otimes !(!(B) \multimap !(A)) \vdash !(!(A) \multimap !(B))}\ !\ \star$$

01-ENC encoding (126ms)

$$\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash A}}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash\ !(A)}\ ! \quad \dfrac{}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash B}}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ !(A) \multimap !(B) \vdash B}\ \star\ \multimap$$
$$\dfrac{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ !(A) \multimap !(B) \vdash B}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash B}\ D_C$$
$$\dfrac{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash B}{!(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(A) \multimap B}\ \star$$
$$\dfrac{!(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(A) \multimap B}{!(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(!(A) \multimap B)}\ !$$
$$\dfrac{}{\cdot :\ !(!(!(A) \multimap !(B)) \,\&\, !(!(B) \multimap !(A))) \vdash !(!(A) \multimap B)}\ \star$$

$\boxed{(17)\ (A) \leftrightarrow (B) \vdash B \to A}$

LJ (37ms)

$$\dfrac{\dfrac{}{B, A \to B, B \to A \vdash B}\ \star \quad \dfrac{}{A, B, A \to B \vdash A}\ \star}{\dfrac{B, A \to B, B \to A \vdash A}{A \to B \wedge B \to A \vdash B \to A}\ \star}\ \supset_L$$

MULTIPLICATIVE encoding (29ms)

Not provable

CALL-BY-NAME encoding (105ms)

$$\dfrac{\dfrac{\dfrac{}{B, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash B}}{B, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash\ !(B)}\ ! \quad \dfrac{}{B, !(A) \multimap B, !(B) \multimap A :\ A \vdash A}}{\dfrac{B, !(A) \multimap B, !(B) \multimap A :\ !(B) \multimap A \vdash A}{B, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash A}\ D_C}\ \multimap$$
$$\dfrac{B, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash A}{\cdot :\ !(!(A) \multimap B\ \&\ !(B) \multimap A) \vdash !(B) \multimap A}\ \star$$

CALL-BY-VALUE encoding (140ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash B}}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash\ !(B)}\ ! \quad \dfrac{\dfrac{}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash A}}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash\ !(A)}\ !}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ !(B) \multimap !(A) \vdash !(A)}\ \star\ \multimap$$
$$\dfrac{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ !(B) \multimap !(A) \vdash !(A)}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(A)}\ D_C$$
$$\dfrac{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(A)}{!(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(B) \multimap !(A)}\ \star$$
$$\dfrac{!(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(B) \multimap !(A)}{!(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(!(B) \multimap !(A))}\ !$$
$$\dfrac{}{\cdot :\ !(!(A) \multimap !(B)) \otimes !(!(B) \multimap !(A)) \vdash !(!(B) \multimap !(A))}\ \star$$

01-ENC encoding (122ms)

$$\dfrac{\dfrac{\dfrac{}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash B}}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash\ !(B)}\ ! \quad \dfrac{}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash A}}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ !(B) \multimap !(A) \vdash A}\ \star\ \multimap$$
$$\dfrac{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ !(B) \multimap !(A) \vdash A}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash A}\ D_C$$
$$\dfrac{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash A}{!(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(B) \multimap A}\ \star$$
$$\dfrac{!(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(B) \multimap A}{!(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(!(B) \multimap A)}\ !$$
$$\dfrac{}{\cdot :\ !(!(!(A) \multimap !(B)) \,\&\, !(!(B) \multimap !(A))) \vdash !(!(B) \multimap A)}\ \star$$

(18) $(A) \leftrightarrow (B), A \vdash B$

LJ (38ms)

$$\dfrac{\dfrac{}{A, A \to B, B \to A \vdash A}\ \star \quad \dfrac{}{A, B, B \to A \vdash B}\ \star}{\dfrac{A, A \to B, B \to A \vdash B}{A, A \to B \wedge B \to A \vdash B}\ \star}\ \supset_L$$

MULTIPLICATIVE encoding (29ms)

Not provable

CALL-BY-NAME encoding (102ms)

$$\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash A}}{A, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap B, !(B) \multimap A :\ B \vdash B}}{\dfrac{A, !(A) \multimap B, !(B) \multimap A :\ !(A) \multimap B \vdash B}{A, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash B}\ D_C}\ \multimap$$
$$\dfrac{}{\cdot :\ !(A), !(!(A) \multimap B \,\&\, !(B) \multimap A) \vdash B}\ \star$$

CALL-BY-VALUE encoding (119ms)

$$\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash A}}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(A)}\ ! \quad \dfrac{\dfrac{}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash B}}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(B)}\ !}{\dfrac{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ !(A) \multimap !(B) \vdash !(B)}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(B)}\ D_C}\ \multimap$$
$$\dfrac{}{\cdot :\ !(A), !(!(A) \multimap !(B)) \otimes !(!(B) \multimap !(A)) \vdash !(B)}\ \star$$

01-ENC encoding (111ms)

$$\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash A}}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(A)}\ ! \quad \dfrac{\dfrac{}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash B}}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ !(B) \vdash B}\ \star}{\dfrac{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ !(A) \multimap !(B) \vdash B}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash B}\ D_C}\ \multimap$$
$$\dfrac{}{\cdot :\ !(A), !(!(!(A) \multimap !(B)) \,\&\, !(!(B) \multimap !(A))) \vdash B}\ \star$$

(19) $(A) \leftrightarrow (B), B \vdash A$

LJ (38ms)

$$\dfrac{\dfrac{}{B, A \to B, B \to A \vdash B}\ \star \quad \dfrac{}{A, B, A \to B \vdash A}\ \star}{\dfrac{B, A \to B, B \to A \vdash A}{B, A \to B \wedge B \to A \vdash A}\ \star}\ \supset_L$$

MULTIPLICATIVE encoding (32ms)

Not provable

CALL-BY-NAME encoding (102ms)

$$\dfrac{\dfrac{\dfrac{}{B, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash B}}{B, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash !(B)}\ ! \quad \dfrac{}{B, !(A) \multimap B, !(B) \multimap A :\ A \vdash A}}{\dfrac{B, !(A) \multimap B, !(B) \multimap A :\ !(B) \multimap A \vdash A}{B, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash A}\ D_C}\ \multimap$$
$$\dfrac{}{\cdot :\ !(B), !(!(A) \multimap B \,\&\, !(B) \multimap A) \vdash A}\ \star$$

CALL-BY-VALUE encoding (118ms)

$$\dfrac{\dfrac{\dfrac{}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash B}}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(B)}\ ! \quad \dfrac{\dfrac{}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash A}}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(A)}\ !}{\dfrac{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ !(B) \multimap !(A) \vdash !(A)}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(A)}\ D_C}\ \multimap$$
$$\dfrac{}{\cdot :\ !(B), !(!(A) \multimap !(B)) \otimes !(!(B) \multimap !(A)) \vdash !(A)}\ \star$$

01-ENC encoding (108ms)

$$\dfrac{\dfrac{\dfrac{}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash B}}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(B)}\ ! \quad \dfrac{\dfrac{}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash A}}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ !(A) \vdash A}\ \star}{\dfrac{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ !(B) \multimap !(A) \vdash A}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash A}\ D_C}\ \multimap$$
$$\dfrac{}{\cdot :\ !(B), !(!(!(A) \multimap !(B)) \,\&\, !(!(B) \multimap !(A))) \vdash A}\ \star$$

$\boxed{(20)\ \cdot \vdash (A) \leftrightarrow (A)}$

LJ (21ms)

$$\frac{}{\cdot \vdash A \to A \wedge A \to A}\ \star$$

MULTIPLICATIVE encoding (34ms)

$$\frac{\dfrac{\dfrac{}{\cdot :\ A \vdash A}}{\cdot :\ \cdot \vdash A \multimap A}\ \star \qquad \dfrac{\dfrac{}{\cdot :\ A \vdash A}}{\cdot :\ \cdot \vdash A \multimap A}\ \star}{\cdot :\ \cdot \vdash A \multimap A \otimes A \multimap A}\ \otimes$$

CALL-BY-NAME encoding (27ms)

$$\frac{\dfrac{}{A :\ \cdot \vdash A} \qquad \dfrac{}{A :\ \cdot \vdash A}}{\cdot :\ \cdot \vdash\ !(A) \multimap A\ \&\ !(A) \multimap A}\ \star$$

CALL-BY-VALUE encoding (47ms)

$$\frac{\dfrac{\dfrac{\dfrac{}{A :\ \cdot \vdash A}}{A :\ \cdot \vdash\ !(A)}\ !}{\cdot :\ \cdot \vdash\ !(A) \multimap\ !(A)}\ \star}{\cdot :\ \cdot \vdash\ !(!(A) \multimap\ !(A))}\ ! \qquad \frac{\dfrac{\dfrac{}{A :\ \cdot \vdash A}}{A :\ \cdot \vdash\ !(A)}\ !}{\cdot :\ \cdot \vdash\ !(A) \multimap\ !(A)}\ \star}{\cdot :\ \cdot \vdash\ !(!(A) \multimap\ !(A)) \otimes\ !(!(A) \multimap\ !(A))}\ \otimes\ !$$

01-ENC encoding (47ms)

$$\frac{\dfrac{\dfrac{\dfrac{}{A :\ \cdot \vdash A}}{\cdot :\ \cdot \vdash\ !(A) \multimap A}\ \star}{\cdot :\ \cdot \vdash\ !(!(A) \multimap A)}\ ! \qquad \frac{\dfrac{\dfrac{}{A :\ \cdot \vdash A}}{\cdot :\ \cdot \vdash\ !(A) \multimap A}\ \star}{\cdot :\ \cdot \vdash\ !(!(A) \multimap A)}\ !}{\cdot :\ \cdot \vdash\ !(!(A) \multimap A)\ \&\ !(!(A) \multimap A)}\ \star$$
$$\frac{\cdot :\ \cdot \vdash\ !(!(A) \multimap A)\ \&\ !(!(A) \multimap A)}{\cdot :\ \cdot \vdash\ !(!(!(A) \multimap A)\ \&\ !(!(A) \multimap A))}\ !$$

$\boxed{(21)\ (A) \leftrightarrow (B) \vdash (B) \leftrightarrow (A)}$

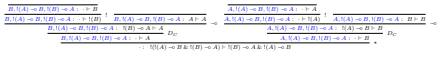
LJ (50ms)

$$\frac{\dfrac{\dfrac{}{B, A \to B, B \to A \vdash B}\ \star \qquad \dfrac{}{A, B, A \to B \vdash A}\ \star}{B, A \to B, B \to A \vdash A}\ \supset L \qquad \dfrac{\dfrac{}{A, A \to B, B \to A \vdash A}\ \star \qquad \dfrac{}{A, B, B \to A \vdash B}\ \star}{A, A \to B, B \to A \vdash B}\ \supset L}{A \to B \wedge B \to A \vdash B \to A \wedge A \to B}\ \star$$

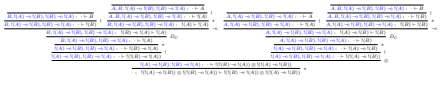
MULTIPLICATIVE encoding (81ms)

$$\frac{\dfrac{\dfrac{\dfrac{}{\cdot :\ B \vdash B} \qquad \dfrac{}{\cdot :\ A \vdash A}}{\cdot :\ B, B \multimap A \vdash A}\ \multimap}{\cdot :\ B \multimap A \vdash B \multimap A}\ \star \qquad \dfrac{\dfrac{\dfrac{}{\cdot :\ A \vdash A} \qquad \dfrac{}{\cdot :\ B \vdash B}}{\cdot :\ A, A \multimap B \vdash B}\ \multimap}{\cdot :\ A \multimap B \vdash A \multimap B}\ \star}{\cdot :\ A \multimap B, B \multimap A \vdash B \multimap A \otimes A \multimap B}\ \otimes$$
$$\frac{}{\cdot :\ A \multimap B \otimes B \multimap A \vdash B \multimap A \otimes A \multimap B}\ \star$$

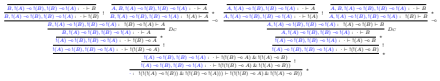
CALL-BY-NAME encoding (138ms)



CALL-BY-VALUE encoding (203ms)



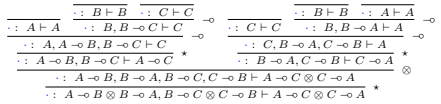
01-ENC encoding (188ms)



(22) $(A) \leftrightarrow (B), (B) \leftrightarrow (C) \vdash (A) \leftrightarrow (C)$

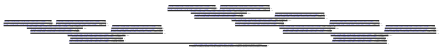
LJ (75ms)

MULTIPLICATIVE encoding (115ms)

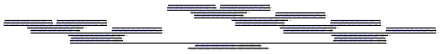


CALL-BY-NAME encoding (218ms)

CALL-BY-VALUE encoding (331ms)



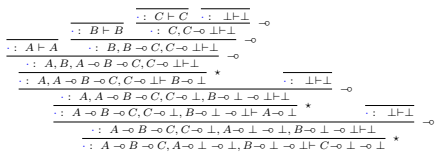
01-ENC encoding (291ms)



(23) $A \to B \to C, \neg\neg A, \neg\neg B \vdash \neg\neg C$

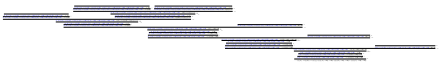
LJ (206ms)

MULTIPLICATIVE encoding (124ms)

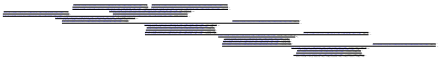


CALL-BY-NAME encoding (221ms)

CALL-BY-VALUE encoding (324ms)



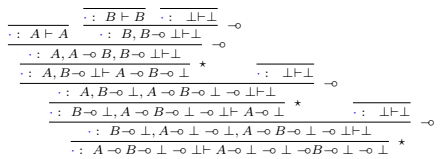
01-ENC encoding (380ms)



(24) $\neg\neg A \to B \vdash \neg\neg A \to \neg\neg B$

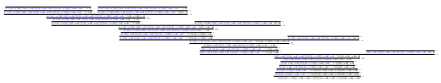
LJ (94ms)

MULTIPLICATIVE encoding (106ms)

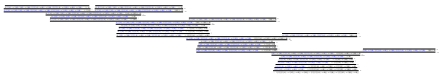


CALL-BY-NAME encoding (199ms)

CALL-BY-VALUE encoding (241ms)



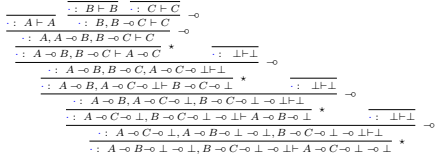
01-ENC encoding (269ms)



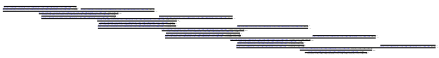
(25) $\neg\neg A \to B, \neg\neg B \to C \vdash \neg\neg A \to C$

LJ (324ms)

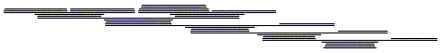
MULTIPLICATIVE encoding (145ms)



CALL-BY-NAME encoding (283ms)



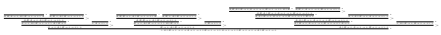
CALL-BY-VALUE encoding (375ms)



01-ENC encoding (421ms)

$(26)\ \cdot \vdash (\neg\neg A \wedge B) \leftrightarrow (\neg\neg A \wedge \neg\neg B)$

LJ (137ms)

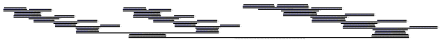


MULTIPLICATIVE encoding (66ms)

Not provable

CALL-BY-NAME encoding (601ms)

CALL-BY-VALUE encoding (584ms)



01-ENC encoding (646ms)

$(27)\ \cdot \vdash (\neg\neg(A) \leftrightarrow (B)) \leftrightarrow (\neg\neg A \to B \wedge \neg\neg B \to A)$

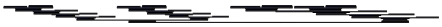
LJ (2703ms)

MULTIPLICATIVE encoding (84ms)

Not provable

CALL-BY-NAME encoding (661ms)

CALL-BY-VALUE encoding (841ms)

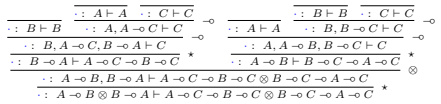


01-ENC encoding (1132ms)

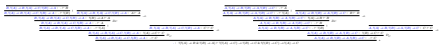
(28) $(A) \leftrightarrow (B) \vdash (A \to C) \leftrightarrow (B \to C)$

LJ (75ms)

MULTIPLICATIVE encoding (95ms)

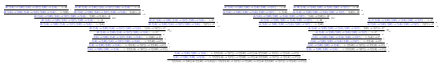


CALL-BY-NAME encoding (173ms)



CALL-BY-VALUE encoding (288ms)

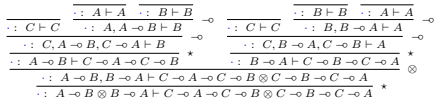
01-ENC encoding (268ms)



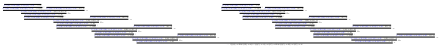
(29) $(A) \leftrightarrow (B) \vdash (C \to A) \leftrightarrow (C \to B)$

LJ (72ms)

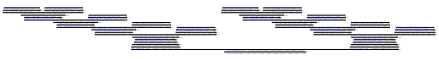
MULTIPLICATIVE encoding (96ms)



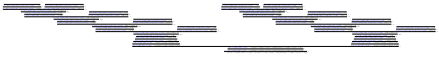
CALL-BY-NAME encoding (250ms)



CALL-BY-VALUE encoding (387ms)

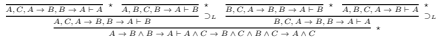


01-ENC encoding (356ms)

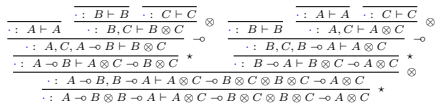


(30) $(A) \leftrightarrow (B) \vdash (A \wedge C) \leftrightarrow (B \wedge C)$

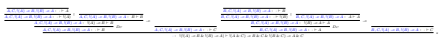
LJ (51ms)



MULTIPLICATIVE encoding (99ms)



CALL-BY-NAME encoding (138ms)

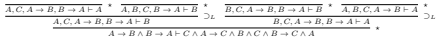


CALL-BY-VALUE encoding (238ms)

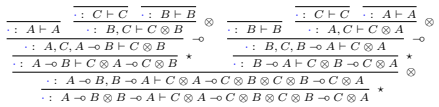
01-ENC encoding (234ms)

(31) $(A) \leftrightarrow (B) \vdash (C \wedge A) \leftrightarrow (C \wedge B)$

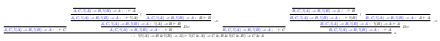
LJ (51ms)



MULTIPLICATIVE encoding (98ms)



CALL-BY-NAME encoding (142ms)

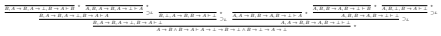


CALL-BY-VALUE encoding (227ms)

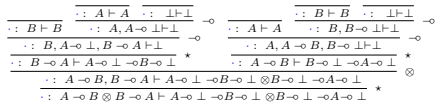
01-ENC encoding (229ms)

(32) $(A) \leftrightarrow (B) \vdash (\neg A) \leftrightarrow (\neg B)$

LJ (74ms)

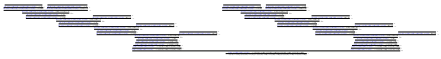


MULTIPLICATIVE encoding (97ms)

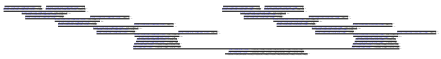


CALL-BY-NAME encoding (223ms)

CALL-BY-VALUE encoding (304ms)

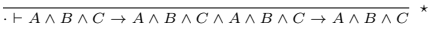


01-ENC encoding (320ms)



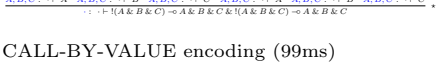
(33) $\cdot \vdash (A \wedge B \wedge C) \leftrightarrow (A \wedge B \wedge C)$

LJ (21ms)

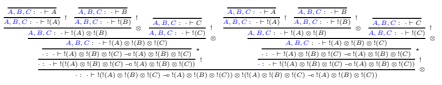


MULTIPLICATIVE encoding (66ms)

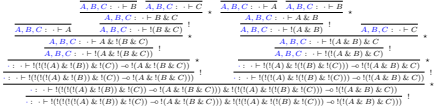
CALL-BY-NAME encoding (34ms)



CALL-BY-VALUE encoding (99ms)



01-ENC encoding (124ms)



$\boxed{(34)\ \cdot \vdash (A \wedge B) \leftrightarrow (B \wedge A)}$

LJ (21ms)

$$\frac{}{\cdot \vdash A \wedge B \to B \wedge A \wedge B \wedge A \to A \wedge B}\ \star$$

MULTIPLICATIVE encoding (73ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\cdot :\ B \vdash B}\quad\dfrac{}{\cdot :\ A \vdash A}}{\cdot :\ A, B \vdash B \otimes A}\otimes}{\cdot :\ \cdot \vdash A \otimes B \multimap B \otimes A}\star \qquad \dfrac{\dfrac{\dfrac{}{\cdot :\ A \vdash A}\quad\dfrac{}{\cdot :\ B \vdash B}}{\cdot :\ A, B \vdash A \otimes B}\otimes}{\cdot :\ \cdot \vdash B \otimes A \multimap A \otimes B}\star}{\cdot :\ \cdot \vdash A \otimes B \multimap B \otimes A \otimes B \otimes A \multimap A \otimes B}\otimes$$

CALL-BY-NAME encoding (35ms)

$$\frac{\dfrac{}{A, B :\ \cdot \vdash B}\quad\dfrac{}{A, B :\ \cdot \vdash A}\quad\dfrac{}{A, B :\ \cdot \vdash A}\quad\dfrac{}{A, B :\ \cdot \vdash B}}{\cdot :\ \cdot \vdash\ !(A \,\&\, B) \multimap B \,\&\, A \,\&\, !(B \,\&\, A) \multimap A \,\&\, B}\ \star$$

CALL-BY-VALUE encoding (107ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \vdash B}}{A, B :\ \cdot \vdash !(B)}!\quad\dfrac{\dfrac{}{A, B :\ \cdot \vdash A}}{A, B :\ \cdot \vdash !(A)}!}{A, B :\ \cdot \vdash !(B) \otimes !(A)}\otimes}{\cdot :\ \cdot \vdash !(A) \otimes !(B) \multimap !(B) \otimes !(A)}\star\quad\dfrac{\dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \vdash A}}{A, B :\ \cdot \vdash !(A)}!\quad\dfrac{\dfrac{}{A, B :\ \cdot \vdash B}}{A, B :\ \cdot \vdash !(B)}!}{A, B :\ \cdot \vdash !(A) \otimes !(B)}\otimes}{\cdot :\ \cdot \vdash !(B) \otimes !(A) \multimap !(A) \otimes !(B)}\star}{\cdot :\ \cdot \vdash !(!(A) \otimes !(B) \multimap !(B) \otimes !(A)) \otimes !(!(B) \otimes !(A) \multimap !(A) \otimes !(B))}\otimes$$

01-ENC encoding (102ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \vdash B}\quad\dfrac{}{A, B :\ \cdot \vdash A}}{A, B :\ \cdot \vdash B \,\&\, A}\star}{A, B :\ \cdot \vdash !(B \,\&\, A)}!}{\cdot :\ \cdot \vdash !(!(A) \,\&\, !(B)) \multimap !(B \,\&\, A)}\star}{\cdot :\ \cdot \vdash !(!(!(A) \,\&\, !(B)) \multimap !(B \,\&\, A))}!\quad\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \vdash A}\quad\dfrac{}{A, B :\ \cdot \vdash B}}{A, B :\ \cdot \vdash A \,\&\, B}\star}{A, B :\ \cdot \vdash !(A \,\&\, B)}!}{\cdot :\ \cdot \vdash !(!(B) \,\&\, !(A)) \multimap !(A \,\&\, B)}\star}{\cdot :\ \cdot \vdash !(!(!(B) \,\&\, !(A)) \multimap !(A \,\&\, B))}!}{\cdot :\ \cdot \vdash !(!(!(!(A) \,\&\, !(B)) \multimap !(B \,\&\, A)) \,\&\, !(!(!(B) \,\&\, !(A)) \multimap !(A \,\&\, B)))}\star}{\cdot :\ \cdot \vdash !(!(!(!(A) \,\&\, !(B)) \multimap !(B \,\&\, A)) \,\&\, !(!(!(B) \,\&\, !(A)) \multimap !(A \,\&\, B)))}!$$

$\boxed{(35)\ \cdot \vdash (A \wedge A) \leftrightarrow (A)}$

LJ (21ms)

$$\frac{}{\cdot \vdash A \wedge A \to A \wedge A \to A \wedge A}\ \star$$

MULTIPLICATIVE encoding (34ms)

Not provable

CALL-BY-NAME encoding (28ms)

$$\frac{\dfrac{}{A :\ \cdot \vdash A}\quad\dfrac{}{A :\ \cdot \vdash A}\quad\dfrac{}{A :\ \cdot \vdash A}}{\cdot :\ \cdot \vdash\ !(A \,\&\, A) \multimap A \,\&\, !(A) \multimap A \,\&\, A}\ \star$$

CALL-BY-VALUE encoding (76ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A :\ \cdot \vdash A}}{A :\ \cdot \vdash !(A)}!}{\cdot :\ \cdot \vdash !(A) \otimes !(A) \multimap !(A)}\star\quad\dfrac{\dfrac{\dfrac{}{A :\ \cdot \vdash A}}{A :\ \cdot \vdash !(A)}!\quad\dfrac{\dfrac{}{A :\ \cdot \vdash A}}{A :\ \cdot \vdash !(A)}!}{A :\ \cdot \vdash !(A) \otimes !(A)}\otimes}{\cdot :\ \cdot \vdash !(A) \otimes !(A) \multimap !(A) \otimes !(A)}\star}{\cdot :\ \cdot \vdash !(!(A) \otimes !(A) \multimap !(A)) \otimes !(!(A) \multimap !(A) \otimes !(A))}\otimes$$

01-ENC encoding (81ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A :\ \cdot \vdash A}}{\cdot :\ \cdot \vdash !(!(A) \,\&\, !(A)) \multimap A}\star}{\cdot :\ \cdot \vdash !(!(!(A) \,\&\, !(A)) \multimap A)}!\quad\dfrac{\dfrac{\dfrac{\dfrac{}{A :\ \cdot \vdash A}\quad\dfrac{}{A :\ \cdot \vdash A}}{A :\ \cdot \vdash A \,\&\, A}\star}{A :\ \cdot \vdash !(A \,\&\, A)}!}{\cdot :\ \cdot \vdash !(A) \multimap !(A \,\&\, A)}\star}{\cdot :\ \cdot \vdash !(!(!(A) \,\&\, !(A)) \multimap A) \,\&\, !(!(A) \multimap !(A \,\&\, A))}!}{\cdot :\ \cdot \vdash !(!(!(!(A) \,\&\, !(A)) \multimap A) \,\&\, !(!(A) \multimap !(A \,\&\, A)))}\star$$

$\boxed{(36)\ A \vdash (A \to B) \leftrightarrow (B)}$

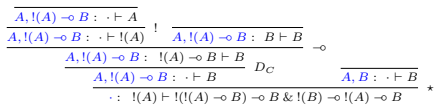
LJ (37ms)

$$\dfrac{\dfrac{\dfrac{}{A, A \to B \vdash A}\ \star \quad \dfrac{}{A, B \vdash B}\ \star}{A, A \to B \vdash B}\ \supset_L}{A \vdash A \to B \to B \wedge B \to A \to B}\ \star$$

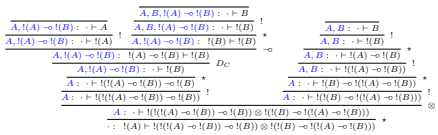
MULTIPLICATIVE encoding (42ms)

Not provable

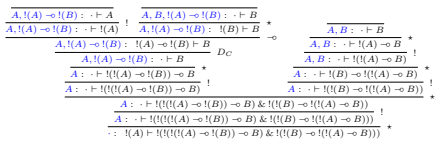
CALL-BY-NAME encoding (73ms)



CALL-BY-VALUE encoding (129ms)



01-ENC encoding (123ms)



$\boxed{(37)\ B \vdash (A \to B) \leftrightarrow (B)}$

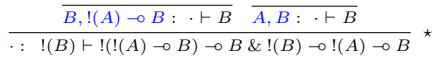
LJ (21ms)

$$\dfrac{}{B \vdash A \to B \to B \wedge B \to A \to B}\ \star$$

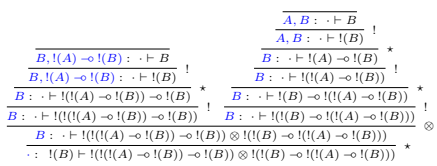
MULTIPLICATIVE encoding (40ms)

Not provable

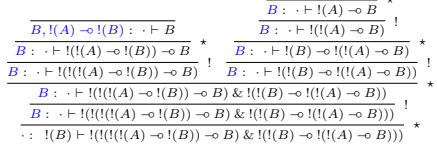
CALL-BY-NAME encoding (39ms)



CALL-BY-VALUE encoding (152ms)

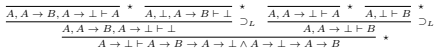


01-ENC encoding (88ms)



$\boxed{(38)\ \neg A \vdash (A \to B) \leftrightarrow (\neg A)}$

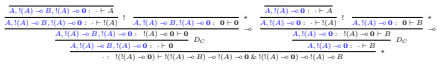
LJ (52ms)



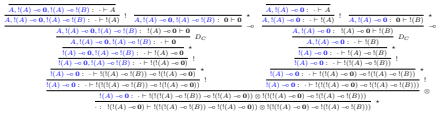
MULTIPLICATIVE encoding (42ms)

Not provable

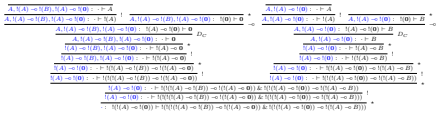
CALL-BY-NAME encoding (101ms)



CALL-BY-VALUE encoding (201ms)

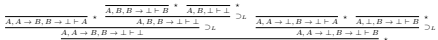


01-ENC encoding (203ms)



$\boxed{(39)\ \neg B \vdash (A \to B) \leftrightarrow (\neg A)}$

LJ (61ms)



MULTIPLICATIVE encoding (43ms)

Not provable

CALL-BY-NAME encoding (185ms)

CALL-BY-VALUE encoding (256ms)

01-ENC encoding (259ms)

$\boxed{(40)\ B \vdash (A \wedge B) \leftrightarrow (A)}$

LJ (21ms)

$$\dfrac{}{B \vdash A \wedge B \to A \wedge A \to A \wedge B}\ \star$$

MULTIPLICATIVE encoding (34ms)

Not provable

CALL-BY-NAME encoding (34ms)

$$\dfrac{\dfrac{}{A, B :\ \cdot \vdash A} \quad \dfrac{}{A, B :\ \cdot \vdash A} \quad \dfrac{}{A, B :\ \cdot \vdash B}\ \star}{\cdot :\ \ !(B) \vdash\ !(A\ \&\ B) \multimap A\ \&\ !(A) \multimap A\ \&\ B}$$

CALL-BY-VALUE encoding (88ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \vdash A}\ !}{A, B :\ \cdot \vdash\ !(A)}}{B :\ \cdot \vdash\ !(A) \otimes\ !(B) \multimap\ !(A)}\ \star \quad \dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \vdash A}\ ! \quad \dfrac{}{A, B :\ \cdot \vdash B}\ !}{A, B :\ \cdot \vdash\ !(A) \otimes\ !(B)}}{B :\ \cdot \vdash\ !(A) \multimap\ !(A) \otimes\ !(B)}\ \star}{B :\ \cdot \vdash\ !(\ !(A) \otimes\ !(B) \multimap\ !(A)) \otimes\ !(\ !(A) \multimap\ !(A) \otimes\ !(B))}\ \otimes}{\cdot :\ \ !(B) \vdash\ !(\ !(A) \otimes\ !(B) \multimap\ !(A)) \otimes\ !(\ !(A) \multimap\ !(A) \otimes\ !(B))}\ \star$$

01-ENC encoding (88ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \vdash A}}{B :\ \cdot \vdash\ !(\ !(A)\ \&\ !(B)) \multimap A}\ \star}{B :\ \cdot \vdash\ !(\ !(\ !(A)\ \&\ !(B)) \multimap A)}\ ! \quad \dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \vdash A} \quad \dfrac{}{A, B :\ \cdot \vdash B}\ \star}{A, B :\ \cdot \vdash A\ \&\ B}}{A, B :\ \cdot \vdash\ !(A\ \&\ B)}\ !}{B :\ \cdot \vdash\ !(\ !(\ !(A)\ \&\ !(B)) \multimap A) \multimap\ !(A) \multimap\ !(A\ \&\ B))}\ \star}{B :\ \cdot \vdash\ !(\ !(\ !(\ !(A)\ \&\ !(B)) \multimap A) \multimap\ !(A) \multimap\ !(A\ \&\ B)))}\ !}{\cdot :\ \ !(B) \vdash\ !(\ !(\ !(\ !(A)\ \&\ !(B)) \multimap A) \multimap\ !(A) \multimap\ !(A\ \&\ B)))}\ \star$$

$\boxed{(41)\ \neg B \vdash (A \wedge B) \leftrightarrow (B)}$

LJ (37ms)

$$\dfrac{\dfrac{\dfrac{}{B, B \to \bot \vdash B}\ \star \quad \dfrac{}{B, \bot \vdash A}\ \star}{B, B \to \bot \vdash A}\ \supset_L}{B \to \bot \vdash A \wedge B \to B \wedge B \to A \wedge B}\ \star$$

MULTIPLICATIVE encoding (41ms)

Not provable

CALL-BY-NAME encoding (78ms)

$$\dfrac{\dfrac{}{A, B, !(B) \multimap \mathbf{0} :\ \cdot \vdash B} \quad \dfrac{\dfrac{\dfrac{}{B, !(B) \multimap \mathbf{0} :\ \cdot \vdash\ !(B)}\ ! \quad \dfrac{}{B, !(B) \multimap \mathbf{0} :\ \mathbf{0} \vdash A}\ \star}{B, !(B) \multimap \mathbf{0} :\ !(B) \multimap \mathbf{0} \vdash A}\ \multimap}{B, !(B) \multimap \mathbf{0} :\ \cdot \vdash A}\ D_C \quad \dfrac{}{B, !(B) \multimap \mathbf{0} :\ \cdot \vdash B}}{\cdot :\ \ !(!(B) \multimap \mathbf{0}) \vdash\ !(A\ \&\ B) \multimap B\ \&\ !(B) \multimap A\ \&\ B}\ \star$$

CALL-BY-VALUE encoding (172ms)

$$\dfrac{\dfrac{\dfrac{}{A, B, !(B) \multimap \mathbf{0} :\ \cdot \vdash B}}{A, B, !(B) \multimap \mathbf{0} :\ \cdot \vdash\ !(B)}\ ! \quad \dfrac{\dfrac{\dfrac{}{B, !(B) \multimap \mathbf{0} :\ \cdot \vdash B}}{B, !(B) \multimap \mathbf{0} :\ \cdot \vdash\ !(B)}\ ! \quad \dfrac{}{B, !(B) \multimap \mathbf{0} :\ \mathbf{0} \vdash\ !(A) \otimes\ !(B)}\ \star}{B, !(B) \multimap \mathbf{0} :\ !(B) \multimap \mathbf{0} \vdash\ !(A) \otimes\ !(B)}\ \multimap}{!(B) \multimap \mathbf{0} :\ \cdot \vdash\ !(A) \otimes\ !(B) \multimap\ !(B)}\ \star \quad \dfrac{\dfrac{\dfrac{}{B, !(B) \multimap \mathbf{0} :\ \cdot \vdash B}}{B, !(B) \multimap \mathbf{0} :\ \cdot \vdash\ !(B)}\ ! \quad \dfrac{}{B, !(B) \multimap \mathbf{0} :\ \mathbf{0} \vdash\ !(A) \otimes\ !(B)}\ \star}{B, !(B) \multimap \mathbf{0} :\ !(B) \multimap \mathbf{0} \vdash\ !(A) \otimes\ !(B)}\ \multimap}{!(B) \multimap \mathbf{0} :\ \cdot \vdash\ !(!(B) \multimap\ !(A) \otimes\ !(B))}\ !}{!(B) \multimap \mathbf{0} :\ \cdot \vdash\ !(A) \otimes\ !(B) \multimap\ !(B) \otimes\ !(!(B) \multimap\ !(A) \otimes\ !(B))}\ \star$$

01-ENC encoding (194ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, B, !(B) \multimap !(\mathbf{0}) :\ \cdot \vdash B}}{!(B) \multimap !(\mathbf{0}) :\ \cdot \vdash\ !(A) \otimes\ !(B) \multimap B}\ \star}{!(B) \multimap !(\mathbf{0}) :\ \cdot \vdash\ !(!(A) \otimes\ !(B)) \multimap B}\ ! \quad \dfrac{\dfrac{\dfrac{}{B, !(B) \multimap !(\mathbf{0}) :\ \cdot \vdash B}}{B, !(B) \multimap !(\mathbf{0}) :\ \cdot \vdash\ !(B)}\ ! \quad \dfrac{}{B, !(B) \multimap !(\mathbf{0}) :\ !(B) \multimap !(\mathbf{0}) \vdash\ !(A\ \&\ B)}\ \star}{B, !(B) \multimap !(\mathbf{0}) :\ \cdot \vdash\ !(A\ \&\ B)}\ D_C}{!(B) \multimap !(\mathbf{0}) :\ \cdot \vdash\ !(A\ \&\ B) \multimap\ !(B)}\ \star \quad \dfrac{\dfrac{\dfrac{}{B, !(B) \multimap !(\mathbf{0}) :\ \cdot \vdash B}}{B, !(B) \multimap !(\mathbf{0}) :\ \cdot \vdash\ !(B)}\ ! \quad \dfrac{}{B, !(B) \multimap !(\mathbf{0}) :\ !(B) \multimap !(\mathbf{0}) \vdash\ !(A\ \&\ B)}\ \star}{B, !(B) \multimap !(\mathbf{0}) :\ !(B) \multimap !(\mathbf{0}) \vdash\ !(A\ \&\ B)}\ \multimap}{!(B) \multimap !(\mathbf{0}) :\ \cdot \vdash\ !(!(B) \multimap\ !(A\ \&\ B))}\ !}{!(B) \multimap !(\mathbf{0}) :\ \cdot \vdash\ !(!(!(A\ \&\ !(B)) \multimap B)\ \&\ !(!(B) \multimap\ !(A\ \&\ B)))}\ !}{\cdot :\ \ !(B) \multimap !(\mathbf{0}) \vdash\ !(!(!(A\ \&\ !(B)) \multimap B)\ \&\ !(!(B) \multimap\ !(A\ \&\ B)))}\ \star$$

(42) $\cdot \vdash A \to \neg\neg A$

LJ (38ms)

$$\dfrac{\dfrac{\dfrac{}{A, A \to \bot \vdash A}\,\star \quad \dfrac{}{A, \bot \vdash \bot}\,\star}{A, A \to \bot \vdash \bot}\,\supset_L}{\cdot \vdash A \to A \to \bot \to \bot}\,\star$$

MULTIPLICATIVE encoding (41ms)

$$\dfrac{\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{}{\cdot :\ \ \bot \vdash \bot}}{\cdot :\ A, A \multimap \bot \vdash \bot}\,\multimap}{\cdot :\ \cdot \vdash A \multimap A \multimap \bot \multimap \bot}\,\star$$

CALL-BY-NAME encoding (66ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(A)}\,! \quad \dfrac{}{A, !(A) \multimap \mathbf{0} :\ \ \mathbf{0} \vdash \mathbf{0}}\,\star}{A, !(A) \multimap \mathbf{0} :\ \ !(A) \multimap \mathbf{0} \vdash \mathbf{0}}\,\multimap}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash \mathbf{0}}\,D_C}{\cdot :\ \cdot \vdash !(A) \multimap !(!(A) \multimap \mathbf{0}) \multimap \mathbf{0}}\,\star$$

CALL-BY-VALUE encoding (84ms)

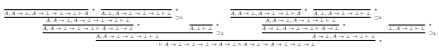
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(A)}\,! \quad \dfrac{}{A, !(A) \multimap \mathbf{0} :\ \ \mathbf{0} \vdash \mathbf{0}}\,\star}{A, !(A) \multimap \mathbf{0} :\ \ !(A) \multimap \mathbf{0} \vdash \mathbf{0}}\,\multimap}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash \mathbf{0}}\,D_C}{A :\ \cdot \vdash !(!(A) \multimap \mathbf{0}) \multimap \mathbf{0}}\,\star}{A :\ \cdot \vdash !(!(!(A) \multimap \mathbf{0}) \multimap \mathbf{0})}\,!}{\cdot :\ \cdot \vdash !(A) \multimap !(!(!(A) \multimap \mathbf{0}) \multimap \mathbf{0})}\,\star}{\cdot :\ \cdot \vdash !(!(A) \multimap !(!(!(A) \multimap \mathbf{0}) \multimap \mathbf{0}))}\,!$$

01-ENC encoding (81ms)

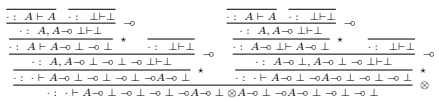
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash A}}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(A)}\,! \quad \dfrac{}{A, !(A) \multimap !(\mathbf{0}) :\ \ !(\mathbf{0}) \vdash \mathbf{0}}\,\star}{A, !(A) \multimap !(\mathbf{0}) :\ \ !(A) \multimap !(\mathbf{0}) \vdash \mathbf{0}}\,\multimap}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash \mathbf{0}}\,D_C}{A :\ \cdot \vdash !(!(A) \multimap !(\mathbf{0})) \multimap \mathbf{0}}\,\star}{A :\ \cdot \vdash !(!(!(A) \multimap !(\mathbf{0})) \multimap \mathbf{0})}\,!}{\cdot :\ \cdot \vdash !(A) \multimap !(!(!(A) \multimap !(\mathbf{0})) \multimap \mathbf{0})}\,\star}{\cdot :\ \cdot \vdash !(!(A) \multimap !(!(!(A) \multimap !(\mathbf{0})) \multimap \mathbf{0}))}\,!$$

(43) $\cdot \vdash (\neg\neg\neg A) \leftrightarrow (\neg A)$

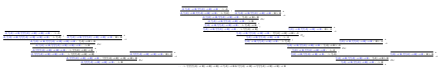
LJ (81ms)



MULTIPLICATIVE encoding (105ms)

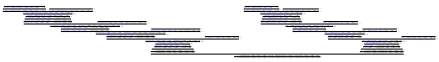


CALL-BY-NAME encoding (244ms)



CALL-BY-VALUE encoding (295ms)

01-ENC encoding (343ms)



(44) $\cdot \vdash \neg A \wedge \neg A$

LJ (38ms)

$$\dfrac{\dfrac{}{A, A \to \bot \vdash A}\ \star \quad \dfrac{}{A, \bot \vdash \bot}\ \star}{\dfrac{A, A \to \bot \vdash \bot}{\cdot \vdash A \wedge A \to \bot \to \bot}\ \star}\ \supset_L$$

MULTIPLICATIVE encoding (41ms)

$$\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{}{\cdot :\ \ \bot \vdash \bot}}{\dfrac{\cdot :\ A, A \multimap \bot \vdash \bot}{\cdot :\ \cdot \vdash A \otimes A \multimap \bot \multimap \bot}\ \star}\ \multimap$$

CALL-BY-NAME encoding (65ms)

$$\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap \mathbf{0} :\ \mathbf{0} \vdash \mathbf{0}}\ \star}{\dfrac{\dfrac{A, !(A) \multimap \mathbf{0} :\ \ !(A) \multimap \mathbf{0} \vdash \mathbf{0}}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash \mathbf{0}}\ D_C}{\cdot :\ \cdot \vdash !(A \,\&\, !(A) \multimap \mathbf{0}) \multimap \mathbf{0}}\ \star}\ \multimap$$

CALL-BY-VALUE encoding (70ms)

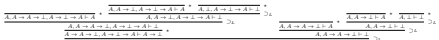
$$\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap \mathbf{0} :\ \mathbf{0} \vdash \mathbf{0}}\ \star}{\dfrac{\dfrac{A, !(A) \multimap \mathbf{0} :\ \ !(A) \multimap \mathbf{0} \vdash \mathbf{0}}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash \mathbf{0}}\ D_C}{\cdot :\ \cdot \vdash !(A) \otimes !(!(A) \multimap \mathbf{0}) \multimap \mathbf{0}}\ \star}\ !}{\cdot :\ \cdot \vdash !(!(A) \otimes !(!(A) \multimap \mathbf{0}) \multimap \mathbf{0})}\ !$$

01-ENC encoding (70ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash A}}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap !(\mathbf{0}) :\ !(\mathbf{0}) \vdash \mathbf{0}}\ \star}{\dfrac{\dfrac{A, !(A) \multimap !(\mathbf{0}) :\ \ !(A) \multimap !(\mathbf{0}) \vdash \mathbf{0}}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash \mathbf{0}}\ D_C}{\cdot :\ \cdot \vdash !(!(A) \,\&\, !(!(A) \multimap !(\mathbf{0}))) \multimap \mathbf{0}}\ \star}\ !}{\cdot :\ \cdot \vdash !(!(!(A) \,\&\, !(!(A) \multimap !(\mathbf{0}))) \multimap \mathbf{0})}\ !$$

(45) $\cdot \vdash \neg(A) \leftrightarrow (\neg A)$

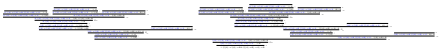
LJ (80ms)



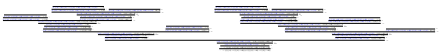
MULTIPLICATIVE encoding (41ms)

Not provable

CALL-BY-NAME encoding (231ms)



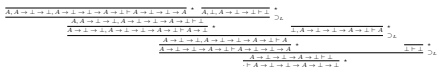
CALL-BY-VALUE encoding (282ms)



01-ENC encoding (300ms)

$\boxed{(46)\ \cdot \vdash \neg\neg\neg\neg A \to A}$

LJ (66ms)



MULTIPLICATIVE encoding (41ms)

Not provable

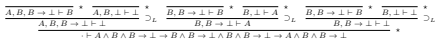
CALL-BY-NAME encoding (198ms)

CALL-BY-VALUE encoding (210ms)

01-ENC encoding (217ms)

$\boxed{(47)\ \cdot \vdash (A \wedge B \wedge \neg B) \leftrightarrow (B \wedge \neg B)}$

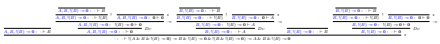
LJ (61ms)



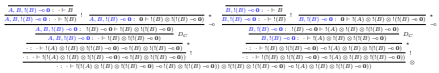
MULTIPLICATIVE encoding (42ms)

Not provable

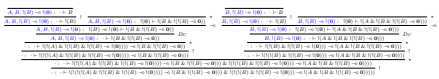
CALL-BY-NAME encoding (121ms)



CALL-BY-VALUE encoding (140ms)

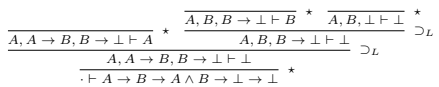


01-ENC encoding (132ms)

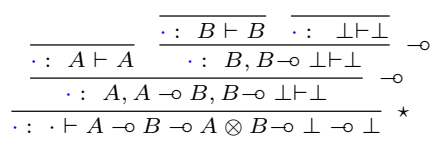


(48) $\cdot \vdash A \to B \to \neg A \wedge \neg B$

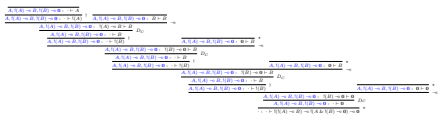
LJ (49ms)



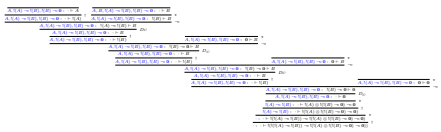
MULTIPLICATIVE encoding (48ms)



CALL-BY-NAME encoding (143ms)



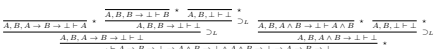
CALL-BY-VALUE encoding (167ms)



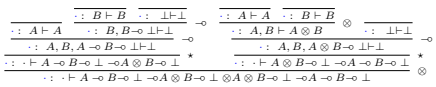
01-ENC encoding (169ms)

(49) $\cdot \vdash (A \to \neg B) \leftrightarrow (\neg A \wedge B)$

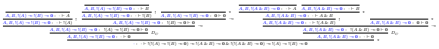
LJ (58ms)



MULTIPLICATIVE encoding (88ms)

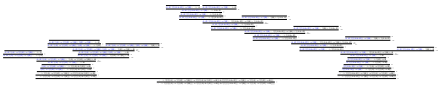


CALL-BY-NAME encoding (118ms)



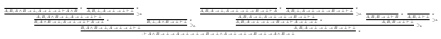
CALL-BY-VALUE encoding (281ms)

01-ENC encoding (318ms)



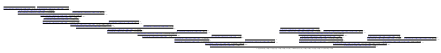
(50) $\cdot \vdash (\neg A \wedge B) \leftrightarrow (\neg\neg A \to \neg B)$

LJ (93ms)

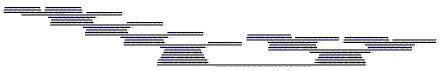


MULTIPLICATIVE encoding (134ms)

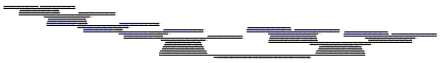
CALL-BY-NAME encoding (333ms)



CALL-BY-VALUE encoding (421ms)

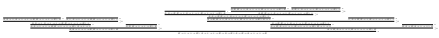


01-ENC encoding (470ms)

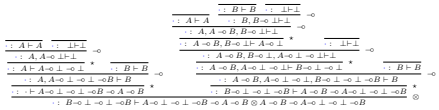


(51) $\neg\neg B \to B \vdash (\neg\neg A \to B) \leftrightarrow (A \to B)$

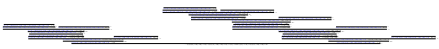
LJ (135ms)



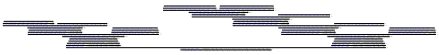
MULTIPLICATIVE encoding (156ms)



CALL-BY-NAME encoding (278ms)



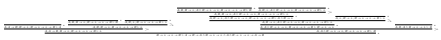
CALL-BY-VALUE encoding (414ms)



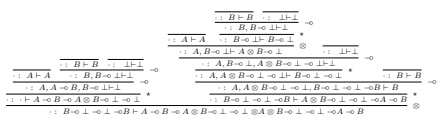
01-ENC encoding (449ms)

(52) $\neg\neg B \to B \vdash (A \to B) \leftrightarrow (\neg A \wedge \neg B)$

LJ (94ms)



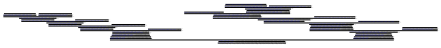
MULTIPLICATIVE encoding (134ms)



CALL-BY-NAME encoding (296ms)

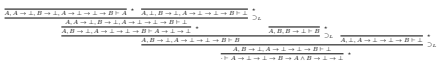
CALL-BY-VALUE encoding (457ms)

01-ENC encoding (489ms)

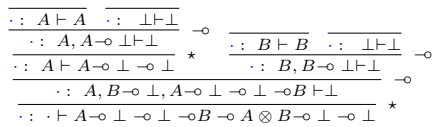


(53) $\cdot \vdash \neg\neg A \to B \to \neg A \wedge \neg B$

LJ (65ms)



MULTIPLICATIVE encoding (75ms)



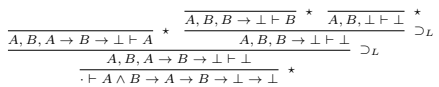
CALL-BY-NAME encoding (178ms)

CALL-BY-VALUE encoding (204ms)

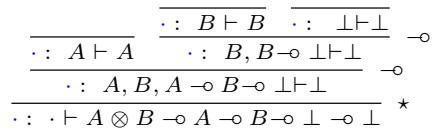
01-ENC encoding (215ms)

$\boxed{(54)\ \cdot \vdash A \wedge B \to \neg A \to \neg B}$

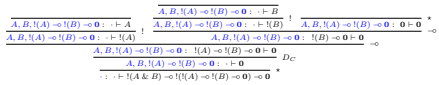
LJ (47ms)



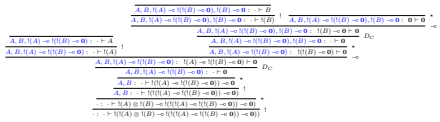
MULTIPLICATIVE encoding (48ms)



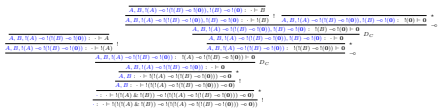
CALL-BY-NAME encoding (77ms)



CALL-BY-VALUE encoding (105ms)

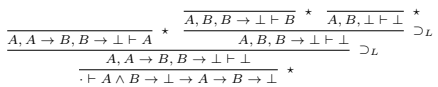


01-ENC encoding (106ms)

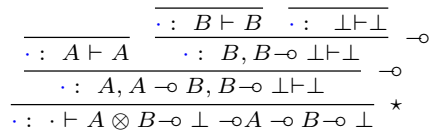


$\boxed{(55)\ \cdot \vdash A \wedge \neg B \to \neg A \to B}$

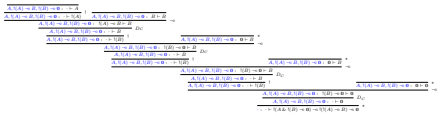
LJ (48ms)



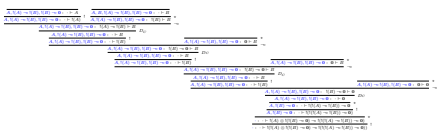
MULTIPLICATIVE encoding (47ms)



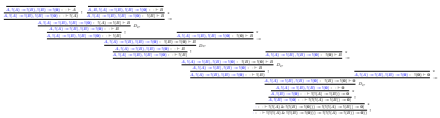
CALL-BY-NAME encoding (144ms)



CALL-BY-VALUE encoding (168ms)

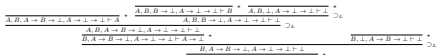


01-ENC encoding (168ms)

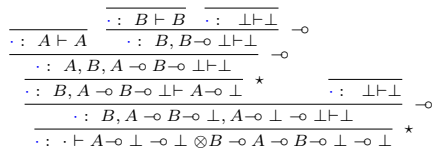


(56) $\cdot \vdash \neg\neg A \wedge B \to \neg A \to \neg B$

LJ (62ms)



MULTIPLICATIVE encoding (77ms)



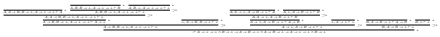
CALL-BY-NAME encoding (182ms)

CALL-BY-VALUE encoding (212ms)

01-ENC encoding (210ms)

(57) $\cdot \vdash (\neg\neg A \wedge \neg B) \leftrightarrow (\neg A \to B)$

LJ (114ms)

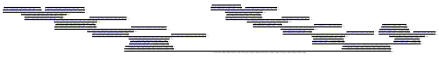


MULTIPLICATIVE encoding (58ms)

Not provable

CALL-BY-NAME encoding (326ms)

CALL-BY-VALUE encoding (421ms)



01-ENC encoding (454ms)

(58) $\cdot \vdash (\neg A \to B) \leftrightarrow (\neg\neg A \wedge \neg B)$

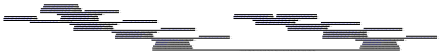
LJ (109ms)

MULTIPLICATIVE encoding (41ms)

Not provable

CALL-BY-NAME encoding (597ms)

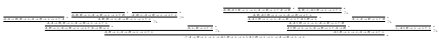
CALL-BY-VALUE encoding (481ms)



01-ENC encoding (501ms)

(59) $\cdot \vdash (\neg\neg A \to B) \leftrightarrow (\neg A \wedge \neg B)$

LJ (112ms)

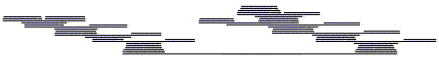


MULTIPLICATIVE encoding (57ms)

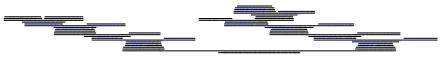
Not provable

CALL-BY-NAME encoding (594ms)

CALL-BY-VALUE encoding (468ms)



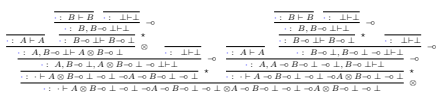
01-ENC encoding (498ms)



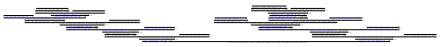
$(60)\ \cdot \vdash (\neg A \wedge \neg B) \leftrightarrow (A \to \neg\neg B)$

LJ (92ms)

MULTIPLICATIVE encoding (124ms)



CALL-BY-NAME encoding (317ms)

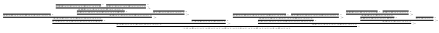


CALL-BY-VALUE encoding (436ms)

01-ENC encoding (468ms)

$(61)\ \cdot \vdash (A \to \neg\neg B) \leftrightarrow (\neg\neg A \to \neg\neg B)$

LJ (169ms)



MULTIPLICATIVE encoding (163ms)

CALL-BY-NAME encoding (412ms)

CALL-BY-VALUE encoding (479ms)

01-ENC encoding (552ms)

3 Alternative Translations

$\boxed{(10)\ \cdot :\ A \multimap \mathbf{0} \vdash A \multimap B}$

encoding (55ms)

$$\dfrac{\dfrac{\overline{\cdot :\ A \vdash A} \quad \overline{\cdot :\ \mathbf{0} \vdash B}\ \star}{\cdot :\ A, A \multimap \mathbf{0} \vdash B}\ \multimap}{\cdot :\ A \multimap \mathbf{0} \vdash A \multimap B}\ \star$$

$\boxed{(11)\ \cdot :\ A \vdash A \multimap \mathbf{0} \multimap B}$

encoding (41ms)

$$\dfrac{\dfrac{\overline{\cdot :\ A \vdash A} \quad \overline{\cdot :\ \mathbf{0} \vdash B}\ \star}{\cdot :\ A, A \multimap \mathbf{0} \vdash B}\ \multimap}{\cdot :\ A \vdash A \multimap \mathbf{0} \multimap B}\ \star$$

$\boxed{(12)\ \cdot :\ B \vdash\ !(A) \multimap B}$

encoding (22ms)

$$\dfrac{\overline{A :\ B \vdash B}}{\cdot :\ B \vdash\ !(A) \multimap B}\ \star$$

$\boxed{(16)\ \cdot :\ A \multimap B \otimes\ !(B \multimap A) \vdash A \multimap B}$

encoding (35ms)

$$\dfrac{\dfrac{\overline{B \multimap A :\ A \vdash A} \quad \overline{B \multimap A :\ B \vdash B}}{B \multimap A :\ A, A \multimap B \vdash B}\ \multimap}{\cdot :\ A \multimap B \otimes\ !(B \multimap A) \vdash A \multimap B}\ \star$$

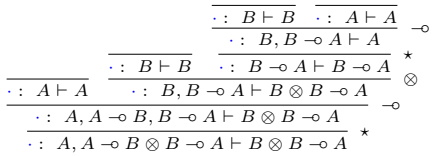
$\boxed{(17)\ \cdot :\ \ !(A \multimap B) \otimes B \multimap A \vdash B \multimap A}$

encoding (38ms)

$$\dfrac{\dfrac{\overline{A \multimap B :\ B \vdash B} \quad \overline{A \multimap B :\ A \vdash A}}{A \multimap B :\ B, B \multimap A \vdash A}\ \multimap}{\cdot :\ \ !(A \multimap B) \otimes B \multimap A \vdash B \multimap A}\ \star$$

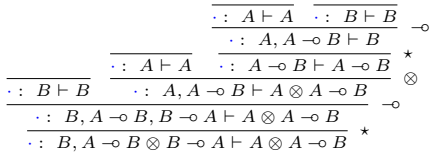
$\boxed{(18)\ \cdot :\ A, A \multimap B \otimes B \multimap A \vdash B \otimes B \multimap A}$

encoding (56ms)



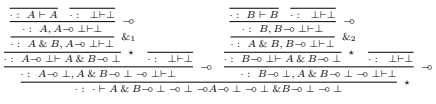
(19) $\cdot : \ B, A \multimap B \otimes B \multimap A \vdash A \otimes A \multimap B$

encoding (55ms)



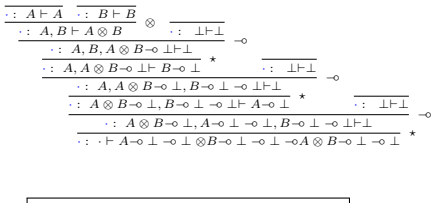
(26a) $\cdot : \ \cdot \vdash A \,\&\, B \multimap \bot \multimap \bot \multimap \bot \multimap A \multimap \bot \multimap \bot \,\&\, B \multimap \bot \multimap \bot$

encoding (107ms)



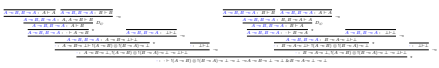
(26b) $\cdot : \ \cdot \vdash A \multimap \bot \multimap \bot \otimes B \multimap \bot \multimap \bot \multimap A \otimes B \multimap \bot \multimap \bot$

encoding (109ms)



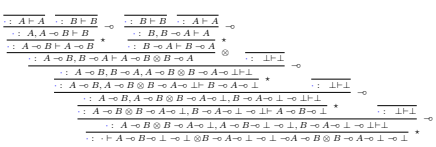
(27a) $\cdot : \ \cdot \vdash !(A \multimap B) \otimes !(B \multimap A) \multimap \bot \multimap \bot \multimap A \multimap B \multimap \bot \multimap \bot \,\&\, B \multimap A \multimap \bot \multimap \bot$

encoding (155ms)



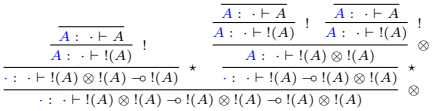
(27b) $\cdot : \ \cdot \vdash A \multimap B \multimap \bot \multimap \bot \otimes B \multimap A \multimap \bot \multimap \bot \multimap A \multimap B \otimes B \multimap A \multimap \bot \multimap \bot$

encoding (168ms)



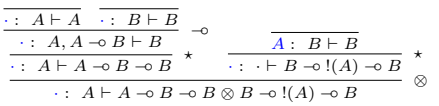
(35) $\cdot : \ \cdot \vdash !(A) \otimes !(A) \multimap !(A) \otimes !(A) \multimap !(A) \otimes !(A)$

encoding (59ms)



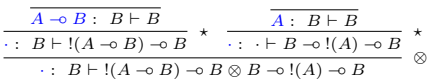
(36) $\cdot : \ A \vdash A \multimap B \multimap B \otimes B \multimap !(A) \multimap B$

encoding (62ms)



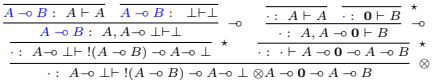
(37) $\cdot : \ B \vdash !(A \multimap B) \multimap B \otimes B \multimap !(A) \multimap B$

encoding (54ms)



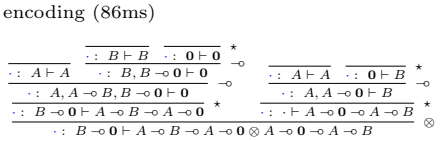
(38) $\cdot : \ A \multimap \bot \vdash !(A \multimap B) \multimap A \multimap \bot \otimes A \multimap \mathbf{0} \multimap A \multimap B$

encoding (74ms)



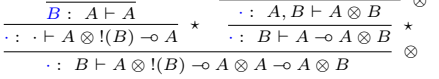
(39) $\cdot : \ B \multimap \mathbf{0} \vdash A \multimap B \multimap A \multimap \mathbf{0} \otimes A \multimap \mathbf{0} \multimap A \multimap B$

encoding (86ms)



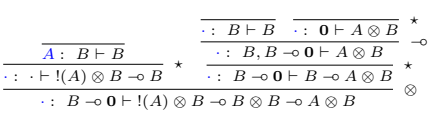
(40) $\cdot : \ B \vdash A \otimes !(B) \multimap A \otimes A \multimap A \otimes B$

encoding (55ms)



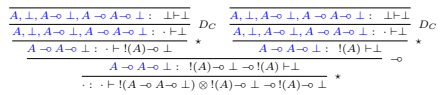
(41) $\cdot : \ B \multimap \mathbf{0} \vdash !(A) \otimes B \multimap B \otimes B \multimap A \otimes B$

encoding (69ms)



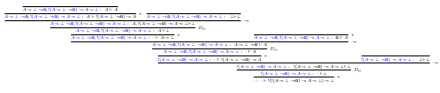
(45) $\cdot : \ \cdot \vdash !(A \multimap A \multimap \bot) \otimes !(A) \multimap \bot \multimap !(A) \multimap \bot$

encoding (65ms)



(46) $\cdot : \cdot \vdash !(!(A \multimap \bot \multimap \mathbf{0}) \multimap A \multimap \bot) \multimap \bot$

encoding (138ms)

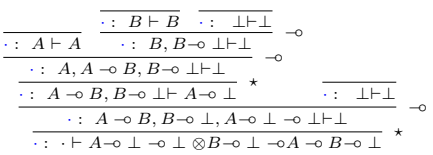


(47) $\cdot : \cdot \vdash A \otimes B \otimes B \multimap \mathbf{0} \multimap B \otimes B \multimap \mathbf{0} \otimes B \otimes B \multimap \mathbf{0} \multimap A \otimes B \otimes B \multimap \mathbf{0}$

encoding (83ms)

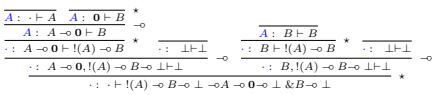
(57a) $\cdot : \cdot \vdash A \multimap \bot \multimap \bot \otimes B \multimap \bot \multimap A \multimap B \multimap \bot$

encoding (71ms)



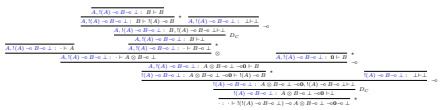
(57b) $\cdot : \cdot \vdash !(A) \multimap B \multimap \bot \multimap A \multimap \mathbf{0} \multimap \bot \,\&\, B \multimap \bot$

encoding (92ms)



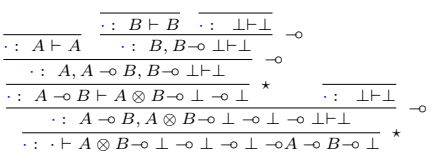
(58a) $\cdot : \cdot \vdash !(!(A) \multimap B \multimap \bot) \multimap A \otimes B \multimap \bot \multimap \mathbf{0} \multimap \bot$

encoding (162ms)



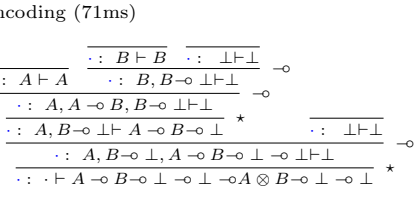
(58b) $\cdot : \cdot \vdash A \otimes B \multimap \bot \multimap \bot \multimap \bot \multimap A \multimap B \multimap \bot$

encoding (69ms)



(59a) $\cdot : \cdot \vdash A \multimap B \multimap \bot \multimap \bot \multimap A \otimes B \multimap \bot \multimap \bot$

encoding (71ms)



(59b) $\cdot : \cdot \vdash A \otimes B \multimap \bot \multimap \mathbf{0} \multimap !(!(A) \multimap B \multimap \bot) \multimap \bot$

encoding (163ms)