

LJ (49ms)

$$\frac{\frac{A, B, A \rightarrow B \rightarrow C \vdash A}{A, B, A \rightarrow B \rightarrow C \vdash C}^* \quad \frac{\frac{A, B, B \rightarrow C \vdash B}{A, B, B \rightarrow C \vdash C}^* \quad \frac{A, B, C \vdash C}{A, B, B \rightarrow C \vdash C}^*}{A, B, A \rightarrow B \rightarrow C \vdash C} \supset_L \quad \frac{A, B, A \rightarrow B \rightarrow C \vdash C}{A \rightarrow B \rightarrow C \vdash B \rightarrow A \rightarrow C}^* \supset_L$$

Multiplicative translation (94ms)

[illegible]

Girard's Translation (128ms)

[illegible]

Positive decoration (208ms)

[illegible]

0/1 focused decoration (189ms)

$$(4) \ A \rightarrow (B \rightarrow C) \vdash (A \wedge B) \rightarrow C$$

LJ (48ms)

$$\frac{\frac{\frac{A, B, A \rightarrow B \rightarrow C \vdash A}{A, B, A \rightarrow B \rightarrow C \vdash B} \star \quad \frac{\frac{A, B, B \rightarrow C \vdash B}{A, B, B \rightarrow C \vdash C} \star \quad \frac{A, B, C \vdash C}{A, B, B \rightarrow C \vdash C} \star}{A, B, A \rightarrow B \rightarrow C \vdash C} \supset_L}{A \rightarrow B \rightarrow C \vdash A \wedge B \rightarrow C} \star$$

Multiplicative translation (97ms)

$$\begin{array}{c}
\frac{\cdot : A \Rightarrow A}{\cdot : B [B \multimap C] \Leftarrow C} I \\
\frac{\cdot : B [B \multimap C] \Leftarrow C}{\cdot : A, B [A \multimap B \multimap C] \Leftarrow C} I \\
\frac{\cdot : A, B [A \multimap B \multimap C] \Leftarrow C}{\cdot : A, B, A \multimap B \multimap C \vdash C} D_L \\
\frac{\cdot : A, B, A \multimap B \multimap C \vdash C}{\cdot : A \multimap B \multimap C \vdash A \otimes B \multimap C} \star
\end{array}$$

Girard's Translation (326ms)

$$\begin{array}{c}
\frac{\overline{A, B, \langle A \rangle \rightarrow \langle B \rangle \Rightarrow C : \cdot \Rightarrow A}}{}^I \quad \frac{\overline{A, B, \langle A \rangle \rightarrow \langle B \rangle \Rightarrow C : \cdot \Rightarrow B}}{}^{D_R} \quad \frac{\overline{A, B, \langle A \rangle \rightarrow \langle B \rangle \Rightarrow C : C \Rightarrow C}}{}^{D_R} \\
\frac{\overline{A, B, \langle A \rangle \rightarrow \langle B \rangle \Rightarrow C : \cdot \vdash A}}{}^{D_R} \quad \frac{\overline{A, B, \langle A \rangle \rightarrow \langle B \rangle \Rightarrow C : \cdot \vdash B}}{}! \quad \frac{\overline{A, B, \langle A \rangle \rightarrow \langle B \rangle \Rightarrow C : C \vdash C}}{}^{D_R} \\
\frac{\overline{A, B, \langle A \rangle \rightarrow \langle B \rangle \Rightarrow C : \cdot \Rightarrow \langle A \rangle}}{}! \quad \frac{\overline{A, B, \langle A \rangle \rightarrow \langle B \rangle \Rightarrow C : \cdot \vdash \langle B \rangle \Rightarrow C}}{}! \quad \frac{\overline{\langle B \rangle \Rightarrow C} \Leftarrow C}{}^{-0} \\
\frac{\overline{A, B, \langle A \rangle \rightarrow \langle B \rangle \Rightarrow C : \cdot \vdash \langle A \rangle \rightarrow \langle B \rangle \Rightarrow C} \Leftarrow C}{\overline{A, B, \langle A \rangle \rightarrow \langle B \rangle \Rightarrow C : \cdot \vdash C}}_{D_C} \\
\frac{\overline{A, B, \langle A \rangle \rightarrow \langle B \rangle \Rightarrow C : \cdot \vdash C}}{}^* \quad \frac{\overline{::!(\langle A \rangle \rightarrow \langle B \rangle \Rightarrow C) \vdash !(A \& B) \Rightarrow C}}{}{}
\end{array}$$

Positive decoration (219ms)

[illegible]

0/1 focused decoration (313ms)

Figure 1 illustrates a complex dependency graph, likely representing a formal logic or a computational system. The graph is organized into layers, with nodes representing formulas and edges representing dependencies. The nodes are labeled with letters (A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z, AA, AB, AC, AD, AE, AF, AG, AH, AI, AJ, AK, AL, AM, AN, AO, AP, AQ, AR, AS, AT, AU, AV, AW, AX, AY, AZ, BA, BB, BC, BD, BE, BF, BG, BH, BI, BJ, BK, BL, BM, BN, BO, BP, BQ, BR, BS, BT, BU, BV, BW, BX, BY, BZ, CA, CB, CC, CD, CE, CF, CG, CH, CI, CJ, CK, CL, CM, CN, CO, CP, CQ, CR, CS, CT, CU, CV, CW, CX, CY, CZ, DA, DB, DC, DD, DE, DF, DG, DH, DI, DJ, DK, DL, DM, DN, DO, DP, DQ, DR, DS, DT, DU, DV, DW, DX, DY, DZ, EA, EB, EC, ED, EE, EF, EG, EH, EI, EJ, EK, EL, EM, EN, EO, EP, EQ, ER, ES, ET, EU, EV, EW, EX, EY, EZ, FA, FB, FC, FD, FE, FF, FG, FH, FI, FJ, FK, FL, FM, FN, FO, FP, FQ, FR, FS, FT, FU, FV, FW, FX, FY, FZ, GA, GB, GC, GD, GE, GF, GG, GH, GI, GJ, GK, GL, GM, GN, GO, GP, GQ, GR, GS, GT, GU, GV, GW, GX, GY, GZ, HA, HB, HC, HD, HE, HF, HG, HH, HI, HJ, HK, HL, HM, HN, HO, HP, HQ, HR, HS, HT, HU, HV, HW, HX, HY, HZ, IA, IB, IC, ID, IE, IF, IG, IH, II, IJ, IK, IL, IM, IN, IO, IP, IQ, IR, IS, IT, IU, IV, IW, IX, IY, IZ, JA, JB, JC, JD, JE, JF, JG, JH, JI, JJ, JK, JL, JM, JN, JO, JP, JQ, JR, JS, JT, JU, JV, JW, JX, JY, JZ, KA, KB, KC, KD, KE, KF, KG, KH, KI, KJ, KK, KL, KM, KN, KO, KP, KQ, KR, KS, KT, KU, KV, KW, KX, KY, KZ, LA, LB, LC, LD, LE, LF, LG, LH, LI, LJ, LK, LL, LM, LN, LO, LP, LQ, LR, LS, LT, LU, LV, LW, LX, LY, LZ, MA, MB, MC, MD, ME, MF, MG, MH, MI, MJ, MK, ML, MM, MN, MO, MP, MQ, MR, MS, MT, MU, MV, MW, MX, MY, MZ, NA, NB, NC, ND, NE, NF, NG, NH, NI, NJ, NK, NL, NM, NN, NO, NP, NQ, NR, NS, NT, NU, NV, NW, NX, NY, NZ, OA, OB, OC, OD, OE, OF, OG, OH, OI, OJ, OK, OL, OM, ON, OO, OP, OQ, OR, OS, OT, OU, OV, OW, OX, OY, OZ, PA, PB, PC, PD, PE, PF, PG, PH, PI, PJ, PK, PL, PM, PN, PO, PP, PQ, PR, PS, PT, PU, PV, PW, PX, PY, PZ, QA, QB, QC, QD, QE, QF, QG, QH, QI, QJ, QK, QL, QM, QN,

(5) $A \wedge B \rightarrow C \vdash A \rightarrow (B \rightarrow C)$

LJ (38ms)

$$\frac{\frac{A, B, A \wedge B \rightarrow C \vdash A \wedge B}{A, B, A \wedge B \rightarrow C \vdash C} \star}{A \wedge B \rightarrow C \vdash A \rightarrow B \rightarrow C} \star \quad \supset_L$$

Multiplicative translation (98ms)

$$\frac{\frac{\frac{\cdot : A \Rightarrow A}{\cdot : A, B \Rightarrow A \otimes B} I \quad \frac{\cdot : B \Rightarrow B}{\cdot : A, B \Rightarrow A \otimes B} I \quad \frac{\frac{\cdot : C \Rightarrow C}{\cdot : C \vdash C} D_R \quad \frac{\cdot : \cdot [C] \Leftarrow C}{\cdot : \cdot [C] \Leftarrow C} R_L}{\frac{\cdot : A, B \Rightarrow A \otimes B \quad \cdot : \cdot [C] \Leftarrow C}{\cdot : A, B \Rightarrow A \otimes B} \otimes} \frac{\frac{\cdot : A, B \Rightarrow A \otimes B \quad \cdot : A, B, A \otimes B \rightarrow C \vdash C}{\cdot : A, B, A \otimes B \rightarrow C \vdash C} D_L \quad \frac{\cdot : A, B, A \otimes B \rightarrow C \vdash C}{\cdot : A \otimes B \multimap C \vdash A \multimap B \multimap C} \star}{\cdot : A, B \Rightarrow A \otimes B} \multimap$$

Girard's Translation (108ms)

$$\begin{array}{c}
\frac{A, B, (A \& B) \rightarrow C : \cdot \Rightarrow A}{A, B, \neg(A \& B) \rightarrow C : \cdot \vdash A} \text{Dr} \quad \frac{A, B, (A \& B) \rightarrow C : \cdot \Rightarrow B}{A, B, \neg(A \& B) \rightarrow C : \cdot \vdash B} \text{Dr} \quad \frac{A, B, \neg(A \& B) \rightarrow C : C \Rightarrow C}{A, B, \neg(A \& B) \rightarrow C : C \Rightarrow C} \text{Dr} \\
\frac{A, B, \neg(A \& B) \rightarrow C : \cdot \vdash A \quad A, B, \neg(A \& B) \rightarrow C : \cdot \vdash B}{A, B, \neg(A \& B) \rightarrow C : \cdot \vdash A \& B} * \\
\frac{A, B, \neg(A \& B) \rightarrow C : \cdot \vdash A \& B}{A, B, \neg(A \& B) \rightarrow C : \cdot \Rightarrow \neg(A \& B)} ! \\
\frac{A, B, \neg(A \& B) \rightarrow C : \cdot \vdash A \quad A, B, \neg(A \& B) \rightarrow C : \cdot \vdash B}{A, B, \neg(A \& B) \rightarrow C : \cdot \vdash A \& B} * \\
\frac{A, B, \neg(A \& B) \rightarrow C : \cdot \vdash A \& B}{A, B, \neg(A \& B) \rightarrow C : \cdot \vdash \neg(A \& B) \rightarrow C} \text{Dc} \\
\frac{A, B, \neg(A \& B) \rightarrow C : \cdot \vdash \neg(A \& B) \rightarrow C}{\therefore \neg(A \& B) \rightarrow C} *
\end{array}$$

Positive decoration (182ms)

[illegible]

0/1 focused decoration (197ms)

[illegible]

$$(6) \quad A \rightarrow B \vdash (B \rightarrow C) \rightarrow (A \rightarrow C)$$

LJ (48ms)

$$\frac{\frac{\frac{A, A \rightarrow B, B \rightarrow C \vdash A}{A, A \rightarrow B, B \rightarrow C \vdash C}^* \quad \frac{\frac{A, B, B \rightarrow C \vdash B}{A, B, B \rightarrow C \vdash C}^* \quad \frac{A, B, C \vdash C}{A, B, B \rightarrow C \vdash C}^*}{A, A \rightarrow B, B \rightarrow C \vdash C} \supset_L \supset_L \frac{A, A \rightarrow B, B \rightarrow C \vdash C}{A \rightarrow B \vdash B \rightarrow C \rightarrow A \rightarrow C}^*$$

Multiplicative translation (106ms)

[illegible]

Girard's Translation (157ms)

[illegible]

Positive decoration (252ms)

[illegible]

0/1 focused decoration (214ms)

$\frac{A, B, \Gamma(A) \Rightarrow (B), \Gamma(B) \Rightarrow \Gamma(C) : \vdash \Rightarrow B}{A, B, \Gamma(A) \Rightarrow (B), \Gamma(B) \Rightarrow \Gamma(C) : \vdash A} \stackrel{I}{Dn}$ $\frac{A, B, \Gamma(A) \Rightarrow (B), \Gamma(B) \Rightarrow \Gamma(C) : \vdash \Rightarrow B}{A, B, \Gamma(A) \Rightarrow (B), \Gamma(B) \Rightarrow \Gamma(C) : \vdash A} \stackrel{Dn}{\vdash}$ $\frac{A, B, \Gamma(A) \Rightarrow (B), \Gamma(B) \Rightarrow \Gamma(C) : \vdash \Rightarrow B}{A, B, \Gamma(A) \Rightarrow (B), \Gamma(B) \Rightarrow \Gamma(C) : \vdash \Gamma(C)} \stackrel{R_L}{\vdash}$	$\frac{A, B, C, \Gamma(A) \Rightarrow (B), \Gamma(B) \Rightarrow \Gamma(C) : \vdash C}{A, B, C, \Gamma(A) \Rightarrow (B), \Gamma(B) \Rightarrow \Gamma(C) : \vdash C} \stackrel{Dn}{\vdash}$ $\frac{A, B, \Gamma(A) \Rightarrow (B), \Gamma(B) \Rightarrow \Gamma(C) : \vdash \Rightarrow B}{A, B, \Gamma(A) \Rightarrow (B), \Gamma(B) \Rightarrow \Gamma(C) : \vdash \Gamma(C)} \stackrel{R_L}{\vdash}$
$\frac{A, B, \Gamma(A) \Rightarrow (B), \Gamma(B) \Rightarrow \Gamma(C) : \vdash \Rightarrow B}{A, B, \Gamma(A) \Rightarrow (B), \Gamma(B) \Rightarrow \Gamma(C) : \vdash \Gamma(C)} \stackrel{D_C}{\vdash}$	
$\frac{A, \Gamma(A) \Rightarrow (B), \Gamma(B) \Rightarrow \Gamma(C) : \vdash \Rightarrow A}{A, \Gamma(A) \Rightarrow (B), \Gamma(B) \Rightarrow \Gamma(C) : \vdash A} \stackrel{I}{Dn}$ $\frac{A, \Gamma(A) \Rightarrow (B), \Gamma(B) \Rightarrow \Gamma(C) : \vdash \Rightarrow A}{A, \Gamma(A) \Rightarrow (B), \Gamma(B) \Rightarrow \Gamma(C) : \vdash A} \stackrel{Dn}{\vdash}$	$\frac{A, \Gamma(A) \Rightarrow (B), \Gamma(B) \Rightarrow \Gamma(C) : \vdash \Rightarrow A}{A, \Gamma(A) \Rightarrow (B), \Gamma(B) \Rightarrow \Gamma(C) : \vdash \Gamma(C)} \stackrel{R_L}{\vdash}$
$\frac{A, \Gamma(A) \Rightarrow (B), \Gamma(B) \Rightarrow \Gamma(C) : \vdash \Rightarrow A}{A, \Gamma(A) \Rightarrow (B), \Gamma(B) \Rightarrow \Gamma(C) : \vdash \Gamma(C)} \stackrel{D_C}{\vdash}$	

$$(7) A \rightarrow B \vdash (C \rightarrow A) \rightarrow (C \rightarrow B)$$

LJ (51ms)

$$\frac{\frac{C, A \rightarrow B, C \rightarrow A \vdash C}{C, A \rightarrow B, C \rightarrow A \vdash A} \star \quad \frac{A, C, A \rightarrow B \vdash A}{B, C, C \rightarrow A \vdash B} \star}{\frac{C, A \rightarrow B, C \rightarrow A \vdash B}{A \rightarrow B \vdash C \rightarrow A \rightarrow C \rightarrow B} \star} \supset_L$$

Multiplicative translation (108ms)

$$\frac{\frac{\frac{\frac{\vdash B \Rightarrow B}{\vdash B \vdash B} I}{\vdash A \Rightarrow A} I \quad \frac{\vdash B \vdash B}{\vdash [B] \Leftarrow B} R_L}{\vdash A [A \rightarrow B] \Leftarrow B} D_L \quad \frac{\vdash A, A \rightarrow B \vdash B}{\vdash A \rightarrow B [A] \Leftarrow B} R_L}{\vdash C \Rightarrow C} I \quad \frac{\vdash C, A \rightarrow B [C \rightarrow A] \Leftarrow B}{\vdash C, A \rightarrow B, C \rightarrow A \vdash B} D_L}{\vdash A \rightarrow B \vdash C \rightarrow A \rightarrow C \rightarrow B} \star$$

Girard's Translation (156ms)

$$\frac{\frac{\frac{C, !!(A) \rightarrow B, !(C) \rightarrow A : \cdot \Rightarrow C}{C, !(A) \rightarrow B, !(C) \rightarrow A : \cdot \vdash C} D_R \quad \frac{C, !(A) \rightarrow B, !(C) \rightarrow A : A \Rightarrow A}{C, !(A) \rightarrow B, !(C) \rightarrow A : A \vdash A} D_R}{\frac{C, !(A) \rightarrow B, !(C) \rightarrow A : \cdot \Rightarrow !(C)}{C, !(A) \rightarrow B, !(C) \rightarrow A : \cdot \vdash !(C)} !} \quad \frac{C, !(A) \rightarrow B, !(C) \rightarrow A : A \Rightarrow A}{C, !(A) \rightarrow B, !(C) \rightarrow A : A \vdash A} R_L}{\frac{C, !(A) \rightarrow B, !(C) \rightarrow A : \cdot \vdash A}{C, !(A) \rightarrow B, !(C) \rightarrow A : \cdot \vdash A} D_C} \quad \frac{C, !(A) \rightarrow B, !(C) \rightarrow A : A \Rightarrow A}{C, !(A) \rightarrow B, !(C) \rightarrow A : A \vdash A} R_L}{\frac{C, !(A) \rightarrow B, !(C) \rightarrow A : \cdot \vdash A}{C, !(A) \rightarrow B, !(C) \rightarrow A : \cdot \vdash A} D_C} \quad \frac{C, !(A) \rightarrow B, !(C) \rightarrow A : A \Rightarrow A}{C, !(A) \rightarrow B, !(C) \rightarrow A : A \vdash A} R_L}{\vdash !(!(A) \rightarrow B) \vdash !(!(C) \rightarrow A) \rightarrow !(C) \rightarrow B} \star$$

Positive decoration (261ms)

$$\frac{\frac{\frac{C, !(A) \rightarrow !(B), !(C) \rightarrow !(A) : \cdot \Rightarrow C}{C, !(A) \rightarrow !(B), !(C) \rightarrow !(A) : \cdot \vdash C} D_R \quad \frac{C, !(A) \rightarrow !(B), !(C) \rightarrow !(A) : A \Rightarrow A}{C, !(A) \rightarrow !(B), !(C) \rightarrow !(A) : A \vdash A} D_R}{\frac{C, !(A) \rightarrow !(B), !(C) \rightarrow !(A) : \cdot \Rightarrow !(C)}{C, !(A) \rightarrow !(B), !(C) \rightarrow !(A) : \cdot \vdash !(C)} !} \quad \frac{C, !(A) \rightarrow !(B), !(C) \rightarrow !(A) : A \Rightarrow A}{C, !(A) \rightarrow !(B), !(C) \rightarrow !(A) : A \vdash A} R_L}{\frac{C, !(A) \rightarrow !(B), !(C) \rightarrow !(A) : \cdot \vdash A}{C, !(A) \rightarrow !(B), !(C) \rightarrow !(A) : \cdot \vdash A} D_C} \quad \frac{C, !(A) \rightarrow !(B), !(C) \rightarrow !(A) : A \Rightarrow A}{C, !(A) \rightarrow !(B), !(C) \rightarrow !(A) : A \vdash A} R_L}{\frac{C, !(A) \rightarrow !(B), !(C) \rightarrow !(A) : \cdot \vdash A}{C, !(A) \rightarrow !(B), !(C) \rightarrow !(A) : \cdot \vdash A} D_C} \quad \frac{C, !(A) \rightarrow !(B), !(C) \rightarrow !(A) : A \Rightarrow A}{C, !(A) \rightarrow !(B), !(C) \rightarrow !(A) : A \vdash A} R_L}{\vdash !(!(A) \rightarrow !(B)) \vdash !(!(C) \rightarrow !(A)) \rightarrow !(C) \rightarrow !(B)} \star$$

0/1 focused decoration (201ms)

$$\frac{\frac{\frac{C, !(A) \rightarrow !(B), !(C) \rightarrow !(A) : \cdot \Rightarrow C}{C, !(A) \rightarrow !(B), !(C) \rightarrow !(A) : \cdot \vdash C} D_R \quad \frac{C, !(A) \rightarrow !(B), !(C) \rightarrow !(A) : A \Rightarrow A}{C, !(A) \rightarrow !(B), !(C) \rightarrow !(A) : A \vdash A} D_R}{\frac{C, !(A) \rightarrow !(B), !(C) \rightarrow !(A) : \cdot \Rightarrow !(C)}{C, !(A) \rightarrow !(B), !(C) \rightarrow !(A) : \cdot \vdash !(C)} !} \quad \frac{C, !(A) \rightarrow !(B), !(C) \rightarrow !(A) : A \Rightarrow A}{C, !(A) \rightarrow !(B), !(C) \rightarrow !(A) : A \vdash A} R_L}{\frac{C, !(A) \rightarrow !(B), !(C) \rightarrow !(A) : \cdot \vdash A}{C, !(A) \rightarrow !(B), !(C) \rightarrow !(A) : \cdot \vdash A} D_C} \quad \frac{C, !(A) \rightarrow !(B), !(C) \rightarrow !(A) : A \Rightarrow A}{C, !(A) \rightarrow !(B), !(C) \rightarrow !(A) : A \vdash A} R_L}{\frac{C, !(A) \rightarrow !(B), !(C) \rightarrow !(A) : \cdot \vdash A}{C, !(A) \rightarrow !(B), !(C) \rightarrow !(A) : \cdot \vdash A} D_C} \quad \frac{C, !(A) \rightarrow !(B), !(C) \rightarrow !(A) : A \Rightarrow A}{C, !(A) \rightarrow !(B), !(C) \rightarrow !(A) : A \vdash A} R_L}{\vdash !(!(A) \rightarrow !(B)) \vdash !(!(C) \rightarrow !(A)) \rightarrow !(C) \rightarrow !(B)} \star$$

$$(8) A \rightarrow B \vdash A \wedge C \rightarrow B \wedge C$$

LJ (34ms)

$$\frac{\frac{A, C, A \rightarrow B \vdash A}{A, C, A \rightarrow B \vdash B} \star \quad \frac{A, B, C \vdash B}{A, C, A \rightarrow B \vdash B} \star}{\frac{A \rightarrow B \vdash A \wedge C \rightarrow B \wedge C} \star} \supset_L$$

Multiplicative translation (91ms)

$$\frac{\frac{\frac{\vdash B \Rightarrow B}{\vdash B, C \Rightarrow B \otimes C} I \quad \frac{\vdash C \Rightarrow C}{\vdash B, C \vdash B \otimes C} I}{\vdash A \Rightarrow A} I \quad \frac{\vdash B, C \vdash B \otimes C}{\vdash C [B] \Leftarrow B \otimes C} D_R}{\vdash A, C [A \rightarrow B] \Leftarrow B \otimes C} R_L \quad \frac{\vdash A, C, A \rightarrow B \vdash B \otimes C}{\vdash A \rightarrow B \vdash A \otimes C \rightarrow B \otimes C} \star$$

Girard's Translation (114ms)

$$\frac{\frac{\frac{A, C, !(A) \rightarrow B : \cdot \Rightarrow A}{A, C, !(A) \rightarrow B : \cdot \vdash A} I \quad \frac{A, C, !(A) \rightarrow B : B \Rightarrow B}{A, C, !(A) \rightarrow B : B \vdash B} I}{\frac{A, C, !(A) \rightarrow B : \cdot \Rightarrow !(A)}{A, C, !(A) \rightarrow B : \cdot \vdash !(A)} !} \quad \frac{A, C, !(A) \rightarrow B : B \Rightarrow B}{A, C, !(A) \rightarrow B : B \vdash B} D_R}{\frac{A, C, !(A) \rightarrow B : \cdot \vdash A}{A, C, !(A) \rightarrow B : \cdot \vdash A} D_C} \quad \frac{A, C, !(A) \rightarrow B : B \Rightarrow B}{A, C, !(A) \rightarrow B : B \vdash B} D_R}{\vdash !(!(A) \rightarrow B) \vdash !(A \& C) \rightarrow B \& C} \star$$

Positive decoration (191ms)

$$\frac{\frac{\frac{A, B, C, !(A) \rightarrow !(B) : \cdot \Rightarrow B}{A, C, !(A) \rightarrow !(B) : \cdot \vdash B} I \quad \frac{A, B, C, !(A) \rightarrow !(B) : \cdot \vdash B}{A, C, !(A) \rightarrow !(B) : \cdot \vdash B} D_R}{\frac{A, C, !(A) \rightarrow !(B) : \cdot \Rightarrow A}{A, C, !(A) \rightarrow !(B) : \cdot \vdash A} !} \quad \frac{A, B, C, !(A) \rightarrow !(B) : \cdot \vdash B}{A, C, !(A) \rightarrow !(B) : \cdot \vdash B} R_L}{\frac{A, C, !(A) \rightarrow !(B) : \cdot \vdash A}{A, C, !(A) \rightarrow !(B) : \cdot \vdash A} D_C} \quad \frac{A, C, !(A) \rightarrow !(B) : \cdot \vdash B}{A, C, !(A) \rightarrow !(B) : \cdot \vdash B} R_L}{\vdash !(!(A) \rightarrow !(B)) \vdash !(!(C) \rightarrow !(B)) \rightarrow !(B) \otimes !(C)} \star$$

0/1 focused decoration (271ms)

$$\frac{\frac{\frac{A, !(A), !(C), !(A) \rightarrow !(B) : \cdot \Rightarrow A}{A, !(A), !(C), !(A) \rightarrow !(B) : \cdot \vdash A} I \quad \frac{A, !(A), !(C), !(A) \rightarrow !(B) : \cdot \vdash A}{A, !(A), !(C), !(A) \rightarrow !(B) : \cdot \vdash A} D_R}{\frac{A, !(A), !(C), !(A) \rightarrow !(B) : \cdot \Rightarrow !(A)}{A, !(A), !(C), !(A) \rightarrow !(B) : \cdot \vdash !(A)} !} \quad \frac{A, !(A), !(C), !(A) \rightarrow !(B) : \cdot \vdash A}{A, !(A), !(C), !(A) \rightarrow !(B) : \cdot \vdash A} R_L}{\frac{A, !(A), !(C), !(A) \rightarrow !(B) : \cdot \vdash A}{A, !(A), !(C), !(A) \rightarrow !(B) : \cdot \vdash A} D_C} \quad \frac{A, !(A), !(C), !(A) \rightarrow !(B) : \cdot \vdash A}{A, !(A), !(C), !(A) \rightarrow !(B) : \cdot \vdash A} R_L}{\vdash !(!(A) \rightarrow !(B)) \vdash !(!(C) \rightarrow !(A)) \rightarrow !(C) \rightarrow !(B)} \star$$

$$(9) \ A \rightarrow B \vdash C \wedge A \rightarrow C \wedge B$$

LJ (35ms)

$$\frac{\frac{A, C, A \rightarrow B \vdash A \quad \star \quad A, B, C \vdash B \quad \star}{A, C, A \rightarrow B \vdash B} \supset_L}{A \rightarrow B \vdash C \wedge A \rightarrow C \wedge B \quad \star}$$

Multiplicative translation (89ms)

$$\frac{\frac{\frac{\frac{\vdash C \Rightarrow C \quad I \quad \vdash B \Rightarrow B \quad I}{\vdash B, C \Rightarrow C \otimes B} \otimes}{\vdash B, C \vdash C \otimes B} D_R}{\vdash C [B] \Leftarrow C \otimes B} R_L}{\vdash A, C [A \multimap B] \Leftarrow C \otimes B} \multimap}{\vdash A, C, A \multimap B \vdash C \otimes B} D_L}{\vdash A \multimap B \vdash C \otimes A \multimap C \otimes B \quad \star}$$

Girard's Translation (111ms)

$$\frac{\frac{\frac{\frac{\frac{\vdash A, C, ! (A) \multimap B : \cdot \Rightarrow A \quad I}{\vdash A, C, ! (A) \multimap B : \cdot \vdash A} D_R}{\vdash A, C, ! (A) \multimap B : \cdot \Rightarrow ! (A)} \quad \frac{\frac{\frac{\vdash A, C, ! (A) \multimap B : B \Rightarrow B \quad I}{\vdash A, C, ! (A) \multimap B : B \vdash B} D_R}{\vdash A, C, ! (A) \multimap B : \cdot [B] \Leftarrow B} R_L}{\vdash A, C, ! (A) \multimap B : \cdot [! (A) \multimap B] \Leftarrow B} \multimap}{\vdash ! (! (A) \multimap B) \vdash ! (C \& A) \multimap C \& B \quad \star}$$

Positive decoration (173ms)

$$\frac{\frac{\frac{\frac{\frac{\vdash A, C, ! (A) \multimap ! (B) : \cdot \Rightarrow C \quad I}{\vdash A, C, ! (A) \multimap ! (B) : \cdot \vdash C} D_R}{\vdash A, C, ! (A) \multimap ! (B) : \cdot \Rightarrow ! (C)} \quad \frac{\frac{\frac{\frac{\vdash A, B, C, ! (A) \multimap ! (B) : \cdot \Rightarrow B \quad I}{\vdash A, B, C, ! (A) \multimap ! (B) : \cdot \vdash B} D_R}{\vdash A, C, ! (A) \multimap ! (B) : \cdot [! (A) \multimap ! (B)] \Leftarrow B} D_C}{\vdash A, C, ! (A) \multimap ! (B) : \cdot \Rightarrow ! (C) \otimes ! (B)} \otimes}{\vdash A, C, ! (A) \multimap ! (B) : \cdot \vdash ! (C) \otimes ! (B)} D_R}{\vdash ! (A) \multimap ! (B) : \cdot \vdash ! (C) \otimes ! (A) \multimap ! (C) \otimes ! (B)} \star}{\vdash ! (A) \multimap ! (B) : \cdot \Rightarrow ! (! (C) \otimes ! (A) \multimap ! (C) \otimes ! (B))} \star}{\vdash ! (A) \multimap ! (B) : \cdot \vdash ! (! (C) \otimes ! (A) \multimap ! (C) \otimes ! (B))} D_R}{\vdash ! (! (A) \multimap ! (B)) \vdash ! (! (C) \otimes ! (A) \multimap ! (C) \otimes ! (B))} \star$$

0/1 focused decoration (257ms)

$$\frac{\frac{\frac{\frac{\frac{\vdash C, ! (A), ! (C), ! (A) \multimap ! (B) : \cdot \Rightarrow C \quad I}{\vdash C, ! (A), ! (C), ! (A) \multimap ! (B) : \cdot \vdash C} D_R}{\vdash ! (A), ! (C), ! (A) \multimap ! (B) : \cdot [! (C)] \Leftarrow C} R_L}{\vdash ! (A), ! (C), ! (A) \multimap ! (B) : \cdot \vdash C} D_C}{\vdash ! (A), ! (C), ! (A) \multimap ! (B) : \cdot \vdash ! (C) \& B} \star}{\vdash ! (A), ! (C), ! (A) \multimap ! (B) : \cdot \Rightarrow ! (! (C) \& B)} D_R}{\vdash ! (A) \multimap ! (B) : \cdot \vdash ! (! (! (C) \& ! (A)) \multimap ! (C \& B))} \star}{\vdash ! (A) \multimap ! (B) : \cdot \Rightarrow ! (! (! (! (C) \& ! (A)) \multimap ! (C \& B))} D_R}{\vdash ! (! (A) \multimap ! (B)) \vdash ! (! (! (! (C) \& ! (A)) \multimap ! (C \& B))} \star$$

$$(10) \ \neg A \vdash A \rightarrow B$$

LJ (34ms)

$$\frac{\frac{A, A \rightarrow \perp \vdash A \quad \star \quad A, \perp \vdash B \quad \star}{A, A \rightarrow \perp \vdash B} \supset_L}{A \rightarrow \perp \vdash A \rightarrow B \quad \star}$$

Multiplicative translation (19ms)

fail

Girard's Translation (79ms)

$$\frac{\frac{\frac{\frac{\vdash A, ! (A) \multimap \mathbf{0} : \cdot \Rightarrow A \quad I}{\vdash A, ! (A) \multimap \mathbf{0} : \cdot \vdash A} D_R}{\vdash A, ! (A) \multimap \mathbf{0} : \cdot \Rightarrow ! (A)} \quad \frac{\frac{\vdash A, ! (A) \multimap \mathbf{0} : \mathbf{0} \vdash B \quad \star}{\vdash A, ! (A) \multimap \mathbf{0} : \cdot [\mathbf{0}] \Leftarrow B} R_L}{\vdash A, ! (A) \multimap \mathbf{0} : \cdot [! (A) \multimap \mathbf{0}] \Leftarrow B} \multimap}{\vdash A, ! (A) \multimap \mathbf{0} : \cdot \vdash B} D_C}{\vdash ! (! (A) \multimap \mathbf{0}) \vdash ! (A) \multimap B \quad \star}$$

Positive decoration (98ms)

$$\frac{\frac{\frac{\frac{\vdash A, ! (A) \multimap \mathbf{0} : \cdot \Rightarrow A \quad I}{\vdash A, ! (A) \multimap \mathbf{0} : \cdot \vdash A} D_R}{\vdash A, ! (A) \multimap \mathbf{0} : \cdot \Rightarrow ! (A)} \quad \frac{\frac{\vdash A, ! (A) \multimap \mathbf{0} : \mathbf{0} \vdash ! (B) \quad \star}{\vdash A, ! (A) \multimap \mathbf{0} : \cdot [\mathbf{0}] \Leftarrow ! (B)} R_L}{\vdash A, ! (A) \multimap \mathbf{0} : \cdot [! (A) \multimap \mathbf{0}] \Leftarrow ! (B)} D_C}{\vdash A, ! (A) \multimap \mathbf{0} : \cdot \vdash ! (B)} \star}{\vdash ! (A) \multimap \mathbf{0} : \cdot \vdash ! (A) \multimap ! (B)} \star}{\vdash ! (A) \multimap \mathbf{0} : \cdot \Rightarrow ! (! (A) \multimap ! (B))} \star}{\vdash ! (A) \multimap \mathbf{0} : \cdot \vdash ! (! (A) \multimap ! (B))} D_R}{\vdash ! (! (A) \multimap \mathbf{0}) \vdash ! (! (A) \multimap ! (B))} \star$$

0/1 focused decoration (98ms)

$$\frac{\frac{\frac{\frac{\vdash A, ! (A) \multimap ! (\mathbf{0}) : \cdot \Rightarrow A \quad I}{\vdash A, ! (A) \multimap ! (\mathbf{0}) : \cdot \vdash A} D_R}{\vdash A, ! (A) \multimap ! (\mathbf{0}) : \cdot \Rightarrow ! (A)} \quad \frac{\frac{\vdash A, ! (A) \multimap ! (\mathbf{0}) : ! (\mathbf{0}) \vdash B \quad \star}{\vdash A, ! (A) \multimap ! (\mathbf{0}) : \cdot [! (\mathbf{0})] \Leftarrow B} R_L}{\vdash A, ! (A) \multimap ! (\mathbf{0}) : \cdot [! (A) \multimap ! (\mathbf{0})] \Leftarrow B} \multimap}{\vdash A, ! (A) \multimap ! (\mathbf{0}) : \cdot \vdash B} D_C}{\vdash ! (A) \multimap ! (\mathbf{0}) : \cdot \vdash ! (A) \multimap B} \star}{\vdash ! (A) \multimap ! (\mathbf{0}) : \cdot \Rightarrow ! (! (A) \multimap B)} \star}{\vdash ! (A) \multimap ! (\mathbf{0}) : \cdot \vdash ! (! (A) \multimap B)} D_R}{\vdash ! (! (A) \multimap ! (\mathbf{0})) \vdash ! (! (A) \multimap B)} \star$$

$$(11) \ A \vdash \neg A \rightarrow B$$

LJ (34ms)

$$\frac{\frac{\overline{A, A \rightarrow \perp \vdash A}^* \quad \overline{A, \perp \vdash B}^*}{A, A \rightarrow \perp \vdash B} \supset_L}{A \vdash A \rightarrow \perp \rightarrow B}^*$$

Multiplicative translation (18ms)

fail

Girard's Translation (82ms)

$$\frac{\frac{\overline{A, !(A) \multimap \mathbf{0} : \cdot \Rightarrow A}^I}{\overline{A, !(A) \multimap \mathbf{0} : \cdot \vdash A}^{D_R}} \quad \frac{\overline{A, !(A) \multimap \mathbf{0} : \mathbf{0} \vdash B}^*}{\overline{A, !(A) \multimap \mathbf{0} : \cdot \Rightarrow !(A)}^!} \quad \frac{\overline{A, !(A) \multimap \mathbf{0} : \cdot \cdot [\mathbf{0}] \Leftarrow B}^{R_L}}{\overline{A, !(A) \multimap \mathbf{0} : \cdot \cdot [!(A) \multimap \mathbf{0}] \Leftarrow B}^{D_C}} \quad \frac{\overline{A, !(A) \multimap \mathbf{0} : \cdot \vdash B}^*}{\overline{\cdot : !(A) \vdash !(!(A) \multimap \mathbf{0}) \multimap B}^*} \quad \multimap$$

Positive decoration (97ms)

$$\frac{\frac{\overline{A, !(A) \multimap \mathbf{0} : \cdot \Rightarrow A}^I}{\overline{A, !(A) \multimap \mathbf{0} : \cdot \vdash A}^{D_R}} \quad \frac{\overline{A, !(A) \multimap \mathbf{0} : \mathbf{0} \vdash !(B)}^*}{\overline{A, !(A) \multimap \mathbf{0} : \cdot \cdot [\mathbf{0}] \Leftarrow !(B)}^{R_L}}}{\overline{A, !(A) \multimap \mathbf{0} : \cdot \cdot [!(A) \multimap \mathbf{0}] \Leftarrow !(B)}^{D_C}} \quad \frac{\overline{A, !(A) \multimap \mathbf{0} : \cdot \vdash !(B)}^*}{\overline{A : \cdot \vdash !(!(A) \multimap \mathbf{0}) \multimap !(B)}^!} \quad \frac{\overline{A : \cdot \Rightarrow !(!(A) \multimap \mathbf{0}) \multimap !(B)}^{D_R}}{\overline{A : \cdot \vdash !(!(A) \multimap \mathbf{0}) \multimap !(B)}^*} \quad \multimap$$

0/1 focused decoration (97ms)

$$\frac{\frac{\overline{A, !(A) \multimap !(\mathbf{0}) : \cdot \Rightarrow A}^I}{\overline{A, !(A) \multimap !(\mathbf{0}) : \cdot \vdash A}^{D_R}} \quad \frac{\overline{A, !(A) \multimap !(\mathbf{0}) : !(\mathbf{0}) \vdash B}^*}{\overline{A, !(A) \multimap !(\mathbf{0}) : \cdot \cdot [!(\mathbf{0})] \Leftarrow B}^{R_L}}}{\overline{A, !(A) \multimap !(\mathbf{0}) : \cdot \cdot [!(A) \multimap !(\mathbf{0})] \Leftarrow B}^{D_C}} \quad \frac{\overline{A, !(A) \multimap !(\mathbf{0}) : \cdot \vdash B}^*}{\overline{A : \cdot \vdash !(!(A) \multimap !(\mathbf{0})) \multimap B}^!} \quad \frac{\overline{A : \cdot \Rightarrow !(!(A) \multimap !(\mathbf{0})) \multimap B}^{D_R}}{\overline{A : \cdot \vdash !(!(A) \multimap !(\mathbf{0})) \multimap B}^*} \quad \multimap$$

$$(12) \ B \vdash A \rightarrow B$$

LJ (19ms)

$$\overline{B \vdash A \rightarrow B}^*$$

Multiplicative translation (20ms)

fail

Girard's Translation (42ms)

$$\frac{\overline{A, B : \cdot \Rightarrow B}^I}{\overline{A, B : \cdot \vdash B}^{D_R}} \quad \frac{\overline{\cdot : !(B) \vdash !(A) \multimap B}^*}{\cdot : !(B) \vdash !(A) \multimap B}^*$$

Positive decoration (74ms)

$$\frac{\overline{A, B : \cdot \Rightarrow B}^I}{\overline{A, B : \cdot \vdash B}^{D_R}} \quad \frac{\overline{A, B : \cdot \Rightarrow !(B)}^!}{\overline{A, B : \cdot \vdash !(B)}^{D_R}} \quad \frac{\overline{B : \cdot \vdash !(A) \multimap !(B)}^*}{\overline{B : \cdot \Rightarrow !(!(A) \multimap !(B))}^!} \quad \frac{\overline{B : \cdot \vdash !(!(A) \multimap !(B))}^{D_R}}{\overline{\cdot : !(B) \vdash !(!(A) \multimap !(B))}^*} \quad \multimap$$

0/1 focused decoration (59ms)

$$\frac{\overline{A, B : \cdot \Rightarrow B}^I}{\overline{A, B : \cdot \vdash B}^{D_R}} \quad \frac{\overline{B : \cdot \vdash !(A) \multimap B}^*}{\overline{B : \cdot \Rightarrow !(!(A) \multimap B)}^!} \quad \frac{\overline{B : \cdot \vdash !(!(A) \multimap B)}^{D_R}}{\overline{\cdot : !(B) \vdash !(!(A) \multimap B)}^*} \quad \multimap$$

(13) $A \rightarrow B \vdash \neg B \rightarrow \neg A$

LJ (47ms)

$$\frac{\frac{A, A \rightarrow B, B \rightarrow \perp \vdash A}{A, A \rightarrow B, B \rightarrow \perp \vdash \perp} \star \quad \frac{\frac{A, B, B \rightarrow \perp \vdash B}{A, B, B \rightarrow \perp \vdash \perp} \star \quad \frac{A, B, \perp \vdash \perp}{A, B, B \rightarrow \perp \vdash \perp} \star}{\frac{A, A \rightarrow B, B \rightarrow \perp \vdash \perp}{A \rightarrow B \vdash B \rightarrow \perp \rightarrow A \rightarrow \perp} \star} \supset_L$$

Multiplicative translation (89ms)

$$\begin{array}{c}
\frac{\vdots \vdash B \Rightarrow B \quad I \quad \vdots \vdash \cdot \vdash [\perp] \Leftarrow \perp}{\vdots \vdash B \vdash [B \multimap \perp] \Leftarrow \perp} \quad \perp \\
\frac{\vdots \vdash B \vdash [B \multimap \perp] \Leftarrow \perp}{\vdots \vdash B, B \multimap \perp \vdash \perp} \quad D_L \\
\frac{\vdots \vdash A \Rightarrow A \quad I \quad \vdots \vdash B \multimap \perp \vdash [B] \Leftarrow \perp}{\vdots \vdash A \Rightarrow A \quad I \quad \vdots \vdash B \multimap \perp \vdash [B] \Leftarrow \perp} \quad R_L \\
\frac{\vdots \vdash A, B \multimap \perp \vdash [A \multimap B] \Leftarrow \perp}{\vdots \vdash A, A \multimap B, B \multimap \perp \vdash \perp} \quad D_L \\
\vdots \vdash A \multimap B \vdash B \multimap \perp \vdash \perp \vdash A \multimap \perp \vdash \perp \quad \star
\end{array}$$

Girard's Translation (137ms)

$$\begin{array}{c}
\frac{\overline{A}, ! (A) \rightarrow B, ! (B) : 0 \vdash \cdot \rightarrow A}{\overline{A}, ! (A) \rightarrow B, ! (B) : 0 \vdash \cdot \vdash A} \text{ }^I \text{ } D_R \quad \frac{\overline{A}, ! (A) \rightarrow B, ! (B) : 0 \vdash \cdot B \rightarrow B}{\overline{A}, ! (A) \rightarrow B, ! (B) : 0 \vdash \cdot B \vdash B} \text{ }^I \text{ } D_R \\
\frac{\overline{A}, ! (A) \rightarrow B, ! (B) : 0 \vdash \cdot \rightarrow ! (B)}{\overline{A}, ! (A) \rightarrow B, ! (B) : 0 \vdash \cdot \rightarrow ! (B)} \text{ }^! \text{ } \quad \frac{\overline{A}, ! (A) \rightarrow B, ! (B) : 0 \vdash \cdot \cdot [B] \Leftarrow B}{\overline{A}, ! (A) \rightarrow B, ! (B) : 0 \vdash \cdot \cdot [B] \Leftarrow B} \text{ }^R \text{ } R_L \\
\frac{\overline{A}, ! (A) \rightarrow B, ! (B) : 0 \vdash \cdot \cdot [! (A) \rightarrow B]}{\overline{A}, ! (A) \rightarrow B, ! (B) : 0 \vdash \cdot \cdot B} \text{ }^! \text{ } D_C \quad \frac{\overline{A}, ! (A) \rightarrow B, ! (B) : 0 \vdash \cdot 0 \vdash 0}{\overline{A}, ! (A) \rightarrow B, ! (B) : 0 \vdash \cdot 0 \vdash 0} \text{ }^* \text{ } R_0 \\
\frac{\overline{A}, ! (A) \rightarrow B, ! (B) : 0 \vdash \cdot \rightarrow ! (B)}{\overline{A}, ! (A) \rightarrow B, ! (B) : 0 \vdash \cdot \cdot [! (B) \rightarrow 0]} \text{ }^! \text{ } \quad \frac{\overline{A}, ! (A) \rightarrow B, ! (B) : 0 \vdash \cdot \cdot [! (B) \rightarrow 0]}{\overline{A}, ! (A) \rightarrow B, ! (B) : 0 \vdash \cdot \cdot [! (B) \rightarrow 0]} \text{ }^! \text{ } D_C \\
\frac{\overline{A}, ! (A) \rightarrow B, ! (B) : 0 \vdash \cdot \cdot 0}{\vdash \cdot \cdot ! (A) \rightarrow B \vdash \cdot ! (B) : 0 \rightarrow ! (A) : 0} \text{ }^* \text{ }
\end{array}$$

Positive decoration (184ms)

[illegible]

0/1 focused decoration (180ms)

[illegible]

(14) $A \rightarrow \neg B \vdash (\neg\neg B) \rightarrow (\neg A)$

LJ (62ms)

$$\frac{\frac{\frac{A, B, B \rightarrow \perp, B \rightarrow \perp \rightarrow \perp \vdash B}{A, B, \perp, B \rightarrow \perp \rightarrow \perp \vdash \perp} \star \quad \frac{A, B, \perp, B \rightarrow \perp \rightarrow \perp \vdash \perp}{A, B, B \rightarrow \perp, B \rightarrow \perp \rightarrow \perp \vdash \perp} \star \quad \frac{A, B, B \rightarrow \perp, B \rightarrow \perp \rightarrow \perp \vdash \perp}{A, B \rightarrow \perp, B \rightarrow \perp \rightarrow \perp \vdash \perp} \star}{\frac{A, A \rightarrow B \rightarrow \perp, B \rightarrow \perp \rightarrow \perp \vdash \perp}{A \rightarrow B \rightarrow \perp \vdash \perp, B \rightarrow \perp \rightarrow \perp \vdash A \rightarrow \perp} \star} \supset_L \quad \frac{A, \perp, B \rightarrow \perp \vdash \perp}{A, B \rightarrow \perp, B \rightarrow \perp \rightarrow \perp \vdash \perp} \star \supset_L$$

Multiplicative translation (96ms)

[illegible]

Girard's Translation (186ms)

[illegible]

Positive decoration (297ms)

[illegible]

0/1 focused decoration (757ms)

[illegible]

$$(15) A \rightarrow B, B \rightarrow A \vdash A \leftrightarrow B$$

LJ (48ms)

$$\frac{\frac{A, A \rightarrow B, B \rightarrow A \vdash A}{A, A \rightarrow B, B \rightarrow A \vdash B} \quad \frac{\overline{A, B, B \rightarrow A \vdash B}^*}{B, A \rightarrow B, B \rightarrow A \vdash B}^* \quad \frac{\overline{B, A \rightarrow B, B \rightarrow A \vdash B}^* \quad \overline{A, B, A \rightarrow B \vdash A}^*}{B, A \rightarrow B, B \rightarrow A \vdash A}^* \quad \frac{A, A \rightarrow B, B \rightarrow A \vdash B \quad B, A \rightarrow B, B \rightarrow A \vdash A}{A \rightarrow B, B \rightarrow A \vdash A \rightarrow B \wedge B \rightarrow A}^* \quad \supset_L \quad \supset_L$$

Multiplicative translation (156ms)

[illegible]

Girard's Translation (181ms)

[illegible]

Positive decoration (527ms)

[illegible]

0/1 focused decoration (331ms)

[illegible]

$$(16) \quad A \leftrightarrow B \vdash A \rightarrow B$$

LJ (34ms)

$$\frac{\frac{A, A \rightarrow B, B \rightarrow A \vdash A}{A, A \rightarrow B, B \rightarrow A \vdash B} \star}{A \rightarrow B \wedge B \rightarrow A \vdash A \rightarrow B} \star \supset_L$$

Multiplicative translation (19ms)

fail

Girard's Translation (91ms)

$$\begin{array}{c}
\frac{A, !(A) \multimap B, !(B) \multimap A : \cdot \Rightarrow A}{A, !(A) \multimap B, !(B) \multimap A : \cdot \vdash A} D_R \quad \frac{A, !(A) \multimap B, !(B) \multimap A : B \Rightarrow B}{A, !(A) \multimap B, !(B) \multimap A : B \vdash B} D_R \\
\frac{A, !(A) \multimap B, !(B) \multimap A : \cdot \Rightarrow !(A)}{A, !(A) \multimap B, !(B) \multimap A : \cdot \vdash !(A)} ! \quad \frac{A, !(A) \multimap B, !(B) \multimap A : \cdot [B] \Leftarrow B}{A, !(A) \multimap B, !(B) \multimap A : \cdot \vdash B} R_L \\
\frac{A, !(A) \multimap B, !(B) \multimap A : \cdot \vdash !(A) \quad A, !(A) \multimap B, !(B) \multimap A : \cdot [B] \Leftarrow B}{A, !(A) \multimap B, !(B) \multimap A : \cdot \vdash B} D_C \\
\frac{A, !(A) \multimap B, !(B) \multimap A : \cdot \vdash B}{\cdot : !(A) \multimap B \ \& \ !(B) \multimap A \vdash !(A) \multimap B} *
\end{array}$$

Positive decoration (139ms)

[illegible]

0/1 focused decoration (137ms)

[illegible]

$$(17) A \leftrightarrow B \vdash B \rightarrow A$$

LJ (35ms)

$$\frac{\frac{B, A \rightarrow B, B \rightarrow A \vdash B}{B, A \rightarrow B, B \rightarrow A \vdash A} \star \quad \frac{A, B, A \rightarrow B \vdash A}{A \rightarrow B \wedge B \rightarrow A \vdash B \rightarrow A} \star}{A \rightarrow B \wedge B \rightarrow A \vdash B \rightarrow A} \supset_L$$

Multiplicative translation (21ms)

fail

Girard's Translation (94ms)

$$\frac{\frac{\frac{B, !(A) \multimap B, !(B) \multimap A : \cdot \Rightarrow B}{B, !(A) \multimap B, !(B) \multimap A : \cdot \vdash B} D_R \quad \frac{B, !(A) \multimap B, !(B) \multimap A : \cdot \Rightarrow !(B)}{B, !(A) \multimap B, !(B) \multimap A : \cdot \vdash !(B)} !}{\frac{B, !(A) \multimap B, !(B) \multimap A : \cdot \vdash B}{B, !(A) \multimap B, !(B) \multimap A : \cdot \vdash !(B)} !} \quad \frac{\frac{B, !(A) \multimap B, !(B) \multimap A : A \Rightarrow A}{B, !(A) \multimap B, !(B) \multimap A : A \vdash A} D_R \quad \frac{B, !(A) \multimap B, !(B) \multimap A : \cdot [A] \Leftarrow A}{B, !(A) \multimap B, !(B) \multimap A : \cdot \vdash A} R_L}{\frac{B, !(A) \multimap B, !(B) \multimap A : \cdot \vdash B}{B, !(A) \multimap B, !(B) \multimap A : \cdot \vdash A} D_C \quad \frac{B, !(A) \multimap B, !(B) \multimap A : \cdot \vdash A}{\vdash : !(A) \multimap B \& !(B) \multimap A \vdash !(B) \multimap A} \star} \multimap$$

Positive decoration (132ms)

$$\frac{\frac{\frac{B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \Rightarrow B}{B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash B} D_R \quad \frac{B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \Rightarrow !(B)}{B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash !(B)} !}{\frac{B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash B}{B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash !(B)} !} \quad \frac{\frac{A, B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \Rightarrow A}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash A} D_R \quad \frac{A, B, !(A) \multimap !(B), !(B) \multimap !(A) : A \vdash A}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot [A] \Leftarrow A} \star}{\frac{A, B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash A}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash A} D_C \quad \frac{A, B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash A}{\vdash : !(A) \multimap !(B) \& !(B) \multimap !(A) \vdash !(B) \multimap !(A)} \star} \multimap$$

0/1 focused decoration (142ms)

$$\frac{\frac{\frac{B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(B) \multimap !(A) : \cdot \Rightarrow B}{B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(B) \multimap !(A) : \cdot \vdash B} D_R \quad \frac{B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(B) \multimap !(A) : \cdot \Rightarrow !(B)}{B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(B) \multimap !(A) : \cdot \vdash !(B)} !}{\frac{B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(B) \multimap !(A) : \cdot \vdash B}{B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(B) \multimap !(A) : \cdot \vdash !(B)} !} \quad \frac{\frac{A, B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(B) \multimap !(A) : \cdot \Rightarrow A}{A, B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(B) \multimap !(A) : \cdot \vdash A} D_R \quad \frac{A, B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(B) \multimap !(A) : A \vdash A}{A, B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(B) \multimap !(A) : \cdot [A] \Leftarrow A} \star}{\frac{A, B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(B) \multimap !(A) : \cdot \vdash A}{A, B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(B) \multimap !(A) : \cdot \vdash A} D_C \quad \frac{A, B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(B) \multimap !(A) : \cdot \vdash A}{\vdash : !(!(A) \multimap !(B)) \& !(!(B) \multimap !(A)) \vdash !(B) \multimap !(A)} \star} \multimap$$

$$(18) A \leftrightarrow B, A \vdash B$$

LJ (34ms)

$$\frac{\frac{A, A \rightarrow B, B \rightarrow A \vdash A}{A, A \rightarrow B, B \rightarrow A \vdash B} \star \quad \frac{A, B, B \rightarrow A \vdash B}{A, A \rightarrow B \wedge B \rightarrow A \vdash B} \star}{A, A \rightarrow B \wedge B \rightarrow A \vdash B} \supset_L$$

Multiplicative translation (24ms)

fail

Girard's Translation (97ms)

$$\frac{\frac{\frac{A, !(A) \multimap B, !(B) \multimap A : \cdot \Rightarrow A}{A, !(A) \multimap B, !(B) \multimap A : \cdot \vdash A} D_R \quad \frac{A, !(A) \multimap B, !(B) \multimap A : \cdot \Rightarrow !(A)}{A, !(A) \multimap B, !(B) \multimap A : \cdot \vdash !(A)} !}{\frac{A, !(A) \multimap B, !(B) \multimap A : \cdot \vdash A}{A, !(A) \multimap B, !(B) \multimap A : \cdot \vdash A} D_C \quad \frac{A, !(A) \multimap B, !(B) \multimap A : \cdot \vdash A}{\vdash : !(A), !(A) \multimap B \& !(B) \multimap A \vdash B} \star} \multimap$$

Positive decoration (115ms)

$$\frac{\frac{\frac{A, B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \Rightarrow A}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash A} D_R \quad \frac{A, B, !(A) \multimap !(B), !(B) \multimap !(A) : A \vdash A}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot [A] \Leftarrow A} \star}{\frac{A, B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash A}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash A} D_C \quad \frac{A, B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash A}{\vdash : !(A), !(A) \multimap B \& !(B) \multimap A \vdash B} \star} \multimap$$

0/1 focused decoration (114ms)

$$\frac{\frac{\frac{A, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(A) \multimap !(B) : \cdot \Rightarrow A}{A, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(A) \multimap !(B) : \cdot \vdash A} D_R \quad \frac{A, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(A) \multimap !(B) : \cdot \Rightarrow !(A)}{A, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(A) \multimap !(B) : \cdot \vdash !(A)} !}{\frac{A, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(A) \multimap !(B) : \cdot \vdash A}{A, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(A) \multimap !(B) : \cdot \vdash A} D_C \quad \frac{A, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(A) \multimap !(B) : \cdot \vdash A}{\vdash : !(A), !(!(A) \multimap !(B)) \& !(!(B) \multimap !(A)) \vdash B} \star} \multimap$$

$$(19) \quad A \leftrightarrow B, B \vdash A$$

LJ (35ms)

$$\frac{\frac{B, A \rightarrow B, B \rightarrow A \vdash B \quad \star \quad A, B, A \rightarrow B \vdash A \quad \star}{B, A \rightarrow B, B \rightarrow A \vdash A} \supset_L}{B, A \rightarrow B \wedge B \rightarrow A \vdash A \quad \star}$$

Multiplicative translation (20ms)

fail

Girard's Translation (92ms)

$$\begin{array}{c}
\frac{B, \mathfrak{!}(A) \multimap B, \mathfrak{!}(B) \multimap A : \cdot \multimap B}{B, \mathfrak{!}(A) \multimap B, \mathfrak{!}(B) \multimap A : \cdot \vdash B} \frac{I}{D_R} \quad \frac{B, \mathfrak{!}(A) \multimap B, \mathfrak{!}(B) \multimap A : A \multimap A}{B, \mathfrak{!}(A) \multimap B, \mathfrak{!}(B) \multimap A : A \vdash A} \frac{I}{D_R} \\
\frac{B, \mathfrak{!}(A) \multimap B, \mathfrak{!}(B) \multimap A : \cdot \multimap \mathfrak{!}(B)}{B, \mathfrak{!}(A) \multimap B, \mathfrak{!}(B) \multimap A : \cdot \vdash \mathfrak{!}(B)} \frac{!}{D_R} \quad \frac{B, \mathfrak{!}(A) \multimap B, \mathfrak{!}(B) \multimap A : \cdot \vdash [A] \Leftarrow A}{B, \mathfrak{!}(A) \multimap B, \mathfrak{!}(B) \multimap A : \cdot \vdash A} \frac{R_L}{-} \\
\frac{B, \mathfrak{!}(A) \multimap B, \mathfrak{!}(B) \multimap A : \cdot \vdash [\mathfrak{!}(B) \multimap A] \Leftarrow A}{B, \mathfrak{!}(A) \multimap B, \mathfrak{!}(B) \multimap A : \cdot \vdash A} \frac{D_C}{\star} \\
\frac{B, \mathfrak{!}(A) \multimap B, \mathfrak{!}(B) \multimap A : \cdot \vdash A}{\cdot : \mathfrak{!}(B), \mathfrak{!}(\mathfrak{!}(A) \multimap B \& \mathfrak{!}(B) \multimap A) \vdash A} \star
\end{array}$$

Positive decoration (113ms)

[illegible]

0/1 focused decoration (115ms)

[illegible]

(20) $\vdash A \leftrightarrow A$

LJ (18ms)

$$\cdot \vdash A \rightarrow A \wedge A \rightarrow A$$

Multiplicative translation (90ms)

$$\frac{\frac{\frac{\vdots : A \Rightarrow A}{\vdots : A \vdash A} D_R \quad \frac{\vdots : A \vdash A}{\vdots : \vdash A \multimap A} \star}{\vdots : \vdash A \multimap A} R_R \quad \frac{\frac{\vdots : A \Rightarrow A}{\vdots : A \vdash A} D_R \quad \frac{\vdots : A \vdash A}{\vdots : \vdash A \multimap A} \star}{\vdots : \vdash A \multimap A} R_R}{\frac{\vdots : \vdash A \multimap A \otimes A \multimap A}{\vdots : \vdash A \multimap A \otimes A \multimap A} D_R} \otimes$$

Girard's Translation (43ms)

$$\frac{\frac{\overline{A : \cdot \Rightarrow A} \quad \overline{A : \cdot \vdash A}}{\cdot : \cdot \vdash !(A) \multimap A \& !(A) \multimap A} \quad \frac{I}{D_R} \quad \frac{\overline{A : \cdot \Rightarrow A} \quad \overline{A : \cdot \vdash A}}{\cdot : \cdot \vdash !(A) \multimap A \& !(A) \multimap A} \quad \frac{I}{D_R} \quad \star$$

Positive decoration (126ms)

$$\frac{\frac{\frac{A : \Rightarrow A}{A : \vdash A} I}{A : \Rightarrow !(A)} D_R \quad !}{\frac{A : \vdash !(A)}{A : \vdash !(A)} D_R} \star \quad \frac{\frac{\frac{A : \Rightarrow A}{A : \vdash A} I}{A : \Rightarrow !(A)} D_R \quad !}{\frac{A : \vdash !(A)}{A : \vdash !(A)} D_R} \star$$

0/1 focused decoration (108ms)

[illegible]

$$(21) A \leftrightarrow B \vdash B \leftrightarrow A$$

LJ (54ms)

$$\frac{\frac{B, A \rightarrow B, B \rightarrow A \vdash B \quad \star}{B, A \rightarrow B, B \rightarrow A \vdash A} \supset_L \quad \frac{A, A \rightarrow B, B \rightarrow A \vdash A \quad \star}{A, A \rightarrow B, B \rightarrow A \vdash B} \supset_L}{A \rightarrow B \wedge B \rightarrow A \vdash B \rightarrow A \wedge A \rightarrow B} \star$$

Multiplicative translation (160ms)

$$\frac{\frac{\frac{\frac{\vdots : B \Rightarrow B}{I} \quad \frac{\vdots : A \Rightarrow A}{D_R} \quad \frac{\vdots : A \vdash A}{R_L}}{\vdots : B [B \multimap A] \Leftarrow A} \multimap \quad \frac{\frac{\vdots : A \Rightarrow A}{I} \quad \frac{\vdots : B \Rightarrow B}{D_R} \quad \frac{\vdots : B \vdash B}{R_L}}{\vdots : A [A \multimap B] \Leftarrow B} \multimap}{\vdots : A \multimap B, B \multimap A \Rightarrow B \multimap A \otimes A \multimap B} \otimes \quad \frac{\vdots : A \multimap B, B \multimap A \vdash B \multimap A \otimes A \multimap B}{\vdots : A \multimap B \otimes B \multimap A \vdash B \multimap A \otimes A \multimap B} \star$$

Girard's Translation (196ms)

$$\frac{\frac{\frac{B, \ulcorner A \urcorner \Rightarrow B, \ulcorner B \urcorner \Rightarrow A : \multimap B}{B, \ulcorner A \urcorner \Rightarrow B, \ulcorner B \urcorner \Rightarrow A : \vdash B} D_R \quad \frac{B, \ulcorner A \urcorner \Rightarrow B, \ulcorner B \urcorner \Rightarrow A : A \multimap A}{B, \ulcorner A \urcorner \Rightarrow B, \ulcorner B \urcorner \Rightarrow A : \vdash A} D_R}{B, \ulcorner A \urcorner \Rightarrow B, \ulcorner B \urcorner \Rightarrow A : \vdash \ulcorner A \urcorner} \vdash \quad \frac{B, \ulcorner A \urcorner \Rightarrow B, \ulcorner B \urcorner \Rightarrow A : \vdash \ulcorner A \urcorner}{B, \ulcorner A \urcorner \Rightarrow B, \ulcorner B \urcorner \Rightarrow A : \vdash A} D_R}{\vdots : \ulcorner A \urcorner \multimap B, \ulcorner B \urcorner \multimap A \Rightarrow \ulcorner B \urcorner \multimap A \otimes \ulcorner A \urcorner \multimap B} \otimes$$

Positive decoration (515ms)

$$\frac{\frac{\frac{\frac{B, \ulcorner A \urcorner \Rightarrow \ulcorner B \urcorner \urcorner \Rightarrow \ulcorner A \urcorner \urcorner : \multimap B}{B, \ulcorner A \urcorner \Rightarrow \ulcorner B \urcorner \urcorner \Rightarrow \ulcorner A \urcorner \urcorner : \vdash B} D_R \quad \frac{B, \ulcorner A \urcorner \Rightarrow \ulcorner B \urcorner \urcorner \Rightarrow \ulcorner A \urcorner \urcorner : A \multimap A}{B, \ulcorner A \urcorner \Rightarrow \ulcorner B \urcorner \urcorner \Rightarrow \ulcorner A \urcorner \urcorner : \vdash A} D_R}{B, \ulcorner A \urcorner \Rightarrow \ulcorner B \urcorner \urcorner \Rightarrow \ulcorner A \urcorner \urcorner : \vdash \ulcorner A \urcorner} \vdash \quad \frac{B, \ulcorner A \urcorner \Rightarrow \ulcorner B \urcorner \urcorner \Rightarrow \ulcorner A \urcorner \urcorner : \vdash \ulcorner A \urcorner}{B, \ulcorner A \urcorner \Rightarrow \ulcorner B \urcorner \urcorner \Rightarrow \ulcorner A \urcorner \urcorner : \vdash A} D_R}{\vdots : \ulcorner A \urcorner \multimap \ulcorner B \urcorner \urcorner, \ulcorner B \urcorner \multimap \ulcorner A \urcorner \urcorner \Rightarrow \ulcorner B \urcorner \multimap \ulcorner A \urcorner \urcorner \otimes \ulcorner A \urcorner \multimap \ulcorner B \urcorner \urcorner} \otimes$$

0/1 focused decoration (1825ms)

$$\frac{\frac{\frac{\frac{B, \ulcorner A \urcorner \Rightarrow \ulcorner B \urcorner \urcorner \Rightarrow \ulcorner A \urcorner \urcorner : \multimap B}{B, \ulcorner A \urcorner \Rightarrow \ulcorner B \urcorner \urcorner \Rightarrow \ulcorner A \urcorner \urcorner : \vdash B} D_R \quad \frac{B, \ulcorner A \urcorner \Rightarrow \ulcorner B \urcorner \urcorner \Rightarrow \ulcorner A \urcorner \urcorner : A \multimap A}{B, \ulcorner A \urcorner \Rightarrow \ulcorner B \urcorner \urcorner \Rightarrow \ulcorner A \urcorner \urcorner : \vdash A} D_R}{B, \ulcorner A \urcorner \Rightarrow \ulcorner B \urcorner \urcorner \Rightarrow \ulcorner A \urcorner \urcorner : \vdash \ulcorner A \urcorner} \vdash \quad \frac{B, \ulcorner A \urcorner \Rightarrow \ulcorner B \urcorner \urcorner \Rightarrow \ulcorner A \urcorner \urcorner : \vdash \ulcorner A \urcorner}{B, \ulcorner A \urcorner \Rightarrow \ulcorner B \urcorner \urcorner \Rightarrow \ulcorner A \urcorner \urcorner : \vdash A} D_R}{\vdots : \ulcorner A \urcorner \multimap \ulcorner B \urcorner \urcorner, \ulcorner B \urcorner \multimap \ulcorner A \urcorner \urcorner \Rightarrow \ulcorner B \urcorner \multimap \ulcorner A \urcorner \urcorner \otimes \ulcorner A \urcorner \multimap \ulcorner B \urcorner \urcorner} \otimes$$

$$(22) A \leftrightarrow B, B \leftrightarrow C \vdash A \leftrightarrow C$$

LJ (150ms)

$$\frac{\frac{\frac{A, B \leftrightarrow B, B \leftrightarrow C \vdash A \leftrightarrow B \quad \star}{A, B \leftrightarrow B, B \leftrightarrow C \vdash A} \supset_L \quad \frac{A, B \leftrightarrow B, B \leftrightarrow C \vdash A \leftrightarrow B \quad \star}{A, B \leftrightarrow B, B \leftrightarrow C \vdash B} \supset_L}{A \leftrightarrow B, B \leftrightarrow C \vdash A \leftrightarrow C} \star$$

Multiplicative translation (228ms)

$$\frac{\frac{\frac{\frac{\vdots : C \Rightarrow C}{I} \quad \frac{\vdots : C \vdash C}{D_R} \quad \frac{\vdots : C \Leftarrow C}{R_L}}{\vdots : B \Rightarrow B} I \quad \frac{\vdots : B \Rightarrow B}{I} \quad \frac{\vdots : A \Rightarrow A}{D_R} \quad \frac{\vdots : A \vdash A}{R_L}}{\vdots : B [B \multimap A] \Leftarrow A} \multimap \quad \frac{\frac{\vdots : A \Rightarrow A}{I} \quad \frac{\vdots : B \Rightarrow B}{D_R} \quad \frac{\vdots : B \vdash B}{R_L}}{\vdots : A [A \multimap B] \Leftarrow B} \multimap}{\vdots : A \multimap B, B \multimap A \Rightarrow B \multimap A \otimes A \multimap B} \otimes \quad \frac{\vdots : A \multimap B, B \multimap A \vdash B \multimap A \otimes A \multimap B}{\vdots : A \multimap B \otimes B \multimap A \vdash B \multimap A \otimes A \multimap B} \star$$

Girard's Translation (Timeout!)

Timeout!

Positive decoration (Timeout!)

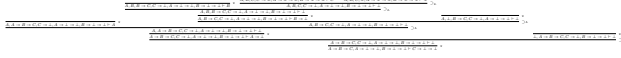
Timeout!

0/1 focused decoration (Timeout!)

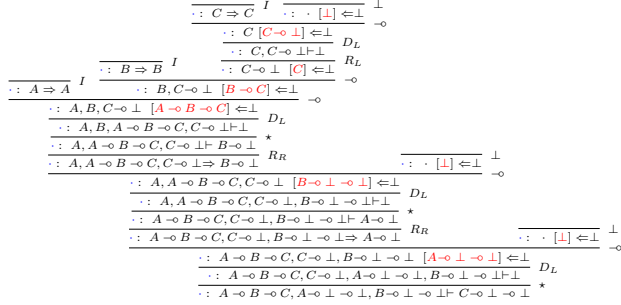
Timeout!

$$(23) A \rightarrow (B \rightarrow C), \neg\neg A, \neg\neg B \vdash \neg\neg C$$

LJ (2318ms)



Multiplicative translation (164ms)



Girard's Translation (Timeout!)

Timeout!

Positive decoration (Timeout!)

Timeout!

0/1 focused decoration (Timeout!)

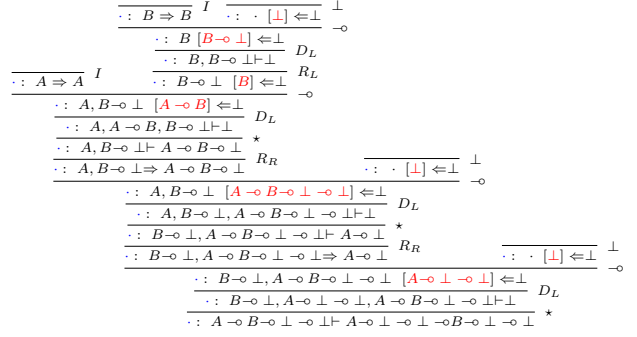
Timeout!

$$(24) \neg\neg(A \rightarrow B) \vdash \neg\neg A \rightarrow \neg\neg B$$

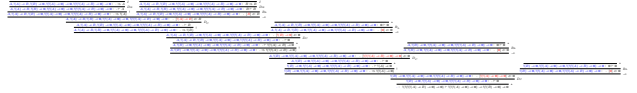
LJ (211ms)



Multiplicative translation (142ms)



Girard's Translation (4694ms)



Positive decoration (5522ms)

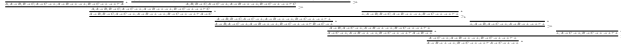


0/1 focused decoration (Timeout!)

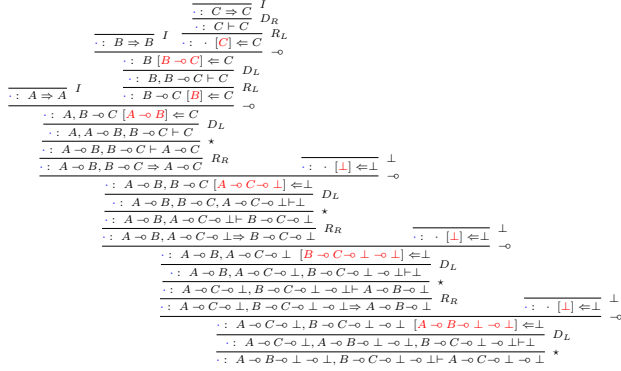
Timeout!

$$(25) \neg\neg(A \rightarrow B), \neg\neg(B \rightarrow C) \vdash \neg\neg(A \rightarrow C)$$

LJ (4063ms)



Multiplicative translation (202ms)



Girard's Translation (Timeout!)

Timeout!

Positive decoration (Timeout!)

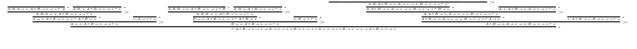
Timeout!

0/1 focused decoration (Timeout!)

Timeout!

$$(26) \cdot \vdash \neg\neg(A \wedge B) \leftrightarrow \neg\neg A \wedge \neg\neg B$$

LJ (140ms)



Multiplicative translation (20ms)

fail

Girard's Translation (27482ms)



Positive decoration (Timeout!)

Timeout!

0/1 focused decoration (Timeout!)

Timeout!

$$(27) \cdot \vdash \neg\neg(A \leftrightarrow B) \leftrightarrow \neg\neg(A \rightarrow B) \wedge \neg\neg(B \rightarrow A)$$

LJ (Timeout!)

Timeout!

Multiplicative translation (23ms)

fail

Girard's Translation (Timeout!)

Timeout!

Positive decoration (**Timeout!**)

Timeout!

0/1 focused decoration (**Timeout!**)

Timeout!

(28) $A \leftrightarrow B \vdash A \rightarrow C \leftrightarrow B \rightarrow C$

LJ (86ms)

$$\frac{\frac{B, A \rightarrow B, A \rightarrow C, B \rightarrow A \supset B \quad A, B, A \rightarrow B, A \rightarrow C \supset A}{B, A \rightarrow B, A \rightarrow C, B \rightarrow A \supset A} \supset_L \quad \frac{B, C, A \rightarrow B, B \rightarrow A \supset C}{A, A \rightarrow B, B \rightarrow A, B \rightarrow C \supset A} * \quad \frac{A, B, B \rightarrow A, B \rightarrow C \supset B \quad A, B, C, B \rightarrow A \supset C}{A, B, B \rightarrow A, B \rightarrow C \supset C} *}{\frac{B, A \rightarrow B, A \rightarrow C, B \rightarrow A \supset C \quad A, A \rightarrow B, B \rightarrow A, B \rightarrow C \supset C}{A \rightarrow B \wedge B \rightarrow A \supset A \rightarrow C \rightarrow B \rightarrow C \wedge B \rightarrow C \rightarrow A \rightarrow C} *} \supset_L$$

Multiplicative translation (227ms)

$$\begin{array}{c}
\frac{\vdots \quad C \Rightarrow C}{\vdots \quad C \vdash C} I_{DR} \quad \frac{\vdots \quad C \Rightarrow C}{\vdots \quad C \vdash C} I_{DR} \\
\vdots \quad A \Rightarrow A \quad I \quad \vdots \quad [C] \Leftarrow C \quad R_L \quad \vdots \quad B \Rightarrow B \quad I \quad \vdots \quad [C] \Leftarrow C \quad R_L \\
\vdots \quad A \vdash [A \multimap C] \Leftarrow C \quad D_L \quad \vdots \quad B \vdash [B \multimap C] \Leftarrow C \quad D_L \\
\vdots \quad A, A \multimap C \vdash C \quad \vdots \quad B, B \multimap C \vdash C \\
\vdots \quad B \Rightarrow B \quad I \quad \vdots \quad A \multimap C [A] \Leftarrow C \quad R_L \quad \vdots \quad A \Rightarrow A \quad I \quad \vdots \quad B \multimap C [B] \Leftarrow C \quad R_L \\
\vdots \quad B, A \multimap C [B \multimap A] \Leftarrow C \quad D_L \quad \vdots \quad A, B \multimap C [A \multimap B] \Leftarrow C \quad D_L \\
\vdots \quad B, A \multimap C, B \multimap A \vdash C \quad \vdots \quad A, A \multimap B, B \multimap C \vdash C \\
\vdots \quad B \multimap A \vdash A \multimap C \multimap B \multimap C \quad * \quad \vdots \quad A \multimap B \vdash B \multimap C \multimap A \multimap C \quad * \\
\vdots \quad B \multimap A \Rightarrow A \multimap C \multimap B \multimap C \quad R_R \quad \vdots \quad A \multimap B \Rightarrow B \multimap C \multimap A \multimap C \quad R_R \\
\vdots \quad A \multimap B, B \multimap A \Rightarrow A \multimap C \multimap B \multimap C \otimes B \multimap C \multimap A \multimap C \quad \otimes \\
\vdots \quad A \multimap B, B \multimap A \vdash A \multimap C \multimap B \multimap C \otimes B \multimap C \multimap A \multimap C \quad D_R \\
\vdots \quad A \multimap B \otimes B \multimap A \vdash A \multimap C \multimap B \multimap C \otimes B \multimap C \multimap A \multimap C \quad *
\end{array}$$

Girard's Translation (Timeout!)

Timeout!

Positive decoration (**Timeout!**)

Timeout!

0/1 focused decoration (**Timeout!**)

Timeout!

$$(30) A \leftrightarrow B \vdash A \wedge C \leftrightarrow B \wedge C$$

LJ (48ms)

$$\frac{\frac{A, C, A \rightarrow B, B \rightarrow A \vdash A}{A, C, A \rightarrow B, B \rightarrow A \vdash B}^* \quad \frac{A, B, C, B \rightarrow A \vdash B}{B, C, A \rightarrow B, B \rightarrow A \vdash A}^*}{\frac{A, C, A \rightarrow B, B \rightarrow A \vdash B}{B, C, A \rightarrow B, B \rightarrow A \vdash A} \supset_L} \supset_L \frac{B, C, A \rightarrow B, B \rightarrow A \vdash B}{B, C, A \rightarrow B, B \rightarrow A \vdash A}^* \supset_L \frac{A, C, A \rightarrow B, B \rightarrow A \vdash A}{A \rightarrow B \wedge B \rightarrow A \vdash A \wedge C \rightarrow B \wedge C \wedge A \wedge C \rightarrow A \wedge C}^*$$

Multiplicative translation (191ms)

$$\begin{array}{c}
\frac{\vdash : B \Rightarrow B \quad I \quad \vdash : C \Rightarrow C}{\vdash : B, C \Rightarrow B \otimes C} I \otimes \\
\vdash : B, C \Rightarrow B \otimes C \\
\vdash : B, C \vdash B \otimes C \quad D_R \\
\vdash : A \Rightarrow A \quad I \quad \vdash : C [B] \Leftarrow B \otimes C \quad R_L \\
\vdash : A \Rightarrow A \quad I \quad \vdash : C [B] \Leftarrow B \otimes C \quad \neg \\
\vdash : A, C [A \Rightarrow B] \Leftarrow B \otimes C \\
\vdash : A, C, A \Rightarrow B \vdash B \otimes C \quad D_L \\
\vdash : A \Rightarrow B \vdash A \otimes C \Rightarrow B \otimes C \quad * \\
\vdash : A \Rightarrow B \vdash A \otimes C \Rightarrow B \otimes C \quad R_R \\
\vdash : A \Rightarrow B, B \Rightarrow A \vdash A \otimes C \Rightarrow B \otimes C \otimes B \otimes C \Rightarrow A \otimes C \\
\vdash : A \Rightarrow B, B \Rightarrow A \vdash A \otimes C \Rightarrow B \otimes C \otimes B \otimes C \Rightarrow A \otimes C \quad D_R \\
\vdash : A \Rightarrow B \otimes B \Rightarrow A \vdash A \otimes C \Rightarrow B \otimes C \otimes B \otimes C \Rightarrow A \otimes C \quad *
\end{array}$$

Girard's Translation (292ms)

$A, C(A) \rightarrow B \text{ 真值表 } \rightarrow A \rightarrow B$ D_1	$A, C(A) \rightarrow B \text{ 真值表 } \rightarrow A \rightarrow B$ D_2	$A, C(A) \rightarrow B \text{ 真值表 } \rightarrow A \rightarrow B$ D_3	$A, C(A) \rightarrow B \text{ 真值表 } \rightarrow A \rightarrow B$ D_4
$A, C(A) \rightarrow B \text{ 真值表 } \rightarrow A \rightarrow B$ D_5	$A, C(A) \rightarrow B \text{ 真值表 } \rightarrow A \rightarrow B$ D_6	$A, C(A) \rightarrow B \text{ 真值表 } \rightarrow A \rightarrow B$ D_7	$A, C(A) \rightarrow B \text{ 真值表 } \rightarrow A \rightarrow B$ D_8
$\vdash (A \rightarrow B) \rightarrow (A \rightarrow B)$			

Positive decoration (3424ms)

The diagram illustrates the decomposition of a polynomial P into a sum of squares (SOS) of other polynomials. It is organized into several horizontal layers, each containing polynomial expressions and their corresponding SOS representations. The polynomials are labeled with subscripts like P_1, P_2, P_3 , etc., and the SOS representations are labeled with subscripts like Q_1, Q_2, Q_3 , etc. The diagram shows the process of finding a certificate of non-negativity for a polynomial by expressing it as a sum of squares of other polynomials.

Layer 1 (Top):

- $P_1 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_2 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_3 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$

Layer 2:

- $P_1 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_2 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_3 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$

Layer 3:

- $P_1 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_2 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_3 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$

Layer 4:

- $P_1 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_2 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_3 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$

Layer 5:

- $P_1 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_2 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_3 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$

Layer 6:

- $P_1 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_2 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_3 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$

Layer 7:

- $P_1 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_2 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_3 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$

Layer 8:

- $P_1 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_2 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_3 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$

Layer 9:

- $P_1 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_2 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_3 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$

Layer 10:

- $P_1 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_2 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_3 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$

Layer 11:

- $P_1 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_2 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_3 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$

Layer 12:

- $P_1 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_2 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_3 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$

Layer 13:

- $P_1 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_2 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_3 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$

Layer 14:

- $P_1 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_2 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_3 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$

Layer 15:

- $P_1 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_2 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_3 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$

Layer 16:

- $P_1 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_2 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_3 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$

Layer 17:

- $P_1 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_2 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_3 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$

Layer 18:

- $P_1 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_2 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_3 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$

Layer 19:

- $P_1 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_2 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_3 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$

Layer 20:

- $P_1 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_2 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_3 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$

Layer 21:

- $P_1 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_2 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_3 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$

Layer 22:

- $P_1 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_2 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_3 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$

Layer 23:

- $P_1 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_2 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_3 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$

Layer 24:

- $P_1 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_2 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_3 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$

Layer 25:

- $P_1 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_2 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$
- $P_3 = x^4 + y^4 + z^4 - x^2y^2 - y^2z^2 - z^2x^2$

Layer 26:

- $P_1 = x^4 + y^4 + z^4 - x^2y^2 - y^2z$

0/1 focused decoration (**Timeout!**)

Timeout!

(31) $A \leftrightarrow B \vdash C \wedge A \leftrightarrow C \wedge B$

LJ (54ms)

$$\frac{\frac{\frac{A, C, A \rightarrow B, B \rightarrow A \vdash A}{A, C, A \rightarrow B, B \rightarrow A \vdash B}^* \quad \frac{A, B, C, B \rightarrow A \vdash B}{B, C, A \rightarrow B, B \rightarrow A \vdash A}^*}{A \rightarrow B \wedge B \rightarrow A \vdash C \wedge A \rightarrow C \wedge B \wedge C \wedge B \rightarrow C \wedge A}^* \supset_L \supset_L$$

Multiplicative translation (209ms)

$$\begin{array}{c}
\frac{\vdash C \Rightarrow C \quad I \quad \vdash B \Rightarrow B}{\vdash B, C \Rightarrow C \otimes B} \otimes \\
\vdash B, C \Rightarrow C \otimes B \quad D_R \\
\vdash B, C \Rightarrow C \otimes B \quad D_R \\
\vdash A \Rightarrow A \quad I \quad \vdash C [B] \Leftarrow C \otimes B \quad R_L \\
\vdash A \Rightarrow A \quad I \quad \vdash B \Rightarrow B \quad I \quad \vdash C [A] \Leftarrow C \otimes A \quad R_L \\
\vdash A, C [A \Rightarrow B] \Leftarrow C \otimes B \quad D_L \\
\vdash A, C, A \Rightarrow B \Leftarrow C \otimes B \quad D_L \\
\vdash A \Rightarrow B \Leftarrow C \otimes A \Rightarrow C \otimes B \quad R_R \\
\vdash A \Rightarrow B \Leftarrow C \otimes A \Rightarrow C \otimes B \quad R_R \\
\vdash A \Rightarrow B, B \Rightarrow A \Rightarrow C \otimes A \Rightarrow C \otimes B \otimes C \otimes B \Leftarrow C \otimes A \quad \otimes \\
\vdash A \Rightarrow B, B \Rightarrow A \Leftarrow C \otimes A \Rightarrow C \otimes B \otimes C \otimes B \Leftarrow C \otimes A \quad D_R \\
\vdash A \Rightarrow B \otimes B \Rightarrow A \Leftarrow C \otimes A \Rightarrow C \otimes B \otimes C \otimes B \Leftarrow C \otimes A \quad *
\end{array}$$

Girard's Translation (353ms)

Positive decoration (3752ms)

0/1 focused decoration (Timeout!)

Timeout!

(32) $A \leftrightarrow B \vdash \neg A \leftrightarrow \neg B$

LJ (83ms)

[illegible]

Multiplicative translation (202ms)

$$\begin{array}{c}
\frac{\vdots, A \Rightarrow A \quad I \quad \vdots, [\perp] \Leftarrow \perp}{\vdots, A [A \Rightarrow \perp] \Leftarrow \perp} \quad \frac{\vdots, B \Rightarrow B \quad I \quad \vdots, [\perp] \Leftarrow \perp}{\vdots, B [B \Rightarrow \perp] \Leftarrow \perp} \quad \frac{\vdots, A, A \Rightarrow \perp \Leftarrow \perp}{\vdots, A \Rightarrow \perp \Leftarrow \perp} \quad D_L \\
\vdots, B \Rightarrow \bar{B} \quad I \quad \vdots, A \Rightarrow \perp \Leftarrow \perp \quad R_L \\
\vdots, B, A \Rightarrow \perp \quad [B \Rightarrow A] \Leftarrow \perp \quad \vdots, A, B \Rightarrow \perp \quad [A \Rightarrow B] \Leftarrow \perp \\
\vdots, B, A \Rightarrow \perp, B \Rightarrow A \Leftarrow \perp \quad \vdots, A, A \Rightarrow B, B \Rightarrow \perp \Leftarrow \perp \\
\vdots, B \Rightarrow A \Leftarrow A \Rightarrow \perp \Rightarrow B \Rightarrow \perp \quad \vdots, A \Rightarrow B \Leftarrow B \Rightarrow \perp \Rightarrow A \Rightarrow \perp \\
\vdots, B \Rightarrow A \Rightarrow A \Rightarrow A \Rightarrow \perp \Rightarrow B \Rightarrow \perp \quad \vdots, A \Rightarrow B \Rightarrow B \Rightarrow \perp \Rightarrow A \Rightarrow \perp \\
\vdots, A \Rightarrow B, B \Rightarrow A \Rightarrow A \Rightarrow \perp \Rightarrow B \Rightarrow \perp \otimes B \Rightarrow \perp \Rightarrow A \Rightarrow \perp \\
\vdots, A \Rightarrow B, B \Rightarrow A \Leftarrow A \Rightarrow \perp \Rightarrow B \Rightarrow \perp \otimes B \Rightarrow \perp \Rightarrow A \Rightarrow \perp \quad D_R \\
\vdots, A \Rightarrow B \otimes B \Rightarrow A \Leftarrow A \Rightarrow \perp \Rightarrow B \Rightarrow \perp \otimes B \Rightarrow \perp \Rightarrow A \Rightarrow \perp \quad *
\end{array}$$

Girard's Translation (12683ms)

Figure 1: A diagram illustrating the construction of the sequence of functions f_n and the corresponding sequence of functions g_n . The diagram shows a sequence of functions f_1, f_2, f_3, \dots and g_1, g_2, g_3, \dots defined on the interval $[0, 1]$. The functions f_n are defined by the recurrence relation $f_{n+1}(x) = f_n(x) + \frac{1}{2^n} \chi_{[0, 1/2^n]}(x)$, where $\chi_{[0, 1/2^n]}(x)$ is the characteristic function of the interval $[0, 1/2^n]$. The functions g_n are defined by the recurrence relation $g_{n+1}(x) = g_n(x) + \frac{1}{2^n} \chi_{[1/2^n, 1/2^{n-1}]}(x)$, where $\chi_{[1/2^n, 1/2^{n-1}]}(x)$ is the characteristic function of the interval $[1/2^n, 1/2^{n-1}]$. The diagram shows that the sequence f_n converges to the function $f(x) = x$ and the sequence g_n converges to the function $g(x) = 1 - x$.

Positive decoration (**Timeout!**)

Timeout!

0/1 focused decoration (**Timeout!**)

Timeout!

$$(34) \cdot \vdash A \wedge B \leftrightarrow B \wedge A$$

LJ (18ms)

$$\overline{\cdot \vdash A \wedge B \rightarrow B \wedge A \wedge B \wedge A \rightarrow A \wedge B}^*$$

Multiplicative translation (131ms)

$$\begin{array}{c}
\frac{\vdots \vdash B \Rightarrow \overline{B} \quad I \quad \vdots \vdash A \Rightarrow \overline{A} \quad I}{\vdots \vdash A, B \Rightarrow \overline{B \otimes A} \quad D_R} \otimes \\
\frac{\vdots \vdash A, B \Rightarrow \overline{B \otimes A} \quad D_R}{\vdots \vdash A, B \vdash \overline{B \otimes A} \quad \star} \\
\frac{\vdots \vdash \vdash A \otimes B \multimap \overline{B \otimes A} \quad \star}{\vdots \vdash \Rightarrow A \otimes B \multimap \overline{B \otimes A} \quad R_R} \\
\frac{\vdots \vdash \Rightarrow A \otimes B \multimap \overline{B \otimes A} \quad R_R \quad \vdots \vdash \Rightarrow A \otimes B \multimap \overline{A \otimes B} \quad R_R}{\vdots \vdash \Rightarrow A \otimes B \multimap \overline{B \otimes A \otimes B \otimes A \otimes B \otimes A \otimes A \otimes B} \quad \otimes} \\
\frac{\vdots \vdash \Rightarrow A \otimes B \multimap \overline{B \otimes A \otimes B \otimes A \otimes B \otimes A \otimes A \otimes B} \quad \otimes}{\vdots \vdash \vdash A \otimes B \multimap \overline{B \otimes A \otimes B \otimes A \otimes B \otimes A \otimes A \otimes B} \quad D_R}
\end{array}$$

Girard's Translation (81ms)

$$\frac{\frac{A, B : \cdot \Rightarrow B}{A, B : \cdot \vdash B} D_R \quad \frac{A, B : \cdot \Rightarrow A}{A, B : \cdot \vdash A} D_R \quad \frac{A, B : \cdot \Rightarrow A}{A, B : \cdot \vdash A} D_R \quad \frac{A, B : \cdot \Rightarrow B}{A, B : \cdot \vdash B} D_R}{\cdot : \cdot \vdash !(A \& B) \multimap B \& A \& !(B \& A) \multimap A \& B} \star$$

Positive decoration (217ms)

[illegible]

0/1 focused decoration (276ms)

[illegible]

LJ (21ms)

LJ (35ms)

$$(40) \quad B \vdash A \wedge B \leftrightarrow A$$

LJ (21ms)

$$\overline{B \vdash A \wedge B \rightarrow A \wedge A \rightarrow A \wedge B}^{\star}$$

Multiplicative translation (22ms)

fail

Girard's Translation (271ms)

$$\frac{\frac{\overline{A, B : \cdot \Rightarrow A}^I}{\overline{A, B : \cdot \vdash A}} D_R \quad \frac{\overline{A, B : \cdot \Rightarrow A}^I}{\overline{A, B : \cdot \vdash A}} D_R \quad \frac{\overline{A, B : \cdot \Rightarrow B}^I}{\overline{A, B : \cdot \vdash B}} D_R}{\cdot : !(B) \vdash !(A \& B) \multimap A \& !(A) \multimap A \& B} \star$$

Positive decoration (595ms)

$$\begin{array}{c}
\frac{\frac{A, B : \multimap A}{A, B : \multimap A} I}{A, B : \multimap A} D_R \quad \frac{\frac{A, B : \multimap A}{A, B : \multimap A} I}{A, B : \multimap A} D_R \quad \frac{\frac{A, B : \multimap A}{A, B : \multimap A} I}{A, B : \multimap A} D_R \\
\frac{\frac{A, B : \multimap A}{A, B : \multimap A} D_R}{A, B : \multimap A} ! \quad \frac{\frac{A, B : \multimap A}{A, B : \multimap A} D_R}{A, B : \multimap A} ! \quad \frac{\frac{A, B : \multimap A}{A, B : \multimap A} D_R}{A, B : \multimap A} ! \\
\frac{\frac{A, B : \multimap A}{A, B : \multimap A} D_R}{A, B : \multimap A} ! \quad \frac{\frac{A, B : \multimap A}{A, B : \multimap A} D_R}{A, B : \multimap A} ! \quad \frac{\frac{A, B : \multimap A}{A, B : \multimap A} D_R}{A, B : \multimap A} ! \\
\frac{\frac{B : \multimap ! (A) \otimes ! (B) \multimap ! (A)}{B : \multimap ! (A) \otimes ! (B) \multimap ! (A)} \star}{B : \multimap ! (A) \otimes ! (B) \multimap ! (A)} ! \quad \frac{\frac{\frac{B : \multimap ! (A) \multimap ! (A) \otimes ! (B)}{B : \multimap ! (A) \multimap ! (A) \otimes ! (B)} \star}{B : \multimap ! (A) \multimap ! (A) \otimes ! (B)} !}{B : \multimap ! (A) \multimap ! (A) \otimes ! (B)} ! \\
\frac{\frac{B : \multimap ! (A) \multimap ! (B) \multimap ! (A) \otimes ! (A) \multimap ! (A) \otimes ! (B)}{B : \multimap ! (A) \multimap ! (B) \multimap ! (A) \otimes ! (A) \multimap ! (A) \otimes ! (B)} \star}{B : \multimap ! (A) \multimap ! (B) \multimap ! (A) \otimes ! (A) \multimap ! (A) \otimes ! (B)} D_R \\
\frac{\frac{B : \multimap ! (A) \multimap ! (B) \multimap ! (A) \otimes ! (A) \multimap ! (A) \otimes ! (B)}{B : \multimap ! (A) \multimap ! (B) \multimap ! (A) \otimes ! (A) \multimap ! (A) \otimes ! (B)} \star}{B : \multimap ! (A) \multimap ! (B) \multimap ! (A) \otimes ! (A) \multimap ! (A) \otimes ! (B)} \star
\end{array}$$

0/1 focused decoration (178ms)

[illegible]

$$(41) \neg B \vdash A \wedge B \leftrightarrow B$$

LJ (33ms)

$$\frac{\frac{B, B \rightarrow \perp \vdash B}{B, B \rightarrow \perp \vdash A} \star}{B \rightarrow \perp \vdash A \wedge B \rightarrow B \wedge B \rightarrow A \wedge B} \star$$

Multiplicative translation (21ms)

fail

Girard's Translation (130ms)

$$\frac{\frac{\frac{B, !!(B) \multimap 0 : \cdot \Rightarrow B}{B, !!(B) \multimap 0 : \cdot \vdash B} I}{B, !!(B) \multimap 0 : \cdot \Rightarrow !!(B)} D_R \quad \frac{B, !!(B) \multimap 0 : \cdot \vdash B}{B, !!(B) \multimap 0 : \cdot \vdash A} \star}{\frac{B, !!(B) \multimap 0 : \cdot \vdash A}{B, !!(B) \multimap 0 : \cdot \vdash A} \star} \multimap$$

Positive decoration (172ms)

$$\frac{\frac{\frac{A, B, !!(B) \multimap 0 : \cdot \Rightarrow B}{A, B, !!(B) \multimap 0 : \cdot \vdash B} I}{A, B, !!(B) \multimap 0 : \cdot \Rightarrow !!(B)} D_R \quad \frac{B, !!(B) \multimap 0 : \cdot \Rightarrow B}{B, !!(B) \multimap 0 : \cdot \vdash B} I}{\frac{A, B, !!(B) \multimap 0 : \cdot \Rightarrow !!(B)}{A, B, !!(B) \multimap 0 : \cdot \vdash !!(B)} \star} \multimap$$

0/1 focused decoration (209ms)

$$\frac{\frac{\frac{B, !!(A), !!(B), !!(B) \multimap !!(0) : \cdot \Rightarrow B}{B, !!(A), !!(B), !!(B) \multimap !!(0) : \cdot \vdash B} I}{!(A), !!(B), !!(B) \multimap !!(0) : \cdot \vdash B} \star}{\frac{!(A), !!(B), !!(B) \multimap !!(0) : \cdot \vdash B}{!(A), !!(B), !!(B) \multimap !!(0) : \cdot \vdash !!(B)} \star} \multimap$$

$$(42) \cdot \vdash A \rightarrow \neg \neg A$$

LJ (41ms)

$$\frac{\frac{A, A \rightarrow \perp \vdash A}{A, A \rightarrow \perp \vdash \perp} \star}{\cdot \vdash A \rightarrow A \rightarrow \perp \rightarrow \perp} \star$$

Multiplicative translation (61ms)

$$\frac{\frac{\cdot : A \Rightarrow A}{\cdot : \cdot [A \multimap \perp] \Leftarrow \perp} I}{\cdot : A, A \multimap \perp \vdash \perp} \multimap \quad \frac{\cdot : A, A \multimap \perp \vdash \perp}{\cdot : \cdot \vdash A \multimap A \multimap \perp \multimap \perp} \star$$

Girard's Translation (83ms)

$$\frac{\frac{A, !(A) \multimap 0 : \cdot \Rightarrow A}{A, !(A) \multimap 0 : \cdot \vdash A} I}{A, !(A) \multimap 0 : \cdot \Rightarrow !(A)} D_R \quad \frac{A, !(A) \multimap 0 : \cdot \vdash A}{A, !(A) \multimap 0 : \cdot \vdash !(A)} \star}{\frac{A, !(A) \multimap 0 : \cdot \vdash !(A)}{A, !(A) \multimap 0 : \cdot \vdash !(A)} \star} \multimap$$

Positive decoration (120ms)

$$\frac{\frac{A, !(A) \multimap 0 : \cdot \Rightarrow A}{A, !(A) \multimap 0 : \cdot \vdash A} I}{A, !(A) \multimap 0 : \cdot \Rightarrow !(A)} D_R \quad \frac{A, !(A) \multimap 0 : \cdot \vdash A}{A, !(A) \multimap 0 : \cdot \vdash !(A)} \star}{\frac{A, !(A) \multimap 0 : \cdot \vdash !(A)}{A, !(A) \multimap 0 : \cdot \vdash !(A)} \star} \multimap$$

0/1 focused decoration (122ms)

$$\frac{\frac{A, !(A) \multimap !(0) : \cdot \Rightarrow A}{A, !(A) \multimap !(0) : \cdot \vdash A} I}{A, !(A) \multimap !(0) : \cdot \Rightarrow !(A)} D_R \quad \frac{A, !(A) \multimap !(0) : \cdot \vdash A}{A, !(A) \multimap !(0) : \cdot \vdash !(A)} \star}{\frac{A, !(A) \multimap !(0) : \cdot \vdash !(A)}{A, !(A) \multimap !(0) : \cdot \vdash !(A)} \star} \multimap$$

$$(45) \cdot \vdash \neg(A \leftrightarrow \neg A)$$

LJ (96ms)

[illegible]

Multiplicative translation (19ms)

fail

Girard's Translation (Timeout!)

Timeout!

Positive decoration (5241ms)

Phylogenetic tree of the 16S rDNA sequences of the 16 bacterial strains. The tree shows relationships between various bacterial species, with some labeled as 'uncultured' or 'strain'. The tree is rooted and shows branching patterns indicating evolutionary relationships.

0/1 focused decoration (Timeout!)

Timeout!

$$(46) \cdot \vdash \neg\neg(\neg\neg A \rightarrow A)$$

LJ (66ms)

$$\begin{array}{c}
\frac{\overline{A, A \rightarrow \perp, A \rightarrow \perp \rightarrow A \rightarrow \perp \rightarrow A \rightarrow \perp \rightarrow A \rightarrow \perp}^* \quad \overline{A, \perp, A \rightarrow \perp \rightarrow \perp \rightarrow \perp}^*}{A, A \rightarrow \perp, A \rightarrow \perp \rightarrow A \rightarrow \perp \rightarrow A \rightarrow \perp \rightarrow A \rightarrow \perp} \supset L \\
\frac{\overline{A, A \rightarrow \perp, A \rightarrow \perp \rightarrow A \rightarrow \perp \rightarrow A \rightarrow \perp \rightarrow A \rightarrow \perp}^*}{\overline{A \rightarrow \perp \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \rightarrow \perp \rightarrow A \rightarrow \perp \rightarrow A \rightarrow \perp}^*} \\
\frac{\overline{A \rightarrow \perp \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \rightarrow \perp \rightarrow A \rightarrow \perp \rightarrow A \rightarrow \perp}^* \quad \overline{\perp, A \rightarrow \perp \rightarrow \perp \rightarrow A \rightarrow \perp \rightarrow A}^*}{\overline{A \rightarrow \perp \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \rightarrow \perp \rightarrow A \rightarrow \perp \rightarrow A \rightarrow \perp}^*} \supset L \\
\frac{\overline{A \rightarrow \perp \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \rightarrow \perp \rightarrow A \rightarrow \perp \rightarrow A \rightarrow \perp}^* \quad \overline{A \rightarrow \perp \rightarrow \perp \rightarrow A \rightarrow \perp \rightarrow A \rightarrow \perp \rightarrow A \rightarrow \perp}^*}{\overline{A \rightarrow \perp \rightarrow \perp \rightarrow A \rightarrow \perp \rightarrow A \rightarrow \perp \rightarrow A \rightarrow \perp}^*} \\
\frac{\overline{A \rightarrow \perp \rightarrow \perp \rightarrow A \rightarrow \perp \rightarrow A \rightarrow \perp \rightarrow A \rightarrow \perp}^* \quad \overline{\perp \rightarrow A \rightarrow \perp \rightarrow \perp \rightarrow A \rightarrow \perp \rightarrow \perp}^*}{\overline{\perp \rightarrow A \rightarrow \perp \rightarrow \perp \rightarrow A \rightarrow \perp \rightarrow \perp}^*} \supset L
\end{array}$$

Multiplicative translation (25ms)

fail

Girard's Translation (185ms)

[illegible]

Positive decoration (250ms)

[illegible]

0/1 focused decoration (427ms)

$$(47) \cdot \vdash A \wedge (B \wedge \neg B) \leftrightarrow B \wedge \neg B$$

LJ (61ms)

$$\frac{\frac{A, B, B \rightarrow \perp \vdash B}{A, B, B \rightarrow \perp \vdash \perp} \star \quad \frac{A, B, \perp \vdash \perp}{A, B, B \rightarrow \perp \vdash A} \supset_L \quad \frac{B, B \rightarrow \perp \vdash B}{B, B \rightarrow \perp \vdash A} \star \quad \frac{B, \perp \vdash A}{B, B \rightarrow \perp \vdash A} \supset_L \quad \frac{B, B \rightarrow \perp \vdash B}{B, B \rightarrow \perp \vdash \perp} \star \quad \frac{B, \perp \vdash \perp}{B, B \rightarrow \perp \vdash \perp} \star}{\vdash A \wedge B \wedge B \rightarrow \perp \rightarrow B \wedge B \rightarrow \perp \wedge B \wedge B \rightarrow \perp \rightarrow A \wedge B \wedge B \rightarrow \perp} \star$$

Multiplicative translation (19ms)

fail

Girard's Translation (234ms)

$$\frac{\frac{\frac{A, B, \neg(B) \rightarrow 0 \rightarrow B}{A, B, \neg(B) \rightarrow 0 \rightarrow \neg(B)} \star \quad \frac{A, B, \neg(B) \rightarrow 0 \rightarrow \neg(B)}{A, B, \neg(B) \rightarrow 0 \rightarrow \neg(B)} \star}{\vdash A \wedge B \wedge B \rightarrow \perp \rightarrow B \wedge B \rightarrow \perp \wedge B \wedge B \rightarrow \perp \rightarrow A \wedge B \wedge B \rightarrow \perp} \star$$

Positive decoration (186ms)

$$\frac{\frac{A, B, \neg(B) \rightarrow 0 \rightarrow B}{A, B, \neg(B) \rightarrow 0 \rightarrow \neg(B)} \star \quad \frac{A, B, \neg(B) \rightarrow 0 \rightarrow \neg(B)}{A, B, \neg(B) \rightarrow 0 \rightarrow \neg(B)} \star}{\vdash A \wedge B \wedge B \rightarrow \perp \rightarrow B \wedge B \rightarrow \perp \wedge B \wedge B \rightarrow \perp \rightarrow A \wedge B \wedge B \rightarrow \perp} \star$$

0/1 focused decoration (875ms)

$$\frac{\frac{\frac{A, B, \neg(B) \rightarrow 0 \rightarrow B}{A, B, \neg(B) \rightarrow 0 \rightarrow \neg(B)} \star \quad \frac{A, B, \neg(B) \rightarrow 0 \rightarrow \neg(B)}{A, B, \neg(B) \rightarrow 0 \rightarrow \neg(B)} \star}{\vdash A \wedge B \wedge B \rightarrow \perp \rightarrow B \wedge B \rightarrow \perp \wedge B \wedge B \rightarrow \perp \rightarrow A \wedge B \wedge B \rightarrow \perp} \star$$

$$(48) \cdot \vdash (A \rightarrow B) \rightarrow \neg(A \wedge \neg B)$$

LJ (48ms)

$$\frac{\frac{A, A \rightarrow B, B \rightarrow \perp \vdash A}{A, A \rightarrow B, B \rightarrow \perp \vdash \perp} \star \quad \frac{A, B, B \rightarrow \perp \vdash B}{A, B, B \rightarrow \perp \vdash \perp} \star \quad \frac{A, B, B \rightarrow \perp \vdash \perp}{A, A \rightarrow B, B \rightarrow \perp \vdash \perp} \star}{\vdash A \rightarrow B \rightarrow A \wedge B \rightarrow \perp \rightarrow \perp} \star$$

Multiplicative translation (94ms)

$$\frac{\frac{\vdash B \Rightarrow B}{\vdash B \Rightarrow B} I \quad \frac{\vdash \cdot [\perp] \Leftarrow \perp}{\vdash \cdot [\perp] \Leftarrow \perp} \perp}{\vdash B \Rightarrow B \Rightarrow \perp} \neg \quad \frac{\vdash B \Rightarrow B \Rightarrow \perp}{\vdash B \Rightarrow B \Rightarrow \perp} D_L \quad \frac{\vdash B \Rightarrow B \Rightarrow \perp}{\vdash B \Rightarrow B \Rightarrow \perp} R_L}{\vdash A \Rightarrow A} I \quad \frac{\vdash B \Rightarrow B \Rightarrow \perp}{\vdash B \Rightarrow B \Rightarrow \perp} \neg \quad \frac{\vdash A, B \Rightarrow \perp \Rightarrow [A \rightarrow B] \Leftarrow \perp}{\vdash A, A \rightarrow B, B \Rightarrow \perp \Rightarrow \perp} D_L}{\vdash \cdot \vdash A \rightarrow B \rightarrow A \otimes B \rightarrow \perp \rightarrow \perp} \star$$

Girard's Translation (146ms)

$$\frac{\frac{A, \neg(A) \rightarrow B, \neg(B) \rightarrow 0 \rightarrow \neg(A)}{A, \neg(A) \rightarrow B, \neg(B) \rightarrow 0 \rightarrow \neg(A)} \star \quad \frac{A, \neg(A) \rightarrow B, \neg(B) \rightarrow 0 \rightarrow \neg(A)}{A, \neg(A) \rightarrow B, \neg(B) \rightarrow 0 \rightarrow \neg(A)} \star}{\vdash A \wedge B \wedge B \rightarrow \perp \rightarrow B \wedge B \rightarrow \perp \wedge B \wedge B \rightarrow \perp \rightarrow A \wedge B \wedge B \rightarrow \perp} \star$$

Positive decoration (181ms)

$$\frac{\frac{A, \neg(A) \rightarrow B, \neg(B) \rightarrow 0 \rightarrow \neg(A)}{A, \neg(A) \rightarrow B, \neg(B) \rightarrow 0 \rightarrow \neg(A)} \star \quad \frac{A, \neg(A) \rightarrow B, \neg(B) \rightarrow 0 \rightarrow \neg(A)}{A, \neg(A) \rightarrow B, \neg(B) \rightarrow 0 \rightarrow \neg(A)} \star}{\vdash A \wedge B \wedge B \rightarrow \perp \rightarrow B \wedge B \rightarrow \perp \wedge B \wedge B \rightarrow \perp \rightarrow A \wedge B \wedge B \rightarrow \perp} \star$$

0/1 focused decoration (370ms)

$$\frac{\frac{\frac{A, \neg(A) \rightarrow B, \neg(B) \rightarrow 0 \rightarrow \neg(A)}{A, \neg(A) \rightarrow B, \neg(B) \rightarrow 0 \rightarrow \neg(A)} \star \quad \frac{A, \neg(A) \rightarrow B, \neg(B) \rightarrow 0 \rightarrow \neg(A)}{A, \neg(A) \rightarrow B, \neg(B) \rightarrow 0 \rightarrow \neg(A)} \star}{\vdash A \wedge B \wedge B \rightarrow \perp \rightarrow B \wedge B \rightarrow \perp \wedge B \wedge B \rightarrow \perp \rightarrow A \wedge B \wedge B \rightarrow \perp} \star$$

$$(49) \cdot \vdash (A \rightarrow \neg B) \leftrightarrow (\neg(A \wedge B))$$

LJ (66ms)

$$\frac{\frac{\frac{A, B, A \rightarrow B \rightarrow \perp \vdash A}{A, B, A \rightarrow B \rightarrow \perp \vdash \perp} *}{A, B, A \rightarrow B \rightarrow \perp \vdash \perp} *}{\vdash A \rightarrow B \rightarrow \perp \vdash A \wedge B \rightarrow \perp \wedge A \wedge B \rightarrow \perp \vdash A \rightarrow B \rightarrow \perp} * \supset_L \frac{\frac{\frac{A, B, B \rightarrow \perp \vdash B}{A, B, B \rightarrow \perp \vdash \perp} *}{A, B, B \rightarrow \perp \vdash \perp} *}{A, B, A \wedge B \rightarrow \perp \vdash A \wedge B} * \supset_L \frac{A, B, A \wedge B \rightarrow \perp \vdash A \wedge B}{A, B, A \wedge B \rightarrow \perp \vdash \perp} * \supset_L$$

Multiplicative translation (161ms)

[illegible]

Girard's Translation (204ms)

[illegible]

Positive decoration (535ms)

Figure 1 illustrates the decomposition of the tensor product of two representations of the Lie algebra $\mathfrak{so}(2, 1)$. The diagram is organized into three main sections: (a), (b), and (c). Each section shows a decomposition of the product of two representations into a direct sum of other representations. The representations are labeled with their dimension and the Lie algebra $\mathfrak{so}(2, 1)$.

Section (a) shows the decomposition of the product of two representations $\mathfrak{so}(2, 1) \otimes \mathfrak{so}(2, 1)$ into a direct sum of representations $\mathfrak{so}(2, 1) \oplus \mathfrak{so}(2, 1) \oplus \mathfrak{so}(2, 1)$. The representations are labeled with their dimension and the Lie algebra $\mathfrak{so}(2, 1)$.

Section (b) shows the decomposition of the product of two representations $\mathfrak{so}(2, 1) \otimes \mathfrak{so}(2, 1)$ into a direct sum of representations $\mathfrak{so}(2, 1) \oplus \mathfrak{so}(2, 1) \oplus \mathfrak{so}(2, 1)$. The representations are labeled with their dimension and the Lie algebra $\mathfrak{so}(2, 1)$.

Section (c) shows the decomposition of the product of two representations $\mathfrak{so}(2, 1) \otimes \mathfrak{so}(2, 1)$ into a direct sum of representations $\mathfrak{so}(2, 1) \oplus \mathfrak{so}(2, 1) \oplus \mathfrak{so}(2, 1)$. The representations are labeled with their dimension and the Lie algebra $\mathfrak{so}(2, 1)$.

0/1 focused decoration (46292ms)

Figure 1 displays three phylogenetic trees (A, B, and C) showing the relationships of the 1000 most abundant OTUs. The OTUs are color-coded by phylum: Bacteroidetes (blue), Proteobacteria (red), Firmicutes (green), and Actinobacteria (yellow). The trees are rooted at the bottom and branch upwards. The OTUs are labeled with their accession numbers and phylum names. The OTUs are grouped into clusters, with some clusters labeled with numbers 1 through 10. The OTUs are also labeled with their relative abundance percentages, ranging from 0.01% to 1.00%.

(50) $\vdash (\neg(A \wedge B)) \leftrightarrow ((\neg\neg A) \rightarrow \neg B)$

LJ (94ms)

[illegible]

Multiplicative translation (245ms)

Figure 1 shows a semantic tableau for the formula $A \supset B$. The tableau is a tree structure with multiple branches. The root node is $A \supset B, I, \perp$. It branches into $A, B \supset A, B, I, \perp$ and $A, B \supset A, B, \perp$. The left branch further branches into $A, B \supset A, B, I, \perp$ and $A, B \supset A, B, \perp$. The right branch branches into $A, B \supset A, B, \perp$ and $A, B \supset A, B, \perp$. The tree continues with various logical rules (D1, D2, R1, R2) and eventually leads to a closed tableau (\perp).

Girard's Translation (2618ms)

[illegible]

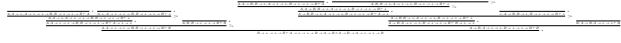
Positive decoration (18580ms)

0/1 focused decoration (Timeout!)

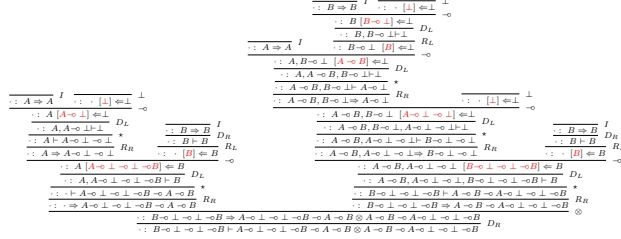
Timeout!

$$(51) \quad \neg\neg B \rightarrow B \vdash (\neg\neg A \rightarrow B) \leftrightarrow (A \rightarrow B)$$

LJ (882ms)



Multiplicative translation (295ms)



Girard's Translation (Timeout!)

Timeout!

Positive decoration (Timeout!)

Timeout!

0/1 focused decoration (Timeout!)

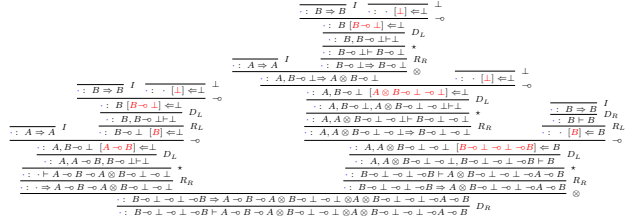
Timeout!

$$(52) \quad \neg\neg B \rightarrow B \vdash (A \rightarrow B) \leftrightarrow \neg(A \wedge \neg B)$$

LJ (112ms)



Multiplicative translation (257ms)



Girard's Translation (Timeout!)

Timeout!

Positive decoration (Timeout!)

Timeout!

0/1 focused decoration (Timeout!)

Timeout!

$$(53) \cdot \vdash (\neg \neg A \rightarrow B) \rightarrow \neg(A \wedge \neg B)$$

LJ (67ms)

$$\frac{\frac{\frac{A, A \rightarrow \perp, B \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \rightarrow B \vdash A}{A, A \rightarrow \perp, B \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \rightarrow B \vdash \perp} \star}{\frac{A, B \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \rightarrow B \vdash A \rightarrow \perp}{A, B \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \rightarrow B \vdash \perp} \star} \supset_L \frac{\frac{A, \perp, A \rightarrow \perp \rightarrow \perp \rightarrow B \vdash \perp}{\vdash A \rightarrow \perp \rightarrow \perp \rightarrow B \rightarrow A \wedge B \rightarrow \perp \rightarrow \perp} \star}{\vdash A \rightarrow \perp \rightarrow \perp \rightarrow B \rightarrow A \wedge B \rightarrow \perp \rightarrow \perp} \star$$

Multiplicative translation (126ms)

$$\frac{\frac{\frac{\vdash : A \Rightarrow A}{\vdash : A [A \multimap \perp] \Leftarrow \perp} I}{\vdash : A, A \multimap \perp \vdash \perp} D_L}{\vdash : A \vdash A \multimap \perp \multimap \perp} \star \frac{\frac{\frac{\vdash : \cdot [\perp] \Leftarrow \perp}{\vdash : B \Rightarrow B} I}{\vdash : B [B \multimap \perp] \Leftarrow \perp} I}{\vdash : B, B \multimap \perp \vdash \perp} D_L}{\vdash : B \multimap \perp [B] \Leftarrow \perp} R_L}{\vdash : A, B \multimap \perp [A \multimap \perp \multimap \perp \multimap B] \Leftarrow \perp} D_L}{\vdash : A, B \multimap \perp, A \multimap \perp \multimap \perp \multimap B \vdash \perp} D_L}{\vdash : \vdash A \multimap \perp \multimap \perp \multimap B \multimap A \otimes B \multimap \perp \multimap \perp} \star$$

Girard's Translation (255ms)

$$\frac{\frac{\frac{A, B, !!(A) \multimap !!(B) \multimap 0 \vdash 0}{A, B, !!(A) \multimap !!(B) \multimap 0 \vdash 0} I}{\frac{A, B, !!(A) \multimap !!(B) \multimap 0 \vdash 0}{A, B, !!(A) \multimap !!(B) \multimap 0 \vdash 0} D_R}{\frac{A, B, !!(A) \multimap !!(B) \multimap 0 \vdash 0}{A, B, !!(A) \multimap !!(B) \multimap 0 \vdash 0} D_C} \star$$

Positive decoration (335ms)

$$\frac{\frac{\frac{A, B, !!(A) \multimap !!(B) \multimap 0 \vdash 0}{A, B, !!(A) \multimap !!(B) \multimap 0 \vdash 0} I}{\frac{A, B, !!(A) \multimap !!(B) \multimap 0 \vdash 0}{A, B, !!(A) \multimap !!(B) \multimap 0 \vdash 0} D_R}{\frac{A, B, !!(A) \multimap !!(B) \multimap 0 \vdash 0}{A, B, !!(A) \multimap !!(B) \multimap 0 \vdash 0} D_C} \star$$

0/1 focused decoration (14764ms)

$$\frac{\frac{\frac{A, B, !!(A) \multimap !!(B) \multimap 0 \vdash 0}{A, B, !!(A) \multimap !!(B) \multimap 0 \vdash 0} I}{\frac{A, B, !!(A) \multimap !!(B) \multimap 0 \vdash 0}{A, B, !!(A) \multimap !!(B) \multimap 0 \vdash 0} D_R}{\frac{A, B, !!(A) \multimap !!(B) \multimap 0 \vdash 0}{A, B, !!(A) \multimap !!(B) \multimap 0 \vdash 0} D_C} \star$$

$$(54) \cdot \vdash A \wedge B \rightarrow \neg(A \rightarrow \neg B)$$

LJ (49ms)

$$\frac{\frac{\frac{A, B, A \rightarrow B \rightarrow \perp \vdash A}{A, B, A \rightarrow B \rightarrow \perp \vdash \perp} \star}{\vdash A \wedge B \rightarrow A \rightarrow B \rightarrow \perp \vdash \perp} \star}{\vdash A \wedge B \rightarrow A \rightarrow B \rightarrow \perp \vdash \perp} \star$$

Multiplicative translation (74ms)

$$\frac{\frac{\frac{\vdash : A \Rightarrow A}{\vdash : A [A \multimap \perp] \Leftarrow \perp} I}{\vdash : A, A \multimap \perp \vdash \perp} D_L}{\vdash : A \vdash A \multimap \perp \multimap \perp} \star \frac{\frac{\frac{\vdash : \cdot [\perp] \Leftarrow \perp}{\vdash : B \Rightarrow B} I}{\vdash : B [B \multimap \perp] \Leftarrow \perp} I}{\vdash : B, B \multimap \perp \vdash \perp} D_L}{\vdash : B \multimap \perp [B] \Leftarrow \perp} R_L}{\vdash : A, B \multimap \perp [A \multimap \perp \multimap \perp \multimap B] \Leftarrow \perp} D_L}{\vdash : A, B \multimap \perp, A \multimap \perp \multimap \perp \multimap B \vdash \perp} D_L}{\vdash : \vdash A \otimes B \multimap A \multimap B \multimap \perp \multimap \perp} \star$$

Girard's Translation (115ms)

$$\frac{\frac{\frac{A, B, !!(A) \multimap !!(B) \multimap 0 \vdash 0}{A, B, !!(A) \multimap !!(B) \multimap 0 \vdash 0} I}{\frac{A, B, !!(A) \multimap !!(B) \multimap 0 \vdash 0}{A, B, !!(A) \multimap !!(B) \multimap 0 \vdash 0} D_R}{\frac{A, B, !!(A) \multimap !!(B) \multimap 0 \vdash 0}{A, B, !!(A) \multimap !!(B) \multimap 0 \vdash 0} D_C} \star$$

Positive decoration (170ms)

$$\frac{\frac{\frac{A, B, !!(A) \multimap !!(B) \multimap 0 \vdash 0}{A, B, !!(A) \multimap !!(B) \multimap 0 \vdash 0} I}{\frac{A, B, !!(A) \multimap !!(B) \multimap 0 \vdash 0}{A, B, !!(A) \multimap !!(B) \multimap 0 \vdash 0} D_R}{\frac{A, B, !!(A) \multimap !!(B) \multimap 0 \vdash 0}{A, B, !!(A) \multimap !!(B) \multimap 0 \vdash 0} D_C} \star$$

0/1 focused decoration (268ms)

$$\frac{\frac{\frac{A, B, !!(A) \multimap !!(B) \multimap 0 \vdash 0}{A, B, !!(A) \multimap !!(B) \multimap 0 \vdash 0} I}{\frac{A, B, !!(A) \multimap !!(B) \multimap 0 \vdash 0}{A, B, !!(A) \multimap !!(B) \multimap 0 \vdash 0} D_R}{\frac{A, B, !!(A) \multimap !!(B) \multimap 0 \vdash 0}{A, B, !!(A) \multimap !!(B) \multimap 0 \vdash 0} D_C} \star$$

LJ (49ms)

Multiplicative translation (92ms)

Girard's Translation (136ms)

Positive decoration (187ms)

0/1 focused decoration (345ms)

LJ (64ms)

Multiplicative translation (97ms)

Girard's Translation (181ms)

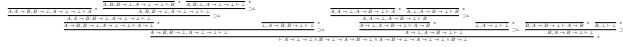
Positive decoration (253ms)

0/1 focused decoration (3946ms)

28

$$(57) \cdot \vdash (\neg\neg A \wedge \neg B) \leftrightarrow (\neg(A \rightarrow B))$$

LJ (385ms)



Multiplicative translation (20ms)

fail

Girard's Translation (**Timeout!**)

Timeout!

Positive decoration (**Timeout!**)

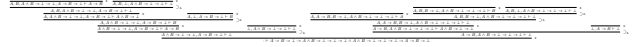
Timeout!

0/1 focused decoration (**Timeout!**)

Timeout!

$$(58) \cdot \vdash (\neg(A \rightarrow B)) \leftrightarrow (\neg\neg(A \wedge \neg B))$$

LJ (118ms)



Multiplicative translation (20ms)

fail

Girard's Translation (**Timeout!**)

Timeout!

Positive decoration (**Timeout!**)

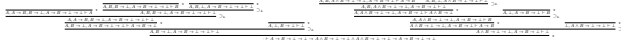
Timeout!

0/1 focused decoration (**Timeout!**)

Timeout!

$$(59) \cdot \vdash (\neg\neg(A \rightarrow B)) \leftrightarrow ((\neg(A \wedge \neg B)))$$

LJ (168ms)



Multiplicative translation (20ms)

fail

Girard's Translation (**Timeout!**)

Timeout!

Positive decoration (**Timeout!**)

Timeout!

0/1 focused decoration (**Timeout!**)

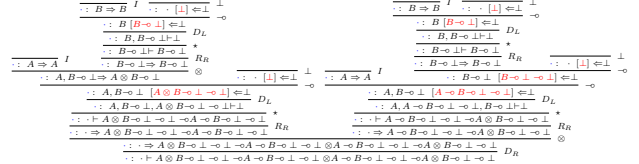
Timeout!

$$(60) \cdot \vdash (\neg(A \wedge \neg B)) \leftrightarrow ((A \rightarrow \neg\neg B))$$

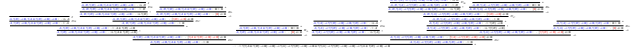
LJ (96ms)



Multiplicative translation (214ms)



Girard's Translation (4004ms)



Positive decoration (8427ms)



0/1 focused decoration (**Timeout!**)

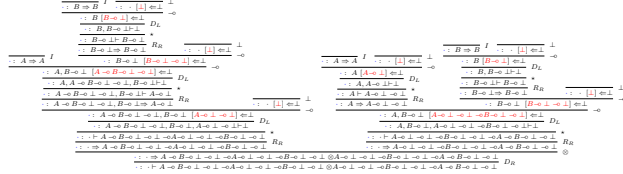
Timeout!

$$(61) \cdot \vdash (A \rightarrow \neg\neg B) \leftrightarrow ((\neg\neg A) \rightarrow \neg\neg B)$$

LJ (9785ms)



Multiplicative translation (288ms)



Girard's Translation (**Timeout!**)

Timeout!

Positive decoration (**Timeout!**)

Timeout!

0/1 focused decoration (**Timeout!**)

Timeout!

2 Alternative Translation

$$(10) \cdot : A \multimap 0 \vdash A \multimap B$$

ILL (106ms)

$$\frac{\frac{\frac{}{\cdot : A \Rightarrow A} I \quad \frac{\frac{}{\cdot : \cdot [0] \Leftarrow B} \star}{\cdot : \cdot [0] \Leftarrow B} R_L}{\cdot : A [A \multimap 0] \Leftarrow B} \multimap \quad \frac{\cdot : A [A \multimap 0] \Leftarrow B}{\cdot : A, A \multimap 0 \vdash B} D_L}{\cdot : A \multimap 0 \vdash A \multimap B} \star$$

$$(11) \cdot : A \vdash (A \multimap 0) \multimap B$$

ILL (105ms)

$$\frac{\frac{\frac{}{\cdot : A \Rightarrow A} I \quad \frac{\frac{}{\cdot : \cdot [0] \Leftarrow B} \star}{\cdot : \cdot [0] \Leftarrow B} R_L}{\cdot : A [A \multimap 0] \Leftarrow B} \multimap \quad \frac{\cdot : A [A \multimap 0] \Leftarrow B}{\cdot : A, A \multimap 0 \vdash B} D_L}{\cdot : A \vdash A \multimap 0 \multimap B} \star$$

$$(12) \cdot : B \vdash (!A) \multimap B$$

ILL (73ms)

$$\frac{\frac{\frac{}{A : B \Rightarrow B} I}{A : B \vdash B} D_R}{\cdot : B \vdash !(A) \multimap B} \star$$

$$(16) \cdot : (A \multimap B) \otimes (B \multimap A) \vdash A \multimap B$$

ILL (131ms)

$$\frac{\frac{\frac{}{B \multimap A : A \Rightarrow A} I \quad \frac{\frac{}{B \multimap A : B \vdash B} D_R}{B \multimap A : B \vdash B} R_L}{B \multimap A : A [A \multimap B] \Leftarrow B} \multimap \quad \frac{B \multimap A : A [A \multimap B] \Leftarrow B}{B \multimap A : A, A \multimap B \vdash B} D_L}{\cdot : A \multimap B \otimes (B \multimap A) \vdash A \multimap B} \star$$

$$(17) \cdot : !(A \multimap B) \otimes (B \multimap A) \vdash B \multimap A$$

ILL (129ms)

$$\frac{\frac{\frac{}{A \multimap B : B \Rightarrow B} I \quad \frac{\frac{}{A \multimap B : A \vdash A} D_R}{A \multimap B : A \vdash A} R_L}{A \multimap B : \cdot [A] \Leftarrow A} \multimap \quad \frac{A \multimap B : B [B \multimap A] \Leftarrow A}{A \multimap B : B, B \multimap A \vdash A} D_L}{\cdot : !(A \multimap B) \otimes B \multimap A \vdash B \multimap A} \star$$

$$(18) \cdot : (A \multimap B), A \vdash B \otimes (B \multimap A)$$

ILL (262ms)

$$\frac{\frac{\frac{\frac{}{A \Rightarrow A} I \quad \frac{\frac{}{B \Rightarrow B} I}{\cdot : B \Rightarrow B} R_L}{\cdot : B \Rightarrow B} \multimap \quad \frac{\frac{}{\cdot : \cdot [A] \Leftarrow A} D_R}{\cdot : \cdot [A] \Leftarrow A} R_L}{\cdot : B [B \multimap A] \Leftarrow A} \multimap \quad \frac{\cdot : B [B \multimap A] \Leftarrow A}{\cdot : B, B \multimap A \vdash A} D_L}{\cdot : B \multimap A \vdash B \multimap A} \star \quad \frac{\frac{}{B \Rightarrow B} I \quad \frac{\frac{}{B \multimap A \Rightarrow B \multimap A} D_R}{B \multimap A \Rightarrow B \multimap A} R_L}{\cdot : B \multimap A \Rightarrow B \otimes B \multimap A} \otimes}{\cdot : A \Rightarrow A} I \quad \frac{\cdot : B \multimap A [B] \Leftarrow B \otimes B \multimap A}{\cdot : A, B \multimap A [A \multimap B] \Leftarrow B \otimes B \multimap A} D_R}{\cdot : A, A \multimap B, B \multimap A \vdash B \otimes B \multimap A} \multimap \quad \frac{\cdot : A, A \multimap B, B \multimap A \vdash B \otimes B \multimap A}{\cdot : A, A \multimap B \otimes B \multimap A \vdash B \otimes B \multimap A} \star$$

$$(19) \cdot : (A \multimap B), B \vdash A \otimes (A \multimap B)$$

ILL (245ms)

$$\frac{\frac{\frac{\frac{}{A \Rightarrow A} I \quad \frac{\frac{}{B \Rightarrow B} I}{\cdot : B \Rightarrow B} D_R}{\cdot : B \Rightarrow B} \multimap \quad \frac{\frac{}{\cdot : \cdot [B] \Leftarrow B} D_R}{\cdot : \cdot [B] \Leftarrow B} R_L}{\cdot : A [A \multimap B] \Leftarrow B} \multimap \quad \frac{\cdot : A [A \multimap B] \Leftarrow B}{\cdot : A, A \multimap B \vdash B} D_L}{\cdot : A \multimap B \vdash A \multimap B} \star \quad \frac{\frac{}{A \Rightarrow A} I \quad \frac{\frac{}{A \multimap B \Rightarrow A \multimap B} D_R}{A \multimap B \Rightarrow A \multimap B} R_L}{\cdot : A \multimap B \Rightarrow A \otimes A \multimap B} \otimes}{\cdot : B \Rightarrow B} I \quad \frac{\cdot : A \multimap B [A] \Leftarrow A \otimes A \multimap B}{\cdot : B, A \multimap B [B \multimap A] \Leftarrow A \otimes A \multimap B} D_R}{\cdot : B, A \multimap B, B \multimap A \vdash A \otimes A \multimap B} \multimap \quad \frac{\cdot : B, A \multimap B, B \multimap A \vdash A \otimes A \multimap B}{\cdot : B, A \multimap B \otimes B \multimap A \vdash A \otimes A \multimap B} \star$$

(26a) $\cdot : \cdot \vdash (\neg\neg(A \& B)) \multimap ((\neg\neg A) \& \neg\neg B)$

ILL (345ms)

$$\begin{array}{lcl}
\vdash A \supset A & I & \vdash \dots, [A] \in \perp \\
\vdash A, A \multimap A \vdash \perp & D_L & \vdash \dots, B \multimap B, [A] \in \perp \\
\vdash A \multimap A, [A] \in \perp & R_L & \vdash \dots, B, B \multimap A \vdash \perp \\
\vdash A \multimap \perp, [A] \in \perp & \&_1 & \vdash \dots, B \multimap \perp, [B] \in \perp \\
\vdash A \multimap A, A \multimap \perp \vdash \perp & D_L & \vdash \dots, B \multimap \perp, [A \& B] \in \perp \\
\vdash A \multimap \perp \& A \& B \multimap \perp & R_R & \vdash \dots, A \& B, B \multimap \perp \vdash \perp \\
\vdash A \multimap \perp \& A \& B \multimap \perp & * & \vdash \dots, B \multimap \perp \& A \& B \multimap \perp \\
\vdash A \multimap \perp, [A \& B \multimap \perp] \in \perp & D_L & \vdash \dots, B \multimap \perp, [A \& B \multimap \perp] \in \perp \\
\vdash A \multimap \perp, A \& B \multimap \perp \vdash \perp & * & \vdash \dots, B \multimap \perp, A \& B \multimap \perp \vdash \perp \\
\vdash A \& B \multimap \perp \vdash \perp \multimap A \multimap \perp \& B \multimap \perp & * &
\end{array}$$

$$(26b) \quad \cdot : \cdot \vdash ((\neg\neg A) \otimes (\neg\neg B)) \multimap \neg\neg(A \otimes B)$$

ILL (234ms)

$$\begin{array}{c}
\frac{\vdots \vdash A \Rightarrow A \quad I \quad \vdots \vdash B \Rightarrow B \quad I}{\vdots \vdash A, B \Rightarrow A \otimes B} \quad \frac{\vdots \vdash \cdot \vdash \perp}{\vdots \vdash \cdot \vdash \perp} \quad \perp \\
\vdots \vdash A, B \vdash [A \otimes B \multimap \perp] \quad \multimap \\
\vdots \vdash A, B, A \otimes B \multimap \perp \vdash \perp \quad \vdots \vdash A, B, A \otimes B \multimap \perp \vdash \perp \quad D_L \\
\vdots \vdash A, A \otimes B \multimap \perp \vdash \perp \quad \vdots \vdash A, A \otimes B \multimap \perp \vdash \perp \quad \star \\
\vdots \vdash A, A \otimes B \multimap \perp \vdash \perp \quad \vdots \vdash A, A \otimes B \multimap \perp \vdash \perp \quad R_R \\
\vdots \vdash A, A \otimes B \multimap \perp \vdash \perp \quad \vdots \vdash A, A \otimes B \multimap \perp \vdash \perp \quad \star \\
\vdots \vdash A, A \otimes B \multimap \perp \vdash \perp \quad \vdots \vdash A, A \otimes B \multimap \perp \vdash \perp \quad D_L \\
\vdots \vdash A, A \otimes B \multimap \perp \vdash \perp \quad \vdots \vdash A, A \otimes B \multimap \perp \vdash \perp \quad \star \\
\vdots \vdash A, A \otimes B \multimap \perp \vdash \perp \quad \vdots \vdash A, A \otimes B \multimap \perp \vdash \perp \quad R_R \\
\vdots \vdash A, A \otimes B \multimap \perp \vdash \perp \quad \vdots \vdash A, A \otimes B \multimap \perp \vdash \perp \quad \star \\
\vdots \vdash A, A \otimes B \multimap \perp \vdash \perp \quad \vdots \vdash A, A \otimes B \multimap \perp \vdash \perp \quad D_L \\
\vdots \vdash A, A \otimes B \multimap \perp \vdash \perp \quad \vdots \vdash A, A \otimes B \multimap \perp \vdash \perp \quad \star
\end{array}$$

$$(27a) \quad \cdot : \cdot \vdash (\neg \neg (! (A \multimap B) \otimes ! (B \multimap A))) \multimap ((\neg \neg (A \multimap B)) \& (\neg \neg (B \multimap A)))$$

ILL (516ms)

[illegible]

$$(27b) \cdot : \cdot \vdash ((\neg\neg(A \multimap B)) \otimes (\neg\neg(B \multimap A))) \multimap (\neg\neg(A \circ\multimap B))$$

ILL (449ms)

[illegible]

$$(35) \cdot : \cdot \vdash ((!A) \otimes (!A)) \multimap !A$$

ILL (270ms)

[illegible]

$$(36) \cdot : A \vdash ((A \multimap B) \multimap B) \otimes (B \multimap (!A) \multimap B)$$

ILL (233ms)

[illegible]

$$(37) \cdot : B \vdash ((!(A \multimap B)) \multimap B) \otimes (B \multimap ((!A) \multimap B))$$

ILL (150ms)

$$\frac{\frac{\frac{A \multimap B : B \Rightarrow B}{A \multimap B : B \vdash B} D_R \quad \frac{\frac{A : B \Rightarrow B}{A : B \vdash B} D_R \quad \frac{\cdot \vdash B \multimap !(A \multimap B) \multimap B}{\cdot \vdash B \multimap !(A \multimap B) \multimap B} \star}{\cdot \vdash B \multimap !(A \multimap B) \multimap B} R_R \quad \frac{\frac{\cdot \vdash B \multimap !(A \multimap B) \multimap B}{\cdot \vdash B \multimap !(A \multimap B) \multimap B} R_R \quad \frac{\cdot \vdash B \multimap !(A \multimap B) \multimap B}{\cdot \vdash B \multimap !(A \multimap B) \multimap B} \star}{\cdot \vdash B \multimap !(A \multimap B) \multimap B \otimes B \multimap !(A \multimap B) \multimap B} \otimes \quad \frac{\cdot \vdash B \multimap !(A \multimap B) \multimap B \otimes B \multimap !(A \multimap B) \multimap B}{\cdot \vdash B \vdash !(A \multimap B) \multimap B \otimes B \multimap !(A \multimap B) \multimap B} D_R$$

(59a) $\cdot : \cdot \vdash (\neg\neg(A \multimap B)) \multimap (\neg(A \otimes (\neg B)))$

ILL (196ms)

$$\begin{array}{c}
\frac{\cdot : B \Rightarrow B}{\cdot : \cdot [\perp] \Leftarrow \perp} I \\
\frac{\cdot : B \Rightarrow B}{\cdot : B [B \multimap \perp] \Leftarrow \perp} D_L \\
\frac{\cdot : B, B \multimap \perp \vdash \perp}{\cdot : B \multimap \perp [B] \Leftarrow \perp} R_L \\
\frac{\cdot : A \Rightarrow A}{\cdot : A, B \multimap \perp [A \multimap B] \Leftarrow \perp} I \\
\frac{\cdot : A, A \multimap B, B \multimap \perp \vdash \perp}{\cdot : A, B \multimap \perp \vdash A \multimap B \multimap \perp} D_L \\
\frac{\cdot : A, B \multimap \perp \vdash A \multimap B \multimap \perp}{\cdot : A, B \multimap \perp \vdash A \multimap B \multimap \perp} R_R \\
\frac{\cdot : A, B \multimap \perp [A \multimap B \multimap \perp \multimap \perp] \Leftarrow \perp}{\cdot : A, B \multimap \perp, A \multimap B \multimap \perp \multimap \perp \vdash \perp} D_L \\
\frac{\cdot : \vdash A \multimap B \multimap \perp \multimap \perp \multimap A \otimes B \multimap \perp \multimap \perp}{\cdot : \vdash A \multimap B \multimap \perp \multimap \perp \multimap A \otimes B \multimap \perp \multimap \perp} *
\end{array}$$

$$(59b) \cdot : \cdot \vdash ((A \otimes \neg B) \multimap 0) \multimap (\neg(!(\neg(!A \multimap B))))$$

ILL (322ms)

[illegible]

ILL (214ms)

35