

Benchmarking Linear Logic: sequents inspired from Kleene's intuitionistic theorems*

October 3, 2018

1 General Information

- Test run on a QEMU Virtual CPU, 2GHz, 64 bits, 2GiB of RAM running Ubuntu.
- Maude version: 2.7.1 built: Oct 2 2017.
- The search procedure considers proofs using up to 4 times the copy rule (focusing on one of the formulas of the classical context). Benchmarks 23 and 25 required the limit to be 5.

2 Translations

$$\boxed{(1) \cdot \vdash A \rightarrow A}$$

LJ (28ms)

$$\cdot \vdash A \rightarrow A \quad *$$

MULTIPLICATIVE encoding (28ms)

$$\frac{\overline{\Gamma : A \vdash A}}{\Gamma : \cdot \vdash A \multimap A} \quad *$$

CALL-BY-NAME encoding (28ms)

$$\frac{\overline{\Gamma : \cdot \vdash A}}{\Gamma : \cdot \vdash !(A) \multimap A} \quad *$$

CALL-BY-VALUE encoding (41ms)

$$\frac{\overline{\Gamma : \cdot \vdash A} \quad !}{\Gamma : \cdot \vdash !(A)} \quad !$$

$$\frac{\Gamma : \cdot \vdash !(A) \multimap !(A)}{\Gamma : \cdot \vdash !(A) \multimap !(A)} \quad !$$

01-ENC encoding (35ms)

$$\frac{\overline{\Gamma : \cdot \vdash A}}{\Gamma : \cdot \vdash !(A) \multimap A} \quad *$$

$$\Gamma : \cdot \vdash !(A) \multimap A \quad !$$

$$\boxed{(2) A \rightarrow B, B \rightarrow C \vdash A \rightarrow C}$$

LJ (46ms)

$$\frac{\overline{A, A \rightarrow B, B \rightarrow C \vdash A} \quad * \quad \frac{\overline{A, B, B \rightarrow C \vdash B} \quad * \quad \overline{A, B, C \vdash C} \quad *}{\overline{A, B, B \rightarrow C \vdash C}} \quad \supset_L}{\overline{A, A \rightarrow B, B \rightarrow C \vdash C}} \quad \supset_L \quad *$$

$$\frac{\overline{A, A \rightarrow B, B \rightarrow C \vdash C}}{A \rightarrow B, B \rightarrow C \vdash A \rightarrow C} \quad *$$

MULTIPLICATIVE encoding (49ms)

$$\frac{\overline{\Gamma : A \vdash A} \quad \frac{\overline{\Gamma : B \vdash B} \quad \overline{\Gamma : C \vdash C}}{\Gamma : B, B \multimap C \vdash C} \quad \multimap}{\Gamma : A, A \multimap B, B \multimap C \vdash C} \quad \multimap}{\Gamma : A \multimap B, B \multimap C \vdash A \multimap C} \quad *$$

CALL-BY-NAME encoding (123ms)

$$\frac{\overline{\Gamma : \cdot \vdash A} \quad ! \quad \overline{\Gamma : B \vdash B}}{\Gamma : \cdot \vdash !(A) \multimap B \vdash B} \quad \multimap \quad D_C$$

$$\frac{\overline{\Gamma : \cdot \vdash B} \quad ! \quad \overline{\Gamma : C \vdash C}}{\Gamma : \cdot \vdash !(B) \multimap C \vdash C} \quad \multimap \quad D_C$$

$$\frac{\Gamma : \cdot \vdash !(A) \multimap B, \Gamma : \cdot \vdash !(B) \multimap C \vdash C}{\Gamma : \cdot \vdash C} \quad D_C$$

$$\Gamma : !(A) \multimap B, !(B) \multimap C \vdash !(A) \multimap C \quad *$$

CALL-BY-VALUE encoding (165ms)

$$\frac{\overline{\Gamma : \cdot \vdash A} \quad ! \quad \overline{\Gamma : \cdot \vdash B}}{\Gamma : \cdot \vdash !(A) \multimap !(B) \vdash B} \quad \multimap \quad D_C \quad \frac{\overline{\Gamma : \cdot \vdash C}}{\Gamma : \cdot \vdash !(C)} \quad !$$

$$\frac{\overline{\Gamma : \cdot \vdash B} \quad ! \quad \overline{\Gamma : \cdot \vdash C}}{\Gamma : \cdot \vdash !(B) \multimap !(C) \vdash C} \quad \multimap \quad D_C$$

$$\frac{\Gamma : \cdot \vdash !(A) \multimap !(B) \vdash B, \Gamma : \cdot \vdash !(B) \multimap !(C) \vdash C}{\Gamma : \cdot \vdash !(A) \multimap !(C)} \quad *$$

$$\Gamma : \cdot \vdash !(A) \multimap !(C) \quad !$$

$$\Gamma : !(A) \multimap !(B), !(B) \multimap !(C) \vdash !(A) \multimap !(C) \quad *$$

01-ENC encoding (146ms)

$$\frac{\overline{\Gamma : \cdot \vdash A} \quad ! \quad \overline{\Gamma : \cdot \vdash B}}{\Gamma : \cdot \vdash !(A) \multimap !(B) \vdash B} \quad \multimap \quad D_C \quad \frac{\overline{\Gamma : \cdot \vdash C}}{\Gamma : \cdot \vdash !(C)} \quad !$$

$$\frac{\overline{\Gamma : \cdot \vdash B} \quad ! \quad \overline{\Gamma : \cdot \vdash C}}{\Gamma : \cdot \vdash !(B) \multimap !(C) \vdash C} \quad \multimap \quad D_C$$

$$\frac{\Gamma : \cdot \vdash !(A) \multimap !(B) \vdash B, \Gamma : \cdot \vdash !(B) \multimap !(C) \vdash C}{\Gamma : \cdot \vdash !(A) \multimap !(C)} \quad *$$

$$\Gamma : \cdot \vdash !(A) \multimap !(C) \quad !$$

$$\Gamma : !(A) \multimap !(B), !(B) \multimap !(C) \vdash !(A) \multimap !(C) \quad *$$

*See <https://github.com/carlosolarte/Benchmarking-Linear-Logic> for details on the encodings used.

(3) $A \rightarrow B \rightarrow C \vdash B \rightarrow A \rightarrow C$

LJ (46ms)

$$\frac{\frac{\frac{A, B, A \rightarrow B \rightarrow C \vdash A}{A, B, A \rightarrow B \rightarrow C \vdash C} \star}{A \rightarrow B \rightarrow C \vdash B \rightarrow A \rightarrow C} \star}{\frac{\frac{A, B, B \rightarrow C \vdash B}{A, B, B \rightarrow C \vdash C} \star}{A, B, C \vdash C} \star} \supset_L$$

MULTIPLICATIVE encoding (47ms)

$$\frac{\frac{\Gamma: A \vdash A \quad \frac{\Gamma: B \vdash B \quad \Gamma: C \vdash C}{\Gamma: B, B \multimap C \vdash C} \multimap}{\Gamma: A, B, A \multimap B \multimap C \vdash C} \multimap}{\Gamma: A \multimap B \multimap C \vdash B \multimap A \multimap C} \star$$

CALL-BY-NAME encoding (71ms)

$$\frac{\frac{\frac{\Gamma; \cdot \vdash A}{\Gamma; \cdot \vdash ! (A)} ! \quad \frac{\frac{\Gamma; \cdot \vdash B}{\Gamma; \cdot \vdash ! (B)} ! \quad \frac{\Gamma; C \vdash C}{\Gamma; ! (B) \multimap C \vdash C} \multimap}{\Gamma; ! (A) \multimap ! (B) \multimap C \vdash C} \multimap \quad D_C}{\Gamma; ! (I(A) \multimap ! (B) \multimap C) \vdash ! (B) \multimap ! (A) \multimap C} \star$$

CALL-BY-VALUE encoding (162ms)

[illegible]

01-ENC encoding (158ms)

$$\begin{array}{c}
\frac{\frac{\Gamma; \cdot \vdash B}{\Gamma; \cdot \vdash \mathbf{!}(B)} \quad \frac{\Gamma; \cdot \vdash C}{\Gamma; \cdot \vdash \mathbf{!}(C) \multimap C} \quad \star}{\Gamma; \mathbf{!}(B) \multimap \mathbf{!}(C) \multimap C} \quad \multimap \\
\frac{\frac{\Gamma; \cdot \vdash A}{\Gamma; \cdot \vdash \mathbf{!}(A)} \quad \frac{\Gamma; \cdot \vdash C}{\Gamma; \cdot \vdash \mathbf{!}(B) \multimap \mathbf{!}(C) \multimap C} \quad D_C}{\Gamma; \mathbf{!}(A) \multimap \mathbf{!}(B) \multimap \mathbf{!}(C) \multimap C} \quad \multimap \\
\frac{\Gamma; \cdot \vdash C}{\Gamma; \cdot \vdash \mathbf{!}(A) \multimap \mathbf{!}(B) \multimap \mathbf{!}(C) \multimap C} \quad D_C \quad \star}{\Gamma; \cdot \vdash \mathbf{!}(A) \multimap \mathbf{!}(B) \multimap \mathbf{!}(C) \multimap C} \quad \star \\
\frac{\Gamma; \cdot \vdash \mathbf{!}(A) \multimap \mathbf{!}(B) \multimap \mathbf{!}(C) \multimap C} \quad \star}{\Gamma; \cdot \vdash \mathbf{!}(A) \multimap \mathbf{!}(B) \multimap \mathbf{!}(A) \multimap \mathbf{!}(C) \multimap C} \quad \star \\
\frac{\Gamma; \cdot \vdash \mathbf{!}(B) \multimap \mathbf{!}(A) \multimap \mathbf{!}(C) \multimap C} \quad \star}{\Gamma; \cdot \vdash \mathbf{!}(B) \multimap \mathbf{!}(B) \multimap \mathbf{!}(A) \multimap \mathbf{!}(C) \multimap C} \quad \star} \\
\frac{\Gamma; \cdot \vdash \mathbf{!}(B) \multimap \mathbf{!}(B) \multimap \mathbf{!}(A) \multimap \mathbf{!}(C) \multimap C} \quad \star}{\Gamma; \mathbf{!}(A) \multimap \mathbf{!}(B) \multimap \mathbf{!}(B) \multimap \mathbf{!}(C) \multimap C} \quad \star}
\end{array}$$

$$(4) \ A \rightarrow B \rightarrow C \vdash A \wedge B \rightarrow C$$

LJ (47ms)

$$\frac{\frac{\overline{A, B, A \rightarrow B \rightarrow C \vdash A}^* \quad \frac{\overline{A, B, B \rightarrow C \vdash B}^* \quad \overline{A, B, C \vdash C}^*}{A, B, B \rightarrow C \vdash C} \supset_L}{\frac{A, B, A \rightarrow B \rightarrow C \vdash C}{A \rightarrow B \rightarrow C \vdash A \wedge B \rightarrow C}^*} \supset_L$$

MULTIPLICATIVE encoding (47ms)

$$\frac{\frac{\Gamma : A \vdash A \quad \frac{\Gamma : B \vdash B \quad \Gamma : C \vdash C}{\Gamma : B, B \multimap C \vdash C} \multimap}{\Gamma : A, B, A \multimap B \multimap C \vdash C} \multimap}{\Gamma : A \multimap B \multimap C \vdash A \otimes B \multimap C} \star$$

CALL-BY-NAME encoding (71ms)

$$\frac{\frac{\frac{\Gamma; \cdot \vdash A}{\Gamma; \cdot \vdash ! (A)}}{\Gamma; \cdot \vdash ! (A)} \quad \frac{\frac{\frac{\Gamma; \cdot \vdash B}{\Gamma; \cdot \vdash ! (B)}}{\Gamma; \cdot \vdash ! (B)} \quad \frac{\Gamma; \cdot \vdash C}{\Gamma; \cdot \vdash C} \quad \multimap}{\frac{\Gamma; \cdot \vdash ! (A) \multimap ! (B) \multimap C \vdash C}{\Gamma; \cdot \vdash ! (A) \multimap ! (B) \multimap C \vdash C} D_C} \quad \multimap \quad \frac{\Gamma; \cdot \vdash C}{\Gamma; \cdot \vdash C} \quad \star}{\Gamma; \cdot \vdash ! (A) \multimap ! (B) \multimap C \vdash ! (A \& B) \multimap C} \star$$

CALL-BY-VALUE encoding (156ms)

$$\begin{array}{c}
\frac{\frac{\frac{\Gamma; \cdot \vdash C}{\Gamma; \cdot \vdash ! (C)} \quad \frac{\Gamma; \cdot \vdash \overline{C}}{\Gamma; \cdot \vdash ! (C)}}{\Gamma; \cdot \vdash ! (C)} \quad \frac{\Gamma; \cdot \vdash B}{\Gamma; \cdot \vdash ! (B)} \quad ! \\
\frac{\Gamma; \cdot \vdash ! (B) \quad \Gamma; \cdot ! (C) \vdash ! (C)}{\Gamma; \cdot ! (B) \multimap ! (C) \vdash ! (C)} \quad ! \\
\frac{\Gamma; \cdot ! (B) \multimap ! (C) \vdash ! (C)}{\Gamma; \cdot \vdash ! (C)} \quad D_C \quad \frac{\Gamma; \cdot \vdash ! (C)}{\Gamma; \cdot \vdash ! (A)} \quad ! \\
\frac{\Gamma; \cdot \vdash ! (A) \quad \Gamma; \cdot ! ! (B) \multimap ! (C) \vdash ! (C)}{\Gamma; \cdot ! (A) \multimap ! ! (B) \multimap ! (C) \vdash ! (C)} \quad ! \\
\frac{\Gamma; \cdot ! (A) \multimap ! ! (B) \multimap ! (C) \vdash ! (C)}{\Gamma; \cdot \vdash ! (C)} \quad D_C \quad \frac{\Gamma; \cdot \vdash ! (C)}{\Gamma; \cdot \vdash ! (A) \otimes ! (B) \multimap ! (C)} \quad * \\
\frac{\Gamma; \cdot \vdash ! (A) \otimes ! (B) \multimap ! (C)}{\Gamma; \cdot \vdash ! ! (A) \otimes ! (B) \multimap ! (C)} \quad ! \\
\frac{\Gamma; \cdot ! ! (A) \otimes ! ! (B) \multimap ! (C) \vdash ! ! (A) \otimes ! (B) \multimap ! (C)}{\Gamma; \cdot ! ! (A) \otimes ! ! (B) \multimap ! (C)} \quad *
\end{array}$$

01-ENC encoding (151ms)

$$\begin{array}{c}
\frac{\frac{\Gamma; \cdot \vdash B}{\Gamma; \cdot \vdash \mathbf{!}(B)} \quad \frac{\Gamma; \cdot \vdash C}{\Gamma; \cdot \vdash \mathbf{!}(C)} \quad \star}{\Gamma; \cdot \vdash \mathbf{!}(B) \multimap \mathbf{!}(C)} \quad \star \\
\frac{\Gamma; \cdot \vdash \mathbf{!}(B) \multimap \mathbf{!}(C)}{\Gamma; \cdot \vdash \mathbf{!}(B)} \quad D_C \\
\frac{\frac{\Gamma; \cdot \vdash A}{\Gamma; \cdot \vdash \mathbf{!}(A)} \quad \frac{\Gamma; \cdot \vdash C}{\Gamma; \cdot \vdash \mathbf{!}(B) \multimap \mathbf{!}(C)} \quad \star}{\Gamma; \cdot \vdash \mathbf{!}(A) \multimap \mathbf{!}(B) \multimap \mathbf{!}(C)} \quad \star \\
\frac{\Gamma; \cdot \vdash \mathbf{!}(A) \multimap \mathbf{!}(B) \multimap \mathbf{!}(C)}{\Gamma; \cdot \vdash \mathbf{!}(A)} \quad D_C \\
\frac{\Gamma; \cdot \vdash C}{\Gamma; \cdot \vdash \mathbf{!}(A) \multimap \mathbf{!}(B) \multimap \mathbf{!}(C)} \quad \star \\
\frac{\Gamma; \cdot \vdash \mathbf{!}(A) \multimap \mathbf{!}(B) \multimap \mathbf{!}(C)}{\Gamma; \cdot \vdash \mathbf{!}(A) \multimap \mathbf{!}(A) \& \mathbf{!}(B)} \quad \star \\
\frac{\Gamma; \cdot \vdash \mathbf{!}(A) \multimap \mathbf{!}(B) \multimap \mathbf{!}(C)}{\Gamma; \cdot \vdash \mathbf{!}(A) \multimap \mathbf{!}(B) \multimap \mathbf{!}(C)} \quad \star
\end{array}$$

$$(5) A \wedge B \rightarrow C \vdash A \rightarrow B \rightarrow C$$

LJ (41ms)

$$\frac{\frac{A, B, A \wedge B \rightarrow C \vdash A \wedge B}{A, B, A \wedge B \rightarrow C \vdash C} \star}{A \wedge B \rightarrow C \vdash A \rightarrow B \rightarrow C} \star \supset_L$$

MULTIPLICATIVE encoding (54ms)

$$\frac{\frac{\frac{\Gamma : A \vdash A}{\Gamma : A, B \vdash A \otimes B} \otimes \frac{\Gamma : C \vdash C}{\Gamma : A, B, A \otimes B \multimap C \vdash C} \multimap}{\Gamma : A \otimes B \multimap C \vdash A \multimap B \multimap C} \star}{\Gamma : A \otimes B \multimap C \vdash A \multimap B \multimap C} \star$$

CALL-BY-NAME encoding (73ms)

$$\frac{\frac{\frac{\Gamma : \cdot \vdash A}{\Gamma : \cdot \vdash A \& B} \star}{\Gamma : \cdot \vdash !(A \& B)} !}{\frac{\Gamma : !(A \& B) \multimap C \vdash C}{\Gamma : \cdot \vdash C} D_C} \multimap \frac{\Gamma : C \vdash C}{\Gamma : !(A \& B) \multimap C \vdash !(A) \multimap !(B) \multimap C} \star$$

CALL-BY-VALUE encoding (205ms)

$$\frac{\frac{\frac{\Gamma : \cdot \vdash A}{\Gamma : \cdot \vdash !(A)} !}{\Gamma : \cdot \vdash !(A) \otimes !(B)} \otimes \frac{\frac{\frac{\Gamma : \cdot \vdash B}{\Gamma : \cdot \vdash !(B)} !}{\Gamma : \cdot \vdash !(C) \vdash !(C)} !}{\Gamma : \cdot \vdash !(C)} !}{\frac{\Gamma : \cdot \vdash !(A) \otimes !(B) \multimap !(C) \vdash !(C)}{\Gamma : \cdot \vdash !(C)} D_C} \star \frac{\Gamma : \cdot \vdash !(B) \multimap !(C)}{\Gamma : \cdot \vdash !(B) \multimap !(C)} !}{\frac{\Gamma : \cdot \vdash !(A) \multimap !(B) \multimap !(C)}{\Gamma : \cdot \vdash !(A) \multimap !(B) \multimap !(C)} !} \star \frac{\Gamma : \cdot \vdash !(A) \multimap !(B) \multimap !(C)}{\Gamma : \cdot \vdash !(A) \multimap !(B) \multimap !(C)} !}{\Gamma : !(A) \otimes !(B) \multimap !(C) \vdash !(A) \multimap !(B) \multimap !(C)} \star$$

01-ENC encoding (201ms)

$$\frac{\frac{\frac{\frac{\Gamma : \cdot \vdash A}{\Gamma : \cdot \vdash A \& B} \star}{\Gamma : \cdot \vdash !(A \& B)} !}{\Gamma : \cdot \vdash !(A \& B) \multimap !(C) \vdash C} \star}{\frac{\Gamma : !(A \& B) \multimap !(C) \vdash C}{\Gamma : \cdot \vdash C} D_C} \star \frac{\Gamma : \cdot \vdash C}{\Gamma : \cdot \vdash !(B) \multimap C} \star}{\frac{\Gamma : \cdot \vdash !(B) \multimap C}{\Gamma : \cdot \vdash !(B) \multimap C} !} \star \frac{\Gamma : \cdot \vdash !(A) \multimap !(B) \multimap C}{\Gamma : \cdot \vdash !(A) \multimap !(B) \multimap C} !}{\Gamma : !(A) \otimes !(B) \multimap !(C) \vdash !(A) \multimap !(B) \multimap C} \star$$

$$(6) A \rightarrow B \vdash B \rightarrow C \rightarrow A \rightarrow C$$

LJ (47ms)

$$\frac{\frac{\frac{A, A \rightarrow B, B \rightarrow C \vdash A}{A, A \rightarrow B, B \rightarrow C \vdash C} \star}{A \rightarrow B \vdash B \rightarrow C \rightarrow A \rightarrow C} \star \supset_L \frac{\frac{\frac{A, B, B \rightarrow C \vdash B}{A, B, B \rightarrow C \vdash C} \star}{A, B, B \rightarrow C \vdash C} \star \supset_L$$

MULTIPLICATIVE encoding (48ms)

$$\frac{\frac{\frac{\Gamma : B \vdash B}{\Gamma : B, B \multimap C \vdash C} \multimap}{\Gamma : A, A \multimap B, B \multimap C \vdash C} \multimap}{\Gamma : A \multimap B \vdash B \multimap C \multimap A \multimap C} \star \multimap$$

CALL-BY-NAME encoding (120ms)

$$\frac{\frac{\frac{\Gamma : \cdot \vdash A}{\Gamma : \cdot \vdash !(A)} !}{\Gamma : \cdot \vdash !(A) \multimap B \vdash B} !}{\frac{\Gamma : !(A) \multimap B \vdash B}{\Gamma : \cdot \vdash B} D_C} \multimap \frac{\frac{\Gamma : \cdot \vdash B}{\Gamma : \cdot \vdash !(B)} !}{\Gamma : \cdot \vdash !(B) \multimap C \vdash C} !}{\frac{\Gamma : !(B) \multimap C \vdash C}{\Gamma : \cdot \vdash C} D_C} \multimap \frac{\Gamma : C \vdash C}{\Gamma : !(A) \multimap B \vdash !(B) \multimap C \multimap A \multimap C} \star$$

CALL-BY-VALUE encoding (168ms)

$$\frac{\frac{\frac{\frac{\Gamma : \cdot \vdash A}{\Gamma : \cdot \vdash !(A)} !}{\Gamma : \cdot \vdash !(A) \multimap !(B) \vdash B} !}{\Gamma : \cdot \vdash !(A) \multimap !(B) \vdash B} !}{\frac{\Gamma : \cdot \vdash !(A) \multimap !(B) \vdash B}{\Gamma : \cdot \vdash B} D_C} \multimap \frac{\frac{\frac{\Gamma : \cdot \vdash B}{\Gamma : \cdot \vdash !(B)} !}{\Gamma : \cdot \vdash !(B) \multimap C \vdash C} !}{\Gamma : \cdot \vdash !(B) \multimap C \vdash C} !}{\frac{\Gamma : \cdot \vdash !(B) \multimap C \vdash C}{\Gamma : \cdot \vdash C} D_C} \multimap \frac{\Gamma : C \vdash C}{\Gamma : !(A) \multimap B \vdash !(B) \multimap C \multimap A \multimap C} \star$$

01-ENC encoding (159ms)

$$\frac{\frac{\frac{\frac{\Gamma : \cdot \vdash A}{\Gamma : \cdot \vdash !(A)} !}{\Gamma : \cdot \vdash !(A) \multimap B \vdash B} !}{\Gamma : \cdot \vdash !(A) \multimap B \vdash B} !}{\frac{\Gamma : \cdot \vdash !(A) \multimap B \vdash B}{\Gamma : \cdot \vdash B} D_C} \multimap \frac{\frac{\Gamma : \cdot \vdash B}{\Gamma : \cdot \vdash !(B)} !}{\Gamma : \cdot \vdash !(B) \multimap C \vdash C} !}{\frac{\Gamma : \cdot \vdash !(B) \multimap C \vdash C}{\Gamma : \cdot \vdash C} D_C} \multimap \frac{\Gamma : C \vdash C}{\Gamma : !(A) \multimap B \vdash !(B) \multimap C \multimap A \multimap C} \star$$

$$(7) \ A \rightarrow B \vdash C \rightarrow A \rightarrow C \rightarrow B$$

LJ (47ms)

$$\frac{\frac{C, A \rightarrow B, C \rightarrow A \vdash C}{C, A \rightarrow B, C \rightarrow A \vdash A} \star \quad \frac{A, C, A \rightarrow B \vdash A}{B, C, C \rightarrow A \vdash B} \star}{\frac{C, A \rightarrow B, C \rightarrow A \vdash B}{A \rightarrow B \vdash C \rightarrow A \rightarrow C \rightarrow B} \star} \supset_L$$

MULTIPLICATIVE encoding (48ms)

$$\frac{\frac{\frac{\Gamma : C \vdash C}{\Gamma : C, A \multimap B, C \multimap A \vdash B} \multimap \quad \frac{\Gamma : A \vdash A \quad \Gamma : B \vdash B}{\Gamma : A, A \multimap B \vdash B} \multimap}{\Gamma : A \multimap B \vdash C \multimap A \multimap C \multimap B} \star$$

CALL-BY-NAME encoding (120ms)

$$\frac{\frac{\frac{\Gamma : \cdot \vdash C}{\Gamma : \cdot \vdash !(C)} ! \quad \frac{\Gamma : A \vdash A}{\Gamma : !(C) \multimap A \vdash A} \multimap}{\frac{\Gamma : \cdot \vdash A}{\Gamma : \cdot \vdash !(A)} ! \quad \frac{\Gamma : B \vdash B}{\Gamma : !(A) \multimap B \vdash B} \multimap}{\frac{\Gamma : \cdot \vdash B}{\Gamma : \cdot \vdash !(B)} ! \quad \frac{\Gamma : !(A) \multimap B \vdash B}{\Gamma : !(A) \multimap B \vdash !(C) \multimap A \multimap B} \star} D_C$$

CALL-BY-VALUE encoding (167ms)

$$\frac{\frac{\frac{\Gamma : \cdot \vdash C}{\Gamma : \cdot \vdash !(C)} ! \quad \frac{\Gamma : \cdot \vdash A}{\Gamma : !(C) \multimap !(A) \vdash A} \star}{\frac{\Gamma : \cdot \vdash A}{\Gamma : \cdot \vdash !(A)} ! \quad \frac{\Gamma : \cdot \vdash B}{\Gamma : !(A) \multimap !(B) \vdash B} \star}{\frac{\Gamma : \cdot \vdash !(A) \otimes !(B) \vdash B}{\Gamma : \cdot \vdash !(A) \otimes !(B) \vdash !(C) \multimap !(A) \multimap !(B)} \star} D_C$$

01-ENC encoding (156ms)

$$\frac{\frac{\frac{\Gamma : \cdot \vdash C}{\Gamma : \cdot \vdash !(C)} ! \quad \frac{\Gamma : \cdot \vdash A}{\Gamma : !(A) \vdash A} \star}{\frac{\Gamma : \cdot \vdash A}{\Gamma : \cdot \vdash !(A)} ! \quad \frac{\Gamma : \cdot \vdash B}{\Gamma : !(A) \multimap !(B) \vdash B} \star}{\frac{\Gamma : \cdot \vdash B}{\Gamma : \cdot \vdash !(B)} ! \quad \frac{\Gamma : !(A) \multimap !(B) \vdash B}{\Gamma : !(A) \multimap !(B) \vdash !(C) \multimap B} \star} D_C$$

$$(8) \ A \rightarrow B \vdash A \wedge C \rightarrow B \wedge C$$

LJ (37ms)

$$\frac{\frac{A, C, A \rightarrow B \vdash A}{A, C, A \rightarrow B \vdash B} \star \quad \frac{A, B, C \vdash B}{A \rightarrow B \vdash A \wedge C \rightarrow B \wedge C} \star}{\supset_L}$$

MULTIPLICATIVE encoding (48ms)

$$\frac{\frac{\frac{\Gamma : B \vdash B}{\Gamma : A \vdash A} \multimap \quad \frac{\Gamma : C \vdash C}{\Gamma : B, C \vdash B \otimes C} \otimes}{\frac{\Gamma : A, C, A \multimap B \vdash B \otimes C}{\Gamma : A \multimap B \vdash A \otimes C \multimap B \otimes C} \star} \multimap$$

CALL-BY-NAME encoding (73ms)

$$\frac{\frac{\frac{\Gamma : \cdot \vdash A}{\Gamma : \cdot \vdash !(A)} ! \quad \frac{\Gamma : B \vdash B}{\Gamma : !(A) \multimap B \vdash B} \multimap}{\frac{\Gamma : \cdot \vdash B}{\Gamma : \cdot \vdash !(B)} ! \quad \frac{\Gamma : \cdot \vdash C}{\Gamma : !(A) \multimap B \vdash C} \star} D_C$$

CALL-BY-VALUE encoding (135ms)

$$\frac{\frac{\frac{\Gamma : \cdot \vdash A}{\Gamma : \cdot \vdash !(A)} ! \quad \frac{\Gamma : \cdot \vdash B}{\Gamma : !(A) \multimap !(B) \vdash B} \star}{\frac{\Gamma : \cdot \vdash B}{\Gamma : \cdot \vdash !(B)} ! \quad \frac{\Gamma : \cdot \vdash C}{\Gamma : !(A) \multimap !(B) \vdash C} \star}{\frac{\Gamma : \cdot \vdash !(B) \otimes !(C)}{\Gamma : \cdot \vdash !(A) \otimes !(B) \otimes !(C)} \otimes} D_C$$

01-ENC encoding (130ms)

$$\frac{\frac{\frac{\Gamma : \cdot \vdash A}{\Gamma : \cdot \vdash !(A)} ! \quad \frac{\Gamma : \cdot \vdash B}{\Gamma : !(A) \multimap !(B) \vdash B} \star}{\frac{\Gamma : \cdot \vdash B}{\Gamma : \cdot \vdash !(B)} ! \quad \frac{\Gamma : \cdot \vdash C}{\Gamma : !(A) \multimap !(B) \vdash C} \star}{\frac{\Gamma : \cdot \vdash B \& C}{\Gamma : \cdot \vdash !(A) \& !(C)} \star} D_C$$

$$(9) \ A \rightarrow B \vdash C \wedge A \rightarrow C \wedge B$$

LJ (37ms)

$$\frac{\frac{A, C, A \rightarrow B \vdash A \quad \star \quad A, B, C \vdash B \quad \star}{A, C, A \rightarrow B \vdash B} \supset_L}{A \rightarrow B \vdash C \wedge A \rightarrow C \wedge B \quad \star}$$

MULTIPLICATIVE encoding (48ms)

$$\frac{\frac{\frac{\Gamma : A \vdash A \quad \frac{\Gamma : C \vdash C \quad \Gamma : B \vdash B}{\Gamma : B, C \vdash C \otimes B} \otimes}{\Gamma : A, C, A \multimap B \vdash C \otimes B} \multimap}{\Gamma : A \multimap B \vdash C \otimes A \multimap C \otimes B} \star$$

CALL-BY-NAME encoding (60ms)

$$\frac{\frac{\frac{\frac{\Gamma : \cdot \vdash A}{\Gamma : \cdot \vdash !(A)} ! \quad \frac{\Gamma : B \vdash B}{\Gamma : !(A) \multimap B \vdash B} \multimap}{\Gamma : \cdot \vdash C \quad \Gamma : \cdot \vdash B}{\Gamma : !(A) \multimap B \vdash !(C \& A) \multimap C \& B} D_C \star$$

CALL-BY-VALUE encoding (137ms)

$$\frac{\frac{\frac{\frac{\frac{\Gamma : \cdot \vdash C}{\Gamma : \cdot \vdash !(C)} ! \quad \frac{\Gamma : \cdot \vdash B}{\Gamma : \cdot \vdash !(B)} !}{\Gamma : \cdot \vdash !(C) \otimes !(B)} \otimes}{\frac{\Gamma : \cdot \vdash A}{\Gamma : \cdot \vdash !(A)} ! \quad \frac{\Gamma : !(C) \otimes !(B)}{\Gamma : !(B) \vdash !(C) \otimes !(B)} \star}{\frac{\Gamma : !(A) \multimap !(B) \vdash !(C) \otimes !(B)}{\Gamma : \cdot \vdash !(C) \otimes !(B)} D_C \multimap}{\frac{\Gamma : \cdot \vdash !(C) \otimes !(A) \multimap !(C) \otimes !(B)}{\Gamma : \cdot \vdash !(C) \otimes !(A) \multimap !(C) \otimes !(B)} !}{\Gamma : !(A) \multimap !(B) \vdash !(C) \otimes !(A) \multimap !(C) \otimes !(B)} \star$$

01-ENC encoding (131ms)

$$\frac{\frac{\frac{\frac{\frac{\Gamma : \cdot \vdash C \quad \Gamma : \cdot \vdash B}{\Gamma : \cdot \vdash C \& B} \star}{\Gamma : \cdot \vdash !(C \& B)} !}{\frac{\Gamma : \cdot \vdash A}{\Gamma : \cdot \vdash !(A)} ! \quad \frac{\Gamma : \cdot \vdash !(C \& B)}{\Gamma : !(B) \vdash !(C \& B)} \star}{\frac{\Gamma : !(A) \multimap !(B) \vdash !(C \& B)}{\Gamma : \cdot \vdash !(C \& B)} D_C \multimap}{\frac{\Gamma : \cdot \vdash !(C) \& !(A) \multimap !(C \& B)}{\Gamma : \cdot \vdash !(C) \& !(A) \multimap !(C \& B)} !}{\Gamma : !(A) \multimap !(B) \vdash !(C) \& !(A) \multimap !(C \& B)} \star$$

$$(10) \ \neg A \vdash A \rightarrow B$$

LJ (37ms)

$$\frac{\frac{A, A \rightarrow \perp \vdash A \quad \star \quad A, \perp \vdash B \quad \star}{A, A \rightarrow \perp \vdash B} \supset_L}{A \rightarrow \perp \vdash A \rightarrow B \quad \star}$$

MULTIPLICATIVE encoding (41ms)

Not provable

CALL-BY-NAME encoding (66ms)

$$\frac{\frac{\frac{\Gamma : \cdot \vdash A}{\Gamma : \cdot \vdash !(A)} ! \quad \frac{\Gamma : \mathbf{0} \vdash B}{\Gamma : !(A) \multimap \mathbf{0} \vdash B} \multimap}{\Gamma : \cdot \vdash B} D_C}{\Gamma : !(A) \multimap \mathbf{0} \vdash !(A) \multimap B} \star$$

CALL-BY-VALUE encoding (117ms)

$$\frac{\frac{\frac{\frac{\Gamma : \cdot \vdash A}{\Gamma : \cdot \vdash !(A)} ! \quad \frac{\Gamma : \mathbf{0} \vdash !(B)}{\Gamma : !(A) \multimap \mathbf{0} \vdash !(B)} \multimap}{\Gamma : \cdot \vdash !(B)} D_C}{\frac{\Gamma : \cdot \vdash !(A) \multimap !(B)}{\Gamma : \cdot \vdash !(A) \multimap !(B)} !}{\Gamma : !(A) \multimap \mathbf{0} \vdash !(A) \multimap !(B)} \star$$

01-ENC encoding (118ms)

$$\frac{\frac{\frac{\frac{\Gamma : \cdot \vdash A}{\Gamma : \cdot \vdash !(A)} ! \quad \frac{\Gamma : !(\mathbf{0}) \vdash B}{\Gamma : !(A) \multimap !(\mathbf{0}) \vdash B} \star}{\Gamma : \cdot \vdash B} D_C}{\frac{\Gamma : \cdot \vdash !(A) \multimap B}{\Gamma : \cdot \vdash !(A) \multimap B} !}{\Gamma : !(A) \multimap !(\mathbf{0}) \vdash !(A) \multimap B} \star$$

$$(11) \ A \vdash \neg A \rightarrow B$$

LJ (37ms)

$$\frac{\frac{\overline{A, A \rightarrow \perp \vdash A} \star \overline{A, \perp \vdash B}}{A, A \rightarrow \perp \vdash B} \supset_L}{A \vdash A \rightarrow \perp \rightarrow B} \star$$

MULTIPLICATIVE encoding (28ms)

Not provable

CALL-BY-NAME encoding (65ms)

$$\frac{\frac{\overline{\Gamma : \cdot \vdash A}}{\Gamma : \cdot \vdash !(A)} ! \quad \frac{\overline{\Gamma : \mathbf{0} \vdash B}}{\Gamma : !(A) \multimap \mathbf{0} \vdash B} \star}{\Gamma : \cdot \vdash B} D_C \multimap}{\Gamma : !(A) \vdash !(A) \multimap \mathbf{0} \multimap B} \star$$

CALL-BY-VALUE encoding (78ms)

$$\frac{\frac{\overline{\Gamma : \cdot \vdash A}}{\Gamma : \cdot \vdash !(A)} ! \quad \frac{\overline{\Gamma : \mathbf{0} \vdash !(B)}}{\Gamma : !(A) \multimap \mathbf{0} \vdash !(B)} \star}{\Gamma : \cdot \vdash !(B)} D_C \multimap}{\Gamma : \cdot \vdash !(A) \multimap \mathbf{0} \multimap !(B)} \star}{\Gamma : \cdot \vdash !(A) \multimap \mathbf{0} \multimap !(B)} !}{\Gamma : !(A) \vdash !(A) \multimap \mathbf{0} \multimap !(B)} \star$$

01-ENC encoding (76ms)

$$\frac{\frac{\overline{\Gamma : \cdot \vdash A}}{\Gamma : \cdot \vdash !(A)} ! \quad \frac{\overline{\Gamma : !(\mathbf{0}) \vdash B}}{\Gamma : !(A) \multimap !(\mathbf{0}) \vdash B} \star}{\Gamma : \cdot \vdash B} D_C \multimap}{\Gamma : \cdot \vdash !(A) \multimap !(\mathbf{0}) \multimap B} \star}{\Gamma : \cdot \vdash !(A) \multimap !(\mathbf{0}) \multimap B} !}{\Gamma : !(A) \vdash !(A) \multimap !(\mathbf{0}) \multimap B} \star$$

$$(12) \ B \vdash A \rightarrow B$$

LJ (21ms)

$$\overline{B \vdash A \rightarrow B} \star$$

MULTIPLICATIVE encoding (21ms)

Not provable

CALL-BY-NAME encoding (27ms)

$$\frac{\overline{\Gamma : \cdot \vdash B}}{\Gamma : !(B) \vdash !(A) \multimap B} \star$$

CALL-BY-VALUE encoding (47ms)

$$\frac{\frac{\overline{\Gamma : \cdot \vdash B}}{\Gamma : \cdot \vdash !(B)} ! \quad \frac{\overline{\Gamma : \cdot \vdash !(A) \multimap !(B)}}{\Gamma : \cdot \vdash !(A) \multimap !(B)} \star}{\Gamma : \cdot \vdash !(A) \multimap !(B)} !}{\Gamma : \cdot \vdash !(A) \multimap !(B)} \star}{\Gamma : !(B) \vdash !(A) \multimap !(B)} \star$$

01-ENC encoding (39ms)

$$\frac{\frac{\overline{\Gamma : \cdot \vdash B}}{\Gamma : \cdot \vdash !(A) \multimap B} \star \quad \frac{\overline{\Gamma : \cdot \vdash !(A) \multimap B}}{\Gamma : \cdot \vdash !(A) \multimap B} !}{\Gamma : !(B) \vdash !(A) \multimap B} \star$$

$$(13) \quad A \rightarrow B \vdash \neg B \rightarrow \neg A$$

LJ (47ms)

$$\frac{\frac{\frac{A, A \rightarrow B, B \rightarrow \perp \vdash A}{A, B, B \rightarrow \perp \vdash A}^* \quad \frac{\frac{A, B, B \rightarrow \perp \vdash B}{A, B, \perp \vdash \perp}^*}{A, B, B \rightarrow \perp \vdash \perp}^* \quad \frac{A, A \rightarrow B, B \rightarrow \perp \vdash A}{A, A \rightarrow B, B \rightarrow \perp \vdash \perp}^*}{A \rightarrow B \vdash B \rightarrow \perp \rightarrow A \rightarrow \perp}^* \quad \supset_L \quad \supset_L$$

MULTIPLICATIVE encoding (47ms)

$$\frac{\frac{\frac{\Gamma : A \vdash A}{\Gamma : B, B \multimap \perp \vdash \perp} \quad \frac{\Gamma : B \vdash B \quad \Gamma : \perp \vdash \perp}{\Gamma : B, B \multimap \perp \vdash \perp}}{\Gamma : A, A \multimap B, B \multimap \perp \vdash \perp} \multimap \quad \frac{\Gamma : A \multimap B \vdash B \multimap \perp \multimap A \multimap \perp}{\Gamma : A \multimap B \vdash B \multimap \perp \multimap A \multimap \perp} \star$$

CALL-BY-NAME encoding (143ms)

$$\begin{array}{c}
\frac{\Gamma : \vdash A \quad !}{\Gamma : \vdash !(A)} \quad \frac{\Gamma : B \vdash B}{\Gamma : !(A) \multimap B \vdash B} D_C \quad \multimap \\
\frac{\Gamma : \vdash B \quad !}{\Gamma : \vdash \neg B} \quad \frac{\Gamma : \mathbf{0} \vdash B}{\Gamma : \neg B \multimap \mathbf{0} \vdash B} * \quad \multimap \\
\frac{\Gamma : \neg(B) \multimap \mathbf{0} \vdash B}{\Gamma : \vdash \neg B} D_C \quad \multimap \\
\frac{\Gamma : \vdash \neg B \quad !}{\Gamma : \vdash \neg \neg(B)} \quad \frac{\Gamma : \mathbf{0} \vdash B}{\Gamma : \neg \neg(B) \multimap \mathbf{0} \vdash B} * \quad \multimap \\
\frac{\Gamma : \neg \neg(B) \multimap \mathbf{0} \vdash B}{\Gamma : \vdash \neg \neg(B)} D_C \quad \multimap \\
\frac{\Gamma : \vdash \neg B \quad !}{\Gamma : \vdash \neg \neg(B)} \quad \frac{\Gamma : \neg(B) \multimap \mathbf{0} \vdash \mathbf{0}}{\Gamma : \vdash \neg \neg(B)} D_C \quad \frac{\Gamma : \mathbf{0} \vdash \mathbf{0}}{\Gamma : \mathbf{0} \vdash \mathbf{0}} * \quad \multimap \\
\frac{\Gamma : \vdash \neg \neg(B) \quad !}{\Gamma : \vdash \neg \neg \neg(B)} \quad \frac{\Gamma : \vdash \mathbf{0}}{\Gamma : \neg \neg \neg(B) \multimap \mathbf{0} \vdash \mathbf{0}} D_C \quad \frac{\Gamma : \mathbf{0} \vdash \mathbf{0}}{\Gamma : \mathbf{0} \vdash \mathbf{0}} * \quad \multimap \\
\frac{\Gamma : \vdash \neg \neg \neg(B)}{\Gamma : !(A) \multimap B \vdash !(B)} \quad \frac{\Gamma : \vdash \mathbf{0}}{\Gamma : !(B) \multimap \mathbf{0} \vdash \mathbf{0}} D_C \quad \frac{\Gamma : \mathbf{0} \vdash \mathbf{0}}{\Gamma : \mathbf{0} \vdash \mathbf{0}} * \quad \multimap \\
\frac{\Gamma : !(A) \multimap B \vdash !(B)}{\Gamma : !(A) \multimap B \vdash !(A)} *
\end{array}$$

CALL-BY-VALUE encoding (174ms)

[illegible]

01-ENC encoding (178ms)

[illegible]

$$(14) \quad A \rightarrow \neg B \vdash \neg\neg B \rightarrow \neg A$$

LJ (64ms)

$$\frac{\frac{\frac{A, B, B \rightarrow \perp, B \rightarrow \perp \rightarrow \perp \vdash B}{A, B, \perp, B \rightarrow \perp \rightarrow \perp \vdash \perp}^*}{A, B, \perp, B \rightarrow \perp \rightarrow \perp \vdash \perp}^*}{A, A \rightarrow B \rightarrow \perp, B \rightarrow \perp \rightarrow \perp \vdash A}^* \quad \frac{\frac{\frac{A, B, B \rightarrow \perp, B \rightarrow \perp \rightarrow \perp \vdash \perp}{A, B, \perp, B \rightarrow \perp \rightarrow \perp \vdash \perp}^*}{A, B \rightarrow \perp, B \rightarrow \perp \rightarrow \perp \vdash \perp}^*}{A, A \rightarrow B \rightarrow \perp, B \rightarrow \perp \rightarrow \perp \vdash \perp}^* \supset_L \quad \frac{A, \perp, B \rightarrow \perp \vdash \perp}^* \supset_L$$

MULTIPLICATIVE encoding (75ms)

$$\frac{\frac{\frac{\Gamma: A \vdash A \quad \Gamma: B \vdash B \quad \Gamma: \perp \vdash \perp}{\Gamma: A \vdash A \quad \Gamma: B, B \multimap \perp \vdash \perp} \multimap}{\Gamma: A, B, A \multimap B \multimap \perp \vdash \perp} \multimap}{\frac{\Gamma: A, A \multimap B \multimap \perp \vdash B \multimap \perp \quad \Gamma: \perp \vdash \perp}{\Gamma: A, A \multimap B \multimap \perp \vdash B \multimap \perp \multimap \perp} *} \multimap} \multimap$$

CALL-BY-NAME encoding (180ms)

[illegible]

CALL-BY-VALUE encoding (217ms)

$$\begin{array}{c}
\frac{\frac{\Gamma; \vdash \neg B}{\Gamma; \vdash \neg \downarrow(B)} \quad \frac{\Gamma; \mathbf{0} \vdash \mathbf{0}}{\Gamma; \mathbf{0} \vdash \mathbf{0}}^*}{\Gamma; \downarrow(B) \rightarrow \mathbf{0} \vdash \mathbf{0}}^* \quad \text{D}_C \quad \text{---}^* \\
\frac{\frac{\frac{\Gamma; \vdash \neg A}{\Gamma; \vdash \neg \downarrow(A)} \quad \frac{\Gamma; \downarrow(B) \rightarrow \mathbf{0} \vdash \mathbf{0}}{\Gamma; \downarrow(B) \rightarrow \mathbf{0} \vdash \mathbf{0}}^*}{\Gamma; \downarrow(A) \rightarrow \downarrow(\downarrow(B) \rightarrow \mathbf{0})}^* \quad \text{D}_C \quad \text{---}^* \\
\frac{\frac{\Gamma; \vdash \mathbf{0}}{\Gamma; \vdash \neg \downarrow(B) \rightarrow \mathbf{0}}^* \quad \frac{\Gamma; \mathbf{0} \vdash \mathbf{0}}{\Gamma; \vdash \neg \downarrow(B) \rightarrow \mathbf{0}}^*}{\Gamma; \vdash \neg \downarrow(\downarrow(B) \rightarrow \mathbf{0})}^* \quad \text{---}^* \\
\frac{\frac{\frac{\Gamma; \vdash \mathbf{0}}{\Gamma; \vdash \neg \downarrow(B) \rightarrow \mathbf{0}}^* \quad \frac{\Gamma; \vdash \neg \downarrow(B) \rightarrow \mathbf{0}}{\Gamma; \vdash \neg \downarrow(\downarrow(B) \rightarrow \mathbf{0})}^*}{\Gamma; \vdash \neg \downarrow(\downarrow(B) \rightarrow \mathbf{0}) \rightarrow \mathbf{0} \vdash \mathbf{0}}^* \quad \text{D}_C \quad \text{---}^* \\
\frac{\frac{\frac{\Gamma; \vdash \mathbf{0}}{\Gamma; \vdash \neg \downarrow(A) \rightarrow \mathbf{0}}^* \quad \frac{\Gamma; \mathbf{0} \vdash \mathbf{0}}{\Gamma; \mathbf{0} \vdash \mathbf{0}}^*}{\Gamma; \vdash \neg \downarrow(\downarrow(A) \rightarrow \mathbf{0})}^* \quad \text{---}^* \\
\frac{\frac{\frac{\Gamma; \vdash \mathbf{0}}{\Gamma; \vdash \neg \downarrow(\downarrow(A) \rightarrow \mathbf{0})}^* \quad \frac{\Gamma; \downarrow(B) \rightarrow \mathbf{0} \vdash \mathbf{0}}{\Gamma; \downarrow(B) \rightarrow \mathbf{0} \vdash \mathbf{0}}^*}{\Gamma; \vdash \neg \downarrow(\downarrow(\downarrow(A) \rightarrow \mathbf{0}) \rightarrow \mathbf{0})}^* \quad \text{D}_C \quad \text{---}^* \\
\frac{\frac{\frac{\Gamma; \vdash \neg \downarrow(\downarrow(\downarrow(A) \rightarrow \mathbf{0}) \rightarrow \mathbf{0}) \rightarrow \mathbf{0} \vdash \mathbf{0}}{\Gamma; \vdash \neg \downarrow(\downarrow(\downarrow(A) \rightarrow \mathbf{0}) \rightarrow \mathbf{0})}^* \quad \frac{\Gamma; \downarrow(B) \rightarrow \mathbf{0} \vdash \mathbf{0}}{\Gamma; \downarrow(B) \rightarrow \mathbf{0} \vdash \mathbf{0}}^*}{\Gamma; \downarrow(A) \rightarrow \downarrow(\downarrow(B) \rightarrow \mathbf{0}) \vdash \downarrow(\downarrow(\downarrow(A) \rightarrow \mathbf{0}) \rightarrow \mathbf{0}) \rightarrow \mathbf{0} \vdash \mathbf{0}}^* \quad \text{---}^* \\
\frac{\Gamma; \downarrow(A) \rightarrow \downarrow(\downarrow(B) \rightarrow \mathbf{0}) \vdash \downarrow(\downarrow(\downarrow(A) \rightarrow \mathbf{0}) \rightarrow \mathbf{0}) \rightarrow \mathbf{0} \vdash \mathbf{0}}{\Gamma; \downarrow(A) \rightarrow \downarrow(\downarrow(B) \rightarrow \mathbf{0}) \vdash \downarrow(\downarrow(\downarrow(A) \rightarrow \mathbf{0}) \rightarrow \mathbf{0}) \rightarrow \mathbf{0} \vdash \mathbf{0}}^*
\end{array}$$

01-ENC encoding (226ms)

[illegible]

$$(15) A \rightarrow B, B \rightarrow A \vdash (A) \leftrightarrow (B)$$

LJ (51ms)

$$\frac{\frac{A, A \rightarrow B, B \rightarrow A \vdash A}{A, A \rightarrow B, B \rightarrow A \vdash B} \star \quad \frac{A, B, B \rightarrow A \vdash B}{A, B, A \rightarrow B \vdash B} \star \quad \frac{B, A \rightarrow B, B \rightarrow A \vdash B}{B, A \rightarrow B, B \rightarrow A \vdash A} \star}{A \rightarrow B, B \rightarrow A \vdash A \rightarrow B \wedge B \rightarrow A} \star \quad \supset_L$$

MULTIPLICATIVE encoding (75ms)

$$\frac{\frac{\Gamma; A \vdash A \quad \Gamma; B \vdash B}{\Gamma; A, A \rightarrow B \vdash B} \multimap \quad \frac{\Gamma; B \vdash B \quad \Gamma; A \vdash A}{\Gamma; B, B \rightarrow A \vdash A} \multimap}{\frac{\Gamma; A \rightarrow B \vdash A \rightarrow B}{\Gamma; A \rightarrow B, B \rightarrow A \vdash A \rightarrow B \otimes B \rightarrow A} \star} \otimes$$

CALL-BY-NAME encoding (138ms)

$$\frac{\frac{\Gamma; \cdot \vdash \overline{A}}{\Gamma; \cdot \vdash !(A)} ! \quad \frac{\Gamma; B \vdash B}{\Gamma; !(A) \rightarrow B \vdash B} \multimap \quad \frac{\Gamma; \cdot \vdash \overline{B}}{\Gamma; \cdot \vdash !(B)} ! \quad \frac{\Gamma; A \vdash A}{\Gamma; !(B) \rightarrow A \vdash A} \multimap}{\frac{\Gamma; \cdot \vdash B}{\Gamma; !(A) \rightarrow B \& !(B) \rightarrow A \vdash !(A) \rightarrow B \& !(B) \rightarrow A} \star} D_C$$

CALL-BY-VALUE encoding (205ms)

$$\frac{\frac{\frac{\Gamma; \cdot \vdash \overline{A}}{\Gamma; \cdot \vdash !(A)} ! \quad \frac{\Gamma; \cdot \vdash \overline{B}}{\Gamma; \cdot \vdash !(B)} !}{\Gamma; !(A) \rightarrow !(B) \vdash !(B)} \star \quad \frac{\frac{\Gamma; \cdot \vdash \overline{B}}{\Gamma; \cdot \vdash !(B)} ! \quad \frac{\Gamma; \cdot \vdash \overline{A}}{\Gamma; \cdot \vdash !(A)} !}{\Gamma; !(B) \rightarrow !(A) \vdash !(A)} \star}{\frac{\Gamma; \cdot \vdash !(A) \rightarrow !(B)}{\Gamma; \cdot \vdash !(A) \rightarrow !(B) \& !(B) \rightarrow A} \star} \otimes$$

01-ENC encoding (191ms)

$$\frac{\frac{\frac{\Gamma; \cdot \vdash \overline{A}}{\Gamma; \cdot \vdash !(A)} ! \quad \frac{\Gamma; \cdot \vdash \overline{B}}{\Gamma; \cdot \vdash !(B)} !}{\Gamma; !(A) \rightarrow !(B) \vdash B} \star \quad \frac{\frac{\Gamma; \cdot \vdash \overline{B}}{\Gamma; \cdot \vdash !(B)} ! \quad \frac{\Gamma; \cdot \vdash \overline{A}}{\Gamma; \cdot \vdash !(A)} !}{\Gamma; !(B) \rightarrow !(A) \vdash A} \star}{\frac{\Gamma; \cdot \vdash B}{\Gamma; \cdot \vdash !(A) \rightarrow B} \star \quad \frac{\Gamma; \cdot \vdash A}{\Gamma; \cdot \vdash !(B) \rightarrow A} \star} \multimap$$

$$(16) (A) \leftrightarrow (B) \vdash A \rightarrow B$$

LJ (37ms)

$$\frac{\frac{A, A \rightarrow B, B \rightarrow A \vdash A}{A, A \rightarrow B, B \rightarrow A \vdash B} \star \quad \frac{A, B, B \rightarrow A \vdash B}{A \rightarrow B \wedge B \rightarrow A \vdash A \rightarrow B} \star}{\supset_L}$$

MULTIPLICATIVE encoding (29ms)

Not provable

CALL-BY-NAME encoding (102ms)

$$\frac{\frac{\Gamma; \cdot \vdash \overline{A}}{\Gamma; \cdot \vdash !(A)} ! \quad \frac{\Gamma; B \vdash B}{\Gamma; !(A) \rightarrow B \vdash B} \multimap}{\frac{\Gamma; \cdot \vdash B}{\Gamma; !(A) \rightarrow B \& !(B) \rightarrow A \vdash !(A) \rightarrow B} \star} D_C$$

CALL-BY-VALUE encoding (132ms)

$$\frac{\frac{\frac{\Gamma; \cdot \vdash \overline{A}}{\Gamma; \cdot \vdash !(A)} ! \quad \frac{\Gamma; \cdot \vdash \overline{B}}{\Gamma; \cdot \vdash !(B)} !}{\Gamma; !(A) \rightarrow !(B) \vdash !(B)} \star \quad \frac{\frac{\Gamma; \cdot \vdash \overline{B}}{\Gamma; \cdot \vdash !(B)} ! \quad \frac{\Gamma; \cdot \vdash \overline{A}}{\Gamma; \cdot \vdash !(A)} !}{\Gamma; !(B) \rightarrow !(A) \vdash !(A)} \star}{\frac{\Gamma; \cdot \vdash !(A) \rightarrow !(B)}{\Gamma; \cdot \vdash !(A) \rightarrow !(B) \& !(B) \rightarrow A} \star} \otimes$$

01-ENC encoding (126ms)

$$\frac{\frac{\frac{\Gamma; \cdot \vdash \overline{A}}{\Gamma; \cdot \vdash !(A)} ! \quad \frac{\Gamma; \cdot \vdash \overline{B}}{\Gamma; \cdot \vdash !(B)} !}{\Gamma; !(A) \rightarrow !(B) \vdash B} \star \quad \frac{\frac{\Gamma; \cdot \vdash \overline{B}}{\Gamma; \cdot \vdash !(B)} ! \quad \frac{\Gamma; \cdot \vdash \overline{A}}{\Gamma; \cdot \vdash !(A)} !}{\Gamma; !(B) \rightarrow !(A) \vdash A} \star}{\frac{\Gamma; \cdot \vdash B}{\Gamma; \cdot \vdash !(A) \rightarrow B} \star \quad \frac{\Gamma; \cdot \vdash A}{\Gamma; \cdot \vdash !(B) \rightarrow A} \star} \multimap$$

$$(17) (A) \leftrightarrow (B) \vdash B \rightarrow A$$

LJ (37ms)

$$\frac{\frac{\overline{B, A \rightarrow B, B \rightarrow A \vdash B}^*}{B, A \rightarrow B, B \rightarrow A \vdash A}^*}{A \rightarrow B \wedge B \rightarrow A \vdash B \rightarrow A}^* \supset_L$$

MULTIPLICATIVE encoding (29ms)

Not provable

CALL-BY-NAME encoding (105ms)

$$\frac{\frac{\frac{\overline{\Gamma : \cdot \vdash B}}{\Gamma : \cdot \vdash !(B)}^!}{\Gamma : !(B) \multimap A \vdash A}^{\multimap}}{\Gamma : \cdot \vdash A}^{D_C} \star$$

$$\frac{\Gamma : !(A) \multimap B \& !(B) \multimap A \vdash !(B) \multimap A}^{\star}$$

CALL-BY-VALUE encoding (140ms)

$$\frac{\frac{\frac{\overline{\Gamma : \cdot \vdash B}}{\Gamma : \cdot \vdash !(B)}^!}{\Gamma : !(B) \multimap !(A) \vdash !(A)}^{\multimap}}{\Gamma : \cdot \vdash !(A)}^{D_C} \star$$

$$\frac{\frac{\Gamma : \cdot \vdash !(A)}{\Gamma : \cdot \vdash !(B) \multimap !(A)}^{\multimap}}{\Gamma : \cdot \vdash !(B) \multimap !(A)}^! \star$$

$$\frac{\Gamma : !(A) \multimap !(B) \otimes !(B) \multimap !(A) \vdash !(B) \multimap !(A)}^{\star}$$

01-ENC encoding (122ms)

$$\frac{\frac{\frac{\overline{\Gamma : \cdot \vdash B}}{\Gamma : \cdot \vdash !(B)}^!}{\Gamma : !(B) \multimap !(A) \vdash A}^{\multimap}}{\Gamma : \cdot \vdash A}^{D_C} \star$$

$$\frac{\frac{\Gamma : \cdot \vdash A}{\Gamma : \cdot \vdash !(B) \multimap A}^{\multimap}}{\Gamma : \cdot \vdash !(B) \multimap A}^! \star$$

$$\frac{\Gamma : !(A) \multimap !(B) \& !(B) \multimap !(A) \vdash !(B) \multimap A}^{\star}$$

$$(18) (A) \leftrightarrow (B), A \vdash B$$

LJ (38ms)

$$\frac{\frac{\overline{A, A \rightarrow B, B \rightarrow A \vdash A}^*}{A, A \rightarrow B, B \rightarrow A \vdash B}^*}{A, A \rightarrow B \wedge B \rightarrow A \vdash B}^* \supset_L$$

MULTIPLICATIVE encoding (29ms)

Not provable

CALL-BY-NAME encoding (102ms)

$$\frac{\frac{\frac{\overline{\Gamma : \cdot \vdash A}}{\Gamma : \cdot \vdash !(A)}^!}{\Gamma : !(A) \multimap B \vdash B}^{\multimap}}{\Gamma : \cdot \vdash B}^{D_C} \multimap$$

$$\frac{\Gamma : !(A), !(A) \multimap B \& !(B) \multimap A \vdash B}^{\star}$$

CALL-BY-VALUE encoding (119ms)

$$\frac{\frac{\frac{\overline{\Gamma : \cdot \vdash A}}{\Gamma : \cdot \vdash !(A)}^!}{\Gamma : !(A) \multimap !(B) \vdash !(B)}^{\multimap}}{\Gamma : \cdot \vdash !(B)}^{D_C} \star$$

$$\frac{\Gamma : !(A), !(A) \multimap !(B) \otimes !(B) \multimap !(A) \vdash !(B)}^{\star}$$

01-ENC encoding (111ms)

$$\frac{\frac{\frac{\overline{\Gamma : \cdot \vdash A}}{\Gamma : \cdot \vdash !(A)}^!}{\Gamma : !(A) \multimap !(B) \vdash B}^{\multimap}}{\Gamma : \cdot \vdash B}^{D_C} \multimap$$

$$\frac{\Gamma : !(A), !(A) \multimap !(B) \& !(B) \multimap !(A) \vdash B}^{\star}$$

$$(19) (A) \leftrightarrow (B), B \vdash A$$

LJ (38ms)

$$\frac{\frac{B, A \rightarrow B, B \rightarrow A \vdash B}{B, A \rightarrow B, B \rightarrow A \vdash A} \star}{B, A \rightarrow B \wedge B \rightarrow A \vdash A} \star \supset_L$$

MULTIPLICATIVE encoding (32ms)

Not provable

CALL-BY-NAME encoding (102ms)

$$\frac{\frac{\frac{\Gamma : \cdot \vdash B}{\Gamma : \cdot \vdash !(B)} !}{\Gamma : !(B) \multimap A \vdash A} \multimap}{\Gamma : \cdot \vdash A} D_C \star$$

CALL-BY-VALUE encoding (118ms)

$$\frac{\frac{\frac{\Gamma : \cdot \vdash B}{\Gamma : \cdot \vdash !(B)} !}{\Gamma : !(B) \multimap !(A) \vdash !(A)} \multimap}{\Gamma : \cdot \vdash !(A)} D_C \star$$

01-ENC encoding (108ms)

$$\frac{\frac{\frac{\Gamma : \cdot \vdash B}{\Gamma : \cdot \vdash !(B)} !}{\Gamma : !(B) \multimap !(A) \vdash A} \multimap}{\Gamma : \cdot \vdash A} D_C \star$$

$$(20) \cdot \vdash (A) \leftrightarrow (A)$$

LJ (21ms)

$$\frac{}{\cdot \vdash A \rightarrow A \wedge A \rightarrow A} \star$$

MULTIPLICATIVE encoding (34ms)

$$\frac{\frac{\Gamma : A \vdash A}{\Gamma : \cdot \vdash A \multimap A} \star}{\Gamma : \cdot \vdash A \multimap A \otimes A \multimap A} \otimes$$

CALL-BY-NAME encoding (27ms)

$$\frac{\frac{\Gamma : \cdot \vdash A}{\Gamma : \cdot \vdash !(A) \multimap A} \multimap}{\Gamma : \cdot \vdash !(A) \multimap A \& !(A) \multimap A} \star$$

CALL-BY-VALUE encoding (47ms)

$$\frac{\frac{\frac{\Gamma : \cdot \vdash A}{\Gamma : \cdot \vdash !(A)} !}{\Gamma : \cdot \vdash !(A) \multimap !(A)} \multimap}{\Gamma : \cdot \vdash !(A) \multimap !(A)} \star$$

01-ENC encoding (47ms)

$$\frac{\frac{\frac{\Gamma : \cdot \vdash A}{\Gamma : \cdot \vdash !(A) \multimap A} \multimap}{\Gamma : \cdot \vdash !(A) \multimap A} \star}{\Gamma : \cdot \vdash !(A) \multimap A \& !(A) \multimap A} \star$$

$$(21) (A) \leftrightarrow (B) \vdash (B) \leftrightarrow (A)$$

LJ (50ms)

$$\frac{\frac{\frac{B, A \rightarrow B, B \rightarrow A \vdash B}{B, A \rightarrow B, B \rightarrow A \vdash A} *}{B, A \rightarrow B, B \rightarrow A \vdash A} *}{A \rightarrow B \wedge B \rightarrow A \vdash B} \supset_L \quad \frac{\frac{\frac{A, A \rightarrow B, B \rightarrow A \vdash A}{A, A \rightarrow B, B \rightarrow A \vdash B} *}{A, A \rightarrow B, B \rightarrow A \vdash B} *}{A \rightarrow B \wedge B \rightarrow A \vdash B} \supset_L$$

MULTIPLICATIVE encoding (81ms)

$$\frac{\frac{\frac{\Gamma: B \vdash B \quad \Gamma: A \vdash A}{\Gamma: B, B \multimap A \vdash A} \multimap \quad \frac{\Gamma: A \vdash A \quad \Gamma: B \vdash B}{\Gamma: A, A \multimap B \vdash B} \multimap}{\frac{\Gamma: B \multimap A \vdash B \multimap A \quad \Gamma: A \multimap B \vdash A \multimap B}{\Gamma: A \multimap B \multimap A \multimap B} \otimes} \frac{\Gamma: A \multimap B, B \multimap A \vdash B \multimap A \otimes A \multimap B}{\Gamma: A \multimap B \otimes B \multimap A \vdash B \multimap A \otimes A \multimap B} \star$$

CALL-BY-NAME encoding (138ms)

$$\frac{\frac{\overline{\Gamma; \cdot \vdash B}}{\Gamma; \cdot \vdash ! (B)} ! \quad \frac{}{\Gamma; A \vdash A}}{\Gamma; \cdot \vdash A} \multimap \quad \frac{\frac{\overline{\Gamma; \cdot \vdash A}}{\Gamma; \cdot \vdash ! (A)} ! \quad \frac{}{\Gamma; B \vdash B}}{\Gamma; \cdot \vdash B} \multimap \quad \frac{\Gamma; ! (B) \multimap A \vdash A \quad \Gamma; \cdot \vdash A}{\Gamma; \cdot \vdash A} D_C \quad \frac{\Gamma; ! (A) \multimap B \vdash B \quad \Gamma; \cdot \vdash B}{\Gamma; \cdot \vdash B} D_C}{\Gamma; ! (! (A) \multimap B \& ! (B) \multimap A) \vdash ! (B) \multimap A \& ! (A) \multimap B}^*$$

CALL-BY-VALUE encoding (203ms)

$$\begin{array}{c}
\frac{\Gamma; \vdash \neg B}{\Gamma; \vdash \neg \mathbf{I}(B)} \quad \frac{\Gamma; \vdash \neg A}{\Gamma; \mathbf{I}(A) \vdash \neg A} \quad * \\
\frac{\Gamma; \mathbf{I}(B) \rightarrow \neg \mathbf{I}(A) \vdash \mathbf{I}(A)}{\Gamma; \vdash \neg \mathbf{I}(A)} \quad D_C \quad \frac{\Gamma; \vdash \neg A}{\Gamma; \vdash \neg \mathbf{I}(A)} \quad \frac{\Gamma; \vdash \neg B}{\Gamma; \mathbf{I}(B) \vdash \neg B} \quad * \\
\frac{\Gamma; \vdash \neg \mathbf{I}(A)}{\Gamma; \vdash \neg \mathbf{I}(\mathbf{I}(B) \rightarrow \neg \mathbf{I}(A))} \quad \frac{\Gamma; \vdash \neg \mathbf{I}(B)}{\Gamma; \vdash \neg \mathbf{I}(\mathbf{I}(A) \rightarrow \neg \mathbf{I}(B))} \quad D_C \\
\frac{\Gamma; \vdash \neg \mathbf{I}(B) \rightarrow \neg \mathbf{I}(A)}{\Gamma; \vdash \neg \mathbf{I}(\mathbf{I}(B) \rightarrow \neg \mathbf{I}(A))} \quad \frac{\Gamma; \vdash \neg \mathbf{I}(A) \rightarrow \neg \mathbf{I}(B)}{\Gamma; \vdash \neg \mathbf{I}(\mathbf{I}(A) \rightarrow \neg \mathbf{I}(B))} \quad \otimes \\
\frac{\Gamma; \vdash \neg \mathbf{I}(\mathbf{I}(B) \rightarrow \neg \mathbf{I}(A)) \otimes \mathbf{I}(\mathbf{I}(A) \rightarrow \neg \mathbf{I}(B))}{\Gamma; \mathbf{I}(\mathbf{I}(A) \rightarrow \neg \mathbf{I}(B)) \otimes \mathbf{I}(\mathbf{I}(B) \rightarrow \neg \mathbf{I}(A)) \vdash \mathbf{I}(B) \rightarrow \neg \mathbf{I}(A) \otimes \mathbf{I}(\mathbf{I}(A) \rightarrow \neg \mathbf{I}(B))} \quad *
\end{array}$$

01-ENC encoding (188ms)

[illegible]

(22) $(A) \leftrightarrow (B), (B) \leftrightarrow (C) \vdash (A) \leftrightarrow (C)$

LJ (75ms)

[illegible]

MULTIPLICATIVE encoding (115ms)

$$\begin{array}{c}
\frac{F: A \vdash A \quad \frac{F: B \vdash B \quad F: C \vdash C}{F: B, B \vdash C \vdash C} \circ}{F: A, A \vdash B, B \vdash C \vdash C} \circ \\
\frac{F: A \vdash B, B \vdash C \vdash C}{F: A \vdash B, B \vdash C \vdash A \odot C} * \\
\frac{F: A \vdash B, B \vdash A \odot A, B \vdash C, C \vdash B \vdash A \odot C \odot C \vdash C \vdash C \vdash A \odot C \odot C \vdash A}{F: A \vdash B \odot B \odot A \odot A, B \vdash C \odot C \odot C \vdash B \vdash A \vdash A \vdash C \odot C \odot C \vdash A} *
\end{array}$$

CALL-BY-NAME encoding (218ms)

[illegible]

CALL-BY-VALUE encoding (331ms)

[illegible]

01-ENC encoding (291ms)

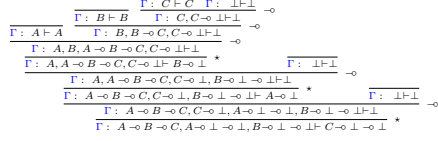
[illegible]

$$(23) A \rightarrow B \rightarrow C, \neg\neg A, \neg\neg B \vdash \neg\neg C$$

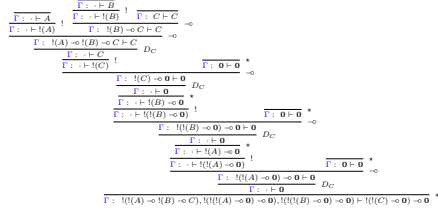
LJ (206ms)



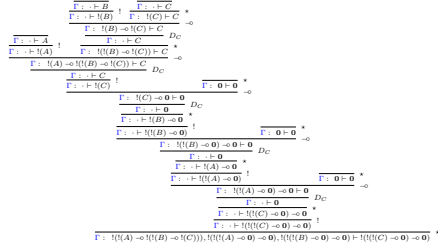
MULTIPLICATIVE encoding (124ms)



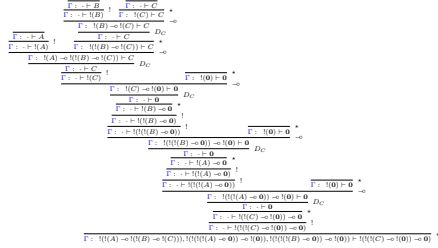
CALL-BY-NAME encoding (221ms)



CALL-BY-VALUE encoding (324ms)



01-ENC encoding (380ms)

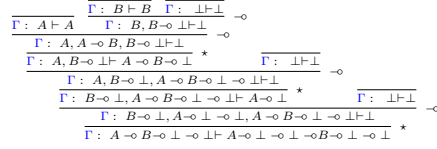


$$(24) \neg\neg A \rightarrow B \vdash \neg\neg A \rightarrow \neg\neg B$$

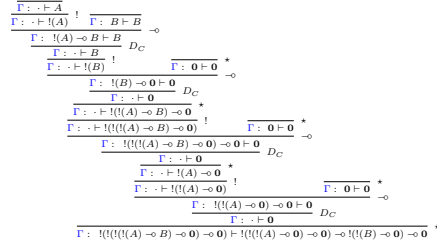
LJ (94ms)



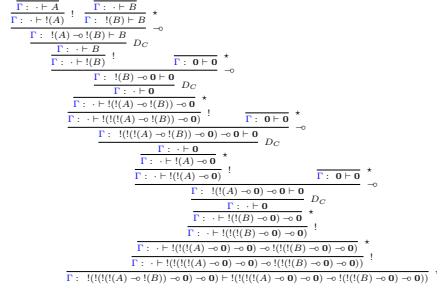
MULTIPLICATIVE encoding (106ms)



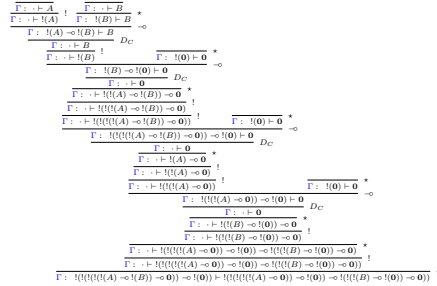
CALL-BY-NAME encoding (199ms)



CALL-BY-VALUE encoding (241ms)



01-ENC encoding (269ms)



$$(25) \neg\neg A \rightarrow B, \neg\neg B \rightarrow C \vdash \neg\neg A \rightarrow C$$

LJ (324ms)

MULTIPLICATIVE encoding (145ms)

$$\begin{array}{c}
\frac{\Gamma: A \vdash A \quad \Gamma: B \vdash B \quad \Gamma: C \vdash C}{\Gamma: B, B \vdash C \vdash C} \multimap \\
\frac{\Gamma: A, A \vdash A, B, B \vdash C \vdash C}{\Gamma: A \vdash B, B \vdash C \vdash A \vdash C} \multimap \quad \frac{\Gamma: \perp \vdash \perp}{\Gamma: A \vdash B, B \vdash C \vdash A \vdash C} \multimap \\
\frac{\Gamma: A \vdash B, B \vdash C \vdash A \vdash C \vdash \perp \vdash \perp}{\Gamma: A \vdash B, A \vdash C \vdash \perp \vdash A \vdash C \vdash \perp \vdash \perp} \multimap \quad \frac{\Gamma: \perp \vdash \perp}{\Gamma: A \vdash B, A \vdash C \vdash \perp \vdash A \vdash C \vdash \perp \vdash \perp} \multimap \\
\frac{\Gamma: A \vdash C \vdash \perp \vdash B \vdash C \vdash \perp \vdash A \vdash B \vdash \perp}{\Gamma: A \vdash C \vdash \perp \vdash A \vdash B \vdash \perp \vdash \perp \vdash B \vdash C \vdash \perp \vdash \perp} \multimap \quad \frac{\Gamma: \perp \vdash \perp}{\Gamma: A \vdash B \vdash \perp \vdash \perp \vdash B \vdash C \vdash \perp \vdash A \vdash C \vdash \perp \vdash \perp} \multimap
\end{array}$$

CALL-BY-NAME encoding (283ms)

[illegible]

CALL-BY-VALUE encoding (375ms)

[illegible]

01-ENC encoding (421ms)

[illegible]

$$(26) \cdot \vdash (\neg\neg A \wedge B) \leftrightarrow (\neg\neg A \wedge \neg\neg B)$$

LJ (137ms)

Schematic representation of the genomic organization of the human HNF1A gene. The diagram shows the gene structure with exons represented by boxes and introns by lines. The gene is divided into two main regions: the 5' region (left) and the 3' region (right). The 5' region contains exons 1 through 5, and the 3' region contains exons 6 through 10. The gene is flanked by the 5' and 3' UTRs. The gene is located on chromosome 12p12. The gene is associated with the HNF1A gene.

MULTIPLICATIVE encoding (66ms)

Not provable

CALL-BY-NAME encoding (601ms)

CALL-BY-VALUE encoding (584ms)

Figure 1 illustrates the hierarchical clustering of 1000 genes into 10 clusters. The tree structure shows the relationship between the genes, with each cluster labeled by a number (1-10) and a list of genes. The genes are listed in a column on the right, with their corresponding cluster number indicated by a small number next to the gene name. The clusters are:

- Cluster 1: 10 genes (1-10)
- Cluster 2: 10 genes (11-20)
- Cluster 3: 10 genes (21-30)
- Cluster 4: 10 genes (31-40)
- Cluster 5: 10 genes (41-50)
- Cluster 6: 10 genes (51-60)
- Cluster 7: 10 genes (61-70)
- Cluster 8: 10 genes (71-80)
- Cluster 9: 10 genes (81-90)
- Cluster 10: 10 genes (91-100)

01-ENC encoding (646ms)

$$(27) \cdot \vdash (\neg\neg(A) \leftrightarrow (B)) \leftrightarrow (\neg\neg A \rightarrow B \wedge \neg\neg B \rightarrow A)$$

LJ (2703ms)

MULTIPLICATIVE encoding (84ms)

Not provable

CALL-BY-NAME encoding (661ms)



CALL-BY-VALUE encoding (841ms)



01-ENC encoding (1132ms)



$$(28) (A) \leftrightarrow (B) \vdash (A \rightarrow C) \leftrightarrow (B \rightarrow C)$$

LJ (75ms)

MULTIPLICATIVE encoding (95ms)

$$\frac{\frac{\frac{\Gamma: B \vdash B \quad \overline{\Gamma: A \vdash A} \quad \overline{\Gamma: C \vdash C}}{\Gamma: B \vdash B \quad \overline{\Gamma: A \vdash A} \quad \overline{\Gamma: C \vdash C}} \multimap \quad \frac{\frac{\frac{\Gamma: A \vdash A \quad \overline{\Gamma: B \vdash B} \quad \overline{\Gamma: C \vdash C}}{\Gamma: A \vdash A \quad \overline{\Gamma: B \vdash B} \quad \overline{\Gamma: C \vdash C}} \multimap}{\frac{\Gamma: B, A \multimap A \multimap C, B \multimap A \multimap C}{\Gamma: B \multimap A \vdash A \multimap C \multimap B \multimap C} *} \quad \frac{\frac{\frac{\frac{\Gamma: A \vdash A \quad \overline{\Gamma: B \vdash B} \quad \overline{\Gamma: C \vdash C}}{\Gamma: A \vdash A \quad \overline{\Gamma: B \vdash B} \quad \overline{\Gamma: C \vdash C}} \multimap \quad \frac{\frac{\frac{\Gamma: B \vdash B \quad \overline{\Gamma: A \vdash A} \quad \overline{\Gamma: C \vdash C}}{\Gamma: B \vdash B \quad \overline{\Gamma: A \vdash A} \quad \overline{\Gamma: C \vdash C}} \multimap}{\frac{\Gamma: A, A \multimap B, B \multimap C \vdash C}{\Gamma: A \multimap B \vdash B \multimap C \multimap A \multimap C} *} \otimes}{\frac{\Gamma: A \multimap B \multimap B \multimap A \vdash A \multimap C \multimap B \multimap C \otimes B \multimap C \multimap C \multimap A \multimap C}{\Gamma: A \multimap B \otimes B \multimap A \vdash A \multimap C \multimap B \multimap C \otimes B \multimap C \multimap C \multimap A \multimap C} *} \otimes$$

CALL-BY-NAME encoding (173ms)

[illegible]

CALL-BY-VALUE encoding (288ms)

[illegible]

01-ENC encoding (268ms)

[illegible]

$$(29) (A) \leftrightarrow (B) \vdash (C \rightarrow A) \leftrightarrow (C \rightarrow B)$$

LJ (72ms)

[illegible]

MULTIPLICATIVE encoding (96ms)

$$\begin{array}{c}
\frac{\frac{\frac{\Gamma; A \vdash A \quad \Gamma; B \vdash B}{\Gamma; C \vdash C} \quad \frac{\Gamma; B \vdash B \quad \Gamma; A \vdash A}{\Gamma; B, B \vdash B} \quad \frac{\Gamma; C \vdash C \quad \Gamma; A \vdash A}{\Gamma; C, A \vdash C} \quad \frac{\Gamma; C, A \vdash C \quad \Gamma; A \vdash B}{\Gamma; A \vdash B \odot C \rightarrow A \rightarrow C \rightarrow B} \quad \frac{\Gamma; B \vdash B \quad \Gamma; A \vdash A}{\Gamma; B \rightarrow A \vdash C \rightarrow B \rightarrow C \rightarrow A} \\
\frac{\Gamma; A \rightarrow B \odot C \rightarrow A \rightarrow C \rightarrow B \odot C \rightarrow B \rightarrow C \rightarrow A}{\Gamma; A \rightarrow B \odot B \rightarrow A \vdash C \rightarrow A \rightarrow C \rightarrow B \odot C \rightarrow B \odot C \rightarrow B \odot C \rightarrow A}
\end{array}$$

CALL-BY-NAME encoding (250ms)

The figure consists of two diagrams, (a) and (b), illustrating the construction of a sequence of sets D_n . Both diagrams show a sequence of sets $D_0, D_1, D_2, D_3, D_4, D_5, D_6, D_7, D_8, D_9, D_{10}$ connected by arrows indicating the construction steps. The sets are defined by a sequence of operations involving the sets A and B .

Diagram (a) shows the construction of D_n for $n \geq 0$. The sets are defined by the following sequence of operations:

- $D_0 = A$
- $D_1 = B \cup A$
- $D_2 = B \cup A \cup B$
- $D_3 = B \cup A \cup B \cup A$
- $D_4 = B \cup A \cup B \cup A \cup B$
- $D_5 = B \cup A \cup B \cup A \cup B \cup A$
- $D_6 = B \cup A \cup B \cup A \cup B \cup A \cup B$
- $D_7 = B \cup A \cup B \cup A \cup B \cup A \cup B \cup A$
- $D_8 = B \cup A \cup B \cup A \cup B \cup A \cup B \cup A \cup B$
- $D_9 = B \cup A \cup B \cup A \cup B \cup A \cup B \cup A \cup B \cup A$
- $D_{10} = B \cup A \cup B \cup A \cup B \cup A \cup B \cup A \cup B \cup A \cup B$

Diagram (b) shows the construction of D_n for $n \geq 0$. The sets are defined by the following sequence of operations:

- $D_0 = A$
- $D_1 = B \cup A$
- $D_2 = B \cup A \cup B$
- $D_3 = B \cup A \cup B \cup A$
- $D_4 = B \cup A \cup B \cup A \cup B$
- $D_5 = B \cup A \cup B \cup A \cup B \cup A$
- $D_6 = B \cup A \cup B \cup A \cup B \cup A \cup B$
- $D_7 = B \cup A \cup B \cup A \cup B \cup A \cup B \cup A$
- $D_8 = B \cup A \cup B \cup A \cup B \cup A \cup B \cup A \cup B$
- $D_9 = B \cup A \cup B \cup A \cup B \cup A \cup B \cup A \cup B \cup A$
- $D_{10} = B \cup A \cup B \cup A \cup B \cup A \cup B \cup A \cup B \cup A \cup B$

CALL-BY-VALUE encoding (387ms)

[illegible]

01-ENC encoding (356ms)

[illegible]

$$(30) (A) \leftrightarrow (B) \vdash (A \wedge C) \leftrightarrow (B \wedge C)$$

LJ (51ms)

$$\frac{\frac{\frac{A, C, A \rightarrow B, B \rightarrow A \vdash A}{A, C, A \rightarrow B, B \rightarrow A \vdash B} \quad \frac{A, B, C, B \rightarrow A \vdash B}{B, C, A \rightarrow B, B \rightarrow A \vdash A}^*}{A \rightarrow B \wedge B \rightarrow A \vdash A \wedge C \rightarrow B \wedge C \wedge B \wedge C \rightarrow A \wedge C}^* \supset_L$$

MULTIPLICATIVE encoding (99ms)

$$\begin{array}{c}
\frac{\Gamma: A \vdash A \quad \frac{\Gamma: B \vdash B \quad \Gamma: C \vdash C}{\Gamma: B \vdash B \quad \Gamma: C \vdash C} \otimes}{\Gamma: A, C, A \rightarrow B \vdash B \vdash C} \multimap \\
\frac{\Gamma: A \vdash A \quad \frac{\Gamma: B \vdash B \quad \Gamma: C \vdash C}{\Gamma: B \vdash B \quad \Gamma: C \vdash C} \otimes}{\Gamma: A, C, A \vdash A \otimes C} \multimap \\
\frac{\Gamma: A, C, A \rightarrow B \vdash B \vdash C}{\Gamma: A \rightarrow B \vdash A \otimes C \rightarrow B \otimes C} * \\
\frac{\Gamma: A \rightarrow B, B \rightarrow A \vdash A \vdash A \otimes C \rightarrow B \otimes C \quad \Gamma: B \rightarrow A \vdash A \otimes C \rightarrow B \otimes C}{\Gamma: A \rightarrow B, B \rightarrow A \vdash A \otimes C \rightarrow B \otimes C \quad \Gamma: B \rightarrow A \vdash A \otimes C \rightarrow B \otimes C} * \\
\frac{\Gamma: A \rightarrow B, B \rightarrow A \vdash A \otimes C \rightarrow B \otimes C \quad \Gamma: B \rightarrow A \vdash A \otimes C \rightarrow B \otimes C}{\Gamma: A \rightarrow B \otimes B \rightarrow A \vdash A \otimes C \rightarrow B \otimes C \quad \Gamma: B \otimes C \rightarrow A \otimes C} *
\end{array}$$

CALL-BY-NAME encoding (138ms)

$$\frac{\frac{\frac{\Gamma; \vdash A}{\Gamma; \vdash ! (A)}}{\Gamma; B \vdash B} ! \quad \frac{\frac{\Gamma; \vdash B}{\Gamma; \vdash ! (B)} ! \quad \frac{\Gamma; A \vdash A}{\Gamma; A \vdash A}}{\frac{\Gamma; ! (A) \multimap B \vdash B}{\Gamma; \vdash B} \multimap} \multimap \quad \frac{\Gamma; \vdash B}{\Gamma; \vdash B} D_C \quad \frac{\frac{\Gamma; \vdash C}{\Gamma; \vdash C} \quad \frac{\Gamma; ! (B) \multimap A \vdash A}{\Gamma; \vdash A} ! \quad \frac{\Gamma; A \vdash A}{\Gamma; A \vdash A}}{\frac{\Gamma; ! (A) \multimap B \vdash B}{\Gamma; \vdash B} \multimap} \multimap \quad \frac{\Gamma; \vdash C}{\Gamma; \vdash C} D_C \quad \frac{\Gamma; \vdash C}{\Gamma; \vdash C} \quad \frac{\Gamma; ! (A) \multimap B \vdash B}{\Gamma; \vdash B} \multimap \quad \frac{\Gamma; ! (B) \multimap A \vdash A}{\Gamma; \vdash A} ! \quad \frac{\Gamma; A \vdash A}{\Gamma; A \vdash A}}{\Gamma; ! (A) \multimap B \vdash B \quad \Gamma; ! (B) \multimap A \vdash A \quad \Gamma; A \vdash A \quad \Gamma; B \vdash B \quad \Gamma; C \vdash C} *$$

CALL-BY-VALUE encoding (238ms)

$$\begin{array}{c}
\frac{\Gamma \vdash A \vdash \star}{\Gamma \vdash !A(A)} \quad \frac{\Gamma \vdash B \vdash \star}{\Gamma \vdash !B(B)} \quad \star \\
\hline
\Gamma \vdash !A(A) \multimap !B(B) \vdash \star \quad Dc \\
\hline
\frac{\Gamma \vdash A \vdash \star}{\Gamma \vdash !A(A)} \quad \frac{\Gamma \vdash B \vdash \star}{\Gamma \vdash !B(B)} \quad \star \\
\hline
\Gamma \vdash !A(A) \multimap !B(B) \vdash \star \quad Dc \\
\hline
\frac{\Gamma \vdash A \vdash \star}{\Gamma \vdash !A(A)} \quad \frac{\Gamma \vdash C \vdash \star}{\Gamma \vdash !C(C)} \quad \star \\
\hline
\Gamma \vdash !A(A) \multimap !B(B) \multimap !C(C) \quad \otimes \\
\hline
\frac{\Gamma \vdash A \vdash \star}{\Gamma \vdash !A(A)} \quad \frac{\Gamma \vdash B \vdash \star}{\Gamma \vdash !B(B)} \quad \frac{\Gamma \vdash C \vdash \star}{\Gamma \vdash !C(C)} \quad \star \\
\hline
\Gamma \vdash !A(A) \multimap !B(B) \multimap !C(C) \quad \otimes \\
\hline
\frac{\Gamma \vdash A \vdash \star}{\Gamma \vdash !A(A)} \quad \frac{\Gamma \vdash B \vdash \star}{\Gamma \vdash !B(B)} \quad \frac{\Gamma \vdash C \vdash \star}{\Gamma \vdash !C(C)} \quad \star \\
\hline
\Gamma \vdash !A(A) \multimap !B(B) \multimap !C(C) \quad \otimes \\
\hline
\frac{\Gamma \vdash A \vdash \star}{\Gamma \vdash !A(A)} \quad \frac{\Gamma \vdash B \vdash \star}{\Gamma \vdash !B(B)} \quad \frac{\Gamma \vdash C \vdash \star}{\Gamma \vdash !C(C)} \quad \star \\
\hline
\Gamma \vdash !A(A) \multimap !B(B) \multimap !C(C) \quad \otimes \\
\hline
\frac{\Gamma \vdash !A(A) \multimap !B(B) \multimap !C(C)}{\Gamma \vdash !A(A) \multimap !B(B) \multimap !C(C)} \quad \otimes
\end{array}$$

01-ENC encoding (234ms)

[illegible]

$$(31) (A) \leftrightarrow (B) \vdash (C \wedge A) \leftrightarrow (C \wedge B)$$

LJ (51ms)

$$\frac{\frac{A, C, A \rightarrow B, B \rightarrow A \vdash A}{A, C, A \rightarrow B, B \rightarrow A \vdash B} \text{ }^*_L \quad \frac{\frac{B, C, A \rightarrow B, B \rightarrow A \vdash B}{B, C, A \rightarrow B, B \rightarrow A \vdash A} \text{ }^*_R \quad \frac{A, B, C, B \rightarrow A \vdash B}{B, C, A \rightarrow B, B \rightarrow A \vdash A} \text{ }^*_L}{\frac{A, C, A \rightarrow B, B \rightarrow A \vdash B \quad B, C, A \rightarrow B, B \rightarrow A \vdash A}{A \rightarrow B \wedge B \rightarrow A \vdash A \vee C \wedge A \rightarrow C \wedge B \wedge C \wedge B \rightarrow C \wedge A} \text{ }^*_R}$$

MULTIPLICATIVE encoding (98ms)

$$\begin{array}{c}
\frac{\frac{\Gamma: A \vdash A \quad \Gamma: C \vdash C \quad \Gamma: B \vdash B}{\Gamma: A, C, B \vdash C \otimes B} \otimes \quad \frac{\Gamma: B \vdash B \quad \Gamma: A, C \vdash C \otimes A}{\Gamma: A, C, B \vdash C \otimes A} \otimes \\
\frac{\Gamma: A, C, A \vdash B \vdash C \otimes B}{\Gamma: A \vdash A \otimes B \vdash C \otimes A \vdash C \otimes B} \circ \quad \frac{\Gamma: B, C, B \vdash A \vdash C \otimes A}{\Gamma: B \vdash A \otimes C \vdash B \vdash C \otimes A} \circ \\
\frac{\Gamma: A \vdash A \otimes B \vdash C \otimes A \vdash C \otimes B}{\Gamma: A \vdash A \otimes B, B \vdash A \otimes C \vdash A \otimes C \vdash B \vdash C \otimes B \vdash C \otimes A} * \quad \frac{\Gamma: B \vdash A \otimes C \vdash B \vdash C \otimes A}{\Gamma: A \vdash A \otimes B, B \vdash A \otimes C \vdash A \otimes C \vdash B \vdash C \otimes B \vdash C \otimes A} * \\
\frac{\Gamma: A \vdash A \otimes B, B \vdash A \otimes C \vdash A \otimes C \vdash B \vdash C \otimes B \vdash C \otimes A}{\Gamma: A \vdash A \otimes B, B \vdash A \otimes C \vdash A \otimes C \vdash B \vdash C \otimes B \vdash C \otimes A} *
\end{array}$$

CALL-BY-NAME encoding (142ms)

$$\frac{\frac{\frac{\Gamma; \cdot \vdash A}{\Gamma; \cdot \vdash !(A)} !}{\Gamma; \cdot \vdash B \multimap B} !}{\Gamma; \cdot \vdash !(A) \multimap B \multimap B} ! \quad \frac{\frac{\frac{\Gamma; \cdot \vdash B}{\Gamma; \cdot \vdash !(B)} !}{\Gamma; A \multimap A} !}{\Gamma; \cdot \vdash A \multimap A} ! \quad \multimap$$

$$\frac{\frac{\Gamma; \cdot \vdash C}{\Gamma; \cdot \vdash B} D_C \quad \frac{\Gamma; \cdot \vdash C}{\Gamma; \cdot \vdash A} D_C}{\Gamma; !(A) \multimap B \& !(B) \multimap A \vdash !(C \& A) \multimap C \& B \& !(C \& B) \multimap C \& A} *$$

CALL-BY-VALUE encoding (227ms)

[illegible]

01-ENC encoding (229ms)

[illegible]

$$(32) (A) \leftrightarrow (B) \vdash (\neg A) \leftrightarrow (\neg B)$$

LJ (74ms)

$$\frac{\frac{A \vee B \text{ 假而 } A \vee B \text{ 真}}{A \vee B \text{ 真}}}{\frac{A \vee B \text{ 真}}{A \vee B \text{ 真}}} \supset_4 \quad \frac{\frac{A \vee B \text{ 真而 } A \vee B \text{ 假}}{A \vee B \text{ 假}}}{\frac{A \vee B \text{ 假}}{A \vee B \text{ 假}}} \supset_4 \quad \frac{\frac{A \vee B \text{ 真而 } A \vee B \text{ 真}}{A \vee B \text{ 真}}}{\frac{A \vee B \text{ 真}}{A \vee B \text{ 真}}} \supset_4 \quad \frac{\frac{A \vee B \text{ 假而 } A \vee B \text{ 假}}{A \vee B \text{ 假}}}{\frac{A \vee B \text{ 假}}{A \vee B \text{ 假}}} \supset_4$$

MULTIPLICATIVE encoding (97ms)

$$\begin{array}{c}
\frac{\Gamma: B \vdash B \quad \frac{\Gamma: A \vdash A \quad \Gamma: \perp \vdash \perp}{\Gamma: A, A \vdash \perp \vdash \perp} \multimap \quad \frac{\Gamma: B \vdash B \quad \Gamma: \perp \vdash \perp}{\Gamma: B, B \vdash \perp \vdash \perp} \multimap \\
\frac{\Gamma: B, A, A \vdash \perp, B \vdash \perp \vdash \perp}{\Gamma: B \vdash A, A \vdash \perp, B \vdash \perp \vdash \perp} \multimap \quad \frac{\Gamma: A, A \vdash \perp, B \vdash \perp \vdash \perp}{\Gamma: A, A \vdash \perp, B \vdash \perp \vdash \perp} \multimap \\
\frac{\Gamma: B \vdash A \vdash A \vdash \perp \vdash \perp \vdash \perp}{\Gamma: A \vdash B \vdash B \vdash \perp \vdash \perp \vdash \perp} * \quad \frac{\Gamma: A \vdash B \vdash B \vdash \perp \vdash \perp \vdash \perp}{\Gamma: A \vdash B \vdash B \vdash \perp \vdash \perp \vdash \perp} * \\
\frac{\Gamma: A \vdash B \vdash B \vdash \perp \vdash \perp \vdash \perp}{\Gamma: A \vdash B \vdash B \vdash \perp \vdash \perp \vdash \perp} \otimes \quad \frac{\Gamma: A \vdash B \vdash B \vdash \perp \vdash \perp \vdash \perp}{\Gamma: A \vdash B \vdash B \vdash \perp \vdash \perp \vdash \perp} \otimes \\
\frac{\Gamma: A \vdash B \vdash B \vdash \perp \vdash \perp \vdash \perp}{\Gamma: A \vdash B \vdash B \vdash \perp \vdash \perp \vdash \perp} * \quad \frac{\Gamma: A \vdash B \vdash B \vdash \perp \vdash \perp \vdash \perp}{\Gamma: A \vdash B \vdash B \vdash \perp \vdash \perp \vdash \perp} *
\end{array}$$

CALL-BY-NAME encoding (223ms)

[illegible]

CALL-BY-VALUE encoding (304ms)

The diagram illustrates the decomposition of a 16-qubit quantum circuit into two 8-qubit sub-circuits. The top half shows the initial decomposition into two 8-qubit blocks, each with a 4-qubit sub-block. The bottom half shows the further decomposition of these 8-qubit blocks into two 4-qubit blocks each, resulting in a total of eight 4-qubit blocks. The diagram uses various quantum gates (CNOT, Toffoli, etc.) and qubit indices (0-15) to illustrate the circuit structure.

01-ENC encoding (320ms)

The figure consists of two diagrams, (a) and (b), illustrating the construction of a 2D grid of points. Both diagrams show a sequence of points $(x_1, y_1), (x_2, y_2), \dots, (x_N, y_N)$ arranged in a grid. The points are connected by arrows, indicating a sequence of points. The labels D_1, D_2, \dots, D_N are placed below the points, indicating distances or weights. In diagram (a), the points are arranged in a grid with x and y axes. In diagram (b), the points are arranged in a grid with x and y axes, but the labels D_1, D_2, \dots, D_N are placed above the points.

$$(33) \cdot \vdash (A \wedge B \wedge C) \leftrightarrow (A \wedge B \wedge C)$$

LJ (21ms)

$$\cdot \vdash A \wedge B \wedge C \rightarrow A \wedge B \wedge C \wedge A \wedge B \wedge C \rightarrow A \wedge B \wedge C \quad \star$$

MULTIPLICATIVE encoding (66ms)

$$\frac{\frac{F_1 : A \multimap A \quad F_2 : B \multimap B}{F_1 : A, B \multimap A \otimes B} \otimes \frac{F_1 : C \multimap C}{F_1 : C \multimap C}}{\frac{F_1 : A, B, C \multimap A \otimes B \otimes C}{F_1 : \neg A \otimes B \otimes C \multimap A \otimes B \otimes C} *} \otimes \frac{\frac{F_1 : A \multimap A \quad F_2 : B \multimap B}{F_1 : A, B \multimap A \otimes B} \otimes \frac{F_1 : C \multimap C}{F_1 : C \multimap C}}{\frac{F_1 : A, B, C \multimap A \otimes B \otimes C}{F_1 : \neg A \otimes B \otimes C \multimap A \otimes B \otimes C} *} \otimes \frac{}{\frac{F_1 : \neg A \otimes B \otimes C \multimap A \otimes B \otimes C \multimap A \otimes B \otimes C \multimap A \otimes B \otimes C}{F_1 : \neg A \otimes B \otimes C \multimap A \otimes B \otimes C \multimap A \otimes B \otimes C} \otimes}$$

CALL-BY-NAME encoding (34ms)

$$\frac{\overline{\Gamma: \cdot \vdash A} \quad \overline{\Gamma: \cdot \vdash B} \quad \overline{\Gamma: \cdot \vdash C} \quad \overline{\Gamma: \cdot \vdash A} \quad \overline{\Gamma: \cdot \vdash B} \quad \overline{\Gamma: \cdot \vdash C}}{\overline{\Gamma: \cdot \vdash !(A \& B \& C) \multimap A \& B \& C \& !(A \& B \& C) \multimap A \& B \& C}}^*$$

CALL-BY-VALUE encoding (99ms)

$$\begin{array}{c}
\frac{\frac{\Gamma; \vdash A}{\Gamma; \vdash \neg(A)} \quad \frac{\Gamma; \vdash B}{\Gamma; \vdash \neg(B)}}{\Gamma; \vdash \neg(A) \otimes \neg(B)} \quad \frac{\Gamma; \vdash C}{\Gamma; \vdash \neg(C)} \quad ! \\
\hline
\Gamma; \vdash \neg(A) \otimes \neg(B) \otimes \neg(C) \\
\hline
\frac{\frac{\Gamma; \vdash \neg(A) \otimes \neg(B) \otimes \neg(C)}{\Gamma; \vdash \neg(A) \otimes \neg(B) \otimes \neg(C)} \quad \frac{\Gamma; \vdash \neg(A) \otimes \neg(B) \otimes \neg(C)}{\Gamma; \vdash \neg(A) \otimes \neg(B) \otimes \neg(C)} \quad * \\
\hline
\frac{\frac{\Gamma; \vdash \neg(A) \otimes \neg(B) \otimes \neg(C)}{\Gamma; \vdash \neg(A) \otimes \neg(B) \otimes \neg(C)} \quad \frac{\Gamma; \vdash \neg(A) \otimes \neg(B) \otimes \neg(C)}{\Gamma; \vdash \neg(A) \otimes \neg(B) \otimes \neg(C)} \quad * \\
\hline
\Gamma; \vdash \neg(A) \otimes \neg(B) \otimes \neg(C) \rightarrow \neg(A) \otimes \neg(B) \otimes \neg(C) \rightarrow \neg(A) \otimes \neg(B) \otimes \neg(C)
\end{array}$$

01-ENC encoding (124ms)

[illegible]

$$(34) \cdot \vdash (A \wedge B) \leftrightarrow (B \wedge A)$$

LJ (21ms)

$$\overline{\cdot \vdash A \wedge B \rightarrow B \wedge A \wedge B \wedge A \rightarrow A \wedge B}^*$$

MULTIPLICATIVE encoding (73ms)

$$\frac{\frac{\Gamma; B \vdash B \quad \Gamma; A \vdash A}{\Gamma; A, B \vdash B \otimes A} \otimes \quad \frac{\Gamma; A \vdash A \quad \Gamma; B \vdash B}{\Gamma; A, B \vdash A \otimes B} \otimes}{\frac{\Gamma; \cdot \vdash A \otimes B \multimap B \otimes A}{\Gamma; \cdot \vdash A \otimes B \multimap B \otimes A \otimes B \multimap A \otimes B} *} \quad \frac{\Gamma; \cdot \vdash B \otimes A \multimap A \otimes B}{\Gamma; \cdot \vdash B \otimes A \multimap A \otimes B} *} \otimes$$

CALL-BY-NAME encoding (35ms)

$$\frac{\overline{\Gamma : \cdot \vdash B} \quad \overline{\Gamma : \cdot \vdash A} \quad \overline{\Gamma : \cdot \vdash A} \quad \overline{\Gamma : \cdot \vdash B}}{\Gamma : \cdot \vdash !(A \& B) \multimap B \& A \& !(B \& A) \multimap A \& B} \star$$

CALL-BY-VALUE encoding (107ms)

$$\frac{\frac{\Gamma; \vdash B}{\Gamma; \vdash ! (B)}! \quad \frac{\Gamma; \vdash A}{\Gamma; \vdash ! (A)}!}{\Gamma; \vdash ! (B) \otimes ! (A)} * \quad \frac{\frac{\Gamma; \vdash A}{\Gamma; \vdash ! (A)}! \quad \frac{\Gamma; \vdash B}{\Gamma; \vdash ! (B)}!}{\Gamma; \vdash ! (A) \otimes ! (B)} * \\ \frac{\frac{\Gamma; \vdash ! (A) \otimes ! (B) \rightarrow ! (B) \rightarrow ! (A)}{\Gamma; \vdash ! (! (A) \otimes ! (B) \rightarrow ! (B) \rightarrow ! (A))}! \quad \frac{\frac{\Gamma; \vdash ! (B) \otimes ! (A) \rightarrow ! (A) \otimes ! (B)}{\Gamma; \vdash ! (! (B) \otimes ! (A) \rightarrow ! (A) \otimes ! (B))}!}{\Gamma; \vdash ! (! (A) \otimes ! (B) \rightarrow ! (B) \rightarrow ! (A)) \otimes ! (! (B) \otimes ! (A) \rightarrow ! (A) \otimes ! (B))}!$$

01-ENC encoding (102ms)

$$\frac{\frac{\frac{\Gamma; \vdash \neg B \quad \Gamma; \vdash \neg A}{\Gamma; \vdash \neg(B \wedge A)} \star}{\Gamma; \vdash \neg!(B \wedge A)} \star \quad \frac{\frac{\frac{\Gamma; \vdash \neg A \quad \Gamma; \vdash \neg B}{\Gamma; \vdash \neg(A \wedge B)} \star}{\Gamma; \vdash \neg!(A \wedge B)} \star}{\frac{\frac{\frac{\Gamma; \vdash \neg!(\neg(A \wedge B)) \rightarrow \neg!(B \wedge A)}{\Gamma; \vdash \neg!(\neg(A \wedge B)) \rightarrow \neg!(B \wedge A)} \star \quad \frac{\frac{\frac{\Gamma; \vdash \neg!(\neg(B \wedge A)) \rightarrow \neg!(A \wedge B)}{\Gamma; \vdash \neg!(\neg(B \wedge A)) \rightarrow \neg!(A \wedge B)} \star}{\Gamma; \vdash \neg!(\neg(A \wedge B)) \rightarrow \neg!(A \wedge B)} \star} \star \quad \frac{\frac{\frac{\frac{\Gamma; \vdash \neg!(\neg(A \wedge B)) \rightarrow \neg!(B \wedge A) \quad \Gamma; \vdash \neg!(\neg(B \wedge A)) \rightarrow \neg!(A \wedge B)}{\Gamma; \vdash \neg!(\neg(A \wedge B)) \rightarrow \neg!(A \wedge B)} \star}{\Gamma; \vdash \neg!(\neg(A \wedge B)) \rightarrow \neg!(B \wedge A) \quad \Gamma; \vdash \neg!(\neg(B \wedge A)) \rightarrow \neg!(A \wedge B)) \star} \star$$

$$(35) \cdot \vdash (A \wedge A) \leftrightarrow (A)$$

LJ (21ms)

$$\cdot \vdash A \wedge A \rightarrow A \wedge A \rightarrow A \wedge A \quad \star$$

MULTIPLICATIVE encoding (34ms)

Not provable

CALL-BY-NAME encoding (28ms)

$$\frac{\overline{\Gamma : \cdot \vdash A} \quad \overline{\Gamma : \cdot \vdash A} \quad \overline{\Gamma : \cdot \vdash A}}{\Gamma : \cdot \vdash !(A \& A) \multimap A \& !(A) \multimap A \& A} \star$$

CALL-BY-VALUE encoding (76ms)

$$\frac{\frac{\frac{\Gamma; \vdash A}{\Gamma; \vdash ! (A)} !}{\frac{\Gamma; \vdash ! (A) \otimes ! (A) \rightarrow ! (A)}{\Gamma; \vdash ! (! (A) \otimes ! (A) \rightarrow ! (A))} *} \quad \frac{\frac{\frac{\Gamma; \vdash A}{\Gamma; \vdash ! (A)} ! \quad \frac{\Gamma; \vdash A}{\Gamma; \vdash ! (A)} !}{\Gamma; \vdash ! (A) \otimes ! (A)} \otimes}{\frac{\frac{\Gamma; \vdash ! (A) \otimes ! (A) \rightarrow ! (A)}{\Gamma; \vdash ! (! (A) \otimes ! (A) \rightarrow ! (A))} ! \quad \frac{\Gamma; \vdash ! (A) \rightarrow ! (A) \otimes ! (A)}{\Gamma; \vdash ! (! (A) \rightarrow ! (A) \otimes ! (A))} *} \otimes} \frac{}{\Gamma; \vdash ! (! (A) \otimes ! (A) \rightarrow ! (A)) \otimes ! (! (A) \rightarrow ! (A) \otimes ! (A))} \otimes$$

01-ENC encoding (81ms)

$$\begin{array}{c}
\frac{\Gamma; \cdot \vdash A \quad \Gamma; \cdot \vdash A}{\Gamma; \cdot \vdash A \& A} \star \\
\frac{\Gamma; \cdot \vdash A \& A}{\Gamma; \cdot \vdash !(\Gamma(A) \& !(\Gamma(A)) \multimap A)} \star \\
\frac{\Gamma; \cdot \vdash !(\Gamma(A) \& !(\Gamma(A)) \multimap A)}{\Gamma; \cdot \vdash !(\Gamma(A) \multimap !(\Gamma(A) \& A))} \star \\
\frac{\Gamma; \cdot \vdash !(\Gamma(A) \multimap !(\Gamma(A) \& A))}{\Gamma; \cdot \vdash !(!(\Gamma(A) \& !(\Gamma(A)) \multimap A) \& !(\Gamma(A) \multimap !(\Gamma(A) \& A)))} \star
\end{array}$$

(36) $A \vdash (A \rightarrow B) \leftrightarrow (B)$

LJ (37ms)

$$\frac{\frac{\overline{A, A \rightarrow B \vdash A}^{\star} \quad \overline{A, B \vdash B}^{\star}}{A, A \rightarrow B \vdash B} \supset_L}{A \vdash A \rightarrow B \rightarrow B \wedge B \rightarrow A \rightarrow B}^{\star}$$

MULTIPLICATIVE encoding (42ms)

Not provable

CALL-BY-NAME encoding (73ms)

$$\frac{\frac{\frac{\Gamma: \cdot \vdash A}{\Gamma: \cdot \vdash ! (A)} ! \quad \frac{\Gamma: B \vdash B}{\Gamma: \cdot \vdash B} \multimap}{\frac{\Gamma: ! (A) \multimap B \vdash B}{\Gamma: \cdot \vdash B} D_C} \quad \frac{\Gamma: \cdot \vdash B}{\Gamma: ! (A) \vdash ! (A) \multimap B \& ! (B) \multimap ! (A) \multimap B} \star$$

CALL-BY-VALUE encoding (129ms)

[illegible]

01-ENC encoding (123ms)

$$\begin{array}{c}
\frac{\Gamma; \cdot \vdash A}{\Gamma; \cdot \vdash \mathbf{!}(A)} \mathbf{!} \quad \frac{\Gamma; \cdot \vdash B}{\Gamma; \cdot \vdash \mathbf{!}(B)} \mathbf{!} \quad \star \\
\hline
\frac{\Gamma; \cdot \vdash \mathbf{!}(A) \rightarrow \mathbf{!}(B) \rightarrow B}{\Gamma; \cdot \vdash B} \text{DC} \quad \circ \\
\hline
\frac{\Gamma; \cdot \vdash B}{\Gamma; \cdot \vdash \mathbf{!}(\mathbf{!}(A) \rightarrow B)} \mathbf{!} \quad \star \\
\hline
\frac{\Gamma; \cdot \vdash \mathbf{!}(\mathbf{!}(A) \rightarrow \mathbf{!}(B)) \rightarrow B}{\Gamma; \cdot \vdash \mathbf{!}(\mathbf{!}(\mathbf{!}(A) \rightarrow \mathbf{!}(B)) \rightarrow B)} \mathbf{!} \quad \star \\
\hline
\frac{\Gamma; \cdot \vdash \mathbf{!}(\mathbf{!}(A) \rightarrow \mathbf{!}(B)) \rightarrow B}{\Gamma; \cdot \vdash \mathbf{!}(\mathbf{!}(A) \rightarrow \mathbf{!}(\mathbf{!}(B) \rightarrow \mathbf{!}(\mathbf{!}(A) \rightarrow B)))} \mathbf{!} \quad \star \\
\hline
\frac{\Gamma; \cdot \vdash \mathbf{!}(\mathbf{!}(\mathbf{!}(A) \rightarrow \mathbf{!}(B)) \rightarrow B) \& \mathbf{!}(\mathbf{!}(B) \rightarrow \mathbf{!}(\mathbf{!}(A) \rightarrow B))}{\Gamma; \cdot \vdash \mathbf{!}(\mathbf{!}(\mathbf{!}(A) \rightarrow \mathbf{!}(B)) \rightarrow B) \& \mathbf{!}(\mathbf{!}(B) \rightarrow \mathbf{!}(\mathbf{!}(A) \rightarrow B))} \mathbf{!} \quad \star \\
\hline
\frac{\Gamma; \cdot \vdash \mathbf{!}(\mathbf{!}(\mathbf{!}(A) \rightarrow \mathbf{!}(B)) \rightarrow B) \& \mathbf{!}(\mathbf{!}(B) \rightarrow \mathbf{!}(\mathbf{!}(A) \rightarrow B))}{\Gamma; \cdot \vdash \mathbf{!}(\mathbf{!}(\mathbf{!}(A) \rightarrow \mathbf{!}(B)) \rightarrow B) \& \mathbf{!}(\mathbf{!}(B) \rightarrow \mathbf{!}(\mathbf{!}(A) \rightarrow B))} \mathbf{!} \quad \star
\end{array}$$

$$(37) \quad B \vdash (A \rightarrow B) \leftrightarrow (B)$$

LJ (21ms)

$$\overline{B \vdash A \rightarrow B \rightarrow B \wedge B \rightarrow A \rightarrow B} \quad \star$$

MULTIPLICATIVE encoding (40ms)

Not provable

CALL-BY-NAME encoding (39ms)

$$\frac{\overline{\Gamma : \cdot \vdash B} \quad \overline{\Gamma : \cdot \vdash B}}{\Gamma : !(B) \vdash !(A) \multimap B \multimap B \& !(B) \multimap !(A) \multimap B}^*$$

CALL-BY-VALUE encoding (152ms)

[illegible]

01-ENC encoding (88ms)

[illegible]

$$(38) \neg A \vdash (A \rightarrow B) \leftrightarrow (\neg A)$$

LJ (52ms)

$$\frac{\frac{A, A \rightarrow B, A \rightarrow \perp \perp A}{A, A \rightarrow B, A \rightarrow \perp \perp \perp}^* \quad \frac{A, \perp, A \rightarrow B \perp \perp}{A, A \rightarrow \perp \perp B}^*}{A, A \rightarrow B, A \rightarrow \perp \perp \perp} \supset_L \quad \frac{\frac{A, A \rightarrow \perp \perp A}{A, A \rightarrow \perp \perp B}^* \quad \frac{A, \perp \perp B}{A, A \rightarrow \perp \perp B}^*}{A, A \rightarrow \perp \perp B} \supset_L$$

MULTIPLICATIVE encoding (42ms)

Not provable

CALL-BY-NAME encoding (101ms)

$$\frac{\frac{\Gamma; \cdot \vdash A}{\Gamma; \cdot \vdash ! (A)} ! \quad \frac{}{\Gamma; 0 \vdash 0} * \quad \frac{\Gamma; \cdot \vdash A}{\Gamma; \cdot \vdash ! (A)} ! \quad \frac{}{\Gamma; 0 \vdash B} *} {\frac{\Gamma; !(A) \rightarrow 0 \vdash 0}{\Gamma; \cdot \vdash 0} D_C \quad \frac{\Gamma; !(A) \rightarrow 0 \vdash B}{\Gamma; \cdot \vdash B} D_C} \frac{}{\Gamma; !(I(A) \rightarrow 0) \vdash !(I(A) \rightarrow 0) \rightarrow !(I(A) \rightarrow 0) \& \& !(I(A) \rightarrow 0) \rightarrow !(I(A) \rightarrow 0) \rightarrow B} *$$

CALL-BY-VALUE encoding (201ms)

[illegible]

01-ENC encoding (203ms)

[illegible]

$$(39) \neg B \vdash (A \rightarrow B) \leftrightarrow (\neg A)$$

LJ (61ms)

$$\frac{\frac{\frac{A, A \rightarrow B, B \rightarrow \perp \vdash A}{A, A \rightarrow B, B \rightarrow \perp \vdash \perp} *}{A, A \rightarrow B, B \rightarrow \perp \vdash \perp} *}{B \rightarrow \perp \vdash A \rightarrow B \rightarrow A \rightarrow \perp \wedge A \rightarrow \perp \vdash A \rightarrow B} * \quad \supset_L \quad \supset_L \quad \frac{\frac{\frac{A, A \rightarrow B, B \rightarrow \perp \vdash A}{A, A \rightarrow B, B \rightarrow \perp \vdash \perp} *}{A, A \rightarrow B, B \rightarrow \perp \vdash \perp} *}{A, A \rightarrow \perp, B \rightarrow \perp \vdash B} * \quad \supset_L$$

MULTIPLICATIVE encoding (43ms)

Not provable

CALL-BY-NAME encoding (185ms)

[illegible]

CALL-BY-VALUE encoding (256ms)

[illegible]

01-ENC encoding (259ms)

[illegible]

$$(40) \quad B \vdash (A \wedge B) \leftrightarrow (A)$$

LJ (21ms)

$$\overline{B \vdash A \wedge B \rightarrow A \wedge A \rightarrow A \wedge B}^{\star}$$

MULTIPLICATIVE encoding (34ms)

Not provable

CALL-BY-NAME encoding (34ms)

$$\frac{\overline{\Gamma : \cdot \vdash A} \quad \overline{\Gamma : \cdot \vdash A} \quad \overline{\Gamma : \cdot \vdash B}}{\Gamma : !(B) \vdash !(A \& B) \multimap A \& !(A) \multimap A \& B}^*$$

CALL-BY-VALUE encoding (88ms)

[illegible]

01-ENC encoding (88ms)

$$\begin{array}{c}
\frac{\Gamma; \vdash A \quad \Gamma; \vdash B}{\Gamma; \vdash A \& B} \star \\
\frac{\Gamma; \vdash \neg A}{\Gamma; \vdash \neg !((A) \& !B)) \rightarrow A} \star \quad \frac{\Gamma; \vdash \neg ! (A \& B) !}{\Gamma; \vdash \neg !((A) \rightarrow ! (A \& B))} \star \\
\frac{\Gamma; \vdash \neg !((!(A) \& !B)) \rightarrow A}{\Gamma; \vdash \neg !((!(A) \& !B)) \rightarrow A} ! \quad \frac{\Gamma; \vdash \neg !((A) \rightarrow ! (A \& B))}{\Gamma; \vdash \neg !((!(A) \rightarrow ! (A \& B)))} \star \\
\frac{\Gamma; \vdash \neg !((!(A) \& !B)) \rightarrow A \& !((A) \rightarrow ! (A \& B))}{\Gamma; \vdash \neg !((!(A) \& !B)) \rightarrow A \& !((A) \rightarrow ! (A \& B)))} ! \\
\frac{\Gamma; \vdash !B \rightarrow !((!(A) \& !B)) \rightarrow A \& !((A) \rightarrow ! (A \& B)))}{\Gamma; \vdash !B \rightarrow !((!(A) \& !B)) \rightarrow A \& !((A) \rightarrow ! (A \& B)))} \star
\end{array}$$

$$(41) \neg B \vdash (A \wedge B) \leftrightarrow (B)$$

LJ (37ms)

$$\frac{\frac{\overline{B, B \rightarrow \perp \vdash B}^* \quad \overline{B, \perp \vdash A}^*}{B, B \rightarrow \perp \vdash A} \supset_L}{B \rightarrow \perp \vdash A \wedge B \rightarrow B \wedge B \rightarrow A \wedge B}^*$$

MULTIPLICATIVE encoding (41ms)

Not provable

CALL-BY-NAME encoding (78ms)

$$\frac{\frac{\frac{\Gamma; \cdot \vdash B}{\Gamma; \cdot \vdash !(B)} ! \quad \frac{\Gamma; \mathbf{0} \vdash A}{\Gamma; \cdot \vdash A} \star}{\frac{\Gamma; \cdot \vdash !(B) \multimap \mathbf{0} \vdash A}{\Gamma; \cdot \vdash A} \multimap}{\frac{\Gamma; \cdot \vdash B}{\Gamma; \cdot \vdash !(B) \multimap \mathbf{0} \vdash A} D_C \quad \frac{\Gamma; \cdot \vdash B}{\Gamma; \cdot \vdash A} \star} \star$$

CALL-BY-VALUE encoding (172ms)

[illegible]

01-ENC encoding (194ms)

[illegible]

(42) $\vdash A \rightarrow \neg\neg A$

LJ (38ms)

$$\frac{\frac{\frac{A, A \rightarrow \perp \vdash A}{\vdash A} \star \quad \frac{A, \perp \vdash \perp}{\vdash \perp} \star}{A, A \rightarrow \perp \vdash \perp} \quad \supset_L}{\vdash A \rightarrow A \rightarrow \perp \rightarrow \perp} \star$$

MULTIPLICATIVE encoding (41ms)

$$\frac{\frac{\Gamma : A \vdash A \quad \Gamma : \perp \vdash \perp}{\Gamma : A, A \multimap \perp \vdash \perp} \multimap \quad \Gamma : \cdot \vdash A \multimap A \multimap \perp \multimap \perp}{\Gamma : \cdot \vdash A \multimap A \multimap \perp \multimap \perp} \star$$

CALL-BY-NAME encoding (66ms)

$$\frac{\frac{\frac{\Gamma : \cdot \vdash \overline{A}}{\Gamma : \cdot \vdash !(A)} \quad ! \quad \frac{}{\Gamma : \mathbf{0} \vdash \mathbf{0}} \quad \star}{\Gamma : !(A) \multimap \mathbf{0} \vdash \mathbf{0}} \quad \multimap}{\Gamma : \cdot \vdash \mathbf{0}} \quad D_C}{\Gamma : \cdot \vdash !(A) \multimap !(A) \multimap \mathbf{0} \multimap \mathbf{0}} \quad \star$$

CALL-BY-VALUE encoding (84ms)

$$\frac{\frac{\frac{\Gamma : \cdot \vdash A}{\Gamma : \cdot \vdash !(A)} !}{\Gamma : !(A) \multimap \mathbf{0} \vdash \mathbf{0}} !}{\Gamma : \cdot \vdash \mathbf{0}} D_C \multimap \frac{\frac{\Gamma : \cdot \vdash !(A)}{\Gamma : \cdot \vdash !(A) \multimap \mathbf{0} \vdash \mathbf{0}} !}{\Gamma : \cdot \vdash !(A) \multimap !(A \multimap \mathbf{0} \multimap \mathbf{0})} \star \frac{\Gamma : \cdot \vdash !(A) \multimap !(A \multimap \mathbf{0} \multimap \mathbf{0})}{\Gamma : \cdot \vdash !(A) \multimap !(A \multimap \mathbf{0} \multimap \mathbf{0})} \star$$

01-ENC encoding (81ms)

$$\frac{\frac{\frac{\Gamma : \cdot \vdash A}{\Gamma : \cdot \vdash !(A)} ! \quad \frac{}{\Gamma : !(0) \vdash 0} \star}{\Gamma : !(A) \multimap !(0) \vdash 0} \multimap \quad \frac{}{\Gamma : \cdot \vdash 0} D_C}{\frac{\frac{\Gamma : \cdot \vdash !(A) \multimap !(0) \vdash 0}{\Gamma : \cdot \vdash !(A) \multimap !(0) \vdash 0} \star}{\Gamma : \cdot \vdash !(A) \multimap !(0) \vdash 0} !} \star$$

$$(43) \cdot \vdash (\neg\neg\neg A) \leftrightarrow (\neg A)$$

LJ (81ms)

[illegible]

MULTIPLICATIVE encoding (105ms)

$$\frac{\frac{\frac{\Gamma: A \vdash A \quad \Gamma: \perp \vdash}{\Gamma: A \vdash A \vdash \perp} \multimap}{\Gamma: A \vdash A \vdash \perp \vdash \perp} \multimap}{\frac{\frac{\Gamma: A \vdash A \vdash \perp \vdash \perp}{\Gamma: A \vdash A \vdash \perp \vdash \perp \vdash \perp} \multimap}{\frac{\Gamma: A \vdash A \vdash \perp \vdash \perp \vdash \perp \vdash \perp}{\Gamma: A \vdash A \vdash \perp \vdash \perp \vdash \perp \vdash \perp \vdash \perp} \multimap} \multimap$$

CALL-BY-NAME encoding (244ms)

[illegible]

CALL-BY-VALUE encoding (295ms)

[illegible]

01-ENC encoding (343ms)

The diagram illustrates the decomposition of a polynomial P into a sum of products of polynomials Q_i and R_i . The polynomials are arranged in a hierarchical structure, with arrows indicating the flow of the decomposition. The polynomials involve variables $x, y, z, w, v, u, t, s, r, q, p, n, m, l, k, j, i, h, g, f, e, d, c, b, a$ and various constants. The diagram is a visual representation of a complex algebraic identity.

(44) $\vdash \neg A \wedge \neg A$

LJ (38ms)

$$\frac{\frac{\frac{A, A \rightarrow \perp \vdash A}{A, A \rightarrow \perp \vdash \perp} \star}{\vdash A \wedge A \rightarrow \perp \rightarrow \perp} \star}{\vdash A \wedge A \rightarrow \perp \rightarrow \perp} \star \supset_L$$

MULTIPLICATIVE encoding (41ms)

$$\frac{\frac{\Gamma : A \vdash A \quad \Gamma : \perp \vdash \perp}{\Gamma : A, A \multimap \perp \vdash \perp} \multimap \quad \Gamma : \cdot \vdash A \otimes A \multimap \perp \multimap \perp}{\Gamma : \cdot \vdash A \otimes A \multimap \perp \multimap \perp} \star$$

CALL-BY-NAME encoding (65ms)

$$\frac{\frac{\frac{\Gamma : \cdot \vdash A}{\Gamma : \cdot \vdash !(A)} \quad ! \quad \frac{}{\Gamma : \mathbf{0} \vdash \mathbf{0}} \quad \star}{\frac{}{\Gamma : !(A) \multimap \mathbf{0} \vdash \mathbf{0}} \quad \multimap} \quad D_C \quad \frac{}{\Gamma : \cdot \vdash \mathbf{0}}}{\Gamma : \cdot \vdash !(A \& !(A) \multimap \mathbf{0}) \multimap \mathbf{0}} \quad \star$$

CALL-BY-VALUE encoding (70ms)

$$\frac{\frac{\frac{\Gamma : \cdot \vdash \overline{A}}{\Gamma : \cdot \vdash !(A)} ! \quad \frac{}{\Gamma : \mathbf{0} \vdash \mathbf{0}} \star}{\frac{}{\Gamma : !(A) \multimap \mathbf{0} \vdash \mathbf{0}} \multimap} \quad \frac{}{\Gamma : \cdot \vdash \mathbf{0}} D_C}{\frac{}{\Gamma : \cdot \vdash !(A) \otimes !(A) \multimap \mathbf{0} \multimap \mathbf{0}} \star} \quad \frac{}{\Gamma : \cdot \vdash !(A) \otimes !(A) \multimap \mathbf{0} \multimap \mathbf{0}} !$$

01-ENC encoding (70ms)

$$\frac{\frac{\frac{\Gamma; \cdot \vdash A}{\Gamma; \cdot \vdash !(A)} \quad ! \quad \frac{\Gamma; !(0) \vdash 0}{\Gamma; !(A) \multimap !(0) \vdash 0} \quad \star}{\Gamma; !(A) \multimap !(0) \vdash 0} \quad \multimap}{\Gamma; \cdot \vdash 0} \quad D_C}{\frac{\Gamma; \cdot \vdash !(A) \& !(A) \multimap !(0)) \quad \star}{\Gamma; \cdot \vdash !(A) \& !(A) \multimap !(0)) \quad 0} \quad \star} \quad !$$

$$(45) \cdot \vdash \neg(A) \leftrightarrow (\neg A)$$

LJ (80ms)

$$\frac{\frac{\frac{A \rightarrow A \rightarrow \perp, A \rightarrow \perp \rightarrow A \rightarrow \perp}{A \rightarrow A \rightarrow \perp \rightarrow A \rightarrow \perp} \supset_L \quad \frac{A \rightarrow \perp, A \rightarrow \perp \rightarrow A \rightarrow \perp}{A \rightarrow \perp \rightarrow A \rightarrow \perp} \supset_L}{\frac{A \rightarrow A \rightarrow \perp \rightarrow A \rightarrow \perp \rightarrow A \rightarrow \perp}{A \rightarrow A \rightarrow \perp \rightarrow A \rightarrow \perp} \supset_L} \supset_L \quad \frac{A \rightarrow A \rightarrow \perp \rightarrow A \rightarrow \perp}{A \rightarrow A \rightarrow \perp \rightarrow A \rightarrow \perp} \supset_L \supset_L$$

MULTIPLICATIVE encoding (41ms)

Not provable

CALL-BY-NAME encoding (231ms)

[illegible]

CALL-BY-VALUE encoding (282ms)

The diagram illustrates a complex proof tree for the formula $A \supset B$. The root node is $A \supset B$. The tree branches into two main paths. The left path involves assumptions $A \supset A$ and $A \supset A$, leading to $A \supset A \supset A$, then $A \supset A \supset A \supset A$, and finally $A \supset A \supset A \supset A \supset A$. The right path involves assumptions $A \supset A$ and $A \supset A$, leading to $A \supset A \supset A$, then $A \supset A \supset A \supset A$, and finally $A \supset A \supset A \supset A \supset A$. The tree concludes with $A \supset B$.

01-ENC encoding (300ms)

[illegible]

$$(46) \cdot \vdash \neg\neg\neg\neg A \rightarrow A$$

LJ (66ms)

[illegible]

MULTIPLICATIVE encoding (41ms)

Not provable

CALL-BY-NAME encoding (198ms)

$$\begin{array}{c}
\frac{\Gamma \vdash \neg \vdash \overline{A}}{\Gamma \vdash \neg \vdash \mathfrak{U}(\mathfrak{U}(A) \rightarrow \neg 0) \rightarrow \neg A}^* \\
\frac{\Gamma \vdash \neg \vdash \mathfrak{U}(\mathfrak{U}(\mathfrak{U}(A) \rightarrow \neg 0) \rightarrow \neg A)}{\Gamma \vdash \neg \vdash \mathfrak{U}(\mathfrak{U}(\mathfrak{U}(A) \rightarrow \neg 0) \rightarrow \neg A) \rightarrow \mathbf{0}}^* \rightarrow \\
\frac{\Gamma \vdash \neg \vdash \mathbf{0}}{\Gamma \vdash \neg \vdash \mathfrak{U}(A) \rightarrow \mathbf{0}} D_C \\
\frac{\Gamma \vdash \neg \vdash \overline{A}}{\Gamma \vdash \neg \vdash \mathfrak{U}(A) \rightarrow \mathbf{0}}^* \\
\frac{\Gamma \vdash \neg \vdash \mathfrak{U}(A) \rightarrow \mathbf{0}}{\Gamma \vdash \neg \vdash \mathfrak{U}(\mathfrak{U}(A) \rightarrow \mathbf{0}) \rightarrow \mathbf{0}}^* \rightarrow \\
\frac{\Gamma \vdash \neg \vdash \mathfrak{U}(\mathfrak{U}(A) \rightarrow \mathbf{0}) \rightarrow \mathbf{0}}{\Gamma \vdash \neg \vdash \mathbf{0}} D_C \\
\frac{\Gamma \vdash \neg \vdash \mathbf{0}}{\Gamma \vdash \neg \vdash \mathfrak{U}(A) \rightarrow \mathbf{0}}^* \\
\frac{\Gamma \vdash \neg \vdash \mathfrak{U}(A) \rightarrow \mathbf{0}}{\Gamma \vdash \neg \vdash \mathfrak{U}(\mathfrak{U}(A) \rightarrow \mathbf{0})}^* \\
\frac{\Gamma \vdash \neg \vdash \mathfrak{U}(\mathfrak{U}(A) \rightarrow \neg 0) \rightarrow \neg A}{\Gamma \vdash \neg \vdash \mathfrak{U}(\mathfrak{U}(A) \rightarrow \neg 0) \rightarrow \neg A}^* \\
\frac{\Gamma \vdash \neg \vdash \mathfrak{U}(\mathfrak{U}(A) \rightarrow \neg 0) \rightarrow \neg A}{\Gamma \vdash \neg \vdash \mathfrak{U}(\mathfrak{U}(A) \rightarrow \neg 0) \rightarrow \neg A}^* \\
\frac{\Gamma \vdash \neg \vdash \mathfrak{U}(\mathfrak{U}(A) \rightarrow \neg 0) \rightarrow \neg A}{\Gamma \vdash \neg \vdash \mathfrak{U}(\mathfrak{U}(\mathfrak{U}(A) \rightarrow \neg 0) \rightarrow \neg A) \rightarrow \mathbf{0}}^* \rightarrow \\
\frac{\Gamma \vdash \neg \vdash \mathfrak{U}(\mathfrak{U}(\mathfrak{U}(A) \rightarrow \neg 0) \rightarrow \neg A) \rightarrow \mathbf{0}}{\Gamma \vdash \neg \vdash \mathbf{0}} D_C \\
\frac{\Gamma \vdash \neg \vdash \mathbf{0}}{\Gamma \vdash \neg \vdash \mathfrak{U}(\mathfrak{U}(\mathfrak{U}(A) \rightarrow \neg 0) \rightarrow \neg A) \rightarrow \mathbf{0}}^*
\end{array}$$

CALL-BY-VALUE encoding (210ms)

[illegible]

01-ENC encoding (217ms)

[illegible]

$$(47) \cdot \vdash (A \wedge B \wedge \neg B) \leftrightarrow (B \wedge \neg B)$$

LJ (61ms)

$$\frac{\frac{A, B, B \rightarrow \perp \vdash B}{A, B, B \rightarrow \perp \vdash \perp}^* \quad \frac{A, B, B \vdash \perp \vdash \perp}{A, B, B \rightarrow \perp \vdash \perp}^* \quad \frac{B, B \rightarrow \perp \vdash B}{B, B \rightarrow \perp \vdash A}^* \quad \frac{B, B \rightarrow \perp \vdash B}{B, B \rightarrow \perp \vdash \perp}^* \quad \frac{B, B \rightarrow \perp \vdash B}{B, B \rightarrow \perp \vdash \perp}^*}{\vdash A \wedge B \wedge B \rightarrow \perp \rightarrow B \wedge B \rightarrow \perp \wedge B \wedge B \rightarrow \perp \rightarrow A \wedge B \wedge B \rightarrow \perp}^* \quad \supset_L \quad \supset_L$$

MULTIPLICATIVE encoding (42ms)

Not provable

CALL-BY-NAME encoding (121ms)

[illegible]

CALL-BY-VALUE encoding (140ms)

[illegible]

01-ENC encoding (132ms)

[illegible]

$$(48) \cdot \vdash A \rightarrow B \rightarrow \neg A \wedge \neg B$$

LJ (49ms)

$$\frac{\frac{\frac{A, A \rightarrow B, B \rightarrow \perp \vdash A}{A, A \rightarrow B, B \rightarrow \perp \vdash \perp} \star \quad \frac{\frac{A, B, B \rightarrow \perp \vdash B}{A, B, \perp \vdash \perp} \star}{A, B, B \rightarrow \perp \vdash \perp} \supset_L}{\frac{A, A \rightarrow B, B \rightarrow \perp \vdash \perp}{\vdash A \rightarrow B \rightarrow A \wedge B \rightarrow \perp \rightarrow \perp} \star} \supset_L$$

MULTIPLICATIVE encoding (48ms)

$$\frac{\frac{\Gamma : A \vdash A}{\Gamma : B, B \multimap \perp \vdash \perp} \multimap \quad \frac{\Gamma : B \vdash B \quad \Gamma : \perp \vdash \perp}{\Gamma : B, B \multimap \perp \vdash \perp} \multimap}{\Gamma : A, A \multimap B, B \multimap \perp \vdash \perp} \multimap \quad \frac{\Gamma : \cdot \vdash A \multimap B \multimap A \otimes B \multimap \perp \multimap \perp}{\Gamma : \cdot \vdash A \multimap B \multimap A \otimes B \multimap \perp \multimap \perp} \star$$

CALL-BY-NAME encoding (143ms)

$$\begin{array}{c}
\frac{\Gamma; \vdash A}{\Gamma; \vdash ! (A)} \dagger \quad \frac{\Gamma; \vdash B}{\Gamma; \vdash \overline{B}} \\
\hline
\frac{\Gamma; !(A) \multimap B \vdash B}{\Gamma; !(A) \multimap B \vdash B} D_C \quad \multimap \\
\hline
\frac{\Gamma; \vdash B}{\Gamma; \vdash ! (B)} \dagger \quad \frac{\Gamma; \mathbf{0} \vdash B}{\Gamma; \mathbf{0} \vdash B} * \\
\hline
\frac{\Gamma; !(B) \multimap \mathbf{0} \vdash B}{\Gamma; !(B) \multimap \mathbf{0} \vdash B} D_C \quad \multimap \\
\hline
\frac{\Gamma; \vdash B}{\Gamma; \vdash ! (B)} \dagger \quad \frac{\Gamma; \mathbf{0} \vdash B}{\Gamma; \mathbf{0} \vdash B} * \\
\hline
\frac{\Gamma; !(B) \multimap \mathbf{0} \vdash B}{\Gamma; !(B) \multimap \mathbf{0} \vdash B} D_C \quad \multimap \\
\hline
\frac{\Gamma; \vdash B}{\Gamma; \vdash ! (B)} \dagger \quad \frac{\Gamma; \mathbf{0} \vdash B}{\Gamma; \mathbf{0} \vdash B} * \\
\hline
\frac{\Gamma; !(B) \multimap \mathbf{0} \vdash \mathbf{0}}{\Gamma; !(B) \multimap \mathbf{0} \vdash \mathbf{0}} D_C \quad \multimap \\
\hline
\frac{\Gamma; \vdash ! (A) \multimap B \vdash ! (A \& ! (B) \multimap \mathbf{0})}{\Gamma; \vdash ! (A) \multimap B \vdash ! (A \& ! (B) \multimap \mathbf{0})} *
\end{array}$$

CALL-BY-VALUE encoding (167ms)

[illegible]

01-ENC encoding (169ms)

$$\begin{array}{c}
\frac{\Gamma \vdash \vdash A \quad !}{\Gamma \vdash \vdash ! (A)} \quad \frac{\Gamma \vdash \vdash B}{\Gamma \vdash \vdash ! (B)} \quad * \\
\frac{\Gamma \vdash ! (A) \rightarrow ! (B) \vdash B}{\Gamma \vdash ! (A) \rightarrow ! (B) \vdash B} \quad D_C \quad \rightarrow \\
\frac{\Gamma \vdash \vdash B \quad !}{\Gamma \vdash \vdash ! (B)} \quad \frac{\Gamma \vdash ! (0) \vdash B}{\Gamma \vdash ! (0) \vdash B} \quad * \\
\frac{\Gamma \vdash ! (B) \rightarrow ! (0) \vdash B}{\Gamma \vdash \vdash ! (B)} \quad D_C \quad \rightarrow \\
\frac{\Gamma \vdash \vdash B \quad !}{\Gamma \vdash \vdash ! (B)} \quad \frac{\Gamma \vdash ! (0) \vdash B}{\Gamma \vdash ! (0) \vdash B} \quad * \\
\frac{\Gamma \vdash ! (B) \rightarrow ! (0) \vdash B}{\Gamma \vdash \vdash ! (B)} \quad D_C \quad \rightarrow \\
\frac{\Gamma \vdash \vdash B \quad !}{\Gamma \vdash \vdash ! (B)} \quad \frac{\Gamma \vdash ! (0) \vdash B}{\Gamma \vdash ! (0) \vdash B} \quad * \\
\frac{\Gamma \vdash \vdash ! (B)}{\Gamma \vdash \vdash ! (B)} \quad \frac{\Gamma \vdash ! (0) \rightarrow ! (0) \vdash 0}{\Gamma \vdash ! (0) \rightarrow ! (0) \vdash 0} \quad D_C \quad \rightarrow \\
\frac{\Gamma \vdash \vdash 0}{\Gamma \vdash \vdash 0} \quad D_C \quad \rightarrow \\
\frac{\Gamma \vdash \vdash ! (! (A) \wedge ! (! (B) \rightarrow ! (0)))}{\Gamma \vdash \vdash ! (! (A) \wedge ! (! (B) \rightarrow ! (0)))} \quad * \\
\frac{\Gamma \vdash \vdash ! (! (A) \rightarrow ! (B)) \rightarrow ! (! (A) \wedge ! (! (B) \rightarrow ! (0)))}{\Gamma \vdash \vdash ! (! (A) \rightarrow ! (B)) \rightarrow ! (! (A) \wedge ! (! (B) \rightarrow ! (0)))} \quad * \\
\frac{\Gamma \vdash \vdash ! (! (A) \rightarrow ! (B)) \rightarrow ! (! (! (A) \rightarrow ! (B)) \rightarrow ! (! (0) \vdash 0))}{\Gamma \vdash \vdash ! (! (A) \rightarrow ! (B)) \rightarrow ! (! (! (A) \rightarrow ! (B)) \rightarrow ! (! (0) \vdash 0))} \quad *
\end{array}$$

$$(49) \cdot \vdash (A \rightarrow \neg B) \leftrightarrow (\neg A \wedge B)$$

LJ (58ms)

[illegible]

MULTIPLICATIVE encoding (88ms)

[illegible]

CALL-BY-NAME encoding (118ms)

[illegible]

CALL-BY-VALUE encoding (281ms)

[illegible]

01-ENC encoding (318ms)

The diagram illustrates the decomposition of a polynomial P into a sum of squares (SOS) of other polynomials. The process is shown in a grid of boxes, with arrows indicating the relationships and substitutions between the polynomials.

Top Row:

- $P_{1,1} = x^2 + y^2 + z^2$
- $P_{1,2} = x^2 + y^2 + z^2$
- $P_{1,3} = x^2 + y^2 + z^2$
- $P_{1,4} = x^2 + y^2 + z^2$
- $P_{1,5} = x^2 + y^2 + z^2$
- $P_{1,6} = x^2 + y^2 + z^2$
- $P_{1,7} = x^2 + y^2 + z^2$
- $P_{1,8} = x^2 + y^2 + z^2$
- $P_{1,9} = x^2 + y^2 + z^2$
- $P_{1,10} = x^2 + y^2 + z^2$
- $P_{1,11} = x^2 + y^2 + z^2$
- $P_{1,12} = x^2 + y^2 + z^2$
- $P_{1,13} = x^2 + y^2 + z^2$
- $P_{1,14} = x^2 + y^2 + z^2$
- $P_{1,15} = x^2 + y^2 + z^2$
- $P_{1,16} = x^2 + y^2 + z^2$
- $P_{1,17} = x^2 + y^2 + z^2$
- $P_{1,18} = x^2 + y^2 + z^2$
- $P_{1,19} = x^2 + y^2 + z^2$
- $P_{1,20} = x^2 + y^2 + z^2$
- $P_{1,21} = x^2 + y^2 + z^2$
- $P_{1,22} = x^2 + y^2 + z^2$
- $P_{1,23} = x^2 + y^2 + z^2$
- $P_{1,24} = x^2 + y^2 + z^2$
- $P_{1,25} = x^2 + y^2 + z^2$
- $P_{1,26} = x^2 + y^2 + z^2$
- $P_{1,27} = x^2 + y^2 + z^2$
- $P_{1,28} = x^2 + y^2 + z^2$
- $P_{1,29} = x^2 + y^2 + z^2$
- $P_{1,30} = x^2 + y^2 + z^2$
- $P_{1,31} = x^2 + y^2 + z^2$
- $P_{1,32} = x^2 + y^2 + z^2$
- $P_{1,33} = x^2 + y^2 + z^2$
- $P_{1,34} = x^2 + y^2 + z^2$
- $P_{1,35} = x^2 + y^2 + z^2$
- $P_{1,36} = x^2 + y^2 + z^2$
- $P_{1,37} = x^2 + y^2 + z^2$
- $P_{1,38} = x^2 + y^2 + z^2$
- $P_{1,39} = x^2 + y^2 + z^2$
- $P_{1,40} = x^2 + y^2 + z^2$
- $P_{1,41} = x^2 + y^2 + z^2$
- $P_{1,42} = x^2 + y^2 + z^2$
- $P_{1,43} = x^2 + y^2 + z^2$
- $P_{1,44} = x^2 + y^2 + z^2$
- $P_{1,45} = x^2 + y^2 + z^2$
- $P_{1,46} = x^2 + y^2 + z^2$
- $P_{1,47} = x^2 + y^2 + z^2$
- $P_{1,48} = x^2 + y^2 + z^2$
- $P_{1,49} = x^2 + y^2 + z^2$
- $P_{1,50} = x^2 + y^2 + z^2$
- $P_{1,51} = x^2 + y^2 + z^2$
- $P_{1,52} = x^2 + y^2 + z^2$
- $P_{1,53} = x^2 + y^2 + z^2$
- $P_{1,54} = x^2 + y^2 + z^2$
- $P_{1,55} = x^2 + y^2 + z^2$
- $P_{1,56} = x^2 + y^2 + z^2$
- $P_{1,57} = x^2 + y^2 + z^2$
- $P_{1,58} = x^2 + y^2 + z^2$
- $P_{1,59} = x^2 + y^2 + z^2$
- $P_{1,60} = x^2 + y^2 + z^2$
- $P_{1,61} = x^2 + y^2 + z^2$
- $P_{1,62} = x^2 + y^2 + z^2$
- $P_{1,63} = x^2 + y^2 + z^2$
- $P_{1,64} = x^2 + y^2 + z^2$
- $P_{1,65} = x^2 + y^2 + z^2$
- $P_{1,66} = x^2 + y^2 + z^2$
- $P_{1,67} = x^2 + y^2 + z^2$
- $P_{1,68} = x^2 + y^2 + z^2$
- $P_{1,69} = x^2 + y^2 + z^2$
- $P_{1,70} = x^2 + y^2 + z^2$
- $P_{1,71} = x^2 + y^2 + z^2$
- $P_{1,72} = x^2 + y^2 + z^2$
- $P_{1,73} = x^2 + y^2 + z^2$
- $P_{1,74} = x^2 + y^2 + z^2$
- $P_{1,75} = x^2 + y^2 + z^2$
- $P_{1,76} = x^2 + y^2 + z^2$
- $P_{1,77} = x^2 + y^2 + z^2$
- $P_{1,78} = x^2 + y^2 + z^2$
- $P_{1,79} = x^2 + y^2 + z^2$
- $P_{1,80} = x^2 + y^2 + z^2$
- $P_{1,81} = x^2 + y^2 + z^2$
- $P_{1,82} = x^2 + y^2 + z^2$
- $P_{1,83} = x^2 + y^2 + z^2$
- $P_{1,84} = x^2 + y^2 + z^2$
- $P_{1,85} = x^2 + y^2 + z^2$
- $P_{1,86} = x^2 + y^2 + z^2$
- $P_{1,87} = x^2 + y^2 + z^2$
- $P_{1,88} = x^2 + y^2 + z^2$
- $P_{1,89} = x^2 + y^2 + z^2$
- $P_{1,90} = x^2 + y^2 + z^2$
- $P_{1,91} = x^2 + y^2 + z^2$
- $P_{1,92} = x^2 + y^2 + z^2$
- $P_{1,93} = x^2 + y^2 + z^2$
- $P_{1,94} = x^2 + y^2 + z^2$
- $P_{1,95} = x^2 + y^2 + z^2$
- $P_{1,96} = x^2 + y^2 + z^2$
- $P_{1,97} = x^2 + y^2 + z^2$
- $P_{1,98} = x^2 + y^2 + z^2$
- $P_{1,99} = x^2 + y^2 + z^2$
- $P_{1,100} = x^2 + y^2 + z^2$
- $P_{1,101} = x^2 + y^2 + z^2$
- $P_{1,102} = x^2 + y^2 + z^2$
- $P_{1,103} = x^2 + y^2 + z^2$
- $P_{1,104} = x^2 + y^2 + z^2$
- $P_{1,105} = x^2 + y^2 + z^2$
- $P_{1,106} = x^2 + y^2 + z^2$
- $P_{1,107} = x^2 + y^2 + z^2$
- $P_{1,108} = x^2 + y^2 + z^2$
- $P_{1,109} = x^2 + y^2 + z^2$
- $P_{1,110} = x^2 + y^2 + z^2$
- $P_{1,111} = x^2 + y^2 + z^2$
- $P_{1,112} = x^2 + y^2 + z^2$
- $P_{1,113} = x^2 + y^2 + z^2$
- $P_{1,114} = x^2 + y^2 + z^2$
- $P_{1,115} = x^2 + y^2 + z^2$
- $P_{1,116} = x^2 + y^2 + z^2$
- $P_{1,117} = x^2 + y^2 + z^2$
- $P_{1,118} = x^2 + y^2 + z^2$
- $P_{1,119} = x^2 + y^2 + z^2$
- $P_{1,120} = x^2 + y^2 + z^2$
- $P_{1,121} = x^2 + y^2 + z^2$
- $P_{1,122} = x^2 + y^2 + z^2$
- $P_{1,123} = x^2 + y^2 + z^2$
- $P_{1,124} = x^2 + y^2 + z^2$
- $P_{1,125} = x^2 + y^2 + z^2$
- $P_{1,126} = x^2 + y^2 + z^2$
- $P_{1,127} = x^2 + y^2 + z^2$
- $P_{1,128} = x^2 + y^2 + z^2$
- $P_{1,129} = x^2 + y^2 + z^2$
- $P_{1,130} = x^2 + y^2 + z^2$
- $P_{1,131} = x^2 + y^2 + z^2$
- $P_{1,132} = x^2 + y^2 + z^2$
- $P_{1,133} = x^2 + y^2 + z^2$
- $P_{1,134} = x^2 + y^2 + z^2$
- $P_{1,135} = x^2 + y^2 + z^2$
- $P_{1,136} = x^2 + y^2 + z^2$
- $P_{1,137} = x^2 + y^2 + z^2$
- $P_{1,138} = x^2 + y^2 + z^2$

$$(50) \cdot \vdash (\neg A \wedge B) \leftrightarrow (\neg\neg A \rightarrow \neg B)$$

LJ (93ms)

MULTIPLICATIVE encoding (134ms)

[illegible]

CALL-BY-NAME encoding (333ms)

The diagram illustrates the decomposition of a polynomial P into a sum of products of polynomials Q_i and R_i . The process is shown as a series of steps, with each step involving a polynomial multiplication or division. The final result is a sum of products of polynomials Q_i and R_i , where the Q_i are polynomials of degree at most d_i and the R_i are polynomials of degree at most d_i .

CALL-BY-VALUE encoding (421ms)

The diagram illustrates the decomposition of a polynomial P into a sum of products of polynomials Q_i and R_i . The decomposition is recursive, with each layer further breaking down the polynomials into simpler components. The layers are labeled with P , Q_i , R_i , and D_i . The diagram uses various symbols like boxes, arrows, and subscripts to indicate the relationships between the polynomials. The final result is a sum of products of polynomials Q_i and R_i , where the Q_i are further decomposed into R_i and D_i .

01-ENC encoding (470ms)

The diagram illustrates the structure of the 2019-2020 academic year, organized into a hierarchical timeline from September to June. It features various events and activities marked along the timeline, with specific dates and descriptions provided for each. The diagram is divided into several sections, each representing a different stage of the year. The top section shows the overall timeline, while the bottom section provides a detailed breakdown of the activities for each month. The diagram is color-coded and includes many small text labels and arrows indicating the flow of the year.

$$(51) \neg\neg B \rightarrow B \vdash (\neg\neg A \rightarrow B) \leftrightarrow (A \rightarrow B)$$

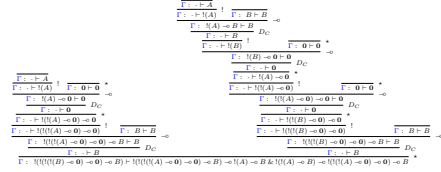
LJ (135ms)



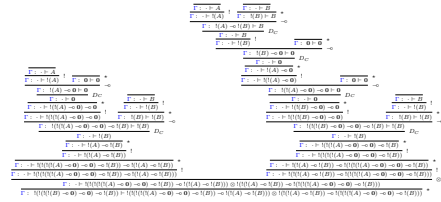
MULTIPLICATIVE encoding (156ms)



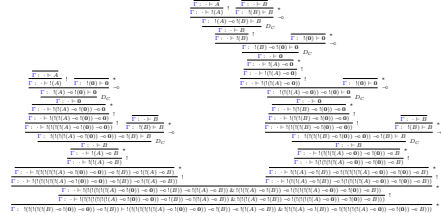
CALL-BY-NAME encoding (278ms)



CALL-BY-VALUE encoding (414ms)



01-ENC encoding (449ms)



$$(52) \neg\neg B \rightarrow B \vdash (A \rightarrow B) \leftrightarrow (\neg A \wedge \neg B)$$

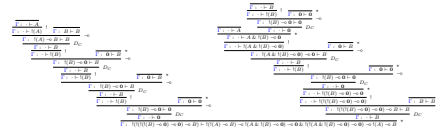
LJ (94ms)



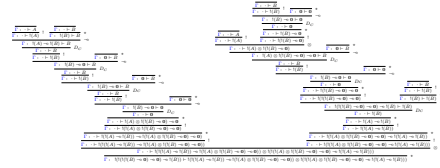
MULTIPLICATIVE encoding (134ms)



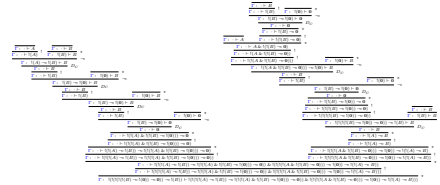
CALL-BY-NAME encoding (296ms)



CALL-BY-VALUE encoding (457ms)



01-ENC encoding (489ms)



$$(53) \cdot \vdash \neg\neg A \rightarrow B \rightarrow \neg A \wedge \neg B$$

LJ (65ms)

$$\begin{array}{c}
 \frac{A, A \rightarrow \perp, B \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \rightarrow \perp \rightarrow B \vdash A}{A, A, B \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \rightarrow \perp \rightarrow B \vdash B} \supset_L \\
 \frac{\frac{A, A \rightarrow \perp, B \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \rightarrow \perp \rightarrow B \vdash A}{A, B \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \rightarrow \perp \rightarrow B \vdash A \rightarrow \perp \rightarrow \perp} \supset^*}{\frac{A, B \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \rightarrow \perp \rightarrow B \vdash A \rightarrow \perp \rightarrow \perp}{A, B \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \rightarrow \perp \rightarrow B \vdash B} \supset_L} \frac{A, A, B \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \rightarrow \perp \rightarrow B \vdash B}{A, B \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \rightarrow \perp \rightarrow B \vdash \perp} \supset_L \\
 \frac{A, B \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \rightarrow \perp \rightarrow B \vdash \perp}{\vdash A \rightarrow \perp \rightarrow \perp \rightarrow B \rightarrow A \wedge B \rightarrow \perp \rightarrow \perp} \supset^*
 \end{array}$$

MULTIPLICATIVE encoding (75ms)

$$\frac{\frac{\frac{\Gamma: A \vdash A \quad \Gamma: \perp \vdash \perp}{\Gamma: A, A \multimap \perp \vdash \perp} \multimap \quad \frac{\Gamma: B \vdash B \quad \Gamma: \perp \vdash \perp}{\Gamma: B, B \multimap \perp \vdash \perp} \multimap}{\frac{\Gamma: A \multimap \perp \multimap \perp \vdash \perp \quad \Gamma: B, B \multimap \perp \vdash \perp}{\Gamma: A, B \multimap \perp, A \multimap \perp \multimap \perp \vdash \perp \multimap B \vdash \perp} \multimap} \frac{}{\Gamma: \vdash A \multimap \perp \multimap \perp \multimap \perp \multimap B \multimap A \otimes B \multimap \perp \multimap \perp} *$$

CALL-BY-NAME encoding (178ms)

$$\begin{array}{c}
\frac{\Gamma \vdash A}{\Gamma \vdash ! (A)} \quad \Gamma \vdash \mathbf{0} \vdash^* \\
\frac{\Gamma \vdash ! (A) \rightarrow \mathbf{0} \vdash^*}{\Gamma \vdash \mathbf{0} \vdash^*} D_C \\
\frac{\Gamma \vdash \mathbf{0} \vdash^*}{\Gamma \vdash ! (! (A) \rightarrow \mathbf{0}) \vdash^*} \\
\frac{\Gamma \vdash \mathbf{0} \vdash^* \quad \Gamma \vdash B \vdash B}{\Gamma \vdash ! (! (! (A) \rightarrow \mathbf{0}) \rightarrow \mathbf{0}) \vdash^*} \\
\frac{\Gamma \vdash ! (! (! (A) \rightarrow \mathbf{0}) \rightarrow \mathbf{0}) \rightarrow B \vdash B}{\Gamma \vdash \mathbf{0} \vdash B} D_C \\
\frac{\Gamma \vdash \mathbf{0} \vdash B}{\Gamma \vdash \mathbf{0} \vdash ! (B)} \quad \Gamma \vdash \mathbf{0} \vdash B^* \\
\frac{\Gamma \vdash \mathbf{0} \vdash ! (B)}{\Gamma \vdash ! (B)} \quad \Gamma \vdash ! (B) \rightarrow \mathbf{0} \vdash B \quad \Gamma \vdash \mathbf{0} \vdash B^* \\
\frac{\Gamma \vdash ! (B) \rightarrow \mathbf{0} \vdash B \quad \Gamma \vdash \mathbf{0} \vdash B^*}{\Gamma \vdash \mathbf{0} \vdash ! (B)} D_C \\
\frac{\Gamma \vdash ! (B) \rightarrow \mathbf{0} \vdash \mathbf{0} \quad \Gamma \vdash \mathbf{0} \vdash \mathbf{0}}{\Gamma \vdash \mathbf{0} \vdash \mathbf{0}} D_C \\
\frac{\Gamma \vdash \mathbf{0} \vdash \mathbf{0} \quad \Gamma \vdash ! (! (! (! (A) \rightarrow \mathbf{0}) \rightarrow \mathbf{0}) \rightarrow \mathbf{0}) \rightarrow \mathbf{0} \vdash \mathbf{0}}{\Gamma \vdash \mathbf{0} \vdash ! (! (! (! (! (A) \rightarrow \mathbf{0}) \rightarrow \mathbf{0}) \rightarrow \mathbf{0}) \rightarrow \mathbf{0}) \rightarrow \mathbf{0}) \vdash \mathbf{0}}
\end{array}$$

CALL-BY-VALUE encoding (204ms)

[illegible]

01-ENC encoding (215ms)

[illegible]

$$(54) \cdot \vdash A \wedge B \rightarrow \neg A \rightarrow \neg B$$

LJ (47ms)

$$\frac{\frac{\frac{A, B, A \rightarrow B \rightarrow \perp \perp A}{A, B, A \rightarrow B \rightarrow \perp \perp \perp}^* \quad \frac{\frac{A, B, B \rightarrow \perp \perp B}{A, B, \perp \perp \perp}^*}{A, B, B \rightarrow \perp \perp \perp}^*}{\vdash A \wedge B \rightarrow A \rightarrow B \rightarrow \perp \rightarrow \perp}^* \supset_L$$

MULTIPLICATIVE encoding (48ms)

$$\frac{\frac{\Gamma : A \vdash A}{\Gamma : A, B, A \multimap B \multimap \perp \vdash \perp} \multimap \quad \frac{\frac{\Gamma : B \vdash B \quad \Gamma : \perp \vdash \perp}{\Gamma : B, B \multimap \perp \vdash \perp} \multimap}{\Gamma : \cdot \vdash A \otimes B \multimap A \multimap B \multimap \perp \multimap \perp} \star$$

CALL-BY-NAME encoding (77ms)

$$\frac{\frac{\overline{\Gamma : \cdot \vdash A}}{\Gamma : \cdot \vdash !(A)} ! \quad \frac{\frac{\overline{\Gamma : \cdot \vdash B}}{\Gamma : \cdot \vdash !(B)} ! \quad \overline{\Gamma : \mathbf{0} \vdash \mathbf{0}}^*}{\overline{\Gamma : \cdot \vdash !(A)} ! \quad \overline{\Gamma : \cdot \vdash !(B)} ! \quad \overline{\Gamma : \mathbf{0} \vdash \mathbf{0}}^*}_{\neg} \quad \frac{\Gamma : \cdot \vdash \mathbf{0}}{\Gamma : \cdot \vdash !(A) \rightarrow \mathbf{0} \vdash \mathbf{0}}_{D_C} \quad \frac{\Gamma : \cdot \vdash \mathbf{0}}{\Gamma : \cdot \vdash !(A \& B) \rightarrow \mathbf{0} \vdash \mathbf{0}}_{\neg}$$

CALL-BY-VALUE encoding (105ms)

$$\begin{array}{c}
\frac{\Gamma; \cdot \vdash B}{\Gamma; \cdot \vdash ! (B)} ! \quad \frac{\Gamma; \mathbf{0} \vdash \mathbf{0}}{\Gamma; \cdot \vdash \mathbf{0}} \star \\
\frac{\Gamma; \cdot \vdash ! (B) \rightarrow \mathbf{0} \vdash \mathbf{0}}{\Gamma; \cdot \vdash \mathbf{0}} D_C \quad \frac{\Gamma; \cdot \vdash \mathbf{0}}{\Gamma; \cdot \vdash ! (A)} ! \\
\frac{\Gamma; \cdot \vdash ! (A)}{\Gamma; \cdot \vdash ! (A)} ! \quad \frac{\Gamma; \cdot \vdash \mathbf{0}}{\Gamma; \cdot \vdash ! (B) \rightarrow \mathbf{0} \vdash \mathbf{0}} \star \\
\frac{\Gamma; \cdot \vdash ! (A) \rightarrow ! (B) \rightarrow \mathbf{0} \vdash \mathbf{0}}{\Gamma; \cdot \vdash ! (A) \rightarrow ! (B) \rightarrow \mathbf{0} \vdash \mathbf{0}} D_C \quad \frac{\Gamma; \cdot \vdash \mathbf{0}}{\Gamma; \cdot \vdash ! (A) \rightarrow ! (B) \rightarrow \mathbf{0} \vdash \mathbf{0}} \star \\
\frac{\Gamma; \cdot \vdash ! (A) \rightarrow ! (B) \rightarrow \mathbf{0} \vdash \mathbf{0}}{\Gamma; \cdot \vdash ! (A) \rightarrow ! (B) \rightarrow \mathbf{0} \vdash \mathbf{0}} ! \quad \frac{\Gamma; \cdot \vdash ! (A) \otimes ! (B) \rightarrow ! (A) \rightarrow ! (B) \rightarrow \mathbf{0} \vdash \mathbf{0}}{\Gamma; \cdot \vdash ! (A) \otimes ! (B) \rightarrow ! (A) \rightarrow ! (B) \rightarrow \mathbf{0} \vdash \mathbf{0}} \star \\
\frac{\Gamma; \cdot \vdash ! (A) \otimes ! (B) \rightarrow ! (A) \rightarrow ! (B) \rightarrow \mathbf{0} \vdash \mathbf{0}}{\Gamma; \cdot \vdash ! (A) \otimes ! (B) \rightarrow ! (A) \rightarrow ! (B) \rightarrow \mathbf{0} \vdash \mathbf{0}} !
\end{array}$$

01-ENC encoding (106ms)

[illegible]

(55) $\vdash A \wedge \neg B \rightarrow \neg A \rightarrow B$

LJ (48ms)

$$\frac{\frac{\frac{A, A \rightarrow B, B \rightarrow \perp \vdash A}{A, A \rightarrow B, B \rightarrow \perp \vdash \perp} \star \quad \frac{\frac{A, B, B \rightarrow \perp \vdash B}{A, B, \perp \vdash \perp} \star}{A, B, B \rightarrow \perp \vdash \perp} \supset_L}{\frac{A, A \rightarrow B, B \rightarrow \perp \vdash \perp}{\vdash A \wedge B \rightarrow \perp \rightarrow A \rightarrow B \rightarrow \perp} \star} \supset_L$$

MULTIPLICATIVE encoding (47ms)

$$\frac{\frac{\frac{\Gamma : A \vdash A}{\Gamma : B, B \multimap \perp \vdash \perp} \multimap}{\Gamma : A, A \multimap B, B \multimap \perp \vdash \perp} \multimap}{\Gamma : \cdot \vdash A \otimes B \multimap \perp \multimap A \multimap B \multimap \perp} \star$$

CALL-BY-NAME encoding (144ms)

$$\begin{array}{c}
\frac{\Gamma; \vdash A}{\Gamma; \vdash ! (A)} \quad ! \\
\frac{}{\Gamma; !(A) \multimap B \vdash B} D_C \quad \multimap \\
\hline
\frac{\Gamma; \vdash B}{\Gamma; \vdash ! (B)} \quad ! \\
\frac{}{\Gamma; ! (B) \multimap \mathbf{0} \vdash B} D_C \quad \multimap \\
\hline
\frac{\Gamma; \vdash B}{\Gamma; \vdash ! (B)} \quad ! \qquad \frac{}{\Gamma; \mathbf{0} \vdash B} * \\
\hline
\frac{\Gamma; ! (B) \multimap \mathbf{0} \vdash B}{\Gamma; \vdash ! (B)} D_C \quad \multimap \qquad \frac{}{\Gamma; \mathbf{0} \vdash B} * \\
\hline
\frac{}{\Gamma; \vdash ! (B)} \quad ! \qquad \frac{}{\Gamma; \mathbf{0} \vdash \mathbf{0}} \quad \frac{}{\Gamma; \mathbf{0} \vdash \mathbf{0}} * \\
\hline
\frac{\Gamma; ! (B) \multimap \mathbf{0} \vdash \mathbf{0}}{\Gamma; \vdash ! (B)} D_C \quad \multimap \qquad \frac{}{\Gamma; \mathbf{0} \vdash \mathbf{0}} * \\
\hline
\frac{}{\Gamma; \vdash ! (A & (B \multimap \mathbf{0}) \multimap ! (A \multimap B))} \quad \wedge
\end{array}$$

CALL-BY-VALUE encoding (168ms)

$$\begin{array}{c}
\frac{\Gamma; \vdash A}{\Gamma; \vdash I(A)} \quad \frac{\Gamma; \vdash B}{\Gamma; \vdash I(B) \rightarrow B}^* \\
\hline
\frac{\Gamma; \vdash I(A) \rightarrow I(B) \rightarrow B}{\Gamma; \vdash I(B)}_{D_C} \quad -o \\
\\
\frac{\Gamma; \vdash B}{\Gamma; \vdash I(B)} \quad \frac{\Gamma; \vdash B}{\Gamma; \vdash I(B) \rightarrow B}^* \\
\hline
\frac{\Gamma; \vdash I(B) \rightarrow O \vdash B}{\Gamma; \vdash I(B)}_{D_C} \quad -o \\
\\
\frac{\Gamma; \vdash B}{\Gamma; \vdash I(B)} \quad \frac{\Gamma; \vdash I(B) \rightarrow O \vdash B}{\Gamma; \vdash I(B)}_{D_C} \quad -o \\
\\
\frac{\Gamma; \vdash B}{\Gamma; \vdash I(B)} \quad \frac{\Gamma; \vdash I(B) \rightarrow O \vdash B}{\Gamma; \vdash I(B)}_{D_C} \quad -o \\
\\
\frac{\Gamma; \vdash B}{\Gamma; \vdash I(B)} \quad \frac{\Gamma; \vdash I(B) \rightarrow O \vdash B}{\Gamma; \vdash I(B)}_{D_C} \quad -o \\
\\
\frac{\Gamma; \vdash I(B) \rightarrow O \vdash B}{\Gamma; \vdash I(B)}_{D_C} \quad -o \\
\\
\frac{\Gamma; \vdash I(B) \rightarrow O \vdash B}{\Gamma; \vdash I(I(A) \rightarrow I(B)) \rightarrow O}^* \\
\hline
\frac{\Gamma; \vdash I(I(A) \rightarrow I(B)) \rightarrow O}{\Gamma; \vdash I(I(A) \otimes I(I(B) \rightarrow O)) \rightarrow I(I(I(A) \rightarrow I(B)) \rightarrow O)}^* \\
\hline
\frac{\Gamma; \vdash I(I(A) \otimes I(I(B) \rightarrow O)) \rightarrow I(I(I(A) \rightarrow I(B)) \rightarrow O)}{\Gamma; \vdash I(I(A) \otimes I(I(B) \rightarrow O)) \rightarrow I(I(I(A) \rightarrow I(B)) \rightarrow O)}^*
\end{array}$$

01-ENC encoding (168ms)

[illegible]

(56) $\vdash \neg\neg A \wedge B \rightarrow \neg A \rightarrow \neg B$

LJ (62ms)

$$\frac{\frac{\frac{A, B, A \rightarrow B \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \vdash A}{A, B, B \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \vdash B} * \quad \frac{A, B, \perp, A \rightarrow \perp \rightarrow \perp \vdash \perp}{A, B, B \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \vdash \perp} *}{\frac{A, B, A \rightarrow B \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \vdash \perp}{B, A \rightarrow B \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \vdash \perp} \supset_k} * \quad \frac{B, A \rightarrow B \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \vdash \perp}{\vdash A \rightarrow \perp \rightarrow \perp \wedge B \rightarrow A \rightarrow B \rightarrow \perp \rightarrow \perp} * \quad \frac{B, \perp, A \rightarrow B \rightarrow \perp \vdash \perp \vdash \perp}{\vdash A \rightarrow \perp \rightarrow \perp \wedge B \rightarrow A \rightarrow B \rightarrow \perp \rightarrow \perp} * \quad \frac{}{\vdash A \rightarrow \perp \rightarrow \perp \wedge B \rightarrow A \rightarrow B \rightarrow \perp \rightarrow \perp} *$$

MULTIPLICATIVE encoding (77ms)

$$\frac{\frac{\frac{\Gamma: A \vdash A \quad \Gamma: B, B \multimap \perp \vdash \perp}{\Gamma: A \vdash A} \multimap}{\Gamma: A, B, A \multimap B \multimap \perp \vdash \perp} \multimap}{\frac{\frac{\Gamma: B, A \multimap B \multimap \perp \vdash A \multimap \perp}{\Gamma: B, A \multimap B \multimap \perp \vdash A \multimap \perp} \star}{\frac{\Gamma: B, A \multimap B \multimap \perp, A \multimap \perp \multimap \perp \vdash \perp}{\Gamma: \cdot \vdash A \multimap \perp \multimap \perp \otimes B \multimap A \multimap B \multimap \perp \multimap \perp} \star} \multimap$$

CALL-BY-NAME encoding (182ms)

$$\begin{array}{c}
\frac{\frac{\Gamma; \vdash A}{\Gamma; \vdash ! (A)} \quad \frac{\frac{\Gamma; \vdash B}{\Gamma; \vdash ! (B)} \quad \frac{\Gamma; \vdash 0}{\Gamma; \vdash 0}^*}{\Gamma; ! (A) \rightarrow ! (B) \rightarrow 0 \vdash 0}^* \\
\frac{\Gamma; ! (A) \rightarrow ! (B) \rightarrow 0 \vdash 0}{\Gamma; \vdash ! (A) \rightarrow 0}^* \quad D_C \\
\frac{\Gamma; \vdash ! (A) \rightarrow 0}{\Gamma; \vdash ! (! (A) \rightarrow 0)}^* \\
\frac{\Gamma; \vdash ! (! (A) \rightarrow 0)}{\Gamma; ! (! (A) \rightarrow 0) \rightarrow 0 \vdash 0}^* \quad D_C \\
\frac{\Gamma; \vdash 0}{\Gamma; \vdash ! (! (A) \rightarrow 0)}^* \quad \frac{\Gamma; \vdash 0}{\Gamma; \vdash ! (! (A) \rightarrow 0)}^* \\
\frac{\Gamma; ! (! (A) \rightarrow 0) \rightarrow 0 \vdash 0}{\Gamma; \vdash ! (! (A) \rightarrow 0)}^* \quad D_C \\
\frac{\Gamma; \vdash ! (A) \rightarrow 0}{\Gamma; \vdash ! (! (A) \rightarrow 0)}^* \quad \frac{\Gamma; \vdash 0}{\Gamma; \vdash ! (! (A) \rightarrow 0)}^* \\
\frac{\Gamma; \vdash ! (! (A) \rightarrow 0)}{\Gamma; ! (! (A) \rightarrow 0) \rightarrow 0 \vdash 0}^* \quad D_C \\
\frac{\Gamma; ! (! (A) \rightarrow 0) \rightarrow 0 \vdash 0}{\Gamma; \vdash ! (! (A) \rightarrow 0) \rightarrow 0 \vdash 0}^* \quad D_C \\
\frac{\Gamma; \vdash ! (! (A) \rightarrow 0) \rightarrow 0 \vdash 0}{\Gamma; \vdash ! (! (A) \rightarrow 0) \rightarrow 0 \vdash ! (! (A) \rightarrow 0) \rightarrow 0}^*
\end{array}$$

CALL-BY-VALUE encoding (212ms)

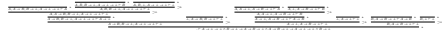
[illegible]

01-ENC encoding (210ms)

$$\begin{array}{c}
\frac{\frac{\Gamma \vdash \vdash B}{\Gamma \vdash \vdash \langle B \rangle} \quad \Gamma \vdash \langle 0 \rangle \vdash 0}{\Gamma \vdash \langle B \rangle \Rightarrow \langle 0 \rangle \vdash 0}^*_{DC} \\
\frac{\frac{\Gamma \vdash \vdash A}{\Gamma \vdash \vdash \langle A \rangle} \quad \Gamma \vdash \langle 0 \rangle \Rightarrow \langle 0 \rangle \vdash 0}{\Gamma \vdash \langle A \rangle \Rightarrow \langle 0 \rangle \Rightarrow \langle 0 \rangle \vdash 0}^*_{DC} \\
\frac{\Gamma \vdash \vdash 0}{\Gamma \vdash \vdash \langle A \rangle \Rightarrow 0}^* \\
\frac{\Gamma \vdash \vdash \langle A \rangle \Rightarrow 0}{\Gamma \vdash \vdash \langle \langle A \rangle \Rightarrow 0 \rangle}^* \\
\frac{\Gamma \vdash \vdash \langle \langle A \rangle \Rightarrow 0 \rangle}{\Gamma \vdash \langle \langle A \rangle \Rightarrow 0 \rangle \Rightarrow \langle 0 \rangle \Rightarrow \langle 0 \rangle \vdash 0}^*_{DC} \\
\frac{\Gamma \vdash \langle \langle A \rangle \Rightarrow 0 \rangle \Rightarrow \langle 0 \rangle \Rightarrow \langle 0 \rangle \vdash 0}{\Gamma \vdash \langle \langle A \rangle \Rightarrow 0 \rangle \Rightarrow \langle 0 \rangle \vdash 0}^*_{DC} \\
\frac{\Gamma \vdash \langle \langle A \rangle \Rightarrow 0 \rangle \Rightarrow \langle 0 \rangle \vdash 0}{\Gamma \vdash \langle 0 \rangle \vdash 0}^*_{DC} \\
\frac{\Gamma \vdash \langle \langle \langle \langle A \rangle \Rightarrow 0 \rangle \Rightarrow \langle 0 \rangle \rangle \Rightarrow \langle 0 \rangle \rangle \vdash 0}{\Gamma \vdash \langle \langle \langle A \rangle \Rightarrow 0 \rangle \Rightarrow \langle 0 \rangle \rangle \vdash 0}^*_{DC} \\
\frac{\Gamma \vdash \langle \langle \langle A \rangle \Rightarrow 0 \rangle \Rightarrow \langle 0 \rangle \rangle \vdash 0}{\Gamma \vdash \langle \langle \langle A \rangle \Rightarrow 0 \rangle \Rightarrow \langle 0 \rangle \rangle \Rightarrow \langle 0 \rangle \rangle \vdash 0}^* \\
\frac{\Gamma \vdash \langle \langle \langle \langle \langle A \rangle \Rightarrow 0 \rangle \Rightarrow \langle 0 \rangle \rangle \Rightarrow \langle 0 \rangle \rangle \Rightarrow \langle 0 \rangle \rangle \vdash 0}{\Gamma \vdash \langle \langle \langle \langle A \rangle \Rightarrow 0 \rangle \Rightarrow \langle 0 \rangle \rangle \Rightarrow \langle 0 \rangle \rangle \vdash 0}^* \\
\frac{\Gamma \vdash \langle \langle \langle \langle \langle A \rangle \Rightarrow 0 \rangle \Rightarrow \langle 0 \rangle \rangle \Rightarrow \langle 0 \rangle \rangle \Rightarrow \langle 0 \rangle \rangle \vdash 0}{\Gamma \vdash \langle \langle \langle \langle A \rangle \Rightarrow 0 \rangle \Rightarrow \langle 0 \rangle \rangle \Rightarrow \langle 0 \rangle \rangle \Rightarrow \langle 0 \rangle \rangle \vdash 0}^*
\end{array}$$

$$(57) \cdot \vdash (\neg\neg A \wedge \neg B) \leftrightarrow (\neg A \rightarrow B)$$

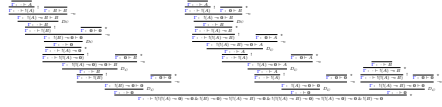
LJ (114ms)



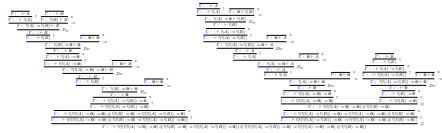
MULTIPLICATIVE encoding (58ms)

Not provable

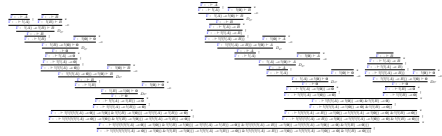
CALL-BY-NAME encoding (326ms)



CALL-BY-VALUE encoding (421ms)



01-ENC encoding (454ms)



$$(58) \cdot \vdash (\neg A \rightarrow B) \leftrightarrow (\neg\neg A \wedge \neg B)$$

LJ (109ms)



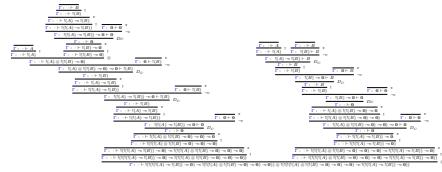
MULTIPLICATIVE encoding (41ms)

Not provable

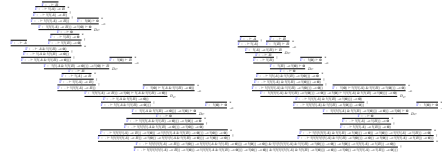
CALL-BY-NAME encoding (597ms)



CALL-BY-VALUE encoding (481ms)



01-ENC encoding (501ms)



$$(59) \cdot \vdash (\neg\neg A \rightarrow B) \leftrightarrow (\neg A \wedge \neg B)$$

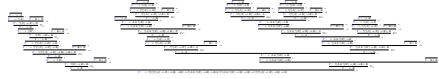
LJ (112ms)



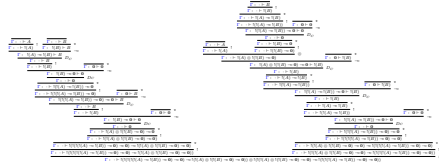
MULTIPLICATIVE encoding (57ms)

Not provable

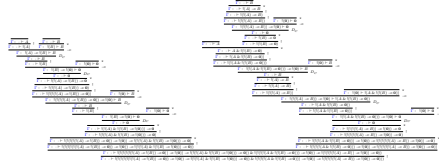
CALL-BY-NAME encoding (594ms)



CALL-BY-VALUE encoding (468ms)

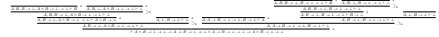


01-ENC encoding (498ms)



$$(60) \cdot \vdash (\neg A \wedge \neg B) \leftrightarrow (A \rightarrow \neg\neg B)$$

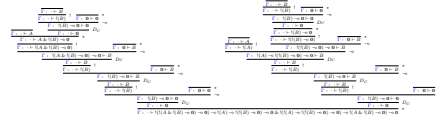
LJ (92ms)



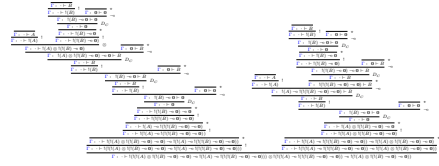
MULTIPLICATIVE encoding (124ms)



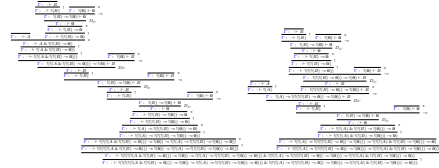
CALL-BY-NAME encoding (317ms)



CALL-BY-VALUE encoding (436ms)



01-ENC encoding (468ms)

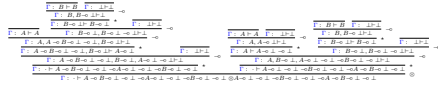


$$(61) \cdot \vdash (A \rightarrow \neg\neg B) \leftrightarrow (\neg\neg A \rightarrow \neg\neg B)$$

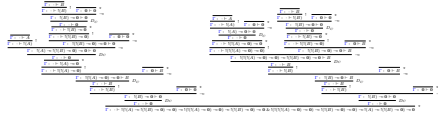
LJ (169ms)



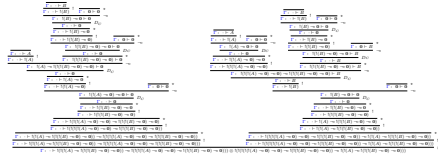
MULTIPLICATIVE encoding (163ms)



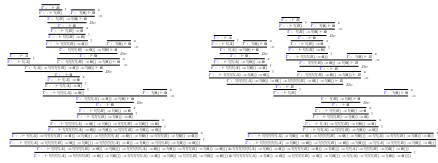
CALL-BY-NAME encoding (412ms)



CALL-BY-VALUE encoding (479ms)



01-ENC encoding (552ms)



3 Alternative Translations

$$(10) \quad \Gamma : A \multimap \mathbf{0} \vdash A \multimap B$$

encoding (55ms)

$$\frac{\frac{\frac{\Gamma : A \vdash A}{\Gamma : A, A \multimap \mathbf{0} \vdash B} \quad \frac{\Gamma : \mathbf{0} \vdash B}{\Gamma : A, A \multimap \mathbf{0} \vdash B} \quad \star}{\Gamma : A \multimap \mathbf{0} \vdash A \multimap B} \quad \multimap$$

$$(11) \quad \Gamma : A \vdash A \multimap \mathbf{0} \multimap B$$

encoding (41ms)

$$\frac{\frac{\frac{\Gamma : A \vdash A}{\Gamma : A, A \multimap \mathbf{0} \vdash B} \quad \frac{\Gamma : \mathbf{0} \vdash B}{\Gamma : A, A \multimap \mathbf{0} \vdash B} \star}{\Gamma : A \vdash A \multimap \mathbf{0} \multimap B} \star$$

$$(12) \quad \Gamma : B \vdash !(A) \multimap B$$

encoding (22ms)

$$\frac{\overline{\Gamma : B \vdash B}}{\Gamma : B \vdash !(A) \multimap B} \star$$

(16) $\Gamma : A \multimap B \otimes !(B \multimap A) \vdash A \multimap B$

encoding (35ms)

$$\frac{\frac{\frac{\Gamma : A \vdash A}{\Gamma : A, A \multimap B \vdash B} \multimap}{\Gamma : A \multimap B \otimes !(B \multimap A) \vdash A \multimap B} \star}{\Gamma : A \multimap B \otimes !(B \multimap A) \vdash A \multimap B} \star$$

(17) $\Gamma : !(A \multimap B) \otimes B \multimap A \vdash B \multimap A$

encoding (38ms)

$$\frac{\frac{\frac{\Gamma : B \vdash B}{\Gamma : B, B \multimap A \vdash A} \multimap}{\Gamma : !(A \multimap B) \otimes B \multimap A \vdash B \multimap A} \star$$

(18) $\Gamma : A, A \multimap B \otimes B \multimap A \vdash B \otimes B \multimap A$

encoding (56ms)

$$\frac{\frac{\frac{\Gamma: A \vdash A}{\Gamma: A, A \multimap B} \quad \frac{\Gamma: B, B \multimap A \vdash B \multimap B \multimap A}{\Gamma: A, A \multimap B, B \multimap A \vdash B \multimap B \multimap A} \star}{\Gamma: A, A \multimap B, B \multimap A \vdash B \otimes B \multimap A} \multimap \quad \frac{\frac{\frac{\frac{\Gamma: B \vdash B}{\Gamma: B, B \multimap A \vdash B} \quad \frac{\Gamma: A \vdash A}{\Gamma: B, B \multimap A \vdash A \vdash A} \circ}{\Gamma: B, B \multimap A \vdash B \multimap A} \star}{\Gamma: B \multimap A \vdash B \multimap B \multimap A} \otimes}{\Gamma: A \vdash A \vdash B \multimap B \multimap A} \multimap$$

(19) $\Gamma : B, A \multimap B \otimes B \multimap A \vdash A \otimes A \multimap B$

encoding (55ms)

$$\frac{\frac{\frac{\Gamma: A \vdash A \quad \Gamma: B \vdash B}{\Gamma: A, A \multimap B \vdash B} \multimap}{\Gamma: B \vdash B} \star}{\frac{\frac{\Gamma: A \vdash A \quad \Gamma: A \multimap B \vdash A \multimap B}{\Gamma: A, A \multimap B \vdash A \multimap B} \otimes}{\Gamma: B, A \multimap B, B \multimap A \vdash A \otimes A \multimap B} \multimap} \star$$

(26a) $\Gamma : \cdot \vdash A \& B \multimap \perp \multimap \perp \multimap A \multimap \perp \multimap \perp \multimap \& B \multimap \perp \multimap \perp$

encoding (107ms)

[illegible]

(26b) $\Gamma : \cdot \vdash A \multimap \perp \multimap \perp \otimes B \multimap \perp \multimap \perp \multimap A \otimes B \multimap \perp \multimap \perp$

encoding (109ms)

$$\frac{\frac{\frac{\Gamma: A \vdash A \quad \Gamma: B \vdash B}{\Gamma: A, B \vdash A \otimes B} \otimes \quad \frac{\Gamma: \perp \vdash \perp}{\Gamma: A, B, A \otimes B \multimap \perp \vdash \perp} \multimap}{\frac{\Gamma: A, A \otimes B \multimap \perp \vdash B \multimap \perp \quad \Gamma: \perp \vdash \perp}{\Gamma: A, A \otimes B \multimap \perp, B \multimap \perp \multimap \perp \vdash \perp} *} \multimap}{\frac{\Gamma: A \otimes B \multimap \perp, B \multimap \perp \multimap \perp \vdash A \multimap \perp \quad \Gamma: \perp \vdash \perp}{\Gamma: \vdash A \otimes B \multimap \perp, B \multimap \perp \multimap \perp \multimap \perp \vdash A \otimes B \multimap \perp \multimap \perp} *} \multimap}$$

$$(27a) \Gamma : \vdash !(\overline{A} \multimap B) \otimes !(B \multimap A) \multimap \perp \multimap \perp \multimap \perp \multimap A \multimap B \multimap \perp \multimap \perp \multimap B \multimap A \multimap \perp \multimap \perp$$

encoding (155ms)

$$\begin{array}{c}
\frac{\frac{\frac{\Gamma, A \vdash A \quad \Gamma, B \vdash B}{\Gamma, A \wedge B \vdash B} \rightarrow \\
\frac{\Gamma, A \wedge B \vdash B}{\Gamma, A \wedge B \vdash B} \text{DC} \\
\frac{\Gamma, \neg \neg A \vdash B^*}{\Gamma, A \rightarrow B \vdash \perp} \rightarrow \\
\frac{\Gamma, A \rightarrow B \vdash \perp}{\Gamma, A \rightarrow B \vdash \perp} \rightarrow \\
\frac{\Gamma, A \rightarrow B \vdash \perp \quad \Gamma(A \rightarrow B) \otimes (B \rightarrow A) \vdash \perp}{\Gamma, A \rightarrow B \vdash \perp \quad \Gamma(A \rightarrow B) \otimes (B \rightarrow A) \vdash \perp} \rightarrow \\
\frac{\Gamma, A \rightarrow B \vdash \perp \quad \Gamma(A \rightarrow B) \otimes (B \rightarrow A) \vdash \perp}{\Gamma, \neg(A \rightarrow B) \otimes (B \rightarrow A) \vdash \perp} \rightarrow \\
\frac{\Gamma, B \vdash B \quad \Gamma, A \vdash A}{\Gamma, B \rightarrow A \vdash A} \rightarrow \\
\frac{\Gamma, B \rightarrow A \vdash A}{\Gamma, \neg \neg B \vdash A^*} \rightarrow \\
\frac{\Gamma, \neg \neg B \vdash A^*}{\Gamma, \neg \neg B \vdash \perp} \rightarrow \\
\frac{\Gamma, B \rightarrow A \vdash \perp \quad \Gamma(B \rightarrow A) \otimes (B \rightarrow A) \vdash \perp}{\Gamma, B \rightarrow A \vdash \perp \quad \Gamma(B \rightarrow A) \otimes (B \rightarrow A) \vdash \perp} \rightarrow \\
\frac{\Gamma, B \rightarrow A \vdash \perp \quad \Gamma(B \rightarrow A) \otimes (B \rightarrow A) \vdash \perp}{\Gamma, \neg(B \rightarrow A) \otimes (B \rightarrow A) \vdash \perp} \rightarrow
\end{array}$$

$$(27b) \quad \Gamma : \vdash A \multimap B \multimap \perp \multimap \perp \otimes B \multimap A \multimap \perp \multimap \perp \multimap A \multimap B \otimes B \multimap A \multimap \perp \multimap \perp$$

encoding (168ms)

[illegible]

(35) $\Gamma : \cdot \vdash !(A) \otimes !(A) \multimap !(A) \otimes !(A) \multimap !(A) \otimes !(A)$

encoding (59ms)

$$\frac{\frac{\Gamma; \cdot \vdash A}{\Gamma; \cdot \vdash !A} !}{\Gamma; \cdot \vdash !A} ! \quad \frac{\frac{\Gamma; \cdot \vdash A}{\Gamma; \cdot \vdash !(A)} ! \quad \frac{\Gamma; \cdot \vdash A}{\Gamma; \cdot \vdash !(A)} !}{\Gamma; \cdot \vdash !(A) \otimes !(A)} \otimes \quad \frac{\Gamma; \cdot \vdash !A \otimes !(A)}{\Gamma; \cdot \vdash !A \otimes !(A)} *$$

encoding (69ms)

$$\begin{array}{c}
\frac{\frac{\Gamma: A \vdash A}{\Gamma: A \vdash A} \quad \frac{\frac{\Gamma: B \vdash B}{\Gamma: B \vdash B} \quad \frac{\Gamma: \perp \vdash \perp}{\Gamma: \perp \vdash \perp}}{\Gamma: B \multimap B} \multimap \quad \frac{\Gamma: A, A \multimap B, B \multimap \perp \vdash \perp}{\Gamma: A, A \multimap B, B \multimap \perp \vdash \perp} \multimap \\
\frac{\Gamma: A \multimap B \vdash A \otimes B \multimap \perp \multimap \perp}{\Gamma: A \multimap B, A \otimes B \multimap \perp \multimap \perp \multimap \perp \vdash \perp} \star \quad \frac{\Gamma: \perp \vdash \perp}{\Gamma: \perp \vdash \perp} \multimap \\
\frac{\Gamma: A \multimap B, A \otimes B \multimap \perp \multimap \perp \multimap \perp \vdash \perp}{\Gamma: \cdot \vdash A \otimes B \multimap \perp \multimap \perp \multimap \perp \multimap A \multimap B \multimap \perp} \star
\end{array}$$

$$(59a) \Gamma: \cdot \vdash A \multimap B \multimap \perp \multimap \perp \multimap A \otimes B \multimap \perp \multimap \perp$$

encoding (71ms)

$$\begin{array}{c}
\frac{\frac{\Gamma: A \vdash A}{\Gamma: A \vdash A} \quad \frac{\frac{\Gamma: B \vdash B}{\Gamma: B \vdash B} \quad \frac{\Gamma: \perp \vdash \perp}{\Gamma: \perp \vdash \perp}}{\Gamma: B \multimap B} \multimap \quad \frac{\Gamma: A, A \multimap B, B \multimap \perp \vdash \perp}{\Gamma: A, A \multimap B, B \multimap \perp \vdash \perp} \multimap \\
\frac{\Gamma: A, B \multimap \perp \vdash A \multimap B \multimap \perp}{\Gamma: A, B \multimap \perp, A \multimap B \multimap \perp \multimap \perp \vdash \perp} \star \quad \frac{\Gamma: \perp \vdash \perp}{\Gamma: \perp \vdash \perp} \multimap \\
\frac{\Gamma: A, B \multimap \perp, A \multimap B \multimap \perp \multimap \perp \vdash \perp}{\Gamma: \cdot \vdash A \multimap B \multimap \perp \multimap \perp \multimap A \otimes B \multimap \perp \multimap \perp} \star
\end{array}$$

$$(59b) \Gamma: \cdot \vdash A \otimes B \multimap \perp \multimap \perp \multimap \perp \multimap \perp \multimap \perp \multimap \perp \multimap \perp \multimap \perp \multimap \perp \multimap \perp$$

encoding (163ms)

$$\begin{array}{c}
\frac{\frac{\Gamma: B \vdash B}{\Gamma: B \vdash B} \star \quad \frac{\Gamma: \perp \vdash \perp}{\Gamma: \perp \vdash \perp}}{\Gamma: B \vdash \mathbf{!}(A) \multimap B} \star \quad \frac{\Gamma: \perp \vdash \perp}{\Gamma: \perp \vdash \perp} \multimap \\
\frac{\Gamma: B, \mathbf{!}(A) \multimap B \multimap \perp \vdash \perp}{\Gamma: B, \mathbf{!}(A) \multimap B \multimap \perp \vdash \perp} D_C \quad \frac{\Gamma: B \vdash \perp}{\Gamma: B \vdash \perp} \star \\
\frac{\Gamma: \cdot \vdash A}{\Gamma: \cdot \vdash A} \quad \frac{\Gamma: \cdot \vdash B \multimap \perp}{\Gamma: \cdot \vdash B \multimap \perp} \star \quad \frac{\Gamma: \mathbf{0} \vdash B}{\Gamma: \mathbf{0} \vdash B} \star \\
\frac{\Gamma: A \otimes B \multimap \perp \multimap \perp \vdash B}{\Gamma: A \otimes B \multimap \perp \multimap \perp \vdash B} \star \quad \frac{\Gamma: \perp \vdash \perp}{\Gamma: \perp \vdash \perp} \multimap \\
\frac{\Gamma: A \otimes B \multimap \perp \multimap \perp \vdash \mathbf{!}(A) \multimap B}{\Gamma: A \otimes B \multimap \perp \multimap \perp \vdash \mathbf{!}(A) \multimap B} \star \quad \frac{\Gamma: \perp \vdash \perp}{\Gamma: \perp \vdash \perp} \multimap \\
\frac{\Gamma: A \otimes B \multimap \perp \multimap \perp \vdash \mathbf{!}(A) \multimap B}{\Gamma: A \otimes B \multimap \perp \multimap \perp \vdash \mathbf{!}(A) \multimap B} D_C \quad \frac{\Gamma: \perp \vdash \perp}{\Gamma: \perp \vdash \perp} \star \\
\frac{\Gamma: A \otimes B \multimap \perp \multimap \perp \vdash \mathbf{!}(A) \multimap B}{\Gamma: \cdot \vdash A \otimes B \multimap \perp \multimap \perp \multimap \perp \multimap \perp \multimap \perp \multimap \perp} \star
\end{array}$$