

Benchmarking Linear Logic: sequents inspired from Kleene's intuitionistic theorems *

September 17, 2018

1 General Information

- Test run on a MacBook Pro, 2,4 GHz Intel Core i7, 8GB RAM.
- Maude version: 2.7.1 built: Oct 2 2017.
- The search procedure considers proofs using up to 4 times the copy rule (focusing on one of the formulas of the classical context). Benchmarks 23 and 25 required the limit to be 5.

2 Translations

$$(1) \cdot \vdash A \rightarrow A$$

LJ (28ms)

$$\cdot \vdash A \rightarrow A \quad \star$$

MULTIPLICATIVE encoding (28ms)

$$\frac{\cdot \vdash A \vdash A}{\cdot \vdash \cdot \vdash A \multimap A} \star$$

CALL-BY-NAME encoding (28ms)

$$\frac{\overline{A : \cdot \vdash A}}{\cdot \vdash \cdot \vdash !(A) \multimap A} \star$$

CALL-BY-VALUE encoding (41ms)

$$\frac{\frac{\overline{A : \cdot \vdash A}}{A : \cdot \vdash !(A)} !}{\cdot \vdash \cdot \vdash !(A) \multimap !(A)} \star$$

01-ENC encoding (35ms)

$$\frac{\overline{A : \cdot \vdash A}}{\cdot \vdash \cdot \vdash !(A) \multimap A} \star$$

$$(2) A \rightarrow B, B \rightarrow C \vdash A \rightarrow C$$

LJ (46ms)

$$\frac{\frac{\overline{A, A \rightarrow B, B \rightarrow C \vdash A} \star \quad \overline{A, B, B \rightarrow C \vdash C} \star}{A, A \rightarrow B, B \rightarrow C \vdash C} \star}{A \rightarrow B, B \rightarrow C \vdash A \rightarrow C} \star$$

MULTIPLICATIVE encoding (49ms)

$$\frac{\frac{\cdot \vdash A \vdash A \quad \frac{\cdot \vdash B \vdash B \quad \cdot \vdash C \vdash C}{\cdot \vdash B, B \multimap C \vdash C} \multimap}{\cdot \vdash A, A \multimap B, B \multimap C \vdash C} \multimap}{\cdot \vdash A \multimap B, B \multimap C \vdash A \multimap C} \star$$

CALL-BY-NAME encoding (123ms)

$$\frac{\frac{\overline{A, !(A) \multimap B, !(B) \multimap C : \cdot \vdash A} \quad \overline{A, !(A) \multimap B, !(B) \multimap C : \cdot \vdash !(A)} !}{\frac{A, !(A) \multimap B, !(B) \multimap C : \cdot \vdash B}{A, !(A) \multimap B, !(B) \multimap C : \cdot \vdash !(B)} !} D_C}{\frac{A, !(A) \multimap B, !(B) \multimap C : \cdot \vdash C}{A, !(A) \multimap B, !(B) \multimap C : \cdot \vdash C} D_C} \multimap$$

CALL-BY-VALUE encoding (165ms)

$$\frac{\frac{\overline{A, !(A) \multimap !(B), !(B) \multimap !(C) : \cdot \vdash A} \quad \overline{A, B, !(A) \multimap !(B), !(B) \multimap !(C) : \cdot \vdash B} \star}{\frac{A, !(A) \multimap !(B), !(B) \multimap !(C) : \cdot \vdash !(A)}{A, !(A) \multimap !(B), !(B) \multimap !(C) : \cdot \vdash B} D_C} \multimap}{\frac{A, !(A) \multimap !(B), !(B) \multimap !(C) : \cdot \vdash B}{A, !(A) \multimap !(B), !(B) \multimap !(C) : \cdot \vdash !(B)} !} D_C$$

01-ENC encoding (146ms)

$$\frac{\frac{\overline{A, !(A) \multimap !(B), !(B) \multimap !(C) : \cdot \vdash A} \quad \overline{A, B, !(A) \multimap !(B), !(B) \multimap !(C) : \cdot \vdash B} \star}{\frac{A, !(A) \multimap !(B), !(B) \multimap !(C) : \cdot \vdash !(A)}{A, !(A) \multimap !(B), !(B) \multimap !(C) : \cdot \vdash B} D_C} \multimap}{\frac{A, !(A) \multimap !(B), !(B) \multimap !(C) : \cdot \vdash B}{A, !(A) \multimap !(B), !(B) \multimap !(C) : \cdot \vdash !(B)} !} D_C$$

*See <https://github.com/carlosolarte/Benchmarking-Linear-Logic> for details on the encodings used.

$$(7) A \rightarrow B \vdash C \rightarrow A \rightarrow C \rightarrow B$$

LJ (47ms)

$$\frac{\frac{C, A \rightarrow B, C \rightarrow A \vdash C}{C, A \rightarrow B, C \rightarrow A \vdash A} \star \quad \frac{A, C, A \rightarrow B \vdash A}{C, A \rightarrow B, C \rightarrow A \vdash A} \star}{\frac{C, A \rightarrow B, C \rightarrow A \vdash A}{C, A \rightarrow B, C \rightarrow A \vdash B} \star} \supset_L \quad \frac{B, C, C \rightarrow A \vdash B}{A \rightarrow B \vdash C \rightarrow A \rightarrow C \rightarrow B} \star \supset_L$$

MULTIPLICATIVE encoding (48ms)

$$\frac{\frac{\cdot : C \vdash C}{\cdot : A \multimap B \vdash C \multimap A \multimap C \multimap B} \star \quad \frac{\cdot : A \vdash A \quad \cdot : B \vdash B}{\cdot : A, A \multimap B \vdash B} \multimap}{\cdot : C, A \multimap B, C \multimap A \vdash B} \multimap \multimap$$

CALL-BY-NAME encoding (120ms)

$$\frac{\frac{C, !!(A) \multimap B, !(C) \multimap A : \cdot \vdash C}{C, !(A) \multimap B, !(C) \multimap A : \cdot \vdash !(C)} ! \quad \frac{C, !(A) \multimap B, !(C) \multimap A : A \vdash A}{C, !(A) \multimap B, !(C) \multimap A : \cdot \vdash A} D_C}{\frac{C, !(A) \multimap B, !(C) \multimap A : \cdot \vdash A}{C, !(A) \multimap B, !(C) \multimap A : \cdot \vdash !(A)} ! \quad \frac{C, !(A) \multimap B, !(C) \multimap A : B \vdash B}{C, !(A) \multimap B, !(C) \multimap A : \cdot \vdash B} D_C} \multimap \quad \frac{C, !(A) \multimap B, !(C) \multimap A : \cdot \vdash B}{\cdot : !(!(A) \multimap B) \vdash !(!(C) \multimap A) \multimap !(C) \multimap B} \star$$

CALL-BY-VALUE encoding (167ms)

$$\frac{\frac{C, !(A) \multimap !(B), !(C) \multimap !(A) : \cdot \vdash C}{C, !(A) \multimap !(B), !(C) \multimap !(A) : \cdot \vdash !(C)} ! \quad \frac{A, C, !(A) \multimap !(B), !(C) \multimap !(A) : \cdot \vdash A}{C, !(A) \multimap !(B), !(C) \multimap !(A) : \cdot \vdash A} \star}{\frac{C, !(A) \multimap !(B), !(C) \multimap !(A) : \cdot \vdash A}{C, !(A) \multimap !(B), !(C) \multimap !(A) : \cdot \vdash !(A)} ! \quad \frac{B, C, !(A) \multimap !(B), !(C) \multimap !(A) : \cdot \vdash B}{C, !(A) \multimap !(B), !(C) \multimap !(A) : \cdot \vdash B} D_C} \multimap \quad \frac{C, !(A) \multimap !(B), !(C) \multimap !(A) : \cdot \vdash B}{C, !(A) \multimap !(B), !(C) \multimap !(A) : \cdot \vdash !(B)} ! \quad \frac{!(A) \multimap !(B), !(C) \multimap !(A) : \cdot \vdash !(C) \multimap !(B)}{!(A) \multimap !(B), !(C) \multimap !(A) : \cdot \vdash !(!(C) \multimap !(B))} ! \quad \frac{!(A) \multimap !(B) : \cdot \vdash !(!(C) \multimap !(A)) \multimap !(!(C) \multimap !(B))}{\cdot : !(!(A) \multimap !(B)) \vdash !(!(C) \multimap !(A)) \multimap !(!(C) \multimap !(B))} \star$$

01-ENC encoding (156ms)

$$\frac{\frac{C, !(A) \multimap !(B), !(C) \multimap !(A) : \cdot \vdash C}{C, !(A) \multimap !(B), !(C) \multimap !(A) : \cdot \vdash !(C)} ! \quad \frac{A, C, !(A) \multimap !(B), !(C) \multimap !(A) : \cdot \vdash A}{C, !(A) \multimap !(B), !(C) \multimap !(A) : \cdot \vdash A} \star}{\frac{C, !(A) \multimap !(B), !(C) \multimap !(A) : \cdot \vdash A}{C, !(A) \multimap !(B), !(C) \multimap !(A) : \cdot \vdash !(A)} ! \quad \frac{B, C, !(A) \multimap !(B), !(C) \multimap !(A) : \cdot \vdash B}{C, !(A) \multimap !(B), !(C) \multimap !(A) : \cdot \vdash B} D_C} \multimap \quad \frac{C, !(A) \multimap !(B), !(C) \multimap !(A) : \cdot \vdash B}{C, !(A) \multimap !(B), !(C) \multimap !(A) : \cdot \vdash !(B)} ! \quad \frac{!(A) \multimap !(B), !(C) \multimap !(A) : \cdot \vdash !(C) \multimap !(B)}{!(A) \multimap !(B), !(C) \multimap !(A) : \cdot \vdash !(!(C) \multimap !(B))} ! \quad \frac{!(A) \multimap !(B) : \cdot \vdash !(!(C) \multimap !(A)) \multimap !(!(C) \multimap !(B))}{\cdot : !(!(A) \multimap !(B)) \vdash !(!(C) \multimap !(A)) \multimap !(!(C) \multimap !(B))} \star$$

$$(8) A \rightarrow B \vdash A \wedge C \rightarrow B \wedge C$$

LJ (37ms)

$$\frac{\frac{A, C, A \rightarrow B \vdash A}{A, C, A \rightarrow B \vdash B} \star \quad \frac{A, B, C \vdash B}{A \rightarrow B \vdash A \wedge C \rightarrow B \wedge C} \star}{\supset_L}$$

MULTIPLICATIVE encoding (48ms)

$$\frac{\frac{\cdot : B \vdash B \quad \cdot : C \vdash C}{\cdot : B, C \vdash B \otimes C} \otimes \quad \frac{\cdot : A \vdash A}{\cdot : A, C, A \multimap B \vdash B \otimes C} \multimap}{\cdot : A \multimap B \vdash A \otimes C \multimap B \otimes C} \star$$

CALL-BY-NAME encoding (73ms)

$$\frac{\frac{A, C, !(A) \multimap B : \cdot \vdash A}{A, C, !(A) \multimap B : \cdot \vdash !(A)} ! \quad \frac{A, C, !(A) \multimap B : B \vdash B}{A, C, !(A) \multimap B : \cdot \vdash B} D_C}{\frac{A, C, !(A) \multimap B : \cdot \vdash B}{\cdot : !(!(A) \multimap B) \vdash !(A \& C) \multimap B \& C} \star}$$

CALL-BY-VALUE encoding (135ms)

$$\frac{\frac{A, C, !(A) \multimap !(B) : \cdot \vdash A}{A, C, !(A) \multimap !(B) : \cdot \vdash !(A)} ! \quad \frac{A, B, C, !(A) \multimap !(B) : \cdot \vdash B}{A, C, !(A) \multimap !(B) : \cdot \vdash B} \star}{\frac{A, C, !(A) \multimap !(B) : \cdot \vdash B}{A, C, !(A) \multimap !(B) : \cdot \vdash !(B)} ! \quad \frac{A, C, !(A) \multimap !(B) : \cdot \vdash B}{A, C, !(A) \multimap !(B) : \cdot \vdash !(B)} ! \quad \frac{A, C, !(A) \multimap !(B) : \cdot \vdash B}{A, C, !(A) \multimap !(B) : \cdot \vdash !(B)} ! \quad \frac{!(A) \multimap !(B) : \cdot \vdash !(A) \otimes !(C) \multimap !(B) \otimes !(C)}{!(A) \multimap !(B) : \cdot \vdash !(!(A) \otimes !(C) \multimap !(B) \otimes !(C))} ! \quad \frac{!(A) \multimap !(B) : \cdot \vdash !(!(A) \otimes !(C) \multimap !(B) \otimes !(C))}{\cdot : !(!(A) \multimap !(B)) \vdash !(!(A) \otimes !(C) \multimap !(B) \otimes !(C))} \star}$$

01-ENC encoding (130ms)

$$\frac{\frac{A, C, !(A) \multimap !(B) : \cdot \vdash A}{A, C, !(A) \multimap !(B) : \cdot \vdash !(A)} ! \quad \frac{A, B, C, !(A) \multimap !(B) : \cdot \vdash B}{A, C, !(A) \multimap !(B) : \cdot \vdash B} \star}{\frac{A, C, !(A) \multimap !(B) : \cdot \vdash B}{A, C, !(A) \multimap !(B) : \cdot \vdash !(B)} ! \quad \frac{A, C, !(A) \multimap !(B) : \cdot \vdash B}{A, C, !(A) \multimap !(B) : \cdot \vdash !(B)} ! \quad \frac{A, C, !(A) \multimap !(B) : \cdot \vdash B}{A, C, !(A) \multimap !(B) : \cdot \vdash !(B)} ! \quad \frac{A, C, !(A) \multimap !(B) : \cdot \vdash B}{A, C, !(A) \multimap !(B) : \cdot \vdash !(B)} ! \quad \frac{!(A) \multimap !(B) : \cdot \vdash !(!(A) \& !(C)) \multimap !(B \& C)}{!(A) \multimap !(B) : \cdot \vdash !(!(A) \& !(C)) \multimap !(B \& C)} ! \quad \frac{!(A) \multimap !(B) : \cdot \vdash !(!(A) \& !(C)) \multimap !(B \& C)}{\cdot : !(!(A) \multimap !(B)) \vdash !(!(A) \& !(C)) \multimap !(B \& C)} \star}$$

$$(9) \ A \rightarrow B \vdash C \wedge A \rightarrow C \wedge B$$

LJ (37ms)

$$\frac{\overline{A, C, A \rightarrow B \vdash A} \star \overline{A, B, C \vdash B} \star}{\overline{A, C, A \rightarrow B \vdash B} \star} \supset_L \frac{\overline{A, C, A \rightarrow B \vdash B} \star}{\overline{A \rightarrow B \vdash C \wedge A \rightarrow C \wedge B} \star}$$

MULTIPLICATIVE encoding (48ms)

$$\frac{\overline{\cdot : A \vdash A} \otimes \frac{\overline{\cdot : C \vdash C} \otimes \overline{\cdot : B \vdash B}}{\overline{\cdot : B, C \vdash C \otimes B} \otimes}{\overline{\cdot : A, C, A \multimap B \vdash C \otimes B} \multimap} \star \frac{\overline{\cdot : A \multimap B \vdash C \otimes A \multimap C \otimes B} \star}{\cdot : A \multimap B \vdash C \otimes A \multimap C \otimes B} \star$$

CALL-BY-NAME encoding (60ms)

$$\frac{\overline{A, C, !(A) \multimap B : \cdot \vdash A} \multimap \frac{\overline{A, C, !(A) \multimap B : \cdot \vdash !(A)} \multimap \frac{\overline{A, C, !(A) \multimap B : ! (A) \multimap B \vdash B} \multimap}{\overline{A, C, !(A) \multimap B : \cdot \vdash B} D_C} \multimap}{\overline{A, C, !(A) \multimap B : \cdot \vdash C} \multimap \frac{\overline{A, C, !(A) \multimap B : ! (A) \multimap B \vdash B} D_C}{\cdot : !(A) \multimap B \vdash !(C \& A) \multimap C \& B} \star}$$

CALL-BY-VALUE encoding (137ms)

$$\frac{\overline{A, C, !(A) \multimap !(B) : \cdot \vdash A} \multimap \frac{\overline{A, C, !(A) \multimap !(B) : \cdot \vdash !(A)} \multimap \frac{\overline{A, B, C, !(A) \multimap !(B) : \cdot \vdash C} \multimap \frac{\overline{A, B, C, !(A) \multimap !(B) : \cdot \vdash !(C)} \multimap \frac{\overline{A, B, C, !(A) \multimap !(B) : \cdot \vdash !(C) \otimes !(B)} \multimap}{\overline{A, C, !(A) \multimap !(B) : ! (A) \multimap !(B) \vdash !(C) \otimes !(B)} D_C} \multimap}{\overline{A, C, !(A) \multimap !(B) : \cdot \vdash !(C) \otimes !(B)} \star} \multimap \frac{\overline{!(A) \multimap !(B) : \cdot \vdash !(C) \otimes !(A) \multimap !(C) \otimes !(B)} \star}{\overline{!(A) \multimap !(B) : \cdot \vdash !(C) \otimes !(A) \multimap !(C) \otimes !(B)} \star} \multimap \frac{\overline{!(A) \multimap !(B) : \cdot \vdash !(C) \otimes !(A) \multimap !(C) \otimes !(B)} \star}{\cdot : !(A) \multimap !(B) \vdash !(C) \otimes !(A) \multimap !(C) \otimes !(B)} \star}$$

01-ENC encoding (131ms)

$$\frac{\overline{A, C, !(A) \multimap !(B) : \cdot \vdash A} \multimap \frac{\overline{A, B, C, !(A) \multimap !(B) : \cdot \vdash C} \multimap \frac{\overline{A, B, C, !(A) \multimap !(B) : \cdot \vdash !(C \& B)} \multimap \frac{\overline{A, B, C, !(A) \multimap !(B) : \cdot \vdash !(C \& B)} \multimap \frac{\overline{A, C, !(A) \multimap !(B) : ! (B) \vdash !(C \& B)} \multimap}{\overline{A, C, !(A) \multimap !(B) : \cdot \vdash !(A)} \multimap \frac{\overline{A, C, !(A) \multimap !(B) : ! (A) \multimap !(B) \vdash !(C \& B)} D_C}{\overline{A, C, !(A) \multimap !(B) : \cdot \vdash !(C \& B)} \star} \multimap \frac{\overline{!(A) \multimap !(B) : \cdot \vdash !(C) \& !(A) \multimap !(C \& B)} \star}{\overline{!(A) \multimap !(B) : \cdot \vdash !(C) \& !(A) \multimap !(C \& B)} \star} \multimap \frac{\overline{!(A) \multimap !(B) : \cdot \vdash !(C) \& !(A) \multimap !(C \& B)} \star}{\cdot : !(A) \multimap !(B) \vdash !(C) \& !(A) \multimap !(C \& B)} \star}$$

$$(10) \ \neg A \vdash A \rightarrow B$$

LJ (37ms)

$$\frac{\overline{A, A \rightarrow \perp \vdash A} \star \overline{A, \perp \vdash B} \star}{\overline{A, A \rightarrow \perp \vdash B} \star} \supset_L \frac{\overline{A, A \rightarrow \perp \vdash B} \star}{\overline{A \rightarrow \perp \vdash A \rightarrow B} \star}$$

MULTIPLICATIVE encoding (41ms)

Not provable

CALL-BY-NAME encoding (66ms)

$$\frac{\overline{A, !(A) \multimap \mathbf{0} : \cdot \vdash A} \multimap \frac{\overline{A, !(A) \multimap \mathbf{0} : \cdot \vdash !(A)} \multimap \frac{\overline{A, !(A) \multimap \mathbf{0} : \mathbf{0} \vdash B} \star}{\overline{A, !(A) \multimap \mathbf{0} : ! (A) \multimap \mathbf{0} \vdash B} D_C} \multimap}{\overline{A, !(A) \multimap \mathbf{0} : \cdot \vdash B} D_C} \multimap \frac{\overline{\cdot : !(A) \multimap \mathbf{0} \vdash !(A) \multimap B} \star}{\cdot : !(A) \multimap \mathbf{0} \vdash !(A) \multimap B} \star$$

CALL-BY-VALUE encoding (117ms)

$$\frac{\overline{A, !(A) \multimap \mathbf{0} : \cdot \vdash A} \multimap \frac{\overline{A, !(A) \multimap \mathbf{0} : \cdot \vdash !(A)} \multimap \frac{\overline{A, !(A) \multimap \mathbf{0} : \mathbf{0} \vdash !(B)} \star}{\overline{A, !(A) \multimap \mathbf{0} : ! (A) \multimap \mathbf{0} \vdash !(B)} D_C} \multimap}{\overline{A, !(A) \multimap \mathbf{0} : \cdot \vdash !(B)} D_C} \multimap \frac{\overline{!(A) \multimap \mathbf{0} : \cdot \vdash !(A) \multimap !(B)} \star}{\overline{!(A) \multimap \mathbf{0} : \cdot \vdash !(A) \multimap !(B)} \star} \multimap \frac{\overline{!(A) \multimap \mathbf{0} : \cdot \vdash !(A) \multimap !(B)} \star}{\cdot : !(A) \multimap \mathbf{0} \vdash !(A) \multimap !(B)} \star$$

01-ENC encoding (118ms)

$$\frac{\overline{A, !(A) \multimap !(\mathbf{0}) : \cdot \vdash A} \multimap \frac{\overline{A, !(A) \multimap !(\mathbf{0}) : \cdot \vdash !(A)} \multimap \frac{\overline{A, !(A) \multimap !(\mathbf{0}) : !(\mathbf{0}) \vdash B} \star}{\overline{A, !(A) \multimap !(\mathbf{0}) : ! (A) \multimap !(\mathbf{0}) \vdash B} D_C} \multimap}{\overline{A, !(A) \multimap !(\mathbf{0}) : \cdot \vdash B} D_C} \multimap \frac{\overline{!(A) \multimap !(\mathbf{0}) : \cdot \vdash !(A) \multimap B} \star}{\overline{!(A) \multimap !(\mathbf{0}) : \cdot \vdash !(A) \multimap B} \star} \multimap \frac{\overline{!(A) \multimap !(\mathbf{0}) : \cdot \vdash !(A) \multimap B} \star}{\cdot : !(A) \multimap !(\mathbf{0}) \vdash !(A) \multimap B} \star$$

$$(11) \ A \vdash \neg A \rightarrow B$$

LJ (37ms)

$$\frac{\frac{A, A \rightarrow \perp \vdash A \quad \star \quad A, \perp \vdash B \quad \star}{A, A \rightarrow \perp \vdash B} \quad \star}{A \vdash A \rightarrow \perp \rightarrow B} \quad \star \quad \supset_L$$

MULTIPLICATIVE encoding (28ms)

Not provable

CALL-BY-NAME encoding (65ms)

$$\frac{\frac{\frac{A, !(A) \multimap \mathbf{0} : \cdot \vdash A}{A, !(A) \multimap \mathbf{0} : \cdot \vdash !(A)} \quad ! \quad \frac{A, !(A) \multimap \mathbf{0} : \mathbf{0} \vdash B}{A, !(A) \multimap \mathbf{0} : \cdot \vdash B} \quad \star}{\frac{A, !(A) \multimap \mathbf{0} : \cdot \vdash B}{A, !(A) \multimap \mathbf{0} : !(A) \multimap \mathbf{0} \vdash B} \quad D_C} \quad \multimap}{\frac{A, !(A) \multimap \mathbf{0} : \cdot \vdash B}{\cdot : !(A) \vdash !(A) \multimap \mathbf{0} \multimap B} \quad \star} \quad \star$$

CALL-BY-VALUE encoding (78ms)

$$\frac{\frac{\frac{A, !(A) \multimap \mathbf{0} : \cdot \vdash A}{A, !(A) \multimap \mathbf{0} : \cdot \vdash !(A)} \quad ! \quad \frac{A, !(A) \multimap \mathbf{0} : \mathbf{0} \vdash !(B)}{A, !(A) \multimap \mathbf{0} : \cdot \vdash !(B)} \quad \star}{\frac{A, !(A) \multimap \mathbf{0} : \cdot \vdash !(B)}{A, !(A) \multimap \mathbf{0} : !(A) \multimap \mathbf{0} \vdash !(B)} \quad D_C} \quad \multimap}{\frac{A, !(A) \multimap \mathbf{0} : \cdot \vdash !(B)}{A : \cdot \vdash !(A) \multimap \mathbf{0} \multimap !(B)} \quad \star}{\frac{A : \cdot \vdash !(A) \multimap \mathbf{0} \multimap !(B)}{\cdot : !(A) \vdash !(A) \multimap \mathbf{0} \multimap !(B)} \quad !} \quad \star$$

01-ENC encoding (76ms)

$$\frac{\frac{\frac{A, !(A) \multimap !(\mathbf{0}) : \cdot \vdash A}{A, !(A) \multimap !(\mathbf{0}) : \cdot \vdash !(A)} \quad ! \quad \frac{A, !(A) \multimap !(\mathbf{0}) : !(\mathbf{0}) \vdash B}{A, !(A) \multimap !(\mathbf{0}) : \cdot \vdash B} \quad \star}{\frac{A, !(A) \multimap !(\mathbf{0}) : \cdot \vdash B}{A, !(A) \multimap !(\mathbf{0}) : !(A) \multimap !(\mathbf{0}) \vdash B} \quad D_C} \quad \multimap}{\frac{A, !(A) \multimap !(\mathbf{0}) : \cdot \vdash B}{A : \cdot \vdash !(A) \multimap !(\mathbf{0}) \multimap B} \quad \star}{\frac{A : \cdot \vdash !(A) \multimap !(\mathbf{0}) \multimap B}{\cdot : !(A) \vdash !(A) \multimap !(\mathbf{0}) \multimap B} \quad !} \quad \star$$

$$(12) \ B \vdash A \rightarrow B$$

LJ (21ms)

$$\frac{B \vdash A \rightarrow B \quad \star}{B \vdash A \rightarrow B} \quad \star$$

MULTIPLICATIVE encoding (21ms)

Not provable

CALL-BY-NAME encoding (27ms)

$$\frac{\frac{A, B : \cdot \vdash B}{\cdot : !(B) \vdash !(A) \multimap B} \quad \star}{\cdot : !(B) \vdash !(A) \multimap B} \quad \star$$

CALL-BY-VALUE encoding (47ms)

$$\frac{\frac{\frac{A, B : \cdot \vdash B}{A, B : \cdot \vdash !(B)} \quad !}{B : \cdot \vdash !(A) \multimap !(B)} \quad \star}{\frac{B : \cdot \vdash !(A) \multimap !(B)}{\cdot : !(B) \vdash !(A) \multimap !(B)} \quad !} \quad \star$$

01-ENC encoding (39ms)

$$\frac{\frac{\frac{A, B : \cdot \vdash B}{B : \cdot \vdash !(A) \multimap B} \quad \star}{B : \cdot \vdash !(A) \multimap B} \quad \star}{\frac{B : \cdot \vdash !(A) \multimap B}{\cdot : !(B) \vdash !(A) \multimap B} \quad !} \quad \star$$

$$(14) \quad A \rightarrow \neg B \vdash \neg\neg B \rightarrow \neg A$$

LJ (64ms)

$$\frac{\frac{\frac{A, B, B \rightarrow \perp, B \rightarrow \perp \rightarrow \perp \vdash B}{A, B, \perp, B \rightarrow \perp \rightarrow \perp \vdash \perp} \star}{A, A \rightarrow B \rightarrow \perp, B \rightarrow \perp \rightarrow \perp \vdash \perp \vdash A} \star}{\frac{A, A \rightarrow B \rightarrow \perp, B \rightarrow \perp \rightarrow \perp \vdash \perp \vdash A}{A \rightarrow B \rightarrow \perp \vdash \perp \rightarrow \perp \rightarrow \perp \vdash A \rightarrow \perp} \star} \supset_L \frac{\frac{\frac{A, B, \perp, B \rightarrow \perp \rightarrow \perp \vdash \perp}{A, B, B \rightarrow \perp, B \rightarrow \perp \rightarrow \perp \vdash \perp} \star}{A, B, B \rightarrow \perp, B \rightarrow \perp \rightarrow \perp \vdash \perp \vdash \perp} \supset_L}{\frac{A, B, B \rightarrow \perp, B \rightarrow \perp \rightarrow \perp \vdash \perp \vdash \perp}{A, B, \perp, B \rightarrow \perp \rightarrow \perp \vdash \perp \vdash \perp} \star} \supset_L$$

MULTIPLICATIVE encoding (75ms)

$$\frac{\frac{\frac{\cdot : A \vdash A}{\cdot : A, B, A \multimap B \multimap \perp \vdash \perp} \quad \frac{\cdot : B \vdash B \quad \cdot : \perp \vdash \perp}{\cdot : B, B \multimap \perp \vdash \perp} \multimap}{\cdot : A, B, A \multimap B \multimap \perp \vdash \perp} \multimap \quad \frac{\cdot : A, A \multimap B \multimap \perp \vdash B \multimap \perp}{\cdot : A, A \multimap B \multimap \perp, B \multimap \perp \multimap \perp \vdash \perp} \star}{\cdot : A, A \multimap B \multimap \perp, B \multimap \perp \multimap \perp \multimap \perp \vdash \perp} \multimap \quad \frac{\cdot : \perp \vdash \perp}{\cdot : A \multimap B \multimap \perp \vdash B \multimap \perp \multimap \perp \multimap \perp \multimap A \multimap \perp} \star$$

CALL-BY-NAME encoding (180ms)

[illegible]

CALL-BY-VALUE encoding (174ms)

[illegible]

01-ENC encoding (178ms)

[illegible]

$$(17) (A) \leftrightarrow (B) \vdash B \rightarrow A$$

LJ (37ms)

$$\frac{\frac{B, A \rightarrow B, B \rightarrow A \vdash B}{B, A \rightarrow B, B \rightarrow A \vdash A} \star}{A \rightarrow B \wedge B \rightarrow A \vdash B \rightarrow A} \star \supset_L$$

MULTIPLICATIVE encoding (29ms)

Not provable

CALL-BY-NAME encoding (105ms)

$$\frac{\frac{B, !(A) \multimap B, !(B) \multimap A : \cdot \vdash B}{B, !(A) \multimap B, !(B) \multimap A : \cdot \vdash !(B)} !}{\frac{B, !(A) \multimap B, !(B) \multimap A : \cdot \vdash A}{B, !(A) \multimap B, !(B) \multimap A : \cdot \vdash A} D_C}{\vdash : !(A) \multimap B \& !(B) \multimap A \vdash !(B) \multimap A} \star \multimap$$

CALL-BY-VALUE encoding (140ms)

$$\frac{\frac{\frac{B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash B}{B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash !(B)} !}{\frac{B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash A}{B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash A} D_C}{\vdash : !(A) \multimap !(B) \otimes !(B) \multimap !(A) \vdash !(B) \multimap !(A)} \star \multimap$$

01-ENC encoding (122ms)

$$\frac{\frac{\frac{B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash B}{B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash !(B)} !}{\frac{B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash A}{B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash A} D_C}{\vdash : !(A) \multimap !(B) \otimes !(B) \multimap !(A) \vdash !(B) \multimap !(A)} \star \multimap$$

$$(18) (A) \leftrightarrow (B), A \vdash B$$

LJ (38ms)

$$\frac{\frac{A, A \rightarrow B, B \rightarrow A \vdash A}{A, A \rightarrow B, B \rightarrow A \vdash B} \star}{A, A \rightarrow B \wedge B \rightarrow A \vdash B} \star \supset_L$$

MULTIPLICATIVE encoding (29ms)

Not provable

CALL-BY-NAME encoding (102ms)

$$\frac{\frac{A, !(A) \multimap B, !(B) \multimap A : \cdot \vdash A}{A, !(A) \multimap B, !(B) \multimap A : \cdot \vdash !(A)} !}{\frac{A, !(A) \multimap B, !(B) \multimap A : \cdot \vdash B}{A, !(A) \multimap B, !(B) \multimap A : \cdot \vdash B} D_C}{\vdash : !(A), !(A) \multimap B \& !(B) \multimap A \vdash B} \star \multimap$$

CALL-BY-VALUE encoding (119ms)

$$\frac{\frac{\frac{A, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash A}{A, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash !(A)} !}{\frac{A, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash B}{A, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash B} D_C}{\vdash : !(A), !(A) \multimap !(B) \otimes !(B) \multimap !(A) \vdash !(B) \multimap !(A)} \star \multimap$$

01-ENC encoding (111ms)

$$\frac{\frac{\frac{A, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash A}{A, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash !(A)} !}{\frac{A, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash B}{A, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash B} D_C}{\vdash : !(A), !(A) \multimap !(B) \otimes !(B) \multimap !(A) \vdash !(B) \multimap !(A)} \star \multimap$$

$$(19) (A) \leftrightarrow (B), B \vdash A$$

LJ (38ms)

$$\frac{\frac{B, A \rightarrow B, B \rightarrow A \vdash B}{B, A \rightarrow B, B \rightarrow A \vdash A} \star}{\frac{B, A \rightarrow B \wedge B \rightarrow A \vdash A}{B, A \rightarrow B \wedge B \rightarrow A \vdash A} \star} \supset_L$$

MULTIPLICATIVE encoding (32ms)

Not provable

CALL-BY-NAME encoding (102ms)

$$\frac{\frac{B, !(A) \multimap B, !(B) \multimap A : \cdot \vdash B}{B, !(A) \multimap B, !(B) \multimap A : \cdot \vdash !(B)} !}{\frac{B, !(A) \multimap B, !(B) \multimap A : \cdot \vdash A}{B, !(A) \multimap B, !(B) \multimap A : \cdot \vdash A} D_C} \multimap$$

CALL-BY-VALUE encoding (118ms)

$$\frac{\frac{\frac{A, B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash A}{B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash B} !}{B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash !(B)} !}{\frac{B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash A}{B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash A} D_C} \multimap$$

01-ENC encoding (108ms)

$$\frac{\frac{B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash B}{B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash !(B)} !}{\frac{B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash A}{B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash A} D_C} \multimap$$

$$(20) \cdot \vdash (A) \leftrightarrow (A)$$

LJ (21ms)

$$\frac{}{\cdot \vdash A \rightarrow A \wedge A \rightarrow A} \star$$

MULTIPLICATIVE encoding (34ms)

$$\frac{\frac{\cdot : A \vdash A}{\cdot : \cdot \vdash A \multimap A} \star}{\cdot : \cdot \vdash A \multimap A \otimes A \multimap A} \otimes$$

CALL-BY-NAME encoding (27ms)

$$\frac{\frac{A : \cdot \vdash A}{\cdot : \cdot \vdash A} \star}{\cdot : \cdot \vdash !(A) \multimap A \& !(A) \multimap A} \star$$

CALL-BY-VALUE encoding (47ms)

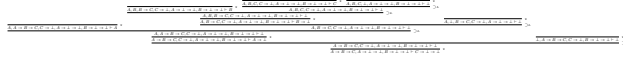
$$\frac{\frac{\frac{A : \cdot \vdash A}{A : \cdot \vdash !(A)} !}{\cdot : \cdot \vdash !(A) \multimap !(A)} \star}{\cdot : \cdot \vdash !(A) \multimap !(A) \otimes !(A) \multimap !(A)} \otimes$$

01-ENC encoding (47ms)

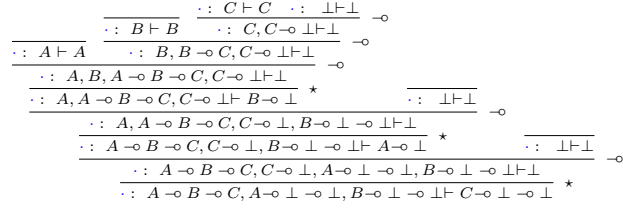
$$\frac{\frac{\frac{A : \cdot \vdash A}{\cdot : \cdot \vdash !(A) \multimap A} \star}{\cdot : \cdot \vdash !(A) \multimap A} !}{\cdot : \cdot \vdash !(A) \multimap A \& !(A) \multimap A} \star$$

$$(23) A \rightarrow B \rightarrow C, \neg\neg A, \neg\neg B \vdash \neg\neg C$$

LJ (206ms)



MULTIPLICATIVE encoding (124ms)



CALL-BY-NAME encoding (221ms)



CALL-BY-VALUE encoding (324ms)



01-ENC encoding (380ms)

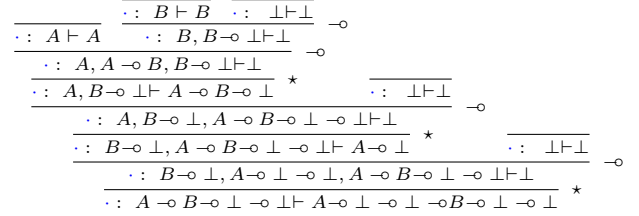


$$(24) \neg\neg A \rightarrow B \vdash \neg\neg A \rightarrow \neg\neg B$$

LJ (94ms)



MULTIPLICATIVE encoding (106ms)



CALL-BY-NAME encoding (199ms)



CALL-BY-VALUE encoding (241ms)

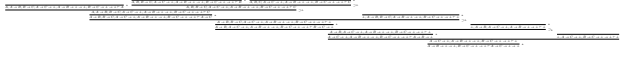


01-ENC encoding (269ms)

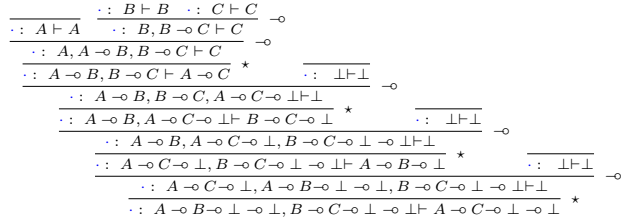


$$(25) \neg\neg A \rightarrow B, \neg\neg B \rightarrow C \vdash \neg\neg A \rightarrow C$$

LJ (324ms)



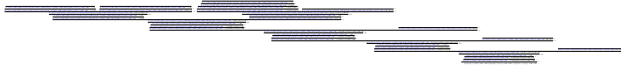
MULTIPLICATIVE encoding (145ms)



CALL-BY-NAME encoding (283ms)



CALL-BY-VALUE encoding (375ms)



01-ENC encoding (421ms)



$$(26) \cdot \vdash (\neg\neg A \wedge B) \leftrightarrow (\neg\neg A \wedge \neg\neg B)$$

LJ (137ms)



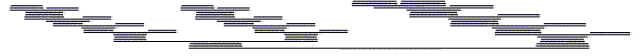
MULTIPLICATIVE encoding (66ms)

Not provable

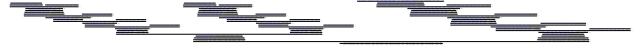
CALL-BY-NAME encoding (601ms)



CALL-BY-VALUE encoding (584ms)

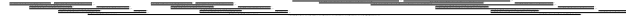


01-ENC encoding (646ms)



$$(27) \cdot \vdash (\neg\neg(A) \leftrightarrow (B)) \leftrightarrow (\neg\neg A \rightarrow B \wedge \neg\neg B \rightarrow A)$$

LJ (2703ms)



MULTIPLICATIVE encoding (84ms)

Not provable

CALL-BY-NAME encoding (661ms)



CALL-BY-VALUE encoding (841ms)

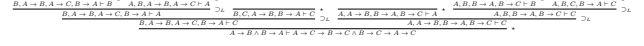


01-ENC encoding (1132ms)

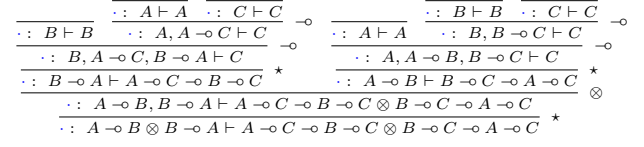


$$(28) (A) \leftrightarrow (B) \vdash (A \rightarrow C) \leftrightarrow (B \rightarrow C)$$

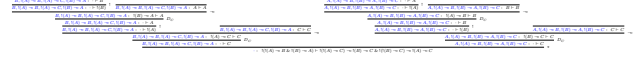
LJ (75ms)



MULTIPLICATIVE encoding (95ms)



CALL-BY-NAME encoding (173ms)



CALL-BY-VALUE encoding (288ms)



01-ENC encoding (268ms)



$$(31) (A) \leftrightarrow (B) \vdash (C \wedge A) \leftrightarrow (C \wedge B)$$

LJ (51ms)

$$\frac{\frac{\frac{A, C, A \rightarrow B, B \rightarrow A \vdash A}{A, C, A \rightarrow B, B \rightarrow A \vdash B}^* \quad \frac{A, B, C, B \rightarrow A \vdash B}{B, C, A \rightarrow B, B \rightarrow A \vdash A}^*}{A \rightarrow B \wedge B \rightarrow A \vdash C \wedge A \rightarrow C \wedge B \wedge C \wedge B \rightarrow C \wedge A}^* \supset_L$$

MULTIPLICATIVE encoding (98ms)

$$\begin{array}{c}
\frac{\vdots C \vdash C \quad \vdots B \vdash B}{\vdots A \vdash A} \otimes \\
\frac{\vdots A, C, A \multimap B \vdash C \otimes B}{\vdots A \multimap B \vdash C \otimes A \multimap C \otimes B} \circ \\
\vdots A \multimap B \vdash C \otimes A \multimap C \otimes B \quad * \\
\vdots A \multimap B, B \multimap A \vdash C \otimes A \multimap C \otimes B \vdash C \otimes A \quad * \\
\vdots A \multimap B \otimes B \multimap A \vdash C \otimes A \multimap C \otimes B \otimes C \otimes B \multimap C \otimes A \quad *
\end{array}$$

CALL-BY-NAME encoding (142ms)

[illegible]

CALL-BY-VALUE encoding (227ms)

[illegible]

01-ENC encoding (229ms)

[illegible]

$$(32) (A) \leftrightarrow (B) \vdash (\neg A) \leftrightarrow (\neg B)$$

LJ (74ms)

$$\frac{\frac{\frac{B, A \rightarrow B, A \rightarrow \perp, B \rightarrow A \vdash B^*}{B, A \rightarrow B, A \rightarrow \perp, A \rightarrow \perp A}^*}{B, A \rightarrow B, A \rightarrow \perp, B \rightarrow A \vdash A}^* \quad \supset_L \quad \frac{\frac{\frac{A, B, B \rightarrow A, B \rightarrow \perp \vdash B^*}{A, B, B \rightarrow A, B \rightarrow \perp \vdash \perp}^*}{A, A \rightarrow B, B \rightarrow A, B \rightarrow \perp \vdash A}^* \quad \supset_L \quad \frac{\frac{B, A \rightarrow B, A \rightarrow \perp, B \rightarrow A \vdash \perp}{B, A \rightarrow B, A \rightarrow \perp, B \rightarrow A \vdash \perp \wedge B \rightarrow \perp \wedge B \rightarrow \perp \wedge A \rightarrow \perp}^*}{A \rightarrow B \wedge B \rightarrow A \vdash A \rightarrow B \rightarrow \perp \wedge B \rightarrow \perp \wedge B \rightarrow \perp \wedge A \rightarrow \perp}^* \quad \supset_L$$

MULTIPLICATIVE encoding (97ms)

$$\begin{array}{c}
\frac{\vdash A \vdash A \quad \vdash \perp \vdash \perp}{\vdash B \vdash B} \quad \circ \\
\vdash B \vdash B, A \vdash A, \vdash \perp \vdash \perp \quad \circ \\
\vdash B, A \vdash A, \vdash B \vdash A \vdash \perp \quad \circ \\
\vdash B \multimap A \vdash A \multimap \perp \multimap B \multimap \perp \quad * \\
\vdash A \multimap B, B \multimap A \vdash A \multimap \perp \multimap B \multimap \perp \multimap A \multimap \perp \quad \otimes \\
\vdash A \multimap B, B \multimap A \vdash A \multimap \perp \multimap B \multimap \perp \multimap A \multimap \perp \quad \otimes \\
\vdash A \multimap B, B \multimap A \vdash A \multimap \perp \multimap B \multimap \perp \multimap A \multimap \perp \quad *
\end{array}$$

CALL-BY-NAME encoding (223ms)

Figure 1: Schematic representation of the 1200 bp DNA fragment. The diagram shows a linear DNA sequence with various restriction sites and fragments labeled. The fragments are numbered 1 through 12. The sequence is 1200 bp long, with a scale bar at the bottom indicating 1000 bp. The fragments are: 1 (100-150 bp), 2 (150-200 bp), 3 (200-250 bp), 4 (250-300 bp), 5 (300-350 bp), 6 (350-400 bp), 7 (400-450 bp), 8 (450-500 bp), 9 (500-550 bp), 10 (550-600 bp), 11 (600-650 bp), and 12 (650-700 bp). The fragments are labeled with their respective sizes in bp.

CALL-BY-VALUE encoding (304ms)

Figure 1: Schematic representation of the experimental design. The diagram shows two groups of participants: 'Control' and 'Experimental'. The 'Control' group is divided into 'Control' and 'Control + Feedback' subgroups. The 'Experimental' group is divided into 'Experimental' and 'Experimental + Feedback' subgroups. Each subgroup is further divided into 'Pre-Test' and 'Post-Test' phases. The 'Control' group is shown in a light blue box, and the 'Experimental' group is shown in a light red box. The 'Control + Feedback' and 'Experimental + Feedback' subgroups are shown in a light green box. The 'Pre-Test' and 'Post-Test' phases are shown in a light yellow box. The diagram illustrates the flow of participants through the different phases and groups, with arrows indicating the progression from Pre-Test to Post-Test.

01-ENC encoding (320ms)

[illegible]

$$(33) \cdot \vdash (A \wedge B \wedge C) \leftrightarrow (A \wedge B \wedge C)$$

LJ (21ms)

$$\cdot \vdash A \wedge B \wedge C \rightarrow A \wedge B \wedge C \wedge A \wedge B \wedge C \rightarrow A \wedge B \wedge C \quad \star$$

MULTIPLICATIVE encoding (66ms)

$$\frac{\frac{\cdot \vdash A \vdash \overline{A} \quad \cdot \vdash B \vdash \overline{B}}{\cdot \vdash A, B \vdash A \otimes B} \otimes \frac{\cdot \vdash C \vdash \overline{C}}{\cdot \vdash C \vdash \overline{C}} \otimes \frac{\cdot \vdash A \vdash \overline{A} \quad \cdot \vdash B \vdash \overline{B}}{\cdot \vdash A, B \vdash A \otimes B} \otimes \frac{\cdot \vdash C \vdash \overline{C}}{\cdot \vdash C \vdash \overline{C}} \otimes \frac{\cdot \vdash A, B, C \vdash A \otimes B \otimes C}{\cdot \vdash A \otimes B \otimes C \multimap A \otimes B \otimes C} \star \frac{\cdot \vdash A, B, C \vdash A \otimes B \otimes C}{\cdot \vdash A \otimes B \otimes C \multimap A \otimes B \otimes C} \star \frac{\cdot \vdash A \otimes B \otimes C \multimap A \otimes B \otimes C}{\cdot \vdash A \otimes B \otimes C \multimap A \otimes B \otimes C} \star$$

CALL-BY-NAME encoding (34ms)

$$\frac{\overline{A, B, C : \cdot \vdash A} \quad \overline{A, B, C : \cdot \vdash B} \quad \overline{A, B, C : \cdot \vdash C} \quad \overline{A, B, C : \cdot \vdash A} \quad \overline{A, B, C : \cdot \vdash B} \quad \overline{A, B, C : \cdot \vdash C}}{\cdot \vdash \cdot \vdash !(A \& B \& C) \multimap A \& B \& C \& !(A \& B \& C) \multimap A \& B \& C} \star$$

CALL-BY-VALUE encoding (99ms)

$$\frac{\frac{\overline{A, B, C : \cdot \vdash A} \quad \overline{A, B, C : \cdot \vdash B}}{\overline{A, B, C : \cdot \vdash !(A) \otimes !(B)}} \otimes \frac{\overline{A, B, C : \cdot \vdash C}}{\overline{A, B, C : \cdot \vdash !(C)}} \otimes \frac{\frac{\overline{A, B, C : \cdot \vdash A} \quad \overline{A, B, C : \cdot \vdash B}}{\overline{A, B, C : \cdot \vdash !(A) \otimes !(B)}} \otimes \frac{\overline{A, B, C : \cdot \vdash C}}{\overline{A, B, C : \cdot \vdash !(C)}} \otimes \frac{\cdot \vdash \cdot \vdash !(A) \otimes !(B) \otimes !(C) \multimap !(A) \otimes !(B) \otimes !(C)}{\cdot \vdash \cdot \vdash !(A) \otimes !(B) \otimes !(C) \multimap !(A) \otimes !(B) \otimes !(C)} \star \frac{\cdot \vdash \cdot \vdash !(A) \otimes !(B) \otimes !(C) \multimap !(A) \otimes !(B) \otimes !(C)}{\cdot \vdash \cdot \vdash !(A) \otimes !(B) \otimes !(C) \multimap !(A) \otimes !(B) \otimes !(C)} \star$$

01-ENC encoding (124ms)

$$\frac{\frac{\overline{A, B, C : \cdot \vdash A} \quad \overline{A, B, C : \cdot \vdash B}}{\overline{A, B, C : \cdot \vdash !(A) \& !(B)}} \star \frac{\overline{A, B, C : \cdot \vdash C}}{\overline{A, B, C : \cdot \vdash !(C)}} \star \frac{\frac{\overline{A, B, C : \cdot \vdash A} \quad \overline{A, B, C : \cdot \vdash B}}{\overline{A, B, C : \cdot \vdash !(A) \& !(B)}} \star \frac{\overline{A, B, C : \cdot \vdash C}}{\overline{A, B, C : \cdot \vdash !(C)}} \star \frac{\cdot \vdash \cdot \vdash !(A) \& !(B) \& !(C) \multimap !(A) \& !(B) \& !(C)}{\cdot \vdash \cdot \vdash !(A) \& !(B) \& !(C) \multimap !(A) \& !(B) \& !(C)} \star \frac{\cdot \vdash \cdot \vdash !(A) \& !(B) \& !(C) \multimap !(A) \& !(B) \& !(C)}{\cdot \vdash \cdot \vdash !(A) \& !(B) \& !(C) \multimap !(A) \& !(B) \& !(C)} \star$$

$$(34) \cdot \vdash (A \wedge B) \leftrightarrow (B \wedge A)$$

LJ (21ms)

$$\cdot \vdash A \wedge B \rightarrow B \wedge A \wedge B \wedge A \rightarrow A \wedge B \quad \star$$

MULTIPLICATIVE encoding (73ms)

$$\frac{\frac{\cdot \vdash B \vdash \overline{B} \quad \cdot \vdash A \vdash \overline{A}}{\cdot \vdash A, B \vdash B \otimes A} \otimes \frac{\frac{\cdot \vdash A \vdash \overline{A} \quad \cdot \vdash B \vdash \overline{B}}{\cdot \vdash A, B \vdash A \otimes B} \otimes \frac{\cdot \vdash \cdot \vdash A \otimes B \multimap B \otimes A}{\cdot \vdash \cdot \vdash A \otimes B \multimap B \otimes A} \star \frac{\cdot \vdash \cdot \vdash B \otimes A \multimap A \otimes B}{\cdot \vdash \cdot \vdash B \otimes A \multimap A \otimes B} \star \frac{\cdot \vdash \cdot \vdash A \otimes B \multimap B \otimes A \otimes B \otimes A \multimap A \otimes B}{\cdot \vdash \cdot \vdash A \otimes B \multimap B \otimes A \otimes B \otimes A \multimap A \otimes B} \otimes$$

CALL-BY-NAME encoding (35ms)

$$\frac{\overline{A, B : \cdot \vdash B} \quad \overline{A, B : \cdot \vdash A} \quad \overline{A, B : \cdot \vdash A} \quad \overline{A, B : \cdot \vdash B}}{\cdot \vdash \cdot \vdash !(A \& B) \multimap B \& A \& !(B \& A) \multimap A \& B} \star$$

CALL-BY-VALUE encoding (107ms)

$$\frac{\frac{\overline{A, B : \cdot \vdash B} \quad \overline{A, B : \cdot \vdash A}}{\overline{A, B : \cdot \vdash !(B) \otimes !(A)}} \otimes \frac{\overline{A, B : \cdot \vdash A} \quad \overline{A, B : \cdot \vdash B}}{\overline{A, B : \cdot \vdash !(A) \otimes !(B)}} \otimes \frac{\cdot \vdash \cdot \vdash !(A) \otimes !(B) \multimap !(B) \otimes !(A)}{\cdot \vdash \cdot \vdash !(A) \otimes !(B) \multimap !(B) \otimes !(A)} \star \frac{\cdot \vdash \cdot \vdash !(A) \otimes !(B) \multimap !(B) \otimes !(A)}{\cdot \vdash \cdot \vdash !(A) \otimes !(B) \multimap !(B) \otimes !(A)} \star \frac{\cdot \vdash \cdot \vdash !(A) \otimes !(B) \multimap !(B) \otimes !(A)}{\cdot \vdash \cdot \vdash !(A) \otimes !(B) \multimap !(B) \otimes !(A)} \star$$

01-ENC encoding (102ms)

$$\frac{\frac{\overline{A, B : \cdot \vdash B} \quad \overline{A, B : \cdot \vdash A}}{\overline{A, B : \cdot \vdash B \& A}} \star \frac{\overline{A, B : \cdot \vdash A} \quad \overline{A, B : \cdot \vdash B}}{\overline{A, B : \cdot \vdash A \& B}} \star \frac{\cdot \vdash \cdot \vdash !(B \& A) \multimap !(A \& B)}{\cdot \vdash \cdot \vdash !(B \& A) \multimap !(A \& B)} \star \frac{\cdot \vdash \cdot \vdash !(B \& A) \multimap !(A \& B)}{\cdot \vdash \cdot \vdash !(B \& A) \multimap !(A \& B)} \star \frac{\cdot \vdash \cdot \vdash !(B \& A) \multimap !(A \& B)}{\cdot \vdash \cdot \vdash !(B \& A) \multimap !(A \& B)} \star$$

$$(35) \cdot \vdash (A \wedge A) \leftrightarrow (A)$$

LJ (21ms)

$$\frac{}{\cdot \vdash A \wedge A \rightarrow A \wedge A \rightarrow A \wedge A} \star$$

MULTIPLICATIVE encoding (34ms)

Not provable

CALL-BY-NAME encoding (28ms)

$$\frac{\overline{A : \cdot \vdash A} \quad \overline{A : \cdot \vdash A} \quad \overline{A : \cdot \vdash A}}{\cdot : \cdot \vdash !(A \& A) \multimap A \& !(A) \multimap A \& A} \star$$

CALL-BY-VALUE encoding (76ms)

$$\frac{\frac{\overline{A : \cdot \vdash A}}{A : \cdot \vdash !(A)} ! \quad \frac{\frac{\overline{A : \cdot \vdash A}}{A : \cdot \vdash !(A)} ! \quad \frac{\overline{A : \cdot \vdash A}}{A : \cdot \vdash !(A)} !}{\cdot : \cdot \vdash !(A) \otimes !(A) \multimap !(A)} \star \quad \frac{\cdot : \cdot \vdash !(A) \otimes !(A) \multimap !(A)}{\cdot : \cdot \vdash !(A) \otimes !(A) \multimap !(A) \otimes !(A)} ! \quad \frac{\cdot : \cdot \vdash !(A) \otimes !(A) \multimap !(A) \otimes !(A)}{\cdot : \cdot \vdash !(A) \otimes !(A) \multimap !(A) \otimes !(A) \otimes !(A)} \otimes$$

01-ENC encoding (81ms)

$$\frac{\frac{\overline{A : \cdot \vdash A} \quad \overline{A : \cdot \vdash A}}{\cdot : \cdot \vdash !(A) \& !(A) \multimap A} \star \quad \frac{\overline{A : \cdot \vdash A \& A}}{A : \cdot \vdash !(A \& A)} !}{\cdot : \cdot \vdash !(A) \& !(A) \multimap A \& !(A \& A)} ! \quad \frac{\cdot : \cdot \vdash !(A) \& !(A) \multimap A \& !(A \& A)}{\cdot : \cdot \vdash !(A) \& !(A) \multimap A \& !(A \& A) \otimes !(A \& A)} \otimes$$

$$(36) A \vdash (A \rightarrow B) \leftrightarrow (B)$$

LJ (37ms)

$$\frac{\overline{A, A \rightarrow B \vdash A} \quad \overline{A, B \vdash B}}{A, A \rightarrow B \vdash B} \star \quad \frac{}{A \vdash A \rightarrow B \rightarrow B \wedge B \rightarrow A \rightarrow B} \star \quad \supset_L$$

MULTIPLICATIVE encoding (42ms)

Not provable

CALL-BY-NAME encoding (73ms)

$$\frac{\overline{A, !(A) \multimap B : \cdot \vdash A}}{A, !(A) \multimap B : \cdot \vdash !(A)} ! \quad \frac{\overline{A, !(A) \multimap B : B \vdash B}}{A, !(A) \multimap B : !(A) \multimap B \vdash B} \multimap \quad \frac{A, !(A) \multimap B : !(A) \multimap B \vdash B}{A, !(A) \multimap B : \cdot \vdash B} D_C \quad \frac{A, B : \cdot \vdash B}{\cdot : !(A) \vdash !(A) \multimap B \multimap B \& !(B) \multimap !(A) \multimap B} \star$$

CALL-BY-VALUE encoding (129ms)

$$\frac{\frac{\overline{A, !(A) \multimap B : \cdot \vdash A}}{A, !(A) \multimap B : \cdot \vdash !(A)} ! \quad \frac{\overline{A, B, !(A) \multimap B : \cdot \vdash B}}{A, B, !(A) \multimap B : !(B) \vdash B} ! \quad \frac{\overline{A, B : \cdot \vdash B}}{A, B : \cdot \vdash !(B)} !}{\frac{A, !(A) \multimap B : !(A) \multimap B \vdash B}{A, !(A) \multimap B : \cdot \vdash B} D_C \quad \frac{A, B : \cdot \vdash !(B) \multimap !(A) \multimap B}{A : \cdot \vdash !(A) \multimap !(B) \multimap !(A) \multimap B} !} \star$$

01-ENC encoding (123ms)

$$\frac{\overline{A, !(A) \multimap B : \cdot \vdash A}}{A, !(A) \multimap B : \cdot \vdash !(A)} ! \quad \frac{\overline{A, B, !(A) \multimap B : \cdot \vdash B}}{A, B, !(A) \multimap B : !(B) \vdash B} ! \quad \frac{\overline{A, B : \cdot \vdash B}}{A, B : \cdot \vdash !(B)} !}{\frac{A, !(A) \multimap B : !(A) \multimap B \vdash B}{A, !(A) \multimap B : \cdot \vdash B} D_C \quad \frac{A, B : \cdot \vdash !(B) \multimap !(A) \multimap B}{A : \cdot \vdash !(A) \multimap !(B) \multimap !(A) \multimap B} !} \star$$

$$(37) \quad B \vdash (A \rightarrow B) \leftrightarrow (B)$$

LJ (21ms)

$$\overline{\overline{B \vdash A \rightarrow B \rightarrow B \wedge B \rightarrow A \rightarrow B}} \quad \star$$

MULTIPLICATIVE encoding (40ms)

Not provable

CALL-BY-NAME encoding (39ms)

$$\frac{\frac{B, !(A) \multimap B : \cdot \vdash B \quad A, B : \cdot \vdash B}{\cdot : !(B) \vdash !(A) \multimap B} \quad \cdot : !(B) \vdash !(A) \multimap B}{\cdot : !(B) \vdash !(A) \multimap B \& !(B) \multimap !(A) \multimap B} \star$$

CALL-BY-VALUE encoding (152ms)

$$\begin{array}{c}
\frac{A, B : \vdash B}{A, B : \vdash \mathbf{!}(B)} \mathbf{!} \\
\frac{B, \mathbf{!}(A) \multimap \mathbf{!}(B) : \vdash B}{B, \mathbf{!}(A) \multimap \mathbf{!}(B) : \vdash \mathbf{!}(B)} \mathbf{!} \\
\frac{B : \vdash \mathbf{!}(\mathbf{!}(A) \multimap \mathbf{!}(B)) \multimap \mathbf{!}(B)}{B : \vdash \mathbf{!}(\mathbf{!}(A) \multimap \mathbf{!}(B)) \multimap \mathbf{!}(B)} \star \\
\frac{B : \vdash \mathbf{!}(\mathbf{!}(A) \multimap \mathbf{!}(B)) \multimap \mathbf{!}(B)}{B : \vdash \mathbf{!}(\mathbf{!}(A) \multimap \mathbf{!}(B)) \multimap \mathbf{!}(B)} \mathbf{!} \\
\frac{B : \vdash \mathbf{!}(\mathbf{!}(A) \multimap \mathbf{!}(B)) \multimap \mathbf{!}(B)}{B : \vdash \mathbf{!}(\mathbf{!}(A) \multimap \mathbf{!}(B)) \multimap \mathbf{!}(B)} \mathbf{!} \\
\frac{B : \vdash \mathbf{!}(\mathbf{!}(A) \multimap \mathbf{!}(B)) \multimap \mathbf{!}(B)}{B : \vdash \mathbf{!}(\mathbf{!}(A) \multimap \mathbf{!}(B)) \multimap \mathbf{!}(B)} \otimes \\
\frac{B : \vdash \mathbf{!}(\mathbf{!}(A) \multimap \mathbf{!}(B)) \multimap \mathbf{!}(B)}{B : \vdash \mathbf{!}(\mathbf{!}(A) \multimap \mathbf{!}(B)) \multimap \mathbf{!}(B)} \star
\end{array}$$

01-ENC encoding (88ms)

[illegible]

$$(38) \neg A \vdash (A \rightarrow B) \leftrightarrow (\neg A)$$

LJ (52ms)

$$\frac{\frac{\frac{A, A \rightarrow B, A \rightarrow \perp \vdash A}{A, A \rightarrow B, A \rightarrow \perp \vdash \perp}^* \quad \frac{A, \perp, A \rightarrow B \vdash \perp}{A, A \rightarrow \perp \vdash B}^*}{A, A \rightarrow \perp \vdash B} \supset_L \quad \frac{\frac{A, A \rightarrow \perp \vdash A}{A, A \rightarrow \perp \vdash B}^* \quad \frac{A, \perp \vdash B}{A, A \rightarrow \perp \vdash B}^*}{A, A \rightarrow \perp \vdash B} \supset_L$$

MULTIPLICATIVE encoding (42ms)

Not provable

CALL-BY-NAME encoding (101ms)

[illegible]

CALL-BY-VALUE encoding (201ms)

[illegible]

01-ENC encoding (203ms)

[illegible]

$$(41) \neg B \vdash (A \wedge B) \leftrightarrow (B)$$

LJ (37ms)

$$\frac{\frac{B, B \rightarrow \perp \vdash B \quad \star \quad B, \perp \vdash A \quad \star}{B, B \rightarrow \perp \vdash A} \supset_L}{B \rightarrow \perp \vdash A \wedge B \rightarrow B \wedge B \rightarrow A \wedge B} \star$$

MULTIPLICATIVE encoding (41ms)

Not provable

CALL-BY-NAME encoding (78ms)

$$\frac{\frac{\frac{B, ! (B) \multimap 0 : \cdot \vdash B}{B, ! (B) \multimap 0 : \cdot \vdash ! (B)} ! \quad \frac{B, ! (B) \multimap 0 : 0 \vdash A}{B, ! (B) \multimap 0 : 0 \vdash A} \star}{\frac{B, ! (B) \multimap 0 : ! (B) \multimap 0 \vdash A}{B, ! (B) \multimap 0 : \cdot \vdash A} D_C} \multimap}{\frac{A, B, ! (B) \multimap 0 : \cdot \vdash B}{A, B, ! (B) \multimap 0 : \cdot \vdash B} \quad \frac{B, ! (B) \multimap 0 : \cdot \vdash A}{B, ! (B) \multimap 0 : \cdot \vdash A} \quad \frac{B, ! (B) \multimap 0 : \cdot \vdash B}{B, ! (B) \multimap 0 : \cdot \vdash B} \star}{\cdot : ! (! (B) \multimap 0) \vdash ! (A \& B) \multimap B \& ! (B) \multimap A \& B} \star$$

CALL-BY-VALUE encoding (172ms)

$$\frac{\frac{\frac{A, B, ! (B) \multimap 0 : \cdot \vdash B}{A, B, ! (B) \multimap 0 : \cdot \vdash ! (B)} ! \quad \frac{B, ! (B) \multimap 0 : \cdot \vdash ! (B)}{B, ! (B) \multimap 0 : \cdot \vdash ! (B)} ! \quad \frac{B, ! (B) \multimap 0 : ! (B) \multimap 0 \vdash ! (A) \otimes ! (B)}{B, ! (B) \multimap 0 : \cdot \vdash ! (A) \otimes ! (B)} \star}{\frac{! (B) \multimap 0 : \cdot \vdash ! (A) \otimes ! (B) \multimap ! (B)}{! (B) \multimap 0 : \cdot \vdash ! (A) \otimes ! (B)} \star \quad \frac{! (B) \multimap 0 : \cdot \vdash ! (B) \multimap ! (A) \otimes ! (B)}{! (B) \multimap 0 : \cdot \vdash ! (B) \multimap ! (A) \otimes ! (B)} \star}{\frac{! (B) \multimap 0 : \cdot \vdash ! (! (A) \otimes ! (B) \multimap ! (B)) \otimes ! (! (B) \multimap ! (A) \otimes ! (B))}{\cdot : ! (! (B) \multimap 0) \vdash ! (! (A) \otimes ! (B) \multimap ! (B)) \otimes ! (! (B) \multimap ! (A) \otimes ! (B))} \otimes} \star$$

01-ENC encoding (194ms)

$$\frac{\frac{\frac{A, B, ! (B) \multimap ! (0) : \cdot \vdash B}{A, B, ! (B) \multimap ! (0) : \cdot \vdash ! (B)} ! \quad \frac{B, ! (B) \multimap ! (0) : \cdot \vdash ! (B)}{B, ! (B) \multimap ! (0) : \cdot \vdash ! (B)} ! \quad \frac{B, ! (B) \multimap ! (0) : ! (B) \multimap ! (0) \vdash ! (A \& B)}{B, ! (B) \multimap ! (0) : \cdot \vdash ! (A \& B)} \star}{\frac{! (B) \multimap ! (0) : \cdot \vdash ! (! (A) \& ! (B)) \multimap B}{! (B) \multimap ! (0) : \cdot \vdash ! (! (A) \& ! (B)) \multimap B} \star \quad \frac{! (B) \multimap ! (0) : \cdot \vdash ! (B) \multimap ! (A \& B)}{! (B) \multimap ! (0) : \cdot \vdash ! (B) \multimap ! (A \& B)} \star}{\frac{! (B) \multimap ! (0) : \cdot \vdash ! (! (! (A) \& ! (B)) \multimap B) \& ! (! (B) \multimap ! (A \& B))}{\cdot : ! (! (B) \multimap ! (0)) \vdash ! (! (! (A) \& ! (B)) \multimap B) \& ! (! (B) \multimap ! (A \& B))} \star} \star$$

$$(42) \cdot \vdash A \rightarrow \neg \neg A$$

LJ (38ms)

$$\frac{\frac{A, A \rightarrow \perp \vdash A \quad \star \quad A, \perp \vdash \perp \quad \star}{A, A \rightarrow \perp \vdash \perp} \supset_L}{\cdot \vdash A \rightarrow A \rightarrow \perp \rightarrow \perp} \star$$

MULTIPLICATIVE encoding (41ms)

$$\frac{\frac{\cdot : A \vdash A \quad \cdot : \perp \vdash \perp}{\cdot : A, A \multimap \perp \vdash \perp} \multimap}{\cdot : \cdot \vdash A \multimap A \multimap \perp \multimap \perp} \star$$

CALL-BY-NAME encoding (66ms)

$$\frac{\frac{A, ! (A) \multimap 0 : \cdot \vdash A}{A, ! (A) \multimap 0 : \cdot \vdash ! (A)} ! \quad \frac{A, ! (A) \multimap 0 : 0 \vdash 0}{A, ! (A) \multimap 0 : 0 \vdash 0} \star}{\frac{A, ! (A) \multimap 0 : ! (A) \multimap 0 \vdash 0}{A, ! (A) \multimap 0 : \cdot \vdash 0} D_C} \multimap}{\cdot : \cdot \vdash ! (A) \multimap ! (! (A) \multimap 0) \multimap 0} \star$$

CALL-BY-VALUE encoding (84ms)

$$\frac{\frac{A, ! (A) \multimap 0 : \cdot \vdash A}{A, ! (A) \multimap 0 : \cdot \vdash ! (A)} ! \quad \frac{A, ! (A) \multimap 0 : 0 \vdash 0}{A, ! (A) \multimap 0 : 0 \vdash 0} \star}{\frac{A, ! (A) \multimap 0 : ! (A) \multimap 0 \vdash 0}{A, ! (A) \multimap 0 : \cdot \vdash 0} D_C} \multimap}{\frac{A : \cdot \vdash ! (! (A) \multimap 0) \multimap 0}{A : \cdot \vdash ! (! (! (A) \multimap 0) \multimap 0)} \star} !}{\cdot : \cdot \vdash ! (A) \multimap ! (! (! (A) \multimap 0) \multimap 0)} \star} !$$

01-ENC encoding (81ms)

$$\frac{\frac{A, ! (A) \multimap ! (0) : \cdot \vdash A}{A, ! (A) \multimap ! (0) : \cdot \vdash ! (A)} ! \quad \frac{A, ! (A) \multimap ! (0) : ! (0) \vdash 0}{A, ! (A) \multimap ! (0) : ! (0) \vdash 0} \star}{\frac{A, ! (A) \multimap ! (0) : ! (A) \multimap ! (0) \vdash 0}{A, ! (A) \multimap ! (0) : \cdot \vdash 0} D_C} \multimap}{\frac{A : \cdot \vdash ! (! (A) \multimap ! (0)) \multimap 0}{A : \cdot \vdash ! (! (! (A) \multimap ! (0)) \multimap 0)} \star} !}{\cdot : \cdot \vdash ! (A) \multimap ! (! (! (A) \multimap ! (0)) \multimap 0)} \star} !$$

$$(45) \cdot \vdash \neg(A) \leftrightarrow (\neg A)$$

LJ (80ms)

$$\frac{\frac{\frac{\frac{A \rightarrow A \rightarrow \perp, A \rightarrow \perp \rightarrow A \vdash A}{A \rightarrow A \rightarrow \perp, A \rightarrow \perp \rightarrow A \vdash A} \text{Id} \quad \frac{A \rightarrow A \rightarrow \perp, A \rightarrow \perp \rightarrow A \vdash A}{A \rightarrow A \rightarrow \perp, A \rightarrow \perp \rightarrow A \vdash A} \text{Id} \quad \frac{A \rightarrow A \rightarrow \perp, A \rightarrow \perp \rightarrow A \vdash A}{A \rightarrow A \rightarrow \perp, A \rightarrow \perp \rightarrow A \vdash A} \text{Id}}{\frac{A \rightarrow A \rightarrow \perp, A \rightarrow \perp \rightarrow A \vdash A}{A \rightarrow A \rightarrow \perp, A \rightarrow \perp \rightarrow A \vdash A} \text{Id}} \text{Id}$$

MULTIPLICATIVE encoding (41ms)

Not provable

CALL-BY-NAME encoding (231ms)

Genomic map of the 1000 Genomes Project on chromosome 10. The map shows the positions of various genomic features, including genes, transcripts, and SNPs. The x-axis represents the genomic position in Mb, ranging from 0 to 100. The y-axis represents the frequency of the variant, ranging from 0 to 100. The map is divided into several regions, each with a different color and pattern. The regions are labeled with their respective gene names and transcript IDs. The map shows a high density of SNPs in the 1000 Genomes Project, particularly in the regions around 10, 20, and 30 Mb.

CALL-BY-VALUE encoding (282ms)

01-ENC encoding (300ms)

Figure 1 is a phylogenetic tree showing the relationships between 16 isolates. The tree is rooted at the bottom and branches upwards. The isolates are labeled with their IDs: 16S, 16S, 16S, 16S, 16S, 16S, 16S, 16S, 16S, 16S, 16S, 16S, 16S, 16S, 16S, and 16S. The tree shows several clusters of isolates with high bootstrap values (e.g., 100%, 99%, 98%, 97%, 96%, 95%, 94%, 93%, 92%, 91%, 90%, 89%, 88%, 87%, 86%, 85%, 84%, 83%, 82%, 81%, 80%, 79%, 78%, 77%, 76%, 75%, 74%, 73%, 72%, 71%, 70%, 69%, 68%, 67%, 66%, 65%, 64%, 63%, 62%, 61%, 60%, 59%, 58%, 57%, 56%, 55%, 54%, 53%, 52%, 51%, 50%, 49%, 48%, 47%, 46%, 45%, 44%, 43%, 42%, 41%, 40%, 39%, 38%, 37%, 36%, 35%, 34%, 33%, 32%, 31%, 30%, 29%, 28%, 27%, 26%, 25%, 24%, 23%, 22%, 21%, 20%, 19%, 18%, 17%, 16%, 15%, 14%, 13%, 12%, 11%, 10%, 9%, 8%, 7%, 6%, 5%, 4%, 3%, 2%, 1%). The tree also shows some isolates with lower bootstrap values (e.g., 60%, 50%, 40%, 30%, 20%, 10%, 5%, 1%).

$$(46) \cdot \vdash \neg\neg\neg\neg A \rightarrow A$$

LJ (66ms)

[illegible]

MULTIPLICATIVE encoding (41ms)

Not provable

CALL-BY-NAME encoding (198ms)

[illegible]

CALL-BY-VALUE encoding (210ms)

[illegible]

01-ENC encoding (217ms)

Figure 1 illustrates the hierarchical clustering of 1000 genes into 10 clusters. The tree structure shows the following clusters and their associated genes:

- Cluster 1:** 1000 genes
- Cluster 2:** 1000 genes
- Cluster 3:** 1000 genes
- Cluster 4:** 1000 genes
- Cluster 5:** 1000 genes
- Cluster 6:** 1000 genes
- Cluster 7:** 1000 genes
- Cluster 8:** 1000 genes
- Cluster 9:** 1000 genes
- Cluster 10:** 1000 genes

$$(47) \cdot \vdash (A \wedge B \wedge \neg B) \leftrightarrow (B \wedge \neg B)$$

LJ (61ms)

$$\frac{\frac{A, B, B \rightarrow \perp \vdash B}{A, B, B \rightarrow \perp \vdash \perp}^* \quad \frac{A, B, \perp \vdash \perp}{A, B, B \rightarrow \perp \vdash \perp}^* \quad \frac{B, B \rightarrow \perp \vdash B}{B, B \rightarrow \perp \vdash A}^* \quad \frac{B, \perp \vdash A}{B, B \rightarrow \perp \vdash A}^* \quad \frac{B, B \rightarrow \perp \vdash B}{B, \perp \vdash \perp}^* \quad \frac{B, \perp \vdash \perp}{B, B \rightarrow \perp \vdash \perp}^*}{\vdash A \wedge B \wedge B \rightarrow \perp \rightarrow B \wedge B \rightarrow \perp \wedge B \wedge B \rightarrow \perp \rightarrow A \wedge B \wedge B \rightarrow \perp}^* \quad \supset_L \quad \supset_L \quad \supset_L$$

MULTIPLICATIVE encoding (42ms)

Not provable

CALL-BY-NAME encoding (121ms)

$$\begin{array}{c}
\frac{\frac{A, B, \text{Int}(x) \rightarrow 0 : \vdash B}{A, B, \text{Int}(x) \rightarrow 0 : \vdash \text{Int}(x)} \quad \frac{B, \text{Int}(x) \rightarrow 0 : \vdash B}{B, \text{Int}(x) \rightarrow 0 : \vdash \text{Int}(x)} + \\
\frac{A, B, \text{Int}(x) \rightarrow 0 : \vdash \text{Int}(x)}{A, B, \text{Int}(x) \rightarrow 0 : \vdash 0} = 0 \\
\frac{A, B, \text{Int}(x) \rightarrow 0 : \vdash \text{Int}(x) \rightarrow 0 \vdash 0}{A, B, \text{Int}(x) \rightarrow 0 : \vdash 0} D_C \\
\frac{A, B, \text{Int}(x) \rightarrow 0 : \vdash 0}{A, B, \text{Int}(x) \rightarrow 0 : \vdash 0} D_C \\
\vdots : \vdash \text{Int}(A \& B \& \text{Int}(x) \rightarrow 0) \rightarrow B \& \text{Int}(x) \rightarrow 0 \& \text{Int}(x) \rightarrow 0 \rightarrow A \& B \& \text{Int}(x) \rightarrow 0
\end{array}$$

CALL-BY-VALUE encoding (140ms)

[illegible]

01-ENC encoding (132ms)

[illegible]

$$(48) \cdot \vdash A \rightarrow B \rightarrow \neg A \wedge \neg B$$

LJ (49ms)

$$\frac{\frac{\frac{A, A \rightarrow B, B \rightarrow \perp \vdash A}{A, A \rightarrow B, B \rightarrow \perp \vdash \perp} \star \quad \frac{\frac{A, B, B \rightarrow \perp \vdash B}{A, B, B \rightarrow \perp \vdash \perp} \star \quad \frac{A, B, \perp \vdash \perp}{A, B, B \rightarrow \perp \vdash \perp} \star}{A, A \rightarrow B, B \rightarrow \perp \vdash \perp} \supset_L \quad \frac{A, A \rightarrow B, B \rightarrow \perp \vdash \perp}{\vdash A \rightarrow B \rightarrow A \wedge B \rightarrow \perp \rightarrow \perp} \star \supset_L$$

MULTIPLICATIVE encoding (48ms)

$$\frac{\frac{\frac{\cdot : A \vdash A}{\cdot : B \vdash B} \quad \frac{\cdot : \perp \vdash \perp}{\cdot : B, B \multimap \perp \vdash \perp}}{\cdot : A, A \multimap B, B \multimap \perp \vdash \perp} \multimap \quad \frac{\cdot : \cdot \vdash A \multimap B \multimap A \otimes B \multimap \perp \multimap \perp}{\cdot : \cdot \vdash A \multimap B \multimap A \otimes B \multimap \perp \multimap \perp} \star$$

CALL-BY-NAME encoding (143ms)

[illegible]

CALL-BY-VALUE encoding (167ms)

[illegible]

01-ENC encoding (169ms)

[illegible]

$$(49) \cdot \vdash (A \rightarrow \neg B) \leftrightarrow (\neg A \wedge B)$$

LJ (58ms)

$$\frac{\overline{A, B, A \rightarrow B \rightarrow \perp \perp}^*}{\overline{A, B, A \rightarrow B \rightarrow \perp \perp}}^* \quad \frac{\overline{A, B, B \rightarrow \perp \perp}^*}{\overline{A, B, B \rightarrow \perp \perp}}^* \quad \frac{\overline{A, B, \perp \perp \perp}^*}{\overline{A, B, \perp \perp \perp}}^* \quad \textcircled{L} \quad \frac{\overline{A, B, A \wedge B \rightarrow \perp \perp}^*}{\overline{A, B, A \wedge B \rightarrow \perp \perp}}^* \quad \frac{\overline{A, B, \perp \perp \perp}^*}{\overline{A, B, \perp \perp \perp}}^* \quad \textcircled{L}$$

MULTIPLICATIVE encoding (88ms)

$$\frac{\frac{\frac{\vdots, B \vdash B \quad \vdots, \perp \vdash \perp}{\vdots, A \vdash A} \quad \vdots, B, B \multimap \perp \vdash \perp}{\vdots, A, B, A \multimap B \vdash \perp} \multimap \quad \frac{\vdots, A, B, A \otimes B \vdash \perp}{\vdots, \perp \vdash \perp} \otimes}{\vdots, A, B, A \multimap B \multimap \perp \vdash \perp} \multimap \quad \frac{\vdots, \vdash A \multimap B \multimap \perp \multimap A \otimes B \multimap \perp}{\vdots, \vdash A \multimap B \multimap \perp \multimap A \otimes B \multimap \perp \multimap A \multimap B \multimap \perp} *$$

CALL-BY-NAME encoding (118ms)

[illegible]

CALL-BY-VALUE encoding (281ms)

$$\begin{aligned}
 & \frac{A_1(A_1+1)(A_1+2)(A_1+3)(A_1+4)(A_1+5)(A_1+6)(A_1+7)(A_1+8)(A_1+9)(A_1+10)(A_1+11)(A_1+12)(A_1+13)(A_1+14)(A_1+15)(A_1+16)(A_1+17)(A_1+18)(A_1+19)(A_1+20)(A_1+21)(A_1+22)(A_1+23)(A_1+24)(A_1+25)(A_1+26)(A_1+27)(A_1+28)(A_1+29)(A_1+30)(A_1+31)(A_1+32)(A_1+33)(A_1+34)(A_1+35)(A_1+36)(A_1+37)(A_1+38)(A_1+39)(A_1+40)(A_1+41)(A_1+42)(A_1+43)(A_1+44)(A_1+45)(A_1+46)(A_1+47)(A_1+48)(A_1+49)(A_1+50)(A_1+51)(A_1+52)(A_1+53)(A_1+54)(A_1+55)(A_1+56)(A_1+57)(A_1+58)(A_1+59)(A_1+60)(A_1+61)(A_1+62)(A_1+63)(A_1+64)(A_1+65)(A_1+66)(A_1+67)(A_1+68)(A_1+69)(A_1+70)(A_1+71)(A_1+72)(A_1+73)(A_1+74)(A_1+75)(A_1+76)(A_1+77)(A_1+78)(A_1+79)(A_1+80)(A_1+81)(A_1+82)(A_1+83)(A_1+84)(A_1+85)(A_1+86)(A_1+87)(A_1+88)(A_1+89)(A_1+90)(A_1+91)(A_1+92)(A_1+93)(A_1+94)(A_1+95)(A_1+96)(A_1+97)(A_1+98)(A_1+99)(A_1+100)(A_1+101)(A_1+102)(A_1+103)(A_1+104)(A_1+105)(A_1+106)(A_1+107)(A_1+108)(A_1+109)(A_1+110)(A_1+111)(A_1+112)(A_1+113)(A_1+114)(A_1+115)(A_1+116)(A_1+117)(A_1+118)(A_1+119)(A_1+120)(A_1+121)(A_1+122)(A_1+123)(A_1+124)(A_1+125)(A_1+126)(A_1+127)(A_1+128)(A_1+129)(A_1+130)(A_1+131)(A_1+132)(A_1+133)(A_1+134)(A_1+135)(A_1+136)(A_1+137)(A_1+138)(A_1+139)(A_1+140)(A_1+141)(A_1+142)(A_1+143)(A_1+144)(A_1+145)(A_1+146)(A_1+147)(A_1+148)(A_1+149)(A_1+150)(A_1+151)(A_1+152)(A_1+153)(A_1+154)(A_1+155)(A_1+156)(A_1+157)(A_1+158)(A_1+159)(A_1+160)(A_1+161)(A_1+162)(A_1+163)(A_1+164)(A_1+165)(A_1+166)(A_1+167)(A_1+168)(A_1+169)(A_1+170)(A_1+171)(A_1+172)(A_1+173)(A_1+174)(A_1+175)(A_1+176)(A_1+177)(A_1+178)(A_1+179)(A_1+180)(A_1+181)(A_1+182)(A_1+183)(A_1+184)(A_1+185)(A_1+186)(A_1+187)(A_1+188)(A_1+189)(A_1+190)(A_1+191)(A_1+192)(A_1+193)(A_1+194)(A_1+195)(A_1+196)(A_1+197)(A_1+198)(A_1+199)(A_1+200)(A_1+201)(A_1+202)(A_1+203)(A_1+204)(A_1+205)(A_1+206)(A_1+207)(A_1+208)(A_1+209)(A_1+210)(A_1+211)(A_1+212)(A_1+213)(A_1+214)(A_1+215)(A_1+216)(A_1+217)(A_1+218)(A_1+219)(A_1+220)(A_1+221)(A_1+222)(A_1+223)(A_1+224)(A_1+225)(A_1+226)(A_1+227)(A_1+228)(A_1+229)(A_1+230)(A_1+231)(A_1+232)(A_1+233)(A_1+234)(A_1+235)(A_1+236)(A_1+237)(A_1+238)(A_1+239)(A_1+240)(A_1+241)(A_1+242)(A_1+243)(A_1+244)(A_1+245)(A_1+246)(A_1+247)(A_1+248)(A_1+249)(A_1+250)(A_1+251)(A_1+252)(A_1+253)(A_1+254)(A_1+255)(A_1+256)(A_1+257)(A_1+258)(A_1+259)(A_1+260)(A_1+261)(A_1+262)(A_1+263)(A_1+264)(A_1+265)(A_1+266)(A_1+267)(A_1+268)(A_1+269)(A_1+270)(A_1+271)(A_1+272)(A_1+273)(A_1+274)(A_1+275)(A_1+276)(A_1+277)(A_1+278)(A_1+279)(A_1+280)(A_1+281)(A_1+282)(A_1+283)(A_1+284)(A_1+285)(A_1+286)(A_1+287)(A_1+288)(A_1+289)(A_1+290)(A_1+291)(A_1+292)(A_1+293)(A_1+294)(A_1+295)(A_1+296)(A_1+297)(A_1+298)(A_1+299)(A_1+300)(A_1+301)(A_1+302)(A_1+303)(A_1+304)(A_1+305)(A_1+306)(A_1+307)(A_1+308)(A_1+309)(A_1+310)(A_1+311)(A_1+312)(A_1+313)(A_1+314)(A_1+315)(A_1+316)(A_1+317)(A_1+318)(A_1+319)(A_1+320)(A_1+321)(A_1+322)(A_1+323)(A_1+324)(A_1+325)(A_1+326)(A_1+327)(A_1+328)(A_1+329)(A_1+330)(A_1+331)(A_1+332)(A_1+333)(A_1+334)(A_1+335)(A_1+336)(A_1+337)(A_1+338)(A_1+339)(A_1+340)(A_1+341)(A_1+342)(A_1+343)(A_1+344)(A_1+345)(A_1+346)(A_1+347)(A_1+348)(A_1+349)(A_1+350)(A_1+351)(A_1+352)(A_1+353)(A_1+354)(A_1+355)(A_1+356)(A_1+357)(A_1+358)(A_1+359)(A_1+360)(A_1+361)(A_1+362)(A_1+363)(A_1+364)(A_1+365)(A_1+366)(A_1+367)(A_1+368)(A_1+369)(A_1+370)(A_1+371)(A_1+372)(A_1+373)(A_1+374)(A_1+375)(A_1+376)(A_1+377)(A_1+378)(A_1+379)(A_1+380)(A_1+381)(A_1+382)(A_1+383)(A_1+384)(A_1+385)(A_1+386)(A_1+387)(A_1+388)(A_1+389)(A_1+390)(A_1+391)(A_1+392)(A_1+393)(A_1+394)(A_1+395)(A_1+396)(A_1+397)(A_1+398)(A_1+399)(A_1+400)(A_1+401)(A_1+402)(A_1+403)(A_1+404)(A_1+405)(A_1+406)(A_1+407)(A_1+408)(A_1+409)(A_1+410)(A_1+411)(A_1+412)(A_1+413)(A_1+414)(A_1+415)(A_1+416)(A_1+417)(A_1+418)(A_1+419)(A_1+420)(A_1+421)(A_1+422)(A_1+423)(A_1+424)(A_1+425)(A_1+426)(A_1+427)(A_1+428)(A_1+429)(A_1+430)(A_1+431)(A_1+432)(A_1+433)(A_1+434)(A_1+435)(A_1+436)(A_1+437)(A_1+438)(A_1+439)(A_1+440)(A_1+441)(A_1+442)(A_1+443)(A_1+444)(A_1+445)(A_1+446)(A_1+447)(A_1+448)(A_1+449)(A_1+450)(A_1+451)(A_1+452)(A_1+453)(A_1+454)(A_1+455)(A_1+456)(A_1+457)(A_1+458)(A_1+459)(A_1+460)(A_1+461)(A_1+462)(A_1+463)(A_1+464)(A_1+465)(A_1+466)(A_1+467)(A_1+468)(A_1+469)(A_1+470)(A_1+471)(A_1+472)(A_1+473)(A_1+474)(A_1+475)(A_1+476)(A_1+477)(A_1+478)(A_1+479)(A_1+480)(A_1+481)(A_1+482)(A_1+483)(A_1+484)(A_1+485)(A_1+486)(A_1+487)(A_1+488)(A_1+489)(A_1+490)(A_1+491)(A_1+492)(A_1+493)(A_1+494)(A_1+495)(A_1+496)(A_1+497)(A_1+498)(A_1+499)(A_1+500)(A_1+501)(A_1+502)(A_1+503)(A_1+504)(A_1+505)(A_1+506)(A_1+507)(A_1+508)(A_1+509)(A_1+510)(A_1+511)(A_1+512)(A_1+513)(A_1+514)(A_1+515)(A_1+516)(A_1+517)(A_1+518)(A_1+519)($$

01-ENC encoding (318ms)

[illegible]

$$(50) \cdot \vdash (\neg A \wedge B) \leftrightarrow (\neg\neg A \rightarrow \neg B)$$

LJ (93ms)

[illegible]

MULTIPLICATIVE encoding (134ms)

[illegible]

CALL-BY-NAME encoding (333ms)

CALL-BY-VALUE encoding (421ms)

The diagram illustrates the 1000 Genomes Project structure. At the top is the '1000 Genomes Project' box. Below it, the project is divided into 'Phase 1' and 'Phase 2'. Phase 1 includes 'YRI', 'CEU', and 'JPT' populations. Phase 2 includes 'JPT+CHB+CEU', 'JPT+CHB+CEU+YRI', and 'JPT+CHB+CEU+YRI+CHL'. The diagram shows the expansion of the reference panel across different populations and phases.

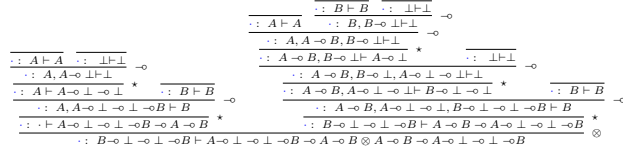
01-ENC encoding (470ms)

$$(51) \neg\neg B \rightarrow B \vdash (\neg\neg A \rightarrow B) \leftrightarrow (A \rightarrow B)$$

LJ (135ms)



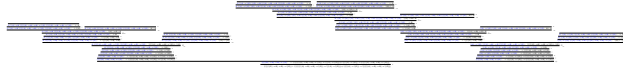
MULTIPLICATIVE encoding (156ms)



CALL-BY-NAME encoding (278ms)



CALL-BY-VALUE encoding (414ms)

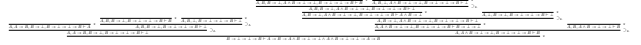


01-ENC encoding (449ms)

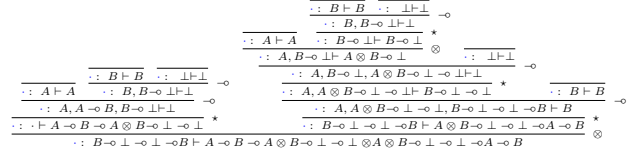


$$(52) \neg\neg B \rightarrow B \vdash (A \rightarrow B) \leftrightarrow (\neg A \wedge \neg B)$$

LJ (94ms)



MULTIPLICATIVE encoding (134ms)



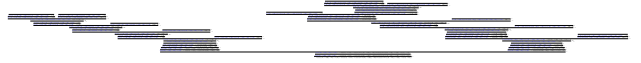
CALL-BY-NAME encoding (296ms)



CALL-BY-VALUE encoding (457ms)



01-ENC encoding (489ms)



$$(53) \cdot \vdash \neg\neg A \rightarrow B \rightarrow \neg A \wedge \neg B$$

LJ (65ms)

$$\frac{\frac{\frac{A, A \rightarrow \perp, B \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \rightarrow B \rightarrow A^*}{A, A \rightarrow \perp, B \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \rightarrow B \rightarrow \perp^*} \supset_L \quad \frac{A, A \rightarrow \perp, B \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \rightarrow B \rightarrow \perp}{A, B \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \rightarrow B \rightarrow A \rightarrow \perp \rightarrow \perp^*} \supset_L \quad \frac{A, B \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \rightarrow B \rightarrow A \rightarrow \perp \rightarrow \perp^*}{A, B \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \rightarrow B \rightarrow B} \supset_L \quad \frac{A, B \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \rightarrow B \rightarrow B}{A, B \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \rightarrow B \rightarrow B \rightarrow \perp^*} \supset_L \quad \frac{A, B \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \rightarrow B \rightarrow B \rightarrow \perp^*}{\vdash A \rightarrow \perp \rightarrow \perp \rightarrow B \rightarrow B \rightarrow A \wedge B \rightarrow \perp \rightarrow \perp^*} \supset_L$$

MULTIPLICATIVE encoding (75ms)

$$\frac{\frac{\frac{\vdots}{\vdots} \vdash A \quad \frac{\vdots}{\vdots} \vdash \perp \vdash \perp}{\vdots \vdash A, A \multimap \perp \vdash \perp} \multimap \quad \frac{\frac{\vdots}{\vdots} \vdash B \vdash B \quad \frac{\vdots}{\vdots} \vdash \perp \vdash \perp}{\vdots \vdash B, B \multimap \perp \vdash \perp} \multimap}{\frac{\frac{\vdots}{\vdots} \vdash A \vdash A \multimap \perp \multimap \perp \quad \frac{\vdots}{\vdots} \vdash B, B \multimap \perp \vdash \perp}{\vdots \vdash A, B \multimap \perp, A \multimap \perp \multimap \perp \multimap B \vdash \perp} \multimap} \star$$

CALL-BY-NAME encoding (178ms)

[illegible]

CALL-BY-VALUE encoding (204ms)

Figure 1. The proposed algorithm for the multi-objective optimization problem. The algorithm starts with an initial population P_0 and iteratively improves it. The process involves several steps: 1. Initialization of P_0 . 2. Calculation of fitness values for each individual in P_0 . 3. Selection of the best individuals based on fitness. 4. Crossover and mutation to generate a new population P_1 . 5. Calculation of fitness values for P_1 . 6. Selection of the best individuals from P_1 . 7. Iteration of steps 2-6 until a stopping criterion is met. 8. Final selection of the best individuals from the final population P_n .

01-ENC encoding (215ms)

Figure 1: Schematic representation of the 1000 Genomes Project. The diagram shows a hierarchical structure of genomic data. At the top, 'Genomes' are represented by blue bars. These are grouped into 'Populations' (indicated by brackets). Within each population, 'Individuals' are shown as smaller blue bars. The diagram illustrates the relationship between the total number of genomes, the number of populations, and the number of individuals sampled within each population. The populations are labeled with their respective sample sizes: African (n=100), East Asian (n=100), European (n=100), South Asian (n=100), and Admixed American (n=100). The total number of genomes is 1000.

$$(54) \cdot \vdash A \wedge B \rightarrow \neg A \rightarrow \neg B$$

LJ (47ms)

$$\frac{\frac{\frac{A, B, A \rightarrow B \rightarrow \perp \perp A}{A, B, A \rightarrow B \rightarrow \perp \perp \perp} \star}{\perp A \wedge B \rightarrow A \rightarrow B \rightarrow \perp \rightarrow \perp} \star}{\frac{\frac{\frac{A, B, B \rightarrow \perp \perp B}{A, B, B \rightarrow \perp \perp \perp} \star}{A, B, B \rightarrow \perp \perp \perp} \star}{A, B, A \rightarrow B \rightarrow \perp \perp \perp} \star} \supset_L \supset_L \supset_L$$

MULTIPLICATIVE encoding (48ms)

$$\frac{\frac{\frac{\cdot \vdash A \vdash A}{\cdot \vdash A \vdash A} \quad \frac{\frac{\cdot \vdash B \vdash B}{\cdot \vdash B, B \multimap \perp \vdash \perp} \quad \frac{\cdot \vdash \perp \vdash \perp}{\cdot \vdash \perp \vdash \perp}}{\cdot \vdash A, B, A \multimap B \multimap \perp \vdash \perp} \multimap \quad \frac{\cdot \vdash A, B, A \multimap B \multimap \perp \vdash \perp}{\cdot \vdash \cdot \vdash A \otimes B \multimap A \multimap B \multimap \perp \multimap \perp} \star$$

CALL-BY-NAME encoding (77ms)

[illegible]

CALL-BY-VALUE encoding (105ms)

[illegible]

01-ENC encoding (106ms)

[illegible]

$$(55) \cdot \vdash A \wedge \neg B \rightarrow \neg A \rightarrow B$$

LJ (48ms)

$$\frac{\frac{\frac{A, A \rightarrow B, B \rightarrow \perp \vdash A}{A, A \rightarrow B, B \rightarrow \perp \vdash \perp} \star \quad \frac{\frac{A, B, B \rightarrow \perp \vdash B}{A, B, B \rightarrow \perp \vdash \perp} \star \quad \frac{A, B, B \rightarrow \perp \vdash \perp}{A, B, B \rightarrow \perp \vdash \perp} \star}{A, A \rightarrow B, B \rightarrow \perp \vdash \perp} \supset_L \quad \frac{A, A \rightarrow B, B \rightarrow \perp \vdash \perp}{\vdash A \wedge B \rightarrow \perp \rightarrow A \rightarrow B \rightarrow \perp} \star \supset_L$$

MULTIPLICATIVE encoding (47ms)

$$\frac{\frac{\frac{\cdot : A \vdash A}{\cdot : A \vdash A} \quad \frac{\frac{\cdot : B \vdash B}{\cdot : B, B \multimap \perp \vdash \perp} \quad \frac{\cdot : \perp \vdash \perp}{\cdot : \perp \vdash \perp}}{\cdot : B, B \multimap \perp \vdash \perp} \multimap \quad \frac{\cdot : A, A \multimap B, B \multimap \perp \vdash \perp}{\cdot : \cdot \vdash A \otimes B \multimap \perp \multimap A \multimap B \multimap \perp} \star$$

CALL-BY-NAME encoding (144ms)

[illegible]

CALL-BY-VALUE encoding (212ms)

[illegible]

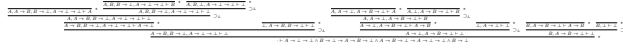
01-ENC encoding (210ms)

01-ENC encoding (168ms)

[illegible]

$$(57) \cdot \vdash (\neg\neg A \wedge \neg B) \leftrightarrow (\neg A \rightarrow B)$$

LJ (114ms)



MULTIPLICATIVE encoding (58ms)

Not provable

CALL-BY-NAME encoding (326ms)



CALL-BY-VALUE encoding (421ms)

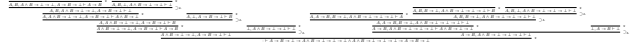


01-ENC encoding (454ms)



$$(58) \cdot \vdash (\neg A \rightarrow B) \leftrightarrow (\neg\neg A \wedge \neg B)$$

LJ (109ms)



MULTIPLICATIVE encoding (41ms)

Not provable

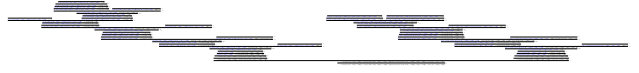
CALL-BY-NAME encoding (597ms)



CALL-BY-VALUE encoding (481ms)

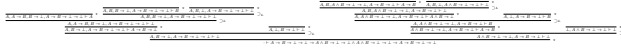


01-ENC encoding (501ms)



$$(59) \cdot \vdash (\neg\neg A \rightarrow B) \leftrightarrow (\neg A \wedge \neg B)$$

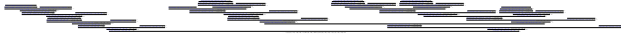
LJ (112ms)



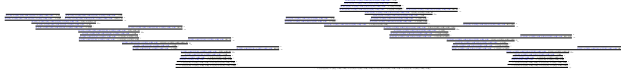
MULTIPLICATIVE encoding (57ms)

Not provable

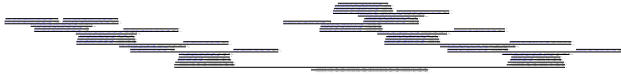
CALL-BY-NAME encoding (594ms)



CALL-BY-VALUE encoding (468ms)

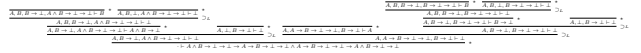


01-ENC encoding (498ms)

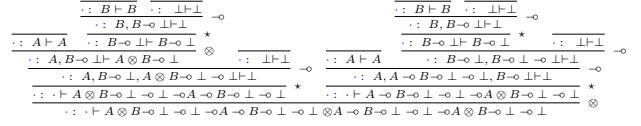


$$(60) \cdot \vdash (\neg A \wedge \neg B) \leftrightarrow (A \rightarrow \neg\neg B)$$

LJ (92ms)



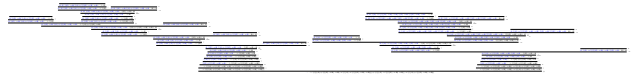
MULTIPLICATIVE encoding (124ms)



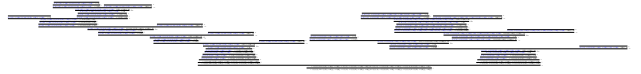
CALL-BY-NAME encoding (317ms)



CALL-BY-VALUE encoding (436ms)

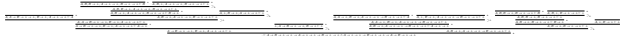


01-ENC encoding (468ms)

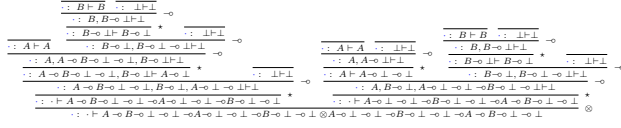


$$(61) \cdot \vdash (A \rightarrow \neg\neg B) \leftrightarrow (\neg\neg A \rightarrow \neg\neg B)$$

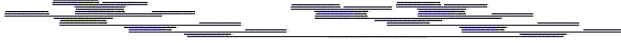
LJ (169ms)



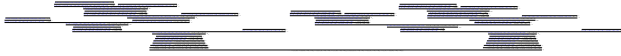
MULTIPLICATIVE encoding (163ms)



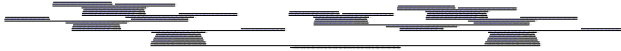
CALL-BY-NAME encoding (412ms)



CALL-BY-VALUE encoding (479ms)



01-ENC encoding (552ms)



3 Alternative Translations

$$(10) \cdot : A \multimap \mathbf{0} \vdash A \multimap B$$

encoding (55ms)

$$\frac{\frac{\cdot : A \vdash A \quad \cdot : \mathbf{0} \vdash B}{\cdot : A, A \multimap \mathbf{0} \vdash B} \star}{\cdot : A \multimap \mathbf{0} \vdash A \multimap B} \multimap$$

$$(11) \cdot : A \vdash A \multimap \mathbf{0} \multimap B$$

encoding (41ms)

$$\frac{\frac{\cdot : A \vdash A \quad \cdot : \mathbf{0} \vdash B}{\cdot : A, A \multimap \mathbf{0} \vdash B} \star}{\cdot : A \vdash A \multimap \mathbf{0} \multimap B} \multimap$$

$$(12) \cdot : B \vdash !(A) \multimap B$$

encoding (22ms)

$$\frac{A : B \vdash B}{\cdot : B \vdash !(A) \multimap B} \star$$

$$(16) \cdot : A \multimap B \otimes !(B \multimap A) \vdash A \multimap B$$

encoding (35ms)

$$\frac{\frac{B \multimap A : A \vdash A \quad B \multimap A : B \vdash B}{B \multimap A : A, A \multimap B \vdash B} \multimap}{\cdot : A \multimap B \otimes !(B \multimap A) \vdash A \multimap B} \star$$

$$(17) \cdot : !(A \multimap B) \otimes B \multimap A \vdash B \multimap A$$

encoding (38ms)

$$\frac{\frac{A \multimap B : B \vdash B \quad A \multimap B : A \vdash A}{A \multimap B : B, B \multimap A \vdash A} \multimap}{\cdot : !(A \multimap B) \otimes B \multimap A \vdash B \multimap A} \star$$

$$(18) \cdot : A, A \multimap B \otimes B \multimap A \vdash B \otimes B \multimap A$$

encoding (56ms)

$$\frac{\frac{\frac{\frac{\cdot : B \vdash B \quad \cdot : A \vdash A}{\cdot : B, B \multimap A \vdash A} \multimap}{\cdot : B \vdash B \quad \cdot : B \multimap A \vdash B \multimap A} \otimes}{\cdot : A \vdash A \quad \cdot : B, B \multimap A \vdash B \otimes B \multimap A} \multimap}{\cdot : A, A \multimap B, B \multimap A \vdash B \otimes B \multimap A} \star$$

$$(19) \cdot : B, A \multimap B \otimes B \multimap A \vdash A \otimes A \multimap B$$

encoding (55ms)

$$\frac{\frac{\frac{\frac{\cdot : A \vdash A \quad \cdot : B \vdash B}{\cdot : A, A \multimap B \vdash B} \multimap}{\cdot : A \vdash A \quad \cdot : A \multimap B \vdash A \multimap B} \otimes}{\cdot : B \vdash B \quad \cdot : A, A \multimap B \vdash A \otimes A \multimap B} \multimap}{\cdot : B, A \multimap B, B \multimap A \vdash A \otimes A \multimap B} \star$$

$$(26a) \cdot : \cdot \vdash A \& B \multimap \perp \multimap \perp \multimap A \multimap \perp \multimap \perp \& B \multimap \perp \multimap \perp$$

encoding (107ms)

$$\frac{\frac{\frac{\cdot : A \vdash A \quad \cdot : \perp \vdash \perp}{\cdot : A, A \multimap \perp \vdash \perp} \multimap}{\cdot : A \& B, A \multimap \perp \vdash \perp} \&_1}{\cdot : A \multimap \perp \vdash A \& B \multimap \perp} \star \quad \frac{\frac{\frac{\cdot : B \vdash B \quad \cdot : \perp \vdash \perp}{\cdot : B, B \multimap \perp \vdash \perp} \multimap}{\cdot : A \& B, B \multimap \perp \vdash \perp} \&_2}{\cdot : B \multimap \perp \vdash A \& B \multimap \perp} \star \quad \frac{\cdot : \perp \vdash \perp}{\cdot : A \multimap \perp, A \& B \multimap \perp \multimap \perp \vdash \perp} \multimap}{\cdot : \cdot \vdash A \& B \multimap \perp \multimap \perp \multimap A \multimap \perp \multimap \perp \& B \multimap \perp \multimap \perp} \star$$

$$(26b) \cdot : \cdot \vdash A \multimap \perp \multimap \perp \otimes B \multimap \perp \multimap \perp \multimap A \otimes B \multimap \perp \multimap \perp$$

encoding (109ms)

$$\frac{\frac{\frac{\cdot : A \vdash A \quad \cdot : B \vdash B}{\cdot : A, B \vdash A \otimes B} \otimes}{\cdot : A, B, A \otimes B \multimap \perp \vdash \perp} \multimap}{\cdot : A, A \otimes B \multimap \perp \vdash B \multimap \perp} \star \quad \frac{\cdot : \perp \vdash \perp}{\cdot : A, A \otimes B \multimap \perp, B \multimap \perp \multimap \perp \vdash \perp} \multimap}{\cdot : A \otimes B \multimap \perp, B \multimap \perp \multimap \perp \vdash A \multimap \perp} \star \quad \frac{\cdot : \perp \vdash \perp}{\cdot : A \otimes B \multimap \perp, A \multimap \perp \multimap \perp, B \multimap \perp \multimap \perp \vdash \perp} \multimap}{\cdot : \cdot \vdash A \multimap \perp \multimap \perp \otimes B \multimap \perp \multimap \perp \multimap A \otimes B \multimap \perp \multimap \perp} \star$$

$$(27a) \cdot : \cdot \vdash !(A \multimap B) \otimes !(B \multimap A) \multimap \perp \multimap \perp \multimap A \multimap B \multimap \perp \multimap \perp \& B \multimap A \multimap \perp \multimap \perp$$

encoding (155ms)

$$(27b) \quad \cdot : \cdot \vdash A \multimap B \multimap \perp \multimap \perp \otimes B \multimap A \multimap \perp \multimap \perp \multimap A \multimap B \otimes B \multimap A \multimap \perp \multimap \perp$$

$$\begin{array}{c}
\frac{\vdash A \vdash A \quad \vdash B \vdash B}{\vdash A, A \multimap B \vdash B} \multimap \quad \frac{\vdash B \vdash B \quad \vdash A \vdash A}{\vdash B, B \multimap A \vdash A} \multimap \\
\vdash A \multimap B \vdash A \multimap B \quad \vdash B \multimap A \vdash B \quad \vdash A \multimap B, B \multimap A \vdash B \multimap B \multimap A \quad \vdash \bot \vdash \bot \\
\vdash A \multimap B, B \multimap A, A \multimap B \otimes B \multimap A \multimap \bot \vdash \bot \\
\vdash A \multimap B \otimes B \otimes B \multimap A \multimap A \multimap \bot \vdash B \multimap A \multimap \bot \quad \vdash \bot \vdash \bot \\
\vdash A \multimap B, A \multimap B \otimes B \multimap A \multimap \bot, B \multimap A \multimap \bot \multimap \bot \vdash \bot \vdash \bot \\
\vdash A \multimap B \otimes B \multimap A \multimap \bot, B \multimap A \multimap \bot \multimap \bot \vdash A \multimap B \multimap \bot \quad \vdash \bot \vdash \bot \\
\vdash A \multimap B \otimes B \multimap A \multimap \bot, A \multimap B \multimap \bot \multimap \bot, B \multimap A \multimap \bot \multimap \bot \vdash \bot \vdash \bot \\
\vdash A \multimap B \otimes A \multimap \bot \multimap \bot \otimes B \multimap A \multimap \bot \multimap \bot \vdash A \multimap B \otimes B \multimap A \multimap \bot \multimap \bot
\end{array}$$

(35) $\cdot : \cdot \vdash !(A) \otimes !(A) \multimap !(A) \otimes !(A) \multimap !(A) \otimes !(A)$

$$\frac{\frac{\overline{A : \vdash A}}{A : \vdash ! (A)} ! \quad \frac{\frac{\overline{A : \vdash A}}{A : \vdash ! (A)} ! \quad \frac{\overline{A : \vdash A}}{A : \vdash ! (A)} !}{\frac{\vdash : \vdash ! (A) \otimes ! (A) \multimap ! (A) \quad \vdash : \vdash ! (A) \multimap ! (A) \otimes ! (A)}{\vdash : \vdash ! (A) \otimes ! (A) \multimap ! (A) \otimes ! (A) \multimap ! (A) \otimes ! (A)} \otimes} \star$$

$$(36) \cdot : A \vdash A \multimap B \multimap B \otimes B \multimap !(A) \multimap B$$

$$\frac{\frac{\frac{\vdots \vdash A \vdash A}{\vdots \vdash A, A \multimap B \vdash B} \multimap \quad \frac{\vdots \vdash B \vdash B}{\vdots \vdash A, A \multimap B \vdash B} \multimap}{\vdots \vdash A \vdash A \multimap B \multimap B} \star \quad \frac{\frac{\vdots \vdash A \vdash A \multimap B \multimap B}{\vdots \vdash \cdot \vdash B \multimap !(A) \multimap B} \star}{\vdots \vdash A \vdash A \multimap B \multimap B \otimes B \multimap !(A) \multimap B} \otimes$$

$$(37) \cdot : B \vdash !(A \multimap B) \multimap B \otimes B \multimap !(A) \multimap B$$

$$\frac{\frac{\overline{A \multimap B : B \vdash B}}{\cdot : B \vdash !(A \multimap B) \multimap B} \star \quad \frac{\overline{A : B \vdash B}}{\cdot : \cdot \vdash B \multimap !(A) \multimap B} \star}{\cdot : B \vdash !(A \multimap B) \multimap B \otimes B \multimap !(A) \multimap B} \otimes$$

$$(38) \cdot : A \multimap \perp \vdash !(A \multimap B) \multimap A \multimap \perp \otimes A \multimap \mathbf{0} \multimap A \multimap B$$

$$\frac{\frac{A \multimap B : A \vdash A \quad A \multimap B : \perp \vdash \perp}{A \multimap B : A, A \multimap \perp \vdash \perp}}{\vdots : A \multimap \perp \vdash ! (A \multimap B) \multimap A \multimap \perp} \multimap \quad \frac{\frac{\vdots : A \vdash A \quad \vdots : \mathbf{0} \vdash B}{\vdots : A, A \multimap \mathbf{0} \vdash B}}{\vdots : \vdash A \multimap \mathbf{0} \multimap A \multimap B} \star \quad \frac{\vdots : A \multimap \perp \vdash ! (A \multimap B) \multimap A \multimap \perp}{\vdots : A \multimap \perp \vdash ! (A \multimap B) \multimap A \multimap \perp \otimes A \multimap \mathbf{0} \multimap A \multimap B} \otimes$$

$$(39) \cdot : B \multimap \mathbf{0} \vdash A \multimap B \multimap A \multimap \mathbf{0} \otimes A \multimap \mathbf{0} \multimap A \multimap B$$

[illegible]

$$(40) \cdot : B \vdash A \otimes !(B) \multimap A \otimes A \multimap A \otimes B$$

$$\frac{\frac{\overline{B : A \vdash A}}{\cdot : \cdot \vdash A \otimes !(B) \multimap A} \star \quad \frac{\frac{\cdot : A \vdash A \quad \cdot : B \vdash B}{\cdot : A, B \vdash A \otimes B} \otimes \quad \frac{\cdot : B \vdash A \multimap A \otimes B}{\cdot : B \vdash A \multimap A \otimes B} \star}{\cdot : B \vdash A \otimes !(B) \multimap A \otimes A \multimap A \otimes B} \otimes$$

$$(41) \cdot : B \multimap \mathbf{0} \vdash !(A) \otimes B \multimap B \otimes B \multimap A \otimes B$$

$$\frac{\frac{\frac{\overline{A : B \multimap B}}{\vdots \cdot \vdash ! (A) \otimes B \multimap B} \star}{\vdots \cdot B \multimap \mathbf{0} \vdash ! (A) \otimes B \multimap B \otimes B \multimap A \otimes B} \star}{\vdots \cdot B \multimap \mathbf{0} \vdash ! (A) \otimes B \multimap B \otimes B \multimap A \otimes B} \otimes} \frac{\frac{\frac{\overline{B : B \multimap B} \quad \vdots \cdot \mathbf{0} \vdash A \otimes B}{\vdots \cdot B, B \multimap \mathbf{0} \vdash A \otimes B} \multimap}{\vdots \cdot B \multimap \mathbf{0} \vdash B \multimap A \otimes B} \star}{\vdots \cdot B \multimap \mathbf{0} \vdash ! (A) \otimes B \multimap B \otimes B \multimap A \otimes B} \otimes} \star$$

$$(45) \quad \cdot : \cdot \vdash !(A \multimap A \multimap \perp) \otimes !(A) \multimap \perp \multimap !(A) \multimap \perp$$

$$\frac{\frac{\frac{A, \perp, A \multimap \perp, A \multimap A \multimap \perp : \perp \vdash \perp}{A, \perp, A \multimap \perp, A \multimap A \multimap \perp : \vdash \perp} \quad D_C}{A \multimap A \multimap \perp : \vdash ! (A) \multimap \perp} \quad * \quad \frac{\frac{\frac{A, \perp, A \multimap \perp, A \multimap A \multimap \perp : \perp \vdash \perp}{A, \perp, A \multimap \perp, A \multimap A \multimap \perp : \vdash \perp} \quad D_C}{A \multimap A \multimap \perp : ! (A) \vdash \perp} \quad * \quad \frac{\vdash ! (A) \multimap \perp : ! (A) \multimap \vdash \vdash ! (A) \vdash \perp}{\vdash \vdash ! (A \multimap A \multimap \perp) \otimes ! (A) \multimap \vdash \vdash ! (A) \multimap \perp} \quad \multimap$$

$$(46) \cdot : \cdot \vdash !(A \multimap \perp \multimap \mathbf{0}) \multimap A \multimap \perp$$

[illegible]

$$(47) \vdash A \otimes B \otimes B \multimap 0 \multimap B \otimes B \multimap 0 \otimes B \otimes B \multimap 0 \multimap A \otimes B \otimes B \multimap 0$$

$$\begin{array}{c}
\frac{\begin{array}{c} \vdots, B \vdash B \\ \vdots, A, 0 \vdash B \otimes B \rightarrow 0 \end{array}^*}{\vdots, A, B, B \rightarrow 0 \vdash B \otimes B \rightarrow 0}^{\circ} \quad \frac{\begin{array}{c} \vdots, B \vdash B \\ \vdots, 0 \vdash A \otimes B \otimes B \rightarrow 0 \end{array}^*}{\vdots, B, B \rightarrow 0 \vdash A \otimes B \otimes B \rightarrow 0}^{\circ} \\
\frac{\vdots, \vdash A \otimes B \otimes B \rightarrow 0 \rightarrow B \otimes B \rightarrow 0}^* \quad \frac{\vdots, \vdash B \otimes B \rightarrow 0 \rightarrow 0 \rightarrow A \otimes B \otimes B \rightarrow 0}^*}{\vdots, \vdash A \otimes B \otimes B \rightarrow 0 \rightarrow B \otimes B \rightarrow 0 \rightarrow 0 \rightarrow A \otimes B \otimes B \rightarrow 0}^{\otimes}
\end{array}$$

(57a) $\cdot : \cdot \vdash A \multimap \perp \multimap \perp \otimes B \multimap \perp \multimap A \multimap B \multimap \perp$

$$\begin{array}{c}
\frac{}{\vdots : A \vdash A} \quad \frac{\vdots : B \vdash B \quad \vdots : \perp \vdash \perp}{\vdots : B, B \multimap \perp \vdash \perp} \multimap \\
\frac{\vdots : A, A \multimap B, B \multimap \perp \vdash \perp}{\vdots : A \multimap B, B \multimap \perp \vdash A \multimap \perp} \multimap \quad \frac{}{\vdots : \perp \vdash \perp} \multimap \\
\frac{\vdots : A \multimap B, B \multimap \perp, A \multimap \perp \multimap \perp \vdash \perp}{\vdots : \vdash A \multimap \perp \multimap \perp \otimes B \multimap \perp \multimap A \multimap B \multimap \perp} \star
\end{array}$$

(57b) $\vdots : \vdash !(A) \multimap B \multimap \perp \multimap A \multimap \mathbf{0} \multimap \perp \& B \multimap \perp$

$$\begin{array}{c}
\frac{\frac{A : \vdash A \quad A : 0 \vdash B}{A : A \multimap 0 \vdash B} \star}{\vdots : A \multimap 0 \vdash !(A) \multimap B} \star \quad \vdots : \perp \vdash \perp \\
\hline
\vdots : A \multimap 0, !(A) \multimap B \multimap \perp \vdash \perp \quad \multimap \quad \frac{\frac{A : B \vdash B}{B : B \vdash (A) \multimap B} \star \quad \vdots : \perp \vdash \perp}{\vdots : B, !(A) \multimap B \multimap \perp \vdash \perp} \multimap \\
\hline
\vdots : \vdash !(A) \multimap B \multimap \perp \multimap A \multimap 0 \multimap \perp \ \& B \multimap \perp \star
\end{array}$$

(58a) $\vdots : \vdash !(A) \multimap B \multimap \perp \multimap A \otimes B \multimap \perp \multimap 0 \multimap \perp$

$$\begin{array}{c}
\frac{\frac{\frac{\mathcal{A}, \mathcal{B}(A) \multimap B \multimap \perp : B \multimap B}{\mathcal{A}, \mathcal{B}(A) \multimap B \multimap \perp : B \multimap \mathcal{B}(A) \multimap B} \quad \frac{\mathcal{A}, \mathcal{B}(A) \multimap B \multimap \perp : \perp \vdash \perp}{\mathcal{A}, \mathcal{B}(A) \multimap B \multimap \perp : B \multimap \perp \vdash \perp}}{\mathcal{A}, \mathcal{B}(A) \multimap B \multimap \perp : B \multimap \perp \vdash \perp} \multimap \\
\frac{\mathcal{A}, \mathcal{B}(A) \multimap B \multimap \perp : \vdash \mathcal{A} \quad \frac{\mathcal{A}, \mathcal{B}(A) \multimap B \multimap \perp : \vdash B \multimap \perp}{\mathcal{A}, \mathcal{B}(A) \multimap B \multimap \perp : \vdash \mathcal{A} \otimes B \multimap \perp} \otimes}{\mathcal{A}, \mathcal{B}(A) \multimap B \multimap \perp : \vdash \mathcal{A} \otimes B \multimap \perp : \vdash B \multimap \perp} \multimap \\
\frac{\mathcal{A}, \mathcal{B}(A) \multimap B \multimap \perp : \mathcal{A} \otimes B \multimap \perp : \multimap \otimes B}{\mathcal{B}(A) \multimap B \multimap \perp : \mathcal{A} \otimes B \multimap \perp : \multimap \otimes \mathcal{B}(A) \multimap B} \multimap \\
\frac{\mathcal{B}(A) \multimap B \multimap \perp : \mathcal{A} \otimes B \multimap \perp : \multimap \otimes \mathcal{B}(A) \multimap B \vdash \perp}{\mathcal{B}(A) \multimap B \multimap \perp : \mathcal{A} \otimes B \multimap \perp : \multimap \otimes \mathcal{B}(A) \multimap B \vdash \perp} \vdash \\
\frac{\mathcal{B}(A) \multimap B \multimap \perp : \mathcal{A} \otimes B \multimap \perp : \multimap \otimes \mathcal{B}(A) \multimap B \vdash \perp : \vdash \mathcal{B}(A) \multimap B \multimap \perp : \multimap \otimes \mathcal{B}(A) \multimap B \vdash \perp}{\vdash : \vdash \mathcal{B}(A) \multimap B \multimap \perp : \mathcal{A} \otimes B \multimap \perp : \multimap \otimes \perp} \vdash \\
\vdash : \vdash \mathcal{B}(A) \multimap B \multimap \perp : \mathcal{A} \otimes B \multimap \perp : \multimap \otimes \perp
\end{array}$$
$$\begin{array}{c}
\frac{\vdots : B \multimap B \quad \vdots : \perp \multimap \perp}{\vdots : A \multimap A} \multimap \\
\frac{\vdots : A, A \multimap B, B \multimap \perp \multimap \perp}{\vdots : A \multimap B \multimap A \otimes B \multimap \perp \multimap \perp} \multimap \quad \frac{\vdots : \perp \multimap \perp}{\vdots : \cdot \multimap A \otimes B \multimap \perp \multimap \perp \multimap \perp \multimap \perp \multimap \perp \multimap A \multimap B \multimap \perp} \star \\
\hline
(59a) \quad \vdots : \cdot \multimap A \multimap B \multimap \perp \multimap \perp \multimap \perp \multimap A \otimes B \multimap \perp \multimap \perp
\end{array}$$
$$\begin{array}{c}
\frac{\cdot \vdash A \vdash A}{\cdot \vdash A, A \multimap B, B \multimap \perp \vdash \perp} \multimap \quad \frac{\cdot \vdash B, B \multimap \perp \vdash \perp}{\cdot \vdash A, B \multimap \perp \vdash A \multimap B \multimap \perp} \multimap \quad \frac{\cdot \vdash A, B \multimap \perp \vdash A \multimap B \multimap \perp}{\cdot \vdash A, B \multimap \perp, A \multimap B \multimap \perp \multimap \perp \vdash \perp} \star \quad \frac{\cdot \vdash \perp \vdash \perp}{\cdot \vdash A, B \multimap \perp, A \multimap B \multimap \perp \multimap \perp \vdash \perp} \star \\
\hline
(59b) \quad \cdot \vdash A \otimes B \multimap \perp \multimap \mathbf{0} \multimap \mathbf{0} !(\mathbf{!}(A) \multimap B \multimap \perp) \multimap \perp
\end{array}$$
[illegible]