Benchmarking Linear Logic: sequents inspired from Kleene's intuitionistic theorems*

October 18, 2018

1 General Information

- Test run on a QEMU Virtual CPU, 2GHz, 64 bits, 2GB of RAM running Ubuntu.
- Maude version: 2.7.1 built: Oct 2 2017.
- The search procedure considers proofs using up to 4 times the copy rule (focusing on one of the formulas of the classical context). Benchmarks 23 and 25 required the limit to be 5.

2 Translations

$\boxed{(1)\ \cdot \vdash A \to A}$

LJ (28ms)

$$\frac{}{\cdot \vdash A \to A}\ \star$$

MULTIPLICATIVE encoding (28ms)

$$\frac{\dfrac{}{\cdot :\ A \vdash A}}{\cdot :\ \cdot \vdash A \multimap A}\ \star$$

CALL-BY-NAME encoding (28ms)

$$\frac{\dfrac{}{A :\ \cdot \vdash A}}{\cdot :\ \cdot \vdash\ !(A) \multimap A}\ \star$$

CALL-BY-VALUE encoding (41ms)

$$\frac{\dfrac{\dfrac{}{A :\ \cdot \vdash A}}{A :\ \cdot \vdash\ !(A)}\ !}{\cdot :\ \cdot \vdash\ !(A) \multimap\ !(A)}\ \star}{\cdot :\ \cdot \vdash\ !(!(A) \multimap\ !(A))}\ !$$

01-ENC encoding (35ms)

$$\frac{\dfrac{\dfrac{}{A :\ \cdot \vdash A}}{\cdot :\ \cdot \vdash\ !(A) \multimap A}\ \star}{\cdot :\ \cdot \vdash\ !(!(A) \multimap A)}\ !$$

$\boxed{(2)\ A \to B, B \to C \vdash A \to C}$

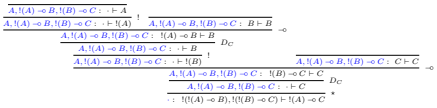
LJ (46ms)

$$\frac{\dfrac{\dfrac{}{A, A \to B, B \to C \vdash A}\ \star \quad \dfrac{\dfrac{}{A, B, B \to C \vdash B}\ \star \quad \dfrac{}{A, B, C \vdash C}\ \star}{A, B, B \to C \vdash C}\ \supset_L}{A, A \to B, B \to C \vdash C}\ \supset_L}{A \to B, B \to C \vdash A \to C}\ \star$$

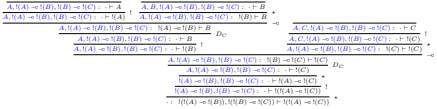
MULTIPLICATIVE encoding (49ms)

$$\frac{\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{\dfrac{}{\cdot :\ B \vdash B} \quad \dfrac{}{\cdot :\ C \vdash C}}{\cdot :\ B, B \multimap C \vdash C}\ \multimap}{\cdot :\ A, A \multimap B, B \multimap C \vdash C}\ \multimap}{\cdot :\ A \multimap B, B \multimap C \vdash A \multimap C}\ \star$$

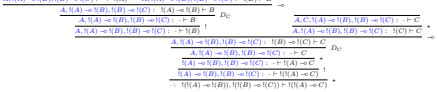
CALL-BY-NAME encoding (123ms)



CALL-BY-VALUE encoding (165ms)



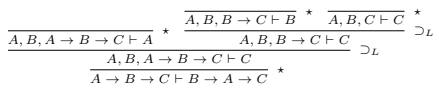
01-ENC encoding (146ms)



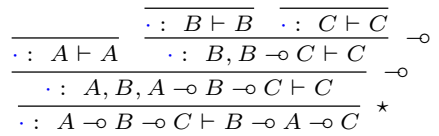
*See `https://github.com/meta-logic/lltp` for details on the encodings used.

(3) $A \to B \to C \vdash B \to A \to C$

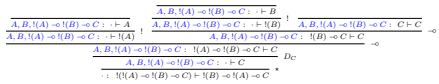
LJ (46ms)



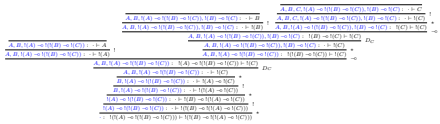
MULTIPLICATIVE encoding (47ms)



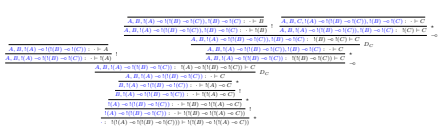
CALL-BY-NAME encoding (71ms)



CALL-BY-VALUE encoding (162ms)

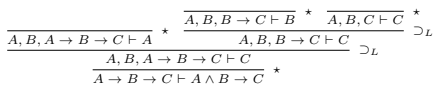


01-ENC encoding (158ms)

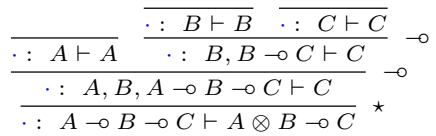


(4) $A \to B \to C \vdash A \wedge B \to C$

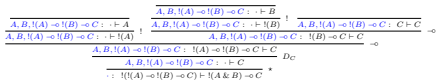
LJ (47ms)



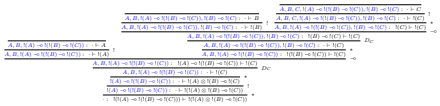
MULTIPLICATIVE encoding (47ms)



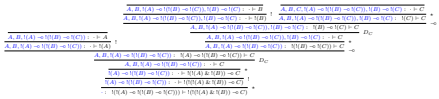
CALL-BY-NAME encoding (71ms)



CALL-BY-VALUE encoding (156ms)



01-ENC encoding (151ms)



(5) $A \wedge B \to C \vdash A \to B \to C$

LJ (41ms)

$$\dfrac{\dfrac{}{A, B, A \wedge B \to C \vdash A \wedge B}\,\star \qquad \dfrac{}{A, B, C \vdash C}\,\star}{\dfrac{A, B, A \wedge B \to C \vdash C}{A \wedge B \to C \vdash A \to B \to C}\,\star}\,\supset_L$$

MULTIPLICATIVE encoding (54ms)

$$\dfrac{\dfrac{\dfrac{}{\cdot :\ A \vdash A} \qquad \dfrac{}{\cdot :\ B \vdash B}}{\cdot :\ A, B \vdash A \otimes B}\,\otimes \qquad \dfrac{}{\cdot :\ C \vdash C}}{\dfrac{\cdot :\ A, B, A \otimes B \multimap C \vdash C}{\cdot :\ A \otimes B \multimap C \vdash A \multimap B \multimap C}\,\star}\,\multimap$$

CALL-BY-NAME encoding (73ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A, B, !(A \,\&\, B) \multimap C :\ \cdot \vdash A} \qquad \dfrac{}{A, B, !(A \,\&\, B) \multimap C :\ \cdot \vdash B}\,\star}{A, B, !(A \,\&\, B) \multimap C :\ \cdot \vdash A \,\&\, B}\,!}{A, B, !(A \,\&\, B) \multimap C :\ \cdot \vdash !(A \,\&\, B)} \qquad \dfrac{}{A, B, !(A \,\&\, B) \multimap C :\ C \vdash C}\,\multimap}{\dfrac{A, B, !(A \,\&\, B) \multimap C :\ !(A \,\&\, B) \multimap C \vdash C}{A, B, !(A \,\&\, B) \multimap C :\ \cdot \vdash C}\,D_C}{\cdot :\ !(!(A \,\&\, B) \multimap C) \vdash !(A) \multimap !(B) \multimap C}\,\star$$

CALL-BY-VALUE encoding (205ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, B, !(A) \otimes !(B) \multimap !(C) :\ \cdot \vdash A}\,!}{A, B, !(A) \otimes !(B) \multimap !(C) :\ \cdot \vdash !(A)} \otimes \dfrac{\dfrac{\dfrac{}{A, B, !(A) \otimes !(B) \multimap !(C) :\ \cdot \vdash B}\,!}{A, B, !(A) \otimes !(B) \multimap !(C) :\ \cdot \vdash !(B)} \otimes \dfrac{}{A, B, C, !(A) \otimes !(B) \multimap !(C) :\ \cdot \vdash C}\,!}{A, B, C, !(A) \otimes !(B) \multimap !(C) :\ \cdot \vdash !(C)}\,\star}{A, B, !(A) \otimes !(B) \multimap !(C) :\ !(A) \otimes !(B) \multimap !(C) \vdash !(C)}\,\multimap}{A, B, !(A) \otimes !(B) \multimap !(C) :\ \cdot \vdash !(C)}\,D_C}{A, !(A) \otimes !(B) \multimap !(C) :\ !(B) \multimap !(C) \vdash !(C)}\,\star}{A, !(A) \otimes !(B) \multimap !(C) :\ \cdot \vdash !(B) \multimap !(C)}\,!}{!(A) \otimes !(B) \multimap !(C) :\ \cdot \vdash !(A) \multimap !(B) \multimap !(C)}\,\star}{\cdot :\ !(!(A) \otimes !(B) \multimap !(C)) \vdash !(!(A) \multimap !(B) \multimap !(C))}\,\star$$

01-ENC encoding (201ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, B, !(!(A \,\&\, B)) \multimap !(C) :\ \cdot \vdash A} \qquad \dfrac{}{A, B, !(!(A \,\&\, B)) \multimap !(C) :\ \cdot \vdash B}\,\star}{A, B, !(!(A \,\&\, B)) \multimap !(C) :\ \cdot \vdash A \,\&\, B}\,!}{A, B, !(!(A \,\&\, B)) \multimap !(C) :\ \cdot \vdash !(!(A \,\&\, B))}\,! \qquad \dfrac{}{A, B, C, !(!(A \,\&\, B)) \multimap !(C) :\ \cdot \vdash C}\,\star}{A, B, !(!(A \,\&\, B)) \multimap !(C) :\ !(!(A \,\&\, B)) \multimap !(C) \vdash C}\,\multimap}{A, B, !(!(A \,\&\, B)) \multimap !(C) :\ \cdot \vdash C}\,D_C}{A, !(!(A \,\&\, B)) \multimap !(C) :\ \cdot \vdash !(B) \multimap C}\,\star}{!(!(A \,\&\, B)) \multimap !(C) :\ \cdot \vdash !(A) \multimap !(B) \multimap C}\,!}{!(!(A \,\&\, B)) \multimap !(C)) \vdash !(!(A) \multimap !(!(B) \multimap C))}\,\star$$

(6) $A \to B \vdash B \to C \to A \to C$

LJ (47ms)

$$\dfrac{\dfrac{}{A, A \to B, B \to C \vdash A}\,\star \qquad \dfrac{\dfrac{}{A, B, B \to C \vdash B}\,\star \qquad \dfrac{}{A, B, C \vdash C}\,\star}{A, B, B \to C \vdash C}\,\supset_L}{A, A \to B, B \to C \vdash C}\,\supset_L$$

$$A \to B \vdash B \to C \to A \to C\ \star$$

MULTIPLICATIVE encoding (48ms)

$$\dfrac{\dfrac{}{\cdot :\ A \vdash A} \qquad \dfrac{\dfrac{}{\cdot :\ B \vdash B} \qquad \dfrac{}{\cdot :\ C \vdash C}}{\cdot :\ B, B \multimap C \vdash C}\,\multimap}{\cdot :\ A, A \multimap B, B \multimap C \vdash C}\,\multimap$$

$$\cdot :\ A \multimap B \vdash B \multimap C \multimap A \multimap C\ \star$$

CALL-BY-NAME encoding (120ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap B, !(B) \multimap C :\ \cdot \vdash A}}{A, !(A) \multimap B, !(B) \multimap C :\ \cdot \vdash !(A)}\,! \qquad \dfrac{}{A, !(A) \multimap B, !(B) \multimap C :\ B \vdash B}\,\multimap}{A, !(A) \multimap B, !(B) \multimap C :\ \cdot \vdash B}\,D_C}{A, !(A) \multimap B, !(B) \multimap C :\ \cdot \vdash !(B)}\,! \qquad \dfrac{}{A, !(A) \multimap B, !(B) \multimap C :\ C \vdash C}\,\multimap}{\dfrac{A, !(A) \multimap B, !(B) \multimap C :\ !(B) \multimap C \vdash C}{A, !(A) \multimap B, !(B) \multimap C :\ \cdot \vdash C}\,D_C}{\cdot :\ !(!(A) \multimap B) \vdash !(!(B) \multimap C) \multimap !(A) \multimap C}\,\star$$

CALL-BY-VALUE encoding (168ms)

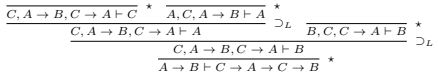
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash A}}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash !(A)}\,! \qquad \dfrac{\dfrac{}{A, B, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash B}\,\star}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash B}\,\multimap}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash !(B)}\,D_C}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ !(B) \multimap !(C) \vdash !(C)}\,\star}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash !(C)}\,! \qquad \dfrac{}{A, C, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash C}\,\star}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ !(A) \multimap !(C) \vdash !(C)}\,\multimap}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash !(C)}\,D_C}{!(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash !(A) \multimap !(C)}\,\star}{!(A) \multimap !(B) :\ \cdot \vdash !(!(B) \multimap !(C)) \multimap !(!(A) \multimap !(C))}\,!}{\cdot :\ !(!(A) \multimap !(B)) \vdash !(!(!(B) \multimap !(C)) \multimap !(!(A) \multimap !(C)))}\,\star$$

01-ENC encoding (159ms)

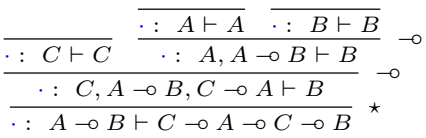
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash A}}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash !(A)}\,! \qquad \dfrac{}{A, B, !(A) \multimap !(B), !(B) \multimap !(C) :\ !(B) \vdash B}\,\star}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ !(A) \multimap !(B) \vdash B}\,\multimap}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash B}\,D_C}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash !(B)}\,! \qquad \dfrac{}{A, C, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash C}\,\star}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ !(B) \multimap !(C) \vdash C}\,\multimap}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash C}\,D_C}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash !(A) \multimap C}\,\star}{!(A) \multimap !(B) :\ \cdot \vdash !(!(B) \multimap !(C)) \multimap !(!(A) \multimap C)}\,!}{\cdot :\ !(!(A) \multimap !(B)) \vdash !(!(!(B) \multimap !(C)) \multimap !(!(A) \multimap C))}\,\star$$

(7) $A \to B \vdash C \to A \to C \to B$

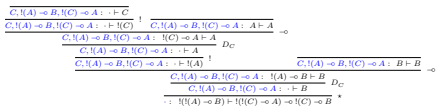
LJ (47ms)



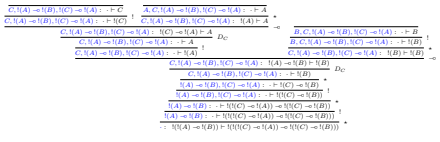
MULTIPLICATIVE encoding (48ms)



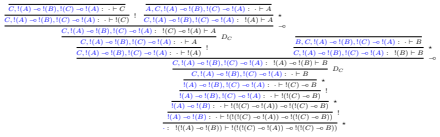
CALL-BY-NAME encoding (120ms)



CALL-BY-VALUE encoding (167ms)

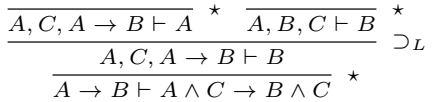


01-ENC encoding (156ms)

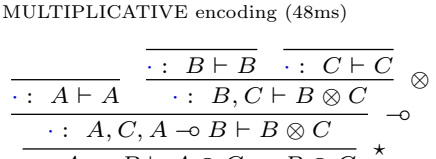


(8) $A \to B \vdash A \wedge C \to B \wedge C$

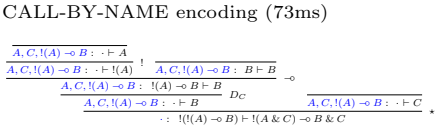
LJ (37ms)



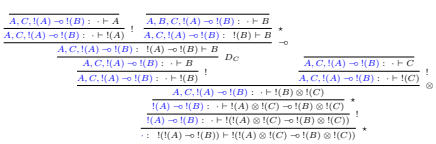
MULTIPLICATIVE encoding (48ms)



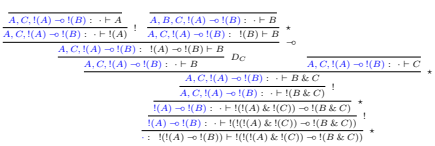
CALL-BY-NAME encoding (73ms)



CALL-BY-VALUE encoding (135ms)



01-ENC encoding (130ms)



(9) $A \to B \vdash C \wedge A \to C \wedge B$

LJ (37ms)

$$\dfrac{\dfrac{\dfrac{}{A, C, A \to B \vdash A}\ \star \quad \dfrac{}{A, B, C \vdash B}\ \star}{A, C, A \to B \vdash B}\ \supset_L}{A \to B \vdash C \wedge A \to C \wedge B}\ \star$$

MULTIPLICATIVE encoding (48ms)

$$\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{\dfrac{}{\cdot :\ C \vdash C} \quad \dfrac{}{\cdot :\ B \vdash B}}{\cdot :\ B, C \vdash C \otimes B}\ \otimes}{\cdot :\ A, C, A \multimap B \vdash C \otimes B}\ \multimap}{\cdot :\ A \multimap B \vdash C \otimes A \multimap C \otimes B}\ \star$$

CALL-BY-NAME encoding (60ms)

$$\dfrac{\dfrac{}{A, C, !(A) \multimap B :\ \cdot \vdash C} \quad \dfrac{\dfrac{\dfrac{}{A, C, !(A) \multimap B :\ \cdot \vdash A}}{A, C, !(A) \multimap B :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, C, !(A) \multimap B :\ B \vdash B}}{A, C, !(A) \multimap B :\ !(A) \multimap B \vdash B}\ \multimap}{A, C, !(A) \multimap B :\ \cdot \vdash B}\ D_C}{\cdot :\ !(!(A) \multimap B) \vdash !(C \,\&\, A) \multimap C \,\&\, B}\ \star$$

CALL-BY-VALUE encoding (137ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A, C, !(A) \multimap !(B) :\ \cdot \vdash A}}{A, C, !(A) \multimap !(B) :\ \cdot \vdash !(A)}\ ! \quad \dfrac{\dfrac{\dfrac{}{A, B, C, !(A) \multimap !(B) :\ \cdot \vdash C}}{A, B, C, !(A) \multimap !(B) :\ \cdot \vdash !(C)}\ ! \quad \dfrac{\dfrac{}{A, B, C, !(A) \multimap !(B) :\ \cdot \vdash B}}{A, B, C, !(A) \multimap !(B) :\ \cdot \vdash !(B)}\ !}{A, B, C, !(A) \multimap !(B) :\ \cdot \vdash !(C) \otimes !(B)}\ \otimes}{A, C, !(A) \multimap !(B) :\ !(A) \multimap !(B) \vdash !(C) \otimes !(B)}\ \multimap}{A, C, !(A) \multimap !(B) :\ \cdot \vdash !(C) \otimes !(B)}\ D_C}{!(A) \multimap !(B) :\ \cdot \vdash !(C) \otimes !(A) \multimap !(C) \otimes !(B)}\ \star}{\cdot :\ !(!(A) \multimap !(B)) \vdash !(!(C) \otimes !(A) \multimap !(C) \otimes !(B))}\ \star$$

01-ENC encoding (131ms)

$$\dfrac{\dfrac{\dfrac{}{A, C, !(A) \multimap !(B) :\ \cdot \vdash A}}{A, C, !(A) \multimap !(B) :\ \cdot \vdash !(A)}\ ! \quad \dfrac{\dfrac{\dfrac{}{A, B, C, !(A) \multimap !(B) :\ \cdot \vdash C} \quad \dfrac{}{A, B, C, !(A) \multimap !(B) :\ \cdot \vdash B}}{A, B, C, !(A) \multimap !(B) :\ \cdot \vdash C \,\&\, B}\ \star}{A, C, !(A) \multimap !(B) :\ !(B) \vdash !(C \,\&\, B)}\ !}{A, C, !(A) \multimap !(B) :\ !(A) \multimap !(B) \vdash !(C \,\&\, B)}\ \multimap}{A, C, !(A) \multimap !(B) :\ \cdot \vdash !(C \,\&\, B)}\ D_C}{!(A) \multimap !(B) :\ \cdot \vdash !(!(C) \,\&\, !(A)) \multimap !(C \,\&\, B)}\ \star}{!(A) \multimap !(B) :\ \cdot \vdash !(!(!(C) \,\&\, !(A)) \multimap !(C \,\&\, B))}\ !}{\cdot :\ !(!(A) \multimap !(B)) \vdash !(!(!(C) \,\&\, !(A)) \multimap !(C \,\&\, B))}\ \star$$

(10) $\neg A \vdash A \to B$

LJ (37ms)

$$\dfrac{\dfrac{\dfrac{}{A, A \to \bot \vdash A}\ \star \quad \dfrac{}{A, \bot \vdash B}\ \star}{A, A \to \bot \vdash B}\ \supset_L}{A \to \bot \vdash A \to B}\ \star$$

MULTIPLICATIVE encoding (41ms)

Not provable

CALL-BY-NAME encoding (66ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap \mathbf{0} :\ \mathbf{0} \vdash B}\ \star}{A, !(A) \multimap \mathbf{0} :\ !(A) \multimap \mathbf{0} \vdash B}\ \multimap}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash B}\ D_C}{\cdot :\ !(!(A) \multimap \mathbf{0}) \vdash !(A) \multimap B}\ \star$$

CALL-BY-VALUE encoding (117ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap \mathbf{0} :\ \mathbf{0} \vdash !(B)}\ \star}{A, !(A) \multimap \mathbf{0} :\ !(A) \multimap \mathbf{0} \vdash !(B)}\ \multimap}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(B)}\ D_C}{!(A) \multimap \mathbf{0} :\ \cdot \vdash !(A) \multimap !(B)}\ \star}{!(A) \multimap \mathbf{0} :\ \cdot \vdash !(!(A) \multimap !(B))}\ !}{\cdot :\ !(!(A) \multimap \mathbf{0}) \vdash !(!(A) \multimap !(B))}\ \star$$

01-ENC encoding (118ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash A}}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap !(\mathbf{0}) :\ !(\mathbf{0}) \vdash B}\ \star}{A, !(A) \multimap !(\mathbf{0}) :\ !(A) \multimap !(\mathbf{0}) \vdash B}\ \multimap}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash B}\ D_C}{!(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(A) \multimap B}\ \star}{!(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(!(A) \multimap B)}\ !}{\cdot :\ !(!(A) \multimap !(\mathbf{0})) \vdash !(!(A) \multimap B)}\ \star$$

(11) $A \vdash \neg A \to B$

LJ (37ms)

$$\dfrac{\dfrac{}{A, A \to \bot \vdash A}\ \star \quad \dfrac{}{A, \bot \vdash B}\ \star}{\dfrac{A, A \to \bot \vdash B}{A \vdash A \to \bot \to B}\ \star}\ \supset_L$$

MULTIPLICATIVE encoding (28ms)

Not provable

CALL-BY-NAME encoding (65ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap \mathbf{0} :\ \mathbf{0} \vdash B}\ \star}{A, !(A) \multimap \mathbf{0} :\ !(A) \multimap \mathbf{0} \vdash B}\ \multimap}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash B}\ D_C}{\cdot :\ !(A) \vdash !(!(A) \multimap \mathbf{0}) \multimap B}\ \star$$

CALL-BY-VALUE encoding (78ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap \mathbf{0} :\ \mathbf{0} \vdash !(B)}\ \star}{A, !(A) \multimap \mathbf{0} :\ !(A) \multimap \mathbf{0} \vdash !(B)}\ \multimap}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(B)}\ D_C}{A :\ \cdot \vdash !(!(A) \multimap \mathbf{0}) \multimap !(B)}\ \star}{A :\ \cdot \vdash !(!(!(A) \multimap \mathbf{0}) \multimap !(B))}\ !}{\cdot :\ !(A) \vdash !(!(!(A) \multimap \mathbf{0}) \multimap !(B))}\ \star$$

01-ENC encoding (76ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash A}}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap !(\mathbf{0}) :\ \mathbf{0} \vdash B}\ \star}{A, !(A) \multimap !(\mathbf{0}) :\ !(A) \multimap !(\mathbf{0}) \vdash B}\ \multimap}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash B}\ D_C}{A :\ \cdot \vdash !(!(A) \multimap !(\mathbf{0})) \multimap B}\ \star}{A :\ \cdot \vdash !(!(!(A) \multimap !(\mathbf{0})) \multimap B)}\ !}{\cdot :\ !(A) \vdash !(!(!(A) \multimap !(\mathbf{0})) \multimap B)}\ \star$$

(12) $B \vdash A \to B$

LJ (21ms)

$$\dfrac{}{B \vdash A \to B}\ \star$$

MULTIPLICATIVE encoding (21ms)

Not provable

CALL-BY-NAME encoding (27ms)

$$\dfrac{\dfrac{}{A, B :\ \cdot \vdash B}}{\cdot :\ !(B) \vdash !(A) \multimap B}\ \star$$

CALL-BY-VALUE encoding (47ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \vdash B}}{A, B :\ \cdot \vdash !(B)}\ !}{B :\ \cdot \vdash !(A) \multimap !(B)}\ \star}{B :\ \cdot \vdash !(!(A) \multimap !(B))}\ !}{\cdot :\ !(B) \vdash !(!(A) \multimap !(B))}\ \star$$

01-ENC encoding (39ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \vdash B}}{B :\ \cdot \vdash !(A) \multimap B}\ \star}{B :\ \cdot \vdash !(!(A) \multimap B)}\ !}{\cdot :\ !(B) \vdash !(!(A) \multimap B)}\ \star$$

(13) $A \to B \vdash \neg B \to \neg A$

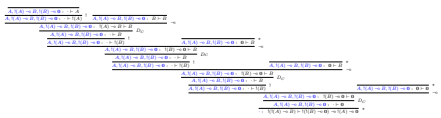
LJ (47ms)

$$\dfrac{\dfrac{}{A, A \to B, B \to \bot \vdash A}\,\star \quad \dfrac{\dfrac{}{A, B, B \to \bot \vdash B}\,\star \quad \dfrac{}{A, B, \bot \vdash \bot}\,\star}{A, B, B \to \bot \vdash \bot}\,\supset_L}{A, A \to B, B \to \bot \vdash \bot}\,\supset_L$$
$$\dfrac{A, A \to B, B \to \bot \vdash \bot}{A \to B \vdash B \to \bot \to A \to \bot}\,\star$$

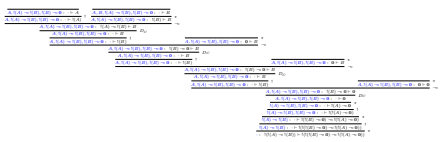
MULTIPLICATIVE encoding (47ms)

$$\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{\dfrac{}{\cdot :\ B \vdash B} \quad \dfrac{}{\cdot :\ \bot \vdash \bot}\,\multimap}{\cdot :\ B, B \multimap \bot \vdash \bot}\,\multimap}{\cdot :\ A, A \multimap B, B \multimap \bot \vdash \bot}\,\multimap$$
$$\dfrac{\cdot :\ A, A \multimap B, B \multimap \bot \vdash \bot}{\cdot :\ A \multimap B \vdash B \multimap \bot \multimap A \multimap \bot}\,\star$$

CALL-BY-NAME encoding (143ms)



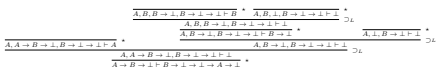
CALL-BY-VALUE encoding (174ms)



01-ENC encoding (178ms)

(14) $A \to \neg B \vdash \neg\neg B \to \neg A$

LJ (64ms)



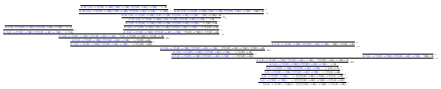
MULTIPLICATIVE encoding (75ms)

$$\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{\dfrac{\dfrac{}{\cdot :\ B \vdash B} \quad \dfrac{}{\cdot :\ \bot \vdash \bot}\,\multimap}{\cdot :\ B, B \multimap \bot \vdash \bot}\,\multimap}{\cdot :\ A, B, A \multimap B \multimap \bot \vdash \bot}\,\star \quad \dfrac{}{\cdot :\ \bot \vdash \bot}\,\multimap}{\cdot :\ A, A \multimap B \multimap \bot \vdash B \multimap \bot, B \multimap \bot \multimap \bot \vdash \bot}\,\multimap$$
$$\dfrac{\cdot :\ A, A \multimap B \multimap \bot \vdash B \multimap \bot, B \multimap \bot \multimap \bot \vdash \bot}{\cdot :\ A \multimap B \multimap \bot \vdash B \multimap \bot \multimap \bot \multimap A \multimap \bot}\,\star$$

CALL-BY-NAME encoding (180ms)

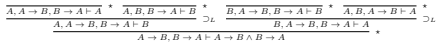
CALL-BY-VALUE encoding (217ms)

01-ENC encoding (226ms)

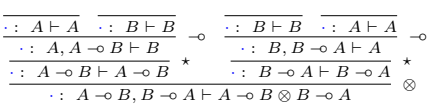


(15) $A \to B, B \to A \vdash (A) \leftrightarrow (B)$

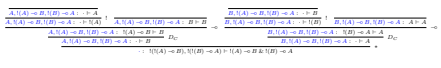
LJ (51ms)



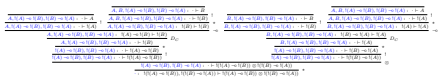
MULTIPLICATIVE encoding (75ms)



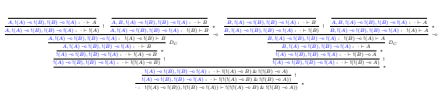
CALL-BY-NAME encoding (138ms)



CALL-BY-VALUE encoding (205ms)

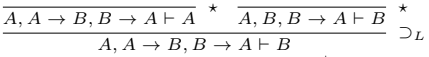


01-ENC encoding (191ms)



(16) $(A) \leftrightarrow (B) \vdash A \to B$

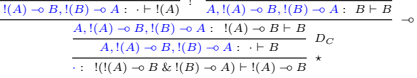
LJ (37ms)



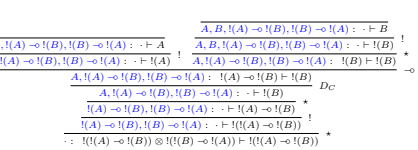
MULTIPLICATIVE encoding (29ms)

Not provable

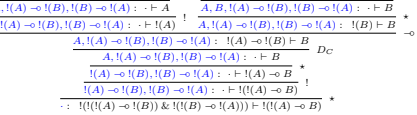
CALL-BY-NAME encoding (102ms)



CALL-BY-VALUE encoding (132ms)



01-ENC encoding (126ms)



(17) $(A) \leftrightarrow (B) \vdash B \to A$

LJ (37ms)

$$\dfrac{\dfrac{}{B, A \to B, B \to A \vdash B}\ \star \quad \dfrac{}{A, B, A \to B \vdash A}\ \star}{\dfrac{B, A \to B, B \to A \vdash A}{A \to B \wedge B \to A \vdash B \to A}\ \star}\ \supset_L$$

MULTIPLICATIVE encoding (29ms)

Not provable

CALL-BY-NAME encoding (105ms)

$$\dfrac{\dfrac{\dfrac{}{B, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash B}}{B, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash !(B)}\ ! \quad \dfrac{}{B, !(A) \multimap B, !(B) \multimap A :\ A \vdash A}}{\dfrac{B, !(A) \multimap B, !(B) \multimap A :\ !(B) \multimap A \vdash A}{\dfrac{B, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash A}{\cdot :\ !(!(A) \multimap B \,\&\, !(B) \multimap A) \vdash !(B) \multimap A}\ \star}\ D_C}\ \multimap$$

CALL-BY-VALUE encoding (140ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash B}}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(B)}\ ! \quad \dfrac{\dfrac{}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash A}}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(A)}\ !}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ !(B) \multimap !(A) \vdash !(A)}\ \star\ \multimap}{\dfrac{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(A)}{!(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(B) \multimap !(A)}\ \star}\ D_C}{\dfrac{!(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(!(B) \multimap !(A))}{\cdot :\ !(!(A) \multimap !(B)) \otimes !(!(B) \multimap !(A)) \vdash !(!(B) \multimap !(A))}\ !}\ \star$$

01-ENC encoding (122ms)

$$\dfrac{\dfrac{\dfrac{}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash B}}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(B)}\ ! \quad \dfrac{\dfrac{}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash A}}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ !(A) \vdash A}\ \star}{\dfrac{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ !(B) \multimap !(A) \vdash A}{\dfrac{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash A}{!(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(B) \multimap A}\ \star}\ D_C}\ \multimap$$
$$\dfrac{\dfrac{!(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(!(B) \multimap A)}{\cdot :\ !(!(!(A) \multimap !(B)) \,\&\, !(!(B) \multimap !(A))) \vdash !(!(B) \multimap A)}\ !}{}\ \star$$

(18) $(A) \leftrightarrow (B), A \vdash B$

LJ (38ms)

$$\dfrac{\dfrac{}{A, A \to B, B \to A \vdash A}\ \star \quad \dfrac{}{A, B, B \to A \vdash B}\ \star}{\dfrac{A, A \to B, B \to A \vdash B}{A, A \to B \wedge B \to A \vdash B}\ \star}\ \supset_L$$

MULTIPLICATIVE encoding (29ms)

Not provable

CALL-BY-NAME encoding (102ms)

$$\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash A}}{A, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap B, !(B) \multimap A :\ B \vdash B}}{\dfrac{A, !(A) \multimap B, !(B) \multimap A :\ !(A) \multimap B \vdash B}{\dfrac{A, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash B}{\cdot :\ !(A), !(!(A) \multimap B \,\&\, !(B) \multimap A) \vdash B}\ \star}\ D_C}\ \multimap$$

CALL-BY-VALUE encoding (119ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash A}}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(A)}\ ! \quad \dfrac{\dfrac{}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash B}}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(B)}\ !}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ !(A) \multimap !(B) \vdash !(B)}\ \star\ \multimap}{\dfrac{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(B)}{\cdot :\ !(A), !(!(A) \multimap !(B)) \otimes !(!(B) \multimap !(A)) \vdash !(B)}\ \star}\ D_C$$

01-ENC encoding (111ms)

$$\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash A}}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(A)}\ ! \quad \dfrac{\dfrac{}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash B}}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ !(A) \vdash B}\ \star}{\dfrac{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ !(A) \multimap !(B) \vdash B}{\dfrac{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash B}{\cdot :\ !(A), !(!(!(A) \multimap !(B)) \,\&\, !(!(B) \multimap !(A))) \vdash B}\ \star}\ D_C}\ \multimap$$

(19) $(A) \leftrightarrow (B), B \vdash A$

LJ (38ms)

$$\dfrac{\dfrac{}{B, A \to B, B \to A \vdash B}\ \star \qquad \dfrac{}{A, B, A \to B \vdash A}\ \star}{\dfrac{B, A \to B, B \to A \vdash A}{B, A \to B \wedge B \to A \vdash A}\ \star}\ \supset_L$$

MULTIPLICATIVE encoding (32ms)

Not provable

CALL-BY-NAME encoding (102ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{B, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash B}}{B, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash\ !(B)}\ ! \qquad \dfrac{}{B, !(A) \multimap B, !(B) \multimap A :\ A \vdash A}}{\dfrac{B, !(A) \multimap B, !(B) \multimap A :\ !(B) \multimap A \vdash A}{B, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash A}\ D_C}\ \multimap}{\cdot :\ !(B), !(!(A) \multimap B \,\&\, !(B) \multimap A) \vdash A}\ \star$$

CALL-BY-VALUE encoding (118ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash B}}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(B)}\ ! \qquad \dfrac{\dfrac{}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash A}}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash\ !(A)}\ !}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ !(B) \multimap !(A) \vdash !(A)}\ \star}{\dfrac{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(A)}{\cdot :\ !(B), !(!(A) \multimap !(B)) \otimes !(!(B) \multimap !(A)) \vdash !(A)}\ \star}\ D_C}\ \multimap$$

01-ENC encoding (108ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash B}}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(B)}\ ! \qquad \dfrac{\dfrac{}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash A}}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ !(A) \vdash A}\ \star}{\dfrac{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ !(B) \multimap !(A) \vdash A}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash A}\ D_C}\ \multimap}{\cdot :\ !(B), !(!(!(A) \multimap !(B)) \,\&\, !(!(B) \multimap !(A))) \vdash A}\ \star$$

(20) $\cdot \vdash (A) \leftrightarrow (A)$

LJ (21ms)

$$\dfrac{}{\cdot \vdash A \to A \wedge A \to A}\ \star$$

MULTIPLICATIVE encoding (34ms)

$$\dfrac{\dfrac{\dfrac{}{\cdot :\ A \vdash A}}{\cdot :\ \cdot \vdash A \multimap A}\ \star \qquad \dfrac{\dfrac{}{\cdot :\ A \vdash A}}{\cdot :\ \cdot \vdash A \multimap A}\ \star}{\cdot :\ \cdot \vdash A \multimap A \otimes A \multimap A}\ \otimes$$

CALL-BY-NAME encoding (27ms)

$$\dfrac{\dfrac{}{A :\ \cdot \vdash A} \qquad \dfrac{}{A :\ \cdot \vdash A}}{\cdot :\ \cdot \vdash\ !(A) \multimap A \,\&\, !(A) \multimap A}\ \star$$

CALL-BY-VALUE encoding (47ms)

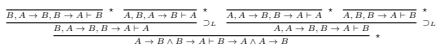
$$\dfrac{\dfrac{\dfrac{\dfrac{}{A :\ \cdot \vdash A}}{A :\ \cdot \vdash\ !(A)}\ ! \qquad \dfrac{\dfrac{}{A :\ \cdot \vdash A}}{A :\ \cdot \vdash\ !(A)}\ !}{\cdot :\ \cdot \vdash\ !(A) \multimap !(A)}\ \star \qquad \dfrac{\dfrac{}{A :\ \cdot \vdash A}}{A :\ \cdot \vdash\ !(A)}\ !}{\cdot :\ \cdot \vdash\ !(A) \multimap !(A))}\ !}{\cdot :\ \cdot \vdash\ !(!(A) \multimap !(A)) \otimes\ !(!(A) \multimap !(A))}\ \otimes$$

01-ENC encoding (47ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A :\ \cdot \vdash A}}{\cdot :\ \cdot \vdash\ !(A) \multimap A}\ \star \qquad \dfrac{\dfrac{}{A :\ \cdot \vdash A}}{\cdot :\ \cdot \vdash\ !(A) \multimap A}\ \star}{\cdot :\ \cdot \vdash\ !(!(A) \multimap A)}\ ! \qquad \dfrac{\dfrac{}{A :\ \cdot \vdash A}}{\cdot :\ \cdot \vdash\ !(!(A) \multimap A)}\ !}{\cdot :\ \cdot \vdash\ !(!(A) \multimap A) \,\&\, !(!(A) \multimap A)}\ \star}{\cdot :\ \cdot \vdash\ !(!(!(A) \multimap A) \,\&\, !(!(A) \multimap A))}\ !$$

(21) $(A) \leftrightarrow (B) \vdash (B) \leftrightarrow (A)$

LJ (50ms)

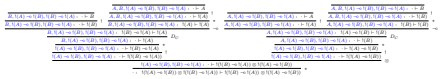


MULTIPLICATIVE encoding (81ms)

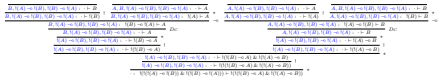
$$\dfrac{\dfrac{\dfrac{}{\cdot :\ B \vdash B} \quad \dfrac{}{\cdot :\ A \vdash A}}{\cdot :\ B,\ B \multimap A \vdash A}\ \multimap \quad \dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{}{\cdot :\ B \vdash B}}{\cdot :\ A,\ A \multimap B \vdash B}\ \multimap}{\dfrac{\cdot :\ B \multimap A \vdash B \multimap A}{\cdot :\ A \multimap B \vdash A \multimap B}\ \star}\ \star}{\dfrac{\cdot :\ A \multimap B,\ B \multimap A \vdash B \multimap A \otimes A \multimap B}{\cdot :\ A \multimap B \otimes B \multimap A \vdash B \multimap A \otimes A \multimap B}\ \star}\ \otimes$$

CALL-BY-NAME encoding (138ms)

CALL-BY-VALUE encoding (203ms)



01-ENC encoding (188ms)



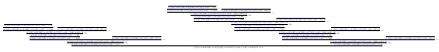
(22) $(A) \leftrightarrow (B), (B) \leftrightarrow (C) \vdash (A) \leftrightarrow (C)$

LJ (75ms)

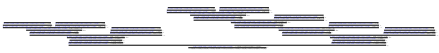
MULTIPLICATIVE encoding (115ms)

$$\dfrac{\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{\dfrac{}{\cdot :\ B \vdash B} \quad \dfrac{}{\cdot :\ C \vdash C}}{\cdot :\ B,\ B \multimap C \vdash C}\ \multimap}{\cdot :\ A,\ A \multimap B,\ B \multimap C \vdash C}\ \multimap \quad \dfrac{\dfrac{}{\cdot :\ C \vdash C} \quad \dfrac{\dfrac{}{\cdot :\ B \vdash B} \quad \dfrac{}{\cdot :\ A \vdash A}}{\cdot :\ B,\ B \multimap A \vdash A}\ \multimap}{\cdot :\ C,\ B \multimap A,\ C \multimap B \vdash A}\ \multimap}{\dfrac{\cdot :\ A \multimap B \vdash A \multimap B}{\cdot :\ A \multimap B,\ B \multimap C \vdash A \multimap C}\ \star \quad \dfrac{\cdot :\ B \multimap A \vdash B \multimap A}{\cdot :\ B \multimap A,\ C \multimap B \vdash C \multimap A}\ \star}\ \otimes}{\dfrac{\cdot :\ A \multimap B,\ B \multimap A,\ B \multimap C,\ C \multimap B \vdash A \multimap C \otimes C \multimap A}{\cdot :\ A \multimap B \otimes B \multimap A,\ B \multimap C \otimes C \multimap B \vdash A \multimap C \otimes C \multimap A}\ \star}$$

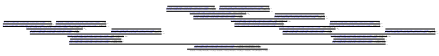
CALL-BY-NAME encoding (218ms)



CALL-BY-VALUE encoding (331ms)



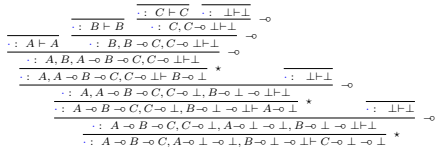
01-ENC encoding (291ms)



(23) $A \to B \to C, \neg\neg A, \neg\neg B \vdash \neg\neg C$

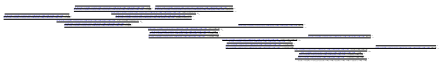
LJ (206ms)

MULTIPLICATIVE encoding (124ms)

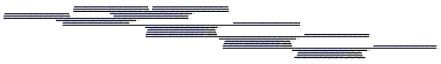


CALL-BY-NAME encoding (221ms)

CALL-BY-VALUE encoding (324ms)



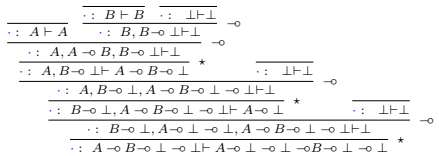
01-ENC encoding (380ms)



(24) $\neg\neg A \to B \vdash \neg\neg A \to \neg\neg B$

LJ (94ms)

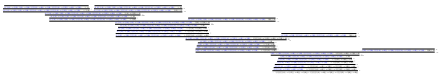
MULTIPLICATIVE encoding (106ms)



CALL-BY-NAME encoding (199ms)

CALL-BY-VALUE encoding (241ms)

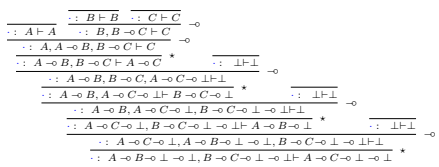
01-ENC encoding (269ms)



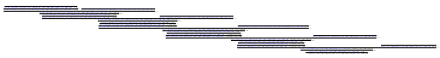
(25) $\neg\neg A \to B, \neg\neg B \to C \vdash \neg\neg A \to C$

LJ (324ms)

MULTIPLICATIVE encoding (145ms)



CALL-BY-NAME encoding (283ms)

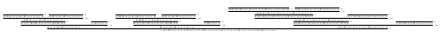


CALL-BY-VALUE encoding (375ms)

01-ENC encoding (421ms)

(26) $\cdot \vdash (\neg\neg A \wedge B) \leftrightarrow (\neg\neg A \wedge \neg\neg B)$

LJ (137ms)



MULTIPLICATIVE encoding (66ms)

Not provable

CALL-BY-NAME encoding (601ms)

CALL-BY-VALUE encoding (584ms)

01-ENC encoding (646ms)

(27) $\cdot \vdash (\neg\neg(A) \leftrightarrow (B)) \leftrightarrow (\neg\neg A \to B \wedge \neg\neg B \to A)$

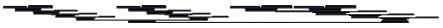
LJ (2703ms)

MULTIPLICATIVE encoding (84ms)

Not provable

CALL-BY-NAME encoding (661ms)

CALL-BY-VALUE encoding (841ms)

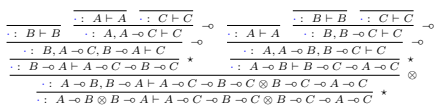


01-ENC encoding (1132ms)

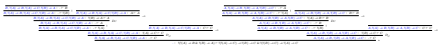
(28) $(A) \leftrightarrow (B) \vdash (A \to C) \leftrightarrow (B \to C)$

LJ (75ms)

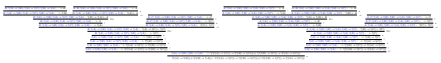
MULTIPLICATIVE encoding (95ms)



CALL-BY-NAME encoding (173ms)



CALL-BY-VALUE encoding (288ms)

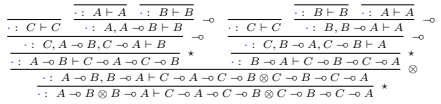


01-ENC encoding (268ms)

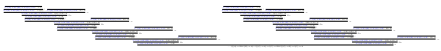
(29) $(A) \leftrightarrow (B) \vdash (C \to A) \leftrightarrow (C \to B)$

LJ (72ms)

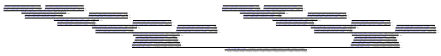
MULTIPLICATIVE encoding (96ms)



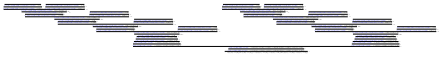
CALL-BY-NAME encoding (250ms)



CALL-BY-VALUE encoding (387ms)



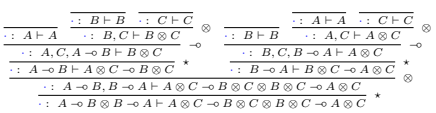
01-ENC encoding (356ms)



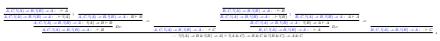
(30) $(A) \leftrightarrow (B) \vdash (A \wedge C) \leftrightarrow (B \wedge C)$

LJ (51ms)

MULTIPLICATIVE encoding (99ms)



CALL-BY-NAME encoding (138ms)

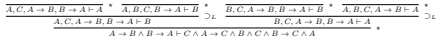


CALL-BY-VALUE encoding (238ms)

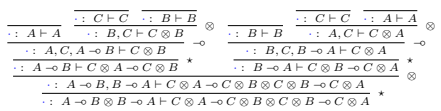
01-ENC encoding (234ms)

(31) $(A) \leftrightarrow (B) \vdash (C \wedge A) \leftrightarrow (C \wedge B)$

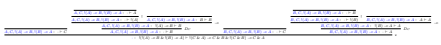
LJ (51ms)



MULTIPLICATIVE encoding (98ms)



CALL-BY-NAME encoding (142ms)

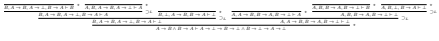


CALL-BY-VALUE encoding (227ms)

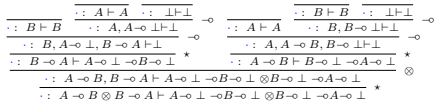
01-ENC encoding (229ms)

(32) $(A) \leftrightarrow (B) \vdash (\neg A) \leftrightarrow (\neg B)$

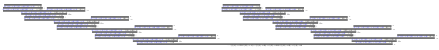
LJ (74ms)



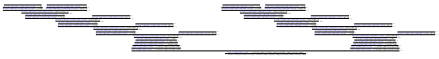
MULTIPLICATIVE encoding (97ms)



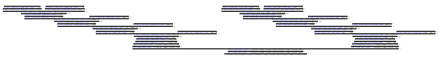
CALL-BY-NAME encoding (223ms)



CALL-BY-VALUE encoding (304ms)

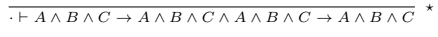


01-ENC encoding (320ms)

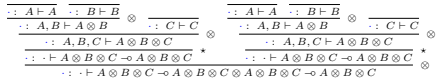


(33) $\cdot \vdash (A \wedge B \wedge C) \leftrightarrow (A \wedge B \wedge C)$

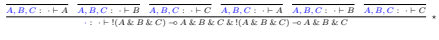
LJ (21ms)



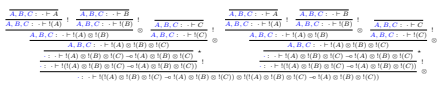
MULTIPLICATIVE encoding (66ms)



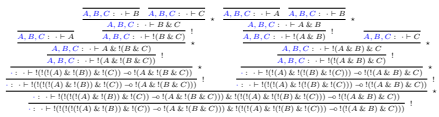
CALL-BY-NAME encoding (34ms)



CALL-BY-VALUE encoding (99ms)



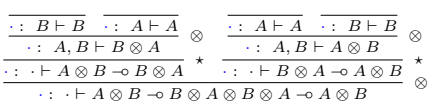
01-ENC encoding (124ms)



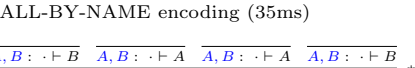
(34) $\cdot \vdash (A \wedge B) \leftrightarrow (B \wedge A)$

LJ (21ms)

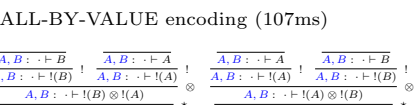
MULTIPLICATIVE encoding (73ms)



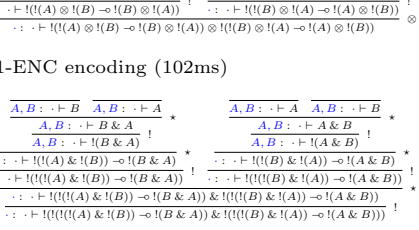
CALL-BY-NAME encoding (35ms)



CALL-BY-VALUE encoding (107ms)



01-ENC encoding (102ms)



$\boxed{(35)\ \cdot \vdash (A \wedge A) \leftrightarrow (A)}$

LJ (21ms)

$$\dfrac{}{\cdot \vdash A \wedge A \to A \wedge A \to A \wedge A}\ \star$$

MULTIPLICATIVE encoding (34ms)

Not provable

CALL-BY-NAME encoding (28ms)

$$\dfrac{\dfrac{}{A:\ \cdot \vdash A} \quad \dfrac{}{A:\ \cdot \vdash A} \quad \dfrac{}{A:\ \cdot \vdash A}}{\cdot:\ \cdot \vdash\ !(A\ \&\ A) \multimap A\ \&\ !(A) \multimap A\ \&\ A}\ \star$$

CALL-BY-VALUE encoding (76ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A:\ \cdot \vdash A}}{A:\ \cdot \vdash\ !(A)}\ !}{\cdot:\ \cdot \vdash\ !(A) \otimes\ !(A) \multimap\ !(A)}\ \star \quad \dfrac{\dfrac{\dfrac{}{A:\ \cdot \vdash A}}{A:\ \cdot \vdash\ !(A)}\ ! \quad \dfrac{\dfrac{}{A:\ \cdot \vdash A}}{A:\ \cdot \vdash\ !(A)}\ !}{A:\ \cdot \vdash\ !(A) \otimes\ !(A)}\ \otimes}{\cdot:\ \cdot \vdash\ !(!(A) \otimes\ !(A) \multimap\ !(A))}\ ! \quad \dfrac{\dfrac{\dfrac{}{A:\ \cdot \vdash A}}{A:\ \cdot \vdash\ !(A)}\ ! \quad \dfrac{\dfrac{}{A:\ \cdot \vdash A}}{A:\ \cdot \vdash\ !(A)}\ !}{A:\ \cdot \vdash\ !(A) \otimes\ !(A)}\ \otimes}{\cdot:\ \cdot \vdash\ !(!(A) \otimes\ !(A) \multimap\ !(A)) \otimes\ !(!(A) \multimap\ !(A) \otimes\ !(A))}$$

01-ENC encoding (81ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A:\ \cdot \vdash A}}{\cdot:\ \cdot \vdash\ !(!(A)\ \&\ !(A)) \multimap A}\ \star \quad \dfrac{\dfrac{\dfrac{}{A:\ \cdot \vdash A} \quad \dfrac{}{A:\ \cdot \vdash A}\ \star}{A:\ \cdot \vdash A\ \&\ A}\ !}{A:\ \cdot \vdash\ !(A\ \&\ A)}}{\cdot:\ \cdot \vdash\ !(A) \multimap\ !(A\ \&\ A)}\ \star}{\cdot:\ \cdot \vdash\ !(!(A)\ \&\ !(A)) \multimap\ !(A) \multimap\ !(A\ \&\ A))}\ ! \quad \dfrac{\dfrac{\dfrac{\dfrac{}{A:\ \cdot \vdash A} \quad \dfrac{}{A:\ \cdot \vdash A}\ \star}{A:\ \cdot \vdash A\ \&\ A}\ !}{A:\ \cdot \vdash\ !(A\ \&\ A)}}{\cdot:\ \cdot \vdash\ !(A) \multimap\ !(A\ \&\ A)}\ \star}{\cdot:\ \cdot \vdash\ !(!(A)\ \&\ !(A)) \multimap\ !(A) \multimap\ !(A\ \&\ A))}\ !}{\cdot:\ \cdot \vdash\ !(!(!(A)\ \&\ !(A)) \multimap A) \&\ !(!(A) \multimap\ !(A\ \&\ A)))}\ !$$

$\boxed{(36)\ A \vdash (A \to B) \leftrightarrow (B)}$

LJ (37ms)

$$\dfrac{\dfrac{\dfrac{}{A, A \to B \vdash A}\ \star \quad \dfrac{}{A, B \vdash B}\ \star}{A, A \to B \vdash B}\ \supset_L}{A \vdash A \to B \to B \wedge B \to A \to B}\ \star$$

MULTIPLICATIVE encoding (42ms)

Not provable

CALL-BY-NAME encoding (73ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap B:\ \cdot \vdash A}}{A, !(A) \multimap B:\ \cdot \vdash\ !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap B:\ B \vdash B}}{A, !(A) \multimap B:\ !(A) \multimap B \vdash B}\ \multimap}{A, !(A) \multimap B:\ \cdot \vdash B}\ D_C \quad \dfrac{}{A, B:\ \cdot \vdash B}}{\cdot:\ !(A) \vdash\ !(!(A) \multimap B) \multimap B\ \&\ !(B) \multimap\ !(A) \multimap B}\ \star$$

CALL-BY-VALUE encoding (129ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap\ !(B):\ \cdot \vdash A}}{A, !(A) \multimap\ !(B):\ \cdot \vdash\ !(A)}\ ! \quad \dfrac{\dfrac{}{A, B, !(A) \multimap\ !(B):\ \cdot \vdash B}}{A, B, !(A) \multimap\ !(B):\ \cdot \vdash\ !(B)}\ !}{A, !(A) \multimap\ !(B):\ !(A) \multimap\ !(B) \vdash\ !(B)}\ \multimap}{A, !(A) \multimap\ !(B):\ \cdot \vdash\ !(B)}\ D_C}{A:\ \cdot \vdash\ !(!(A) \multimap\ !(B)) \multimap\ !(B)}\ \star}{A:\ \cdot \vdash\ !(!(A) \multimap\ !(B)) \otimes\ !(!(B) \multimap\ !(!(A) \multimap\ !(B)))}\ \star \quad \dfrac{\dfrac{\dfrac{\dfrac{}{A, B:\ \cdot \vdash B}}{A, B:\ \cdot \vdash\ !(B)}\ ! \quad \dfrac{\dfrac{}{A, B:\ \cdot \vdash\ !(A) \multimap\ !(B)}}{A, B:\ \cdot \vdash\ !(!(A) \multimap\ !(B))}\ !}{A:\ \cdot \vdash\ !(B) \multimap\ !(!(A) \multimap\ !(B))}\ \star}{A:\ \cdot \vdash\ !(!(B) \multimap\ !(!(A) \multimap\ !(B)))}\ !}{A:\ \cdot \vdash\ !(!(!(A) \multimap\ !(B)) \otimes\ !(!(B) \multimap\ !(!(A) \multimap\ !(B))))}\ \otimes$$

01-ENC encoding (123ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(B):\ \cdot \vdash A}}{A, !(A) \multimap !(B):\ \cdot \vdash\ !(A)}\ ! \quad \dfrac{\dfrac{}{A, B, !(A) \multimap !(B):\ \cdot \vdash B}}{A, !(A) \multimap !(B):\ !(B) \vdash B}\ !}{A, !(A) \multimap !(B):\ !(A) \multimap !(B) \vdash B}\ \multimap}{A, !(A) \multimap !(B):\ \cdot \vdash B}\ D_C}{A:\ \cdot \vdash\ !(!(A) \multimap !(B)) \multimap B}\ \star}{A:\ \cdot \vdash\ !(!(!(A) \multimap !(B)) \multimap B) \&\ !(!(B) \multimap !(!(A) \multimap B))}\ ! \quad \dfrac{\dfrac{\dfrac{\dfrac{}{A, B:\ \cdot \vdash B}}{A, B:\ \cdot \vdash\ !(!(A) \multimap B)}\ ! \quad \dfrac{\dfrac{}{A, B:\ \cdot \vdash\ !(A) \multimap B}}{A, B:\ \cdot \vdash\ !(!(A) \multimap B)}\ !}{A:\ \cdot \vdash\ !(B) \multimap\ !(!(A) \multimap B)}\ \star}{A:\ \cdot \vdash\ !(!(B) \multimap\ !(!(A) \multimap B))}\ !}{A:\ \cdot \vdash\ !(!(!(A) \multimap !(B)) \multimap B) \&\ !(!(B) \multimap !(!(A) \multimap B)))}\ !}{\cdot:\ !(A) \vdash\ !(!(!(A) \multimap !(B)) \multimap B) \&\ !(!(B) \multimap !(!(A) \multimap B)))}\ \star$$

(37) $B \vdash (A \to B) \leftrightarrow (B)$

LJ (21ms)

$$\frac{}{B \vdash A \to B \to B \wedge B \to A \to B}\ \star$$

MULTIPLICATIVE encoding (40ms)

Not provable

CALL-BY-NAME encoding (39ms)

$$\frac{\dfrac{}{B, !(A) \multimap B :\ \cdot \vdash B}\quad \dfrac{}{A, B :\ \cdot \vdash B}}{\cdot :\ \ !(B) \vdash !(!(A) \multimap B) \multimap B \,\&\, !(B) \multimap !(A) \multimap B}\ \star$$

CALL-BY-VALUE encoding (152ms)

$$\frac{\dfrac{\dfrac{\dfrac{}{B, !(A) \multimap !(B) :\ \cdot \vdash B}}{B, !(A) \multimap !(B) :\ \cdot \vdash !(B)}\ !}{B :\ \cdot \vdash !(!(A) \multimap !(B)) \multimap !(B)}\ \star}{B :\ \cdot \vdash !(!(!(A) \multimap !(B)) \multimap !(B))}\ ! \quad \frac{\dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \vdash B}}{A, B :\ \cdot \vdash !(B)}\ !}{B :\ \cdot \vdash !(A) \multimap !(B)}\ \star}{B :\ \cdot \vdash !(!(A) \multimap !(B))}\ ! \quad \frac{\dfrac{\dfrac{}{B :\ \cdot \vdash B}}{B :\ \cdot \vdash !(B)}\ !}{B :\ \cdot \vdash !(B) \multimap !(!(A) \multimap !(B))}\ \star}{B :\ \cdot \vdash !(!(!(A) \multimap !(B)) \multimap !(B)) \otimes !(!(B) \multimap !(!(A) \multimap !(B)))}\ !\ \otimes$$
$$\frac{}{\cdot :\ \ !(B) \vdash !(!(!(A) \multimap !(B)) \multimap !(B)) \otimes !(!(B) \multimap !(!(A) \multimap !(B)))}\ \star$$

01-ENC encoding (88ms)

$$\frac{\dfrac{\dfrac{\dfrac{\dfrac{}{B, !(A) \multimap !(B) :\ \cdot \vdash B}}{B :\ \cdot \vdash !(!(A) \multimap !(B)) \multimap B}\ \star}{B :\ \cdot \vdash !(!(!(A) \multimap !(B)) \multimap B)}\ ! \quad \dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \vdash B}}{B :\ \cdot \vdash !(A) \multimap B}\ \star}{B :\ \cdot \vdash !(!(B) \multimap !(!(A) \multimap B))}\ !}{B :\ \cdot \vdash !(!(!(A) \multimap !(B)) \multimap B) \,\&\, !(!(B) \multimap !(!(A) \multimap B)))}\ \star}{B :\ \cdot \vdash !(!(!(!(A) \multimap !(B)) \multimap B) \,\&\, !(!(B) \multimap !(!(A) \multimap B)))}\ !$$
$$\frac{}{\cdot :\ \ !(B) \vdash !(!(!(!(A) \multimap !(B)) \multimap B) \,\&\, !(!(B) \multimap !(!(A) \multimap B)))}\ \star$$

(38) $\neg A \vdash (A \to B) \leftrightarrow (\neg A)$

LJ (52ms)

$$\frac{\dfrac{\dfrac{}{A, A \to B, A \to \bot \vdash A}\ \star \quad \dfrac{}{A, \bot, A \to B \vdash \bot}\ \star}{A, A \to B, A \to \bot \vdash \bot}\ \supset_L \quad \dfrac{\dfrac{}{A, A \to \bot \vdash A}\ \star \quad \dfrac{}{A, \bot \vdash B}\ \star}{A, A \to \bot \vdash B}\ \supset_L}{A \to \bot \vdash A \to B \to A \to \bot \wedge A \to \bot \to A \to B}\ \star$$

MULTIPLICATIVE encoding (42ms)

Not provable

CALL-BY-NAME encoding (101ms)

$$\frac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap B, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}}{A, !(A) \multimap B, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap B, !(A) \multimap \mathbf{0} :\ \mathbf{0} \vdash \mathbf{0}}\ \star}{A, !(A) \multimap B, !(A) \multimap \mathbf{0} :\ !(A) \multimap \mathbf{0} \vdash \mathbf{0}}\ \multimap \quad \frac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap \mathbf{0} :\ \mathbf{0} \vdash B}\ \star}{A, !(A) \multimap \mathbf{0} :\ !(A) \multimap \mathbf{0} \vdash B}\ \multimap}{A, !(A) \multimap B :\ \cdot \vdash \mathbf{0}}\ D_C \quad \frac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash B}\ D_C$$
$$\frac{}{\cdot :\ \ !(!(A) \multimap \mathbf{0}) \vdash !(!(A) \multimap B) \multimap !(A) \multimap \mathbf{0} \,\&\, !(!(A) \multimap \mathbf{0}) \multimap !(A) \multimap B}\ \star$$

CALL-BY-VALUE encoding (201ms)

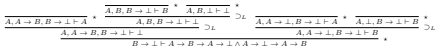
$$\frac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0}, !(A) \multimap !(B) :\ \cdot \vdash A}}{A, !(A) \multimap \mathbf{0}, !(A) \multimap !(B) :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap \mathbf{0}, !(A) \multimap !(B) :\ \mathbf{0} \vdash \mathbf{0}}\ \star}{A, !(A) \multimap \mathbf{0}, !(A) \multimap !(B) :\ !(A) \multimap \mathbf{0} \vdash \mathbf{0}}\ \multimap}{A, !(A) \multimap \mathbf{0}, !(A) \multimap !(B) :\ \cdot \vdash \mathbf{0}}\ D_C}{!(A) \multimap \mathbf{0}, !(A) \multimap !(B) :\ \cdot \vdash !(A) \multimap \mathbf{0}}\ ! \quad \frac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap \mathbf{0} :\ \mathbf{0} \vdash !(B)}\ \star}{A, !(A) \multimap \mathbf{0} :\ !(A) \multimap \mathbf{0} \vdash !(B)}\ \multimap}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(A) \multimap !(B)}\ D_C}{!(A) \multimap \mathbf{0} :\ \cdot \vdash !(A) \multimap !(B)}\ !$$
$$\frac{\dfrac{!(A) \multimap \mathbf{0} :\ \cdot \vdash !(!(A) \multimap !(B)) \multimap !(A) \multimap \mathbf{0})}{!(A) \multimap \mathbf{0} :\ \cdot \vdash !(!(A) \multimap !(B)) \multimap !(A) \multimap \mathbf{0})}\ ! \quad \dfrac{!(A) \multimap \mathbf{0} :\ \cdot \vdash !(!(A) \multimap !(B)) \multimap !(A) \multimap \mathbf{0})}{!(A) \multimap \mathbf{0} :\ \cdot \vdash !(!(A) \multimap !(B)) \multimap !(A) \multimap \mathbf{0})}\ !}{!(A) \multimap \mathbf{0} :\ \cdot \vdash !(!(A) \multimap !(B)) \multimap !(A) \multimap \mathbf{0}) \otimes !(!(A) \multimap \mathbf{0}) \multimap !(!(A) \multimap !(B)))}\ \otimes$$
$$\frac{}{\cdot :\ \ !(!(A) \multimap \mathbf{0}) \vdash !(!(!(A) \multimap !(B)) \multimap !(A) \multimap \mathbf{0}) \otimes !(!(A) \multimap \mathbf{0}) \multimap !(!(A) \multimap !(B)))}\ \star$$

01-ENC encoding (203ms)

$$\frac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash A}}{A, !(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ !(\mathbf{0}) \vdash \mathbf{0}}\ \star}{A, !(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ !(A) \multimap !(\mathbf{0}) \vdash \mathbf{0}}\ \multimap}{A, !(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash \mathbf{0}}\ D_C}{!(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(A) \multimap \mathbf{0}}\ ! \quad \frac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash A}}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap !(\mathbf{0}) :\ !(A) \multimap !(\mathbf{0}) \vdash B}\ \star}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(A) \multimap B}\ D_C}{!(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(A) \multimap B}\ !$$
$$\frac{\dfrac{!(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(!(A) \multimap !(B)) \multimap !(A) \multimap \mathbf{0}) \,\&\, !(!(A) \multimap !(\mathbf{0})) \multimap !(A) \multimap B))}{!(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(!(A) \multimap !(B)) \multimap !(A) \multimap \mathbf{0}) \,\&\, !(!(A) \multimap !(\mathbf{0})) \multimap !(A) \multimap B))}\ ! \quad \dfrac{!(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(!(A) \multimap !(B)) \multimap !(A) \multimap \mathbf{0}) \,\&\, !(!(A) \multimap !(\mathbf{0})) \multimap !(A) \multimap B))}{!(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(!(A) \multimap !(B)) \multimap !(A) \multimap \mathbf{0}) \,\&\, !(!(A) \multimap !(\mathbf{0})) \multimap !(A) \multimap B))}\ !}{!(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(!(!(A) \multimap !(B)) \multimap !(A) \multimap \mathbf{0}) \,\&\, !(!(A) \multimap !(\mathbf{0})) \multimap !(A) \multimap B))}\ \star$$

(39) $\neg B \vdash (A \to B) \leftrightarrow (\neg A)$

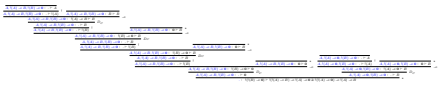
LJ (61ms)



MULTIPLICATIVE encoding (43ms)

Not provable

CALL-BY-NAME encoding (185ms)



CALL-BY-VALUE encoding (256ms)

01-ENC encoding (259ms)

(40) $B \vdash (A \wedge B) \leftrightarrow (A)$

LJ (21ms)

$$\dfrac{}{B \vdash A \wedge B \to A \wedge A \to A \wedge B}\ \star$$

MULTIPLICATIVE encoding (34ms)

Not provable

CALL-BY-NAME encoding (34ms)

$$\dfrac{\dfrac{}{A, B :\ \cdot \vdash A}\quad \dfrac{}{A, B :\ \cdot \vdash A}\quad \dfrac{}{A, B :\ \cdot \vdash B}}{\cdot :\ \ !(B) \vdash\ !(A \,\&\, B) \multimap A \,\&\, !(A) \multimap A \,\&\, B}\ \star$$

CALL-BY-VALUE encoding (88ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \vdash A}}{A, B :\ \cdot \vdash !(A)}\ !}{B :\ \cdot \vdash !(A) \otimes !(B) \multimap !(A)}\ \star}{B :\ \cdot \vdash !(!(A) \otimes !(B) \multimap !(A))}\ ! \qquad \dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \vdash A}}{A, B :\ \cdot \vdash !(A)}\ ! \quad \dfrac{\dfrac{}{A, B :\ \cdot \vdash B}}{A, B :\ \cdot \vdash !(B)}\ !}{A, B :\ \cdot \vdash !(A) \otimes !(B)}\ \otimes}{\dfrac{B :\ \cdot \vdash !(!(A) \otimes !(B) \multimap !(A)) \otimes !(!(A) \multimap !(A) \otimes !(B))}{\cdot :\ \ !(B) \vdash !(!(A) \otimes !(B) \multimap !(A)) \otimes !(!(A) \multimap !(A) \otimes !(B))}\ \star}\ \otimes$$

01-ENC encoding (88ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \vdash A}}{B :\ \cdot \vdash !(!(A) \,\&\, !(B)) \multimap A}\ \star}{B :\ \cdot \vdash !(!(!(A) \,\&\, !(B)) \multimap A)}\ ! \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \vdash A}\quad \dfrac{}{A, B :\ \cdot \vdash B}}{A, B :\ \cdot \vdash A \,\&\, B}\ \star}{A, B :\ \cdot \vdash !(A \,\&\, B)}\ !}{B :\ \cdot \vdash !(A) \multimap !(A \,\&\, B)}\ \star}{B :\ \cdot \vdash !(!(!(A) \,\&\, !(B)) \multimap A) \,\&\, !(A) \multimap !(A \,\&\, B))}\ \star}{B :\ \cdot \vdash !(!(!(!(A) \,\&\, !(B)) \multimap A) \,\&\, !(A) \multimap !(A \,\&\, B)))}\ !}{\cdot :\ \ !(B) \vdash !(!(!(!(A) \,\&\, !(B)) \multimap A) \,\&\, !(A) \multimap !(A \,\&\, B)))}\ \star$$

(41) $\neg B \vdash (A \wedge B) \leftrightarrow (B)$

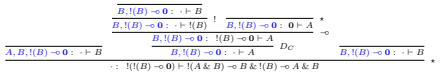
LJ (37ms)

$$\dfrac{\dfrac{\dfrac{}{B, B \to \bot \vdash B}\ \star \quad \dfrac{}{B, \bot \vdash A}\ \star}{B, B \to \bot \vdash A}\ \supset_L}{B \to \bot \vdash A \wedge B \to B \wedge B \to A \wedge B}\ \star$$

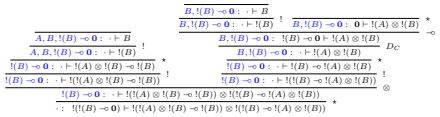
MULTIPLICATIVE encoding (41ms)

Not provable

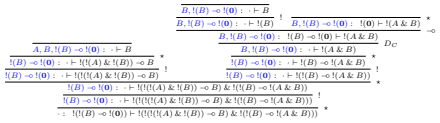
CALL-BY-NAME encoding (78ms)



CALL-BY-VALUE encoding (172ms)



01-ENC encoding (194ms)



(42) $\cdot \vdash A \to \neg\neg A$

LJ (38ms)

$$\dfrac{\dfrac{\dfrac{}{A, A \to \bot \vdash A}\ \star \quad \dfrac{}{A, \bot \vdash \bot}\ \star}{A, A \to \bot \vdash \bot}\ \supset_L}{\cdot \vdash A \to A \to \bot \to \bot}\ \star$$

MULTIPLICATIVE encoding (41ms)

$$\dfrac{\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{}{\cdot :\ \ \bot \vdash \bot}}{\cdot :\ A, A \multimap \bot \vdash \bot}\ \multimap}{\cdot :\ \cdot \vdash A \multimap A \multimap \bot \multimap \bot}\ \star$$

CALL-BY-NAME encoding (66ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap \mathbf{0} :\ \mathbf{0} \vdash \mathbf{0}}\ \star}{A, !(A) \multimap \mathbf{0} :\ !(A) \multimap \mathbf{0} \vdash \mathbf{0}}\ \multimap}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash \mathbf{0}}\ D_C}{\cdot :\ \cdot \vdash !(A) \multimap !(!(A) \multimap \mathbf{0}) \multimap \mathbf{0}}\ \star$$

CALL-BY-VALUE encoding (84ms)

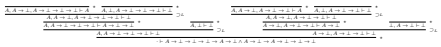
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap \mathbf{0} :\ \mathbf{0} \vdash \mathbf{0}}\ \star}{A, !(A) \multimap \mathbf{0} :\ !(A) \multimap \mathbf{0} \vdash \mathbf{0}}\ \multimap}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash \mathbf{0}}\ D_C}{A :\ \cdot \vdash !(!(A) \multimap \mathbf{0}) \multimap \mathbf{0}}\ \star}{A :\ \cdot \vdash !(!(!(A) \multimap \mathbf{0}) \multimap \mathbf{0})}\ !}{\cdot :\ \cdot \vdash !(A) \multimap !(!(!(A) \multimap \mathbf{0}) \multimap \mathbf{0})}\ \star}{\cdot :\ \cdot \vdash !(!(A) \multimap !(!(!(A) \multimap \mathbf{0}) \multimap \mathbf{0}))}\ !$$

01-ENC encoding (81ms)

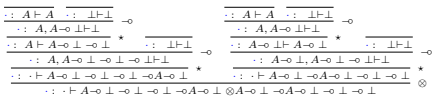
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash A}}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap !(\mathbf{0}) :\ !(\mathbf{0}) \vdash \mathbf{0}}\ \star}{A, !(A) \multimap !(\mathbf{0}) :\ !(A) \multimap !(\mathbf{0}) \vdash \mathbf{0}}\ \multimap}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash \mathbf{0}}\ D_C}{A :\ \cdot \vdash !(!(A) \multimap !(\mathbf{0})) \multimap \mathbf{0}}\ \star}{A :\ \cdot \vdash !(!(!(A) \multimap !(\mathbf{0})) \multimap \mathbf{0})}\ !}{\cdot :\ \cdot \vdash !(A) \multimap !(!(!(A) \multimap !(\mathbf{0})) \multimap \mathbf{0})}\ \star}{\cdot :\ \cdot \vdash !(!(A) \multimap !(!(!(A) \multimap !(\mathbf{0})) \multimap \mathbf{0}))}\ !$$

$\boxed{(43)\ \cdot \vdash (\neg\neg\neg A) \leftrightarrow (\neg A)}$

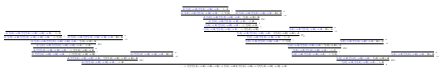
LJ (81ms)



MULTIPLICATIVE encoding (105ms)



CALL-BY-NAME encoding (244ms)



CALL-BY-VALUE encoding (295ms)

01-ENC encoding (343ms)

$\boxed{(44)\ \cdot \vdash \neg A \wedge \neg A}$

LJ (38ms)

$$\dfrac{\dfrac{}{A, A \to \bot \vdash A}\ \star \quad \dfrac{}{A, \bot \vdash \bot}\ \star}{\dfrac{A, A \to \bot \vdash \bot}{\cdot \vdash A \wedge A \to \bot \to \bot}\ \star}\ \supset_L$$

MULTIPLICATIVE encoding (41ms)

$$\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{}{\cdot :\ \bot \vdash \bot}}{\dfrac{\cdot :\ A, A \multimap \bot \vdash \bot}{\cdot :\ \cdot \vdash A \otimes A \multimap \bot \multimap \bot}\ \star}\ \multimap$$

CALL-BY-NAME encoding (65ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap \mathbf{0} :\ \mathbf{0} \vdash \mathbf{0}}\ \star}{A, !(A) \multimap \mathbf{0} :\ !(A) \multimap \mathbf{0} \vdash \mathbf{0}}\ \multimap}{\dfrac{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash \mathbf{0}}{\cdot :\ \cdot \vdash !(A \,\&\, !(A) \multimap \mathbf{0}) \multimap \mathbf{0}}\ \star}\ D_C$$

CALL-BY-VALUE encoding (70ms)

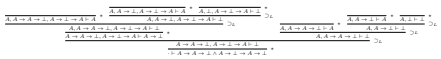
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap \mathbf{0} :\ \mathbf{0} \vdash \mathbf{0}}\ \star}{A, !(A) \multimap \mathbf{0} :\ !(A) \multimap \mathbf{0} \vdash \mathbf{0}}\ \multimap}{\dfrac{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash \mathbf{0}}{\cdot :\ \cdot \vdash !(A) \otimes !(!(A) \multimap \mathbf{0}) \multimap \mathbf{0}}\ \star}\ D_C}{\cdot :\ \cdot \vdash !(!(A) \otimes !(!(A) \multimap \mathbf{0}) \multimap \mathbf{0})}\ !$$

01-ENC encoding (70ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash A}}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap !(\mathbf{0}) :\ !(\mathbf{0}) \vdash \mathbf{0}}\ \star}{A, !(A) \multimap !(\mathbf{0}) :\ !(A) \multimap !(\mathbf{0}) \vdash \mathbf{0}}\ \multimap}{\dfrac{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash \mathbf{0}}{\cdot :\ \cdot \vdash !(!(A) \,\&\, !(!(A) \multimap !(\mathbf{0}))) \multimap \mathbf{0}}\ \star}\ D_C}{\cdot :\ \cdot \vdash !(!(!(A) \,\&\, !(!(A) \multimap !(\mathbf{0}))) \multimap \mathbf{0})}\ !$$

$\boxed{(45)\ \cdot \vdash \neg(A) \leftrightarrow (\neg A)}$

LJ (80ms)



MULTIPLICATIVE encoding (41ms)

Not provable

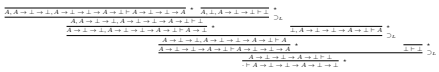
CALL-BY-NAME encoding (231ms)

CALL-BY-VALUE encoding (282ms)

01-ENC encoding (300ms)

$\boxed{(46)\ \cdot \vdash \neg\neg\neg\neg A \to A}$

LJ (66ms)



MULTIPLICATIVE encoding (41ms)

Not provable

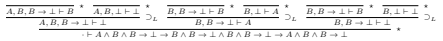
CALL-BY-NAME encoding (198ms)

CALL-BY-VALUE encoding (210ms)

01-ENC encoding (217ms)

$\boxed{(47)\ \cdot \vdash (A \wedge B \wedge \neg B) \leftrightarrow (B \wedge \neg B)}$

LJ (61ms)

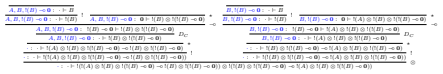


MULTIPLICATIVE encoding (42ms)

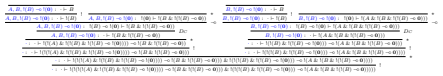
Not provable

CALL-BY-NAME encoding (121ms)

CALL-BY-VALUE encoding (140ms)

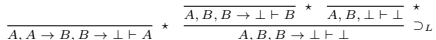


01-ENC encoding (132ms)



$\boxed{(48)\ \cdot \vdash A \to B \to \neg A \wedge \neg B}$

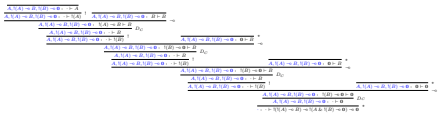
LJ (49ms)



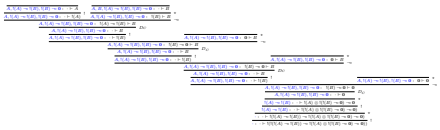
MULTIPLICATIVE encoding (48ms)

$$\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{\dfrac{}{\cdot :\ B \vdash B} \quad \dfrac{}{\cdot :\ \bot \vdash \bot}\ \multimap}{\cdot :\ A, A \multimap B, B \multimap \bot \vdash \bot}\ \multimap}{\cdot :\ \cdot \vdash A \multimap B \multimap A \otimes B \multimap \bot \multimap \bot}\ \star$$

CALL-BY-NAME encoding (143ms)



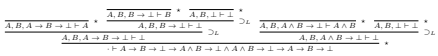
CALL-BY-VALUE encoding (167ms)



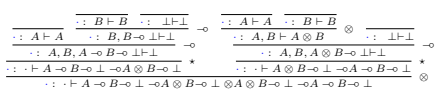
01-ENC encoding (169ms)

(49) $\cdot \vdash (A \to \neg B) \leftrightarrow (\neg A \wedge B)$

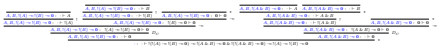
LJ (58ms)



MULTIPLICATIVE encoding (88ms)

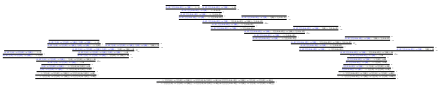


CALL-BY-NAME encoding (118ms)



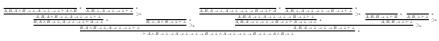
CALL-BY-VALUE encoding (281ms)

01-ENC encoding (318ms)

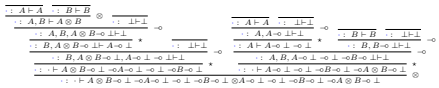


(50) $\cdot \vdash (\neg A \wedge B) \leftrightarrow (\neg\neg A \to \neg B)$

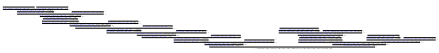
LJ (93ms)



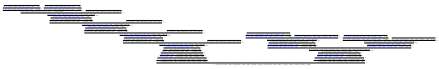
MULTIPLICATIVE encoding (134ms)



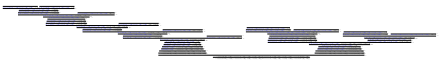
CALL-BY-NAME encoding (333ms)



CALL-BY-VALUE encoding (421ms)

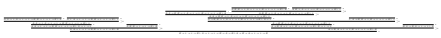


01-ENC encoding (470ms)

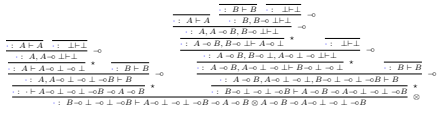


(51) $\neg\neg B \to B \vdash (\neg\neg A \to B) \leftrightarrow (A \to B)$

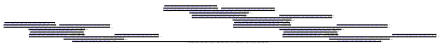
LJ (135ms)



MULTIPLICATIVE encoding (156ms)

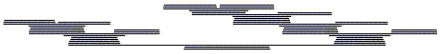


CALL-BY-NAME encoding (278ms)



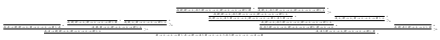
CALL-BY-VALUE encoding (414ms)

01-ENC encoding (449ms)

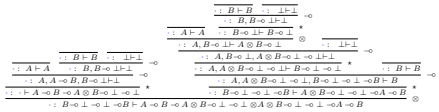


(52) $\neg\neg B \to B \vdash (A \to B) \leftrightarrow (\neg A \wedge \neg B)$

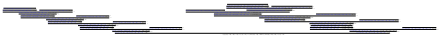
LJ (94ms)



MULTIPLICATIVE encoding (134ms)

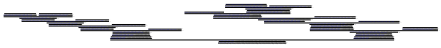


CALL-BY-NAME encoding (296ms)



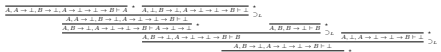
CALL-BY-VALUE encoding (457ms)

01-ENC encoding (489ms)



(53) $\cdot \vdash \neg\neg A \to B \to \neg A \wedge \neg B$

LJ (65ms)



MULTIPLICATIVE encoding (75ms)

$$\dfrac{\dfrac{\dfrac{}{\cdot :\ A \vdash A}\quad \dfrac{}{\cdot :\ \ \bot\vdash\bot}}{\cdot :\ A, A\multimap \bot\vdash\bot}\multimap \quad \dfrac{\dfrac{}{\cdot :\ B \vdash B}\quad \dfrac{}{\cdot :\ \ \bot\vdash\bot}}{\cdot :\ B, B\multimap \bot\vdash\bot}\multimap}{\cdot :\ A\vdash A\multimap \bot \multimap \bot}\star \qquad \multimap}{\cdot :\ A, B\multimap \bot, A\multimap \bot \multimap \bot \multimap B\vdash\bot}\multimap$$
$$\dfrac{}{\cdot :\ \cdot\vdash A\multimap \bot \multimap \bot \multimap B\multimap A\otimes B\multimap \bot \multimap \bot}\star$$

CALL-BY-NAME encoding (178ms)

CALL-BY-VALUE encoding (204ms)

01-ENC encoding (215ms)

(54) $\cdot \vdash A \wedge B \to \neg A \to \neg B$

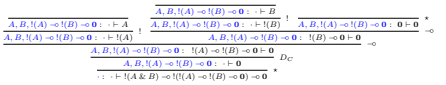
LJ (47ms)

$$\dfrac{\dfrac{}{A, B, A \to B \to \bot \vdash A}\star \quad \dfrac{\dfrac{}{A, B, B \to \bot \vdash B}\star \quad \dfrac{}{A, B, \bot \vdash \bot}\star}{A, B, B \to \bot \vdash \bot}\supset_L}{A, B, A \to B \to \bot \vdash \bot}\supset_L$$
$$\dfrac{}{\cdot \vdash A \wedge B \to A \to B \to \bot \to \bot}\star$$

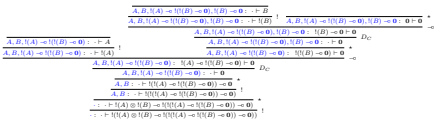
MULTIPLICATIVE encoding (48ms)

$$\dfrac{\dfrac{}{\cdot :\ A \vdash A}\quad \dfrac{\dfrac{}{\cdot :\ B \vdash B}\quad \dfrac{}{\cdot :\ \ \bot\vdash\bot}}{\cdot :\ A, B, B\multimap \bot\vdash\bot}\multimap}{\cdot :\ A, B, A\multimap B\multimap \bot\vdash\bot}\multimap$$
$$\dfrac{}{\cdot :\ \cdot\vdash A\otimes B\multimap A\multimap B\multimap \bot\multimap \bot}\star$$

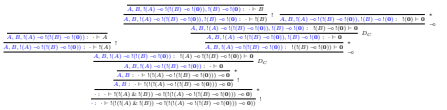
CALL-BY-NAME encoding (77ms)



CALL-BY-VALUE encoding (105ms)



01-ENC encoding (106ms)



(55) $\cdot \vdash A \wedge \neg B \to \neg A \to B$

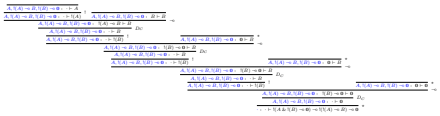
LJ (48ms)

$$\dfrac{\dfrac{}{A, A \to B, B \to \bot \vdash A}\,\star \quad \dfrac{\dfrac{}{A, B, B \to \bot \vdash B}\,\star \quad \dfrac{}{A, B, \bot \vdash \bot}\,\star}{A, B, B \to \bot \vdash \bot}\,\supset_L}{A, A \to B, B \to \bot \vdash \bot}\,\supset_L$$
$$\cdot \vdash A \wedge B \to \bot \to A \to B \to \bot \;\star$$

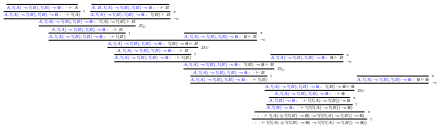
MULTIPLICATIVE encoding (47ms)

$$\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{\dfrac{}{\cdot :\ B \vdash B} \quad \dfrac{}{\cdot :\ \bot \vdash \bot}\,\multimap}{\cdot :\ B, B \multimap \bot \vdash \bot}\,\multimap}{\cdot :\ A, A \multimap B, B \multimap \bot \vdash \bot}\,\multimap$$
$$\cdot :\ \cdot \vdash A \otimes B \multimap \bot \multimap A \multimap B \multimap \bot \;\star$$

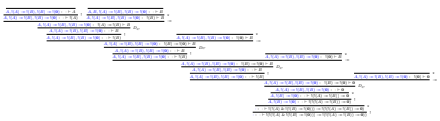
CALL-BY-NAME encoding (144ms)



CALL-BY-VALUE encoding (168ms)

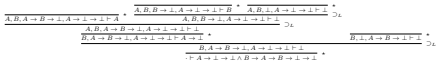


01-ENC encoding (168ms)



(56) $\cdot \vdash \neg\neg A \wedge B \to \neg A \to \neg B$

LJ (62ms)



MULTIPLICATIVE encoding (77ms)

$$\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{\dfrac{\dfrac{}{\cdot :\ B \vdash B} \quad \dfrac{}{\cdot :\ \bot \vdash \bot}\,\multimap}{\cdot :\ B, B \multimap \bot \vdash \bot}\,\multimap}{\cdot :\ A, B, A \multimap B \multimap \bot \vdash \bot}\,\multimap}{\cdot :\ B, A \multimap B \multimap \bot \vdash A \multimap \bot}\,\star \quad \dfrac{}{\cdot :\ \bot \vdash \bot}\,\multimap$$
$$\cdot :\ \cdot \vdash A \multimap \bot \multimap \bot \otimes B \multimap A \multimap B \multimap \bot \multimap \bot \;\star$$

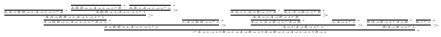
CALL-BY-NAME encoding (182ms)

CALL-BY-VALUE encoding (212ms)

01-ENC encoding (210ms)

(57) $\cdot \vdash (\neg\neg A \wedge \neg B) \leftrightarrow (\neg A \to B)$

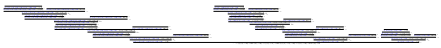
LJ (114ms)



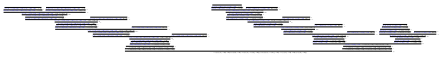
MULTIPLICATIVE encoding (58ms)

Not provable

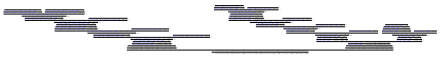
CALL-BY-NAME encoding (326ms)



CALL-BY-VALUE encoding (421ms)



01-ENC encoding (454ms)



(58) $\cdot \vdash (\neg A \to B) \leftrightarrow (\neg\neg A \wedge \neg B)$

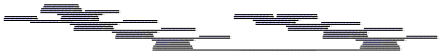
LJ (109ms)

MULTIPLICATIVE encoding (41ms)

Not provable

CALL-BY-NAME encoding (597ms)

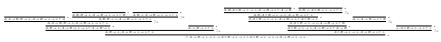
CALL-BY-VALUE encoding (481ms)



01-ENC encoding (501ms)

$(59)\ \cdot \vdash (\neg\neg A \to B) \leftrightarrow (\neg A \wedge \neg B)$

LJ (112ms)

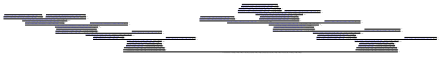


MULTIPLICATIVE encoding (57ms)

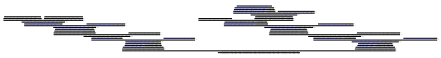
Not provable

CALL-BY-NAME encoding (594ms)

CALL-BY-VALUE encoding (468ms)

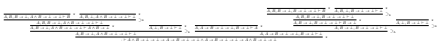


01-ENC encoding (498ms)

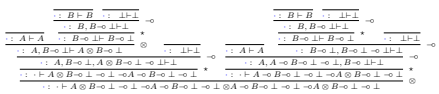


$(60)\ \cdot \vdash (\neg A \wedge \neg B) \leftrightarrow (A \to \neg\neg B)$

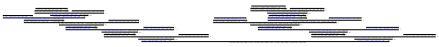
LJ (92ms)



MULTIPLICATIVE encoding (124ms)

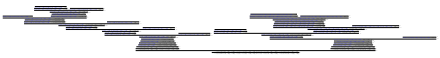


CALL-BY-NAME encoding (317ms)



CALL-BY-VALUE encoding (436ms)

01-ENC encoding (468ms)



(61) $\cdot \vdash (A \to \neg\neg B) \leftrightarrow (\neg\neg A \to \neg\neg B)$

LJ (169ms)

MULTIPLICATIVE encoding (163ms)

CALL-BY-NAME encoding (412ms)

CALL-BY-VALUE encoding (479ms)

01-ENC encoding (552ms)

3 Alternative Translations

(10) $\cdot : A \multimap \mathbf{0} \vdash A \multimap B$

encoding (55ms)

$$\dfrac{\dfrac{}{\cdot : A \vdash A} \quad \dfrac{}{\cdot : \mathbf{0} \vdash B}\,\star}{\dfrac{\cdot : A, A \multimap \mathbf{0} \vdash B}{\cdot : A \multimap \mathbf{0} \vdash A \multimap B}\,\star}\,\multimap$$

(11) $\cdot : A \vdash A \multimap \mathbf{0} \multimap B$

encoding (41ms)

$$\dfrac{\dfrac{}{\cdot : A \vdash A} \quad \dfrac{}{\cdot : \mathbf{0} \vdash B}\,\star}{\dfrac{\cdot : A, A \multimap \mathbf{0} \vdash B}{\cdot : A \vdash A \multimap \mathbf{0} \multimap B}\,\star}\,\multimap$$

(12) $\cdot : B \vdash !(A) \multimap B$

encoding (22ms)

$$\dfrac{\dfrac{}{A : B \vdash B}}{\cdot : B \vdash !(A) \multimap B}\,\star$$

(16) $\cdot : A \multimap B \otimes !(B \multimap A) \vdash A \multimap B$

encoding (35ms)

$$\dfrac{\dfrac{}{B \multimap A : A \vdash A} \quad \dfrac{}{B \multimap A : B \vdash B}}{\dfrac{B \multimap A : A, A \multimap B \vdash B}{\cdot : A \multimap B \otimes !(B \multimap A) \vdash A \multimap B}\,\star}\,\multimap$$

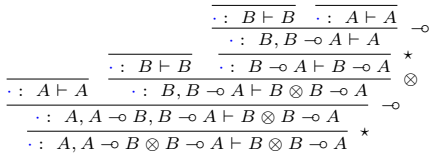
(17) $\cdot : !(A \multimap B) \otimes B \multimap A \vdash B \multimap A$

encoding (38ms)

$$\dfrac{\dfrac{}{A \multimap B : B \vdash B} \quad \dfrac{}{A \multimap B : A \vdash A}}{\dfrac{A \multimap B : B, B \multimap A \vdash A}{\cdot : !(A \multimap B) \otimes B \multimap A \vdash B \multimap A}\,\star}\,\multimap$$

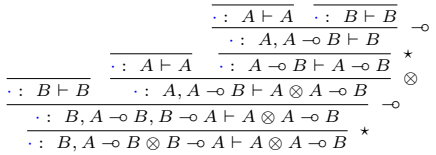
(18) $\cdot : A, A \multimap B \otimes B \multimap A \vdash B \otimes B \multimap A$

encoding (56ms)



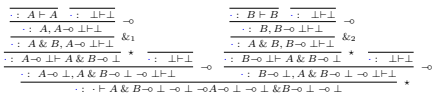
(19) $\cdot : \ B, A \multimap B \otimes B \multimap A \vdash A \otimes A \multimap B$

encoding (55ms)



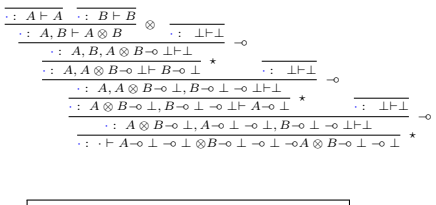
(26a) $\cdot : \ \cdot \vdash A \,\&\, B \multimap \bot \multimap \bot \multimap \bot \multimap A \multimap \bot \multimap \bot \,\&\, B \multimap \bot \multimap \bot$

encoding (107ms)



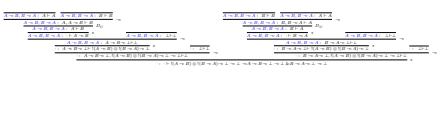
(26b) $\cdot : \ \cdot \vdash A \multimap \bot \multimap \bot \otimes B \multimap \bot \multimap \bot \multimap A \otimes B \multimap \bot \multimap \bot$

encoding (109ms)



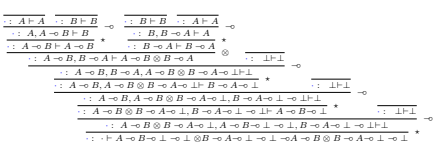
(27a) $\cdot : \ \cdot \vdash !(A \multimap B) \otimes !(B \multimap A) \multimap \bot \multimap \bot \multimap \bot \multimap A \multimap B \multimap \bot \multimap \bot \,\&\, B \multimap A \multimap \bot \multimap \bot$

encoding (155ms)



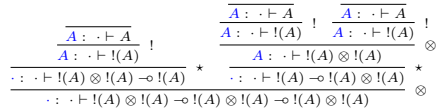
(27b) $\cdot : \ \cdot \vdash A \multimap B \multimap \bot \multimap \bot \otimes B \multimap A \multimap \bot \multimap \bot \multimap A \multimap B \otimes B \multimap A \multimap \bot \multimap \bot$

encoding (168ms)



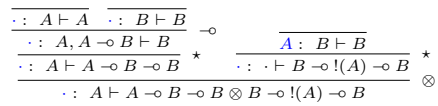
(35) $\cdot : \ \cdot \vdash !(A) \otimes !(A) \multimap !(A) \otimes !(A) \multimap !(A) \otimes !(A)$

encoding (59ms)



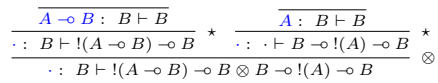
(36) $\cdot : \ A \vdash A \multimap B \multimap B \otimes B \multimap !(A) \multimap B$

encoding (62ms)



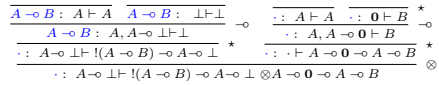
(37) $\cdot : \ B \vdash !(A \multimap B) \multimap B \otimes B \multimap !(A) \multimap B$

encoding (54ms)



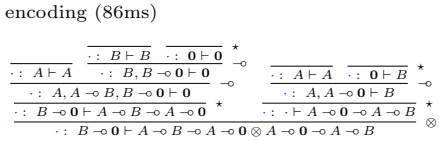
(38) $\cdot : \ A \multimap \bot \vdash !(A \multimap B) \multimap A \multimap \bot \otimes A \multimap \mathbf{0} \multimap A \multimap B$

encoding (74ms)



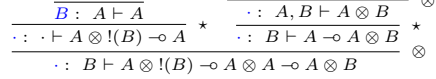
(39) $\cdot : \ B \multimap \mathbf{0} \vdash A \multimap B \multimap A \multimap \mathbf{0} \otimes A \multimap \mathbf{0} \multimap A \multimap B$

encoding (86ms)



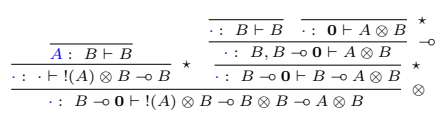
(40) $\cdot : \ B \vdash A \otimes !(B) \multimap A \otimes A \multimap A \otimes B$

encoding (55ms)



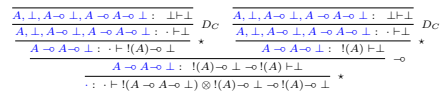
(41) $\cdot : \ B \multimap \mathbf{0} \vdash !(A) \otimes B \multimap B \otimes B \multimap A \otimes B$

encoding (69ms)



(45) $\cdot : \ \cdot \vdash !(A \multimap A \multimap \bot) \otimes !(A) \multimap \bot \multimap !(A) \multimap \bot$

encoding (65ms)

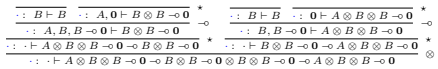


(46) $\cdot : \ \cdot \vdash !(!(A \multimap \bot \multimap \mathbf{0}) \multimap A \multimap \bot) \multimap \bot$

encoding (138ms)

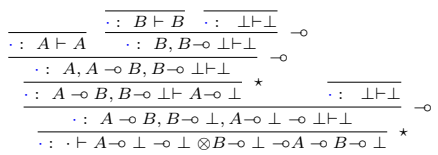
(47) $\cdot : \ \cdot \vdash A \otimes B \otimes B \multimap \mathbf{0} \multimap B \otimes B \multimap \mathbf{0} \otimes B \otimes B \multimap \mathbf{0} \multimap A \otimes B \otimes B \multimap \mathbf{0}$

encoding (83ms)



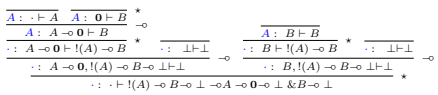
(57a) $\cdot : \ \cdot \vdash A \multimap \bot \multimap \bot \otimes B \multimap \bot \multimap A \multimap B \multimap \bot$

encoding (71ms)



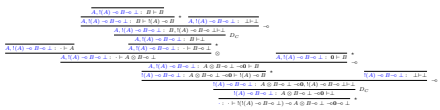
(57b) $\cdot : \ \cdot \vdash !(A) \multimap B \multimap \bot \multimap A \multimap \mathbf{0} \multimap \bot \ \& B \multimap \bot$

encoding (92ms)



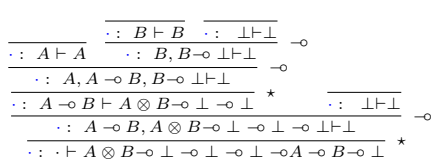
(58a) $\cdot : \ \cdot \vdash !(!(A) \multimap B \multimap \bot) \multimap A \otimes B \multimap \bot \multimap \mathbf{0} \multimap \bot$

encoding (162ms)



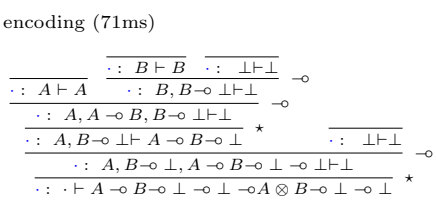
(58b) $\cdot : \ \cdot \vdash A \otimes B \multimap \bot \multimap \bot \multimap \bot \multimap A \multimap B \multimap \bot$

encoding (69ms)



(59a) $\cdot : \ \cdot \vdash A \multimap B \multimap \bot \multimap \bot \multimap A \otimes B \multimap \bot \multimap \bot$

encoding (71ms)



(59b) $\cdot : \ \cdot \vdash A \otimes B \multimap \bot \multimap \mathbf{0} \multimap !(!(A) \multimap B \multimap \bot) \multimap \bot$

encoding (163ms)