1 Translations

(1) $\cdot \vdash A \to A$

LJ (31ms)

$$\dfrac{}{\cdot \vdash A \to A}\ \star$$

Multiplicative translation (46ms)

$$\dfrac{\dfrac{\dfrac{}{\cdot :\ A \Rightarrow A}\ I}{\cdot :\ A \vdash A}\ D_R}{\cdot :\ \cdot \vdash A \multimap A}\ \star$$

Girard's Translation (43ms)

$$\dfrac{\dfrac{\dfrac{}{A :\ \cdot \Rightarrow A}\ I}{A :\ \cdot \vdash A}\ D_R}{\cdot :\ \cdot \vdash\ !(A) \multimap A}\ \star$$

Positive decoration (83ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A :\ \cdot \Rightarrow A}\ I}{A :\ \cdot \vdash A}\ D_R}{A :\ \cdot \Rightarrow\ !(A)}\ !}{A :\ \cdot \vdash\ !(A)}\ D_R}{\cdot :\ \cdot \vdash\ !(A) \multimap\ !(A)}\ \star}{\cdot :\ \cdot \Rightarrow\ !(!(A) \multimap\ !(A))}\ !}{\cdot :\ \cdot \vdash\ !(!(A) \multimap\ !(A))}\ D_R$$

0/1 focused decoration (59ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A :\ \cdot \Rightarrow A}\ I}{A :\ \cdot \vdash A}\ D_R}{\cdot :\ \cdot \vdash\ !(A) \multimap A}\ \star}{\cdot :\ \cdot \Rightarrow\ !(!(A) \multimap A)}\ !}{\cdot :\ \cdot \vdash\ !(!(A) \multimap A)}\ D_R$$

(2) $A \to B, B \to C \vdash A \to C$

LJ (48ms)

$$\dfrac{\dfrac{\dfrac{}{A, A \to B, B \to C \vdash A}\ \star \quad \dfrac{\dfrac{}{A, B, B \to C \vdash B}\ \star \quad \dfrac{}{A, B, C \vdash C}\ \star}{A, B, B \to C \vdash C}\ \supset_L}{A, A \to B, B \to C \vdash C}\ \supset_L}{A \to B, B \to C \vdash A \to C}\ \star$$

Multiplicative translation (111ms)

$$\dfrac{\dfrac{\dfrac{}{\cdot :\ A \Rightarrow A}\ I \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{\cdot :\ B \Rightarrow B}\ I \qquad \dfrac{\dfrac{\dfrac{}{\cdot :\ C \Rightarrow C}\ I}{\cdot :\ C \vdash C}\ D_R}{\cdot :\ \cdot\ [C] \Leftarrow C}\ R_L}{\cdot :\ B\ [B \multimap C] \Leftarrow C}\ \multimap}{\cdot :\ B, B \multimap C \vdash C}\ D_L \qquad \dfrac{\dfrac{}{\cdot :\ B \multimap C\ [B] \Leftarrow C}\ R_L}{\cdot :\ A, B \multimap C\ [A \multimap B] \Leftarrow C}\ \multimap}{\cdot :\ A, A \multimap B, B \multimap C \vdash C}\ D_L}{\cdot :\ A \multimap B, B \multimap C \vdash A \multimap C}\ \star$$

Girard's Translation (168ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap B, !(B) \multimap C :\ \cdot \Rightarrow A}\ I}{A, !(A) \multimap B, !(B) \multimap C :\ \cdot \vdash A}\ D_R}{A, !(A) \multimap B, !(B) \multimap C :\ \cdot \Rightarrow\ !(A)}\ ! \qquad \dfrac{\dfrac{}{A, !(A) \multimap B, !(B) \multimap C :\ B \Rightarrow B}\ I}{A, !(A) \multimap B, !(B) \multimap C :\ B \vdash B}\ D_R}{A, !(A) \multimap B, !(B) \multimap C :\ \cdot\ [B] \Leftarrow B}\ R_L}{A, !(A) \multimap B, !(B) \multimap C :\ \cdot\ [!(A) \multimap B] \Leftarrow B}\ \multimap}{A, !(A) \multimap B, !(B) \multimap C :\ \cdot \vdash B}\ D_C}{A, !(A) \multimap B, !(B) \multimap C :\ \cdot \Rightarrow\ !(B)}\ ! \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap B, !(B) \multimap C :\ C \Rightarrow C}\ I}{A, !(A) \multimap B, !(B) \multimap C :\ C \vdash C}\ D_R}{A, !(A) \multimap B, !(B) \multimap C :\ \cdot\ [C] \Leftarrow C}\ R_L}{A, !(A) \multimap B, !(B) \multimap C :\ \cdot\ [!(B) \multimap C] \Leftarrow C}\ \multimap}{A, !(A) \multimap B, !(B) \multimap C :\ \cdot \vdash C}\ D_C}{\cdot :\ !(!(A) \multimap B), !(!(B) \multimap C) \vdash\ !(A) \multimap C}\ \star$$

Positive decoration (226ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, B, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \Rightarrow B}\ I}{A, B, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash B}\ D_R}{A, B, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \Rightarrow\ !(B)}\ ! \qquad \dfrac{\dfrac{\dfrac{}{A, B, C, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \Rightarrow C}\ I}{A, B, C, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash C}\ D_R}{A, B, !(A) \multimap !(B), !(B) \multimap !(C) :\ !(C) \vdash C}\ \star}{A, B, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot\ [!(C)] \Leftarrow C}\ R_L}{A, B, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot\ [!(B) \multimap !(C)] \Leftarrow C}\ \multimap}{A, B, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash C}\ D_C}{A, B, !(A) \multimap !(B), !(B) \multimap !(C) :\ !(B) \vdash C}\ \star}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot\ [!(B)] \Leftarrow C}\ R_L}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot\ [!(A) \multimap !(B)] \Leftarrow C}\ \multimap}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash C}\ D_C}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \Rightarrow\ !(C)}\ !}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash\ !(C)}\ D_R}{!(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash\ !(A) \multimap\ !(C)}\ \star}{!(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \Rightarrow\ !(!(A) \multimap\ !(C))}\ !}{!(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash\ !(!(A) \multimap\ !(C))}\ D_R}{\cdot :\ !(!(A) \multimap !(B)), !(!(B) \multimap !(C)) \vdash\ !(!(A) \multimap !(C))}\ \star$$

0/1 focused decoration (185ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, B, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \Rightarrow B}\ I}{A, B, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash B}\ D_R}{A, B, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \Rightarrow\ !(B)}\ ! \qquad \dfrac{\dfrac{\dfrac{}{A, B, C, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \Rightarrow C}\ I}{A, B, C, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash C}\ D_R}{A, B, !(A) \multimap !(B), !(B) \multimap !(C) :\ !(C) \vdash C}\ \star}{A, B, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot\ [!(C)] \Leftarrow C}\ R_L}{A, B, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot\ [!(B) \multimap !(C)] \Leftarrow C}\ \multimap}{A, B, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash C}\ D_C}{A, B, !(A) \multimap !(B), !(B) \multimap !(C) :\ !(B) \vdash C}\ \star}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot\ [!(B)] \Leftarrow C}\ R_L}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot\ [!(A) \multimap !(B)] \Leftarrow C}\ \multimap}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash C}\ D_C}{!(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash\ !(A) \multimap C}\ \star}{!(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \Rightarrow\ !(!(A) \multimap C)}\ !}{!(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash\ !(!(A) \multimap C)}\ D_R}{\cdot :\ !(!(A) \multimap !(B)), !(!(B) \multimap !(C)) \vdash\ !(!(A) \multimap C)}\ \star$$

(3) $A \to (B \to C) \vdash B \to (A \to C)$

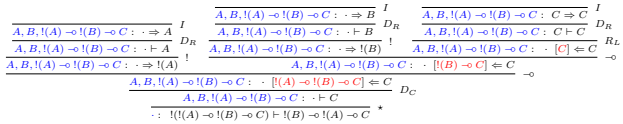
LJ (49ms)

$$\dfrac{\dfrac{\dfrac{}{A, B, A \to B \to C \vdash A}\star \quad \dfrac{\dfrac{}{A, B, B \to C \vdash B}\star \quad \dfrac{}{A, B, C \vdash C}\star}{A, B, B \to C \vdash C}\supset_L}{A, B, A \to B \to C \vdash C}\supset_L}{A \to B \to C \vdash B \to A \to C}\star$$

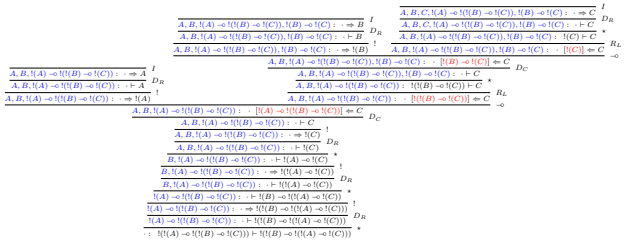
Multiplicative translation (94ms)

$$\dfrac{\dfrac{}{\cdot :\ A \Rightarrow A}I \quad \dfrac{\dfrac{}{\cdot :\ B \Rightarrow B}I \quad \dfrac{\dfrac{\dfrac{}{\cdot :\ C \Rightarrow C}I}{\cdot :\ C \vdash C}D_R}{\cdot :\ \cdot\ [C] \Leftarrow C}R_L}{\cdot :\ B\ [B \multimap C] \Leftarrow C}\multimap}{\cdot :\ A, B\ [A \multimap B \multimap C] \Leftarrow C}\multimap}{\cdot :\ A, B, A \multimap B \multimap C \vdash C}D_L}{\cdot :\ A \multimap B \multimap C \vdash B \multimap A \multimap C}\star$$

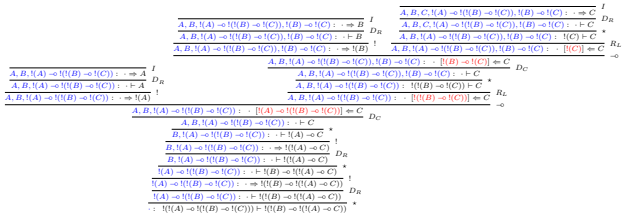
Girard's Translation (128ms)



Positive decoration (208ms)



0/1 focused decoration (189ms)



(4) $A \to (B \to C) \vdash (A \wedge B) \to C$

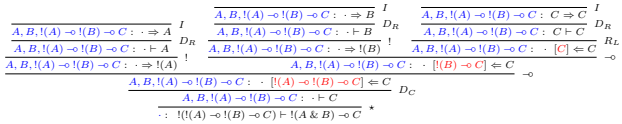
LJ (48ms)

$$\dfrac{\dfrac{\dfrac{}{A, B, A \to B \to C \vdash A}\star \quad \dfrac{\dfrac{}{A, B, B \to C \vdash B}\star \quad \dfrac{}{A, B, C \vdash C}\star}{A, B, B \to C \vdash C}\supset_L}{A, B, A \to B \to C \vdash C}\supset_L}{A \to B \to C \vdash A \wedge B \to C}\star$$

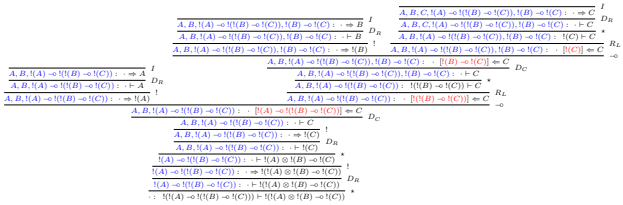
Multiplicative translation (97ms)

$$\dfrac{\dfrac{}{\cdot :\ A \Rightarrow A}I \quad \dfrac{\dfrac{}{\cdot :\ B \Rightarrow B}I \quad \dfrac{\dfrac{\dfrac{}{\cdot :\ C \Rightarrow C}I}{\cdot :\ C \vdash C}D_R}{\cdot :\ \cdot\ [C] \Leftarrow C}R_L}{\cdot :\ B\ [B \multimap C] \Leftarrow C}\multimap}{\cdot :\ A, B\ [A \multimap B \multimap C] \Leftarrow C}\multimap}{\cdot :\ A, B, A \multimap B \multimap C \vdash C}D_L}{\cdot :\ A \multimap B \multimap C \vdash A \otimes B \multimap C}\star$$

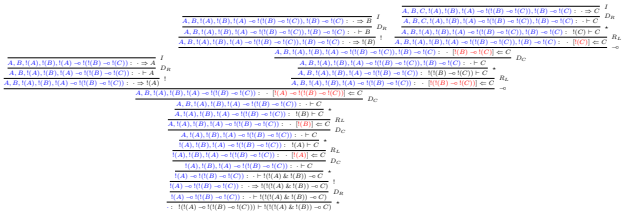
Girard's Translation (326ms)



Positive decoration (219ms)



0/1 focused decoration (313ms)



(5) $A \wedge B \to C \vdash A \to (B \to C)$

LJ (38ms)

$$\dfrac{\dfrac{}{A, B, A \wedge B \to C \vdash A \wedge B}\ \star \quad \dfrac{}{A, B, C \vdash C}\ \star}{\dfrac{A, B, A \wedge B \to C \vdash C}{A \wedge B \to C \vdash A \to B \to C}\ \star}\ \supset L$$

Multiplicative translation (98ms)

$$\dfrac{\dfrac{}{\cdot :\ A \Rightarrow A}\ I \quad \dfrac{}{\cdot :\ B \Rightarrow B}\ I}{\dfrac{\cdot :\ A, B \Rightarrow A \otimes B}{\dfrac{\cdot :\ A, B\ [A \otimes B \multimap C] \Leftarrow C}{\dfrac{\cdot :\ A, B, A \otimes B \multimap C \vdash C}{\cdot :\ A \otimes B \multimap C \vdash A \multimap B \multimap C}\ \star}\ D_L}\ R_L}\ \otimes \quad \dfrac{\dfrac{\dfrac{}{\cdot :\ C \Rightarrow C}\ I}{\cdot :\ C \vdash C}\ D_R}{\cdot :\ \cdot\ [C] \Leftarrow C}\ R_L}\ \multimap$$

Girard's Translation (108ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, B, !(A \,\&\, B) \multimap C :\ \cdot \Rightarrow A}\ I}{A, B, !(A \,\&\, B) \multimap C :\ \cdot \vdash A}\ D_R \quad \dfrac{\dfrac{}{A, B, !(A \,\&\, B) \multimap C :\ \cdot \Rightarrow B}\ I}{A, B, !(A \,\&\, B) \multimap C :\ \cdot \vdash B}\ D_R}{A, B, !(A \,\&\, B) \multimap C :\ \cdot \vdash A \,\&\, B}\ \star}{A, B, !(A \,\&\, B) \multimap C :\ \cdot \Rightarrow !(A \,\&\, B)}\ !}{A, B, !(A \,\&\, B) \multimap C :\ \cdot\ [!(A \,\&\, B) \multimap C] \Leftarrow C}\ D_C \quad \dfrac{\dfrac{\dfrac{}{A, B, !(A \,\&\, B) \multimap C :\ C \Rightarrow C}\ I}{A, B, !(A \,\&\, B) \multimap C :\ C \vdash C}\ D_R}{A, B, !(A \,\&\, B) \multimap C :\ \cdot\ [C] \Leftarrow C}\ R_L}{\cdot :\ !(!(A \,\&\, B) \multimap C) \vdash !(A) \multimap !(B) \multimap C}\ \star}\ \multimap$$

Positive decoration (182ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, B, !(A) \otimes !(B) \multimap !(C) :\ \cdot \Rightarrow A}\ I}{A, B, !(A) \otimes !(B) \multimap !(C) :\ \cdot \vdash A}\ D_R}{A, B, !(A) \otimes !(B) \multimap !(C) :\ \cdot \Rightarrow !(A)}\ !}{A, B, !(A) \otimes !(B) \multimap !(C) :\ \cdot \Rightarrow !(A) \otimes !(B)}\ \otimes \quad \dfrac{\dfrac{\dfrac{}{A, B, !(A) \otimes !(B) \multimap !(C) :\ \cdot \Rightarrow B}\ I}{A, B, !(A) \otimes !(B) \multimap !(C) :\ \cdot \vdash B}\ D_R}{A, B, !(A) \otimes !(B) \multimap !(C) :\ \cdot \vdash !(A) \otimes !(B)}\ !}{A, B, !(A) \otimes !(B) \multimap !(C) :\ \cdot\ [!(A) \otimes !(B) \multimap !(C)] \Leftarrow C}\ D_C \quad \dfrac{\dfrac{\dfrac{\dfrac{}{A, B, C, !(A) \otimes !(B) \multimap !(C) :\ \cdot \Rightarrow C}\ I}{A, B, C, !(A) \otimes !(B) \multimap !(C) :\ C \vdash C}\ D_R}{A, B, !(A) \otimes !(B) \multimap !(C) :\ !(C) \vdash C}\ \star}{A, B, !(A) \otimes !(B) \multimap !(C) :\ \cdot\ [!(C)] \Leftarrow C}\ R_L}{A, B, !(A) \otimes !(B) \multimap !(C) :\ \cdot \vdash C}\ !}{A, !(A) \otimes !(B) \multimap !(C) :\ \cdot \vdash !(B) \multimap !(C)}\ D_R}{A, !(A) \otimes !(B) \multimap !(C) :\ \cdot \Rightarrow !(!(B) \multimap !(C))}\ !}{!(A) \otimes !(B) \multimap !(C) :\ \cdot \vdash !(A) \multimap !(!(B) \multimap !(C))}\ \star}{!(A) \otimes !(B) \multimap !(C) :\ \cdot \Rightarrow !(!(A) \multimap !(!(B) \multimap !(C)))}\ !}{!(A) \otimes !(B) \multimap !(C) :\ \cdot \vdash !(A) \multimap !(!(B) \multimap !(C))}\ D_R}{\cdot :\ !(!(A) \otimes !(B) \multimap !(C)) \vdash !(!(A) \multimap !(!(B) \multimap !(C)))}\ \star}\ \multimap$$

0/1 focused decoration (197ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, B, !(!(A \,\&\, B)) \multimap !(C) :\ \cdot \Rightarrow A}\ I}{A, B, !(!(A \,\&\, B)) \multimap !(C) :\ \cdot \vdash A}\ D_R \quad \dfrac{\dfrac{}{A, B, !(!(A \,\&\, B)) \multimap !(C) :\ \cdot \Rightarrow B}\ I}{A, B, !(!(A \,\&\, B)) \multimap !(C) :\ \cdot \vdash B}\ D_R}{A, B, !(!(A \,\&\, B)) \multimap !(C) :\ \cdot \vdash A \,\&\, B}\ \star}{A, B, !(!(A \,\&\, B)) \multimap !(C) :\ \cdot \Rightarrow !(A \,\&\, B)}\ !}{A, B, !(!(A \,\&\, B)) \multimap !(C) :\ \cdot\ [!(!(A \,\&\, B)) \multimap !(C)] \Leftarrow C}\ D_C \quad \dfrac{\dfrac{\dfrac{\dfrac{}{A, B, C, !(!(A \,\&\, B)) \multimap !(C) :\ \cdot \Rightarrow C}\ I}{A, B, C, !(!(A \,\&\, B)) \multimap !(C) :\ C \vdash C}\ D_R}{A, B, !(!(A \,\&\, B)) \multimap !(C) :\ !(C) \vdash C}\ \star}{A, B, !(!(A \,\&\, B)) \multimap !(C) :\ \cdot\ [!(C)] \Leftarrow C}\ R_L}{A, B, !(!(A \,\&\, B)) \multimap !(C) :\ \cdot \vdash C}\ !}{A, !(!(A \,\&\, B)) \multimap !(C) :\ \cdot \vdash !(B) \multimap C}\ \star}{A, !(!(A \,\&\, B)) \multimap !(C) :\ \cdot \Rightarrow !(!(B) \multimap C)}\ !}{!(!(A \,\&\, B)) \multimap !(C) :\ \cdot \vdash !(A) \multimap !(!(B) \multimap C)}\ \star}{!(!(A \,\&\, B)) \multimap !(C) :\ \cdot \Rightarrow !(!(A) \multimap !(!(B) \multimap C))}\ !}{!(!(A \,\&\, B)) \multimap !(C) :\ \cdot \vdash !(!(A) \multimap !(!(B) \multimap C))}\ D_R}{\cdot :\ !(!(!(A \,\&\, B)) \multimap !(C)) \vdash !(!(A) \multimap !(!(B) \multimap C))}\ \star}\ \multimap$$

(6) $A \to B \vdash (B \to C) \to (A \to C)$

LJ (48ms)

$$\dfrac{\dfrac{}{A, A \to B, B \to C \vdash A}\ \star \quad \dfrac{\dfrac{}{A, B, B \to C \vdash B}\ \star \quad \dfrac{}{A, B, C \vdash C}\ \star}{A, B, B \to C \vdash C}\ \supset L}{\dfrac{A, A \to B, B \to C \vdash C}{A \to B \vdash B \to C \to A \to C}\ \star}\ \supset L$$

Multiplicative translation (106ms)

$$\dfrac{\dfrac{}{\cdot :\ A \Rightarrow A}\ I \quad \dfrac{\dfrac{\dfrac{\dfrac{}{\cdot :\ B \Rightarrow B}\ I}{\cdot :\ B \vdash B}\ D_R \quad \dfrac{\dfrac{}{\cdot :\ C \Rightarrow C}\ I}{\cdot :\ C \vdash C}\ D_R}{\cdot :\ \cdot\ [C] \Leftarrow C}\ R_L}{\cdot :\ B\ [B \multimap C] \Leftarrow C}\ \multimap}{\dfrac{\cdot :\ B, B \multimap C \vdash C}{\cdot :\ B \multimap C\ [B] \Leftarrow C}\ R_L}\ D_L}{\dfrac{\cdot :\ A, B \multimap C\ [A \multimap B] \Leftarrow C}{\cdot :\ A, A \multimap B, B \multimap C \vdash C}\ D_L}{\cdot :\ A \multimap B \vdash B \multimap C \multimap A \multimap C}\ \star}\ \multimap$$

Girard's Translation (157ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap B, !(B) \multimap C :\ \cdot \Rightarrow A}\ I}{A, !(A) \multimap B, !(B) \multimap C :\ \cdot \vdash A}\ D_R \quad \dfrac{\dfrac{}{A, !(A) \multimap B, !(B) \multimap C :\ B \Rightarrow B}\ I}{A, !(A) \multimap B, !(B) \multimap C :\ B \vdash B}\ D_R}{A, !(A) \multimap B, !(B) \multimap C :\ \cdot \Rightarrow !(A)}\ !}{A, !(A) \multimap B, !(B) \multimap C :\ \cdot\ [!(A) \multimap B] \Leftarrow B}\ D_C \quad \dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap B, !(B) \multimap C :\ C \Rightarrow C}\ I}{A, !(A) \multimap B, !(B) \multimap C :\ C \vdash C}\ D_R}{A, !(A) \multimap B, !(B) \multimap C :\ \cdot\ [C] \Leftarrow C}\ R_L}{A, !(A) \multimap B, !(B) \multimap C :\ \cdot \Rightarrow !(B)}\ !}{A, !(A) \multimap B, !(B) \multimap C :\ \cdot\ [!(B) \multimap C] \Leftarrow C}\ D_C}{A, !(A) \multimap B, !(B) \multimap C :\ \cdot \vdash C}\ !}{\cdot :\ !(!(A) \multimap B) \vdash !(!(B) \multimap C) \multimap !(A) \multimap C}\ \star}\ \multimap$$

Positive decoration (252ms)

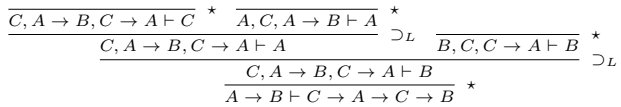
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \Rightarrow A}\ I}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash A}\ D_R}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \Rightarrow !(A)}\ !}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot\ [!(A) \multimap !(B)] \Leftarrow C}\ D_C \quad \dfrac{\dfrac{\dfrac{\dfrac{}{A, B, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \Rightarrow B}\ I}{A, B, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash B}\ D_R}{A, B, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \Rightarrow !(B)}\ !}{A, B, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot\ [!(B) \multimap !(C)] \Leftarrow C}\ D_C \quad \dfrac{\dfrac{\dfrac{\dfrac{}{A, B, C, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \Rightarrow C}\ I}{A, B, C, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash C}\ D_R}{A, B, !(A) \multimap !(B), !(B) \multimap !(C) :\ !(C) \vdash C}\ \star}{A, B, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot\ [!(C)] \Leftarrow C}\ R_L}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash C}\ !}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash !(A)}\ D_R}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \Rightarrow !(!(A))}\ !}{!(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash !(A) \multimap !(!(A))}\ \star}{!(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \Rightarrow !(!(A) \multimap !(!(A)))}\ !}{!(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash !(!(A) \multimap !(!(A)))}\ D_R}{!(A) \multimap !(B) :\ \cdot \vdash !(!(B) \multimap !(C)) \multimap !(!(A) \multimap !(!(A)))}\ \star}{!(A) \multimap !(B) :\ \cdot \Rightarrow !(!(!(B) \multimap !(C)) \multimap !(!(A) \multimap !(!(A))))}\ !}{!(A) \multimap !(B) :\ \cdot \vdash !(!(!(B) \multimap !(C)) \multimap !(!(A) \multimap !(!(A))))}\ D_R}{\cdot :\ !(!(A) \multimap !(B)) \vdash !(!(!(B) \multimap !(C)) \multimap !(!(A) \multimap !(!(A))))}\ \star}\ \multimap$$

0/1 focused decoration (214ms)

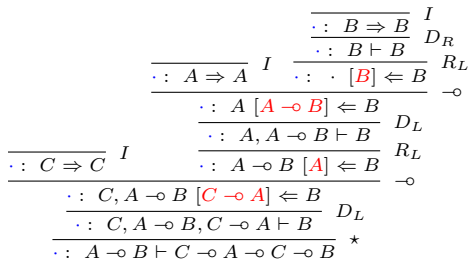
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, B, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \Rightarrow A}\ I}{A, B, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash A}\ D_R}{A, B, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \Rightarrow !(A)}\ !}{A, B, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot\ [!(A) \multimap !(B)] \Leftarrow C}\ D_C \quad \dfrac{\dfrac{\dfrac{\dfrac{}{A, B, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \Rightarrow B}\ I}{A, B, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash B}\ D_R}{A, B, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \Rightarrow !(B)}\ !}{A, B, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot\ [!(B) \multimap !(C)] \Leftarrow C}\ D_C \quad \dfrac{\dfrac{\dfrac{\dfrac{}{A, B, C, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \Rightarrow C}\ I}{A, B, C, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash C}\ D_R}{A, B, !(A) \multimap !(B), !(B) \multimap !(C) :\ !(C) \vdash C}\ \star}{A, B, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot\ [!(C)] \Leftarrow C}\ R_L}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash C}\ !}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash !(A) \multimap C}\ \star}{!(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \Rightarrow !(!(A) \multimap C)}\ !}{!(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash !(!(A) \multimap C)}\ D_R}{!(A) \multimap !(B) :\ \cdot \vdash !(!(B) \multimap !(C)) \multimap !(!(A) \multimap C)}\ \star}{!(A) \multimap !(B) :\ \cdot \Rightarrow !(!(!(B) \multimap !(C)) \multimap !(!(A) \multimap C))}\ !}{!(A) \multimap !(B) :\ \cdot \vdash !(!(!(B) \multimap !(C)) \multimap !(!(A) \multimap C))}\ D_R}{\cdot :\ !(!(A) \multimap !(B)) \vdash !(!(!(B) \multimap !(C)) \multimap !(!(A) \multimap C))}\ \star}\ \multimap$$

(7) $A \to B \vdash (C \to A) \to (C \to B)$

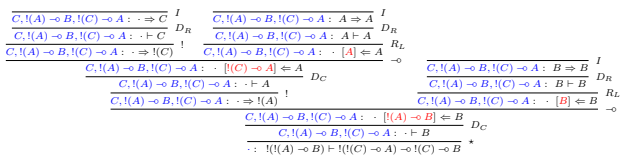
LJ (51ms)



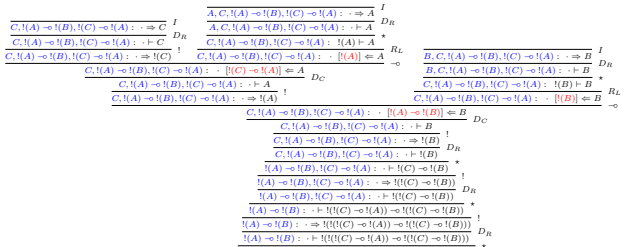
Multiplicative translation (108ms)



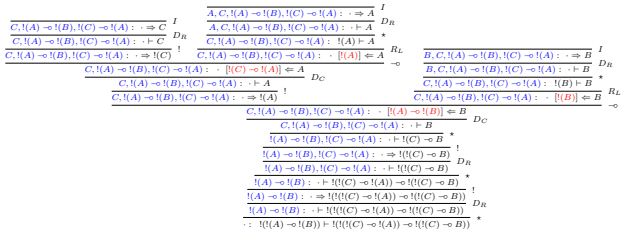
Girard's Translation (156ms)



Positive decoration (261ms)

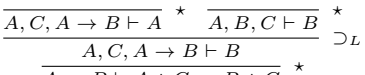


0/1 focused decoration (201ms)

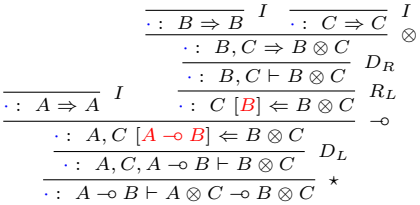


(8) $A \to B \vdash A \wedge C \to B \wedge C$

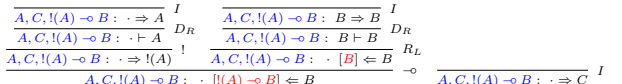
LJ (34ms)



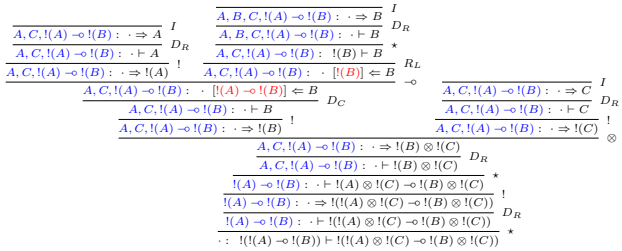
Multiplicative translation (91ms)



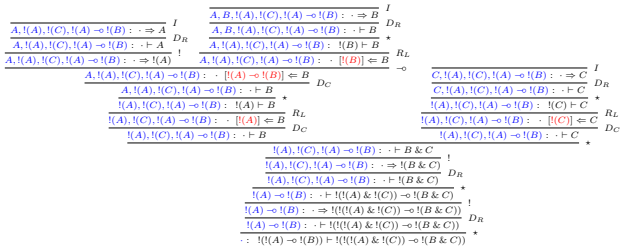
Girard's Translation (114ms)



Positive decoration (191ms)



0/1 focused decoration (271ms)



(9) $A \to B \vdash C \wedge A \to C \wedge B$

LJ (35ms)

$$\dfrac{\dfrac{}{A, C, A \to B \vdash A}\ \star \quad \dfrac{}{A, B, C \vdash B}\ \star}{\dfrac{A, C, A \to B \vdash B}{A \to B \vdash C \wedge A \to C \wedge B}\ \star}\ \supset_L$$

Multiplicative translation (89ms)

$$\dfrac{\dfrac{}{\cdot :\ A \Rightarrow A}\ I \qquad \dfrac{\dfrac{\dfrac{}{\cdot :\ C \Rightarrow C}\ I \quad \dfrac{}{\cdot :\ B \Rightarrow B}\ I}{\cdot :\ B, C \Rightarrow C \otimes B}\ \otimes}{\cdot :\ B, C \vdash C \otimes B}\ D_R}{\cdot :\ C\ [B] \Leftarrow C \otimes B}\ R_L \qquad \dfrac{}{\cdot :\ A\ [A \multimap B] \Leftarrow C \otimes B}\ -\!\circ$$

$$\dfrac{\cdot :\ A, C\ [A \multimap B] \Leftarrow C \otimes B}{\cdot :\ A, C, A \multimap B \vdash C \otimes B}\ D_L$$

$$\dfrac{\cdot :\ A, C, A \multimap B \vdash C \otimes B}{\cdot :\ A \multimap B \vdash C \otimes A \multimap C \otimes B}\ \star$$

Girard's Translation (111ms)

$$\dfrac{\dfrac{\dfrac{}{A, C, !(A) \multimap B :\ \cdot \Rightarrow C}\ I}{A, C, !(A) \multimap B :\ \cdot \vdash C}\ D_R \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{A, C, !(A) \multimap B :\ \cdot \Rightarrow A}\ I}{A, C, !(A) \multimap B :\ \cdot \vdash A}\ D_R \quad \dfrac{\dfrac{}{A, C, !(A) \multimap B :\ B \Rightarrow B}\ I}{A, C, !(A) \multimap B :\ B \vdash B}\ D_R}{A, C, !(A) \multimap B :\ \cdot \Rightarrow !(A)}\ !}{A, C, !(A) \multimap B :\ \cdot\ [B] \Leftarrow B}\ R_L \qquad \dfrac{}{A, C, !(A) \multimap B :\ \cdot\ [!(A) \multimap B] \Leftarrow B}\ -\!\circ$$

$$\dfrac{A, C, !(A) \multimap B :\ \cdot \Rightarrow C}{A, C, !(A) \multimap B :\ \cdot \vdash C}\ D_R \qquad \dfrac{A, C, !(A) \multimap B :\ \cdot\ [!(A) \multimap B] \Leftarrow B}{A, C, !(A) \multimap B :\ \cdot \vdash B}\ D_C$$

$$\dfrac{A, C, !(A) \multimap B :\ \cdot \vdash C \qquad A, C, !(A) \multimap B :\ \cdot \vdash B}{A, C, !(A) \multimap B :\ \cdot \Rightarrow !(C) \otimes !(B)}\ \otimes$$

$$\dfrac{A, C, !(A) \multimap B :\ \cdot \Rightarrow !(C) \otimes !(B)}{A, C, !(A) \multimap B :\ \cdot \vdash !(C) \otimes !(B)}\ D_R$$

$$\dfrac{A, C, !(A) \multimap B :\ \cdot \vdash !(C) \otimes !(B)}{!(A) \multimap B :\ \cdot \vdash !(C) \otimes !(A) \multimap !(C) \otimes !(B)}\ \star$$

$$\dfrac{!(A) \multimap B :\ \cdot \vdash !(C) \otimes !(A) \multimap !(C) \otimes !(B)}{!(A) \multimap B :\ \cdot \Rightarrow !(!(C) \otimes !(A) \multimap !(C) \otimes !(B))}\ !$$

$$\dfrac{!(A) \multimap B :\ \cdot \Rightarrow !(!(C) \otimes !(A) \multimap !(C) \otimes !(B))}{\cdot :\ !(!(A) \multimap !(B)) \vdash !(!(C) \otimes !(A) \multimap !(C) \otimes !(B))}\ D_R$$

$$\cdot :\ !(!(A) \multimap !(B)) \vdash !(!(C) \otimes !(A) \multimap !(C) \otimes !(B))\ \star$$

Positive decoration (173ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A, C, !(A) \multimap !(B) :\ \cdot \Rightarrow C}\ I}{A, C, !(A) \multimap !(B) :\ \cdot \vdash C}\ D_R \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, C, !(A) \multimap !(B) :\ \cdot \Rightarrow A}\ I}{A, C, !(A) \multimap !(B) :\ \cdot \vdash A}\ D_R \quad \dfrac{\dfrac{}{A, B, C, !(A) \multimap !(B) :\ \cdot \Rightarrow B}\ I}{A, B, C, !(A) \multimap !(B) :\ \cdot \vdash B}\ D_R}{A, C, !(A) \multimap !(B) :\ \cdot \Rightarrow !(A)}\ ! \quad \dfrac{}{A, C, !(A) \multimap !(B) :\ !(B) \vdash B}\ \star}{A, C, !(A) \multimap !(B) :\ \cdot\ [!(B)] \Leftarrow B}\ R_L \qquad \dfrac{}{A, C, !(A) \multimap !(B) :\ \cdot\ [!(A) \multimap !(B)] \Leftarrow B}\ -\!\circ$$

$$\dfrac{A, C, !(A) \multimap !(B) :\ \cdot \Rightarrow !(C)}{A, C, !(A) \multimap !(B) :\ \cdot \vdash !(C)}\ ! \qquad \dfrac{A, C, !(A) \multimap !(B) :\ \cdot \vdash B}{A, C, !(A) \multimap !(B) :\ \cdot \Rightarrow !(B)}\ !$$

$$\dfrac{A, C, !(A) \multimap !(B) :\ \cdot \Rightarrow !(C) \otimes !(B)}{A, C, !(A) \multimap !(B) :\ \cdot \vdash !(C) \otimes !(B)}\ D_R$$

$$\dfrac{A, C, !(A) \multimap !(B) :\ \cdot \vdash !(C) \otimes !(B)}{!(A) \multimap !(B) :\ \cdot \vdash !(C) \otimes !(A) \multimap !(C) \otimes !(B)}\ \star$$

$$\dfrac{!(A) \multimap !(B) :\ \cdot \vdash !(C) \otimes !(A) \multimap !(C) \otimes !(B)}{!(A) \multimap !(B) :\ \cdot \Rightarrow !(!(C) \otimes !(A) \multimap !(C) \otimes !(B))}\ !$$

$$\dfrac{!(A) \multimap !(B) :\ \cdot \Rightarrow !(!(C) \otimes !(A) \multimap !(C) \otimes !(B))}{\cdot :\ !(!(A) \multimap !(B)) \vdash !(!(C) \otimes !(A) \multimap !(C) \otimes !(B))}\ D_R$$

$$\cdot :\ !(!(A) \multimap !(B)) \vdash !(!(C) \otimes !(A) \multimap !(C) \otimes !(B))\ \star$$

0/1 focused decoration (257ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{C, !(A), !(C), !(A) \multimap !(B) :\ \cdot \Rightarrow C}\ I}{C, !(A), !(C), !(A) \multimap !(B) :\ \cdot \vdash C}\ D_R}{!(A), !(C), !(A) \multimap !(B) :\ !(C) \vdash C}\ \star}{!(A), !(C), !(A) \multimap !(B) :\ \cdot\ [!(C)] \Leftarrow C}\ R_L}{!(A), !(C), !(A) \multimap !(B) :\ \cdot \vdash C}\ D_C \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A), !(C), !(A) \multimap !(B) :\ \cdot \Rightarrow A}\ I}{A, !(A), !(C), !(A) \multimap !(B) :\ \cdot \vdash A}\ D_R}{A, !(A), !(C), !(A) \multimap !(B) :\ \cdot \Rightarrow !(A)}\ !}{A, !(A), !(C), !(A) \multimap !(B) :\ \cdot\ [!(A) \multimap !(B)] \Leftarrow B}\ R_L}{A, !(A), !(C), !(A) \multimap !(B) :\ \cdot \vdash B}\ D_C \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{A, B, !(A), !(C), !(A) \multimap !(B) :\ \cdot \Rightarrow B}\ I}{A, B, !(A), !(C), !(A) \multimap !(B) :\ \cdot \vdash B}\ D_R}{A, B, !(A), !(C), !(A) \multimap !(B) :\ !(B) \vdash B}\ \star}{A, !(A), !(C), !(A) \multimap !(B) :\ \cdot\ [!(B)] \Leftarrow B}\ R_L}{!(A), !(C), !(A) \multimap !(B) :\ \cdot \vdash B}\ D_C$$

$$\dfrac{!(A), !(C), !(A) \multimap !(B) :\ \cdot \vdash C \qquad !(A), !(C), !(A) \multimap !(B) :\ \cdot \vdash B}{!(A), !(C), !(A) \multimap !(B) :\ \cdot \vdash C \& B}\ \&$$

$$\dfrac{!(A), !(C), !(A) \multimap !(B) :\ \cdot \vdash C \& B}{!(A), !(C), !(A) \multimap !(B) :\ \cdot \Rightarrow !(C \& B)}\ !$$

$$\dfrac{!(A), !(C), !(A) \multimap !(B) :\ \cdot \Rightarrow !(C \& B)}{!(A), !(C), !(A) \multimap !(B) :\ \cdot \vdash !(C \& B)}\ D_R$$

$$\dfrac{!(A), !(C), !(A) \multimap !(B) :\ \cdot \vdash !(C \& B)}{!(A) \multimap !(B) :\ \cdot \vdash !(!(C) \& !(A)) \multimap !(C \& B)}\ \star$$

$$\dfrac{!(A) \multimap !(B) :\ \cdot \vdash !(!(C) \& !(A)) \multimap !(C \& B)}{!(A) \multimap !(B) :\ \cdot \Rightarrow !(!(!(C) \& !(A)) \multimap !(C \& B))}\ !$$

$$\dfrac{!(A) \multimap !(B) :\ \cdot \Rightarrow !(!(!(C) \& !(A)) \multimap !(C \& B))}{\cdot :\ !(!(A) \multimap !(B)) \vdash !(!(!(C) \& !(A)) \multimap !(C \& B))}\ D_R$$

$$\cdot :\ !(!(A) \multimap !(B)) \vdash !(!(!(C) \& !(A)) \multimap !(C \& B))\ \star$$

(10) $\neg A \vdash A \to B$

LJ (34ms)

$$\dfrac{\dfrac{}{A, A \to \bot \vdash A}\ \star \quad \dfrac{}{A, \bot \vdash B}\ \star}{\dfrac{A, A \to \bot \vdash B}{A \to \bot \vdash A \to B}\ \star}\ \supset_L$$

Multiplicative translation (19ms)

fail

Girard's Translation (79ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \Rightarrow A}\ I}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}\ D_R}{A, !(A) \multimap \mathbf{0} :\ \cdot \Rightarrow !(A)}\ ! \qquad \dfrac{}{A, !(A) \multimap \mathbf{0} :\ \mathbf{0} \vdash B}\ \star}{A, !(A) \multimap \mathbf{0} :\ \cdot\ [\mathbf{0}] \Leftarrow B}\ R_L \qquad -\!\circ$$

$$\dfrac{A, !(A) \multimap \mathbf{0} :\ \cdot\ [!(A) \multimap \mathbf{0}] \Leftarrow B}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash B}\ D_C$$

$$\cdot :\ !(!(A) \multimap \mathbf{0}) \vdash !(A) \multimap B\ \star$$

Positive decoration (98ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \Rightarrow A}\ I}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}\ D_R}{A, !(A) \multimap \mathbf{0} :\ \cdot \Rightarrow !(A)}\ ! \qquad \dfrac{}{A, !(A) \multimap \mathbf{0} :\ \mathbf{0} \vdash !(B)}\ \star}{A, !(A) \multimap \mathbf{0} :\ \cdot\ [\mathbf{0}] \Leftarrow !(B)}\ R_L \qquad -\!\circ$$

$$\dfrac{A, !(A) \multimap \mathbf{0} :\ \cdot\ [!(A) \multimap \mathbf{0}] \Leftarrow !(B)}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(B)}\ D_C$$

$$\dfrac{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(B)}{!(A) \multimap \mathbf{0} :\ \cdot \vdash !(A) \multimap !(B)}\ \star$$

$$\dfrac{!(A) \multimap \mathbf{0} :\ \cdot \vdash !(A) \multimap !(B)}{!(A) \multimap \mathbf{0} :\ \cdot \Rightarrow !(!(A) \multimap !(B))}\ !$$

$$\dfrac{!(A) \multimap \mathbf{0} :\ \cdot \Rightarrow !(!(A) \multimap !(B))}{!(A) \multimap \mathbf{0} :\ \cdot \vdash !(!(A) \multimap !(B))}\ D_R$$

$$\cdot :\ !(!(A) \multimap \mathbf{0}) \vdash !(!(A) \multimap !(B))\ \star$$

0/1 focused decoration (98ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \Rightarrow A}\ I}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash A}\ D_R}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \Rightarrow !(A)}\ ! \qquad \dfrac{}{A, !(A) \multimap !(\mathbf{0}) :\ !(\mathbf{0}) \vdash B}\ \star}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot\ [!(\mathbf{0})] \Leftarrow B}\ R_L \qquad -\!\circ$$

$$\dfrac{A, !(A) \multimap !(\mathbf{0}) :\ \cdot\ [!(A) \multimap !(\mathbf{0})] \Leftarrow B}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash B}\ D_C$$

$$\dfrac{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash B}{!(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(A) \multimap B}\ \star$$

$$\dfrac{!(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(A) \multimap B}{!(A) \multimap !(\mathbf{0}) :\ \cdot \Rightarrow !(!(A) \multimap B)}\ !$$

$$\dfrac{!(A) \multimap !(\mathbf{0}) :\ \cdot \Rightarrow !(!(A) \multimap B)}{!(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(!(A) \multimap B)}\ D_R$$

$$\cdot :\ !(!(A) \multimap !(\mathbf{0})) \vdash !(!(A) \multimap B)\ \star$$

(11) $A \vdash \neg A \to B$

LJ (34ms)

$$\dfrac{\dfrac{}{A, A \to \bot \vdash A}\ \star \quad \dfrac{}{A, \bot \vdash B}\ \star}{\dfrac{A, A \to \bot \vdash B}{A \vdash A \to \bot \to B}\ \star}\ \supset_L$$

Multiplicative translation (18ms)

fail

Girard's Translation (82ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \Rightarrow A}\ I}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}\ D_R}{A, !(A) \multimap \mathbf{0} :\ \cdot \Rightarrow\ !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap \mathbf{0} :\ \mathbf{0} \vdash B}\ \star}{A, !(A) \multimap \mathbf{0} :\ \cdot\ [\mathbf{0}] \Leftarrow B}\ R_L\ \multimap}{\dfrac{A, !(A) \multimap \mathbf{0} :\ \cdot\ [!(A) \multimap \mathbf{0}] \Leftarrow B}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash B}\ D_C}{\cdot :\ \ !(A) \vdash !(!(A) \multimap \mathbf{0}) \multimap B}\ \star$$

Positive decoration (97ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \Rightarrow A}\ I}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}\ D_R}{A, !(A) \multimap \mathbf{0} :\ \cdot \Rightarrow\ !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap \mathbf{0} :\ \mathbf{0} \vdash\ !(B)}\ \star}{A, !(A) \multimap \mathbf{0} :\ \cdot\ [\mathbf{0}] \Leftarrow\ !(B)}\ R_L\ \multimap}{\dfrac{A, !(A) \multimap \mathbf{0} :\ \cdot\ [!(A) \multimap \mathbf{0}] \Leftarrow\ !(B)}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash\ !(B)}\ D_C}{A :\ \cdot \vdash\ !(!(A) \multimap \mathbf{0}) \multimap\ !(B)}\ \star}{A :\ \cdot \Rightarrow\ !(!(!(A) \multimap \mathbf{0}) \multimap\ !(B))}\ !}{A :\ \cdot \vdash\ !(!(!(A) \multimap \mathbf{0}) \multimap\ !(B))}\ D_R}{\cdot :\ \ !(A) \vdash\ !(!(!(A) \multimap \mathbf{0}) \multimap\ !(B))}\ \star$$

0/1 focused decoration (97ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap\ !(\mathbf{0}) :\ \cdot \Rightarrow A}\ I}{A, !(A) \multimap\ !(\mathbf{0}) :\ \cdot \vdash A}\ D_R}{A, !(A) \multimap\ !(\mathbf{0}) :\ \cdot \Rightarrow\ !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap\ !(\mathbf{0}) :\ !(\mathbf{0}) \vdash B}\ \star}{A, !(A) \multimap\ !(\mathbf{0}) :\ \cdot\ [!(\mathbf{0})] \Leftarrow B}\ R_L\ \multimap}{\dfrac{A, !(A) \multimap\ !(\mathbf{0}) :\ \cdot\ [!(A) \multimap\ !(\mathbf{0})] \Leftarrow B}{A, !(A) \multimap\ !(\mathbf{0}) :\ \cdot \vdash B}\ D_C}{A :\ \cdot \vdash\ !(!(A) \multimap\ !(\mathbf{0})) \multimap B}\ \star}{A :\ \cdot \Rightarrow\ !(!(!(A) \multimap\ !(\mathbf{0})) \multimap B)}\ !}{A :\ \cdot \vdash\ !(!(!(A) \multimap\ !(\mathbf{0})) \multimap B)}\ D_R}{\cdot :\ \ !(A) \vdash\ !(!(!(A) \multimap\ !(\mathbf{0})) \multimap B)}\ \star$$

(12) $B \vdash A \to B$

LJ (19ms)

$$\dfrac{}{B \vdash A \to B}\ \star$$

Multiplicative translation (20ms)

fail

Girard's Translation (42ms)

$$\dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \Rightarrow B}\ I}{A, B :\ \cdot \vdash B}\ D_R}{\cdot :\ \ !(B) \vdash\ !(A) \multimap B}\ \star$$

Positive decoration (74ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \Rightarrow B}\ I}{A, B :\ \cdot \vdash B}\ D_R}{A, B :\ \cdot \Rightarrow\ !(B)}\ !}{A, B :\ \cdot \vdash\ !(B)}\ D_R}{B :\ \cdot \vdash\ !(A) \multimap\ !(B)}\ \star}{B :\ \cdot \Rightarrow\ !(!(A) \multimap\ !(B))}\ !}{B :\ \cdot \vdash\ !(!(A) \multimap\ !(B))}\ D_R}{\cdot :\ \ !(B) \vdash\ !(!(A) \multimap\ !(B))}\ \star$$

0/1 focused decoration (59ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \Rightarrow B}\ I}{A, B :\ \cdot \vdash B}\ D_R}{B :\ \cdot \vdash\ !(A) \multimap B}\ \star}{B :\ \cdot \Rightarrow\ !(!(A) \multimap B)}\ !}{B :\ \cdot \vdash\ !(!(A) \multimap B)}\ D_R}{\cdot :\ \ !(B) \vdash\ !(!(A) \multimap B)}\ \star$$

(13) $A \to B \vdash \neg B \to \neg A$

LJ (47ms)

$$\dfrac{\dfrac{}{A, A \to B, B \to \bot \vdash A}\ \star \quad \dfrac{\dfrac{}{A, B, B \to \bot \vdash B}\ \star \quad \dfrac{}{A, B, \bot \vdash \bot}\ \star}{A, B, B \to \bot \vdash \bot}\ \supset_L}{\dfrac{A, A \to B, B \to \bot \vdash \bot}{A \to B \vdash B \to \bot \to A \to \bot}\ \star}\ \supset_L$$

Multiplicative translation (89ms)

$$\dfrac{\dfrac{}{\cdot :\ A \Rightarrow A}\ I \quad \dfrac{\dfrac{\dfrac{\dfrac{}{\cdot :\ B \Rightarrow B}\ I \quad \dfrac{}{\cdot :\ \cdot\ [\bot] \Leftarrow \bot}\ \bot}{\cdot :\ B\ [B \multimap \bot] \Leftarrow \bot}\ \multimap}{\cdot :\ B, B \multimap \bot \vdash \bot}\ D_L}{\cdot :\ B \multimap \bot\ \ [B] \Leftarrow \bot}\ R_L}{\cdot :\ A, B \multimap \bot\ \ [A \multimap B] \Leftarrow \bot}\ \multimap}{\cdot :\ A, A \multimap B, B \multimap \bot \vdash \bot}\ D_L}{\cdot :\ A \multimap B \vdash B \multimap \bot \multimap A \multimap \bot}\ \star$$

Girard's Translation (137ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap B, !(B) \multimap \mathbf{0} :\ \cdot \Rightarrow A}\ I}{A, !(A) \multimap B, !(B) \multimap \mathbf{0} :\ \cdot \vdash A}\ D_R}{A, !(A) \multimap B, !(B) \multimap \mathbf{0} :\ \cdot \Rightarrow\ !(A)}\ !}{A, !(A) \multimap B, !(B) \multimap \mathbf{0} :\ \cdot\ [!(A) \multimap B] \Leftarrow B}\ D_C}{A, !(A) \multimap B, !(B) \multimap \mathbf{0} :\ \cdot \vdash B}\ !}{A, !(A) \multimap B, !(B) \multimap \mathbf{0} :\ \cdot\ [!(B) \multimap \mathbf{0}] \Leftarrow \mathbf{0}}\ D_C}{A, !(A) \multimap B, !(B) \multimap \mathbf{0} :\ \cdot \vdash \mathbf{0}}\ \star}{\cdot :\ !(!(A) \multimap B) \vdash !(!(B) \multimap \mathbf{0}) \multimap !(A) \multimap \mathbf{0}}\ \star$$

$$\dfrac{\dfrac{\dfrac{}{A, B, !(A) \multimap !(B), !(B) \multimap \mathbf{0} :\ \cdot \Rightarrow B}\ I}{A, B, !(A) \multimap !(B), !(B) \multimap \mathbf{0} :\ \cdot \vdash B}\ D_R}{A, B, !(A) \multimap !(B), !(B) \multimap \mathbf{0} :\ \cdot \Rightarrow\ !(B)}\ !$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, B, !(A) \multimap !(B), !(B) \multimap \mathbf{0} :\ \cdot \Rightarrow B}\ I}{A, B, !(A) \multimap !(B), !(B) \multimap \mathbf{0} :\ \cdot \vdash B}\ D_R}{A, B, !(A) \multimap !(B), !(B) \multimap \mathbf{0} :\ \cdot \Rightarrow\ !(B)}\ !}{A, B, !(A) \multimap !(B), !(B) \multimap \mathbf{0} :\ \cdot\ [!(B) \multimap \mathbf{0}] \Leftarrow \mathbf{0}}\ D_C}{A, B, !(A) \multimap !(B), !(B) \multimap \mathbf{0} :\ \cdot \vdash \mathbf{0}}\ \star}{A, !(A) \multimap !(B), !(B) \multimap \mathbf{0} :\ \cdot \vdash\ !(B)}\ \star}{A, !(A) \multimap !(B), !(B) \multimap \mathbf{0} :\ \cdot\ [!(B)] \Leftarrow \mathbf{0}}\ R_L}{A, !(A) \multimap !(B), !(B) \multimap \mathbf{0} :\ \cdot\ [!(A) \multimap !(B)] \Leftarrow \mathbf{0}}\ D_C}{A, !(A) \multimap !(B), !(B) \multimap \mathbf{0} :\ \cdot \vdash \mathbf{0}}\ \star}{!(A) \multimap !(B), !(B) \multimap \mathbf{0} :\ \cdot \vdash\ !(A) \multimap \mathbf{0}}\ \star}{!(A) \multimap !(B), !(B) \multimap \mathbf{0} :\ \cdot \Rightarrow\ !(!(A) \multimap \mathbf{0})}\ !}{!(A) \multimap !(B), !(B) \multimap \mathbf{0} :\ \cdot \vdash\ !(!(A) \multimap \mathbf{0})}\ D_R$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(B), !(B) \multimap \mathbf{0} :\ \cdot \Rightarrow A}\ I}{A, !(A) \multimap !(B), !(B) \multimap \mathbf{0} :\ \cdot \vdash A}\ D_R}{A, !(A) \multimap !(B), !(B) \multimap \mathbf{0} :\ \cdot \Rightarrow\ !(A)}\ !}{A, !(A) \multimap !(B), !(B) \multimap \mathbf{0} :\ \cdot\ [!(A) \multimap !(B)] \Leftarrow \mathbf{0}}\ D_C}{A, !(A) \multimap !(B), !(B) \multimap \mathbf{0} :\ \cdot \vdash \mathbf{0}}\ \star}{!(A) \multimap !(B), !(B) \multimap \mathbf{0} :\ \cdot \vdash\ !(A) \multimap \mathbf{0}}\ \star}{!(A) \multimap !(B), !(B) \multimap \mathbf{0} :\ \cdot \Rightarrow\ !(!(A) \multimap \mathbf{0})}\ !}{!(A) \multimap !(B), !(B) \multimap \mathbf{0} :\ \cdot \vdash\ !(!(A) \multimap \mathbf{0})}\ D_R}{!(A) \multimap !(B) :\ \cdot \vdash\ !(!(B) \multimap \mathbf{0}) \multimap !(!(A) \multimap \mathbf{0})}\ \star}{!(A) \multimap !(B) :\ \cdot \Rightarrow\ !(!(!(B) \multimap \mathbf{0}) \multimap !(!(A) \multimap \mathbf{0}))}\ !}{!(A) \multimap !(B) :\ \cdot \vdash\ !(!(!(B) \multimap \mathbf{0}) \multimap !(!(A) \multimap \mathbf{0}))}\ D_R}{\cdot :\ !(!(A) \multimap !(B)) \vdash !(!(!(B) \multimap \mathbf{0}) \multimap !(!(A) \multimap \mathbf{0}))}\ \star$$

0/1 focused decoration (180ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, B, !(A) \multimap !(B), !(B) \multimap !(\mathbf{0}) :\ \cdot \Rightarrow B}\ I}{A, B, !(A) \multimap !(B), !(B) \multimap !(\mathbf{0}) :\ \cdot \vdash B}\ D_R}{A, B, !(A) \multimap !(B), !(B) \multimap !(\mathbf{0}) :\ \cdot \Rightarrow\ !(B)}\ !}{A, B, !(A) \multimap !(B), !(B) \multimap !(\mathbf{0}) :\ \cdot\ [!(B) \multimap !(\mathbf{0})] \Leftarrow \mathbf{0}}\ D_C}{A, B, !(A) \multimap !(B), !(B) \multimap !(\mathbf{0}) :\ \cdot \vdash \mathbf{0}}\ \star}{A, !(A) \multimap !(B), !(B) \multimap !(\mathbf{0}) :\ \cdot \vdash\ !(B) \vdash \mathbf{0}}\ \star}{A, !(A) \multimap !(B), !(B) \multimap !(\mathbf{0}) :\ \cdot\ [!(B)] \Leftarrow \mathbf{0}}\ R_L}{A, !(A) \multimap !(B), !(B) \multimap !(\mathbf{0}) :\ \cdot\ [!(A) \multimap !(B)] \Leftarrow \mathbf{0}}\ D_C}{A, !(A) \multimap !(B), !(B) \multimap !(\mathbf{0}) :\ \cdot \vdash \mathbf{0}}\ \star}{!(A) \multimap !(B), !(B) \multimap !(\mathbf{0}) :\ \cdot \vdash\ !(A) \multimap \mathbf{0}}\ \star}{!(A) \multimap !(B), !(B) \multimap !(\mathbf{0}) :\ \cdot \Rightarrow\ !(!(A) \multimap \mathbf{0})}\ !}{!(A) \multimap !(B), !(B) \multimap !(\mathbf{0}) :\ \cdot \vdash\ !(!(A) \multimap \mathbf{0})}\ D_R$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(B), !(B) \multimap !(\mathbf{0}) :\ \cdot \Rightarrow A}\ I}{A, !(A) \multimap !(B), !(B) \multimap !(\mathbf{0}) :\ \cdot \vdash A}\ D_R}{A, !(A) \multimap !(B), !(B) \multimap !(\mathbf{0}) :\ \cdot \Rightarrow\ !(A)}\ !}{A, !(A) \multimap !(B), !(B) \multimap !(\mathbf{0}) :\ \cdot\ [!(A) \multimap !(B)] \Leftarrow \mathbf{0}}\ D_C}{A, !(A) \multimap !(B), !(B) \multimap !(\mathbf{0}) :\ \cdot \vdash \mathbf{0}}\ \star}{!(A) \multimap !(B), !(B) \multimap !(\mathbf{0}) :\ \cdot \vdash\ !(A) \multimap \mathbf{0}}\ \star}{!(A) \multimap !(B), !(B) \multimap !(\mathbf{0}) :\ \cdot \Rightarrow\ !(!(A) \multimap \mathbf{0})}\ !}{!(A) \multimap !(B), !(B) \multimap !(\mathbf{0}) :\ \cdot \vdash\ !(!(A) \multimap \mathbf{0})}\ D_R}{!(A) \multimap !(B) :\ \cdot \vdash\ !(!(B) \multimap !(\mathbf{0})) \multimap !(!(A) \multimap \mathbf{0})}\ \star}{!(A) \multimap !(B) :\ \cdot \Rightarrow\ !(!(!(B) \multimap !(\mathbf{0})) \multimap !(!(A) \multimap \mathbf{0}))}\ !}{!(A) \multimap !(B) :\ \cdot \vdash\ !(!(!(B) \multimap !(\mathbf{0})) \multimap !(!(A) \multimap \mathbf{0}))}\ D_R}{\cdot :\ !(!(A) \multimap !(B)) \vdash !(!(!(B) \multimap !(\mathbf{0})) \multimap !(!(A) \multimap \mathbf{0}))}\ \star$$

(14) $A \to \neg B \vdash (\neg\neg B) \to (\neg A)$

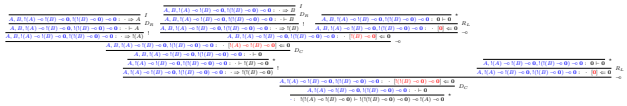
LJ (62ms)

$$\dfrac{\dfrac{}{A, A \to B \to \bot, B \to \bot \to \bot \vdash A}\ \star \quad \dfrac{\dfrac{\dfrac{}{A, B, B \to \bot, B \to \bot \to \bot \vdash B}\ \star \quad \dfrac{}{A, B, \bot, B \to \bot \to \bot \vdash \bot}\ \star}{A, B, B \to \bot, B \to \bot \to \bot \vdash \bot}\ \supset_L}{A, B, B \to \bot, B \to \bot \to \bot \vdash \bot}\ \supset_L \quad \dfrac{}{A, \bot, B \to \bot \vdash \bot}\ \star}{A, A \to B \to \bot, B \to \bot \to \bot \vdash \bot}\ \supset_L}{\dfrac{A, A \to B \to \bot, B \to \bot \to \bot \vdash \bot}{A \to B \to \bot \vdash B \to \bot \to \bot \to A \to \bot}\ \star}\ \supset_L$$

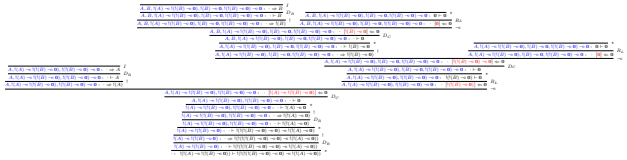
Multiplicative translation (96ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\cdot :\ A \Rightarrow A}\ I \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\cdot :\ B \Rightarrow B}\ I \quad \dfrac{}{\cdot :\ \cdot\ [\bot] \Leftarrow \bot}\ \bot}{\cdot :\ B\ [B \multimap \bot] \Leftarrow \bot}\ \multimap}{\cdot :\ A, B\ [A \multimap B \multimap \bot] \Leftarrow \bot}\ D_L}{\cdot :\ A, B, A \multimap B \multimap \bot \vdash \bot}\ D_L}{\cdot :\ A, A \multimap B \multimap \bot \vdash B \multimap \bot}\ \star}{\cdot :\ A, A \multimap B \multimap \bot \Rightarrow B \multimap \bot}\ R_R \quad \dfrac{}{\cdot :\ \cdot\ [\bot] \Leftarrow \bot}\ \bot}{\cdot :\ A, A \multimap B \multimap \bot\ \ [B \multimap \bot \multimap \bot] \Leftarrow \bot}\ \multimap}{\cdot :\ A, A \multimap B \multimap \bot, B \multimap \bot \multimap \bot \vdash \bot}\ D_L}{\cdot :\ A \multimap B \multimap \bot \vdash B \multimap \bot \multimap \bot \multimap A \multimap \bot}\ \star$$

Girard's Translation (186ms)



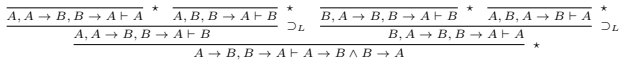
Positive decoration (297ms)



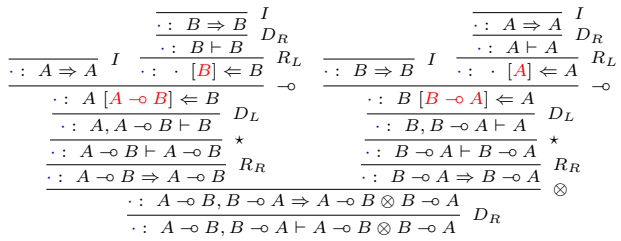
0/1 focused decoration (757ms)

(15) $A \to B, B \to A \vdash A \leftrightarrow B$

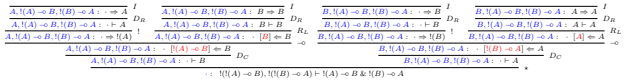
LJ (48ms)



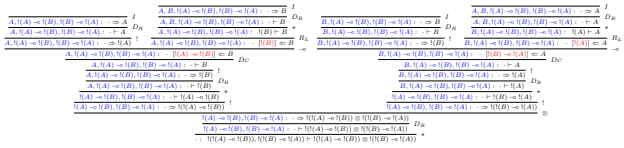
Multiplicative translation (156ms)



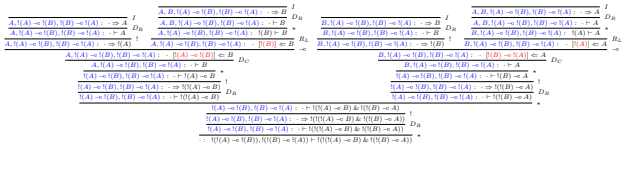
Girard's Translation (181ms)



Positive decoration (527ms)

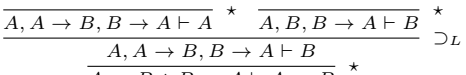


0/1 focused decoration (331ms)



(16) $A \leftrightarrow B \vdash A \to B$

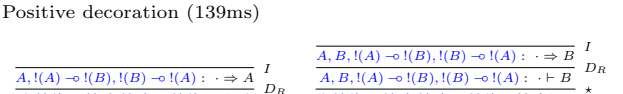
LJ (34ms)



Multiplicative translation (19ms)

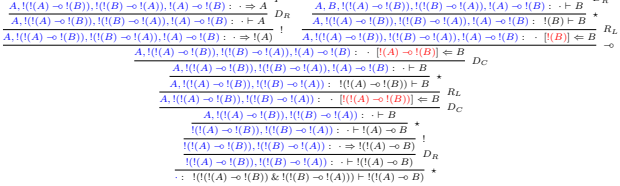
fail

Girard's Translation (91ms)



Positive decoration (139ms)

0/1 focused decoration (137ms)



(17) $A \leftrightarrow B \vdash B \to A$

LJ (35ms)

$$\dfrac{\dfrac{}{B, A \to B, B \to A \vdash B}\ \star \quad \dfrac{}{A, B, A \to B \vdash A}\ \star}{\dfrac{B, A \to B, B \to A \vdash A}{A \to B \wedge B \to A \vdash B \to A}\ \star}\ \supset_L$$

Multiplicative translation (21ms)

fail

Girard's Translation (94ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{B, !(A) \multimap B, !(B) \multimap A :\ \cdot \Rightarrow B}\ I}{B, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash B}\ D_R}{B, !(A) \multimap B, !(B) \multimap A :\ \cdot \Rightarrow !(B)}\ !}{B, !(A) \multimap B, !(B) \multimap A :\ \cdot\ [!(B) \multimap A] \Leftarrow A}\ D_C}{B, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash A}\ \star}{\cdot :\ !(!(A) \multimap B \,\&\, !(B) \multimap A) \vdash !(B) \multimap A}$$

$$\dfrac{\dfrac{}{B, !(A) \multimap B, !(B) \multimap A :\ A \Rightarrow A}\ I}{B, !(A) \multimap B, !(B) \multimap A :\ A \vdash A}\ D_R \qquad \dfrac{}{B, !(A) \multimap B, !(B) \multimap A :\ \cdot\ [A] \Leftarrow A}\ R_L \multimap$$

Positive decoration (132ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \Rightarrow B}\ I}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash B}\ D_R}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \Rightarrow !(B)}\ !}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot\ [!(B) \multimap !(A)] \Leftarrow A}\ D_C}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash A}\ D_R}{!(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(B) \multimap !(A)}\ \star}{!(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \Rightarrow !(!(B) \multimap !(A))}\ !}{!(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(!(B) \multimap !(A))}\ D_R}{\cdot :\ !(!(A) \multimap !(B)) \otimes !(!(B) \multimap !(A)) \vdash !(!(B) \multimap !(A))}\ \star$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \Rightarrow A}\ I}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash A}\ D_R}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ !(A) \vdash A}\ \star}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot\ [!(A)] \Leftarrow A}\ R_L \multimap}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot\ [!(B) \multimap !(A)] \Leftarrow A}\ D_C}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash A}\ D_R}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \Rightarrow !(A)}\ !}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(A)}\ D_R}{!(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(B) \multimap !(A)}\ \star}{!(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \Rightarrow !(!(B) \multimap !(A))}\ !}{!(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(!(B) \multimap !(A))}\ D_R}{\cdot :\ !(!(A) \multimap !(B)) \otimes !(!(B) \multimap !(A)) \vdash !(!(B) \multimap !(A))}\ \star$$

0/1 focused decoration (142ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(B) \multimap !(A) :\ \cdot \Rightarrow B}\ I}{B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(B) \multimap !(A) :\ \cdot \vdash B}\ D_R}{B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(B) \multimap !(A) :\ \cdot \Rightarrow !(B)}\ !}{B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(B) \multimap !(A) :\ \cdot\ [!(B) \multimap !(A)] \Leftarrow A}\ D_C}{B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(B) \multimap !(A) :\ \cdot \vdash A}\ \star}{B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)) :\ \cdot\ [!(!(B) \multimap !(A))] \Leftarrow A}\ R_L}{B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)) :\ \cdot \vdash A}\ D_C}{!(!(A) \multimap !(B)), !(!(B) \multimap !(A)) :\ \cdot \vdash !(!(B) \multimap A)}\ \star}{!(!(A) \multimap !(B)), !(!(B) \multimap !(A)) :\ \cdot \Rightarrow !(!(B) \multimap A)}\ !}{!(!(A) \multimap !(B)), !(!(B) \multimap !(A)) :\ \cdot \vdash !(!(B) \multimap A)}\ D_R}{\cdot :\ !(!(!(A) \multimap !(B)) \,\&\, !(!(B) \multimap !(A))) \vdash !(!(B) \multimap A)}\ \star$$

$$\dfrac{\dfrac{}{B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(B) \multimap !(A) :\ \cdot \Rightarrow B}\ I}{B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(B) \multimap !(A) :\ \cdot \vdash B}\ D_R \qquad \dfrac{}{A, B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(B) \multimap !(A) :\ \cdot \vdash A}\ I\ D_R$$

$$\dfrac{}{B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(B) \multimap !(A) :\ !(A) \vdash A}\ \star \qquad \dfrac{}{B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(B) \multimap !(A) :\ \cdot\ [!(A)] \Leftarrow A}\ R_L \multimap$$

(18) $A \leftrightarrow B, A \vdash B$

LJ (34ms)

$$\dfrac{\dfrac{}{A, A \to B, B \to A \vdash A}\ \star \quad \dfrac{}{A, B, B \to A \vdash B}\ \star}{\dfrac{A, A \to B, B \to A \vdash B}{A, A \to B \wedge B \to A \vdash B}\ \star}\ \supset_L$$

Multiplicative translation (24ms)

fail

Girard's Translation (97ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap B, !(B) \multimap A :\ \cdot \Rightarrow A}\ I}{A, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash A}\ D_R}{A, !(A) \multimap B, !(B) \multimap A :\ \cdot \Rightarrow !(A)}\ !}{A, !(A) \multimap B, !(B) \multimap A :\ \cdot\ [!(A) \multimap B] \Leftarrow B}\ D_C}{A, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash B}\ \star}{\cdot :\ !(A), !(!(A) \multimap B \,\&\, !(B) \multimap A) \vdash B}$$

$$\dfrac{\dfrac{}{A, !(A) \multimap B, !(B) \multimap A :\ B \Rightarrow B}\ I}{A, !(A) \multimap B, !(B) \multimap A :\ B \vdash B}\ D_R \qquad \dfrac{}{A, !(A) \multimap B, !(B) \multimap A :\ \cdot\ [B] \Leftarrow B}\ R_L \multimap$$

Positive decoration (115ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \Rightarrow A}\ I}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash A}\ D_R}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \Rightarrow !(A)}\ !}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot\ [!(A) \multimap !(B)] \Leftarrow B}\ D_C}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash B}\ D_R}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \Rightarrow !(B)}\ !}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(B)}\ D_R}{\cdot :\ !(A), !(!(A) \multimap !(B)) \otimes !(!(B) \multimap !(A)) \vdash !(B)}\ \star$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \Rightarrow B}\ I}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash B}\ D_R}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ !(B) \vdash B}\ \star}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot\ [!(B)] \Leftarrow B}\ R_L \multimap}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot\ [!(A) \multimap !(B)] \Leftarrow B}\ D_C}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash B}\ D_R}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \Rightarrow !(B)}\ !}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(B)}\ D_R}{\cdot :\ !(A), !(!(A) \multimap !(B)) \otimes !(!(B) \multimap !(A)) \vdash !(B)}\ \star$$

0/1 focused decoration (114ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(A) \multimap !(B) :\ \cdot \Rightarrow A}\ I}{A, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(A) \multimap !(B) :\ \cdot \vdash A}\ D_R}{A, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(A) \multimap !(B) :\ \cdot \Rightarrow !(A)}\ !}{A, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(A) \multimap !(B) :\ \cdot\ [!(A) \multimap !(B)] \Leftarrow B}\ D_C}{A, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(A) \multimap !(B) :\ \cdot \vdash B}\ D_R}{A, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)) :\ !(!(A) \multimap !(B)) \vdash B}\ \star}{A, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)) :\ \cdot\ [!(!(A) \multimap !(B))] \Leftarrow B}\ R_L}{A, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)) :\ \cdot \vdash B}\ D_C}{\cdot :\ !(A), !(!(!(A) \multimap !(B)) \,\&\, !(!(B) \multimap !(A))) \vdash B}\ \star$$

$$\dfrac{\dfrac{}{A, B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(A) \multimap !(B) :\ \cdot \Rightarrow B}\ I}{A, B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(A) \multimap !(B) :\ \cdot \vdash B}\ D_R \qquad \dfrac{}{A, B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(A) \multimap !(B) :\ !(B) \vdash B}\ \star$$

$$\dfrac{}{A, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(A) \multimap !(B) :\ \cdot\ [!(A) \multimap !(B)] \Leftarrow B}\ R_L \multimap$$

(19) $A \leftrightarrow B, B \vdash A$

LJ (35ms)

$$\dfrac{\dfrac{}{B, A \to B, B \to A \vdash B}\ \star \quad \dfrac{}{A, B, A \to B \vdash A}\ \star}{\dfrac{B, A \to B, B \to A \vdash A}{B, A \to B \wedge B \to A \vdash A}\ \star}\ \supset_L$$

Multiplicative translation (20ms)

fail

Girard's Translation (92ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{B, !(A) \multimap B, !(B) \multimap A :\ \cdot \Rightarrow B}\ I}{B, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash B}\ D_R}{B, !(A) \multimap B, !(B) \multimap A :\ \cdot \Rightarrow\ !(B)}\ !}{B, !(A) \multimap B, !(B) \multimap A :\ \cdot\ [!(B) \multimap A] \Leftarrow A}\ R_L \quad \dfrac{\dfrac{}{B, !(A) \multimap B, !(B) \multimap A :\ A \Rightarrow A}\ I}{B, !(A) \multimap B, !(B) \multimap A :\ A \vdash A}\ D_R}{B, !(A) \multimap B, !(B) \multimap A :\ \cdot\ [!(B) \multimap A] \Leftarrow A}\ \multimap}{B, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash A}\ D_C}{\cdot :\ !(B), !(!(A) \multimap B \,\&\, !(B) \multimap A) \vdash A}\ \star$$

Positive decoration (113ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \Rightarrow B}\ I}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash B}\ D_R}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \Rightarrow\ !(B)}\ !}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot\ [!(B) \multimap !(A)] \Leftarrow A}\ R_L \quad \dfrac{\dfrac{}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \Rightarrow A}\ I}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash A}\ D_R}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ !(A) \vdash A}\ \star}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot\ [!(A)] \Leftarrow A}\ \multimap}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot\ [!(B) \multimap !(A)] \Leftarrow A}\ D_C}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash A}\ !}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \Rightarrow !(A)}\ D_R}{\cdot :\ !(B), !(!(A) \multimap !(B)) \otimes !(!(B) \multimap !(A)) \vdash !(A)}\ \star$$

0/1 focused decoration (115ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(B) \multimap !(A) :\ \cdot \Rightarrow B}\ I}{B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(B) \multimap !(A) :\ \cdot \vdash B}\ D_R}{B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(B) \multimap !(A) :\ \cdot \Rightarrow\ !(B)}\ !}{B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(B) \multimap !(A) :\ \cdot\ [!(B) \multimap !(A)] \Leftarrow A}\ R_L \quad \dfrac{\dfrac{}{A, B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(B) \multimap !(A) :\ \cdot \Rightarrow A}\ I}{A, B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(B) \multimap !(A) :\ \cdot \vdash A}\ D_R}{B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(B) \multimap !(A) :\ !(A) \vdash A}\ \star}{B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(B) \multimap !(A) :\ \cdot\ [!(A)] \Leftarrow A}\ \multimap}{B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(B) \multimap !(A) :\ \cdot\ [!(B) \multimap !(A)] \Leftarrow A}\ D_C}{B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)), !(B) \multimap !(A) :\ \cdot \vdash A}\ !}{B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)) :\ !(!(B) \multimap !(A)) \vdash A}\ \star}{B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)) :\ \cdot\ [!(!(B) \multimap !(A))] \Leftarrow A}\ R_L}{B, !(!(A) \multimap !(B)), !(!(B) \multimap !(A)) :\ \cdot \vdash A}\ D_C}{\cdot :\ !(B), !(!(!(A) \multimap !(B)) \,\&\, !(!(B) \multimap !(A))) \vdash A}\ \star$$

(20) $\cdot \vdash A \leftrightarrow A$

LJ (18ms)

$$\dfrac{}{\cdot \vdash A \to A \wedge A \to A}\ \star$$

Multiplicative translation (90ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\cdot :\ A \Rightarrow A}\ I}{\cdot :\ A \vdash A}\ D_R}{\cdot :\ \cdot \vdash A \multimap A}\ \star}{\cdot :\ \cdot \Rightarrow A \multimap A}\ R_R \quad \dfrac{\dfrac{\dfrac{}{\cdot :\ A \Rightarrow A}\ I}{\cdot :\ A \vdash A}\ D_R}{\cdot :\ \cdot \vdash A \multimap A}\ \star}{\cdot :\ \cdot \Rightarrow A \multimap A}\ R_R}{\cdot :\ \cdot \Rightarrow A \multimap A \otimes A \multimap A}\ \otimes$$
$$\dfrac{}{\cdot :\ \cdot \vdash A \multimap A \otimes A \multimap A}\ D_R$$

Girard's Translation (43ms)

$$\dfrac{\dfrac{\dfrac{}{A :\ \cdot \Rightarrow A}\ I}{A :\ \cdot \vdash A}\ D_R \quad \dfrac{\dfrac{}{A :\ \cdot \Rightarrow A}\ I}{A :\ \cdot \vdash A}\ D_R}{\cdot :\ \cdot \vdash\ !(A) \multimap A \,\&\, !(A) \multimap A}\ \star$$

Positive decoration (126ms)

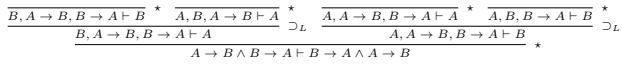
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A :\ \cdot \Rightarrow A}\ I}{A :\ \cdot \vdash A}\ D_R}{A :\ \cdot \Rightarrow\ !(A)}\ !}{A :\ \cdot \vdash\ !(A)}\ D_R}{\cdot :\ \cdot \vdash\ !(A) \multimap\ !(A)}\ \star \quad \dfrac{\dfrac{\dfrac{\dfrac{}{A :\ \cdot \Rightarrow A}\ I}{A :\ \cdot \vdash A}\ D_R}{A :\ \cdot \Rightarrow\ !(A)}\ !}{A :\ \cdot \vdash\ !(A)}\ D_R}{\cdot :\ \cdot \vdash\ !(A) \multimap\ !(A)}\ \star}{\cdot :\ \cdot \Rightarrow\ !(!(A) \multimap\ !(A))}\ ! \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A :\ \cdot \Rightarrow A}\ I}{A :\ \cdot \vdash A}\ D_R}{A :\ \cdot \Rightarrow\ !(A)}\ !}{A :\ \cdot \vdash\ !(A)}\ D_R}{\cdot :\ \cdot \vdash\ !(A) \multimap\ !(A)}\ \star}{\cdot :\ \cdot \Rightarrow\ !(!(A) \multimap\ !(A))}\ !}{\cdot :\ \cdot \Rightarrow\ !(!(A) \multimap\ !(A)) \otimes\ !(!(A) \multimap\ !(A))}\ \otimes$$
$$\dfrac{}{\cdot :\ \cdot \vdash\ !(!(A) \multimap\ !(A)) \otimes\ !(!(A) \multimap\ !(A))}\ D_R$$

0/1 focused decoration (108ms)

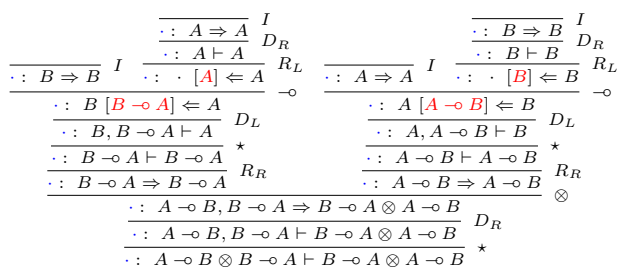
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A :\ \cdot \Rightarrow A}\ I}{A :\ \cdot \vdash A}\ D_R}{\cdot :\ \cdot \vdash\ !(A) \multimap A}\ \star}{\cdot :\ \cdot \Rightarrow\ !(!(A) \multimap A)}\ ! \quad \dfrac{\dfrac{\dfrac{\dfrac{}{A :\ \cdot \Rightarrow A}\ I}{A :\ \cdot \vdash A}\ D_R}{\cdot :\ \cdot \vdash\ !(A) \multimap A}\ \star}{\cdot :\ \cdot \Rightarrow\ !(!(A) \multimap A)}\ !}{\cdot :\ \cdot \vdash\ !(!(A) \multimap A)}\ D_R \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A :\ \cdot \Rightarrow A}\ I}{A :\ \cdot \vdash A}\ D_R}{\cdot :\ \cdot \vdash\ !(A) \multimap A}\ \star}{\cdot :\ \cdot \Rightarrow\ !(!(A) \multimap A)}\ ! \quad \dfrac{\dfrac{\dfrac{\dfrac{}{A :\ \cdot \Rightarrow A}\ I}{A :\ \cdot \vdash A}\ D_R}{\cdot :\ \cdot \vdash\ !(A) \multimap A}\ \star}{\cdot :\ \cdot \Rightarrow\ !(!(A) \multimap A)}\ !}{\cdot :\ \cdot \vdash\ !(!(A) \multimap A)}\ D_R}{\cdot :\ \cdot \vdash\ !(!(A) \multimap A) \,\&\, !(!(A) \multimap A)}\ \star$$
$$\dfrac{\dfrac{}{\cdot :\ \cdot \Rightarrow\ !(!(!(A) \multimap A) \,\&\, !(!(A) \multimap A))}\ !}{\cdot :\ \cdot \vdash\ !(!(!(A) \multimap A) \,\&\, !(!(A) \multimap A))}\ D_R$$

(21) $A \leftrightarrow B \vdash B \leftrightarrow A$

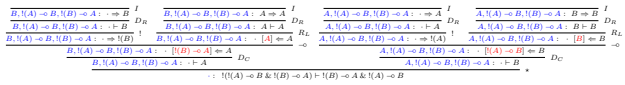
LJ (54ms)



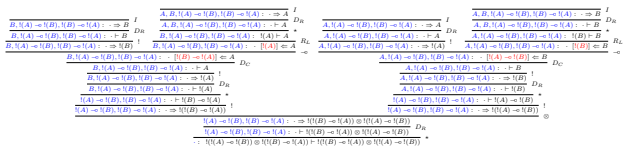
Multiplicative translation (160ms)



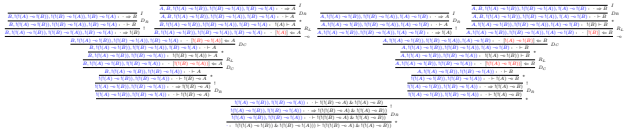
Girard's Translation (196ms)



Positive decoration (515ms)

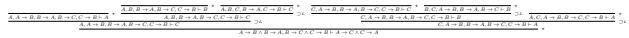


0/1 focused decoration (1825ms)

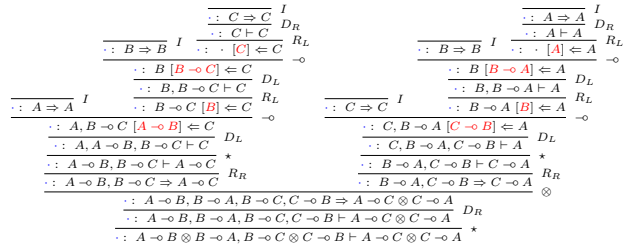


(22) $A \leftrightarrow B, B \leftrightarrow C \vdash A \leftrightarrow C$

LJ (150ms)



Multiplicative translation (228ms)



Girard's Translation (Timeout!)

Timeout!

Positive decoration (Timeout!)

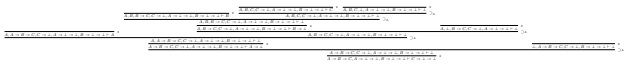
Timeout!

0/1 focused decoration (Timeout!)

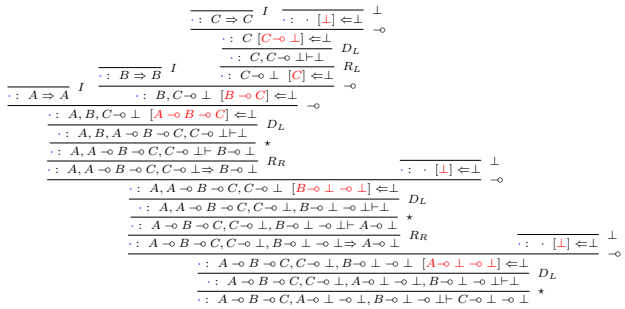
Timeout!

(23) $A \to (B \to C), \neg\neg A, \neg\neg B \vdash \neg\neg C$

LJ (2318ms)



Multiplicative translation (164ms)



Girard's Translation (Timeout!)

Timeout!

Positive decoration (Timeout!)

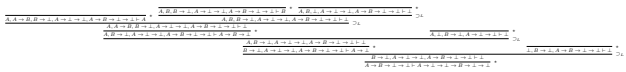
Timeout!

0/1 focused decoration (Timeout!)

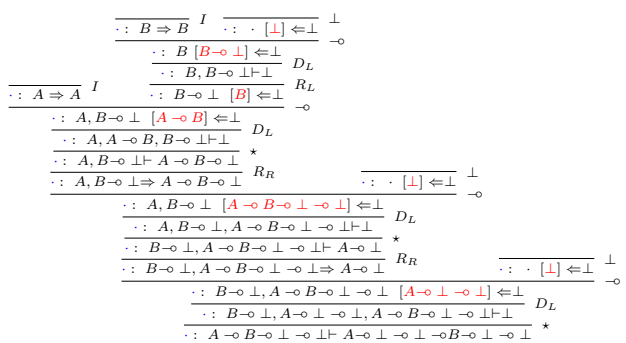
Timeout!

(24) $\neg\neg(A \to B) \vdash \neg\neg A \to \neg\neg B$

LJ (211ms)



Multiplicative translation (142ms)



Girard's Translation (4694ms)

Positive decoration (5522ms)

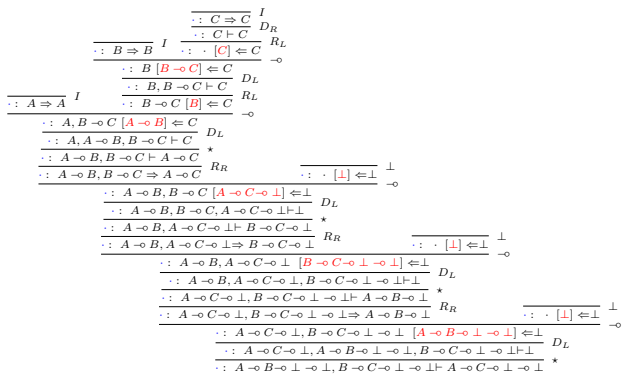
0/1 focused decoration (Timeout!)

Timeout!

(25) $\neg\neg(A \to B), \neg\neg(B \to C) \vdash \neg\neg(A \to C)$

LJ (4063ms)

Multiplicative translation (202ms)



Girard's Translation (Timeout!)

Timeout!

Positive decoration (Timeout!)

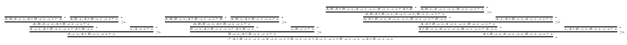
Timeout!

0/1 focused decoration (Timeout!)

Timeout!

(26) $\cdot \vdash \neg\neg(A \wedge B) \leftrightarrow \neg\neg A \wedge \neg\neg B$

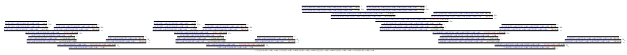
LJ (140ms)



Multiplicative translation (20ms)

fail

Girard's Translation (27482ms)



Positive decoration (Timeout!)

Timeout!

0/1 focused decoration (Timeout!)

Timeout!

(27) $\cdot \vdash \neg\neg(A \leftrightarrow B) \leftrightarrow \neg\neg(A \to B) \wedge \neg\neg(B \to A)$

LJ (Timeout!)

Timeout!

Multiplicative translation (23ms)

fail

Girard's Translation (Timeout!)

Timeout!

Positive decoration (Timeout!)

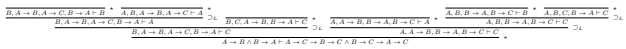
Timeout!

0/1 focused decoration (Timeout!)

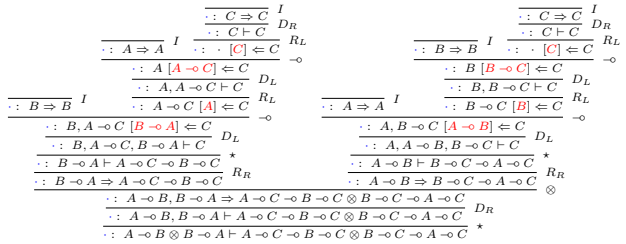
Timeout!

(28) $A \leftrightarrow B \vdash A \to C \leftrightarrow B \to C$

LJ (86ms)



Multiplicative translation (227ms)



Girard's Translation (Timeout!)

Timeout!

Positive decoration (Timeout!)

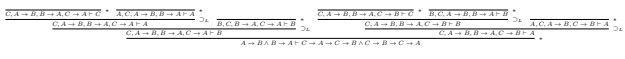
Timeout!

0/1 focused decoration (Timeout!)

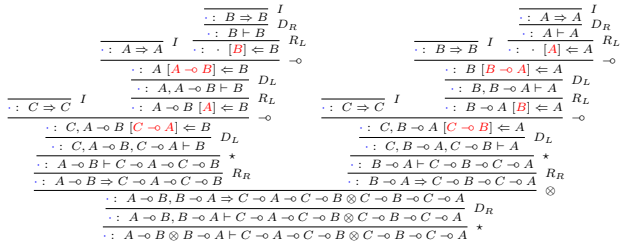
Timeout!

(29) $A \leftrightarrow B \vdash C \to A \leftrightarrow C \to B$

LJ (86ms)



Multiplicative translation (240ms)



Girard's Translation (Timeout!)

Timeout!

Positive decoration (Timeout!)

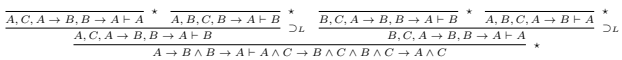
Timeout!

0/1 focused decoration (Timeout!)

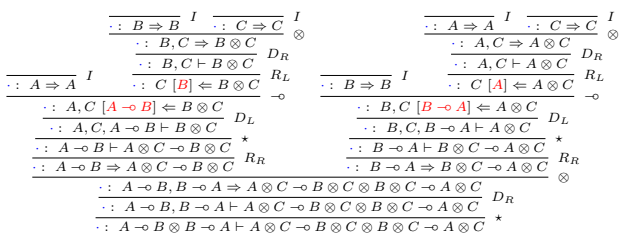
Timeout!

(30) $A \leftrightarrow B \vdash A \wedge C \leftrightarrow B \wedge C$

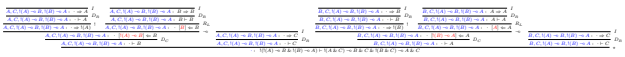
LJ (48ms)



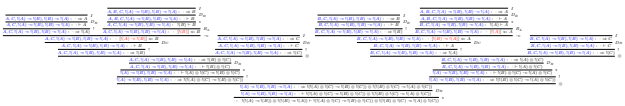
Multiplicative translation (191ms)



Girard's Translation (292ms)



Positive decoration (3424ms)

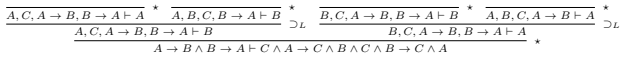


0/1 focused decoration (Timeout!)

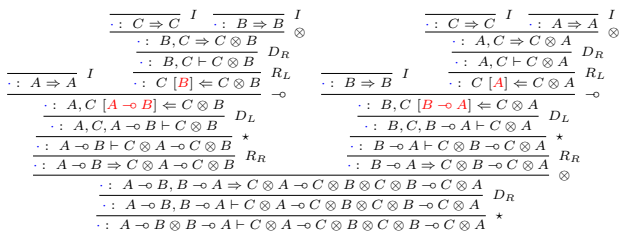
Timeout!

(31) $A \leftrightarrow B \vdash C \wedge A \leftrightarrow C \wedge B$

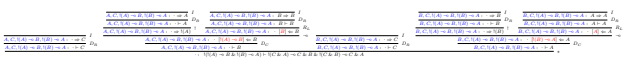
LJ (54ms)



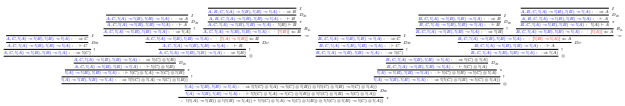
Multiplicative translation (209ms)



Girard's Translation (353ms)



Positive decoration (3752ms)

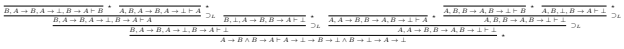


0/1 focused decoration (Timeout!)

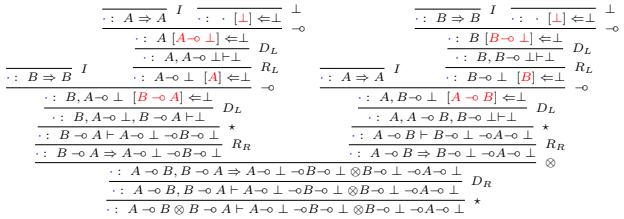
Timeout!

(32) $A \leftrightarrow B \vdash \neg A \leftrightarrow \neg B$

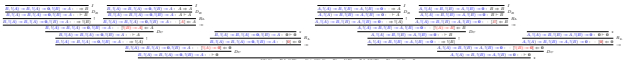
LJ (83ms)



Multiplicative translation (202ms)



Girard's Translation (12683ms)



Positive decoration (Timeout!)

Timeout!

0/1 focused decoration (Timeout!)

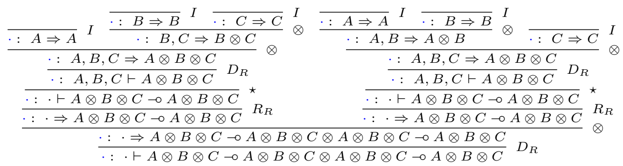
Timeout!

(33) $\cdot \vdash (A \wedge B) \wedge C \leftrightarrow A \wedge (B \wedge C)$

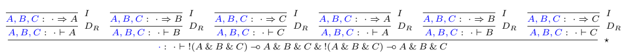
LJ (21ms)

$$\dfrac{}{\cdot \vdash A \wedge B \wedge C \to A \wedge B \wedge C \wedge A \wedge B \wedge C \to A \wedge B \wedge C}\ \star$$

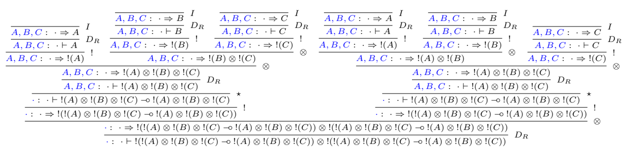
Multiplicative translation (166ms)



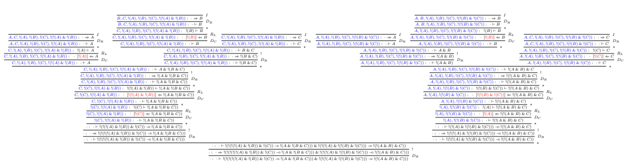
Girard's Translation (123ms)



Positive decoration (281ms)



0/1 focused decoration (609ms)

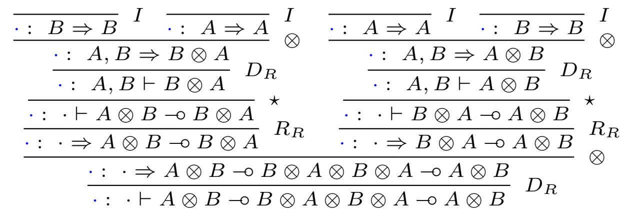


(34) $\cdot \vdash A \wedge B \leftrightarrow B \wedge A$

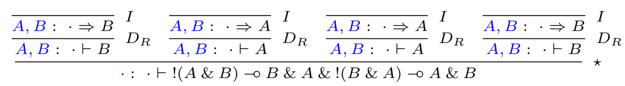
LJ (18ms)

$$\dfrac{}{\cdot \vdash A \wedge B \to B \wedge A \wedge B \wedge A \to A \wedge B}\ \star$$

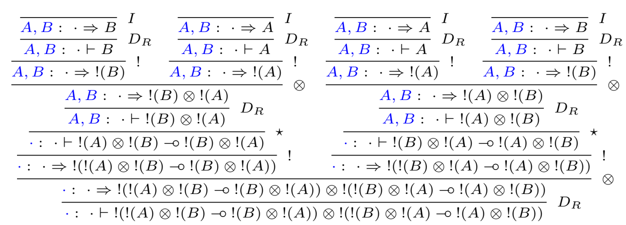
Multiplicative translation (131ms)



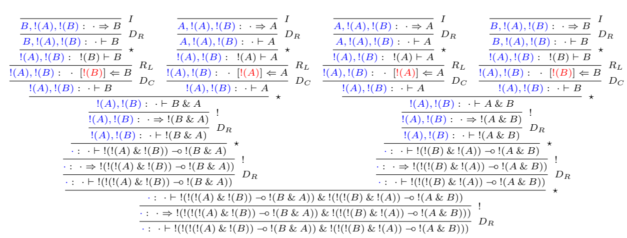
Girard's Translation (81ms)



Positive decoration (217ms)



0/1 focused decoration (276ms)



(35) $\cdot \vdash A \wedge A \leftrightarrow A$

LJ (21ms)

$$\dfrac{}{\cdot \vdash A \wedge A \to A \wedge A \to A \wedge A}\ \star$$

Multiplicative translation (18ms)

fail

Girard's Translation (50ms)

$$\dfrac{\dfrac{\dfrac{}{A:\ \cdot \Rightarrow A}\ I}{A:\ \cdot \vdash A}\ D_R \quad \dfrac{\dfrac{}{A:\ \cdot \Rightarrow A}\ I}{A:\ \cdot \vdash A}\ D_R \quad \dfrac{\dfrac{}{A:\ \cdot \Rightarrow A}\ I}{A:\ \cdot \vdash A}\ D_R}{\cdot:\ \cdot \vdash\ !(A \,\&\, A) \multimap A \,\&\, !(A) \multimap A \,\&\, A}\ \star$$

Positive decoration (180ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A:\ \cdot \Rightarrow A}\ I}{A:\ \cdot \vdash A}\ D_R}{A:\ \cdot \Rightarrow\ !(A)}\ !}{A:\ \cdot \vdash\ !(A)}\ D_R}{\cdot:\ \cdot \vdash\ !(A) \otimes\ !(A) \multimap\ !(A)}\ \star}{\cdot:\ \cdot \Rightarrow\ !(!(A) \otimes\ !(A) \multimap\ !(A))}\ ! \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A:\ \cdot \Rightarrow A}\ I}{A:\ \cdot \vdash A}\ D_R}{A:\ \cdot \Rightarrow\ !(A)}\ !}{A:\ \cdot \Rightarrow\ !(A) \otimes\ !(A)}\ D_R}{\cdot:\ \cdot \vdash\ !(A) \multimap\ !(A) \otimes\ !(A)}\ \star}{\cdot:\ \cdot \Rightarrow\ !(!(A) \multimap\ !(A) \otimes\ !(A))}\ !}{\cdot:\ \cdot \Rightarrow\ !(!(A) \otimes\ !(A) \multimap\ !(A)) \otimes\ !(!(A) \multimap\ !(A) \otimes\ !(A))}\ \otimes$$
$$\dfrac{\cdot:\ \cdot \Rightarrow\ !(!(A) \otimes\ !(A) \multimap\ !(A)) \otimes\ !(!(A) \multimap\ !(A) \otimes\ !(A))}{\cdot:\ \cdot \vdash\ !(!(A) \otimes\ !(A) \multimap\ !(A)) \otimes\ !(!(A) \multimap\ !(A) \otimes\ !(A))}\ D_R$$

0/1 focused decoration (153ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A):\ \cdot \Rightarrow A}\ I}{A, !(A):\ \cdot \vdash A}\ D_R}{!(A):\ !(A) \vdash A}\ \star}{!(A):\ \cdot\ [!(A)] \Leftarrow A}\ R_L}{!(A):\ \cdot \vdash A}\ D_C}{\cdot:\ \cdot \vdash\ !(!(A) \,\&\, !(A)) \multimap A}\ \star}{\cdot:\ \cdot \Rightarrow\ !(!(!(A) \,\&\, !(A)) \multimap A)}\ ! \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A:\ \cdot \Rightarrow A}\ I}{A:\ \cdot \vdash A}\ D_R}{A:\ \cdot \vdash A \,\&\, A}\ \star}{A:\ \cdot \Rightarrow\ !(A \,\&\, A)}\ !}{A:\ \cdot \vdash\ !(A \,\&\, A)}\ D_R}{\cdot:\ \cdot \vdash\ !(A) \multimap\ !(A \,\&\, A)}\ \star}{\cdot:\ \cdot \Rightarrow\ !(!(A) \multimap\ !(A \,\&\, A))}\ !}{\cdot:\ \cdot \vdash\ !(!(!(A) \,\&\, !(A)) \multimap A) \,\&\, !(!(A) \multimap\ !(A \,\&\, A))}\ D_R$$
$$\dfrac{\cdot:\ \cdot \vdash\ !(!(!(A) \,\&\, !(A)) \multimap A) \,\&\, !(!(A) \multimap\ !(A \,\&\, A))}{\cdot:\ \cdot \Rightarrow\ !(!(!(!(A) \,\&\, !(A)) \multimap A) \,\&\, !(!(A) \multimap\ !(A \,\&\, A)))}\ !$$
$$\dfrac{\cdot:\ \cdot \Rightarrow\ !(!(!(!(A) \,\&\, !(A)) \multimap A) \,\&\, !(!(A) \multimap\ !(A \,\&\, A)))}{\cdot:\ \cdot \vdash\ !(!(!(!(A) \,\&\, !(A)) \multimap A) \,\&\, !(!(A) \multimap\ !(A \,\&\, A)))}\ D_R$$

(36) $A \vdash A \to B \leftrightarrow B$

LJ (35ms)

$$\dfrac{\dfrac{\dfrac{}{A, A \to B \vdash A}\ \star \quad \dfrac{}{A, B \vdash B}\ \star}{A, A \to B \vdash B}\ \supset_L}{A \vdash A \to B \to B \wedge B \to A \to B}\ \star$$

Multiplicative translation (19ms)

fail

Girard's Translation (107ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap B:\ \cdot \Rightarrow A}\ I}{A, !(A) \multimap B:\ \cdot \vdash A}\ D_R}{A, !(A) \multimap B:\ \cdot \Rightarrow\ !(A)}\ ! \quad \dfrac{\dfrac{}{A, !(A) \multimap B:\ B \Rightarrow B}\ I}{A, !(A) \multimap B:\ B \vdash B}\ D_R}{A, !(A) \multimap B:\ \cdot\ [B] \Leftarrow B}\ R_L}{A, !(A) \multimap B:\ \cdot\ [!(A) \multimap B] \Leftarrow B}\ \multimap \quad \dfrac{\dfrac{}{A, B:\ \cdot \Rightarrow B}\ I}{A, B:\ \cdot \vdash B}\ D_R}{A, !(A) \multimap B:\ \cdot \vdash B}\ D_C$$
$$\dfrac{A, !(A) \multimap B:\ \cdot \vdash B}{\cdot:\ \ !(A) \vdash\ !(!(A) \multimap B) \multimap B \,\&\, !(B) \multimap\ !(A) \multimap B}\ \star$$

Positive decoration (209ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(B):\ \cdot \Rightarrow A}\ I}{A, !(A) \multimap !(B):\ \cdot \vdash A}\ D_R}{A, !(A) \multimap !(B):\ \cdot \Rightarrow\ !(A)}\ ! \quad \dfrac{\dfrac{}{A, B, !(A) \multimap !(B):\ \cdot \Rightarrow B}\ I}{A, B, !(A) \multimap !(B):\ \cdot \vdash B}\ D_R}{A, !(A) \multimap !(B):\ \cdot\ [!(B)] \Leftarrow B}\ R_L}{A, !(A) \multimap !(B):\ \cdot\ [!(A) \multimap !(B)] \Leftarrow B}\ \multimap \quad \dfrac{\dfrac{}{A, B:\ \cdot \Rightarrow B}\ I}{A, B:\ \cdot \vdash B}\ D_R}{A, !(A) \multimap !(B):\ \cdot \vdash B}\ D_C$$
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(B):\ \cdot \Rightarrow !(B)}\ D_R}{A, !(A) \multimap !(B):\ \cdot \vdash !(B)}\ !}{A:\ \cdot \vdash\ !(!(A) \multimap !(B)) \multimap !(B)}\ \star}{A:\ \cdot \Rightarrow\ !(!(A) \multimap !(B)) \multimap !(B)}\ !}{A:\ \cdot \Rightarrow\ !(!(A) \multimap !(B)) \otimes\ !(!(B) \multimap\ !(!(A) \multimap !(B)))}\ \otimes}{A:\ \cdot \vdash\ !(!(A) \multimap !(B)) \multimap !(B)) \otimes\ !(!(B) \multimap\ !(!(A) \multimap !(B)))}\ D_R}{\cdot:\ \ !(A) \vdash\ !(!(!(A) \multimap !(B)) \multimap !(B)) \otimes\ !(!(B) \multimap\ !(!(A) \multimap !(B)))}\ \star$$
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, B:\ \cdot \Rightarrow B}\ I}{A, B:\ \cdot \vdash B}\ D_R}{A, B:\ \cdot \vdash\ !(A) \multimap B}\ \star}{A, B:\ \cdot \Rightarrow\ !(!(A) \multimap B)}\ !}{A, B:\ \cdot \vdash\ !(!(A) \multimap B)}\ D_R}{A:\ \cdot \vdash\ !(B) \multimap\ !(!(A) \multimap B)}\ \star}{A:\ \cdot \Rightarrow\ !(!(B) \multimap\ !(!(A) \multimap B))}\ !}{A:\ \cdot \vdash\ !(!(B) \multimap\ !(!(A) \multimap B))}\ D_R$$

0/1 focused decoration (184ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(B):\ \cdot \Rightarrow A}\ I}{A, !(A) \multimap !(B):\ \cdot \vdash A}\ D_R}{A, !(A) \multimap !(B):\ \cdot \Rightarrow\ !(A)}\ ! \quad \dfrac{\dfrac{}{A, B, !(A) \multimap !(B):\ \cdot \Rightarrow B}\ I}{A, B, !(A) \multimap !(B):\ \cdot \vdash B}\ D_R}{A, !(A) \multimap !(B):\ \cdot\ [!(B)] \Leftarrow B}\ R_L}{A, !(A) \multimap !(B):\ \cdot\ [!(A) \multimap !(B)] \Leftarrow B}\ \multimap \quad \dfrac{\dfrac{}{A, B:\ \cdot \Rightarrow B}\ I}{A, B:\ \cdot \vdash B}\ D_R}{A, !(A) \multimap !(B):\ \cdot \vdash B}\ D_C$$
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(B):\ \cdot \Rightarrow !(B)}\ D_R}{A, !(A) \multimap !(B):\ \cdot \vdash !(B)}\ !}{A:\ \cdot \vdash\ !(!(A) \multimap !(B)) \multimap !(B)}\ \star}{A:\ \cdot \Rightarrow\ !(!(A) \multimap !(B)) \multimap !(B)}\ !}{A:\ \cdot \Rightarrow\ !(!(A) \multimap !(B)) \otimes\ !(!(B) \multimap\ !(!(A) \multimap !(B)))}\ \otimes}{A:\ \cdot \vdash\ !(!(A) \multimap !(B)) \multimap !(B)) \otimes\ !(!(B) \multimap\ !(!(A) \multimap !(B)))}\ D_R}{\cdot:\ \ !(A) \vdash\ !(!(!(A) \multimap !(B)) \multimap !(B)) \otimes\ !(!(B) \multimap\ !(!(A) \multimap !(B)))}\ \star$$
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, B:\ \cdot \Rightarrow B}\ I}{A, B:\ \cdot \vdash B}\ D_R}{A, B:\ \cdot \vdash\ !(A) \multimap B}\ \star}{A, B:\ \cdot \Rightarrow\ !(!(A) \multimap B)}\ !}{A, B:\ \cdot \vdash\ !(!(A) \multimap B)}\ D_R}{A:\ \cdot \vdash\ !(B) \multimap\ !(!(A) \multimap B)}\ \star}{A:\ \cdot \Rightarrow\ !(!(B) \multimap\ !(!(A) \multimap B))}\ !}{A:\ \cdot \vdash\ !(!(B) \multimap\ !(!(A) \multimap B))}\ D_R$$
$$\dfrac{A:\ \cdot \vdash\ !(!(!(A) \multimap !(B)) \multimap B) \,\&\, !(!(B) \multimap\ !(!(A) \multimap B))}{A:\ \cdot \Rightarrow\ !(!(!(!(A) \multimap !(B)) \multimap B) \,\&\, !(!(B) \multimap\ !(!(A) \multimap B)))}\ !$$
$$\dfrac{A:\ \cdot \Rightarrow\ !(!(!(!(A) \multimap !(B)) \multimap B) \,\&\, !(!(B) \multimap\ !(!(A) \multimap B)))}{\cdot:\ \ !(A) \vdash\ !(!(!(!(A) \multimap !(B)) \multimap B) \,\&\, !(!(B) \multimap\ !(!(A) \multimap B)))}\ D_R$$

(37) $B \vdash A \to B \leftrightarrow B$

LJ (18ms)

$$\frac{}{B \vdash A \to B \to B \wedge B \to A \to B}\ \star$$

Multiplicative translation (19ms)

fail

Girard's Translation (67ms)

$$\dfrac{\dfrac{\dfrac{}{B, !(A) \multimap B :\ \cdot \Rightarrow B}\ I}{B, !(A) \multimap B :\ \cdot \vdash B}\ D_R \quad \dfrac{\dfrac{}{A, B :\ \cdot \Rightarrow B}\ I}{A, B :\ \cdot \vdash B}\ D_R}{\cdot :\ \ !(B) \vdash !(!(A) \multimap B) \multimap B \,\&\, !(B) \multimap !(!A) \multimap B}\ \star$$

Positive decoration (151ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{B, !(A) \multimap !(B) :\ \cdot \Rightarrow B}\ I}{B, !(A) \multimap !(B) :\ \cdot \vdash B}\ D_R}{B, !(A) \multimap !(B) :\ \cdot \Rightarrow !(B)}\ !}{B, !(A) \multimap !(B) :\ \cdot \vdash !(B)}\ D_R}{B :\ \cdot \vdash !(!(A) \multimap !(B)) \multimap !(B)}\ \star}{B :\ \cdot \Rightarrow !(!(!(A) \multimap !(B)) \multimap !(B))}\ ! \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \Rightarrow B}\ I}{A, B :\ \cdot \vdash B}\ D_R}{A, B :\ \cdot \Rightarrow !(B)}\ !}{A, B :\ \cdot \vdash !(B)}\ D_R}{B :\ \cdot \vdash !(!(A) \multimap !(B))}\ \star}{B :\ \cdot \Rightarrow !(!(A) \multimap !(B))}\ ! \quad \dfrac{}{B :\ \cdot \vdash !(B) \multimap !(!(A) \multimap !(B))}\ \star}{B :\ \cdot \Rightarrow !(!(!(A) \multimap !(B)) \multimap !(B)) \otimes !(!(B) \multimap !(!(A) \multimap !(B)))}\ !\ \otimes$$

$$\dfrac{\dfrac{B :\ \cdot \vdash !(!(!(A) \multimap !(B)) \multimap !(B)) \otimes !(!(B) \multimap !(!(A) \multimap !(B)))}{\cdot :\ \ !(B) \vdash !(!(!(A) \multimap !(B)) \multimap !(B)) \otimes !(!(B) \multimap !(!(A) \multimap !(B)))}\ D_R}{}\ \star$$

0/1 focused decoration (139ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{B, !(A) \multimap !(B) :\ \cdot \Rightarrow B}\ I}{B, !(A) \multimap !(B) :\ \cdot \vdash B}\ D_R}{B :\ \cdot \vdash !(!(A) \multimap !(B)) \multimap B}\ \star}{B :\ \cdot \Rightarrow !(!(!(A) \multimap !(B)) \multimap B)}\ !}{B :\ \cdot \vdash !(!(!(A) \multimap !(B)) \multimap B)}\ D_R}{B :\ \cdot \vdash !(!(!(A) \multimap !(B)) \multimap B) \,\&\, !(!(B) \multimap !(!(A) \multimap !(B)))}\ \star \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \Rightarrow B}\ I}{A, B :\ \cdot \vdash B}\ D_R}{B :\ \cdot \vdash !(A) \multimap B}\ \star}{B :\ \cdot \Rightarrow !(!(A) \multimap B)}\ !}{B :\ \cdot \vdash !(!(A) \multimap B)}\ D_R}{B :\ \cdot \vdash !(B) \multimap !(!(A) \multimap B)}\ \star}{B :\ \cdot \Rightarrow !(!(B) \multimap !(!(A) \multimap B))}\ !}{B :\ \cdot \vdash !(!(B) \multimap !(!(A) \multimap B))}\ D_R$$

$$\dfrac{\dfrac{B :\ \cdot \Rightarrow !(!(!(A) \multimap !(B)) \multimap B) \,\&\, !(!(B) \multimap !(!(A) \multimap !(B)))}{B :\ \cdot \vdash !(!(!(A) \multimap !(B)) \multimap B) \,\&\, !(!(B) \multimap !(!(A) \multimap !(B)))}\ D_R}{\cdot :\ \ !(B) \vdash !(!(!(A) \multimap !(B)) \multimap B) \,\&\, !(!(B) \multimap !(!(A) \multimap !(B)))}\ \star$$

(38) $\neg A \vdash A \to B \leftrightarrow \neg A$

LJ (54ms)

$$\dfrac{\dfrac{\dfrac{}{A, A \to B, A \to \bot \vdash A}\ \star \quad \dfrac{}{A, \bot, A \to B \vdash \bot}\ \star}{A, A \to B, A \to \bot \vdash \bot}\ \supset_L \quad \dfrac{\dfrac{}{A, A \to \bot \vdash A}\ \star \quad \dfrac{}{A, \bot \vdash B}\ \star}{A, A \to \bot \vdash B}\ \supset_L}{A \to \bot \vdash A \to B \to A \to \bot \wedge A \to \bot \to A \to B}\ \star$$

Multiplicative translation (21ms)

fail

Girard's Translation (157ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap B, !(A) \multimap \mathbf{0} :\ \cdot \Rightarrow A}\ I}{A, !(A) \multimap B, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}\ D_R \quad \dfrac{}{A, !(A) \multimap B, !(A) \multimap \mathbf{0} :\ \cdot\ [\mathbf{0}] \Leftarrow \mathbf{0}}\ \star}{A, !(A) \multimap B, !(A) \multimap \mathbf{0} :\ \cdot \Rightarrow !(A)}\ !}{A, !(A) \multimap B, !(A) \multimap \mathbf{0} :\ \cdot\ [!(A) \multimap \mathbf{0}] \Leftarrow \mathbf{0}}\ R_L \multimap}{A, !(A) \multimap B, !(A) \multimap \mathbf{0} :\ \cdot \vdash \mathbf{0}}\ D_C \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \Rightarrow A}\ I}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}\ D_R \quad \dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot\ [\mathbf{0}] \Leftarrow B}\ \star}{A, !(A) \multimap \mathbf{0} :\ \cdot \Rightarrow !(A)}\ !}{A, !(A) \multimap \mathbf{0} :\ \cdot\ [!(A) \multimap \mathbf{0}] \Leftarrow B}\ R_L \multimap}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash B}\ D_C}{\cdot :\ \ !(!(A) \multimap \mathbf{0}) \vdash !(!(A) \multimap B) \multimap !(A) \multimap \mathbf{0} \,\&\, !(!(A) \multimap \mathbf{0}) \multimap !(A) \multimap B}\ \star$$

Positive decoration (333ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0}, !(A) \multimap !(B) :\ \cdot \Rightarrow A}\ I}{A, !(A) \multimap \mathbf{0}, !(A) \multimap !(B) :\ \cdot \vdash A}\ D_R \quad \dfrac{}{A, !(A) \multimap \mathbf{0}, !(A) \multimap !(B) :\ \cdot\ [\mathbf{0}] \Leftarrow \mathbf{0}}\ \star}{A, !(A) \multimap \mathbf{0}, !(A) \multimap !(B) :\ \cdot \Rightarrow !(A)}\ !}{A, !(A) \multimap \mathbf{0}, !(A) \multimap !(B) :\ \cdot\ [!(A) \multimap \mathbf{0}] \Leftarrow \mathbf{0}}\ R_L \multimap}{A, !(A) \multimap \mathbf{0}, !(A) \multimap !(B) :\ \cdot \vdash \mathbf{0}}\ D_C}{!(A) \multimap \mathbf{0}, !(A) \multimap !(B) :\ \cdot \vdash !(A) \multimap \mathbf{0}}\ \star}{!(A) \multimap \mathbf{0}, !(A) \multimap !(B) :\ \cdot \Rightarrow !(!(A) \multimap \mathbf{0})}\ !}{!(A) \multimap \mathbf{0}, !(A) \multimap !(B) :\ \cdot \vdash !(!(A) \multimap \mathbf{0})}\ D_R}{!(A) \multimap \mathbf{0} :\ \cdot \vdash !(!(A) \multimap !(B)) \multimap !(!(A) \multimap \mathbf{0})}\ \star}{!(A) \multimap \mathbf{0} :\ \cdot \Rightarrow !(!(!(A) \multimap !(B)) \multimap !(!(A) \multimap \mathbf{0}))}\ ! \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0}, !(A) \multimap !(B) :\ \cdot \Rightarrow A}\ I}{A, !(A) \multimap \mathbf{0}, !(A) \multimap !(B) :\ \cdot \vdash A}\ D_R \quad \dfrac{}{A, !(A) \multimap \mathbf{0}, !(A) \multimap !(B) :\ \cdot\ [\mathbf{0}] \Leftarrow \mathbf{0}}\ \star}{A, !(A) \multimap \mathbf{0}, !(A) \multimap !(B) :\ \cdot \Rightarrow !(A)}\ !}{A, !(A) \multimap \mathbf{0}, !(A) \multimap !(B) :\ \cdot\ [!(A) \multimap \mathbf{0}] \Leftarrow \mathbf{0}}\ R_L \multimap}{A, !(A) \multimap \mathbf{0}, !(A) \multimap !(B) :\ \cdot \vdash \mathbf{0}}\ D_C}{!(A) \multimap \mathbf{0}, !(A) \multimap !(B) :\ \cdot \vdash !(A) \multimap \mathbf{0}}\ \star}{!(A) \multimap \mathbf{0}, !(A) \multimap !(B) :\ \cdot \Rightarrow !(!(A) \multimap \mathbf{0})}\ !}{!(A) \multimap \mathbf{0}, !(A) \multimap !(B) :\ \cdot \vdash !(!(A) \multimap \mathbf{0})}\ D_R}{!(A) \multimap \mathbf{0} :\ \cdot \vdash !(!(A) \multimap !(B)) \multimap !(!(A) \multimap \mathbf{0})}\ \star}{!(A) \multimap \mathbf{0} :\ \cdot \Rightarrow !(!(!(A) \multimap !(B)) \multimap !(!(A) \multimap \mathbf{0}))}\ !\ \otimes$$

$$\dfrac{\dfrac{!(A) \multimap \mathbf{0} :\ \cdot \Rightarrow !(!(!(A) \multimap !(B)) \multimap !(!(A) \multimap \mathbf{0})) \otimes !(!(!(A) \multimap \mathbf{0}) \multimap !(!(A) \multimap !(B)))}{!(A) \multimap \mathbf{0} :\ \cdot \vdash !(!(!(A) \multimap !(B)) \multimap !(!(A) \multimap \mathbf{0})) \otimes !(!(!(A) \multimap \mathbf{0}) \multimap !(!(A) \multimap !(B)))}\ D_R}{\cdot :\ \ !(!(A) \multimap \mathbf{0}) \vdash !(!(!(A) \multimap !(B)) \multimap !(!(A) \multimap \mathbf{0})) \otimes !(!(!(A) \multimap \mathbf{0}) \multimap !(!(A) \multimap !(B)))}\ \star$$

0/1 focused decoration (319ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ \cdot \Rightarrow A}\ I}{A, !(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash A}\ D_R \quad \dfrac{}{A, !(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ \cdot\ [!(\mathbf{0})] \Leftarrow \mathbf{0}}\ \star}{A, !(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ \cdot \Rightarrow !(A)}\ !}{A, !(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ \cdot\ [!(A) \multimap !(\mathbf{0})] \Leftarrow \mathbf{0}}\ R_L \multimap}{A, !(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash \mathbf{0}}\ D_C}{!(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(A) \multimap \mathbf{0}}\ \star}{!(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ \cdot \Rightarrow !(!(A) \multimap \mathbf{0})}\ !}{!(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(!(A) \multimap \mathbf{0})}\ D_R}{!(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(!(A) \multimap !(B)) \multimap !(!(A) \multimap \mathbf{0})}\ \star}{!(A) \multimap !(\mathbf{0}) :\ \cdot \Rightarrow !(!(!(A) \multimap !(B)) \multimap !(!(A) \multimap \mathbf{0}))}\ ! \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ \cdot \Rightarrow A}\ I}{A, !(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash A}\ D_R \quad \dfrac{}{A, !(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ \cdot\ [!(A) \multimap !(\mathbf{0})] \Leftarrow \mathbf{0}}\ \star}{A, !(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ \cdot \Rightarrow !(A)}\ !}{A, !(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ \cdot\ [!(A) \multimap !(\mathbf{0})] \Leftarrow B}\ R_L \multimap}{A, !(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash B}\ D_C}{!(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(A) \multimap B}\ \star}{!(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ \cdot \Rightarrow !(!(A) \multimap B)}\ !}{!(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(!(A) \multimap B)}\ D_R}{!(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(!(A) \multimap !(B)) \multimap !(!(A) \multimap B)}\ \star}{!(A) \multimap !(\mathbf{0}) :\ \cdot \Rightarrow !(!(!(A) \multimap !(B)) \multimap !(!(A) \multimap B))}\ ! \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ \cdot \Rightarrow A}\ I}{A, !(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash A}\ D_R \quad \dfrac{}{A, !(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ \cdot\ [!(\mathbf{0})] \Leftarrow B}\ \star}{A, !(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ \cdot \Rightarrow !(A)}\ !}{A, !(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ \cdot\ [!(A) \multimap !(\mathbf{0})] \Leftarrow B}\ R_L \multimap}{A, !(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash B}\ D_C}{!(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(A) \multimap B}\ \star}{!(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ \cdot \Rightarrow !(!(A) \multimap B)}\ !}{!(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(!(A) \multimap B)}\ D_R}{!(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(!(A) \multimap !(B)) \multimap !(!(A) \multimap B)}\ \star}{!(A) \multimap !(\mathbf{0}) :\ \cdot \Rightarrow !(!(!(A) \multimap !(B)) \multimap !(!(A) \multimap B))}\ !}{!(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(!(!(A) \multimap !(B)) \multimap !(!(A) \multimap B))}\ D_R$$

$$\dfrac{\dfrac{\dfrac{!(A) \multimap !(\mathbf{0}) :\ \cdot \Rightarrow !(!(!(A) \multimap !(B)) \multimap !(!(A) \multimap B)) \,\&\, !(!(!(A) \multimap !(\mathbf{0})) \multimap !(!(A) \multimap B))}{!(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(!(!(A) \multimap !(B)) \multimap !(!(A) \multimap B)) \,\&\, !(!(!(A) \multimap !(\mathbf{0})) \multimap !(!(A) \multimap B))}\ D_R}{\cdot :\ \ !(!(A) \multimap !(\mathbf{0})) \vdash !(!(!(A) \multimap !(B)) \multimap !(!(A) \multimap B)) \,\&\, !(!(!(A) \multimap !(\mathbf{0})) \multimap !(!(A) \multimap B))}\ \star$$

(39) $\neg B \vdash A \to B \leftrightarrow \neg A$

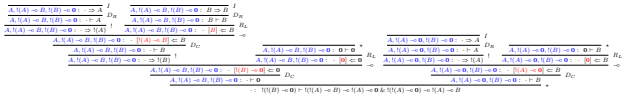
LJ (67ms)

$$\dfrac{\dfrac{}{A, A \to B, B \to \bot \vdash A}\ \star \quad \dfrac{\dfrac{}{A, B, B \to \bot \vdash B}\ \star \quad \dfrac{}{A, B, \bot \vdash \bot}\ \star}{A, B, B \to \bot \vdash \bot}\ \supset_L \quad \dfrac{}{A, A \to \bot, B \to \bot \vdash A}\ \star \quad \dfrac{}{A, \bot, B \to \bot \vdash B}\ \star}{A, A \to B, B \to \bot \vdash \bot}\ \supset_L \quad \dfrac{}{A, A \to \bot, B \to \bot \vdash B}\ \star}{B \to \bot \vdash A \to B \to A \to \bot \wedge A \to \bot \to A \to B}\ \star$$

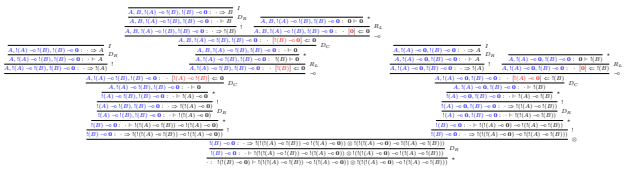
Multiplicative translation (22ms)

fail

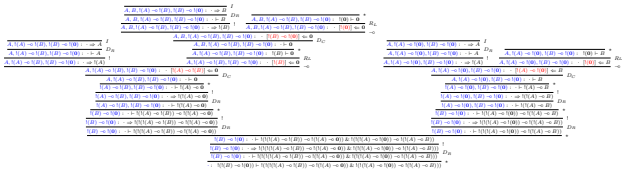
Girard's Translation (271ms)



Positive decoration (595ms)



0/1 focused decoration (626ms)



(40) $B \vdash A \wedge B \leftrightarrow A$

LJ (21ms)

$$\dfrac{}{B \vdash A \wedge B \to A \wedge A \to A \wedge B}\ \star$$

Multiplicative translation (18ms)

fail

Girard's Translation (67ms)

$$\dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \Rightarrow A}\ I}{A, B :\ \cdot \vdash A}\ D_R \quad \dfrac{\dfrac{}{A, B :\ \cdot \Rightarrow A}\ I}{A, B :\ \cdot \vdash A}\ D_R \quad \dfrac{\dfrac{}{A, B :\ \cdot \Rightarrow B}\ I}{A, B :\ \cdot \vdash B}\ D_R}{\cdot :\ !(B) \vdash !(A \,\&\, B) \multimap A \,\&\, !(A) \multimap A \,\&\, B}\ \star$$

Positive decoration (165ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \Rightarrow A}\ I}{A, B :\ \cdot \vdash A}\ D_R}{A, B :\ \cdot \Rightarrow !(A)}\ !}{A, B :\ \cdot \vdash !(A)}\ D_R}{B :\ \cdot \vdash !(A) \otimes !(B) \multimap !(A)}\ \star}{B :\ \cdot \Rightarrow !(!(A) \otimes !(B) \multimap !(A))}\ ! \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \Rightarrow A}\ I}{A, B :\ \cdot \vdash A}\ D_R}{A, B :\ \cdot \Rightarrow !(A)}\ !}{A, B :\ \cdot \vdash !(A) \otimes !(B)}\ D_R}{B :\ \cdot \vdash !(A) \multimap !(A) \otimes !(B)}\ \star}{B :\ \cdot \Rightarrow !(!(A) \multimap !(A) \otimes !(B))}\ !}{B :\ \cdot \Rightarrow !(!(A) \otimes !(B) \multimap !(A)) \otimes !(!(A) \multimap !(A) \otimes !(B))}\ \otimes$$
$$\dfrac{B :\ \cdot \vdash !(!(A) \otimes !(B) \multimap !(A)) \otimes !(!(A) \multimap !(A) \otimes !(B))}{\cdot :\ !(B) \vdash !(!(A) \otimes !(B) \multimap !(A)) \otimes !(!(A) \multimap !(A) \otimes !(B))}\ D_R \quad \star$$

0/1 focused decoration (178ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, B, !(A), !(B) :\ \cdot \Rightarrow A}\ I}{A, B, !(A), !(B) :\ \cdot \vdash A}\ D_R}{B, !(A), !(B) :\ !(A) \vdash A}\ \star}{B, !(A), !(B) :\ \cdot\ [!(A)] \Leftarrow A}\ R_L}{B, !(A), !(B) :\ \cdot \vdash A}\ D_C}{B :\ \cdot \vdash !(!(A) \,\&\, !(B)) \multimap A}\ \star}{B :\ \cdot \Rightarrow !(!(!(A) \,\&\, !(B)) \multimap A)}\ ! \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \Rightarrow A}\ I}{A, B :\ \cdot \vdash A}\ D_R}{A, B :\ \cdot \vdash A \,\&\, B}\ \star}{A, B :\ \cdot \Rightarrow !(A \,\&\, B)}\ !}{A, B :\ \cdot \vdash !(A \,\&\, B)}\ D_R}{B :\ \cdot \vdash !(A) \multimap !(A \,\&\, B)}\ \star}{B :\ \cdot \Rightarrow !(!(A) \multimap !(A \,\&\, B))}\ !}{B :\ \cdot \Rightarrow !(!(!(A) \,\&\, !(B)) \multimap A) \,\&\, !(!(A) \multimap !(A \,\&\, B)))}\ \&$$
$$\dfrac{B :\ \cdot \vdash !(!(!(A) \,\&\, !(B)) \multimap A) \,\&\, !(!(A) \multimap !(A \,\&\, B)))}{\cdot :\ !(B) \vdash !(!(!(A) \,\&\, !(B)) \multimap A) \,\&\, !(!(A) \multimap !(A \,\&\, B)))}\ D_R \quad \star$$

(41) $\neg B \vdash A \wedge B \leftrightarrow B$

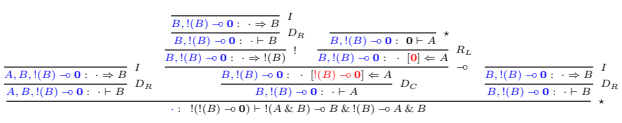
LJ (33ms)

$$\dfrac{\dfrac{\dfrac{}{B, B \to \bot \vdash B}\,\star \quad \dfrac{}{B, \bot \vdash A}\,\star}{B, B \to \bot \vdash A}\,\supset_L}{B \to \bot \vdash A \wedge B \to B \wedge B \to A \wedge B}\,\star$$

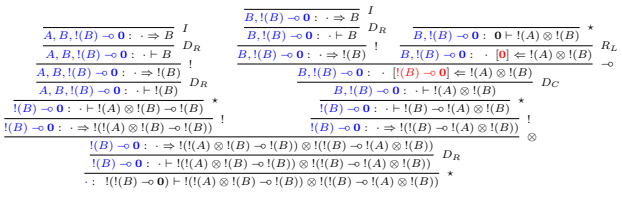
Multiplicative translation (21ms)

fail

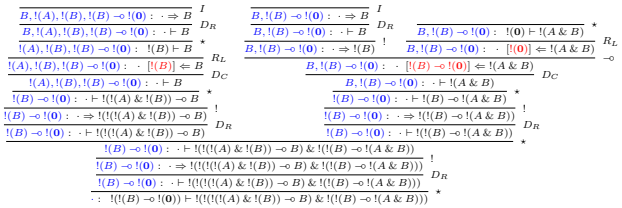
Girard's Translation (130ms)



Positive decoration (172ms)



0/1 focused decoration (209ms)



(42) $\cdot \vdash A \to \neg\neg A$

LJ (41ms)

$$\dfrac{\dfrac{\dfrac{}{A, A \to \bot \vdash A}\,\star \quad \dfrac{}{A, \bot \vdash \bot}\,\star}{A, A \to \bot \vdash \bot}\,\supset_L}{\cdot \vdash A \to A \to \bot \to \bot}\,\star$$

Multiplicative translation (61ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\cdot :\ A \Rightarrow A}\,I \quad \dfrac{}{\cdot :\ \cdot\ [\bot] \Leftarrow \bot}\,\bot}{\cdot :\ A\ [A \multimap \bot] \Leftarrow \bot}\,\multimap}{\cdot :\ A, A \multimap \bot \vdash \bot}\,D_L}{\cdot :\ \cdot \vdash A \multimap A \multimap \bot \multimap \bot}\,\star$$

Girard's Translation (83ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \Rightarrow A}\,I}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}\,D_R}{A, !(A) \multimap \mathbf{0} :\ \cdot \Rightarrow\ !(A)}\,! \quad \dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \mathbf{0} \vdash \mathbf{0}}\,\star}{A, !(A) \multimap \mathbf{0} :\ \cdot\ [\mathbf{0}] \Leftarrow \mathbf{0}}\,R_L}{A, !(A) \multimap \mathbf{0} :\ \cdot\ [!(A) \multimap \mathbf{0}] \Leftarrow \mathbf{0}}\,\multimap}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash \mathbf{0}}\,D_C}{\cdot :\ \cdot \vdash\ !(A) \multimap\ !(!(A) \multimap \mathbf{0}) \multimap \mathbf{0}}\,\star$$

Positive decoration (120ms)

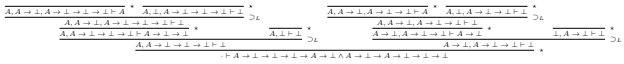
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \Rightarrow A}\,I}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}\,D_R}{A, !(A) \multimap \mathbf{0} :\ \cdot \Rightarrow\ !(A)}\,! \quad \dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \mathbf{0} \vdash \mathbf{0}}\,\star}{A, !(A) \multimap \mathbf{0} :\ \cdot\ [\mathbf{0}] \Leftarrow \mathbf{0}}\,R_L}{A, !(A) \multimap \mathbf{0} :\ \cdot\ [!(A) \multimap \mathbf{0}] \Leftarrow \mathbf{0}}\,\multimap}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash \mathbf{0}}\,D_C}{A :\ \cdot \vdash\ !(!(A) \multimap \mathbf{0}) \multimap \mathbf{0}}\,\star}{A :\ \cdot \Rightarrow\ !(!(A) \multimap \mathbf{0}) \multimap \mathbf{0})}\,!}{A :\ \cdot \vdash\ !(!(!(A) \multimap \mathbf{0}) \multimap \mathbf{0})}\,D_R}{\cdot :\ \cdot \vdash\ !(A) \multimap\ !(!(!(A) \multimap \mathbf{0}) \multimap \mathbf{0})}\,\star}{\cdot :\ \cdot \Rightarrow\ !(!(A) \multimap\ !(!(!(A) \multimap \mathbf{0}) \multimap \mathbf{0}))}\,!}{\cdot :\ \cdot \vdash\ !(!(A) \multimap\ !(!(!(A) \multimap \mathbf{0}) \multimap \mathbf{0}))}\,D_R$$

0/1 focused decoration (122ms)

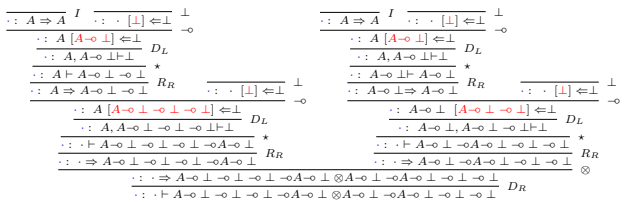
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \Rightarrow A}\,I}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash A}\,D_R}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \Rightarrow\ !(A)}\,! \quad \dfrac{\dfrac{}{A, !(A) \multimap !(\mathbf{0}) :\ !(\mathbf{0}) \vdash \mathbf{0}}\,\star}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot\ [!(\mathbf{0})] \Leftarrow \mathbf{0}}\,R_L}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot\ [!(A) \multimap !(\mathbf{0})] \Leftarrow \mathbf{0}}\,\multimap}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash \mathbf{0}}\,D_C}{A :\ \cdot \vdash\ !(!(A) \multimap !(\mathbf{0})) \multimap \mathbf{0}}\,\star}{A :\ \cdot \Rightarrow\ !(!(!(A) \multimap !(\mathbf{0})) \multimap \mathbf{0})}\,!}{A :\ \cdot \vdash\ !(!(!(A) \multimap !(\mathbf{0})) \multimap \mathbf{0})}\,D_R}{\cdot :\ \cdot \vdash\ !(A) \multimap\ !(!(!(A) \multimap !(\mathbf{0})) \multimap \mathbf{0})}\,\star}{\cdot :\ \cdot \Rightarrow\ !(!(A) \multimap\ !(!(!(A) \multimap !(\mathbf{0})) \multimap \mathbf{0}))}\,!}{\cdot :\ \cdot \vdash\ !(!(A) \multimap\ !(!(!(A) \multimap !(\mathbf{0})) \multimap \mathbf{0}))}\,D_R$$

(43) $\cdot \vdash \neg\neg\neg A \leftrightarrow \neg A$

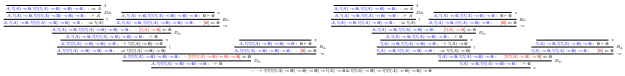
LJ (96ms)



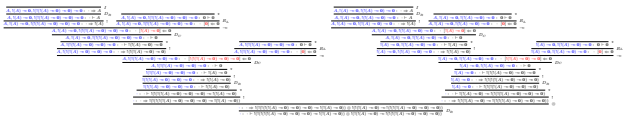
Multiplicative translation (183ms)



Girard's Translation (271ms)



Positive decoration (528ms)



0/1 focused decoration (7431ms)

(44) $\cdot \vdash \neg(A \wedge \neg A)$

LJ (34ms)

$$\dfrac{\dfrac{}{A, A \to \bot \vdash A}\ \star \qquad \dfrac{}{A, \bot \vdash \bot}\ \star}{\dfrac{A, A \to \bot \vdash \bot}{\cdot \vdash A \wedge A \to \bot \to \bot}\ \star}\ \supset_L$$

Multiplicative translation (59ms)

$$\dfrac{\dfrac{\dfrac{}{\cdot :\ A \Rightarrow A}\ I \qquad \dfrac{}{\cdot :\ \cdot\ [\bot] \Leftarrow \bot}\ \bot}{\cdot :\ A\ [A \multimap \bot] \Leftarrow \bot}\ \multimap}{\cdot :\ A, A \multimap \bot \vdash \bot}\ D_L}{\cdot :\ \cdot \vdash A \otimes A \multimap \bot \multimap \bot}\ \star$$

Girard's Translation (88ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \Rightarrow A}\ I}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}\ D_R}{A, !(A) \multimap \mathbf{0} :\ \cdot \Rightarrow\ !(A)}\ ! \qquad \dfrac{}{A, !(A) \multimap \mathbf{0} :\ \mathbf{0} \vdash \mathbf{0}}\ \star}{A, !(A) \multimap \mathbf{0} :\ \cdot\ [\mathbf{0}] \Leftarrow \mathbf{0}}\ R_L \multimap}{A, !(A) \multimap \mathbf{0} :\ \cdot\ [!(A) \multimap \mathbf{0}] \Leftarrow \mathbf{0}}\ D_C}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash \mathbf{0}}$$

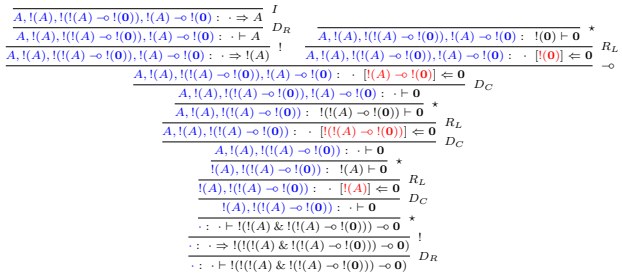
$$\dfrac{}{\cdot :\ \cdot \vdash\ !(A\ \&\ !(A) \multimap \mathbf{0}) \multimap \mathbf{0}}\ \star$$

Positive decoration (102ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \Rightarrow A}\ I}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}\ D_R}{A, !(A) \multimap \mathbf{0} :\ \cdot \Rightarrow\ !(A)}\ ! \qquad \dfrac{}{A, !(A) \multimap \mathbf{0} :\ \mathbf{0} \vdash \mathbf{0}}\ \star}{A, !(A) \multimap \mathbf{0} :\ \cdot\ [\mathbf{0}] \Leftarrow \mathbf{0}}\ R_L \multimap}{A, !(A) \multimap \mathbf{0} :\ \cdot\ [!(A) \multimap \mathbf{0}] \Leftarrow \mathbf{0}}\ D_C}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash \mathbf{0}}$$

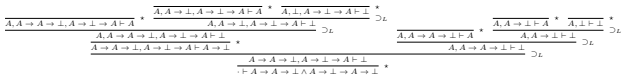
$$\dfrac{\dfrac{\dfrac{\dfrac{}{\cdot :\ \cdot \vdash\ !(A) \otimes\ !(!(A) \multimap \mathbf{0}) \multimap \mathbf{0}}\ \star}{\cdot :\ \cdot \Rightarrow\ !(!(A) \otimes\ !(!(A) \multimap \mathbf{0}) \multimap \mathbf{0})}\ !}{\cdot :\ \cdot \vdash\ !(!(A) \otimes\ !(!(A) \multimap \mathbf{0}) \multimap \mathbf{0})}\ D_R$$

0/1 focused decoration (141ms)



$(45)\ \cdot \vdash \neg(A \leftrightarrow \neg A)$

LJ (96ms)



Multiplicative translation (19ms)

fail

Girard's Translation (Timeout!)

Timeout!

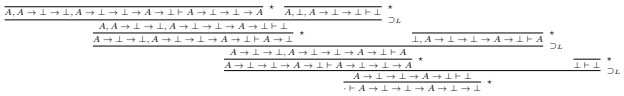
Positive decoration (5241ms)

0/1 focused decoration (Timeout!)

Timeout!

$(46)\ \cdot \vdash \neg\neg(\neg\neg A \to A)$

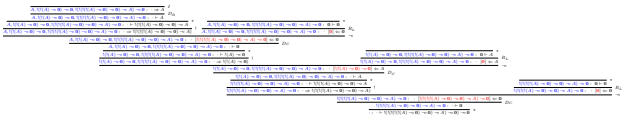
LJ (66ms)



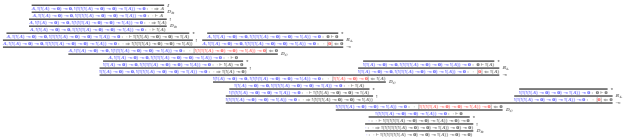
Multiplicative translation (25ms)

fail

Girard's Translation (185ms)



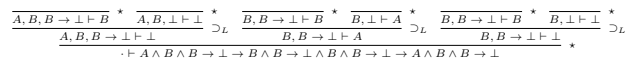
Positive decoration (250ms)



0/1 focused decoration (427ms)

(47) $\cdot \vdash A \wedge (B \wedge \neg B) \leftrightarrow B \wedge \neg B$

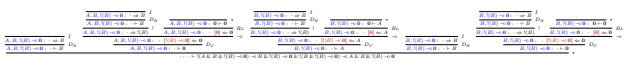
LJ (61ms)



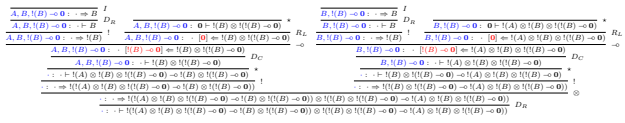
Multiplicative translation (19ms)

fail

Girard's Translation (234ms)



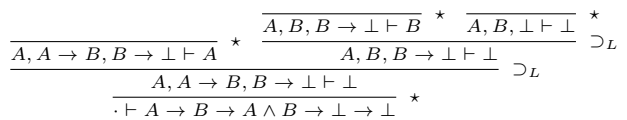
Positive decoration (186ms)



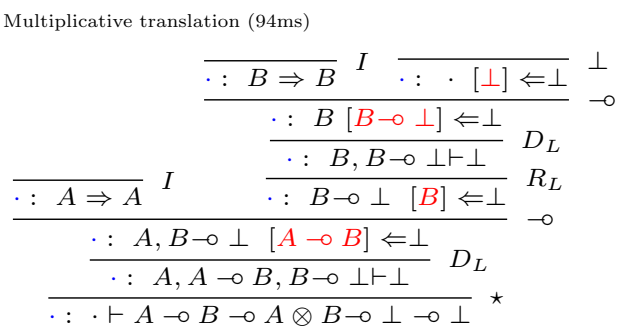
0/1 focused decoration (875ms)

(48) $\cdot \vdash (A \to B) \to \neg(A \wedge \neg B)$

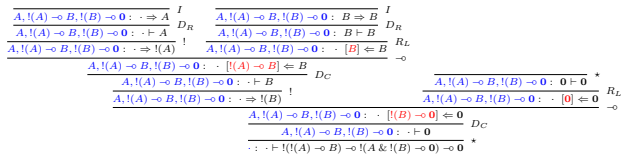
LJ (48ms)



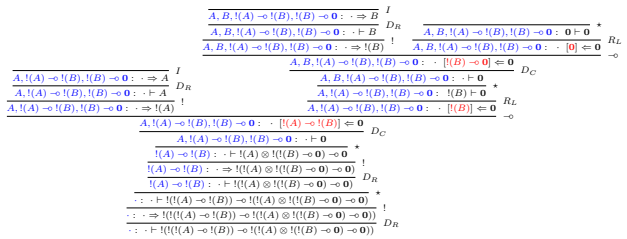
Multiplicative translation (94ms)



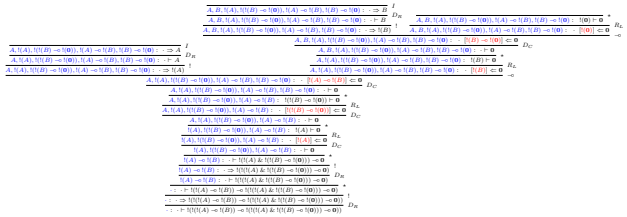
Girard's Translation (146ms)



Positive decoration (181ms)

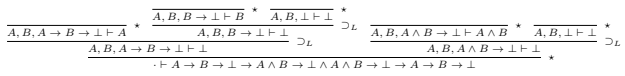


0/1 focused decoration (370ms)

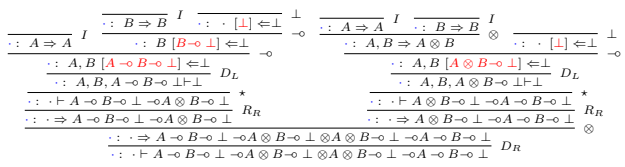


(49) $\cdot \vdash (A \to \neg B) \leftrightarrow (\neg(A \wedge B))$

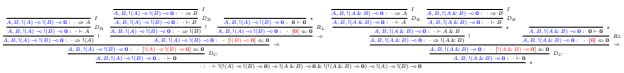
LJ (66ms)



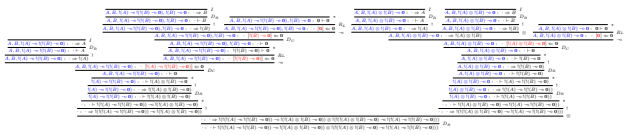
Multiplicative translation (161ms)



Girard's Translation (204ms)



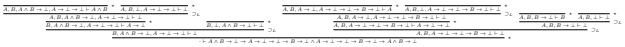
Positive decoration (535ms)



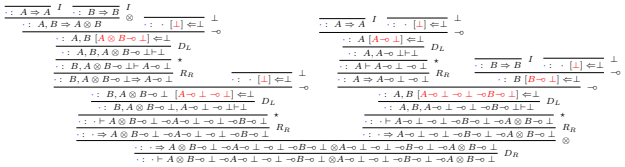
0/1 focused decoration (46292ms)

(50) $\cdot \vdash (\neg(A \wedge B)) \leftrightarrow ((\neg\neg A) \to \neg B)$

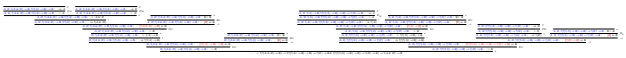
LJ (94ms)



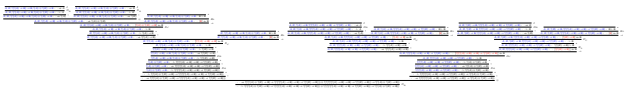
Multiplicative translation (245ms)



Girard's Translation (2618ms)



Positive decoration (18580ms)



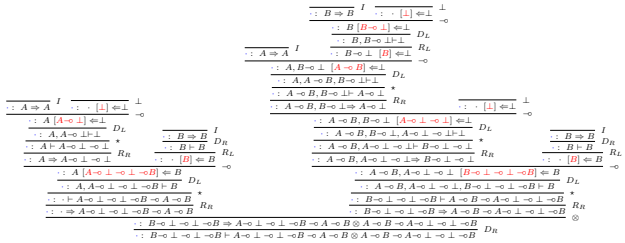
0/1 focused decoration (Timeout!)

Timeout!

(51) $\neg\neg B \to B \vdash (\neg\neg A \to B) \leftrightarrow (A \to B)$

LJ (882ms)

Multiplicative translation (295ms)



Girard's Translation (Timeout!)

Timeout!

Positive decoration (Timeout!)

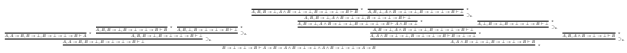
Timeout!

0/1 focused decoration (Timeout!)

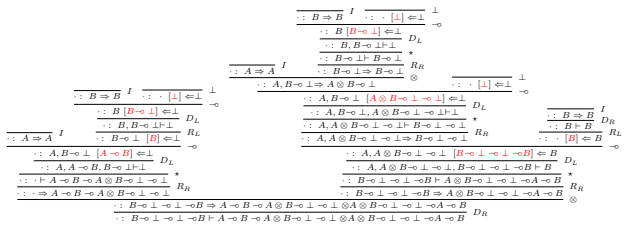
Timeout!

(52) $\neg\neg B \to B \vdash (A \to B) \leftrightarrow \neg(A \wedge \neg B)$

LJ (112ms)



Multiplicative translation (257ms)



Girard's Translation (Timeout!)

Timeout!

Positive decoration (Timeout!)

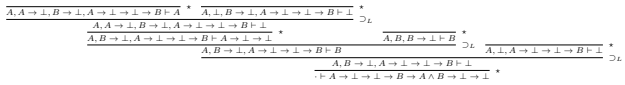
Timeout!

0/1 focused decoration (Timeout!)

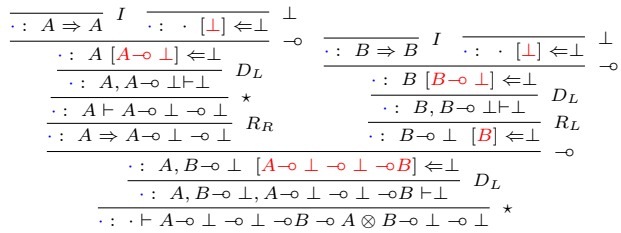
Timeout!

(53) $\cdot \vdash (\neg\neg A \to B) \to \neg(A \wedge \neg B)$

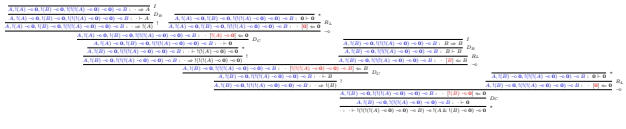
LJ (67ms)



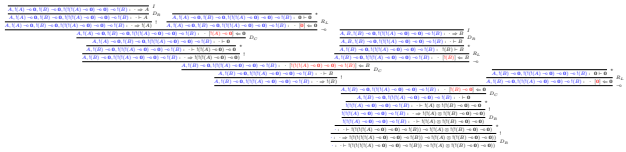
Multiplicative translation (126ms)



Girard's Translation (255ms)



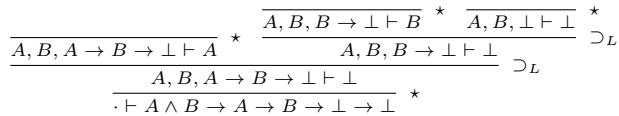
Positive decoration (335ms)



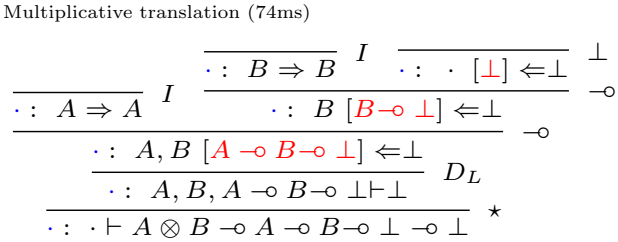
0/1 focused decoration (14764ms)

(54) $\cdot \vdash A \wedge B \to \neg(A \to \neg B)$

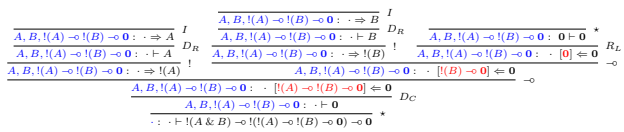
LJ (49ms)



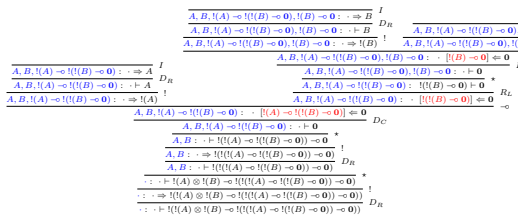
Multiplicative translation (74ms)



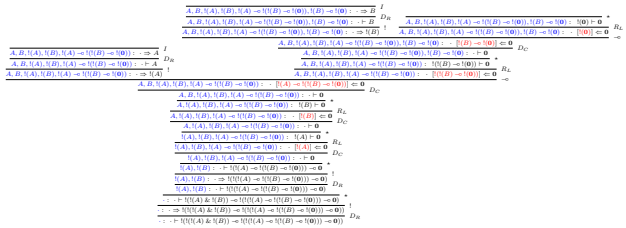
Girard's Translation (115ms)



Positive decoration (170ms)

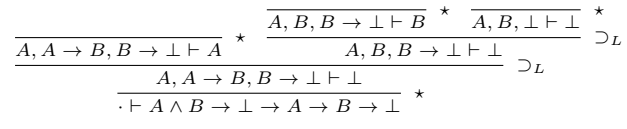


0/1 focused decoration (268ms)

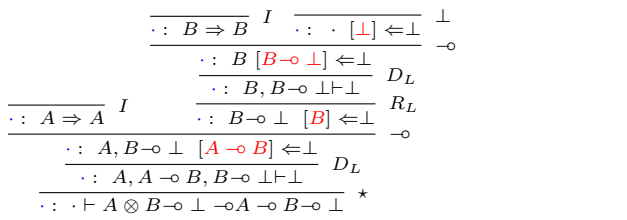


(55) $\cdot \vdash (A \wedge \neg B) \to \neg(A \to B)$

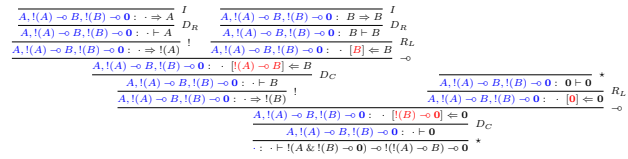
LJ (49ms)



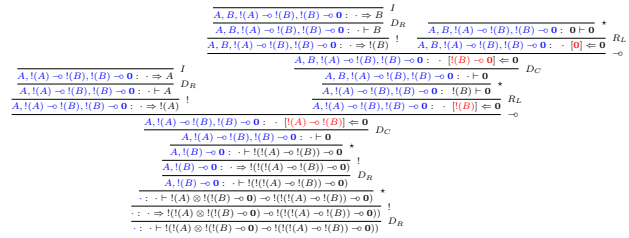
Multiplicative translation (92ms)



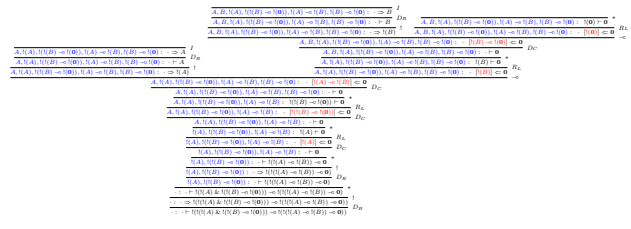
Girard's Translation (136ms)



Positive decoration (187ms)

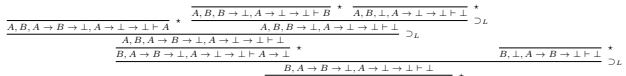


0/1 focused decoration (345ms)

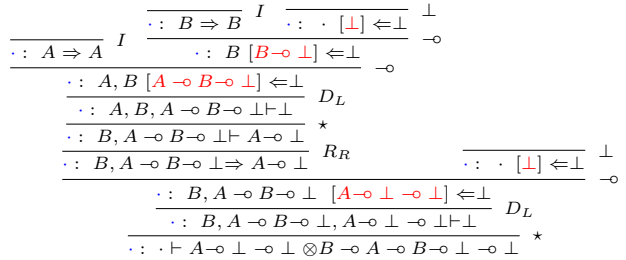


(56) $\cdot \vdash \neg\neg A \wedge B \to \neg(A \to \neg B)$

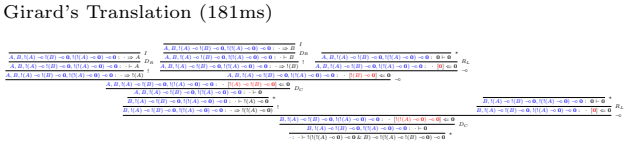
LJ (64ms)



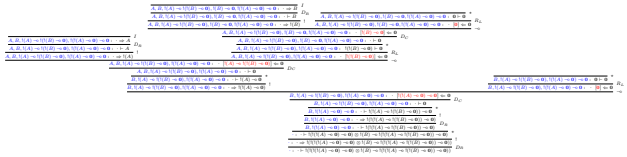
Multiplicative translation (97ms)



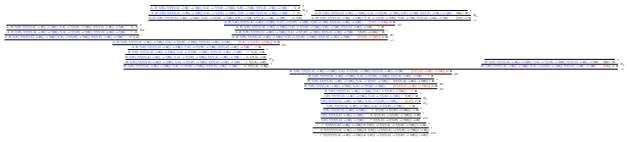
Girard's Translation (181ms)



Positive decoration (253ms)

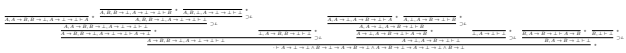


0/1 focused decoration (3946ms)



(57) $\cdot \vdash (\neg\neg A \wedge \neg B) \leftrightarrow (\neg(A \to B))$

LJ (385ms)



Multiplicative translation (20ms)

fail

Girard's Translation (Timeout!)

Timeout!

Positive decoration (Timeout!)

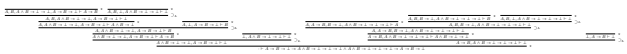
Timeout!

0/1 focused decoration (Timeout!)

Timeout!

(58) $\cdot \vdash (\neg(A \to B)) \leftrightarrow (\neg\neg(A \wedge \neg B))$

LJ (118ms)



Multiplicative translation (20ms)

fail

Girard's Translation (Timeout!)

Timeout!

Positive decoration (Timeout!)

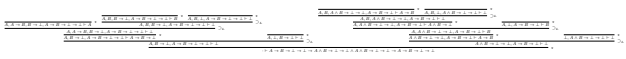
Timeout!

0/1 focused decoration (Timeout!)

Timeout!

$(59)\ \cdot \vdash (\neg\neg(A \to B)) \leftrightarrow ((\neg(A \wedge \neg B)))$

LJ (168ms)



Multiplicative translation (20ms)

fail

Girard's Translation (Timeout!)

Timeout!

Positive decoration (Timeout!)

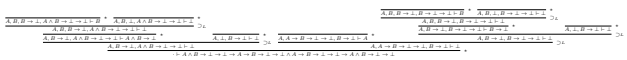
Timeout!

0/1 focused decoration (Timeout!)

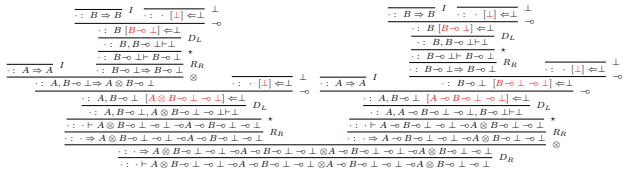
Timeout!

$(60)\ \cdot \vdash (\neg(A \wedge \neg B)) \leftrightarrow ((A \to \neg\neg B))$

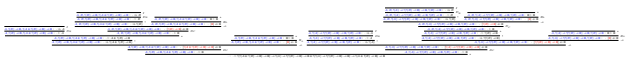
LJ (96ms)



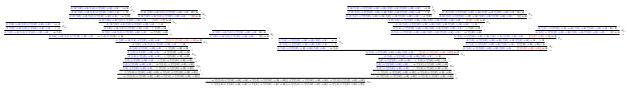
Multiplicative translation (214ms)



Girard's Translation (4004ms)



Positive decoration (8427ms)

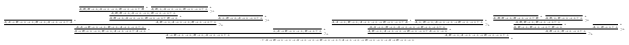


0/1 focused decoration (Timeout!)

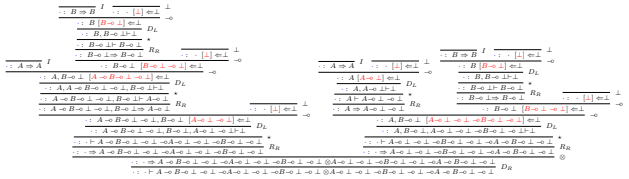
Timeout!

$$\boxed{(61)\ \cdot \vdash (A \to \neg\neg B) \leftrightarrow ((\neg\neg A) \to \neg\neg B)}$$

LJ (9785ms)



Multiplicative translation (288ms)



Girard's Translation (Timeout!)

Timeout!

Positive decoration (Timeout!)

Timeout!

0/1 focused decoration (Timeout!)

Timeout!

2 Alternative Translation

(10) $\cdot : A \multimap 0 \vdash A \multimap B$

ILL (106ms)

$$\dfrac{\dfrac{\dfrac{}{\cdot :\ A \Rightarrow A}\ I \quad \dfrac{\dfrac{}{\cdot :\ \mathbf{0} \vdash B}\ \star}{\cdot :\ \cdot\ [\mathbf{0}] \Leftarrow B}\ R_L}{\cdot :\ A\ [A \multimap \mathbf{0}] \Leftarrow B}\ \multimap}{\dfrac{\cdot :\ A, A \multimap \mathbf{0} \vdash B}{\cdot :\ A \multimap \mathbf{0} \vdash A \multimap B}\ \star}\ D_L$$

(11) $\cdot : A \vdash (A \multimap 0) \multimap B$

ILL (105ms)

$$\dfrac{\dfrac{\dfrac{}{\cdot :\ A \Rightarrow A}\ I \quad \dfrac{\dfrac{}{\cdot :\ \mathbf{0} \vdash B}\ \star}{\cdot :\ \cdot\ [\mathbf{0}] \Leftarrow B}\ R_L}{\cdot :\ A\ [A \multimap \mathbf{0}] \Leftarrow B}\ \multimap}{\dfrac{\cdot :\ A, A \multimap \mathbf{0} \vdash B}{\cdot :\ A \vdash A \multimap \mathbf{0} \multimap B}\ \star}\ D_L$$

(12) $\cdot : B \vdash (!A) \multimap B$

ILL (73ms)

$$\dfrac{\dfrac{\dfrac{}{A :\ B \Rightarrow B}\ I}{A :\ B \vdash B}\ D_R}{\cdot :\ B \vdash\ !(A) \multimap B}\ \star$$

(16) $\cdot : (A \multimap B) \otimes !(B \multimap A) \vdash A \multimap B$

ILL (131ms)

$$\dfrac{\dfrac{\dfrac{}{B \multimap A :\ A \Rightarrow A}\ I \quad \dfrac{\dfrac{\dfrac{}{B \multimap A :\ B \Rightarrow B}\ I}{B \multimap A :\ B \vdash B}\ D_R}{B \multimap A :\ \cdot\ [B] \Leftarrow B}\ R_L}{B \multimap A :\ A\ [A \multimap B] \Leftarrow B}\ \multimap}{\dfrac{B \multimap A :\ A, A \multimap B \vdash B}{\cdot :\ A \multimap B \otimes\ !(B \multimap A) \vdash A \multimap B}\ \star}\ D_L$$

(17) $\cdot : !(A \multimap B) \otimes (B \multimap A) \vdash B \multimap A$

ILL (129ms)

$$\dfrac{\dfrac{\dfrac{}{A \multimap B :\ B \Rightarrow B}\ I \quad \dfrac{\dfrac{\dfrac{}{A \multimap B :\ A \Rightarrow A}\ I}{A \multimap B :\ A \vdash A}\ D_R}{A \multimap B :\ \cdot\ [A] \Leftarrow A}\ R_L}{A \multimap B :\ B\ [B \multimap A] \Leftarrow A}\ \multimap}{\dfrac{A \multimap B :\ B, B \multimap A \vdash A}{\cdot :\ !(A \multimap B) \otimes B \multimap A \vdash B \multimap A}\ \star}\ D_L$$

(18) $\cdot : (A \multimap\!\!\circ\ B), A \vdash B \otimes (B \multimap A)$

ILL (262ms)

$$\dfrac{\dfrac{\dfrac{}{\cdot :\ A \Rightarrow A}\ I \quad \dfrac{\dfrac{\dfrac{}{\cdot :\ B \Rightarrow B}\ I \quad \dfrac{\dfrac{\dfrac{}{\cdot :\ A \Rightarrow A}\ I}{\cdot :\ A \vdash A}\ D_R}{\cdot :\ \cdot\ [A] \Leftarrow A}\ R_L}{\cdot :\ B\ [B \multimap A] \Leftarrow A}\ \multimap}{\cdot :\ B, B \multimap A \vdash A}\ D_L}{\dfrac{\cdot :\ B \multimap A \vdash B \multimap A}{\cdot :\ B \multimap A \Rightarrow B \multimap A}\ \star}\ R_R}{\dfrac{\cdot :\ B, B \multimap A \Rightarrow B \otimes B \multimap A}{\cdot :\ B, B \multimap A \vdash B \otimes B \multimap A}\ D_R}\ \otimes}{\dfrac{\dfrac{\dfrac{}{\cdot :\ A \Rightarrow A}\ I \quad \dfrac{\cdot :\ B \multimap A\ [B] \Leftarrow B \otimes B \multimap A}{\cdot :\ A, B \multimap A\ [A \multimap B] \Leftarrow B \otimes B \multimap A}\ D_L}{\cdot :\ A, A \multimap B, B \multimap A \vdash B \otimes B \multimap A}\ \star}{\cdot :\ A, A \multimap B \otimes B \multimap A \vdash B \otimes B \multimap A}\ R_L}\ \multimap$$

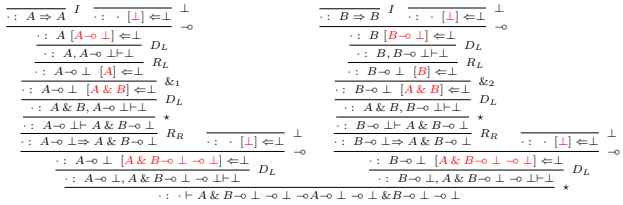
(19) $\cdot : (A \multimap\!\!\circ\ B), B \vdash A \otimes (A \multimap B)$

ILL (245ms)

$$\dfrac{\dfrac{\dfrac{}{\cdot :\ B \Rightarrow B}\ I \quad \dfrac{\dfrac{\dfrac{}{\cdot :\ A \Rightarrow A}\ I \quad \dfrac{\dfrac{\dfrac{}{\cdot :\ B \Rightarrow B}\ I}{\cdot :\ B \vdash B}\ D_R}{\cdot :\ \cdot\ [B] \Leftarrow B}\ R_L}{\cdot :\ A\ [A \multimap B] \Leftarrow B}\ \multimap}{\cdot :\ A, A \multimap B \vdash B}\ D_L}{\dfrac{\cdot :\ A \multimap B \vdash A \multimap B}{\cdot :\ A \multimap B \Rightarrow A \multimap B}\ \star}\ R_R}{\dfrac{\cdot :\ A, A \multimap B \Rightarrow A \otimes A \multimap B}{\cdot :\ A, A \multimap B \vdash A \otimes A \multimap B}\ D_R}\ \otimes}{\dfrac{\dfrac{\dfrac{}{\cdot :\ A \Rightarrow A}\ I \quad \dfrac{\cdot :\ A \multimap B\ [A] \Leftarrow A \otimes A \multimap B}{\cdot :\ B, A \multimap B\ [B \multimap A] \Leftarrow A \otimes A \multimap B}\ D_L}{\cdot :\ B, A \multimap B, B \multimap A \vdash A \otimes A \multimap B}\ \star}{\cdot :\ B, A \multimap B \otimes B \multimap A \vdash A \otimes A \multimap B}\ R_L}\ \multimap$$

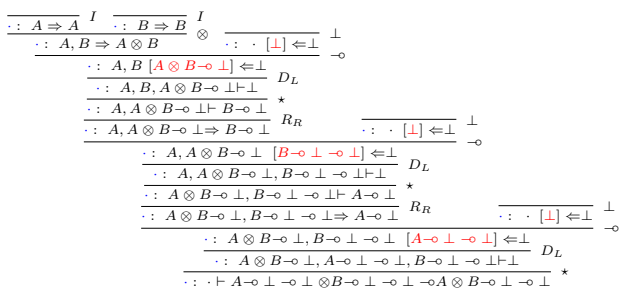
(26a) $\cdot : \cdot \vdash (\neg\neg(A \,\&\, B)) \multimap ((\neg\neg A) \,\&\, \neg\neg B)$

ILL (345ms)



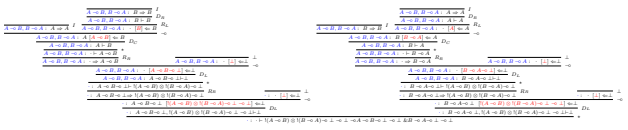
(26b) $\cdot : \cdot \vdash ((\neg\neg A) \otimes (\neg\neg B)) \multimap \neg\neg(A \otimes B)$

ILL (234ms)



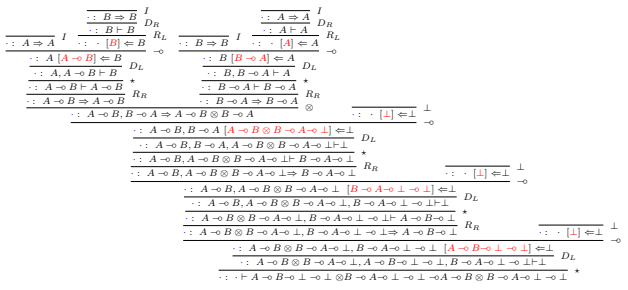
(27a) $\cdot : \cdot \vdash (\neg\neg(!(A \multimap B) \otimes !(B \multimap A))) \multimap ((\neg\neg(A \multimap B)) \,\&\, (\neg\neg(B \multimap A)))$

ILL (516ms)



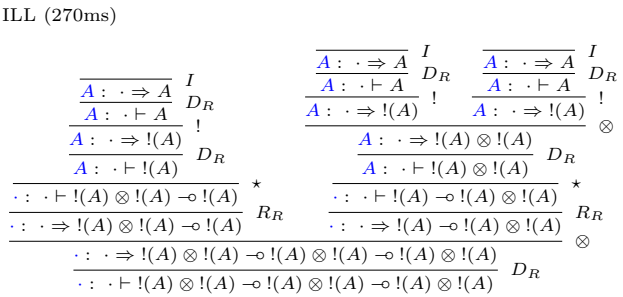
(27b) $\cdot : \cdot \vdash ((\neg\neg(A \multimap B)) \otimes (\neg\neg(B \multimap A))) \multimap (\neg\neg(A \multimap\!\circ B))$

ILL (449ms)



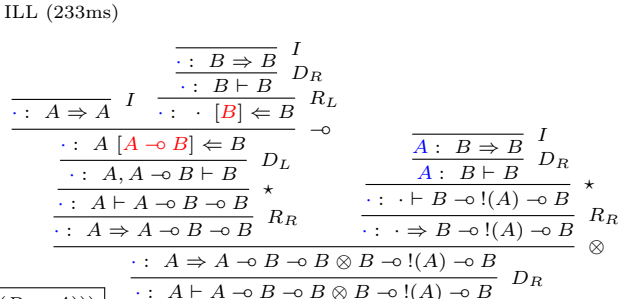
(35) $\cdot : \cdot \vdash ((!A) \otimes (!A)) \circ\!\multimap !A$

ILL (270ms)



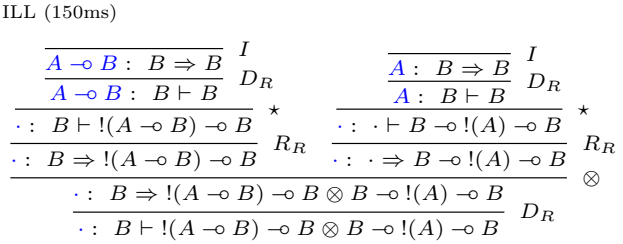
(36) $\cdot : A \vdash ((A \multimap B) \multimap B) \otimes (B \multimap ((!A) \multimap B))$

ILL (233ms)



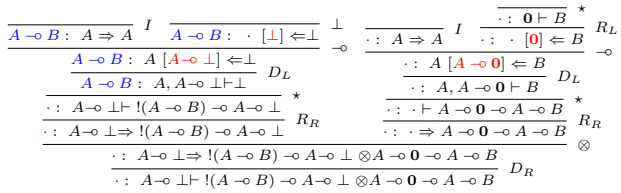
(37) $\cdot : B \vdash ((!(A \multimap B)) \multimap B) \otimes (B \multimap ((!A) \multimap B))$

ILL (150ms)



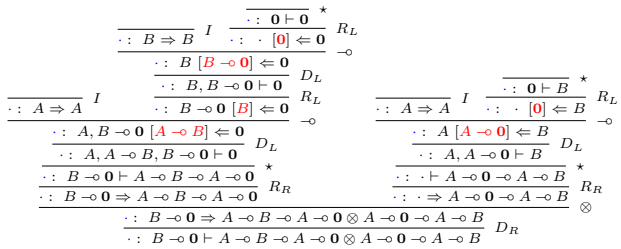
(38) $\cdot \neg A \vdash ((!(A \multimap B) \multimap \neg A)) \otimes ((A \multimap 0) \multimap (A \multimap B))$

ILL (221ms)



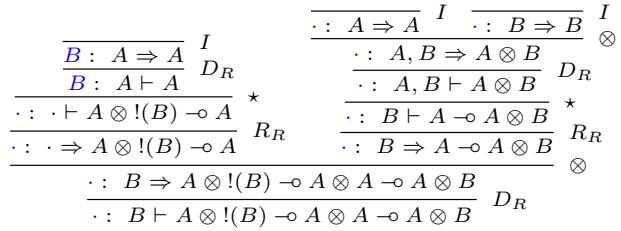
(39) $\cdot B \multimap 0 \vdash (A \multimap B) \multimap\!\circ (A \multimap 0)$

ILL (310ms)



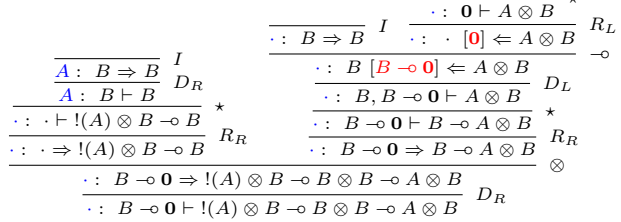
(40) $\cdot : B \vdash ((A \otimes !B) \multimap A) \otimes (A \multimap (A \otimes B))$

ILL (207ms)



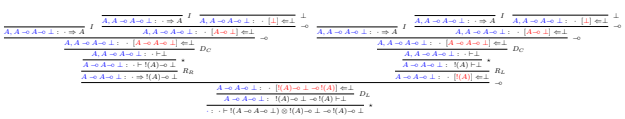
(41) $\cdot : B \multimap 0 \vdash (((!A) \otimes B) \multimap B) \otimes (B \multimap (A \otimes B))$

ILL (193ms)



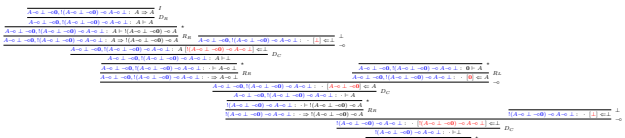
(45) $\cdot : \cdot \vdash \neg((!(A \multimap \neg A)) \otimes ((\neg(!A)) \multimap\!\circ !A))$

ILL (243ms)



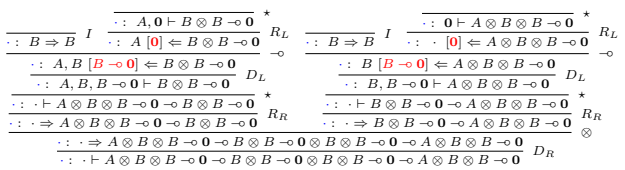
(46) $\cdot \vdash \neg(!(\neg(!(\neg A \multimap 0) \multimap A)))$

ILL (318ms)



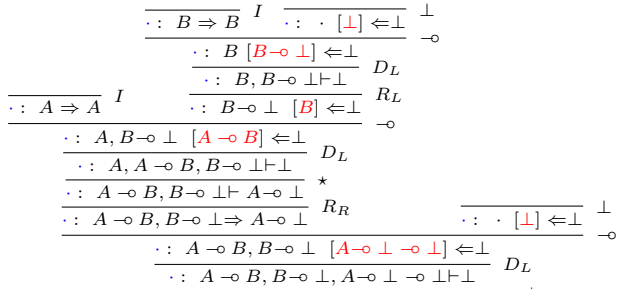
(47) $\cdot : \cdot \vdash (A \otimes (B \otimes (B \multimap 0))) \multimap\!\circ (B \otimes (B \multimap 0))$

ILL (266ms)



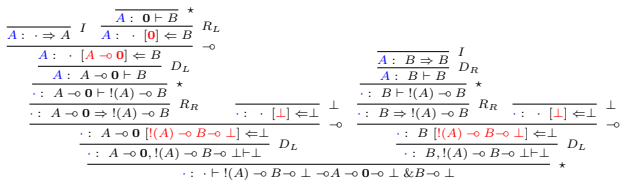
(57a) $\cdot : \cdot \vdash ((\neg\neg A) \otimes (\neg B)) \multimap \neg(A \multimap B)$

ILL (199ms)



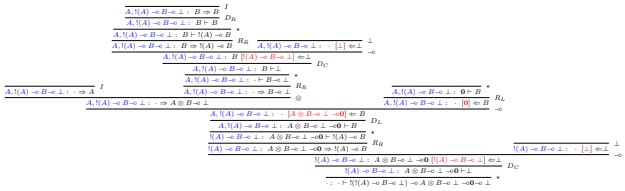
(57b) $\cdot : \cdot \vdash (\neg((!A) \multimap B)) \multimap ((\neg(A \multimap 0)) \,\&\, \neg B)$

ILL (257ms)



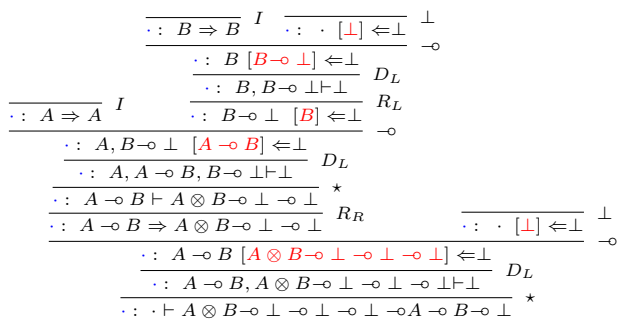
(58a) $\cdot : \cdot \vdash (!(\neg((!A) \multimap B))) \multimap \neg((A \otimes \neg B) \multimap 0)$

ILL (315ms)



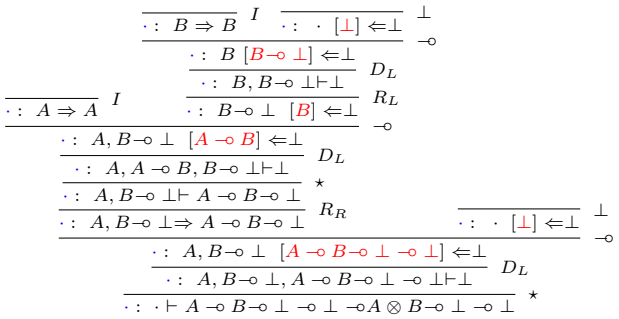
(58b) $\cdot : \cdot \vdash (\neg\neg(A \otimes (\neg B))) \multimap \neg(A \multimap B)$

ILL (214ms)



(59a) $\cdot : \cdot \vdash (\neg\neg(A \multimap B)) \multimap (\neg(A \otimes (\neg B)))$

ILL (196ms)



(59b) $\cdot : \cdot \vdash ((A \otimes \neg B) \multimap 0) \multimap (\neg(!(\neg(!A \multimap B))))$

ILL (322ms)