Benchmarking Linear Logic: sequents inspired from Kleene's intuitionistic theorems *

September 17, 2018

1 General Information

- Test run on a MacBook Pro, 2,4 GHz Intel Core i7, 8GB RAM.
- Maude version: 2.7.1 built: Oct 2 2017.
- The search procedure considers proofs using up to 4 times the copy rule (focusing on one of the formulas of the classical context). Benchmarks 23 and 25 required the limit to be 5.

2 Translations

$\boxed{(1)\ \cdot \vdash A \to A}$

LJ (28ms)

$$\frac{}{\cdot \vdash A \to A}\ \star$$

MULTIPLICATIVE encoding (28ms)

$$\frac{\dfrac{}{\cdot :\ A \vdash A}}{\cdot :\ \cdot \vdash A \multimap A}\ \star$$

CALL-BY-NAME encoding (28ms)

$$\frac{\dfrac{}{A :\ \cdot \vdash A}}{\cdot :\ \cdot \vdash\ !(A) \multimap A}\ \star$$

CALL-BY-VALUE encoding (41ms)

$$\frac{\dfrac{\dfrac{}{A :\ \cdot \vdash A}}{A :\ \cdot \vdash\ !(A)}\ !}{\cdot :\ \cdot \vdash\ !(A) \multimap\ !(A)}\ \star}{\cdot :\ \cdot \vdash\ !(!(A) \multimap\ !(A))}\ !$$

01-ENC encoding (35ms)

$$\frac{\dfrac{}{A :\ \cdot \vdash A}}{\cdot :\ \cdot \vdash\ !(A) \multimap A}\ \star}{\cdot :\ \cdot \vdash\ !(!(A) \multimap A)}\ !$$

$\boxed{(2)\ A \to B, B \to C \vdash A \to C}$

LJ (46ms)

$$\frac{\dfrac{}{A, A \to B, B \to C \vdash A}\ \star \quad \dfrac{\dfrac{}{A, B, B \to C \vdash B}\ \star \quad \dfrac{}{A, B, C \vdash C}\ \star}{A, B, B \to C \vdash C}\ \supset_L}{A, A \to B, B \to C \vdash C}\ \supset_L}{A \to B, B \to C \vdash A \to C}\ \star$$

MULTIPLICATIVE encoding (49ms)

$$\frac{\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{\dfrac{}{\cdot :\ B \vdash B} \quad \dfrac{}{\cdot :\ C \vdash C}}{\cdot :\ B, B \multimap C \vdash C}\ \multimap}{\cdot :\ A, A \multimap B, B \multimap C \vdash C}\ \multimap}{\cdot :\ A \multimap B, B \multimap C \vdash A \multimap C}\ \star$$

CALL-BY-NAME encoding (123ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A,!(A)\multimap B,!(B)\multimap C :\ \cdot \vdash A}}{A,!(A)\multimap B,!(B)\multimap C :\ \cdot \vdash\ !(A)}\ ! \quad \dfrac{}{A,!(A)\multimap B,!(B)\multimap C :\ B \vdash B}}{A,!(A)\multimap B,!(B)\multimap C :\ !(A)\multimap B \vdash B}\ \multimap}{A,!(A)\multimap B,!(B)\multimap C :\ \cdot \vdash B}\ D_C}{A,!(A)\multimap B,!(B)\multimap C :\ \cdot \vdash\ !(B)}\ ! \quad \dfrac{}{A,!(A)\multimap B,!(B)\multimap C :\ C \vdash C}}{A,!(A)\multimap B,!(B)\multimap C :\ !(B)\multimap C \vdash C}\ \multimap}{A,!(A)\multimap B,!(B)\multimap C :\ \cdot \vdash C}\ D_C}{\cdot :\ !(!(A)\multimap B),!(!(B)\multimap C) \vdash\ !(A)\multimap C}\ \star$$

CALL-BY-VALUE encoding (165ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A,!(A)\multimap\ !(B),!(B)\multimap\ !(C) :\ \cdot \vdash A}}{A,!(A)\multimap\ !(B),!(B)\multimap\ !(C) :\ \cdot \vdash\ !(A)}\ ! \quad \dfrac{}{A,B,!(A)\multimap\ !(B),!(B)\multimap\ !(C) :\ \cdot \vdash B}}{A,!(A)\multimap\ !(B),!(B)\multimap\ !(C) :\ !(A)\multimap\ !(B) \vdash B}\ \star}{A,!(A)\multimap\ !(B),!(B)\multimap\ !(C) :\ \cdot \vdash B}\ D_C}{A,!(A)\multimap\ !(B),!(B)\multimap\ !(C) :\ \cdot \vdash\ !(B)}\ ! \quad \dfrac{}{A,C,!(A)\multimap\ !(B),!(B)\multimap\ !(C) :\ \cdot \vdash C}}{A,C,!(A)\multimap\ !(B),!(B)\multimap\ !(C) :\ \cdot \vdash\ !(C)}\ ! \quad \dfrac{}{A,!(A)\multimap\ !(B),!(B)\multimap\ !(C) :\ !(C) \vdash\ !(C)}}{A,!(A)\multimap\ !(B),!(B)\multimap\ !(C) :\ !(B)\multimap\ !(C) \vdash\ !(C)}\ \star}{A,!(A)\multimap\ !(B),!(B)\multimap\ !(C) :\ \cdot \vdash\ !(C)}\ D_C}{!(A)\multimap\ !(B),!(B)\multimap\ !(C) :\ \cdot \vdash\ !(A)\multimap\ !(C)}\ \star}{!(A)\multimap\ !(B),!(B)\multimap\ !(C) :\ \cdot \vdash\ !(!(A)\multimap\ !(C))}\ !}{\cdot :\ !(!(A)\multimap\ !(B)),!(!(B)\multimap\ !(C)) \vdash\ !(!(A)\multimap\ !(C))}\ \star$$

01-ENC encoding (146ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A,!(A)\multimap\ !(B),!(B)\multimap\ !(C) :\ \cdot \vdash A}}{A,!(A)\multimap\ !(B),!(B)\multimap\ !(C) :\ \cdot \vdash\ !(A)}\ ! \quad \dfrac{}{A,B,!(A)\multimap\ !(B),!(B)\multimap\ !(C) :\ \cdot \vdash B}}{A,!(A)\multimap\ !(B),!(B)\multimap\ !(C) :\ !(A)\multimap\ !(B) \vdash B}\ \star}{A,!(A)\multimap\ !(B),!(B)\multimap\ !(C) :\ \cdot \vdash B}\ D_C}{A,!(A)\multimap\ !(B),!(B)\multimap\ !(C) :\ \cdot \vdash\ !(B)}\ ! \quad \dfrac{}{A,C,!(A)\multimap\ !(B),!(B)\multimap\ !(C) :\ \cdot \vdash C}}{A,!(A)\multimap\ !(B),!(B)\multimap\ !(C) :\ !(B)\multimap\ !(C) \vdash C}\ \star}{A,!(A)\multimap\ !(B),!(B)\multimap\ !(C) :\ \cdot \vdash C}\ D_C}{!(A)\multimap\ !(B),!(B)\multimap\ !(C) :\ \cdot \vdash\ !(A)\multimap C}\ \star}{!(A)\multimap\ !(B),!(B)\multimap\ !(C) :\ \cdot \vdash\ !(!(A)\multimap C)}\ !}{\cdot :\ !(!(A)\multimap\ !(B)),!(!(B)\multimap\ !(C)) \vdash\ !(!(A)\multimap C)}\ \star$$

*See `https://github.com/carlosolarte/Benchmarking-Linear-Logic` for details on the encodings used.

$\boxed{(3)\ A \to B \to C \vdash B \to A \to C}$

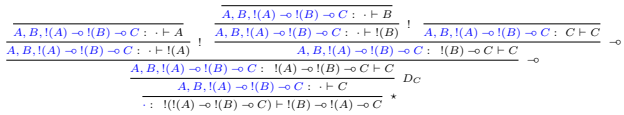
LJ (46ms)

$$\dfrac{\dfrac{\dfrac{}{A,B,A \to B \to C \vdash A}\star \quad \dfrac{\dfrac{}{A,B,B \to C \vdash B}\star \quad \dfrac{}{A,B,C \vdash C}\star}{A,B,B \to C \vdash C}\supset_L}{A,B,A \to B \to C \vdash C}\supset_L}{A \to B \to C \vdash B \to A \to C}\star$$

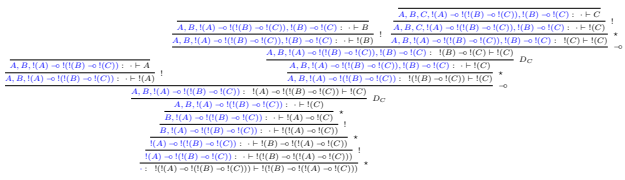
MULTIPLICATIVE encoding (47ms)

$$\dfrac{\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{\dfrac{}{\cdot :\ B \vdash B} \quad \dfrac{}{\cdot :\ C \vdash C}}{\cdot :\ B, B \multimap C \vdash C}\multimap}{\cdot :\ A, B, A \multimap B \multimap C \vdash C}\multimap}{\cdot :\ A \multimap B \multimap C \vdash B \multimap A \multimap C}\star$$

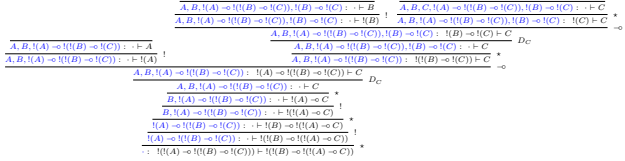
CALL-BY-NAME encoding (71ms)



CALL-BY-VALUE encoding (162ms)



01-ENC encoding (158ms)



$\boxed{(4)\ A \to B \to C \vdash A \wedge B \to C}$

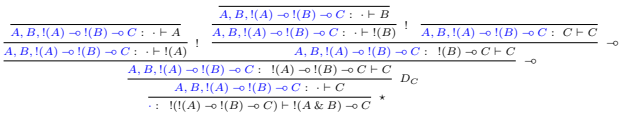
LJ (47ms)

$$\dfrac{\dfrac{\dfrac{}{A,B,A \to B \to C \vdash A}\star \quad \dfrac{\dfrac{}{A,B,B \to C \vdash B}\star \quad \dfrac{}{A,B,C \vdash C}\star}{A,B,B \to C \vdash C}\supset_L}{A,B,A \to B \to C \vdash C}\supset_L}{A \to B \to C \vdash A \wedge B \to C}\star$$

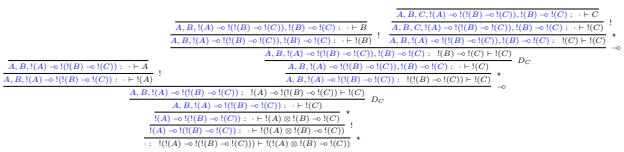
MULTIPLICATIVE encoding (47ms)

$$\dfrac{\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{\dfrac{}{\cdot :\ B \vdash B} \quad \dfrac{}{\cdot :\ C \vdash C}}{\cdot :\ B, B \multimap C \vdash C}\multimap}{\cdot :\ A, B, A \multimap B \multimap C \vdash C}\multimap}{\cdot :\ A \multimap B \multimap C \vdash A \otimes B \multimap C}\star$$

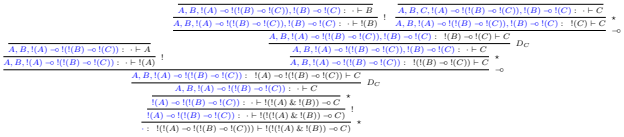
CALL-BY-NAME encoding (71ms)



CALL-BY-VALUE encoding (156ms)



01-ENC encoding (151ms)



$\boxed{(5)\ A \wedge B \to C \vdash A \to B \to C}$

LJ (41ms)

$$\dfrac{\dfrac{\overline{A, B, A \wedge B \to C \vdash A \wedge B}\ \star \quad \overline{A, B, C \vdash C}\ \star}{A, B, A \wedge B \to C \vdash C}\ \supset L}{A \wedge B \to C \vdash A \to B \to C}\ \star$$

MULTIPLICATIVE encoding (54ms)

$$\dfrac{\dfrac{\dfrac{\overline{\cdot :\ A \vdash A} \quad \overline{\cdot :\ B \vdash B}}{\cdot :\ A, B \vdash A \otimes B}\ \otimes \quad \overline{\cdot :\ C \vdash C}}{\cdot :\ A, B, A \otimes B \multimap C \vdash C}\ \multimap}{\cdot :\ A \otimes B \multimap C \vdash A \multimap B \multimap C}\ \star$$

CALL-BY-NAME encoding (73ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\overline{A, B, !(A \,\&\, B) \multimap C :\ \cdot \vdash A} \quad \overline{A, B, !(A \,\&\, B) \multimap C :\ \cdot \vdash B}\ \star}{A, B, !(A \,\&\, B) \multimap C :\ \cdot \vdash A \,\&\, B}}{A, B, !(A \,\&\, B) \multimap C :\ \cdot \vdash !(A \,\&\, B)}\ ! \quad \overline{A, B, !(A \,\&\, B) \multimap C :\ C \vdash C}}{A, B, !(A \,\&\, B) \multimap C :\ !(A \,\&\, B) \multimap C \vdash C}\ \multimap}{A, B, !(A \,\&\, B) \multimap C :\ \cdot \vdash C}\ D_C}{\cdot :\ !(!(A \,\&\, B) \multimap C) \vdash !(A) \multimap !(B) \multimap C}\ \star$$

CALL-BY-VALUE encoding (205ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\overline{A, B, !(A) \otimes !(B) \multimap !(C) :\ \cdot \vdash A}}{A, B, !(A) \otimes !(B) \multimap !(C) :\ \cdot \vdash !(A)}\ ! \quad \dfrac{\overline{A, B, !(A) \otimes !(B) \multimap !(C) :\ \cdot \vdash B}}{A, B, !(A) \otimes !(B) \multimap !(C) :\ \cdot \vdash !(B)}\ !}{A, B, !(A) \otimes !(B) \multimap !(C) :\ \cdot \vdash !(A) \otimes !(B)}\ \otimes \quad \dfrac{\overline{A, B, C, !(A) \otimes !(B) \multimap !(C) :\ \cdot \vdash C}}{A, B, !(A) \otimes !(B) \multimap !(C) :\ \cdot \vdash !(C)}\ !}{A, B, !(A) \otimes !(B) \multimap !(C) :\ !(A) \otimes !(B) \multimap !(C) \vdash !(C)}\ \star \multimap}{A, B, !(A) \otimes !(B) \multimap !(C) :\ \cdot \vdash !(C)}\ D_C}{A, !(A) \otimes !(B) \multimap !(C) :\ \cdot \vdash !(B) \multimap !(C)}\ \star}{A, !(A) \otimes !(B) \multimap !(C) :\ \cdot \vdash !(!(B) \multimap !(C))}\ !}{!(A) \otimes !(B) \multimap !(C) :\ \cdot \vdash !(A) \multimap !(!(B) \multimap !(C))}\ \star}{!(A) \otimes !(B) \multimap !(C) :\ \cdot \vdash !(!(A) \multimap !(!(B) \multimap !(C)))}\ !}{\cdot :\ !(!(A) \otimes !(B) \multimap !(C)) \vdash !(!(A) \multimap !(!(B) \multimap !(C)))}\ \star$$

01-ENC encoding (201ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\overline{A, B, !(!(A \,\&\, B)) \multimap !(C) :\ \cdot \vdash A} \quad \overline{A, B, !(!(A \,\&\, B)) \multimap !(C) :\ \cdot \vdash B}\ \star}{A, B, !(!(A \,\&\, B)) \multimap !(C) :\ \cdot \vdash A \,\&\, B}}{A, B, !(!(A \,\&\, B)) \multimap !(C) :\ \cdot \vdash !(A \,\&\, B)}\ ! \quad \overline{A, B, C, !(!(A \,\&\, B)) \multimap !(C) :\ \cdot \vdash C}}{A, B, !(!(A \,\&\, B)) \multimap !(C) :\ !(!(A \,\&\, B)) \multimap !(C) \vdash C}\ \star \multimap}{A, B, !(!(A \,\&\, B)) \multimap !(C) :\ \cdot \vdash C}\ D_C}{A, !(!(A \,\&\, B)) \multimap !(C) :\ \cdot \vdash !(B) \multimap C}\ \star}{!(!(A \,\&\, B)) \multimap !(C) :\ \cdot \vdash !(A) \multimap !(!(B) \multimap C)}\ !}{!(!(A \,\&\, B)) \multimap !(C) :\ \cdot \vdash !(A) \multimap !(!(B) \multimap !(C))}\ \star}{\cdot :\ !(!(!(A \,\&\, B)) \multimap !(C)) \vdash !(!(A) \multimap !(!(B) \multimap C))}\ \star$$

$\boxed{(6)\ A \to B \vdash B \to C \to A \to C}$

LJ (47ms)

$$\dfrac{\dfrac{\dfrac{\overline{A, A \to B, B \to C \vdash A}\ \star \quad \dfrac{\overline{A, B, B \to C \vdash B}\ \star \quad \overline{A, B, C \vdash C}\ \star}{A, B, B \to C \vdash C}\ \supset L}{A, A \to B, B \to C \vdash C}\ \supset L}{A \to B \vdash B \to C \to A \to C}\ \star$$

MULTIPLICATIVE encoding (48ms)

$$\dfrac{\dfrac{\dfrac{\overline{\cdot :\ A \vdash A} \quad \dfrac{\overline{\cdot :\ B \vdash B} \quad \overline{\cdot :\ C \vdash C}}{\cdot :\ B, B \multimap C \vdash C}\ \multimap}{\cdot :\ A, A \multimap B, B \multimap C \vdash C}\ \multimap}{\cdot :\ A \multimap B \vdash B \multimap C \multimap A \multimap C}\ \star$$

CALL-BY-NAME encoding (120ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\overline{A, !(A) \multimap B, !(B) \multimap C :\ \cdot \vdash A}}{A, !(A) \multimap B, !(B) \multimap C :\ \cdot \vdash !(A)}\ ! \quad \dfrac{\overline{A, !(A) \multimap B, !(B) \multimap C :\ B \vdash B}}{A, !(A) \multimap B, !(B) \multimap C :\ !(A) \multimap B \vdash B}\ \multimap}{A, !(A) \multimap B, !(B) \multimap C :\ !(A) \multimap B \vdash !(B)}\ D_C}{A, !(A) \multimap B, !(B) \multimap C :\ \cdot \vdash !(B)}\ ! \quad \overline{A, !(A) \multimap B, !(B) \multimap C :\ C \vdash C}}{A, !(A) \multimap B, !(B) \multimap C :\ !(B) \multimap C \vdash C}\ \multimap}{A, !(A) \multimap B, !(B) \multimap C :\ \cdot \vdash C}\ D_C}{\cdot :\ !(!(A) \multimap B) \vdash !(!(B) \multimap C) \multimap !(A) \multimap C}\ \star$$

CALL-BY-VALUE encoding (168ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\overline{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash A}}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash !(A)}\ ! \quad \dfrac{\overline{A, B, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash B}\ \star}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash !(B)}\ !}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ !(A) \multimap !(B) \vdash B}\ \multimap \quad \dfrac{\overline{A, C, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash C}}{A, C, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash !(C)}\ !}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ !(A) \multimap !(B) \vdash !(C)}\ \star \multimap}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash !(C)}\ D_C}{!(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash !(A) \multimap !(C)}\ \star}{!(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash !(!(A) \multimap !(C))}\ !}{!(A) \multimap !(B) :\ \cdot \vdash !(!(B) \multimap !(C)) \multimap !(!(A) \multimap !(C))}\ \star}{!(A) \multimap !(B) :\ \cdot \vdash !(!(!(B) \multimap !(C)) \multimap !(!(A) \multimap !(C)))}\ !}{\cdot :\ !(!(A) \multimap !(B)) \vdash !(!(!(B) \multimap !(C)) \multimap !(!(A) \multimap !(C)))}\ \star$$

01-ENC encoding (159ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\overline{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash A}}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash !(A)}\ ! \quad \dfrac{\overline{A, B, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash B}\ \star}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash !(B)}\ !}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ !(A) \multimap !(B) \vdash B}\ \multimap \quad \dfrac{\overline{A, C, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash C}}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash !(C)}\ !}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ !(B) \multimap !(C) \vdash !(C)}\ \star \multimap}{A, !(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash !(C)}\ D_C}{!(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash !(A) \multimap !(C)}\ \star}{!(A) \multimap !(B), !(B) \multimap !(C) :\ \cdot \vdash !(!(A) \multimap !(C))}\ !}{!(A) \multimap !(B) :\ \cdot \vdash !(!(!(B) \multimap !(C)) \multimap !(!(A) \multimap !(C)))}\ \star}{\cdot :\ !(!(A) \multimap !(B)) \vdash !(!(!(B) \multimap !(C)) \multimap !(!(A) \multimap !(C)))}\ !$$

(7) $A \to B \vdash C \to A \to C \to B$

LJ (47ms)

$$\dfrac{\dfrac{\dfrac{}{C, A \to B, C \to A \vdash C}\,\star \quad \dfrac{}{A, C, A \to B \vdash A}\,\star}{C, A \to B, C \to A \vdash A}\,\supset_L \quad \dfrac{}{B, C, C \to A \vdash B}\,\star}{\dfrac{C, A \to B, C \to A \vdash B}{A \to B \vdash C \to A \to C \to B}\,\star}\,\supset_L$$

MULTIPLICATIVE encoding (48ms)

$$\dfrac{\dfrac{\dfrac{}{\cdot :\ C \vdash C} \quad \dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{}{\cdot :\ B \vdash B}}{\cdot :\ A, A \multimap B \vdash B}\,\multimap}{\cdot :\ C, A \multimap B, C \multimap A \vdash B}\,\multimap}{\cdot :\ A \multimap B \vdash C \multimap A \multimap C \multimap B}\,\star$$

CALL-BY-NAME encoding (120ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{C, !(A) \multimap B, !(C) \multimap A :\ \cdot \vdash C}}{C, !(A) \multimap B, !(C) \multimap A :\ \cdot \vdash !(C)}\,! \quad \dfrac{}{C, !(A) \multimap B, !(C) \multimap A :\ A \vdash A}}{C, !(A) \multimap B, !(C) \multimap A :\ !(C) \multimap A \vdash A}\,\multimap}{C, !(A) \multimap B, !(C) \multimap A :\ \cdot \vdash A}\,D_C \quad \dfrac{}{C, !(A) \multimap B, !(C) \multimap A :\ B \vdash B}}{\dfrac{C, !(A) \multimap B, !(C) \multimap A :\ !(A) \multimap B \vdash B}{C, !(A) \multimap B, !(C) \multimap A :\ \cdot \vdash B}\,D_C}\,\multimap$$
$$\cdot :\ !(!(A) \multimap B) \vdash !(!(C) \multimap A) \multimap !(C) \multimap B\,\star$$

CALL-BY-VALUE encoding (167ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{C, !(A) \multimap !(B), !(C) \multimap !(A) :\ \cdot \vdash C}}{C, !(A) \multimap !(B), !(C) \multimap !(A) :\ \cdot \vdash !(C)}\,! \quad \dfrac{\dfrac{}{A, C, !(A) \multimap !(B), !(C) \multimap !(A) :\ \cdot \vdash A}}{C, !(A) \multimap !(B), !(C) \multimap !(A) :\ !(C) \multimap !(A) \vdash A}\,\star}{C, !(A) \multimap !(B), !(C) \multimap !(A) :\ \cdot \vdash A}\,\multimap}{C, !(A) \multimap !(B), !(C) \multimap !(A) :\ \cdot \vdash !(A)}\,D_C \quad \dfrac{\dfrac{\dfrac{}{B, C, !(A) \multimap !(B), !(C) \multimap !(A) :\ \cdot \vdash B}}{B, C, !(A) \multimap !(B), !(C) \multimap !(A) :\ \cdot \vdash !(B)}\,! \quad \dfrac{}{C, !(A) \multimap !(B), !(C) \multimap !(A) :\ !(B) \vdash !(B)}\,\star}{C, !(A) \multimap !(B), !(C) \multimap !(A) :\ \cdot \vdash !(B)}\,\multimap}{C, !(A) \multimap !(B), !(C) \multimap !(A) :\ !(A) \multimap !(B) \vdash !(B)}\,D_C$$
$$\dfrac{C, !(A) \multimap !(B), !(C) \multimap !(A) :\ \cdot \vdash !(B)}{!(A) \multimap !(B), !(C) \multimap !(A) :\ \cdot \vdash !(C) \multimap !(B)}\,\star$$
$$\dfrac{!(A) \multimap !(B), !(C) \multimap !(A) :\ \cdot \vdash !(C) \multimap !(B))}{!(A) \multimap !(B) :\ \cdot \vdash !(!(C) \multimap !(A)) \multimap !(!(C) \multimap !(B))}\,\star$$
$$\dfrac{!(A) \multimap !(B) :\ \cdot \vdash !(!(!(C) \multimap !(A)) \multimap !(!(C) \multimap !(B)))}{\cdot :\ !(!(A) \multimap !(B)) \vdash !(!(!(C) \multimap !(A)) \multimap !(!(C) \multimap !(B)))}\,!\,\star$$

01-ENC encoding (156ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{C, !(A) \multimap !(B), !(C) \multimap !(A) :\ \cdot \vdash C}}{C, !(A) \multimap !(B), !(C) \multimap !(A) :\ \cdot \vdash !(C)}\,! \quad \dfrac{\dfrac{}{A, C, !(A) \multimap !(B), !(C) \multimap !(A) :\ \cdot \vdash A}}{C, !(A) \multimap !(B), !(C) \multimap !(A) :\ !(C) \multimap !(A) \vdash A}\,\star}{C, !(A) \multimap !(B), !(C) \multimap !(A) :\ \cdot \vdash A}\,\multimap}{C, !(A) \multimap !(B), !(C) \multimap !(A) :\ \cdot \vdash !(A)}\,D_C \quad \dfrac{\dfrac{\dfrac{}{B, C, !(A) \multimap !(B), !(C) \multimap !(A) :\ \cdot \vdash B}}{B, C, !(A) \multimap !(B), !(C) \multimap !(A) :\ \cdot \vdash !(B)}\,! \quad \dfrac{}{C, !(A) \multimap !(B), !(C) \multimap !(A) :\ !(B) \vdash B}\,\star}{C, !(A) \multimap !(B), !(C) \multimap !(A) :\ \cdot \vdash !(B)}\,\multimap}{C, !(A) \multimap !(B), !(C) \multimap !(A) :\ !(A) \multimap !(B) \vdash B}\,D_C$$
$$\dfrac{C, !(A) \multimap !(B), !(C) \multimap !(A) :\ \cdot \vdash B}{!(A) \multimap !(B), !(C) \multimap !(A) :\ \cdot \vdash !(C) \multimap B}\,\star$$
$$\dfrac{!(A) \multimap !(B), !(C) \multimap !(A) :\ \cdot \vdash !(C) \multimap B)}{!(A) \multimap !(B) :\ \cdot \vdash !(!(C) \multimap !(A)) \multimap !(!(C) \multimap B)}\,!$$
$$\dfrac{!(A) \multimap !(B) :\ \cdot \vdash !(!(!(C) \multimap !(A)) \multimap !(!(C) \multimap B))}{\cdot :\ !(!(A) \multimap !(B)) \vdash !(!(!(C) \multimap !(A)) \multimap !(!(C) \multimap B))}\,\star$$

(8) $A \to B \vdash A \wedge C \to B \wedge C$

LJ (37ms)

$$\dfrac{\dfrac{\dfrac{}{A, C, A \to B \vdash A}\,\star \quad \dfrac{}{A, B, C \vdash B}\,\star}{A, C, A \to B \vdash B}\,\supset_L}{A \to B \vdash A \wedge C \to B \wedge C}\,\star$$

MULTIPLICATIVE encoding (48ms)

$$\dfrac{\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{\dfrac{}{\cdot :\ B \vdash B} \quad \dfrac{}{\cdot :\ C \vdash C}}{\cdot :\ B, C \vdash B \otimes C}\,\otimes}{\cdot :\ A, C, A \multimap B \vdash B \otimes C}\,\multimap}{\cdot :\ A \multimap B \vdash A \otimes C \multimap B \otimes C}\,\star$$

CALL-BY-NAME encoding (73ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A, C, !(A) \multimap B :\ \cdot \vdash A}}{A, C, !(A) \multimap B :\ \cdot \vdash !(A)}\,! \quad \dfrac{}{A, C, !(A) \multimap B :\ B \vdash B}}{A, C, !(A) \multimap B :\ !(A) \multimap B \vdash B}\,\multimap}{A, C, !(A) \multimap B :\ \cdot \vdash B}\,D_C \quad \dfrac{}{A, C, !(A) \multimap B :\ \cdot \vdash C}}{\cdot :\ !(!(A) \multimap B) \vdash !(A \,\&\, C) \multimap B \,\&\, C}\,\star$$

CALL-BY-VALUE encoding (135ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, C, !(A) \multimap !(B) :\ \cdot \vdash A}}{A, C, !(A) \multimap !(B) :\ \cdot \vdash !(A)}\,! \quad \dfrac{\dfrac{}{A, B, C, !(A) \multimap !(B) :\ \cdot \vdash B}}{A, C, !(A) \multimap !(B) :\ !(B) \vdash B}\,\star}{A, C, !(A) \multimap !(B) :\ !(A) \multimap !(B) \vdash B}\,\multimap}{A, C, !(A) \multimap !(B) :\ \cdot \vdash B}\,D_C \quad \dfrac{\dfrac{\dfrac{}{A, C, !(A) \multimap !(B) :\ \cdot \vdash C}}{A, C, !(A) \multimap !(B) :\ \cdot \vdash !(C)}\,! \quad \dfrac{}{A, C, !(A) \multimap !(B) :\ \cdot \vdash !(B) \otimes !(C)}\,\otimes}{A, C, !(A) \multimap !(B) :\ \cdot \vdash !(B) \otimes !(C)}\,\star$$
$$\dfrac{!(A) \multimap !(B) :\ \cdot \vdash !(A) \otimes !(C) \multimap !(B) \otimes !(C)}{!(A) \multimap !(B) :\ \cdot \vdash !(!(A) \otimes !(C) \multimap !(B) \otimes !(C))}\,!$$
$$\cdot :\ !(!(A) \multimap !(B)) \vdash !(!(A) \otimes !(C) \multimap !(B) \otimes !(C))\,\star$$

01-ENC encoding (130ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, C, !(A) \multimap !(B) :\ \cdot \vdash A}}{A, C, !(A) \multimap !(B) :\ \cdot \vdash !(A)}\,! \quad \dfrac{\dfrac{}{A, B, C, !(A) \multimap !(B) :\ \cdot \vdash B}}{A, C, !(A) \multimap !(B) :\ !(B) \vdash B}\,\star}{A, C, !(A) \multimap !(B) :\ !(A) \multimap !(B) \vdash B}\,\multimap}{A, C, !(A) \multimap !(B) :\ \cdot \vdash B}\,D_C \quad \dfrac{}{A, C, !(A) \multimap !(B) :\ \cdot \vdash C}}{\dfrac{A, C, !(A) \multimap !(B) :\ \cdot \vdash B \,\&\, C}{A, C, !(A) \multimap !(B) :\ \cdot \vdash !(B \,\&\, C)}\,!}\,\star$$
$$\dfrac{!(A) \multimap !(B) :\ \cdot \vdash !(!(A) \,\&\, !(C)) \multimap !(B \,\&\, C)}{!(A) \multimap !(B) :\ \cdot \vdash !(!(!(A) \,\&\, !(C)) \multimap !(B \,\&\, C))}\,\star$$
$$\cdot :\ !(!(A) \multimap !(B)) \vdash !(!(!(A) \,\&\, !(C)) \multimap !(B \,\&\, C))\,!$$

(9) $A \to B \vdash C \wedge A \to C \wedge B$

LJ (37ms)

$$\dfrac{\dfrac{\dfrac{}{A, C, A \to B \vdash A}\ \star \quad \dfrac{}{A, B, C \vdash B}\ \star}{A, C, A \to B \vdash B}\ \supset L}{A \to B \vdash C \wedge A \to C \wedge B}\ \star$$

MULTIPLICATIVE encoding (48ms)

$$\dfrac{\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{\dfrac{}{\cdot :\ C \vdash C} \quad \dfrac{}{\cdot :\ B \vdash B}}{\cdot :\ B, C \vdash C \otimes B}\ \otimes}{\cdot :\ A, C, A \multimap B \vdash C \otimes B}\ \multimap}{\cdot :\ A \multimap B \vdash C \otimes A \multimap C \otimes B}\ \star$$

CALL-BY-NAME encoding (60ms)

$$\dfrac{\dfrac{}{A, C, !(A) \multimap B :\ \cdot \vdash C} \quad \dfrac{\dfrac{\dfrac{\dfrac{}{A, C, !(A) \multimap B :\ \cdot \vdash A}}{A, C, !(A) \multimap B :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, C, !(A) \multimap B :\ B \vdash B}}{A, C, !(A) \multimap B :\ !(A) \multimap B \vdash B}\ \multimap}{A, C, !(A) \multimap B :\ \cdot \vdash B}\ D_C}{\cdot :\ !(!(A) \multimap B) \vdash !(A) \multimap C \,\&\, B}\ \star$$

CALL-BY-VALUE encoding (137ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, C, !(A) \multimap !(B) :\ \cdot \vdash A}}{A, C, !(A) \multimap !(B) :\ \cdot \vdash !(A)}\ ! \quad \dfrac{\dfrac{\dfrac{}{A, B, C, !(A) \multimap !(B) :\ \cdot \vdash C}}{A, B, C, !(A) \multimap !(B) :\ \cdot \vdash !(C)}\ ! \quad \dfrac{}{A, B, C, !(A) \multimap !(B) :\ \cdot \vdash B}}{A, B, C, !(A) \multimap !(B) :\ \cdot \vdash !(C) \otimes !(B)}\ \otimes}{A, C, !(A) \multimap !(B) :\ !(B) \vdash !(C) \otimes !(B)}\ \multimap}{A, C, !(A) \multimap !(B) :\ !(A) \multimap !(B) \vdash !(C) \otimes !(B)}\ D_C}{!(A) \multimap !(B) :\ \cdot \vdash !(C) \otimes !(A) \multimap !(C) \otimes !(B)}\ \star}{!(A) \multimap !(B) :\ \cdot \vdash !(!(C) \otimes !(A) \multimap !(C) \otimes !(B))}\ !}{\cdot :\ !(!(A) \multimap !(B)) \vdash !(!(C) \otimes !(A) \multimap !(C) \otimes !(B))}\ \star$$

01-ENC encoding (131ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A, C, !(A) \multimap !(B) :\ \cdot \vdash A}}{A, C, !(A) \multimap !(B) :\ \cdot \vdash !(A)}\ ! \quad \dfrac{\dfrac{\dfrac{\dfrac{}{A, B, C, !(A) \multimap !(B) :\ \cdot \vdash C} \quad \dfrac{}{A, B, C, !(A) \multimap !(B) :\ \cdot \vdash B}}{A, B, C, !(A) \multimap !(B) :\ \cdot \vdash C \,\&\, B}\ \star}{A, B, C, !(A) \multimap !(B) :\ \cdot \vdash !(C \,\&\, B)}\ !}{A, C, !(A) \multimap !(B) :\ !(B) \vdash !(C \,\&\, B)}\ \star}{A, C, !(A) \multimap !(B) :\ !(A) \multimap !(B) \vdash !(C \,\&\, B)}\ \multimap}{A, C, !(A) \multimap !(B) :\ \cdot \vdash !(C \,\&\, B)}\ D_C}{!(A) \multimap !(B) :\ \cdot \vdash !(!(C) \,\&\, !(A)) \multimap !(C \,\&\, B)}\ \star}{!(A) \multimap !(B) :\ \cdot \vdash !(!(!(C) \,\&\, !(A)) \multimap !(C \,\&\, B))}\ !}{\cdot :\ !(!(A) \multimap !(B)) \vdash !(!(!(C) \,\&\, !(A)) \multimap !(C \,\&\, B))}\ \star$$

(10) $\neg A \vdash A \to B$

LJ (37ms)

$$\dfrac{\dfrac{\dfrac{}{A, A \to \bot \vdash A}\ \star \quad \dfrac{}{A, \bot \vdash B}\ \star}{A, A \to \bot \vdash B}\ \supset L}{A \to \bot \vdash A \to B}\ \star$$

MULTIPLICATIVE encoding (41ms)

Not provable

CALL-BY-NAME encoding (66ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap \mathbf{0} :\ \mathbf{0} \vdash B}\ \star}{A, !(A) \multimap \mathbf{0} :\ !(A) \multimap \mathbf{0} \vdash B}\ \multimap}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash B}\ D_C}{\cdot :\ !(!(A) \multimap \mathbf{0}) \vdash !(A) \multimap B}\ \star$$

CALL-BY-VALUE encoding (117ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap \mathbf{0} :\ \mathbf{0} \vdash !(B)}\ \star}{A, !(A) \multimap \mathbf{0} :\ !(A) \multimap \mathbf{0} \vdash !(B)}\ \multimap}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(B)}\ D_C}{!(A) \multimap \mathbf{0} :\ \cdot \vdash !(A) \multimap !(B)}\ \star}{!(A) \multimap \mathbf{0} :\ \cdot \vdash !(!(A) \multimap !(B))}\ !}{\cdot :\ !(!(A) \multimap \mathbf{0}) \vdash !(!(A) \multimap !(B))}\ \star$$

01-ENC encoding (118ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash A}}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap !(\mathbf{0}) :\ !(\mathbf{0}) \vdash B}\ \star}{A, !(A) \multimap !(\mathbf{0}) :\ !(A) \multimap !(\mathbf{0}) \vdash B}\ \multimap}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash B}\ D_C}{!(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(A) \multimap B}\ \star}{!(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(!(A) \multimap B)}\ !}{\cdot :\ !(!(A) \multimap !(\mathbf{0})) \vdash !(!(A) \multimap B)}\ \star$$

(11) $A \vdash \neg A \to B$

LJ (37ms)

$$\dfrac{\dfrac{}{A, A \to \bot \vdash A}\ \star \qquad \dfrac{}{A, \bot \vdash B}\ \star}{\dfrac{A, A \to \bot \vdash B}{A \vdash A \to \bot \to B}\ \star}\ \supset L$$

MULTIPLICATIVE encoding (28ms)

Not provable

CALL-BY-NAME encoding (65ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(A)}\ ! \qquad \dfrac{}{A, !(A) \multimap \mathbf{0} :\ \mathbf{0} \vdash B}\ \star}{A, !(A) \multimap \mathbf{0} :\ !(A) \multimap \mathbf{0} \vdash B}\ \multimap}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash B}\ D_C}{\cdot :\ !(A) \vdash !(!(A) \multimap \mathbf{0}) \multimap B}\ \star$$

CALL-BY-VALUE encoding (78ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(A)}\ ! \qquad \dfrac{}{A, !(A) \multimap \mathbf{0} :\ \mathbf{0} \vdash !(B)}\ \star}{A, !(A) \multimap \mathbf{0} :\ !(A) \multimap \mathbf{0} \vdash !(B)}\ \multimap}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(B)}\ D_C}{A :\ \cdot \vdash !(!(A) \multimap \mathbf{0}) \multimap !(B)}\ \star}{A :\ \cdot \vdash !(!(!(A) \multimap \mathbf{0}) \multimap !(B))}\ !}{\cdot :\ !(A) \vdash !(!(!(A) \multimap \mathbf{0}) \multimap !(B))}\ \star$$

01-ENC encoding (76ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash A}}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(A)}\ ! \qquad \dfrac{}{A, !(A) \multimap !(\mathbf{0}) :\ !(\mathbf{0}) \vdash B}\ \star}{A, !(A) \multimap !(\mathbf{0}) :\ !(A) \multimap !(\mathbf{0}) \vdash B}\ \multimap}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash B}\ D_C}{A :\ \cdot \vdash !(!(A) \multimap !(\mathbf{0})) \multimap B}\ \star}{A :\ \cdot \vdash !(!(!(A) \multimap !(\mathbf{0})) \multimap B)}\ !}{\cdot :\ !(A) \vdash !(!(!(A) \multimap !(\mathbf{0})) \multimap B)}\ \star$$

(12) $B \vdash A \to B$

LJ (21ms)

$$\dfrac{}{B \vdash A \to B}\ \star$$

MULTIPLICATIVE encoding (21ms)

Not provable

CALL-BY-NAME encoding (27ms)

$$\dfrac{\dfrac{}{A, B :\ \cdot \vdash B}}{\cdot :\ !(B) \vdash !(A) \multimap B}\ \star$$

CALL-BY-VALUE encoding (47ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \vdash B}}{A, B :\ \cdot \vdash !(B)}\ !}{B :\ \cdot \vdash !(A) \multimap !(B)}\ \star}{B :\ \cdot \vdash !(!(A) \multimap !(B))}\ !}{\cdot :\ !(B) \vdash !(!(A) \multimap !(B))}\ \star$$

01-ENC encoding (39ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \vdash B}}{B :\ \cdot \vdash !(A) \multimap B}\ \star}{B :\ \cdot \vdash !(!(A) \multimap B)}\ !}{\cdot :\ !(B) \vdash !(!(A) \multimap B)}\ \star$$

(13) $A \to B \vdash \neg B \to \neg A$

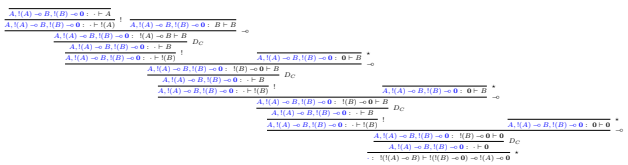
LJ (47ms)

$$\dfrac{\dfrac{\dfrac{}{A, A \to B, B \to \bot \vdash A}\star \quad \dfrac{\dfrac{}{A, B, B \to \bot \vdash B}\star \quad \dfrac{}{A, B, \bot \vdash \bot}\star}{A, B, B \to \bot \vdash \bot}\supset_L}{A, A \to B, B \to \bot \vdash \bot}\supset_L}{A \to B \vdash B \to \bot \to A \to \bot}\star$$

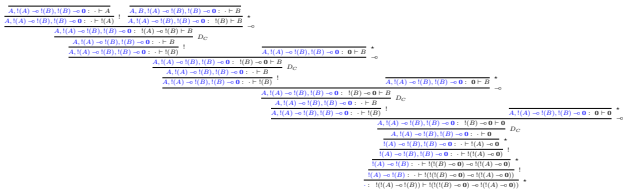
MULTIPLICATIVE encoding (47ms)

$$\dfrac{\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{\dfrac{}{\cdot :\ B \vdash B} \quad \dfrac{}{\cdot :\ \ \bot \vdash \bot}}{\cdot :\ B, B \multimap \bot \vdash \bot}\multimap}{\cdot :\ A, A \multimap B, B \multimap \bot \vdash \bot}\multimap}{\cdot :\ A \multimap B \vdash B \multimap \bot \multimap A \multimap \bot}\star$$

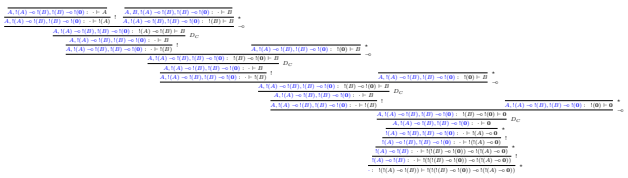
CALL-BY-NAME encoding (143ms)



CALL-BY-VALUE encoding (174ms)



01-ENC encoding (178ms)



(14) $A \to \neg B \vdash \neg\neg B \to \neg A$

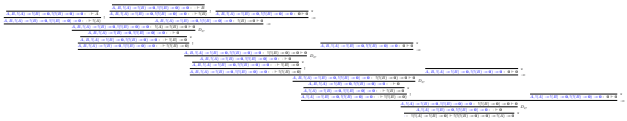
LJ (64ms)

$$\dfrac{\dfrac{}{A, A \to B \to \bot, B \to \bot \to \bot \vdash A}\star \quad \dfrac{\dfrac{\dfrac{}{A, B, B \to \bot, B \to \bot \to \bot \vdash \bot}\star \quad \dfrac{}{A, B, \bot, B \to \bot \to \bot \vdash \bot}\star}{A, B, B \to \bot, B \to \bot \to \bot \vdash \bot}\supset_L \quad \dfrac{}{A, \bot, B \to \bot \vdash \bot}\star}{A, B, B \to \bot, B \to \bot \to \bot \vdash \bot}\supset_L}{A, A \to B \to \bot, B \to \bot \to \bot \vdash \bot}\supset_L}{A \to B \to \bot \vdash B \to \bot \to \bot \to A \to \bot}\star$$

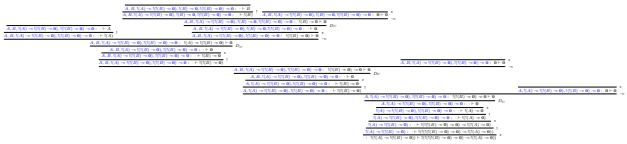
MULTIPLICATIVE encoding (75ms)

$$\dfrac{\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{\dfrac{}{\cdot :\ B \vdash B} \quad \dfrac{}{\cdot :\ \ \bot \vdash \bot}}{\cdot :\ B, B \multimap \bot \vdash \bot}\multimap}{\cdot :\ A, B, A \multimap B \multimap \bot \vdash \bot}\multimap \quad \dfrac{}{\cdot :\ \ \bot \vdash \bot}}{\cdot :\ A, A \multimap B \multimap \bot, B \multimap \bot \multimap \bot \vdash \bot}\star}{\cdot :\ A \multimap B \multimap \bot \vdash B \multimap \bot \multimap \bot \multimap A \multimap \bot}\multimap$$

CALL-BY-NAME encoding (180ms)



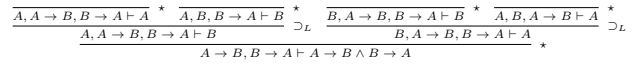
CALL-BY-VALUE encoding (217ms)



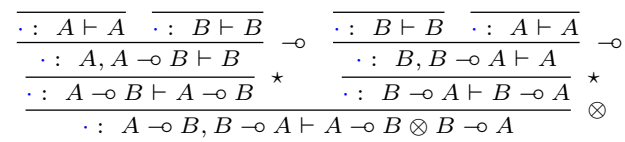
01-ENC encoding (226ms)

(15) $A \to B, B \to A \vdash (A) \leftrightarrow (B)$

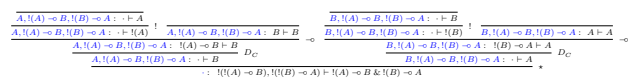
LJ (51ms)



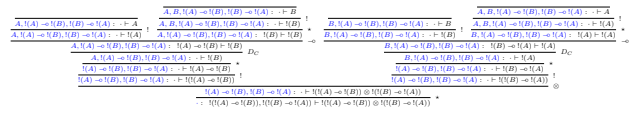
MULTIPLICATIVE encoding (75ms)



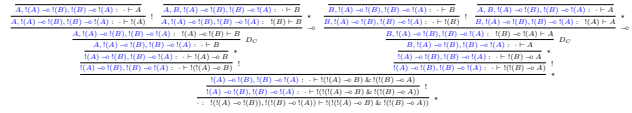
CALL-BY-NAME encoding (138ms)



CALL-BY-VALUE encoding (205ms)

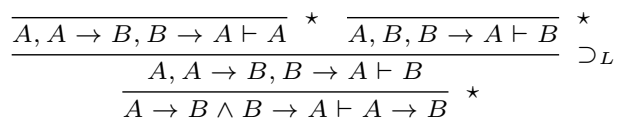


01-ENC encoding (191ms)



(16) $(A) \leftrightarrow (B) \vdash A \to B$

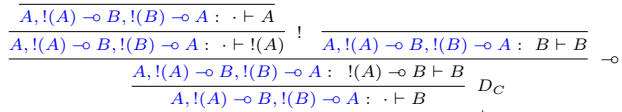
LJ (37ms)



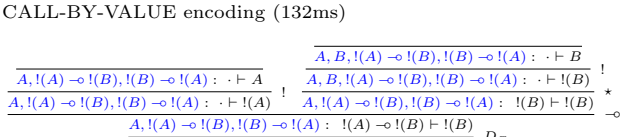
MULTIPLICATIVE encoding (29ms)

Not provable

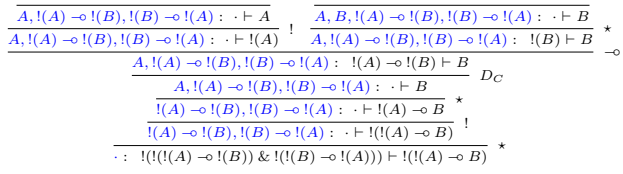
CALL-BY-NAME encoding (102ms)



CALL-BY-VALUE encoding (132ms)



01-ENC encoding (126ms)



(17) $(A) \leftrightarrow (B) \vdash B \to A$

LJ (37ms)

$$\dfrac{\dfrac{}{B, A \to B, B \to A \vdash B}\,\star \quad \dfrac{}{A, B, A \to B \vdash A}\,\star}{\dfrac{B, A \to B, B \to A \vdash A}{A \to B \wedge B \to A \vdash B \to A}\,\star}\,\supset_L$$

MULTIPLICATIVE encoding (29ms)

Not provable

CALL-BY-NAME encoding (105ms)

$$\dfrac{\dfrac{\dfrac{}{B, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash B}}{B, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash !(B)}\,! \quad \dfrac{}{B, !(A) \multimap B, !(B) \multimap A :\ A \vdash A}}{\dfrac{B, !(A) \multimap B, !(B) \multimap A :\ !(B) \multimap A \vdash A}{\dfrac{B, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash A}{\cdot :\ !(!(A) \multimap B \,\&\, !(B) \multimap A) \vdash !(B) \multimap A}\,\star}\,D_C}\,\multimap$$

CALL-BY-VALUE encoding (140ms)

$$\dfrac{\dfrac{\dfrac{}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash B}}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(B)}\,! \quad \dfrac{\dfrac{}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash A}}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) :\ !(A) \vdash !(A)}\,!}{\dfrac{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ !(B) \multimap !(A) \vdash !(A)}{\dfrac{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(A)}{\dfrac{!(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(B) \multimap !(A)}{\dfrac{!(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(!(B) \multimap !(A))}{\cdot :\ !(!(A) \multimap !(B)) \otimes !(!(B) \multimap !(A)) \vdash !(!(B) \multimap !(A))}\,\star}\,!}\,\star}\,D_C}\,\multimap$$

01-ENC encoding (122ms)

$$\dfrac{\dfrac{\dfrac{}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash B}}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(B)}\,! \quad \dfrac{\dfrac{}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash A}}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ !(A) \vdash A}\,\star}{\dfrac{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ !(B) \multimap !(A) \vdash A}{\dfrac{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash A}{\dfrac{!(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(B) \multimap A}{\dfrac{!(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(!(B) \multimap A)}{\cdot :\ !(!(!(A) \multimap !(B)) \,\&\, !(!(B) \multimap !(A))) \vdash !(!(B) \multimap A)}\,\star}\,!}\,\star}\,D_C}\,\multimap$$

(18) $(A) \leftrightarrow (B), A \vdash B$

LJ (38ms)

$$\dfrac{\dfrac{}{A, A \to B, B \to A \vdash A}\,\star \quad \dfrac{}{A, B, B \to A \vdash B}\,\star}{\dfrac{A, A \to B, B \to A \vdash B}{A, A \to B \wedge B \to A \vdash B}\,\star}\,\supset_L$$

MULTIPLICATIVE encoding (29ms)

Not provable

CALL-BY-NAME encoding (102ms)

$$\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash A}}{A, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash !(A)}\,! \quad \dfrac{}{A, !(A) \multimap B, !(B) \multimap A :\ B \vdash B}}{\dfrac{A, !(A) \multimap B, !(B) \multimap A :\ !(A) \multimap B \vdash B}{\dfrac{A, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash B}{\cdot :\ !(A), !(!(A) \multimap B \,\&\, !(B) \multimap A) \vdash B}\,\star}\,D_C}\,\multimap$$

CALL-BY-VALUE encoding (119ms)

$$\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash A}}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(A)}\,! \quad \dfrac{\dfrac{}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash B}}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) :\ !(B) \vdash !(B)}\,!}{\dfrac{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ !(A) \multimap !(B) \vdash !(B)}{\dfrac{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(B)}{\cdot :\ !(A), !(!(A) \multimap !(B)) \otimes !(!(B) \multimap !(A)) \vdash !(B)}\,\star}\,D_C}\,\multimap$$

01-ENC encoding (111ms)

$$\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash A}}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(A)}\,! \quad \dfrac{\dfrac{}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash B}}{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ !(B) \vdash B}\,\star}{\dfrac{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ !(A) \multimap !(B) \vdash B}{\dfrac{A, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash B}{\cdot :\ !(A), !(!(!(A) \multimap !(B)) \,\&\, !(!(B) \multimap !(A))) \vdash B}\,\star}\,D_C}\,\multimap$$

(19) $(A) \leftrightarrow (B), B \vdash A$

LJ (38ms)

$$\dfrac{\dfrac{}{B, A \to B, B \to A \vdash B}\ \star \qquad \dfrac{}{A, B, A \to B \vdash A}\ \star}{\dfrac{B, A \to B, B \to A \vdash A}{B, A \to B \wedge B \to A \vdash A}\ \star}\ \supset_L$$

MULTIPLICATIVE encoding (32ms)

Not provable

CALL-BY-NAME encoding (102ms)

$$\dfrac{\dfrac{\dfrac{}{B, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash B}}{B, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash\ !(B)}\ ! \qquad \dfrac{}{B, !(A) \multimap B, !(B) \multimap A :\ A \vdash A}}{\dfrac{B, !(A) \multimap B, !(B) \multimap A :\ !(B) \multimap A \vdash A}{\dfrac{B, !(A) \multimap B, !(B) \multimap A :\ \cdot \vdash A}{\cdot :\ !(B), !(!(A) \multimap B \,\&\, !(B) \multimap A) \vdash A}\ \star}\ D_C}\ \multimap$$

CALL-BY-VALUE encoding (118ms)

$$\dfrac{\dfrac{\dfrac{}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash B}}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash\ !(B)}\ ! \qquad \dfrac{\dfrac{}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash A}}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash\ !(A)}\ !}{\dfrac{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ !(B) \multimap !(A) \vdash !(A)}{\dfrac{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash !(A)}{\cdot :\ !(B), !(!(A) \multimap !(B)) \otimes !(!(B) \multimap !(A)) \vdash !(A)}\ \star}\ D_C}\ \multimap$$

01-ENC encoding (108ms)

$$\dfrac{\dfrac{\dfrac{}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash B}}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash\ !(B)}\ ! \qquad \dfrac{\dfrac{}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash A}}{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ !(A) \vdash A}\ \star}{\dfrac{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ !(B) \multimap !(A) \vdash A}{\dfrac{B, !(A) \multimap !(B), !(B) \multimap !(A) :\ \cdot \vdash A}{\cdot :\ !(B), !(!(!(A) \multimap !(B)) \,\&\, !(!(B) \multimap !(A))) \vdash A}\ \star}\ D_C}\ \multimap$$

(20) $\cdot \vdash (A) \leftrightarrow (A)$

LJ (21ms)

$$\dfrac{}{\cdot \vdash A \to A \wedge A \to A}\ \star$$

MULTIPLICATIVE encoding (34ms)

$$\dfrac{\dfrac{\dfrac{}{\cdot :\ A \vdash A}}{\cdot :\ \cdot \vdash A \multimap A}\ \star \qquad \dfrac{\dfrac{}{\cdot :\ A \vdash A}}{\cdot :\ \cdot \vdash A \multimap A}\ \star}{\cdot :\ \cdot \vdash A \multimap A \otimes A \multimap A}\ \otimes$$

CALL-BY-NAME encoding (27ms)

$$\dfrac{\dfrac{}{A :\ \cdot \vdash A} \qquad \dfrac{}{A :\ \cdot \vdash A}}{\cdot :\ \cdot \vdash\ !(A) \multimap A \,\&\, !(A) \multimap A}\ \star$$

CALL-BY-VALUE encoding (47ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A :\ \cdot \vdash A}}{A :\ \cdot \vdash\ !(A)}\ ! \qquad \dfrac{\dfrac{}{A :\ \cdot \vdash A}}{A :\ \cdot \vdash\ !(A)}\ !}{\cdot :\ \cdot \vdash\ !(A) \multimap\ !(A)}\ \star \qquad \dfrac{\dfrac{}{A :\ \cdot \vdash A}}{A :\ \cdot \vdash\ !(A)}\ ! \qquad \dfrac{\dfrac{}{A :\ \cdot \vdash A}}{A :\ \cdot \vdash\ !(A)}\ !}{\cdot :\ \cdot \vdash\ !(!(A) \multimap\ !(A))}\ ! \qquad \dfrac{\dfrac{}{A :\ \cdot \vdash A}}{A :\ \cdot \vdash\ !(A)}\ !}{\cdot :\ \cdot \vdash\ !(!(A) \multimap\ !(A)) \otimes !(!(A) \multimap\ !(A))}\ \otimes$$

01-ENC encoding (47ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A :\ \cdot \vdash A}}{\cdot :\ \cdot \vdash\ !(A) \multimap A}\ \star \qquad \dfrac{\dfrac{}{A :\ \cdot \vdash A}}{\cdot :\ \cdot \vdash\ !(A) \multimap A}\ \star}{\cdot :\ \cdot \vdash\ !(!(A) \multimap A)}\ ! \qquad \dfrac{\dfrac{}{A :\ \cdot \vdash A}}{\cdot :\ \cdot \vdash\ !(A) \multimap A}\ \star}{\cdot :\ \cdot \vdash\ !(!(A) \multimap A) \,\&\, !(!(A) \multimap A)}\ ! \qquad \dfrac{\dfrac{}{A :\ \cdot \vdash A}}{\cdot :\ \cdot \vdash\ !(A) \multimap A}\ \star}{\cdot :\ \cdot \vdash\ !(!(!(A) \multimap A) \,\&\, !(!(A) \multimap A))}\ !$$

(21) $(A) \leftrightarrow (B) \vdash (B) \leftrightarrow (A)$

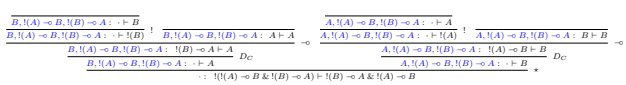
LJ (50ms)

$$\dfrac{\dfrac{B, A \to B, B \to A \vdash B}{B, A \to B, B \to A \vdash A}\,\star \quad \dfrac{A, B, A \to B \vdash A}{B, A \to B, B \to A \vdash A}\,\star}{A \to B \wedge B \to A \vdash B \to A \wedge A \to B}\,\star$$

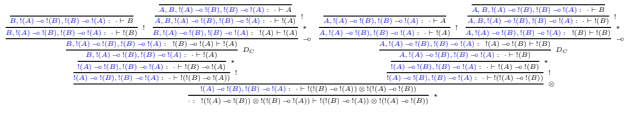
MULTIPLICATIVE encoding (81ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\cdot :\ B \vdash B \quad \cdot :\ A \vdash A}{\cdot :\ B, B \multimap A \vdash A}\,\multimap \quad \dfrac{\cdot :\ A \vdash A \quad \cdot :\ B \vdash B}{\cdot :\ A, A \multimap B \vdash B}\,\multimap}{\cdot :\ B \multimap A \vdash B \multimap A}\,\star \quad \dfrac{\cdot :\ A \vdash A \quad \cdot :\ B \vdash B}{\cdot :\ A, A \multimap B \vdash B}\,\multimap}{\cdot :\ A \multimap B, B \multimap A \vdash B \multimap A \otimes A \multimap B}\,\otimes}{\cdot :\ A \multimap B \otimes B \multimap A \vdash B \multimap A \otimes A \multimap B}\,\star$$

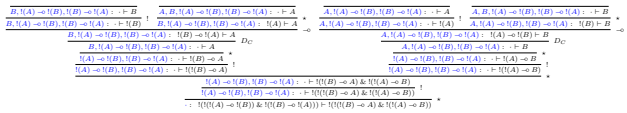
CALL-BY-NAME encoding (138ms)



CALL-BY-VALUE encoding (203ms)



01-ENC encoding (188ms)



(22) $(A) \leftrightarrow (B), (B) \leftrightarrow (C) \vdash (A) \leftrightarrow (C)$

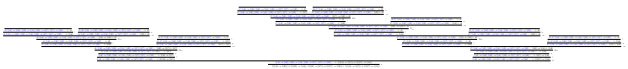
LJ (75ms)

MULTIPLICATIVE encoding (115ms)

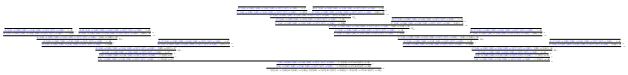
$$\dfrac{\dfrac{\dfrac{\dfrac{\cdot :\ B \vdash B \quad \cdot :\ C \vdash C}{\cdot :\ B, B \multimap C \vdash C}\,\multimap}{\cdot :\ A, A \multimap B, B \multimap C \vdash C}\,\star \quad \dfrac{\dfrac{\cdot :\ B \vdash B \quad \cdot :\ A \vdash A}{\cdot :\ B, B \multimap A \vdash A}\,\multimap}{\cdot :\ C, B \multimap A, C \multimap B \vdash A}\,\star}{\cdot :\ A \multimap B, B \multimap C, C \multimap B \vdash A \multimap C \otimes C \multimap A}\,\otimes}{\cdot :\ A \multimap B \otimes B \multimap A, B \multimap C \otimes C \multimap B \vdash A \multimap C \otimes C \multimap A}\,\star$$

CALL-BY-NAME encoding (218ms)

CALL-BY-VALUE encoding (331ms)

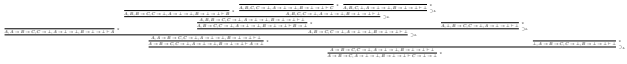


01-ENC encoding (291ms)



$\boxed{(23)\ A \to B \to C, \neg\neg A, \neg\neg B \vdash \neg\neg C}$

LJ (206ms)



MULTIPLICATIVE encoding (124ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{\dfrac{}{\cdot :\ B \vdash B} \quad \dfrac{\dfrac{}{\cdot :\ C \vdash C} \quad \dfrac{}{\cdot :\ \bot \vdash \bot}}{\cdot :\ C, C \multimap \bot \vdash \bot}}{\cdot :\ B, B \multimap C, C \multimap \bot \vdash \bot}}{\cdot :\ A, B, A \multimap B \multimap C, C \multimap \bot \vdash \bot} \quad \dfrac{}{\cdot :\ \bot \vdash \bot}}{\cdot :\ A, A \multimap B \multimap C, C \multimap \bot, B \multimap \bot \multimap \bot \vdash \bot} \quad \dfrac{}{\cdot :\ \bot \vdash \bot}}{\cdot :\ A \multimap B \multimap C, C \multimap \bot, A \multimap \bot \multimap \bot, B \multimap \bot \multimap \bot \vdash \bot}$$

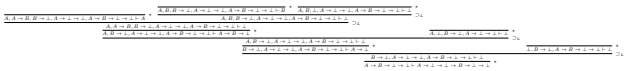
CALL-BY-NAME encoding (221ms)

CALL-BY-VALUE encoding (324ms)

01-ENC encoding (380ms)

$\boxed{(24)\ \neg\neg A \to B \vdash \neg\neg A \to \neg\neg B}$

LJ (94ms)



MULTIPLICATIVE encoding (106ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{\dfrac{}{\cdot :\ B \vdash B} \quad \dfrac{}{\cdot :\ \bot \vdash \bot}}{\cdot :\ B, B \multimap \bot \vdash \bot}}{\cdot :\ A, A \multimap B, B \multimap \bot \vdash \bot} \quad \dfrac{}{\cdot :\ \bot \vdash \bot}}{\cdot :\ A, B \multimap \bot, A \multimap B \multimap \bot \multimap \bot \vdash \bot} \quad \dfrac{}{\cdot :\ \bot \vdash \bot}}{\cdot :\ B \multimap \bot, A \multimap B \multimap \bot \multimap \bot \vdash A \multimap \bot} \quad \dfrac{}{\cdot :\ \bot \vdash \bot}}{\cdot :\ B \multimap \bot, A \multimap \bot \multimap \bot, A \multimap B \multimap \bot \multimap \bot \vdash \bot}$$

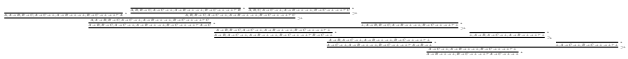
CALL-BY-NAME encoding (199ms)

CALL-BY-VALUE encoding (241ms)

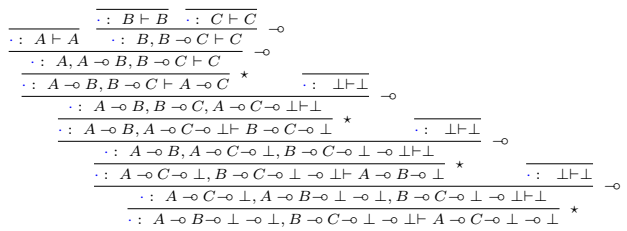
01-ENC encoding (269ms)

(25) $\neg\neg A \to B, \neg\neg B \to C \vdash \neg\neg A \to C$

LJ (324ms)

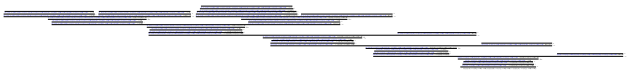


MULTIPLICATIVE encoding (145ms)

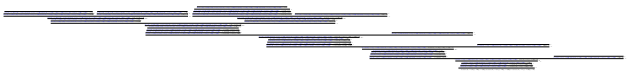


CALL-BY-NAME encoding (283ms)

CALL-BY-VALUE encoding (375ms)

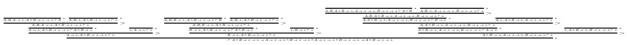


01-ENC encoding (421ms)



(26) $\cdot \vdash (\neg\neg A \wedge B) \leftrightarrow (\neg\neg A \wedge \neg\neg B)$

LJ (137ms)

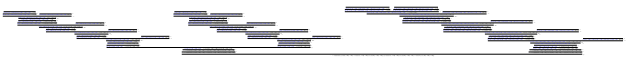


MULTIPLICATIVE encoding (66ms)

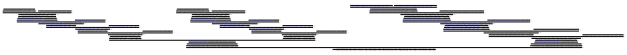
Not provable

CALL-BY-NAME encoding (601ms)

CALL-BY-VALUE encoding (584ms)



01-ENC encoding (646ms)



$(27)\ \cdot \vdash (\neg\neg(A) \leftrightarrow (B)) \leftrightarrow (\neg\neg A \to B \wedge \neg\neg B \to A)$

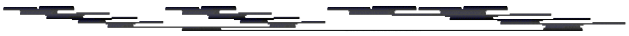
LJ (2703ms)

MULTIPLICATIVE encoding (84ms)

Not provable

CALL-BY-NAME encoding (661ms)

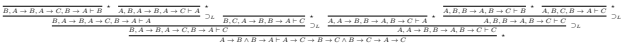
CALL-BY-VALUE encoding (841ms)



01-ENC encoding (1132ms)

$(28)\ (A) \leftrightarrow (B) \vdash (A \to C) \leftrightarrow (B \to C)$

LJ (75ms)



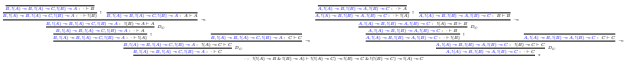
MULTIPLICATIVE encoding (95ms)

$$\dfrac{\dfrac{}{\cdot :\ B \vdash B}\quad\dfrac{\dfrac{}{\cdot :\ A \vdash A}\quad\dfrac{}{\cdot :\ C \vdash C}}{\cdot :\ A, A \multimap C \vdash C}\multimap}{\cdot :\ B, A \multimap C, B \multimap A \vdash C}\multimap \qquad \dfrac{\dfrac{}{\cdot :\ A \vdash A}\quad\dfrac{}{\cdot :\ B \vdash B}\quad\dfrac{}{\cdot :\ C \vdash C}}{\cdot :\ B, B \multimap C \vdash C}\multimap}{\cdot :\ A, A \multimap B, B \multimap C \vdash C}\multimap$$

$$\dfrac{\cdot :\ B \multimap A \vdash A \multimap C \multimap B \multimap C}{\cdot :\ A \multimap B \vdash B \multimap C \multimap A \multimap C}\star \qquad \dfrac{\cdot :\ A \multimap B \vdash B \multimap C \multimap A \multimap C}{\cdot :\ A \multimap B, B \multimap A \vdash A \multimap C \multimap B \multimap C \otimes B \multimap C \multimap A \multimap C}\otimes$$

$$\dfrac{\cdot :\ A \multimap B, B \multimap A \vdash A \multimap C \multimap B \multimap C \otimes B \multimap C \multimap A \multimap C}{\cdot :\ A \multimap B \otimes B \multimap A \vdash A \multimap C \multimap B \multimap C \otimes B \multimap C \multimap A \multimap C}\star$$

CALL-BY-NAME encoding (173ms)

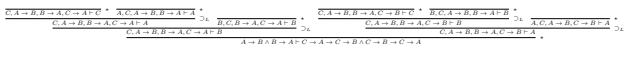


CALL-BY-VALUE encoding (288ms)

01-ENC encoding (268ms)

$\boxed{(29)\ (A) \leftrightarrow (B) \vdash (C \to A) \leftrightarrow (C \to B)}$

LJ (72ms)

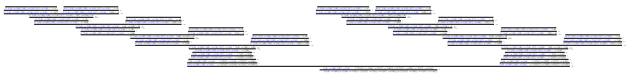


MULTIPLICATIVE encoding (96ms)

$$\dfrac{\dfrac{}{\cdot : C \vdash C} \quad \dfrac{\dfrac{}{\cdot : A \vdash A} \quad \dfrac{}{\cdot : B \vdash B}}{\cdot : A, A \multimap B \vdash B}\ \multimap}{\cdot : C, A \multimap B, C \multimap A \vdash B}\ \multimap \qquad \dfrac{\dfrac{}{\cdot : C \vdash C} \quad \dfrac{\dfrac{}{\cdot : B \vdash B} \quad \dfrac{}{\cdot : A \vdash A}}{\cdot : B, B \multimap A \vdash A}\ \multimap}{\cdot : C, B \multimap A, C \multimap B \vdash A}\ \multimap}{\dfrac{\cdot : A \multimap B \vdash C \multimap A \multimap C \multimap B}{\cdot : A \multimap B, B \multimap A \vdash C \multimap A \multimap C \multimap B \otimes C \multimap B \multimap C \multimap A}\ \star \qquad \dfrac{\cdot : B \multimap A \vdash C \multimap B \multimap C \multimap A}{\cdot : A \multimap B \otimes B \multimap A \vdash C \multimap A \multimap C \multimap B \otimes C \multimap B \multimap C \multimap A}\ \star}\ \otimes$$

CALL-BY-NAME encoding (250ms)

CALL-BY-VALUE encoding (387ms)



01-ENC encoding (356ms)

$\boxed{(30)\ (A) \leftrightarrow (B) \vdash (A \wedge C) \leftrightarrow (B \wedge C)}$

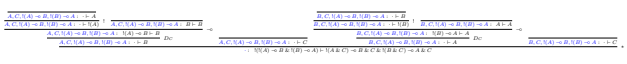
LJ (51ms)

$$\dfrac{\dfrac{A, C, A \to B, B \to A \vdash A}{A, C, A \to B, B \to A \vdash B}\ \star \quad \dfrac{A, B, C, B \to A \vdash B}{}\ \star}{A \to B \wedge B \to A \vdash A \wedge C \to B \wedge C \wedge B \wedge C \to A \wedge C}\ \supset_L \qquad \dfrac{\dfrac{B, C, A \to B, B \to A \vdash B}{B, C, A \to B, B \to A \vdash A}\ \star \quad \dfrac{A, B, C, A \to B \vdash A}{}\ \star}{}\ \supset_L$$

MULTIPLICATIVE encoding (99ms)

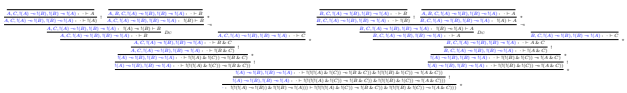
$$\dfrac{\dfrac{}{\cdot : A \vdash A} \quad \dfrac{\dfrac{}{\cdot : B \vdash B} \quad \dfrac{}{\cdot : C \vdash C}}{\cdot : B, C \vdash B \otimes C}\ \otimes}{\cdot : A, C, A \multimap B \vdash B \otimes C}\ \multimap}{\dfrac{\cdot : A \multimap B \vdash A \otimes C \multimap B \otimes C}{\cdot : A \multimap B, B \multimap A \vdash A \otimes C \multimap B \otimes C \otimes B \otimes C \multimap A \otimes C}\ \star \qquad \dfrac{\dfrac{}{\cdot : B \vdash B} \quad \dfrac{\dfrac{}{\cdot : A \vdash A} \quad \dfrac{}{\cdot : C \vdash C}}{\cdot : A, C \vdash A \otimes C}\ \otimes}{\cdot : B, C, B \multimap A \vdash A \otimes C}\ \multimap}{\cdot : B \multimap A \vdash B \otimes C \multimap A \otimes C}\ \star}\ \otimes$$

CALL-BY-NAME encoding (138ms)



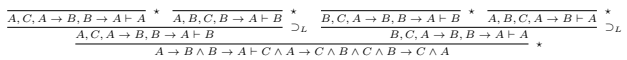
CALL-BY-VALUE encoding (238ms)

01-ENC encoding (234ms)

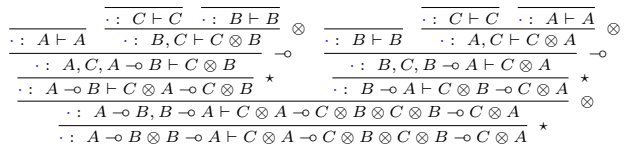


(31) $(A) \leftrightarrow (B) \vdash (C \wedge A) \leftrightarrow (C \wedge B)$

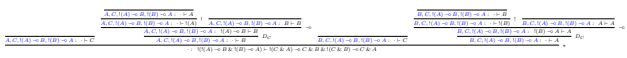
LJ (51ms)



MULTIPLICATIVE encoding (98ms)

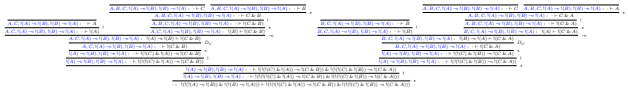


CALL-BY-NAME encoding (142ms)



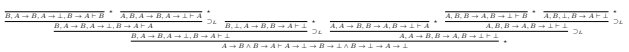
CALL-BY-VALUE encoding (227ms)

01-ENC encoding (229ms)

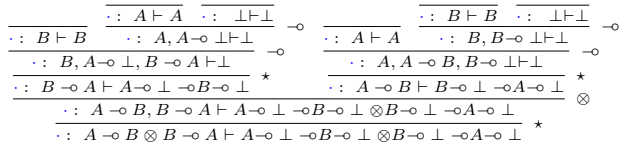


(32) $(A) \leftrightarrow (B) \vdash (\neg A) \leftrightarrow (\neg B)$

LJ (74ms)

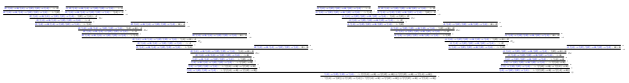


MULTIPLICATIVE encoding (97ms)

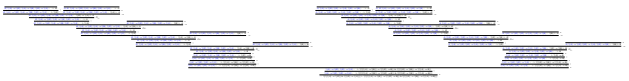


CALL-BY-NAME encoding (223ms)

CALL-BY-VALUE encoding (304ms)



01-ENC encoding (320ms)



$\boxed{(33)\ \cdot \vdash (A \wedge B \wedge C) \leftrightarrow (A \wedge B \wedge C)}$

LJ (21ms)

$$\frac{}{\cdot \vdash A \wedge B \wedge C \to A \wedge B \wedge C \wedge A \wedge B \wedge C \to A \wedge B \wedge C}\ \star$$

MULTIPLICATIVE encoding (66ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\cdot :\ A \vdash A}\quad\dfrac{}{\cdot :\ B \vdash B}}{\cdot :\ A, B \vdash A \otimes B}\otimes\quad\dfrac{}{\cdot :\ C \vdash C}}{\cdot :\ A, B, C \vdash A \otimes B \otimes C}\otimes}{\cdot :\ \cdot \vdash A \otimes B \otimes C \multimap A \otimes B \otimes C}\star\quad\dfrac{\dfrac{\dfrac{}{\cdot :\ A \vdash A}\quad\dfrac{}{\cdot :\ B \vdash B}}{\cdot :\ A, B \vdash A \otimes B}\otimes\quad\dfrac{}{\cdot :\ C \vdash C}}{\cdot :\ A, B, C \vdash A \otimes B \otimes C}\otimes}{\cdot :\ \cdot \vdash A \otimes B \otimes C \multimap A \otimes B \otimes C}\star}{\cdot :\ \cdot \vdash A \otimes B \otimes C \multimap A \otimes B \otimes C \otimes A \otimes B \otimes C \multimap A \otimes B \otimes C}\otimes$$

CALL-BY-NAME encoding (34ms)

$$\dfrac{\overline{A, B, C :\ \cdot \vdash A}\quad\overline{A, B, C :\ \cdot \vdash B}\quad\overline{A, B, C :\ \cdot \vdash C}\quad\overline{A, B, C :\ \cdot \vdash A}\quad\overline{A, B, C :\ \cdot \vdash B}\quad\overline{A, B, C :\ \cdot \vdash C}}{\cdot :\ \cdot \vdash\ !(A \,\&\, B \,\&\, C) \multimap A \,\&\, B \,\&\, C \,\&\, !(A \,\&\, B \,\&\, C) \multimap A \,\&\, B \,\&\, C}\ \star$$

CALL-BY-VALUE encoding (99ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\overline{A, B, C :\ \cdot \vdash A}}{A, B, C :\ \cdot \vdash\ !(A)}\,!\quad\dfrac{\overline{A, B, C :\ \cdot \vdash B}}{A, B, C :\ \cdot \vdash\ !(B)}\,!}{A, B, C :\ \cdot \vdash\ !(A) \otimes\ !(B)}\otimes\quad\dfrac{\overline{A, B, C :\ \cdot \vdash C}}{A, B, C :\ \cdot \vdash\ !(C)}\,!}{A, B, C :\ \cdot \vdash\ !(A) \otimes\ !(B) \otimes\ !(C)}\otimes}{\cdot :\ \cdot \vdash\ !(A) \otimes\ !(B) \otimes\ !(C) \multimap\ !(A) \otimes\ !(B) \otimes\ !(C)}\star}{\cdot :\ \cdot \vdash\ !(!(A) \otimes\ !(B) \otimes\ !(C) \multimap\ !(A) \otimes\ !(B) \otimes\ !(C))}\,!\quad\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\overline{A, B, C :\ \cdot \vdash A}}{A, B, C :\ \cdot \vdash\ !(A)}\,!\quad\dfrac{\overline{A, B, C :\ \cdot \vdash B}}{A, B, C :\ \cdot \vdash\ !(B)}\,!}{A, B, C :\ \cdot \vdash\ !(A) \otimes\ !(B)}\otimes\quad\dfrac{\overline{A, B, C :\ \cdot \vdash C}}{A, B, C :\ \cdot \vdash\ !(C)}\,!}{A, B, C :\ \cdot \vdash\ !(A) \otimes\ !(B) \otimes\ !(C)}\otimes}{\cdot :\ \cdot \vdash\ !(A) \otimes\ !(B) \otimes\ !(C) \multimap\ !(A) \otimes\ !(B) \otimes\ !(C)}\star}{\cdot :\ \cdot \vdash\ !(!(A) \otimes\ !(B) \otimes\ !(C) \multimap\ !(A) \otimes\ !(B) \otimes\ !(C))}\,!}{\cdot :\ \cdot \vdash\ !(!(A) \otimes\ !(B) \otimes\ !(C) \multimap\ !(A) \otimes\ !(B) \otimes\ !(C)) \otimes\ !(!(A) \otimes\ !(B) \otimes\ !(C) \multimap\ !(A) \otimes\ !(B) \otimes\ !(C))}\otimes$$

01-ENC encoding (124ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\overline{A, B, C :\ \cdot \vdash A}}{A, B, C :\ \cdot \vdash\ !(B \,\&\, !(C))}\,!}{A, B, C :\ \cdot \vdash A \,\&\, !(B \,\&\, !(C))}\star}{A, B, C :\ \cdot \vdash\ !(A \,\&\, !(B \,\&\, !(C))}\,!}{\cdot :\ \cdot \vdash\ !(!(A) \,\&\, !(B)) \,\&\, !(C)) \multimap\ !(A \,\&\, !(B \,\&\, !(C)))}\star}{\cdot :\ \cdot \vdash\ !(!(!(A) \,\&\, !(B)) \,\&\, !(C)) \multimap\ !(A \,\&\, !(B \,\&\, !(C)))}\,!\quad\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\overline{A, B, C :\ \cdot \vdash A}}{A, B, C :\ \cdot \vdash\ !(A)}\,!\quad\dfrac{\overline{A, B, C :\ \cdot \vdash B}}{A, B, C :\ \cdot \vdash\ !(B)}\,!}{A, B, C :\ \cdot \vdash A \,\&\, B}\star}{A, B, C :\ \cdot \vdash\ !(A \,\&\, B)}\,!\quad\dfrac{\overline{A, B, C :\ \cdot \vdash C}}{A, B, C :\ \cdot \vdash\ !(!(A \,\&\, B) \,\&\, !(C))}\,!}{A, B, C :\ \cdot \vdash\ !(!(A \,\&\, B) \,\&\, !(C)) \multimap\ !(!(A \,\&\, B) \,\&\, !(C))}\star}{\cdot :\ \cdot \vdash\ !(!(!(A) \,\&\, !(B)) \,\&\, !(C)) \multimap\ !(!(A \,\&\, B) \,\&\, !(C)))}\,!}{\cdot :\ \cdot \vdash\ !(!(!(!(A) \,\&\, !(B)) \,\&\, !(C)) \multimap\ !(A \,\&\, !(B \,\&\, !(C))) \,\&\, !(!(!(A) \,\&\, !(B) \,\&\, !(C))) \multimap\ !(!(A \,\&\, B) \,\&\, !(C)))}\star}{\cdot :\ \cdot \vdash\ !(!(!(!(!(A) \,\&\, !(B)) \,\&\, !(C)) \multimap\ !(A \,\&\, !(B \,\&\, !(C))) \,\&\, !(!(!(A) \,\&\, !(B) \,\&\, !(C))) \multimap\ !(!(A \,\&\, B) \,\&\, !(C)))}\,!$$

$\boxed{(34)\ \cdot \vdash (A \wedge B) \leftrightarrow (B \wedge A)}$

LJ (21ms)

$$\frac{}{\cdot \vdash A \wedge B \to B \wedge A \wedge B \wedge A \to A \wedge B}\ \star$$

MULTIPLICATIVE encoding (73ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\cdot :\ B \vdash B}\quad\dfrac{}{\cdot :\ A \vdash A}}{\cdot :\ A, B \vdash B \otimes A}\otimes}{\cdot :\ \cdot \vdash A \otimes B \multimap B \otimes A}\star\quad\dfrac{\dfrac{\dfrac{}{\cdot :\ A \vdash A}\quad\dfrac{}{\cdot :\ B \vdash B}}{\cdot :\ A, B \vdash A \otimes B}\otimes}{\cdot :\ \cdot \vdash B \otimes A \multimap A \otimes B}\star}{\cdot :\ \cdot \vdash A \otimes B \multimap B \otimes A \otimes B \otimes A \multimap A \otimes B}\otimes$$

CALL-BY-NAME encoding (35ms)

$$\dfrac{\overline{A, B :\ \cdot \vdash B}\quad\overline{A, B :\ \cdot \vdash A}\quad\overline{A, B :\ \cdot \vdash A}\quad\overline{A, B :\ \cdot \vdash B}}{\cdot :\ \cdot \vdash\ !(A \,\&\, B) \multimap B \,\&\, A \,\&\, !(B \,\&\, A) \multimap A \,\&\, B}\ \star$$

CALL-BY-VALUE encoding (107ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\overline{A, B :\ \cdot \vdash B}}{A, B :\ \cdot \vdash\ !(B)}\,!\quad\dfrac{\overline{A, B :\ \cdot \vdash A}}{A, B :\ \cdot \vdash\ !(A)}\,!}{A, B :\ \cdot \vdash\ !(B) \otimes\ !(A)}\otimes}{\cdot :\ \cdot \vdash\ !(A) \otimes\ !(B) \multimap\ !(B) \otimes\ !(A)}\star}{\cdot :\ \cdot \vdash\ !(!(A) \otimes\ !(B) \multimap\ !(B) \otimes\ !(A))}\,!\quad\dfrac{\dfrac{\dfrac{\dfrac{\overline{A, B :\ \cdot \vdash A}}{A, B :\ \cdot \vdash\ !(A)}\,!\quad\dfrac{\overline{A, B :\ \cdot \vdash B}}{A, B :\ \cdot \vdash\ !(B)}\,!}{A, B :\ \cdot \vdash\ !(A) \otimes\ !(B)}\otimes}{\cdot :\ \cdot \vdash\ !(A) \otimes\ !(B) \multimap\ !(B) \otimes\ !(A)}\star}{\cdot :\ \cdot \vdash\ !(!(B) \otimes\ !(A) \multimap\ !(A) \otimes\ !(B))}\,!}{\cdot :\ \cdot \vdash\ !(!(A) \otimes\ !(B) \multimap\ !(B) \otimes\ !(A)) \otimes\ !(!(B) \otimes\ !(A) \multimap\ !(A) \otimes\ !(B))}\otimes$$

01-ENC encoding (102ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\overline{A, B :\ \cdot \vdash B}\quad\overline{A, B :\ \cdot \vdash A}}{A, B :\ \cdot \vdash B \,\&\, A}\star}{A, B :\ \cdot \vdash\ !(B \,\&\, A)}\,!}{\cdot :\ \cdot \vdash\ !(!(A) \,\&\, !(B)) \multimap\ !(B \,\&\, A)}\star}{\cdot :\ \cdot \vdash\ !(!(!(A) \,\&\, !(B)) \multimap\ !(B \,\&\, A))}\,!\quad\dfrac{\dfrac{\dfrac{\dfrac{\overline{A, B :\ \cdot \vdash A}\quad\overline{A, B :\ \cdot \vdash B}}{A, B :\ \cdot \vdash A \,\&\, B}\star}{A, B :\ \cdot \vdash\ !(A \,\&\, B)}\,!}{\cdot :\ \cdot \vdash\ !(!(B) \,\&\, !(A)) \multimap\ !(A \,\&\, B)}\star}{\cdot :\ \cdot \vdash\ !(!(!(B) \,\&\, !(A)) \multimap\ !(A \,\&\, B))}\,!}{\cdot :\ \cdot \vdash\ !(!(!(A) \,\&\, !(B)) \multimap\ !(B \,\&\, A)) \,\&\, !(!(!(B) \,\&\, !(A)) \multimap\ !(A \,\&\, B))}\star$$

$$\dfrac{\cdot :\ \cdot \vdash\ !(!(!(!(A) \,\&\, !(B)) \multimap\ !(B \,\&\, A)) \,\&\, !(!(!(B) \,\&\, !(A)) \multimap\ !(A \,\&\, B)))}{}\,!$$

$(35)\ \cdot \vdash (A \wedge A) \leftrightarrow (A)$

LJ (21ms)

$$\dfrac{}{\cdot \vdash A \wedge A \to A \wedge A \to A \wedge A}\ \star$$

MULTIPLICATIVE encoding (34ms)

Not provable

CALL-BY-NAME encoding (28ms)

$$\dfrac{\dfrac{}{A :\ \cdot \vdash A}\quad \dfrac{}{A :\ \cdot \vdash A}\quad \dfrac{}{A :\ \cdot \vdash A}}{\cdot :\ \cdot \vdash\ !(A \,\&\, A) \multimap A \,\&\, !(A) \multimap A \,\&\, A}\ \star$$

CALL-BY-VALUE encoding (76ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A :\ \cdot \vdash A}}{A :\ \cdot \vdash\ !(A)}\ !}{\cdot :\ \cdot \vdash\ !(A) \otimes\ !(A) \multimap\ !(A)}\ \star \quad \dfrac{\dfrac{\dfrac{}{A :\ \cdot \vdash A}\quad \dfrac{}{A :\ \cdot \vdash A}}{A :\ \cdot \vdash\ !(A) \otimes\ !(A)}\ !}{\cdot :\ \cdot \vdash\ !(A) \multimap\ !(A) \otimes\ !(A)}\ \star}{\cdot :\ \cdot \vdash\ !(!(A) \otimes\ !(A) \multimap\ !(A)) \otimes\ !(!(A) \multimap\ !(A) \otimes\ !(A))}\ \otimes$$

01-ENC encoding (81ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A :\ \cdot \vdash A}}{\cdot :\ \cdot \vdash\ !(!(A) \,\&\, !(A)) \multimap A}\ \star}{\cdot :\ \cdot \vdash\ !(!(!(A) \,\&\, !(A)) \multimap A)}\ ! \quad \dfrac{\dfrac{\dfrac{}{A :\ \cdot \vdash A}\quad \dfrac{}{A :\ \cdot \vdash A}}{A :\ \cdot \vdash A \,\&\, A}\ \star}{\cdot :\ \cdot \vdash\ !(A \,\&\, A)}\ !}{\cdot :\ \cdot \vdash\ !(!(!(A) \,\&\, !(A)) \multimap A) \,\&\, !(!(A) \multimap\ !(A \,\&\, A))}\ \star}{\cdot :\ \cdot \vdash\ !(!(!(!(A) \,\&\, !(A)) \multimap A) \,\&\, !(!(A) \multimap\ !(A \,\&\, A)))}\ !$$

$(36)\ A \vdash (A \to B) \leftrightarrow (B)$

LJ (37ms)

$$\dfrac{\dfrac{\dfrac{}{A, A \to B \vdash A}\ \star \quad \dfrac{}{A, B \vdash B}\ \star}{A, A \to B \vdash B}\ \supset L}{A \vdash A \to B \to B \wedge B \to A \to B}\ \star$$

MULTIPLICATIVE encoding (42ms)

Not provable

CALL-BY-NAME encoding (73ms)

$$\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap B :\ \cdot \vdash A}}{A, !(A) \multimap B :\ \cdot \vdash\ !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap B :\ B \vdash B}}{\dfrac{A, !(A) \multimap B :\ !(A) \multimap B \vdash B}{A, !(A) \multimap B :\ \cdot \vdash B}\ D_C \qquad \dfrac{}{A, B :\ \cdot \vdash B}}{\cdot :\ !(A) \vdash\ !(!(A) \multimap B) \multimap B \,\&\, !(B) \multimap\ !(A) \multimap B}\ \star\ \multimap$$

CALL-BY-VALUE encoding (129ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap\ !(B) :\ \cdot \vdash A}}{A, !(A) \multimap\ !(B) :\ \cdot \vdash\ !(A)}\ ! \quad \dfrac{\dfrac{}{A, B, !(A) \multimap\ !(B) :\ \cdot \vdash B}}{A, !(A) \multimap\ !(B) :\ !(B) \vdash\ !(B)}\ !}{\dfrac{A, !(A) \multimap\ !(B) :\ !(A) \multimap\ !(B) \vdash\ !(B)}{A, !(A) \multimap\ !(B) :\ \cdot \vdash\ !(B)}\ D_C}\ \multimap \quad \dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \vdash B}}{A, B :\ \cdot \vdash\ !(B)}\ ! \quad \dfrac{}{A, B :\ \cdot \vdash\ !(A) \multimap\ !(B)}\ \star}{A, B :\ \cdot \vdash\ !(!(A) \multimap\ !(B))}\ !}{\dfrac{A :\ \cdot \vdash\ !(!(A) \multimap\ !(B)) \multimap\ !(B)}{A :\ \cdot \vdash\ !(!(!(A) \multimap\ !(B)) \multimap\ !(B)) \otimes\ !(!(B) \multimap\ !(!(A) \multimap\ !(B)))}\ !}{\cdot :\ !(A) \vdash\ !(!(!(A) \multimap\ !(B)) \multimap\ !(B)) \otimes\ !(!(B) \multimap\ !(!(A) \multimap\ !(B)))}\ \star}\ \otimes$$

01-ENC encoding (123ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap\ !(B) :\ \cdot \vdash A}}{A, !(A) \multimap\ !(B) :\ \cdot \vdash\ !(A)}\ ! \quad \dfrac{\dfrac{}{A, B, !(A) \multimap\ !(B) :\ \cdot \vdash B}}{A, !(A) \multimap\ !(B) :\ !(B) \vdash B}\ !}{\dfrac{A, !(A) \multimap\ !(B) :\ !(A) \multimap\ !(B) \vdash B}{A, !(A) \multimap\ !(B) :\ \cdot \vdash B}\ D_C}\ \multimap \quad \dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \vdash B}}{A, B :\ \cdot \vdash\ !(A) \multimap B}\ \star \quad \dfrac{}{A, B :\ \cdot \vdash\ !(!(A) \multimap B)}\ !}{A :\ \cdot \vdash\ !(B) \multimap\ !(!(A) \multimap B)}\ \star \quad \dfrac{}{A :\ \cdot \vdash\ !(!(B) \multimap\ !(!(A) \multimap B))}\ !}{\dfrac{A :\ \cdot \vdash\ !(!(!(A) \multimap\ !(B)) \multimap B) \,\&\, !(!(B) \multimap\ !(!(A) \multimap B))}{A :\ \cdot \vdash\ !(!(!(!(A) \multimap\ !(B)) \multimap B) \,\&\, !(!(B) \multimap\ !(!(A) \multimap B)))}\ !}{\cdot :\ !(A) \vdash\ !(!(!(!(A) \multimap\ !(B)) \multimap B) \,\&\, !(!(B) \multimap\ !(!(A) \multimap B)))}\ \star}\ \multimap$$

(37) $B \vdash (A \to B) \leftrightarrow (B)$

LJ (21ms)

$$\dfrac{}{B \vdash A \to B \to B \wedge B \to A \to B}\ \star$$

MULTIPLICATIVE encoding (40ms)

Not provable

CALL-BY-NAME encoding (39ms)

$$\dfrac{\dfrac{}{B, !(A) \multimap B :\ \cdot \vdash B} \quad \dfrac{}{A, B :\ \cdot \vdash B}\ \star}{\cdot :\ \ !(B) \vdash !(!(A) \multimap B) \multimap B \,\&\, !(B) \multimap !(A) \multimap B}\ \star$$

CALL-BY-VALUE encoding (152ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{B, !(A) \multimap !(B) :\ \cdot \vdash B}\ !}{B, !(A) \multimap !(B) :\ \cdot \vdash !(B)}\ \star}{B :\ \cdot \vdash !(!(A) \multimap !(B)) \multimap !(B)}\ \star \quad \dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \vdash B}\ !}{A, B :\ \cdot \vdash !(B)}\ \star}{B :\ \cdot \vdash !(B) \multimap !(!(A) \multimap !(B))}\ !}{B :\ \cdot \vdash !(!(!(A) \multimap !(B)) \multimap !(B))}\ \star \quad \dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \vdash B}\ !}{B :\ \cdot \vdash !(A) \multimap !(B)}\ \star}{B :\ \cdot \vdash !(!(A) \multimap !(B))}\ !}{B :\ \cdot \vdash !(!(!(A) \multimap !(B)) \multimap !(B)) \otimes !(!(B) \multimap !(!(A) \multimap !(B)))}\ \otimes$$
$$\dfrac{}{\cdot :\ \ !(B) \vdash !(!(!(A) \multimap !(B)) \multimap !(B)) \otimes !(!(B) \multimap !(!(A) \multimap !(B)))}\ \star$$

01-ENC encoding (88ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{B, !(A) \multimap !(B) :\ \cdot \vdash B}}{B :\ \cdot \vdash !(A) \multimap !(B)) \multimap B}\ \star \quad \dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \vdash B}\ \star}{B :\ \cdot \vdash !(A) \multimap B)}\ !}{B :\ \cdot \vdash !(B) \multimap !(!(A) \multimap B)}\ \star}{B :\ \cdot \vdash !(!(!(A) \multimap !(B)) \multimap B) \,\&\, !(!(B) \multimap !(!(A) \multimap B))}\ !}{B :\ \cdot \vdash !(!(!(!(A) \multimap !(B)) \multimap B) \,\&\, !(!(B) \multimap !(!(A) \multimap B)))}\ \star$$
$$\dfrac{}{\cdot :\ \ !(B) \vdash !(!(!(!(A) \multimap !(B)) \multimap B) \,\&\, !(!(B) \multimap !(!(A) \multimap B)))}\ \star$$

(38) $\neg A \vdash (A \to B) \leftrightarrow (\neg A)$

LJ (52ms)

$$\dfrac{\dfrac{}{A, A \to B, A \to \bot \vdash A}\ \star \quad \dfrac{}{A, \bot, A \to B \vdash \bot}\ \star}{A, A \to B, A \to \bot \vdash \bot}\ \supset_L \qquad \dfrac{\dfrac{}{A, A \to \bot \vdash A}\ \star \quad \dfrac{}{A, \bot \vdash B}\ \star}{A, A \to \bot \vdash B}\ \supset_L$$
$$\dfrac{}{A \to \bot \vdash A \to B \to A \to \bot \wedge A \to \bot \to A \to B}\ \star$$

MULTIPLICATIVE encoding (42ms)

Not provable

CALL-BY-NAME encoding (101ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap B, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}}{A, !(A) \multimap B, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap B, !(A) \multimap \mathbf{0} :\ \mathbf{0} \vdash \mathbf{0}}\ \star}{A, !(A) \multimap B, !(A) \multimap \mathbf{0} :\ \cdot \vdash \mathbf{0}}\ D_C}{\cdot :\ \ !(!(A) \multimap \mathbf{0}) \vdash !(!(A) \multimap B) \multimap !(A) \multimap \mathbf{0} \,\&\, !(!(A) \multimap \mathbf{0}) \multimap !(A) \multimap B}\ \star \quad \dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap \mathbf{0} :\ !(A) \multimap \mathbf{0} \vdash B}\ \star}{A, !(A) \multimap \mathbf{0} :\ !(A) \multimap \mathbf{0} \vdash B}\ D_C}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash B}\ \star$$

CALL-BY-VALUE encoding (201ms)

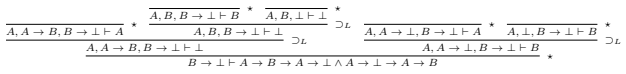
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0}, !(A) \multimap !(B) :\ \cdot \vdash A}}{A, !(A) \multimap \mathbf{0}, !(A) \multimap !(B) :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap \mathbf{0}, !(A) \multimap !(B) :\ !(A) \multimap \mathbf{0} \vdash \mathbf{0}}\ \star}{A, !(A) \multimap \mathbf{0}, !(A) \multimap !(B) :\ \cdot \vdash \mathbf{0}}\ D_C}{!(A) \multimap \mathbf{0}, !(A) \multimap !(B) :\ \cdot \vdash !(A) \multimap \mathbf{0}}\ \star}{!(A) \multimap \mathbf{0}, !(A) \multimap !(B) :\ \cdot \vdash !(!(A) \multimap \mathbf{0})}\ !}{!(A) \multimap \mathbf{0} :\ \cdot \vdash !(!(A) \multimap !(B)) \multimap !(!(A) \multimap \mathbf{0}))}\ \star \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap \mathbf{0} :\ \mathbf{0} \vdash !(B)}\ \star}{A, !(A) \multimap \mathbf{0} :\ !(A) \multimap \mathbf{0} \vdash !(B)}\ D_C}{A, !(A) \multimap \mathbf{0} :\ !(A) \multimap !(B)}\ \star}{!(A) \multimap \mathbf{0} :\ \cdot \vdash !(A) \multimap !(B)}\ !}{!(A) \multimap \mathbf{0} :\ \cdot \vdash !(!(A) \multimap \mathbf{0}) \multimap !(!(A) \multimap !(B))}\ \star}{!(A) \multimap \mathbf{0} :\ \cdot \vdash !(!(!(A) \multimap !(B)) \multimap !(!(A) \multimap \mathbf{0})) \otimes !(!(!(A) \multimap \mathbf{0}) \multimap !(!(A) \multimap !(B)))}\ \otimes$$
$$\dfrac{}{\cdot :\ \ !(!(A) \multimap \mathbf{0}) \vdash !(!(!(A) \multimap !(B)) \multimap !(!(A) \multimap \mathbf{0})) \otimes !(!(!(A) \multimap \mathbf{0}) \multimap !(!(A) \multimap !(B)))}\ \star$$

01-ENC encoding (203ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash A}}{A, !(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ !(A) \multimap !(\mathbf{0}) \vdash \mathbf{0}}\ \star}{A, !(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash \mathbf{0}}\ D_C}{!(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(A) \multimap \mathbf{0}}\ \star}{!(A) \multimap !(B), !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(!(A) \multimap \mathbf{0})}\ !}{!(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(!(A) \multimap !(B)) \multimap !(!(A) \multimap \mathbf{0}))}\ \star \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash A}}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap !(\mathbf{0}) :\ !(A) \multimap !(\mathbf{0}) \vdash B}\ \star}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(A) \multimap B)}\ D_C}{!(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(A) \multimap B)}\ \star}{!(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(!(A) \multimap B)}\ !}{!(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(!(A) \multimap !(\mathbf{0})) \multimap !(!(A) \multimap B))}\ \star}{!(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(!(!(A) \multimap !(B)) \multimap !(!(A) \multimap \mathbf{0})) \,\&\, !(!(!(A) \multimap !(\mathbf{0})) \multimap !(!(A) \multimap B))}\ !}{\cdot :\ \ !(!(A) \multimap !(\mathbf{0})) \vdash !(!(!(A) \multimap !(B)) \multimap !(!(A) \multimap \mathbf{0})) \,\&\, !(!(!(A) \multimap !(\mathbf{0})) \multimap !(!(A) \multimap B))}\ \star$$

(39) $\neg B \vdash (A \to B) \leftrightarrow (\neg A)$

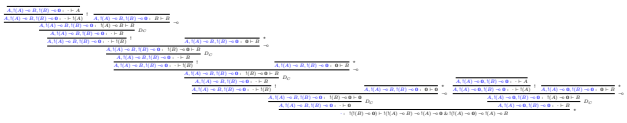
LJ (61ms)



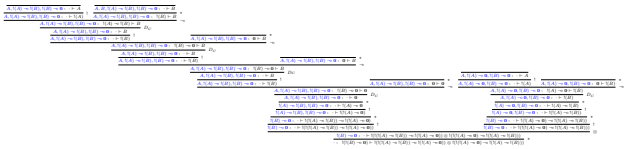
MULTIPLICATIVE encoding (43ms)

Not provable

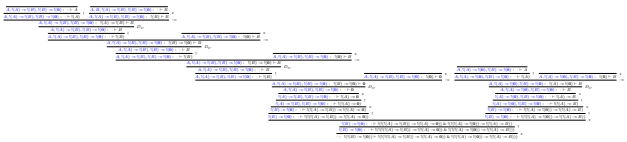
CALL-BY-NAME encoding (185ms)



CALL-BY-VALUE encoding (256ms)



01-ENC encoding (259ms)



(40) $B \vdash (A \wedge B) \leftrightarrow (A)$

LJ (21ms)

$$\dfrac{}{B \vdash A \wedge B \to A \wedge A \to A \wedge B}\ \star$$

MULTIPLICATIVE encoding (34ms)

Not provable

CALL-BY-NAME encoding (34ms)

$$\dfrac{\dfrac{}{A, B :\ \cdot \vdash A} \quad \dfrac{}{A, B :\ \cdot \vdash A} \quad \dfrac{}{A, B :\ \cdot \vdash B}}{\cdot :\ \ !(B) \vdash\ !(A \,\&\, B) \multimap A \,\&\, !(A) \multimap A \,\&\, B}\ \star$$

CALL-BY-VALUE encoding (88ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \vdash A}}{A, B :\ \cdot \vdash !(A)}\ !}{B :\ \cdot \vdash !(A) \otimes !(B) \multimap !(A)}\ \star}{B :\ \cdot \vdash !(!(A) \otimes !(B) \multimap !(A))}\ ! \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \vdash A}}{A, B :\ \cdot \vdash !(A)}\ ! \quad \dfrac{\dfrac{}{A, B :\ \cdot \vdash B}}{A, B :\ \cdot \vdash !(B)}\ !}{A, B :\ \cdot \vdash !(A) \otimes !(B)}\ \otimes}{B :\ \cdot \vdash !(A) \multimap !(A) \otimes !(B)}\ \star}{B :\ \cdot \vdash !(!(A) \multimap !(A) \otimes !(B))}\ !$$

$$\dfrac{B :\ \cdot \vdash !(!(A) \otimes !(B) \multimap !(A)) \otimes !(!(A) \multimap !(A) \otimes !(B))}{\cdot :\ \ !(B) \vdash !(!(A) \otimes !(B) \multimap !(A)) \otimes !(!(A) \multimap !(A) \otimes !(B))}\ \star$$

01-ENC encoding (88ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \vdash A}}{B :\ \cdot \vdash !(!(A) \,\&\, !(B)) \multimap A}\ \star}{B :\ \cdot \vdash !(!(!(A) \,\&\, !(B)) \multimap A)}\ ! \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{A, B :\ \cdot \vdash A} \quad \dfrac{}{A, B :\ \cdot \vdash B}}{A, B :\ \cdot \vdash A \,\&\, B}\ !}{A, B :\ \cdot \vdash !(A \,\&\, B)}\ !}{B :\ \cdot \vdash !(A) \multimap !(A \,\&\, B)}\ \star}{B :\ \cdot \vdash !(!(!(A) \,\&\, !(B)) \multimap A) \,\&\, !(!(A) \multimap !(A \,\&\, B))}\ \star$$

$$\dfrac{B :\ \cdot \vdash !(!(!(!(A) \,\&\, !(B)) \multimap A) \,\&\, !(!(A) \multimap !(A \,\&\, B)))}{\cdot :\ \ !(B) \vdash !(!(!(!(A) \,\&\, !(B)) \multimap A) \,\&\, !(!(A) \multimap !(A \,\&\, B)))}\ \star$$

$\boxed{(41)\ \neg B \vdash (A \wedge B) \leftrightarrow (B)}$

LJ (37ms)

$$\dfrac{\dfrac{\dfrac{}{B, B \to \bot \vdash B}\ \star \quad \dfrac{}{B, \bot \vdash A}\ \star}{B, B \to \bot \vdash A}\ \supset_L}{B \to \bot \vdash A \wedge B \to B \wedge B \to A \wedge B}\ \star$$

MULTIPLICATIVE encoding (41ms)

Not provable

CALL-BY-NAME encoding (78ms)

$$\dfrac{\dfrac{}{A, B, !(B) \multimap \mathbf{0} :\ \cdot \vdash B} \quad \dfrac{\dfrac{\dfrac{\dfrac{}{B, !(B) \multimap \mathbf{0} :\ \cdot \vdash B}\ !}{B, !(B) \multimap \mathbf{0} :\ \cdot \vdash !(B)} \quad \dfrac{}{B, !(B) \multimap \mathbf{0} :\ \mathbf{0} \vdash A}\ \star}{B, !(B) \multimap \mathbf{0} :\ !(B) \multimap \mathbf{0} \vdash A}\ \multimap}{B, !(B) \multimap \mathbf{0} :\ \cdot \vdash A}\ D_C \quad \dfrac{}{B, !(B) \multimap \mathbf{0} :\ \cdot \vdash B}}{\cdot :\ !(!(B) \multimap \mathbf{0}) \vdash !(A \,\&\, B) \multimap B \,\&\, !(B) \multimap A \,\&\, B}\ \star$$

CALL-BY-VALUE encoding (172ms)

$$\dfrac{\dfrac{\dfrac{}{A, B, !(B) \multimap \mathbf{0} :\ \cdot \vdash B}\ !}{A, B, !(B) \multimap \mathbf{0} :\ \cdot \vdash !(B)}}{\dfrac{!(B) \multimap \mathbf{0} :\ \cdot \vdash !(A) \otimes !(B) \multimap !(B)}{!(B) \multimap \mathbf{0} :\ \cdot \vdash !(!(A) \otimes !(B) \multimap !(B))}\ !}\ \star \quad \dfrac{\dfrac{\dfrac{\dfrac{}{B, !(B) \multimap \mathbf{0} :\ \cdot \vdash B}\ !}{B, !(B) \multimap \mathbf{0} :\ \cdot \vdash !(B)} \quad \dfrac{}{B, !(B) \multimap \mathbf{0} :\ \mathbf{0} \vdash !(A) \otimes !(B)}\ \star}{B, !(B) \multimap \mathbf{0} :\ !(B) \multimap \mathbf{0} \vdash !(A) \otimes !(B)}\ \multimap}{B, !(B) \multimap \mathbf{0} :\ \cdot \vdash !(A) \otimes !(B)}\ D_C}{!(B) \multimap \mathbf{0} :\ \cdot \vdash !(!(A) \otimes !(B) \multimap !(B))}\ !}\ \otimes$$
$$\dfrac{!(B) \multimap \mathbf{0} :\ \cdot \vdash !(!(A) \otimes !(B) \multimap !(B)) \otimes !(!(B) \multimap !(A) \otimes !(B))}{\cdot :\ !(!(B) \multimap \mathbf{0}) \vdash !(!(A) \otimes !(B) \multimap !(B)) \otimes !(!(B) \multimap !(A) \otimes !(B))}\ \star$$

01-ENC encoding (194ms)

$$\dfrac{\dfrac{\dfrac{}{A, B, !(B) \multimap !(\mathbf{0}) :\ \cdot \vdash B}}{\dfrac{!(\mathbf{0}) \multimap !(B) :\ \cdot \vdash !(!(A) \,\&\, !(B)) \multimap B}{!(B) \multimap !(\mathbf{0}) :\ \cdot \vdash !(!(!(A) \,\&\, !(B)) \multimap B)}\ !}\ \star \quad \dfrac{\dfrac{\dfrac{\dfrac{}{B, !(B) \multimap !(\mathbf{0}) :\ \cdot \vdash B}\ !}{B, !(B) \multimap !(\mathbf{0}) :\ \cdot \vdash !(B)} \quad \dfrac{}{B, !(B) \multimap !(\mathbf{0}) :\ !(\mathbf{0}) \vdash !(A \,\&\, B)}\ \star}{B, !(B) \multimap !(\mathbf{0}) :\ !(B) \multimap !(\mathbf{0}) \vdash !(A \,\&\, B)}\ \multimap}{B, !(B) \multimap !(\mathbf{0}) :\ \cdot \vdash !(A \,\&\, B)}\ D_C}{!(B) \multimap !(\mathbf{0}) :\ \cdot \vdash !(!(A) \,\&\, !(B)) \multimap !(A \,\&\, B))}\ !}\ \star$$
$$\dfrac{!(B) \multimap !(\mathbf{0}) :\ \cdot \vdash !(!(!(A) \,\&\, !(B)) \multimap B) \,\&\, !(!(B) \multimap !(A \,\&\, B))}{!(B) \multimap !(\mathbf{0}) :\ \cdot \vdash !(!(!(!(A) \,\&\, !(B)) \multimap B) \,\&\, !(!(B) \multimap !(A \,\&\, B)))}\ !$$
$$\dfrac{}{\cdot :\ !(!(B) \multimap !(\mathbf{0})) \vdash !(!(!(!(A) \,\&\, !(B)) \multimap B) \,\&\, !(!(B) \multimap !(A \,\&\, B)))}\ \star$$

$\boxed{(42)\ \cdot \vdash A \to \neg\neg A}$

LJ (38ms)

$$\dfrac{\dfrac{}{A, A \to \bot \vdash A}\ \star \quad \dfrac{}{A, \bot \vdash \bot}\ \star}{\dfrac{A, A \to \bot \vdash \bot}{\cdot \vdash A \to A \to \bot \to \bot}\ \star}\ \supset_L$$

MULTIPLICATIVE encoding (41ms)

$$\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{}{\cdot :\ \bot \vdash \bot}}{\cdot :\ A, A \multimap \bot \vdash \bot}\ \multimap}{\cdot :\ \cdot \vdash A \multimap A \multimap \bot \multimap \bot}\ \star$$

CALL-BY-NAME encoding (66ms)

$$\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap \mathbf{0} :\ \mathbf{0} \vdash \mathbf{0}}\ \star}{A, !(A) \multimap \mathbf{0} :\ !(A) \multimap \mathbf{0} \vdash \mathbf{0}}\ \multimap}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash \mathbf{0}}\ D_C}{\cdot :\ \cdot \vdash !(A) \multimap !(!(A) \multimap \mathbf{0}) \multimap \mathbf{0}}\ \star$$

CALL-BY-VALUE encoding (84ms)

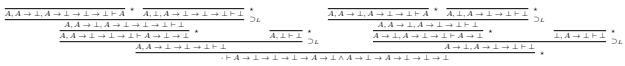
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap \mathbf{0} :\ \mathbf{0} \vdash \mathbf{0}}\ \star}{A, !(A) \multimap \mathbf{0} :\ !(A) \multimap \mathbf{0} \vdash \mathbf{0}}\ \multimap}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash \mathbf{0}}\ D_C}{A :\ \cdot \vdash !(!(A) \multimap \mathbf{0}) \multimap \mathbf{0}}\ \star}{A :\ \cdot \vdash !(!(!(A) \multimap \mathbf{0}) \multimap \mathbf{0})}\ !}{\cdot :\ \cdot \vdash !(A) \multimap !(!(!(A) \multimap \mathbf{0}) \multimap \mathbf{0})}\ \star}{\cdot :\ \cdot \vdash !(!(A) \multimap !(!(!(A) \multimap \mathbf{0}) \multimap \mathbf{0}))}\ !$$

01-ENC encoding (81ms)

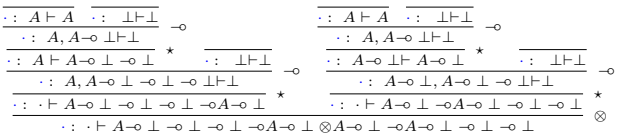
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash A}}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap !(\mathbf{0}) :\ !(\mathbf{0}) \vdash \mathbf{0}}\ \star}{A, !(A) \multimap !(\mathbf{0}) :\ !(A) \multimap !(\mathbf{0}) \vdash \mathbf{0}}\ \multimap}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash \mathbf{0}}\ D_C}{A :\ \cdot \vdash !(!(A) \multimap !(\mathbf{0})) \multimap \mathbf{0}}\ \star}{A :\ \cdot \vdash !(!(!(A) \multimap !(\mathbf{0})) \multimap \mathbf{0})}\ !}{\cdot :\ \cdot \vdash !(A) \multimap !(!(!(A) \multimap !(\mathbf{0})) \multimap \mathbf{0})}\ \star}{\cdot :\ \cdot \vdash !(!(A) \multimap !(!(!(A) \multimap !(\mathbf{0})) \multimap \mathbf{0}))}\ !$$

$(43)\ \cdot \vdash (\neg\neg\neg A) \leftrightarrow (\neg A)$

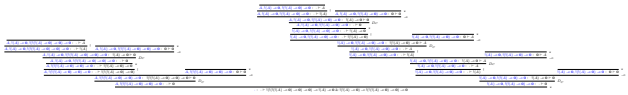
LJ (81ms)



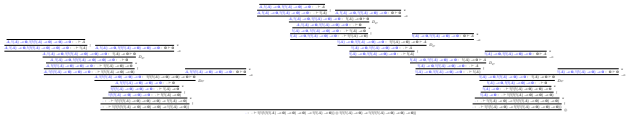
MULTIPLICATIVE encoding (105ms)



CALL-BY-NAME encoding (244ms)



CALL-BY-VALUE encoding (295ms)



01-ENC encoding (343ms)

$(44)\ \cdot \vdash \neg A \wedge \neg A$

LJ (38ms)

$$\dfrac{\dfrac{}{A, A \to \bot \vdash A}\ \star \quad \dfrac{}{A, \bot \vdash \bot}\ \star}{\dfrac{A, A \to \bot \vdash \bot}{\cdot \vdash A \wedge A \to \bot \to \bot}\ \star}\ \supset_L$$

MULTIPLICATIVE encoding (41ms)

$$\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{}{\cdot :\ \ \bot \vdash \bot}}{\dfrac{\cdot :\ A, A \multimap \bot \vdash \bot}{\cdot :\ \cdot \vdash A \otimes A \multimap \bot \multimap \bot}\ \star}\ \multimap$$

CALL-BY-NAME encoding (65ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap \mathbf{0} :\ \ \mathbf{0} \vdash \mathbf{0}}\ \star}{A, !(A) \multimap \mathbf{0} :\ \ !(A) \multimap \mathbf{0} \vdash \mathbf{0}}\ \multimap}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash \mathbf{0}}\ D_C}{\cdot :\ \cdot \vdash !(A \,\&\, !(A) \multimap \mathbf{0}) \multimap \mathbf{0}}\ \star$$

CALL-BY-VALUE encoding (70ms)

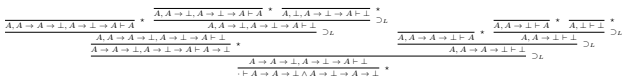
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash A}}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap \mathbf{0} :\ \ \mathbf{0} \vdash \mathbf{0}}\ \star}{A, !(A) \multimap \mathbf{0} :\ \ !(A) \multimap \mathbf{0} \vdash \mathbf{0}}\ \multimap}{A, !(A) \multimap \mathbf{0} :\ \cdot \vdash \mathbf{0}}\ D_C}{\cdot :\ \cdot \vdash !(A) \otimes !(!(A) \multimap \mathbf{0}) \multimap \mathbf{0}}\ \star}{\cdot :\ \cdot \vdash !(!(A) \otimes !(!(A) \multimap \mathbf{0}) \multimap \mathbf{0})}\ !$$

01-ENC encoding (70ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash A}}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash !(A)}\ ! \quad \dfrac{}{A, !(A) \multimap !(\mathbf{0}) :\ \ !(\mathbf{0}) \vdash \mathbf{0}}\ \star}{A, !(A) \multimap !(\mathbf{0}) :\ \ !(A) \multimap !(\mathbf{0}) \vdash \mathbf{0}}\ \multimap}{A, !(A) \multimap !(\mathbf{0}) :\ \cdot \vdash \mathbf{0}}\ D_C}{\cdot :\ \cdot \vdash !(!(A) \,\&\, !(!(A) \multimap !(\mathbf{0}))) \multimap \mathbf{0}}\ \star}{\cdot :\ \cdot \vdash !(!(!(A) \,\&\, !(!(A) \multimap !(\mathbf{0}))) \multimap \mathbf{0})}\ !$$

$(45)\ \cdot \vdash \neg(A) \leftrightarrow (\neg A)$

LJ (80ms)



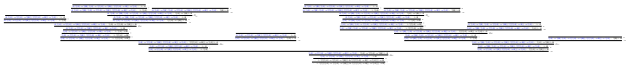
MULTIPLICATIVE encoding (41ms)

Not provable

CALL-BY-NAME encoding (231ms)

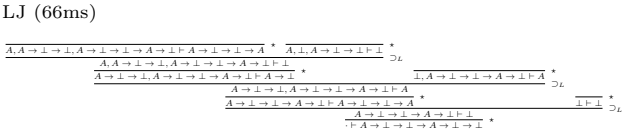
CALL-BY-VALUE encoding (282ms)

01-ENC encoding (300ms)



$(46)\ \cdot \vdash \neg\neg\neg\neg A \rightarrow A$

LJ (66ms)



MULTIPLICATIVE encoding (41ms)

Not provable

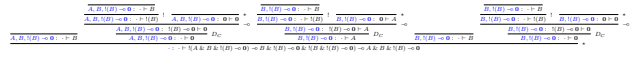
CALL-BY-NAME encoding (198ms)

CALL-BY-VALUE encoding (210ms)

01-ENC encoding (217ms)

$$\boxed{(47)\ \cdot \vdash (A \wedge B \wedge \neg B) \leftrightarrow (B \wedge \neg B)}$$

LJ (61ms)

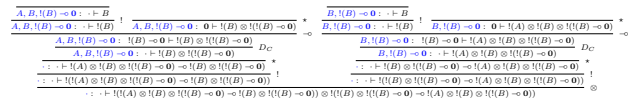


MULTIPLICATIVE encoding (42ms)

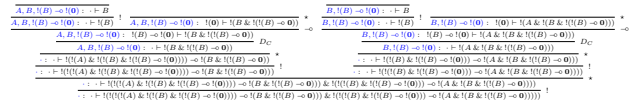
Not provable

CALL-BY-NAME encoding (121ms)

CALL-BY-VALUE encoding (140ms)

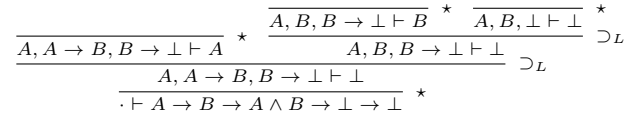


01-ENC encoding (132ms)

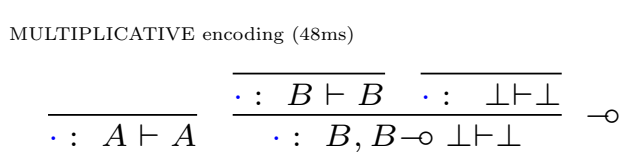


$$\boxed{(48)\ \cdot \vdash A \to B \to \neg A \wedge \neg B}$$

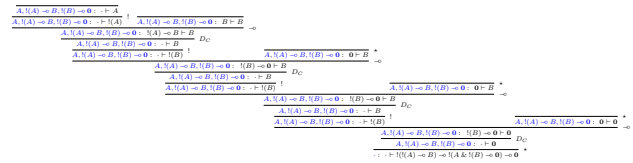
LJ (49ms)



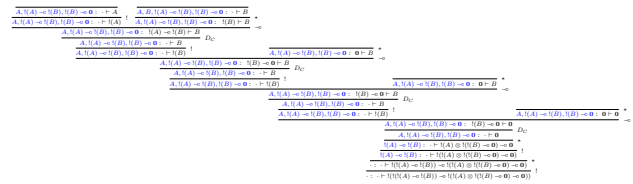
MULTIPLICATIVE encoding (48ms)



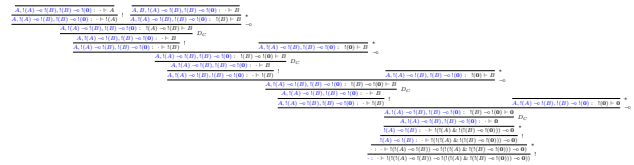
CALL-BY-NAME encoding (143ms)



CALL-BY-VALUE encoding (167ms)

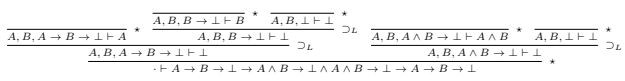


01-ENC encoding (169ms)

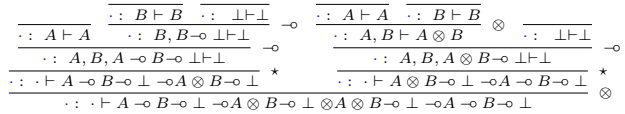


$\boxed{(49)\ \cdot \vdash (A \to \neg B) \leftrightarrow (\neg A \wedge B)}$

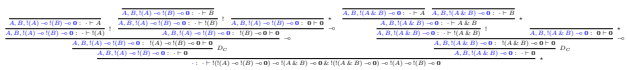
LJ (58ms)



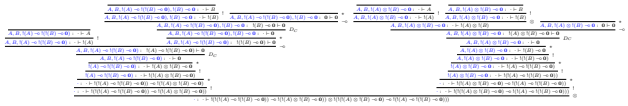
MULTIPLICATIVE encoding (88ms)



CALL-BY-NAME encoding (118ms)



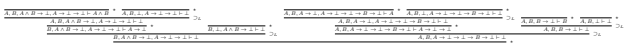
CALL-BY-VALUE encoding (281ms)



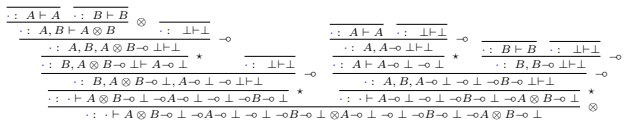
01-ENC encoding (318ms)

$\boxed{(50)\ \cdot \vdash (\neg A \wedge B) \leftrightarrow (\neg\neg A \to \neg B)}$

LJ (93ms)

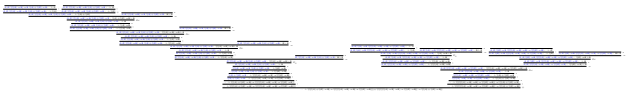


MULTIPLICATIVE encoding (134ms)

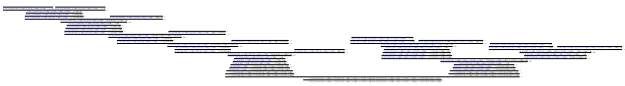


CALL-BY-NAME encoding (333ms)

CALL-BY-VALUE encoding (421ms)

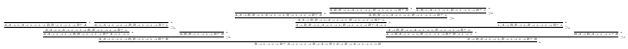


01-ENC encoding (470ms)

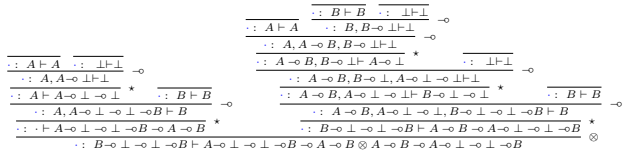


(51) $\neg\neg B \to B \vdash (\neg\neg A \to B) \leftrightarrow (A \to B)$

LJ (135ms)

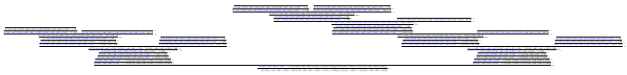


MULTIPLICATIVE encoding (156ms)



CALL-BY-NAME encoding (278ms)

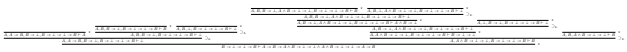
CALL-BY-VALUE encoding (414ms)



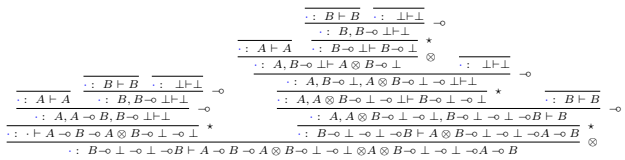
01-ENC encoding (449ms)

(52) $\neg\neg B \to B \vdash (A \to B) \leftrightarrow (\neg A \wedge \neg B)$

LJ (94ms)



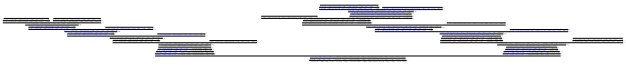
MULTIPLICATIVE encoding (134ms)



CALL-BY-NAME encoding (296ms)

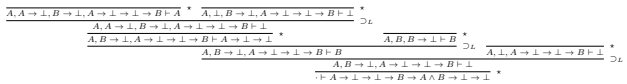
CALL-BY-VALUE encoding (457ms)

01-ENC encoding (489ms)



$$\boxed{(53)\ \cdot \vdash \neg\neg A \to B \to \neg A \wedge \neg B}$$

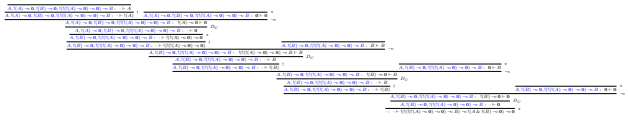
LJ (65ms)



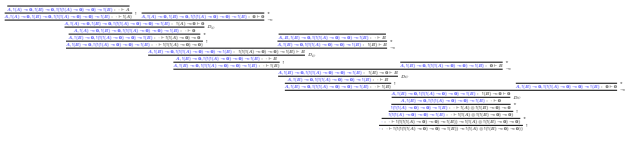
MULTIPLICATIVE encoding (75ms)

$$\dfrac{\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{}{\cdot :\ \ \perp \vdash \perp}}{\cdot :\ A, A \multimap \perp \vdash \perp} \multimap \quad \dfrac{\dfrac{}{\cdot :\ B \vdash B} \quad \dfrac{}{\cdot :\ \ \perp \vdash \perp}}{\cdot :\ B, B \multimap \perp \vdash \perp} \multimap}{\dfrac{\cdot :\ A, B \multimap \perp, A \multimap \perp \multimap \perp \multimap B \vdash \perp}{\cdot :\ \cdot \vdash A \multimap \perp \multimap \perp \multimap B \multimap A \otimes B \multimap \perp \multimap \perp} \star} \multimap} \star$$

CALL-BY-NAME encoding (178ms)



CALL-BY-VALUE encoding (204ms)



01-ENC encoding (215ms)

$$\boxed{(54)\ \cdot \vdash A \wedge B \to \neg A \to \neg B}$$

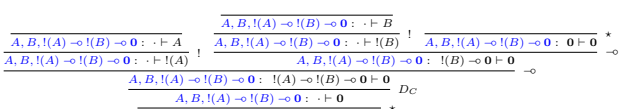
LJ (47ms)

$$\dfrac{\dfrac{\dfrac{}{A, B, A \to B \to \perp \vdash A} \star \quad \dfrac{\dfrac{}{A, B, B \to \perp \vdash B} \star \quad \dfrac{}{A, B, \perp \vdash \perp} \star}{A, B, B \to \perp \vdash \perp} \supset_L}{A, B, A \to B \to \perp \vdash \perp} \supset_L}{\cdot \vdash A \wedge B \to A \to B \to \perp \to \perp} \star$$

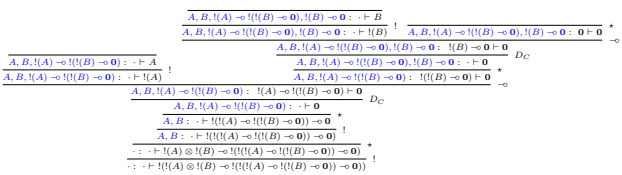
MULTIPLICATIVE encoding (48ms)

$$\dfrac{\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{\dfrac{}{\cdot :\ B \vdash B} \quad \dfrac{}{\cdot :\ \ \perp \vdash \perp}}{\cdot :\ B, B \multimap \perp \vdash \perp} \multimap}{\cdot :\ A, B, A \multimap B \multimap \perp \vdash \perp} \multimap}{\cdot :\ \cdot \vdash A \otimes B \multimap A \multimap B \multimap \perp \multimap \perp} \star$$

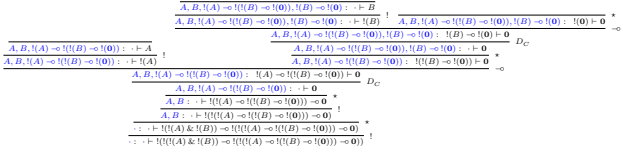
CALL-BY-NAME encoding (77ms)



CALL-BY-VALUE encoding (105ms)



01-ENC encoding (106ms)



$\boxed{(55)\ \cdot \vdash A \wedge \neg B \to \neg A \to B}$

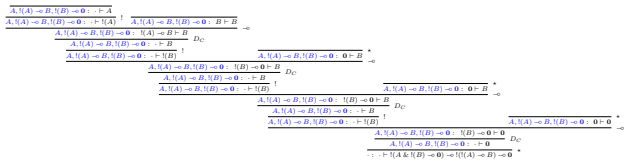
LJ (48ms)

$$\dfrac{\dfrac{\dfrac{}{A, A \to B, B \to \bot \vdash A}\ \star \quad \dfrac{\dfrac{}{A, B, B \to \bot \vdash B}\ \star \quad \dfrac{}{A, B, \bot \vdash \bot}\ \star}{A, B, B \to \bot \vdash \bot}\ \supset_L}{A, A \to B, B \to \bot \vdash \bot}\ \supset_L}{\cdot \vdash A \wedge B \to \bot \to A \to B \to \bot}\ \star$$

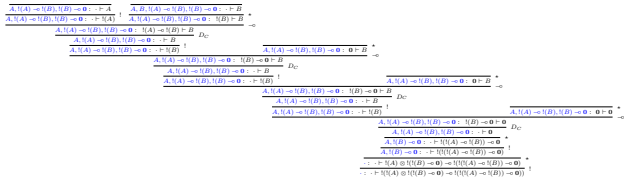
MULTIPLICATIVE encoding (47ms)

$$\dfrac{\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{\dfrac{}{\cdot :\ B \vdash B} \quad \dfrac{}{\cdot :\ \bot \vdash \bot}}{\cdot :\ B, B \multimap \bot \vdash \bot}}{\cdot :\ A, A \multimap B, B \multimap \bot \vdash \bot}}{\cdot :\ \cdot \vdash A \otimes B \multimap \bot \multimap A \multimap B \multimap \bot}\ \star$$

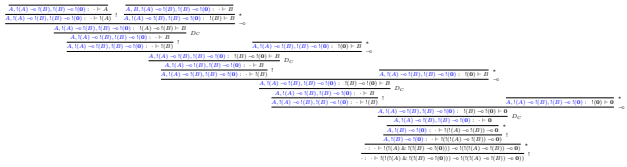
CALL-BY-NAME encoding (144ms)



CALL-BY-VALUE encoding (168ms)



01-ENC encoding (168ms)



$\boxed{(56)\ \cdot \vdash \neg\neg A \wedge B \to \neg A \to \neg B}$

LJ (62ms)

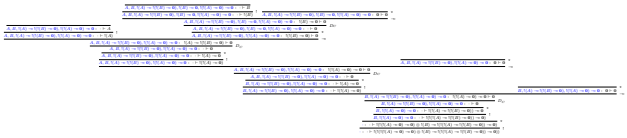
$$\dfrac{\dfrac{\dfrac{}{A, B, A \to B \to \bot, A \to \bot \to \bot \vdash A}\ \star \quad \dfrac{\dfrac{}{A, B, B \to \bot, A \to \bot \to \bot \vdash B}\ \star \quad \dfrac{}{A, B, \bot, A \to \bot \to \bot \vdash \bot}\ \star}{A, B, B \to \bot, A \to \bot \to \bot \vdash \bot}\ \supset_L}{A, B, A \to B \to \bot, A \to \bot \to \bot \vdash \bot}\ \supset_L \quad \dfrac{}{B, \bot, A \to B \to \bot \vdash \bot}\ \star}{B, A \to B \to \bot, A \to \bot \to \bot \vdash \bot}\ \supset_L$$
$$\cdot \vdash A \to \bot \to \bot \wedge B \to A \to B \to \bot \to \bot\ \star$$

MULTIPLICATIVE encoding (77ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{\dfrac{}{\cdot :\ B \vdash B} \quad \dfrac{}{\cdot :\ \bot \vdash \bot}}{\cdot :\ B, B \multimap \bot \vdash \bot}}{\cdot :\ A, B, A \multimap B \multimap \bot \vdash \bot}}{\cdot :\ B, A \multimap B \multimap \bot \vdash A \multimap \bot}\ \star \quad \dfrac{}{\cdot :\ \bot \vdash \bot}}{\cdot :\ B, A \multimap B \multimap \bot, A \multimap \bot \multimap \bot \vdash \bot}}{\cdot :\ \cdot \vdash A \multimap \bot \multimap \bot \otimes B \multimap A \multimap B \multimap \bot \multimap \bot}\ \star$$

CALL-BY-NAME encoding (182ms)

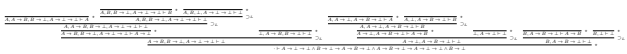
CALL-BY-VALUE encoding (212ms)



01-ENC encoding (210ms)

$\boxed{(57)\ \cdot \vdash (\neg\neg A \wedge \neg B) \leftrightarrow (\neg A \to B)}$

LJ (114ms)



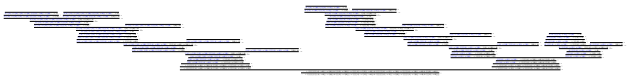
MULTIPLICATIVE encoding (58ms)

Not provable

CALL-BY-NAME encoding (326ms)

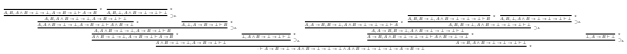
CALL-BY-VALUE encoding (421ms)

01-ENC encoding (454ms)



$\boxed{(58)\ \cdot \vdash (\neg A \to B) \leftrightarrow (\neg\neg A \wedge \neg B)}$

LJ (109ms)

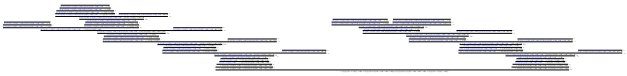


MULTIPLICATIVE encoding (41ms)

Not provable

CALL-BY-NAME encoding (597ms)

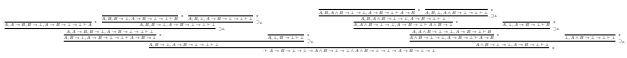
CALL-BY-VALUE encoding (481ms)



01-ENC encoding (501ms)

$\boxed{(59)\ \cdot \vdash (\neg\neg A \to B) \leftrightarrow (\neg A \wedge \neg B)}$

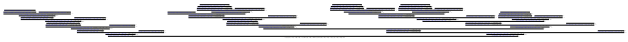
LJ (112ms)



MULTIPLICATIVE encoding (57ms)

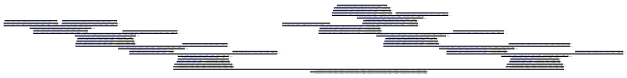
Not provable

CALL-BY-NAME encoding (594ms)



CALL-BY-VALUE encoding (468ms)

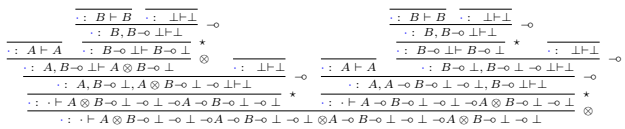
01-ENC encoding (498ms)



$\boxed{(60)\ \cdot \vdash (\neg A \wedge \neg B) \leftrightarrow (A \to \neg\neg B)}$

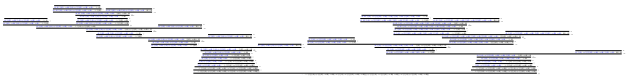
LJ (92ms)

MULTIPLICATIVE encoding (124ms)

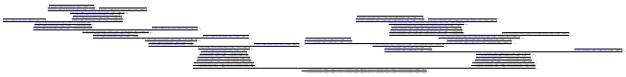


CALL-BY-NAME encoding (317ms)

CALL-BY-VALUE encoding (436ms)

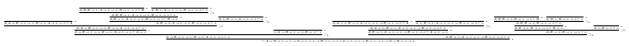


01-ENC encoding (468ms)

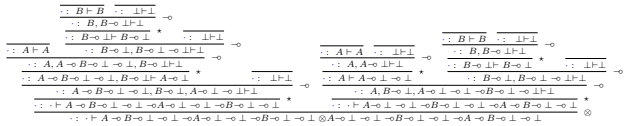


$$\boxed{(61)\ \cdot \vdash (A \to \neg\neg B) \leftrightarrow (\neg\neg A \to \neg\neg B)}$$

LJ (169ms)

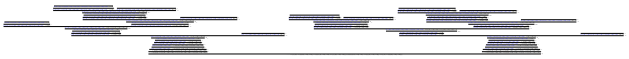


MULTIPLICATIVE encoding (163ms)



CALL-BY-NAME encoding (412ms)

CALL-BY-VALUE encoding (479ms)



01-ENC encoding (552ms)

3 Alternative Translations

$$(10)\ \cdot :\ A \multimap \mathbf{0} \vdash A \multimap B$$

encoding (55ms)

$$\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{}{\cdot :\ \mathbf{0} \vdash B}\ \star}{\dfrac{\cdot :\ A, A \multimap \mathbf{0} \vdash B}{\cdot :\ A \multimap \mathbf{0} \vdash A \multimap B}\ \star}\ \multimap$$

$$(11)\ \cdot :\ A \vdash A \multimap \mathbf{0} \multimap B$$

encoding (41ms)

$$\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{}{\cdot :\ \mathbf{0} \vdash B}\ \star}{\dfrac{\cdot :\ A, A \multimap \mathbf{0} \vdash B}{\cdot :\ A \vdash A \multimap \mathbf{0} \multimap B}\ \star}\ \multimap$$

$$(12)\ \cdot :\ B \vdash\ !(A) \multimap B$$

encoding (22ms)

$$\dfrac{\dfrac{}{A :\ B \vdash B}}{\cdot :\ B \vdash\ !(A) \multimap B}\ \star$$

$$(16)\ \cdot :\ A \multimap B \otimes\ !(B \multimap A) \vdash A \multimap B$$

encoding (35ms)

$$\dfrac{\dfrac{}{B \multimap A :\ A \vdash A} \quad \dfrac{}{B \multimap A :\ B \vdash B}}{\dfrac{B \multimap A :\ A, A \multimap B \vdash B}{\cdot :\ A \multimap B \otimes\ !(B \multimap A) \vdash A \multimap B}\ \star}\ \multimap$$

$$(17)\ \cdot :\ !(A \multimap B) \otimes B \multimap A \vdash B \multimap A$$

encoding (38ms)

$$\dfrac{\dfrac{}{A \multimap B :\ B \vdash B} \quad \dfrac{}{A \multimap B :\ A \vdash A}}{\dfrac{A \multimap B :\ B, B \multimap A \vdash A}{\cdot :\ !(A \multimap B) \otimes B \multimap A \vdash B \multimap A}\ \star}\ \multimap$$

$$(18)\ \cdot :\ A, A \multimap B \otimes B \multimap A \vdash B \otimes B \multimap A$$

encoding (56ms)

$$\dfrac{\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{\dfrac{}{\cdot :\ B \vdash B} \quad \dfrac{}{\cdot :\ B \multimap A \vdash B \multimap A}\ \star}{\cdot :\ B, B \multimap A \vdash B \otimes B \multimap A}\ \otimes}{\cdot :\ A, A \multimap B, B \multimap A \vdash B \otimes B \multimap A}\ \multimap}{\cdot :\ A, A \multimap B \otimes B \multimap A \vdash B \otimes B \multimap A}\ \star \qquad \dfrac{\dfrac{}{\cdot :\ B \vdash B} \quad \dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{}{\cdot :\ A \multimap B \vdash A \multimap B}\ \star}{\cdot :\ A, A \multimap B \vdash A \otimes A \multimap B}\ \otimes}{\cdot :\ B, A \multimap B, B \multimap A \vdash A \otimes A \multimap B}\ \multimap$$

$$(19)\ \cdot :\ B, A \multimap B \otimes B \multimap A \vdash A \otimes A \multimap B$$

encoding (55ms)

$$\dfrac{\dfrac{\dfrac{}{\cdot :\ B \vdash B} \quad \dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{}{\cdot :\ A \multimap B \vdash A \multimap B}\ \star}{\cdot :\ A, A \multimap B \vdash A \otimes A \multimap B}\ \otimes}{\cdot :\ B, A \multimap B, B \multimap A \vdash A \otimes A \multimap B}\ \multimap}{\cdot :\ B, A \multimap B \otimes B \multimap A \vdash A \otimes A \multimap B}\ \star \qquad \dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{}{\cdot :\ B \vdash B}}{\cdot :\ A, B \vdash A \otimes B}\ \otimes \quad \dfrac{}{\cdot :\ \bot \vdash \bot}\ \multimap$$

$$(26a)\ \cdot :\ \cdot \vdash A \,\&\, B \multimap \bot \multimap \bot \multimap A \multimap \bot \multimap \bot \,\&\, B \multimap \bot \multimap \bot$$

encoding (107ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{}{\cdot :\ \bot \vdash \bot}\ \multimap}{\cdot :\ A, A \multimap \bot \vdash \bot}}{\cdot :\ A \,\&\, B, A \multimap \bot \vdash \bot}\ \&_1}{\cdot :\ A \multimap \bot \vdash A \,\&\, B \multimap \bot}\ \star \quad \dfrac{}{\cdot :\ \bot \vdash \bot}\ \multimap}{\cdot :\ A \multimap \bot, A \,\&\, B \multimap \bot \multimap \bot \vdash \bot}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\cdot :\ B \vdash B} \quad \dfrac{}{\cdot :\ \bot \vdash \bot}\ \multimap}{\cdot :\ B, B \multimap \bot \vdash \bot}}{\cdot :\ A \,\&\, B, B \multimap \bot \vdash \bot}\ \&_2}{\cdot :\ B \multimap \bot \vdash A \,\&\, B \multimap \bot}\ \star \quad \dfrac{}{\cdot :\ \bot \vdash \bot}\ \multimap}{\cdot :\ B \multimap \bot, A \,\&\, B \multimap \bot \multimap \bot \vdash \bot}$$

$$\dfrac{\cdot :\ \cdot \vdash A \,\&\, B \multimap \bot \multimap \bot \multimap A \multimap \bot \multimap \bot \,\&\, B \multimap \bot \multimap \bot}{}\ \star$$

$$(26b)\ \cdot :\ \cdot \vdash A \multimap \bot \multimap \bot \otimes B \multimap \bot \multimap \bot \multimap A \otimes B \multimap \bot \multimap \bot$$

encoding (109ms)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\cdot :\ A \vdash A} \quad \dfrac{}{\cdot :\ B \vdash B}}{\cdot :\ A, B \vdash A \otimes B}\ \otimes \quad \dfrac{}{\cdot :\ \bot \vdash \bot}}{\cdot :\ A, B, A \otimes B \multimap \bot \vdash \bot}\ \multimap}{\cdot :\ A, A \otimes B \multimap \bot \vdash B \multimap \bot}\ \star \quad \dfrac{}{\cdot :\ \bot \vdash \bot}}{\cdot :\ A, A \otimes B \multimap \bot, B \multimap \bot \multimap \bot \vdash \bot}\ \multimap$$

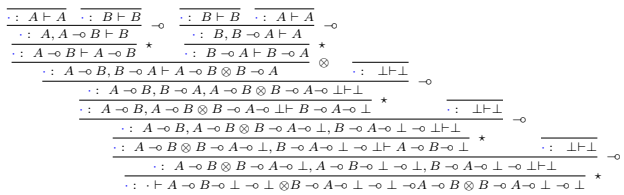
$$\dfrac{\dfrac{\dfrac{}{\cdot :\ A \otimes B \multimap \bot, B \multimap \bot \multimap \bot \vdash A \multimap \bot}\ \star \quad \dfrac{}{\cdot :\ \bot \vdash \bot}}{\cdot :\ A \otimes B \multimap \bot, A \multimap \bot \multimap \bot, B \multimap \bot \multimap \bot \vdash \bot}\ \multimap}{\cdot :\ \cdot \vdash A \multimap \bot \multimap \bot \otimes B \multimap \bot \multimap \bot \multimap A \otimes B \multimap \bot \multimap \bot}\ \star$$

$$(27a)\ \cdot :\ \cdot \vdash\ !(A \multimap B) \otimes\ !(B \multimap A) \multimap \bot \multimap \bot \multimap A \multimap B \multimap \bot \multimap \bot \,\&\, B \multimap A \multimap \bot \multimap \bot$$

encoding (155ms)

(27b) $\cdot : \ \cdot \vdash A \multimap B \multimap \bot \multimap \bot \otimes B \multimap A \multimap \bot \multimap \bot \multimap A \multimap B \otimes B \multimap A \multimap \bot \multimap \bot$

encoding (168ms)



(35) $\cdot : \ \cdot \vdash !(A) \otimes !(A) \multimap !(A) \otimes !(A) \multimap !(A) \otimes !(A)$

encoding (59ms)

$$\dfrac{\dfrac{\overline{A : \ \cdot \vdash A}}{A : \ \cdot \vdash !(A)}\,! \qquad \dfrac{\dfrac{\overline{A : \ \cdot \vdash A}}{A : \ \cdot \vdash !(A)}\,! \qquad \dfrac{\overline{A : \ \cdot \vdash A}}{A : \ \cdot \vdash !(A)}\,!}{A : \ \cdot \vdash !(A) \otimes !(A)}\,\otimes}{\cdot : \ \cdot \vdash !(A) \otimes !(A) \multimap !(A)}\,\star \qquad \dfrac{}{\cdot : \ \cdot \vdash !(A) \multimap !(A) \otimes !(A)}\,\star}{\cdot : \ \cdot \vdash !(A) \otimes !(A) \multimap !(A) \otimes !(A) \multimap !(A) \otimes !(A)}\,\otimes$$

(36) $\cdot : \ A \vdash A \multimap B \multimap B \otimes B \multimap !(A) \multimap B$

encoding (62ms)

$$\dfrac{\dfrac{\overline{\cdot : \ A \vdash A} \qquad \overline{\cdot : \ B \vdash B}}{\cdot : \ A, A \multimap B \vdash B}\,\multimap}{\cdot : \ A \vdash A \multimap B \multimap B}\,\star \qquad \dfrac{\overline{A : \ B \vdash B}}{\cdot : \ \cdot \vdash B \multimap !(A) \multimap B}\,\star}{\cdot : \ A \vdash A \multimap B \multimap B \otimes B \multimap !(A) \multimap B}\,\otimes$$

(37) $\cdot : \ B \vdash !(A \multimap B) \multimap B \otimes B \multimap !(A) \multimap B$

encoding (54ms)

$$\dfrac{\overline{A \multimap B : \ B \vdash B}}{\cdot : \ B \vdash !(A \multimap B) \multimap B}\,\star \qquad \dfrac{\overline{A : \ B \vdash B}}{\cdot : \ \cdot \vdash B \multimap !(A) \multimap B}\,\star}{\cdot : \ B \vdash !(A \multimap B) \multimap B \otimes B \multimap !(A) \multimap B}\,\otimes$$

(38) $\cdot : \ A \multimap \bot \vdash !(A \multimap B) \multimap A \multimap \bot \otimes A \multimap \mathbf{0} \multimap A \multimap B$

encoding (74ms)

$$\dfrac{\dfrac{\overline{A \multimap B : \ A \vdash A} \qquad \overline{A \multimap B : \ \bot \vdash \bot}}{A \multimap B : \ A, A \multimap \bot \vdash \bot}\,\multimap}{\cdot : \ A \multimap \bot \vdash !(A \multimap B) \multimap A \multimap \bot}\,\star \qquad \dfrac{\dfrac{\overline{\cdot : \ A \vdash A} \qquad \overline{\cdot : \ \mathbf{0} \vdash B}}{\cdot : \ A, A \multimap \mathbf{0} \vdash B}\,\multimap}{\cdot : \ \cdot \vdash A \multimap \mathbf{0} \multimap A \multimap B}\,\star}{\cdot : \ A \multimap \bot \vdash !(A \multimap B) \multimap A \multimap \bot \otimes A \multimap \mathbf{0} \multimap A \multimap B}\,\otimes$$

(39) $\cdot : \ B \multimap \mathbf{0} \vdash A \multimap B \multimap A \multimap \mathbf{0} \otimes A \multimap \mathbf{0} \multimap A \multimap B$

encoding (86ms)

$$\dfrac{\dfrac{\overline{\cdot : \ A \vdash A}}{\cdot : \ A \vdash A}\,\star \qquad \dfrac{\dfrac{\overline{\cdot : \ B \vdash B} \qquad \overline{\cdot : \ \mathbf{0} \vdash \mathbf{0}}}{\cdot : \ B, B \multimap \mathbf{0} \vdash \mathbf{0}}\,\multimap}{\cdot : \ A, A \multimap B, B \multimap \mathbf{0} \vdash \mathbf{0}}\,\multimap}{\cdot : \ B \multimap \mathbf{0} \vdash A \multimap B \multimap A \multimap \mathbf{0}}\,\star \qquad \dfrac{\dfrac{\overline{\cdot : \ A \vdash A} \qquad \overline{\cdot : \ \mathbf{0} \vdash B}}{\cdot : \ A, A \multimap \mathbf{0} \vdash B}\,\multimap}{\cdot : \ \cdot \vdash A \multimap \mathbf{0} \multimap A \multimap B}\,\star}{\cdot : \ B \multimap \mathbf{0} \vdash A \multimap B \multimap A \multimap \mathbf{0} \otimes A \multimap \mathbf{0} \multimap A \multimap B}\,\otimes$$

(40) $\cdot : \ B \vdash A \otimes !(B) \multimap A \otimes A \multimap A \otimes B$

encoding (55ms)

$$\dfrac{\overline{B : \ A \vdash A}}{\cdot : \ \cdot \vdash A \otimes !(B) \multimap A}\,\star \qquad \dfrac{\overline{\cdot : \ A \vdash A} \qquad \overline{\cdot : \ B \vdash B}}{\cdot : \ A, B \vdash A \otimes B}\,\otimes}{\cdot : \ B \vdash A \otimes !(B) \multimap A \otimes A \multimap A \otimes B}\,\star}{\cdot : \ B \vdash A \otimes !(B) \multimap A \otimes A \multimap A \otimes B}\,\otimes$$

(41) $\cdot : \ B \multimap \mathbf{0} \vdash !(A) \otimes B \multimap B \otimes B \multimap A \otimes B$

encoding (69ms)

$$\dfrac{\overline{A : \ B \vdash B}}{\cdot : \ \cdot \vdash !(A) \otimes B \multimap B}\,\star \qquad \dfrac{\dfrac{\overline{\cdot : \ B \vdash B} \qquad \overline{\cdot : \ \mathbf{0} \vdash A \otimes B}}{\cdot : \ B, B \multimap \mathbf{0} \vdash A \otimes B}\,\multimap}{\cdot : \ B \multimap \mathbf{0} \vdash B \multimap A \otimes B}\,\star}{\cdot : \ B \multimap \mathbf{0} \vdash !(A) \otimes B \multimap B \otimes B \multimap A \otimes B}\,\otimes$$

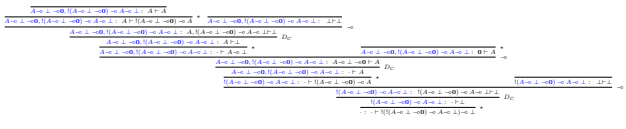
(45) $\cdot : \ \cdot \vdash !(A \multimap A \multimap \bot) \otimes !(A) \multimap \bot \multimap !(A) \multimap \bot$

encoding (65ms)

$$\dfrac{\dfrac{\overline{A, \bot, A \multimap \bot, A \multimap A \multimap \bot : \ \bot \vdash \bot}}{A, \bot, A \multimap \bot, A \multimap A \multimap \bot : \ \cdot \vdash \bot}\,D_C}{A \multimap A \multimap \bot : \ \cdot \vdash !(A) \multimap \bot}\,\star \qquad \dfrac{\dfrac{\overline{A, \bot, A \multimap \bot, A \multimap A \multimap \bot : \ \bot \vdash \bot}}{A, \bot, A \multimap \bot, A \multimap A \multimap \bot : \ \cdot \vdash \bot}\,D_C}{A \multimap A \multimap \bot : \ !(A) \vdash \bot}\,\star}{A \multimap A \multimap \bot : \ !(A) \multimap \bot \multimap !(A) \vdash \bot}\,\multimap}{\cdot : \ \cdot \vdash !(A \multimap A \multimap \bot) \otimes !(A) \multimap \bot \multimap !(A) \multimap \bot}\,\star$$

(46) $\cdot : \ \cdot \vdash !(!(A \multimap \bot \multimap \mathbf{0}) \multimap A \multimap \bot) \multimap \bot$

encoding (138ms)



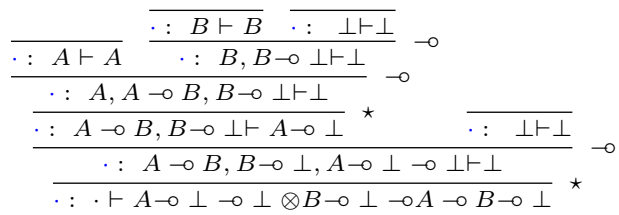
(47) $\cdot : \ \cdot \vdash A \otimes B \otimes B \multimap \mathbf{0} \multimap B \otimes \mathbf{0} \otimes B \otimes B \multimap \mathbf{0} \multimap A \otimes B \otimes B \multimap \mathbf{0}$

encoding (83ms)

$$\dfrac{\dfrac{\overline{\cdot : \ B \vdash B} \qquad \overline{\cdot : \ A, \mathbf{0} \vdash B \otimes B \multimap \mathbf{0}}}{\cdot : \ A, B, B \multimap \mathbf{0} \vdash B \otimes B \multimap \mathbf{0}}\,\multimap}{\cdot : \ \cdot \vdash A \otimes B \otimes B \multimap \mathbf{0} \multimap B \otimes B \multimap \mathbf{0}}\,\star \qquad \dfrac{\dfrac{\overline{\cdot : \ B \vdash B} \qquad \overline{\cdot : \ \mathbf{0} \vdash A \otimes B \otimes B \multimap \mathbf{0}}}{\cdot : \ B, B \multimap \mathbf{0} \vdash A \otimes B \otimes B \multimap \mathbf{0}}\,\multimap}{\cdot : \ \cdot \vdash B \otimes B \multimap \mathbf{0} \multimap A \otimes B \otimes B \multimap \mathbf{0}}\,\star}{\cdot : \ \cdot \vdash A \otimes B \otimes B \multimap \mathbf{0} \multimap B \otimes B \multimap \mathbf{0} \otimes B \otimes B \multimap \mathbf{0} \multimap A \otimes B \otimes B \multimap \mathbf{0}}\,\otimes$$

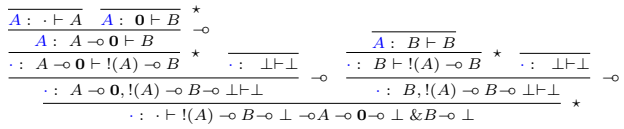
(57a) $\cdot : \ \cdot \vdash A \multimap \bot \multimap \bot \otimes B \multimap \bot \multimap A \multimap B \multimap \bot$

encoding (71ms)



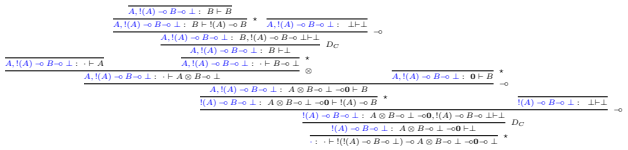
(57b) $\cdot : \ \cdot \vdash !(A) \multimap B \multimap \bot \multimap A \multimap \mathbf{0} \multimap \bot \ \& B \multimap \bot$

encoding (92ms)



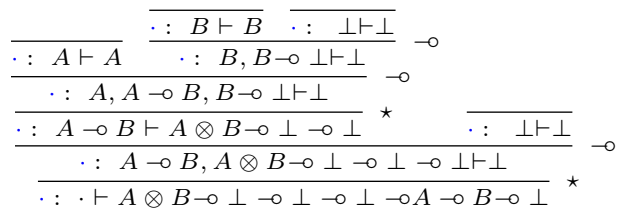
(58a) $\cdot : \ \cdot \vdash !(!(A) \multimap B \multimap \bot) \multimap A \otimes B \multimap \bot \multimap \mathbf{0} \multimap \bot$

encoding (162ms)



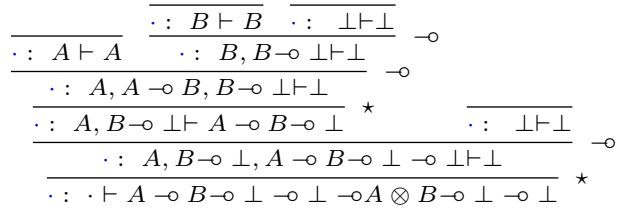
(58b) $\cdot : \ \cdot \vdash A \otimes B \multimap \bot \multimap \bot \multimap \bot \multimap A \multimap B \multimap \bot$

encoding (69ms)



(59a) $\cdot : \ \cdot \vdash A \multimap B \multimap \bot \multimap \bot \multimap A \otimes B \multimap \bot \multimap \bot$

encoding (71ms)



(59b) $\cdot : \ \cdot \vdash A \otimes B \multimap \bot \multimap \mathbf{0} \multimap !(!(A) \multimap B \multimap \bot) \multimap \bot$

encoding (163ms)