

Benchmarking Linear Logic: sequents inspired from Kleene's intuitionistic theorems*

October 10, 2018

1 General Information

- Test run on a QEMU Virtual CPU, 2GHz, 64 bits, 2GB of RAM running Ubuntu.
- Timeout in all the cases was 2 minutes.
- Maude version: 2.7.1 built: Oct 2 2017.
- The search procedure considers proofs using up to 4 times the copy rule (focusing on one of the formulas of the classical context). Benchmarks 23 and 25 required the limit to be 5.

2 Translations

(1) $\cdot \vdash A \rightarrow A$

LJ (28ms)

$$\frac{}{\cdot \vdash A \rightarrow A} \star$$

MULTIPLICATIVE encoding (28ms)

$$\frac{\overline{\cdot : A \vdash A}}{\cdot : \cdot \vdash A \multimap A} \star$$

CALL-BY-NAME encoding (28ms)

$$\frac{\overline{A : \cdot \vdash A}}{\cdot : \cdot \vdash !(A) \multimap A} \star$$

CALL-BY-VALUE encoding (41ms)

$$\frac{\frac{\overline{A : \cdot \vdash A}}{A : \cdot \vdash !(A)} !}{\cdot : \cdot \vdash !(A) \multimap !(A)} \star$$
$$\frac{}{\cdot : \cdot \vdash !(A) \multimap !(A)} !$$

01-ENC encoding (35ms)

$$\frac{\overline{A : \cdot \vdash A}}{\cdot : \cdot \vdash !(A) \multimap A} \star$$
$$\frac{}{\cdot : \cdot \vdash !(A) \multimap A} !$$

*See <https://github.com/carlosolarte/Benchmarking-Linear-Logic> for details on the encodings used.

LJ (46ms)

$$\frac{\frac{\frac{A, A \rightarrow B, B \rightarrow C \vdash A}{A, A \rightarrow B, B \rightarrow C \vdash C} \star}{A, A \rightarrow B, B \rightarrow C \vdash C} \supset_L \quad \frac{\frac{A, B, B \rightarrow C \vdash B}{A, B, B \rightarrow C \vdash C} \star}{A, B, B \rightarrow C \vdash C} \supset_L}{A, A \rightarrow B, B \rightarrow C \vdash C} \supset_L \quad \frac{A, A \rightarrow B, B \rightarrow C \vdash C}{A \rightarrow B, B \rightarrow C \vdash A \rightarrow C} \star$$

$$\frac{\frac{\frac{\cdot : A \vdash A}{\cdot : B, B \multimap C \vdash C} \multimap \quad \frac{\cdot : B \vdash B \quad \cdot : C \vdash C}{\cdot : B, B \multimap C \vdash C} \multimap}{\cdot : A, A \multimap B, B \multimap C \vdash C} \multimap}{\cdot : A \multimap B, B \multimap C \vdash A \multimap C} \star$$
$$\frac{\frac{\frac{A, \neg(A) \rightarrow B, \neg(B) \rightarrow C : \vdash A}{A, \neg(A) \rightarrow B, \neg(B) \rightarrow C : \vdash \neg(A)} \quad \frac{A, \neg(A) \rightarrow B, \neg(B) \rightarrow C : \vdash B \vdash B}{\frac{A, \neg(A) \rightarrow B, \neg(B) \rightarrow C : \vdash B \vdash B}{A, \neg(A) \rightarrow B, \neg(B) \rightarrow C : \vdash B} D_C}{\frac{A, \neg(A) \rightarrow B, \neg(B) \rightarrow C : \vdash B}{A, \neg(A) \rightarrow B, \neg(B) \rightarrow C : \vdash \neg(B)} \quad \frac{A, \neg(A) \rightarrow B, \neg(B) \rightarrow C : \vdash B}{A, \neg(A) \rightarrow B, \neg(B) \rightarrow C : \vdash \neg(B)} \quad \frac{A, \neg(A) \rightarrow B, \neg(B) \rightarrow C : \vdash \neg(B)}{A, \neg(A) \rightarrow B, \neg(B) \rightarrow C : \vdash \neg(A)} \quad \frac{A, \neg(A) \rightarrow B, \neg(B) \rightarrow C : \vdash \neg(A)}{\vdash : \neg(A) \rightarrow B, \neg(B) \rightarrow C \vdash \neg(A) \rightarrow C} D_C$$
[illegible]

LJ (46ms)

$$\frac{\frac{\overline{A, B, A \rightarrow B \rightarrow C \vdash A}^* \quad \frac{\overline{A, B, B \rightarrow C \vdash B}^* \quad \overline{A, B, C \vdash C}^*}{A, B, B \rightarrow C \vdash C} \supset_L}{\frac{A, B, A \rightarrow B \rightarrow C \vdash C}{A \rightarrow B \rightarrow C \vdash B \rightarrow A \rightarrow C}^*} \supset_L$$

$$\frac{\frac{\frac{\cdot : A \vdash A}{\cdot : B \vdash B} \quad \frac{\cdot : C \vdash C}{\cdot : B, B \multimap C \vdash C} \multimap}{\cdot : A, B, A \multimap B \multimap C \vdash C} \multimap}{\cdot : A \multimap B \multimap C \vdash B \multimap A \multimap C} \star$$
$$\begin{array}{c}
\frac{\frac{A, B, !\langle A \rangle \multimap !\langle B \rangle \multimap C : \vdash A}{A, B, !\langle A \rangle \multimap !\langle B \rangle \multimap C : \vdash !A} \quad \frac{\frac{A, B, !\langle A \rangle \multimap !\langle B \rangle \multimap C : \vdash \bar{B}}{A, B, !\langle A \rangle \multimap !\langle B \rangle \multimap C : \vdash \bar{B}} \quad \frac{A, B, !\langle A \rangle \multimap !\langle B \rangle \multimap C : \vdash !\langle B \rangle}{A, B, !\langle A \rangle \multimap !\langle B \rangle \multimap C : \vdash !\langle B \rangle}}{\frac{A, B, !\langle A \rangle \multimap !\langle B \rangle \multimap C : \vdash !A \quad A, B, !\langle A \rangle \multimap !\langle B \rangle \multimap C : \vdash \bar{B} \quad A, B, !\langle A \rangle \multimap !\langle B \rangle \multimap C : \vdash !\langle B \rangle}{A, B, !\langle A \rangle \multimap !\langle B \rangle \multimap C : \vdash !\langle B \rangle} \multimap} \\
\frac{A, B, !\langle A \rangle \multimap !\langle B \rangle \multimap C : \vdash !\langle A \rangle \multimap !\langle B \rangle \multimap C \multimap C}{A, B, !\langle A \rangle \multimap !\langle B \rangle \multimap C : \vdash !\langle A \rangle \multimap !\langle B \rangle \multimap C} D_C \multimap \\
\vdash : !\langle !\langle A \rangle \multimap !\langle B \rangle \multimap C \rangle \vdash !\langle B \rangle \multimap !\langle A \rangle \multimap C *
\end{array}$$
[illegible][illegible]

$$(4) \ A \rightarrow B \rightarrow C \vdash A \wedge B \rightarrow C$$

LJ (47ms)

$$\frac{\frac{\frac{A, B, A \rightarrow B \rightarrow C \vdash A}{A, B, A \rightarrow B \rightarrow C \vdash C} \star}{A \rightarrow B \rightarrow C \vdash A \wedge B \rightarrow C} \star}{\frac{\frac{A, B, B \rightarrow C \vdash B}{A, B, C \vdash C} \star}{A, B, B \rightarrow C \vdash C} \star} \supset_L \supset_L$$

MULTIPLICATIVE encoding (47ms)

$$\frac{\frac{\frac{\cdot : A \vdash A}{\cdot : B, B \multimap C \vdash C} \multimap \quad \frac{\cdot : B \vdash B \quad \cdot : C \vdash C}{\cdot : B, B \multimap C \vdash C} \multimap}{\cdot : A, B, A \multimap B \multimap C \vdash C} \multimap}{\cdot : A \multimap B \multimap C \vdash A \otimes B \multimap C} \star$$

CALL-BY-NAME encoding (71ms)

$$\frac{\frac{\frac{A, B, !(A) \multimap !(B) \multimap C : \vdash A}{A, B, !(A) \multimap !(B) \multimap C : \vdash !(A)} \quad \frac{\frac{A, B, !(A) \multimap !(B) \multimap C : \vdash B}{A, B, !(A) \multimap !(B) \multimap C : \vdash !(B)} \quad \frac{A, B, !(A) \multimap !(B) \multimap C : \vdash C}{A, B, !(A) \multimap !(B) \multimap C : \vdash C}}{\frac{A, B, !(A) \multimap !(B) \multimap C : \vdash A \quad A, B, !(A) \multimap !(B) \multimap C : \vdash !(A) \quad A, B, !(A) \multimap !(B) \multimap C : \vdash !(B) \quad A, B, !(A) \multimap !(B) \multimap C : \vdash C}{A, B, !(A) \multimap !(B) \multimap C : \vdash C}} \multimap \quad \frac{A, B, !(A) \multimap !(B) \multimap C : \vdash C}{A, B, !(A) \multimap !(B) \multimap C : \vdash C} \quad D_C \quad \frac{A, B, !(A) \multimap !(B) \multimap C : \vdash C}{\vdash : !(A) \multimap !(B) \multimap C \vdash !(A \& B) \multimap C} *$$

CALL-BY-VALUE encoding (156ms)

01-ENC encoding (151ms)

[illegible]

(5) $A \wedge B \rightarrow C \vdash A \rightarrow B \rightarrow C$

LJ (41ms)

$$\frac{\frac{A, B, A \wedge B \rightarrow C \vdash A \wedge B}{A, B, A \wedge B \rightarrow C \vdash C}^*}{A \wedge B \rightarrow C \vdash A \rightarrow B \rightarrow C}^* \supset_L$$

MULTIPLICATIVE encoding (54ms)

$$\frac{\frac{\frac{\vdots : A \vdash A}{\vdots : A, B \vdash A \otimes B}}{\vdots : A, B, A \otimes B \multimap C \vdash C} \otimes \frac{\vdots : C \vdash C}{\vdots : A \otimes B \multimap C \vdash A \multimap B \multimap C} \multimap \star$$

CALL-BY-NAME encoding (73ms)

$$\begin{array}{c}
\frac{A, B, \uparrow(A \& B) \multimap C : \vdash A \quad A, B, \uparrow(A \& B) \multimap C : \vdash B}{A, B, \uparrow(A \& B) \multimap C : \vdash A \& B} * \\
\frac{A, B, \uparrow(A \& B) \multimap C : \vdash A \& B}{A, B, \uparrow(A \& B) \multimap C : \vdash \uparrow(A \& B)} ! \quad \frac{A, B, \uparrow(A \& B) \multimap C : C \vdash C}{A, B, \uparrow(A \& B) \multimap C : \vdash \uparrow(A \& B) \multimap C} \rightarrow \\
\frac{A, B, \uparrow(A \& B) \multimap C : \vdash \uparrow(A \& B) \multimap C \vdash C}{A, B, \uparrow(A \& B) \multimap C : \vdash C} D_C \\
\therefore \uparrow(\uparrow(A \& B) \multimap C) \vdash \uparrow(A) \multimap \uparrow(B) \multimap C *
\end{array}$$

CALL-BY-VALUE encoding (205ms)

[illegible]

01-ENC encoding (201ms)

[illegible]

(6) $A \rightarrow B \vdash B \rightarrow C \rightarrow A \rightarrow C$

LJ (47ms)

$$\frac{\frac{\frac{A, A \rightarrow B, B \rightarrow C \vdash A}{A, B, B \rightarrow C \vdash A} \star \quad \frac{\frac{A, B, B \rightarrow C \vdash B}{A, B, B \rightarrow C \vdash B} \star \quad \frac{A, B, B \rightarrow C \vdash C}{A, B, B \rightarrow C \vdash C} \star}{A, B, B \rightarrow C \vdash C} \supset_L \quad \frac{A, A \rightarrow B, B \rightarrow C \vdash C}{A \rightarrow B \vdash B \rightarrow C \rightarrow A \rightarrow C} \star}{A, A \rightarrow B, B \rightarrow C \vdash A} \star \supset_L$$

MULTIPLICATIVE encoding (48ms)

$$\frac{\frac{\frac{\cdot : A \vdash A}{\cdot : B \vdash B} \quad \frac{\cdot : C \vdash C}{\cdot : B, B \multimap C \vdash C} \multimap}{\cdot : A, A \multimap B, B \multimap C \vdash C} \multimap}{\cdot : A \multimap B \vdash B \multimap C \multimap A \multimap C} \star$$

CALL-BY-NAME encoding (120ms)

$$\begin{array}{c}
\frac{A, \neg(A) \rightarrow B, \neg(B) \rightarrow C; \vdash \neg A}{A, \neg(A) \rightarrow B, \neg(B) \rightarrow C; \vdash \neg(A)} \quad \frac{A, \neg(A) \rightarrow B, \neg(B) \rightarrow C; \vdash \neg B}{A, \neg(A) \rightarrow B, \neg(B) \rightarrow C; \neg(A) \rightarrow B \rightarrow B} \quad \Rightarrow \\
\frac{A, \neg(A) \rightarrow B, \neg(B) \rightarrow C; \neg(A) \rightarrow B \rightarrow B}{A, \neg(A) \rightarrow B, \neg(B) \rightarrow C; \vdash B} \quad D_C \\
\frac{A, \neg(A) \rightarrow B, \neg(B) \rightarrow C; \vdash B}{A, \neg(A) \rightarrow B, \neg(B) \rightarrow C; \vdash \neg(B)} \quad \frac{A, \neg(A) \rightarrow B, \neg(B) \rightarrow C; \vdash \neg(B)}{A, \neg(A) \rightarrow B, \neg(B) \rightarrow C; \vdash C \rightarrow C} \quad \Rightarrow \\
\frac{A, \neg(A) \rightarrow B, \neg(B) \rightarrow C; \vdash \neg(B)}{A, \neg(A) \rightarrow B, \neg(B) \rightarrow C; \vdash C} \quad D_C \\
\vdash \neg(\neg(A) \rightarrow B) \rightarrow \neg(\neg(B) \rightarrow C) \rightarrow \neg(A) \rightarrow C
\end{array}$$

CALL-BY-VALUE encoding (168ms)

[illegible]

01-ENC encoding (159ms)

[illegible]

$$(7) \quad A \rightarrow B \vdash C \rightarrow A \rightarrow C \rightarrow B$$

LJ (47ms)

$$\frac{\frac{\frac{C, A \rightarrow B, C \rightarrow A \vdash C}{C, A \rightarrow B, C \rightarrow A \vdash A}^* \quad \frac{A, C, A \rightarrow B \vdash A}{B, C, C \rightarrow A \vdash B}^*}{\frac{C, A \rightarrow B, C \rightarrow A \vdash B}{A \rightarrow B \vdash C \rightarrow A \rightarrow C \rightarrow B}^*} \supset_L$$

MULTIPLICATIVE encoding (48ms)

$$\frac{\frac{\frac{\cdot : C \vdash C}{\cdot : C \vdash C} \quad \frac{\frac{\cdot : A \vdash A}{\cdot : A \vdash A} \quad \frac{\cdot : B \vdash B}{\cdot : B \vdash B}}{\cdot : A, A \multimap B \vdash B} \multimap}{\cdot : C, A \multimap B, C \multimap A \vdash B} \multimap}{\cdot : A \multimap B \vdash C \multimap A \multimap C \multimap B} \star$$

CALL-BY-NAME encoding (120ms)

[illegible]

CALL-BY-VALUE encoding (167ms)

[illegible]

01-ENC encoding (156ms)

[illegible]

$$\boxed{(8) \ A \rightarrow B \vdash A \wedge C \rightarrow B \wedge C}$$

LJ (37ms)

$$\frac{\frac{\overline{A, C, A \rightarrow B \vdash A} \star \overline{A, B, C \vdash B}}{A, C, A \rightarrow B \vdash B} \star}{A \rightarrow B \vdash A \wedge C \rightarrow B \wedge C} \star \supset_L$$

MULTIPLICATIVE encoding (48ms)

$$\frac{\frac{\overline{\cdot : B \vdash B} \quad \overline{\cdot : C \vdash C}}{\cdot : A \vdash A \quad \cdot : B, C \vdash B \otimes C} \otimes}{\cdot : A, C, A \multimap B \vdash B \otimes C} \multimap}{\cdot : A \multimap B \vdash A \otimes C \multimap B \otimes C} \star$$

CALL-BY-NAME encoding (73ms)

$$\frac{\frac{\overline{A, C, !!(A) \multimap B : \vdash A} \quad \overline{A, C, !(A) \multimap B : B \vdash B}}{\overline{A, C, !(A) \multimap B : !(A) \multimap B \vdash B} \multimap} \multimap}{\overline{A, C, !(A) \multimap B : \vdash B} \quad \overline{A, C, !(A) \multimap B : B \vdash B} \quad D_C}{\overline{A, C, !(A) \multimap B : \vdash C} \star} \star$$

CALL-BY-VALUE encoding (135ms)

$$\frac{\frac{\overline{A, C, !(A) \multimap !(B) : \vdash A} \quad \overline{A, B, C, !(A) \multimap !(B) : \vdash B} \star}{\overline{A, C, !(A) \multimap !(B) : \vdash !(A)} \quad \overline{A, C, !(A) \multimap !(B) : !(B) \vdash B} \star}{\overline{A, C, !(A) \multimap !(B) : \vdash B} \quad D_C}{\overline{A, C, !(A) \multimap !(B) : \vdash C} \quad \overline{A, C, !(A) \multimap !(B) : \vdash !(C)} \quad !}{\overline{A, C, !(A) \multimap !(B) : \vdash !(B) \otimes !(C)} \quad !}{\overline{!(A) \multimap !(B) : \vdash !(A) \otimes !(C) \multimap !(B) \otimes !(C)} \star} \star$$

01-ENC encoding (130ms)

$$\frac{\frac{\overline{A, C, !(A) \multimap !(B) : \vdash A} \quad \overline{A, B, C, !(A) \multimap !(B) : \vdash B} \star}{\overline{A, C, !(A) \multimap !(B) : \vdash !(A)} \quad \overline{A, C, !(A) \multimap !(B) : !(B) \vdash B} \star}{\overline{A, C, !(A) \multimap !(B) : \vdash B} \quad D_C}{\overline{A, C, !(A) \multimap !(B) : \vdash C} \quad \overline{A, C, !(A) \multimap !(B) : \vdash B \& C} \quad !}{\overline{A, C, !(A) \multimap !(B) : \vdash !(B \& C)} \quad !}{\overline{!(A) \multimap !(B) : \vdash !(!(A) \& !(C)) \multimap !(B \& C)} \star} \star$$

$$\boxed{(9) \ A \rightarrow B \vdash C \wedge A \rightarrow C \wedge B}$$

LJ (37ms)

$$\frac{\frac{\overline{A, C, A \rightarrow B \vdash A} \star \overline{A, B, C \vdash B}}{A, C, A \rightarrow B \vdash B} \star}{A \rightarrow B \vdash C \wedge A \rightarrow C \wedge B} \star \supset_L$$

MULTIPLICATIVE encoding (48ms)

$$\frac{\frac{\overline{\cdot : C \vdash C} \quad \overline{\cdot : B \vdash B}}{\cdot : A \vdash A \quad \cdot : B, C \vdash C \otimes B} \otimes}{\cdot : A, C, A \multimap B \vdash C \otimes B} \multimap}{\cdot : A \multimap B \vdash C \otimes A \multimap C \otimes B} \star$$

CALL-BY-NAME encoding (60ms)

$$\frac{\frac{\overline{A, C, !(A) \multimap B : \vdash A} \quad \overline{A, C, !(A) \multimap B : B \vdash B}}{\overline{A, C, !(A) \multimap B : !(A) \multimap B \vdash B} \multimap} \multimap}{\overline{A, C, !(A) \multimap B : \vdash C} \quad \overline{A, C, !(A) \multimap B : B \vdash B} \quad D_C}{\overline{A, C, !(A) \multimap B : \vdash B} \star} \star$$

CALL-BY-VALUE encoding (137ms)

$$\frac{\frac{\overline{A, B, C, !(A) \multimap !(B) : \vdash C} \quad \overline{A, B, C, !(A) \multimap !(B) : \vdash B} \star}{\overline{A, B, C, !(A) \multimap !(B) : \vdash !(C)} \quad \overline{A, B, C, !(A) \multimap !(B) : !(B) \vdash B} \star}{\overline{A, B, C, !(A) \multimap !(B) : \vdash B} \quad D_C}{\overline{A, B, C, !(A) \multimap !(B) : \vdash C} \quad \overline{A, B, C, !(A) \multimap !(B) : \vdash !(C) \otimes !(B)} \quad !}{\overline{!(A) \multimap !(B) : \vdash !(!(C) \& !(A)) \multimap !(C) \otimes !(B)} \star} \star$$

01-ENC encoding (131ms)

$$\frac{\frac{\overline{A, B, C, !(A) \multimap !(B) : \vdash C} \quad \overline{A, B, C, !(A) \multimap !(B) : \vdash B} \star}{\overline{A, B, C, !(A) \multimap !(B) : \vdash C \& B} \quad !}{\overline{A, B, C, !(A) \multimap !(B) : \vdash !(C \& B)} \quad !}{\overline{A, B, C, !(A) \multimap !(B) : \vdash B} \quad D_C}{\overline{A, B, C, !(A) \multimap !(B) : \vdash C} \quad \overline{A, B, C, !(A) \multimap !(B) : \vdash !(C \& B)} \quad !}{\overline{!(A) \multimap !(B) : \vdash !(!(C) \& !(A)) \multimap !(C \& B)} \star} \star$$

$$(10) \neg A \vdash A \rightarrow B$$

LJ (37ms)

$$\frac{\frac{A, A \rightarrow \perp \vdash A}{A, A \rightarrow \perp \vdash B} \star}{A \rightarrow \perp \vdash A \rightarrow B} \star \supset_L$$

MULTIPLICATIVE encoding (41ms)

Not provable

CALL-BY-NAME encoding (66ms)

$$\frac{\frac{A, !(A) \multimap \mathbf{0} : \cdot \vdash A}{A, !(A) \multimap \mathbf{0} : \cdot \vdash !(A)} ! \quad \frac{A, !(A) \multimap \mathbf{0} : \mathbf{0} \vdash B}{A, !(A) \multimap \mathbf{0} : \cdot \vdash B} \star}{\frac{A, !(A) \multimap \mathbf{0} : \cdot \vdash B}{\cdot : !(A) \multimap \mathbf{0} \vdash !(A) \multimap B} D_C} \multimap$$

CALL-BY-VALUE encoding (117ms)

$$\frac{\frac{A, !(A) \multimap \mathbf{0} : \cdot \vdash A}{A, !(A) \multimap \mathbf{0} : \cdot \vdash !(A)} ! \quad \frac{A, !(A) \multimap \mathbf{0} : \mathbf{0} \vdash !(B)}{A, !(A) \multimap \mathbf{0} : \cdot \vdash !(B)} \star}{\frac{A, !(A) \multimap \mathbf{0} : \cdot \vdash !(B)}{!(A) \multimap \mathbf{0} : \cdot \vdash !(A) \multimap !(B)} D_C} \multimap$$

$$\frac{\frac{!(A) \multimap \mathbf{0} : \cdot \vdash !(A) \multimap !(B)}{!(A) \multimap \mathbf{0} : \cdot \vdash !(!(A) \multimap !(B))} !}{\cdot : !(A) \multimap \mathbf{0} \vdash !(A) \multimap !(B)} \star$$

01-ENC encoding (118ms)

$$\frac{\frac{A, !(A) \multimap !(\mathbf{0}) : \cdot \vdash A}{A, !(A) \multimap !(\mathbf{0}) : \cdot \vdash !(A)} ! \quad \frac{A, !(A) \multimap !(\mathbf{0}) : !(\mathbf{0}) \vdash B}{A, !(A) \multimap !(\mathbf{0}) : \cdot \vdash B} \star}{\frac{A, !(A) \multimap !(\mathbf{0}) : \cdot \vdash B}{!(A) \multimap !(\mathbf{0}) : \cdot \vdash !(A) \multimap B} D_C} \multimap$$

$$\frac{\frac{!(A) \multimap !(\mathbf{0}) : \cdot \vdash !(A) \multimap B}{!(A) \multimap !(\mathbf{0}) : \cdot \vdash !(!(A) \multimap B)} !}{\cdot : !(A) \multimap \mathbf{0} \vdash !(A) \multimap B} \star$$

$$(11) A \vdash \neg A \rightarrow B$$

LJ (37ms)

$$\frac{\frac{A, A \rightarrow \perp \vdash A}{A, A \rightarrow \perp \vdash B} \star}{A \vdash A \rightarrow \perp \rightarrow B} \star \supset_L$$

MULTIPLICATIVE encoding (28ms)

Not provable

CALL-BY-NAME encoding (65ms)

$$\frac{\frac{A, !(A) \multimap \mathbf{0} : \cdot \vdash A}{A, !(A) \multimap \mathbf{0} : \cdot \vdash !(A)} ! \quad \frac{A, !(A) \multimap \mathbf{0} : \mathbf{0} \vdash B}{A, !(A) \multimap \mathbf{0} : \cdot \vdash B} \star}{\frac{A, !(A) \multimap \mathbf{0} : \cdot \vdash B}{\cdot : !(A) \vdash !(A) \multimap B} D_C} \multimap$$

CALL-BY-VALUE encoding (78ms)

$$\frac{\frac{A, !(A) \multimap \mathbf{0} : \cdot \vdash A}{A, !(A) \multimap \mathbf{0} : \cdot \vdash !(A)} ! \quad \frac{A, !(A) \multimap \mathbf{0} : \mathbf{0} \vdash !(B)}{A, !(A) \multimap \mathbf{0} : \cdot \vdash !(B)} \star}{\frac{A, !(A) \multimap \mathbf{0} : \cdot \vdash !(B)}{!(A) \multimap \mathbf{0} : \cdot \vdash !(A) \multimap !(B)} D_C} \multimap$$

$$\frac{\frac{!(A) \multimap \mathbf{0} : \cdot \vdash !(A) \multimap !(B)}{!(A) \multimap \mathbf{0} : \cdot \vdash !(!(A) \multimap !(B))} !}{\cdot : !(A) \vdash !(A) \multimap !(B)} \star$$

01-ENC encoding (76ms)

$$\frac{\frac{A, !(A) \multimap !(\mathbf{0}) : \cdot \vdash A}{A, !(A) \multimap !(\mathbf{0}) : \cdot \vdash !(A)} ! \quad \frac{A, !(A) \multimap !(\mathbf{0}) : !(\mathbf{0}) \vdash B}{A, !(A) \multimap !(\mathbf{0}) : \cdot \vdash B} \star}{\frac{A, !(A) \multimap !(\mathbf{0}) : \cdot \vdash B}{!(A) \multimap !(\mathbf{0}) : \cdot \vdash !(A) \multimap B} D_C} \multimap$$

$$\frac{\frac{!(A) \multimap !(\mathbf{0}) : \cdot \vdash !(A) \multimap B}{!(A) \multimap !(\mathbf{0}) : \cdot \vdash !(!(A) \multimap B)} !}{\cdot : !(A) \vdash !(A) \multimap B} \star$$

$$(12) \ B \vdash A \rightarrow B$$

LJ (21ms)

$$\overline{B \vdash A \rightarrow B} \star$$

MULTIPLICATIVE encoding (21ms)

Not provable

CALL-BY-NAME encoding (27ms)

$$\frac{\overline{A, B : \cdot \vdash B}}{\cdot : !(B) \vdash !(A) \multimap B} \star$$

CALL-BY-VALUE encoding (47ms)

$$\frac{\frac{\overline{A, B : \cdot \vdash B}}{A, B : \cdot \vdash !(B)} !}{B : \cdot \vdash !(A) \multimap !(B)} \star$$

$$\frac{\frac{B : \cdot \vdash !(A) \multimap !(B)}{B : \cdot \vdash !(A) \multimap !(B)} !}{\cdot : !(B) \vdash !(A) \multimap !(B)} \star$$

01-ENC encoding (39ms)

$$\frac{\overline{A, B : \cdot \vdash B}}{B : \cdot \vdash !(A) \multimap B} \star$$

$$\frac{\frac{B : \cdot \vdash !(A) \multimap B}{B : \cdot \vdash !(A) \multimap B} !}{\cdot : !(B) \vdash !(A) \multimap B} \star$$

$$(13) \ A \rightarrow B \vdash \neg B \rightarrow \neg A$$

LJ (47ms)

$$\frac{\frac{\overline{A, B, B \rightarrow \perp \vdash A} \star \quad \frac{\overline{A, B, B \rightarrow \perp \vdash B} \star \quad \overline{A, B, \perp \vdash \perp} \star}{A, B, B \rightarrow \perp \vdash \perp} \supset_L}{A \rightarrow B \vdash B \rightarrow \perp \rightarrow A \rightarrow \perp} \star$$

MULTIPLICATIVE encoding (47ms)

$$\frac{\frac{\cdot : A \vdash A \quad \frac{\cdot : B \vdash B \quad \cdot : \perp \vdash \perp}{\cdot : B, B \multimap \perp \vdash \perp} \multimap}{\cdot : A, A \multimap B, B \multimap \perp \vdash \perp} \multimap}{\cdot : A \multimap B \vdash B \multimap \perp \multimap A \multimap \perp} \star$$

CALL-BY-NAME encoding (143ms)

CALL-BY-VALUE encoding (174ms)

01-ENC encoding (178ms)

$$(16) (A) \leftrightarrow (B) \vdash A \rightarrow B$$

LJ (37ms)

$$\frac{\overline{A, A \rightarrow B, B \rightarrow A \vdash A}^* \quad \overline{A, B, B \rightarrow A \vdash B}^*}{\overline{A, A \rightarrow B, B \rightarrow A \vdash B}^*} \supset_L$$

$$\frac{\overline{A, A \rightarrow B, B \rightarrow A \vdash B}^*}{A \rightarrow B \wedge B \rightarrow A \vdash A \rightarrow B}^*$$

MULTIPLICATIVE encoding (29ms)

Not provable

CALL-BY-NAME encoding (102ms)

$$\frac{\overline{A, !!(A) \multimap B, !(B) \multimap A : \cdot \vdash A} \quad \overline{A, !(A) \multimap B, !(B) \multimap A : B \vdash B}^*}{\overline{A, !(A) \multimap B, !(B) \multimap A : \cdot \vdash !(A)} \quad \overline{A, !(A) \multimap B, !(B) \multimap A : !!(A) \multimap B \vdash B}^*} \multimap$$

$$\frac{\overline{A, !(A) \multimap B, !(B) \multimap A : \cdot \vdash !(A)} \quad \overline{A, !(A) \multimap B, !(B) \multimap A : !!(A) \multimap B \vdash B}^*}{\overline{A, !(A) \multimap B, !(B) \multimap A : \cdot \vdash B}^{D_C}} \multimap$$

$$\therefore !!(A) \multimap B \& !(B) \multimap A \vdash !(A) \multimap B^*$$

CALL-BY-VALUE encoding (132ms)

$$\frac{\overline{A, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash A} \quad \overline{A, B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash B}^*}{\overline{A, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash !(A)} \quad \overline{A, !(A) \multimap !(B), !(B) \multimap !(A) : !(B) \vdash !(B)}^*} \multimap$$

$$\frac{\overline{A, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash !(A)} \quad \overline{A, !(A) \multimap !(B), !(B) \multimap !(A) : !(B) \vdash !(B)}^*}{\overline{A, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash !(B)}^{D_C}} \multimap$$

$$\frac{\overline{!(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash !(A) \multimap !(B)}^*}{\overline{!(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash !(!(A) \multimap !(B))}^*} \multimap$$

$$\therefore !!(A) \multimap !(B) \otimes !!(B) \multimap !(A) \vdash !(!(A) \multimap !(B))^*$$

01-ENC encoding (126ms)

$$\frac{\overline{A, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash A} \quad \overline{A, B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash B}^*}{\overline{A, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash !(A)} \quad \overline{A, !(A) \multimap !(B), !(B) \multimap !(A) : !(B) \vdash B}^*} \multimap$$

$$\frac{\overline{A, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash !(A)} \quad \overline{A, !(A) \multimap !(B), !(B) \multimap !(A) : !(B) \vdash B}^*}{\overline{A, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash B}^{D_C}} \multimap$$

$$\frac{\overline{!(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash !(A) \multimap B}^*}{\overline{!(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash !(!(A) \multimap B)}^*} \multimap$$

$$\therefore !!(!(A) \multimap !(B)) \& !!(B) \multimap !(A) \vdash !(!(A) \multimap B)^*$$

$$(17) (A) \leftrightarrow (B) \vdash B \rightarrow A$$

LJ (37ms)

$$\frac{\overline{B, A \rightarrow B, B \rightarrow A \vdash B}^* \quad \overline{A, B, A \rightarrow B \vdash A}^*}{\overline{B, A \rightarrow B, B \rightarrow A \vdash A}^*} \supset_L$$

$$\frac{\overline{B, A \rightarrow B, B \rightarrow A \vdash A}^*}{A \rightarrow B \wedge B \rightarrow A \vdash B \rightarrow A}^*$$

MULTIPLICATIVE encoding (29ms)

Not provable

CALL-BY-NAME encoding (105ms)

$$\frac{\overline{B, !(A) \multimap B, !(B) \multimap A : \cdot \vdash B} \quad \overline{B, !(A) \multimap B, !(B) \multimap A : A \vdash A}^*}{\overline{B, !(A) \multimap B, !(B) \multimap A : \cdot \vdash !(B)} \quad \overline{B, !(A) \multimap B, !(B) \multimap A : !!(B) \multimap A \vdash A}^*} \multimap$$

$$\frac{\overline{B, !(A) \multimap B, !(B) \multimap A : \cdot \vdash !(B)} \quad \overline{B, !(A) \multimap B, !(B) \multimap A : !!(B) \multimap A \vdash A}^*}{\overline{B, !(A) \multimap B, !(B) \multimap A : \cdot \vdash A}^{D_C}} \multimap$$

$$\therefore !!(A) \multimap B \& !(B) \multimap A \vdash !(B) \multimap A^*$$

CALL-BY-VALUE encoding (140ms)

$$\frac{\overline{B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash B} \quad \overline{A, B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash A}^*}{\overline{B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash !(B)} \quad \overline{B, !(A) \multimap !(B), !(B) \multimap !(A) : !(A) \vdash !(A)}^*} \multimap$$

$$\frac{\overline{B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash !(B)} \quad \overline{B, !(A) \multimap !(B), !(B) \multimap !(A) : !(A) \vdash !(A)}^*}{\overline{B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash !(A)}^{D_C}} \multimap$$

$$\frac{\overline{!(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash !(B) \multimap !(A)}^*}{\overline{!(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash !(!(B) \multimap !(A))}^*} \multimap$$

$$\therefore !!(A) \multimap !(B) \otimes !!(B) \multimap !(A) \vdash !(!(B) \multimap !(A))^*$$

01-ENC encoding (122ms)

$$\frac{\overline{B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash B} \quad \overline{A, B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash A}^*}{\overline{B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash !(B)} \quad \overline{B, !(A) \multimap !(B), !(B) \multimap !(A) : !(A) \vdash A}^*} \multimap$$

$$\frac{\overline{B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash !(B)} \quad \overline{B, !(A) \multimap !(B), !(B) \multimap !(A) : !(A) \vdash A}^*}{\overline{B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash A}^{D_C}} \multimap$$

$$\frac{\overline{!(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash !(B) \multimap A}^*}{\overline{!(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash !(!(B) \multimap A)}^*} \multimap$$

$$\therefore !!(!(A) \multimap !(B)) \& !!(B) \multimap !(A) \vdash !(!(B) \multimap A)^*$$

$$(18) (A) \leftrightarrow (B), A \vdash B$$

LJ (38ms)

$$\frac{\frac{A, A \rightarrow B, B \rightarrow A \vdash A}{A, A \rightarrow B, B \rightarrow A \vdash B}^* \quad \frac{A, B, B \rightarrow A \vdash B}{A, A \rightarrow B, B \rightarrow A \vdash B}^*}{A, A \rightarrow B \wedge B \rightarrow A \vdash B}^* \supset_L$$

MULTIPLICATIVE encoding (29ms)

Not provable

CALL-BY-NAME encoding (102ms)

$$\frac{\frac{\frac{A, !!(A) \multimap B, !(B) \multimap A : \cdot \vdash A}{A, !(A) \multimap B, !(B) \multimap A : \cdot \vdash !(A)}^! \quad \frac{A, !(A) \multimap B, !(B) \multimap A : B \vdash B}{A, !(A) \multimap B, !(B) \multimap A : \cdot \vdash B}^*}{\frac{A, !(A) \multimap B, !(B) \multimap A : !(A) \multimap B \vdash B}{A, !(A) \multimap B, !(B) \multimap A : \cdot \vdash B}^{D_C} \quad \vdash : !(A), !(!(A) \multimap B \& !(B) \multimap A) \vdash B}^* \multimap$$

CALL-BY-VALUE encoding (119ms)

$$\frac{\frac{\frac{A, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash A}{A, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash !(A)}^! \quad \frac{A, B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash B}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash !(B)}^*}{\frac{A, !(A) \multimap !(B), !(B) \multimap !(A) : !(A) \multimap !(B) \vdash !(B)}{A, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash !(B)}^{D_C} \quad \vdash : !(A), !(!(A) \multimap !(B)) \otimes !(!(B) \multimap !(A)) \vdash !(B)}^* \multimap$$

01-ENC encoding (111ms)

$$\frac{\frac{\frac{A, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash A}{A, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash !(A)}^! \quad \frac{A, B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash B}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash !(B)}^*}{\frac{A, !(A) \multimap !(B), !(B) \multimap !(A) : !(A) \multimap !(B) \vdash B}{A, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash B}^{D_C} \quad \vdash : !(A), !(!(A) \multimap !(B)) \& !(!(B) \multimap !(A)) \vdash B}^* \multimap$$

$$(19) (A) \leftrightarrow (B), B \vdash A$$

LJ (38ms)

$$\frac{\frac{B, A \rightarrow B, B \rightarrow A \vdash B}{B, A \rightarrow B, B \rightarrow A \vdash A}^* \quad \frac{A, B, A \rightarrow B \vdash A}{B, A \rightarrow B, B \rightarrow A \vdash A}^*}{B, A \rightarrow B \wedge B \rightarrow A \vdash A}^* \supset_L$$

MULTIPLICATIVE encoding (32ms)

Not provable

CALL-BY-NAME encoding (102ms)

$$\frac{\frac{\frac{B, !(A) \multimap B, !(B) \multimap A : \cdot \vdash B}{B, !(A) \multimap B, !(B) \multimap A : \cdot \vdash !(B)}^! \quad \frac{B, !(A) \multimap B, !(B) \multimap A : A \vdash A}{B, !(A) \multimap B, !(B) \multimap A : \cdot \vdash A}^*}{\frac{B, !(A) \multimap B, !(B) \multimap A : !(B) \multimap A \vdash A}{B, !(A) \multimap B, !(B) \multimap A : \cdot \vdash A}^{D_C} \quad \vdash : !(B), !(!(A) \multimap B \& !(B) \multimap A) \vdash A}^* \multimap$$

CALL-BY-VALUE encoding (118ms)

$$\frac{\frac{\frac{B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash B}{B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash !(B)}^! \quad \frac{A, B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash A}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash !(A)}^*}{\frac{B, !(A) \multimap !(B), !(B) \multimap !(A) : !(B) \multimap !(A) \vdash !(A)}{B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash !(A)}^{D_C} \quad \vdash : !(B), !(!(A) \multimap !(B)) \otimes !(!(B) \multimap !(A)) \vdash !(A)}^* \multimap$$

01-ENC encoding (108ms)

$$\frac{\frac{\frac{B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash B}{B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash !(B)}^! \quad \frac{A, B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash A}{A, B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash !(A)}^*}{\frac{B, !(A) \multimap !(B), !(B) \multimap !(A) : !(B) \multimap !(A) \vdash A}{B, !(A) \multimap !(B), !(B) \multimap !(A) : \cdot \vdash A}^{D_C} \quad \vdash : !(B), !(!(A) \multimap !(B)) \& !(!(B) \multimap !(A)) \vdash A}^* \multimap$$

$$(20) \cdot \vdash (A) \leftrightarrow (A)$$

LJ (21ms)

$$\frac{}{\cdot \vdash A \rightarrow A \wedge A \rightarrow A} \star$$

MULTIPLICATIVE encoding (34ms)

$$\frac{\frac{\frac{}{\cdot \vdash A \vdash A} \star}{\cdot \vdash \cdot \vdash A \multimap A} \star}{\cdot \vdash \cdot \vdash A \multimap A \otimes A \multimap A} \otimes$$

CALL-BY-NAME encoding (27ms)

$$\frac{\frac{\overline{A : \cdot \vdash A} \quad \overline{A : \cdot \vdash A}}{\cdot \vdash \cdot \vdash !(A) \multimap A \& !(A) \multimap A} \star$$

CALL-BY-VALUE encoding (47ms)

$$\frac{\frac{\frac{\overline{A : \cdot \vdash A} !}{\cdot \vdash \cdot \vdash !(A)} \star}{\cdot \vdash \cdot \vdash !(A) \multimap !(A)} !}{\cdot \vdash \cdot \vdash !(A) \multimap !(A)} ! \quad \frac{\frac{\frac{\overline{A : \cdot \vdash A} !}{\cdot \vdash \cdot \vdash !(A)} \star}{\cdot \vdash \cdot \vdash !(A) \multimap !(A)} !}{\cdot \vdash \cdot \vdash !(A) \multimap !(A)} ! \quad \frac{}{\cdot \vdash \cdot \vdash !(A) \multimap !(A) \otimes !(A) \multimap !(A)} \otimes$$

01-ENC encoding (47ms)

$$\frac{\frac{\frac{\overline{A : \cdot \vdash A} \star}{\cdot \vdash \cdot \vdash !(A) \multimap A} \star}{\cdot \vdash \cdot \vdash !(A) \multimap A} !}{\cdot \vdash \cdot \vdash !(A) \multimap A \& !(A) \multimap A} \star \quad \frac{\frac{\frac{\overline{A : \cdot \vdash A} \star}{\cdot \vdash \cdot \vdash !(A) \multimap A} \star}{\cdot \vdash \cdot \vdash !(A) \multimap A} !}{\cdot \vdash \cdot \vdash !(A) \multimap A \& !(A) \multimap A} !$$

$$(21) (A) \leftrightarrow (B) \vdash (B) \leftrightarrow (A)$$

LJ (50ms)

$$\frac{\frac{B, A \rightarrow B, B \rightarrow A \vdash B \star}{B, A \rightarrow B, B \rightarrow A \vdash A} \star}{A \rightarrow B \wedge B \rightarrow A \vdash B \rightarrow A \wedge A \rightarrow B} \supset_L$$

MULTIPLICATIVE encoding (81ms)

$$\frac{\frac{\frac{\cdot \vdash B \vdash B \quad \cdot \vdash A \vdash A}{\cdot \vdash B, B \multimap A \vdash A} \multimap}{\cdot \vdash B \multimap A \vdash B \multimap A} \star}{\cdot \vdash A \multimap B, B \multimap A \vdash B \multimap A \otimes A \multimap B} \otimes \quad \frac{}{\cdot \vdash A \multimap B \otimes B \multimap A \vdash B \multimap A \otimes A \multimap B} \star$$

CALL-BY-NAME encoding (138ms)

$$\frac{\frac{\frac{\overline{B(A) \multimap B(B) \multimap A} \multimap B}{B(A) \multimap B(B) \multimap A} !}{\cdot \vdash \cdot \vdash B(A) \multimap B(B) \multimap A} !}{\cdot \vdash \cdot \vdash B(A) \multimap B(B) \multimap A} ! \quad \frac{\frac{\frac{\overline{A(B) \multimap B(B) \multimap A} \multimap B}{A(B) \multimap B(B) \multimap A} !}{\cdot \vdash \cdot \vdash A(B) \multimap B(B) \multimap A} !}{\cdot \vdash \cdot \vdash A(B) \multimap B(B) \multimap A} ! \quad \frac{}{\cdot \vdash \cdot \vdash B(A) \multimap B(B) \multimap A \otimes A(B) \multimap B(B) \multimap A} \otimes$$

CALL-BY-VALUE encoding (203ms)

$$\frac{\frac{\frac{\frac{\frac{\overline{B(A) \multimap B(B) \multimap A} \multimap B}{B(A) \multimap B(B) \multimap A} !}{\cdot \vdash \cdot \vdash B(A) \multimap B(B) \multimap A} !}{\cdot \vdash \cdot \vdash B(A) \multimap B(B) \multimap A} !}{\cdot \vdash \cdot \vdash B(A) \multimap B(B) \multimap A} ! \quad \frac{\frac{\frac{\frac{\frac{\overline{A(B) \multimap B(B) \multimap A} \multimap B}{A(B) \multimap B(B) \multimap A} !}{\cdot \vdash \cdot \vdash A(B) \multimap B(B) \multimap A} !}{\cdot \vdash \cdot \vdash A(B) \multimap B(B) \multimap A} !}{\cdot \vdash \cdot \vdash A(B) \multimap B(B) \multimap A} ! \quad \frac{}{\cdot \vdash \cdot \vdash B(A) \multimap B(B) \multimap A \otimes A(B) \multimap B(B) \multimap A} \otimes$$

01-ENC encoding (188ms)

$$\frac{\frac{\frac{\frac{\frac{\frac{\overline{B(A) \multimap B(B) \multimap A} \multimap B}{B(A) \multimap B(B) \multimap A} !}{\cdot \vdash \cdot \vdash B(A) \multimap B(B) \multimap A} !}{\cdot \vdash \cdot \vdash B(A) \multimap B(B) \multimap A} !}{\cdot \vdash \cdot \vdash B(A) \multimap B(B) \multimap A} ! \quad \frac{\frac{\frac{\frac{\frac{\frac{\overline{A(B) \multimap B(B) \multimap A} \multimap B}{A(B) \multimap B(B) \multimap A} !}{\cdot \vdash \cdot \vdash A(B) \multimap B(B) \multimap A} !}{\cdot \vdash \cdot \vdash A(B) \multimap B(B) \multimap A} !}{\cdot \vdash \cdot \vdash A(B) \multimap B(B) \multimap A} ! \quad \frac{}{\cdot \vdash \cdot \vdash B(A) \multimap B(B) \multimap A \otimes A(B) \multimap B(B) \multimap A} \otimes$$

$$(22) (A \leftrightarrow (B), (B \leftrightarrow (C) \vdash (A \leftrightarrow (C))$$

LJ (75ms)

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MULTIPLICATIVE encoding (115ms)

$$\frac{\frac{\frac{\vdash B \vdash B \quad \vdash C \vdash C}{\vdash A \vdash A} \multimap \quad \frac{\frac{\vdash B \vdash B \quad \vdash A \vdash A}{\vdash C \vdash C} \multimap}{\vdash A, A \multimap B, B \multimap C \vdash C} \multimap \quad \frac{\frac{\vdash C \vdash C \quad \vdash B, B \multimap A \vdash A}{\vdash C, B \multimap A, C \multimap B \vdash A} \multimap}{\vdash A \multimap B, B \multimap C \vdash A \multimap C} * \quad \frac{\vdash A \multimap B, B \multimap A, B \multimap C, C \multimap B \vdash A \multimap C \otimes C \multimap A}{\vdash A \multimap B \otimes B \multimap A, B \multimap C \otimes C \multimap B \vdash A \multimap C \otimes C \multimap A} *$$

CALL-BY-NAME encoding (218ms)

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CALL-BY-VALUE encoding (331ms)

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01-ENC encoding (291ms)

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$$(23) A \rightarrow B \rightarrow C, \neg\neg A, \neg\neg B \vdash \neg\neg C$$

LJ (206ms)

=====

MULTIPLICATIVE encoding (124ms)

$$\frac{\frac{\frac{\vdash C \vdash C \quad \vdash \bot \vdash \bot}{\vdash B \vdash B} \multimap \quad \frac{\vdash C \multimap \bot \vdash \bot}{\vdash C, C \multimap \bot \vdash \bot} \multimap}{\vdash A \vdash A} \multimap \quad \frac{\frac{\vdash B, B \multimap C, C \multimap \bot \vdash \bot}{\vdash A, B, A \multimap B \multimap C, C \multimap \bot \vdash \bot} \multimap}{\vdash A, A \multimap B \multimap C, C \multimap \bot \vdash B \multimap \bot} * \quad \frac{\vdash \bot \vdash \bot}{\vdash A, A \multimap B \multimap C, C \multimap \bot, B \multimap \bot \multimap \bot \vdash \bot} \multimap \quad \frac{\vdash A \multimap B \multimap C, C \multimap \bot, B \multimap \bot \multimap \bot \vdash \bot}{\vdash A \multimap B \multimap C, C \multimap \bot, A \multimap \bot \multimap \bot \vdash \bot} * \quad \frac{\vdash A \multimap B \multimap C, C \multimap \bot, A \multimap \bot \multimap \bot \vdash \bot, B \multimap \bot \multimap \bot \vdash \bot}{\vdash A \multimap B \multimap C, A \multimap \bot \multimap \bot, B \multimap \bot \multimap \bot \vdash C \multimap \bot \multimap \bot} *$$

CALL-BY-NAME encoding (221ms)

=====

CALL-BY-VALUE encoding (324ms)

=====

01-ENC encoding (380ms)

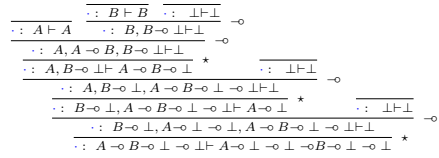
=====

$$(24) \neg\neg A \rightarrow B \vdash \neg\neg A \rightarrow \neg\neg B$$

LJ (94ms)



MULTIPLICATIVE encoding (106ms)



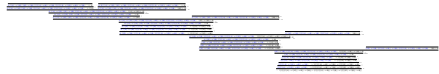
CALL-BY-NAME encoding (199ms)



CALL-BY-VALUE encoding (241ms)



01-ENC encoding (269ms)

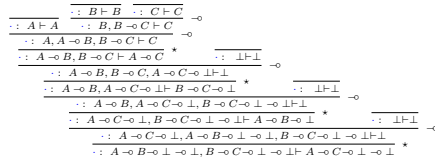


(25) $\neg\neg A \rightarrow B, \neg\neg B \rightarrow C \vdash \neg\neg A \rightarrow C$

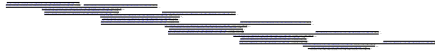
LJ (324ms)



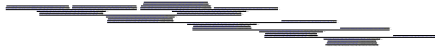
MULTIPLICATIVE encoding (145ms)



CALL-BY-NAME encoding (283ms)



CALL-BY-VALUE encoding (375ms)



01-ENC encoding (421ms)



$$(26) \cdot \vdash (\neg\neg A \wedge B) \leftrightarrow (\neg\neg A \wedge \neg\neg B)$$

LJ (137ms)



MULTIPLICATIVE encoding (66ms)

Not provable

CALL-BY-NAME encoding (601ms)



CALL-BY-VALUE encoding (584ms)



01-ENC encoding (646ms)



$$(27) \cdot \vdash (\neg\neg(A) \leftrightarrow (B)) \leftrightarrow (\neg\neg A \rightarrow B \wedge \neg\neg B \rightarrow A)$$

LJ (2703ms)



MULTIPLICATIVE encoding (84ms)

Not provable

CALL-BY-NAME encoding (661ms)



CALL-BY-VALUE encoding (841ms)



01-ENC encoding (1132ms)

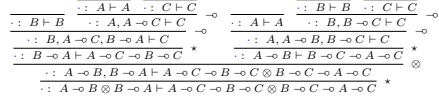


$$(28) (A) \leftrightarrow (B) \vdash (A \rightarrow C) \leftrightarrow (B \rightarrow C)$$

LJ (75ms)



MULTIPLICATIVE encoding (95ms)



CALL-BY-NAME encoding (173ms)



CALL-BY-VALUE encoding (288ms)



01-ENC encoding (268ms)

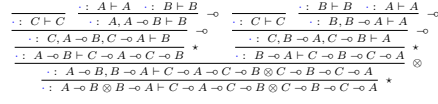


$$(29) (A) \leftrightarrow (B) \vdash (C \rightarrow A) \leftrightarrow (C \rightarrow B)$$

LJ (72ms)



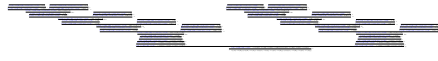
MULTIPLICATIVE encoding (96ms)



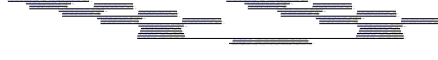
CALL-BY-NAME encoding (250ms)



CALL-BY-VALUE encoding (387ms)



01-ENC encoding (356ms)



$$(30) \quad (A) \leftrightarrow (B) \vdash (A \wedge C) \leftrightarrow (B \wedge C)$$

LJ (51ms)

$$\frac{\frac{\frac{A, C, A \rightarrow B, B \rightarrow A \vdash A}{A, C, A \rightarrow B, B \rightarrow A \vdash A \vdash B}^* \quad \frac{\frac{A, B, C, B \rightarrow A \vdash B}{A, B, C, A \rightarrow B \vdash B}^* \quad \frac{\frac{B, C, A \rightarrow B, B \rightarrow A \vdash B}{B, C, A \rightarrow B, B \rightarrow A \vdash A \vdash B}^* \quad \frac{\frac{A, B, C, A \rightarrow B \vdash A}{A, B, C, A \rightarrow B \vdash A \vdash A}^*}{\frac{A, C, A \rightarrow B, B \rightarrow A \vdash A \vdash B \quad B, C, A \rightarrow B, B \rightarrow A \vdash A \vdash A}{A \rightarrow B \wedge B \rightarrow A \vdash A \vdash A \wedge C \rightarrow B \wedge C \wedge B \wedge C \rightarrow A \wedge C}^*} \supset_L \supset_L$$

MULTIPLICATIVE encoding (99ms)

$$\begin{array}{c}
\frac{\frac{\frac{\cdot}{\cdot} \vdash B \vdash B \quad \cdot \vdash C \vdash C}{\cdot \vdash A \vdash A} \otimes \quad \frac{\frac{\cdot}{\cdot} \vdash B \vdash B \quad \cdot \vdash C \vdash C}{\cdot \vdash A \vdash A} \otimes \\
\frac{\cdot \vdash A, C, A \multimap B \vdash B \otimes C}{\cdot \vdash A \multimap B \vdash A \otimes C \multimap B \otimes C} * \quad \frac{\cdot \vdash B \vdash B \quad \cdot \vdash A, C \vdash A \otimes C}{\cdot \vdash B, C, B \multimap A \vdash A \otimes C} \multimap \\
\frac{\cdot \vdash A \multimap B \vdash A \otimes C \multimap B \otimes C}{\cdot \vdash A \multimap B, B \multimap A \vdash A \otimes C \multimap B \otimes C \otimes B \otimes C \multimap A \otimes C} * \quad \frac{\cdot \vdash B \multimap A \vdash B \otimes C \multimap A \otimes C}{\cdot \vdash A \multimap B, B \multimap A \vdash A \otimes C \multimap B \otimes C \otimes B \otimes C \multimap A \otimes C} * \\
\frac{\cdot \vdash A \multimap B, B \multimap A \vdash A \otimes C \multimap B \otimes C \otimes B \otimes C \multimap A \otimes C}{\cdot \vdash A \multimap B, B \multimap A \vdash A \otimes C \multimap B \otimes C \otimes B \otimes C \multimap A \otimes C} *
\end{array}$$

CALL-BY-NAME encoding (138ms)

[illegible]

CALL-BY-VALUE encoding (238ms)

The diagram illustrates the relationships between various mathematical concepts and their corresponding mathematical symbols. It is organized into several horizontal layers. The top layer includes 'Mathematical Concepts' and 'Mathematical Symbols'. The middle layer shows 'Mathematical Concepts' and 'Mathematical Symbols' with arrows indicating relationships. The bottom layer shows 'Mathematical Concepts' and 'Mathematical Symbols' with arrows indicating relationships. The diagram is a complex web of connections between these concepts and symbols.

01-ENC encoding (234ms)

The diagram illustrates the experimental design for two groups: Control and Experimental. Both groups follow a similar sequence of events: Pretest, Training, and Posttest. The Training phase for the Experimental group is marked with a red 'X' and a red arrow pointing to the Posttest phase, indicating a comparison or intervention point. The Control group's Training phase is marked with a green 'X' and a green arrow pointing to the Posttest phase. The diagram also shows the timing of the Pretest and Posttest phases relative to the Training phase.

$$(31) (A) \leftrightarrow (B) \vdash (C \wedge A) \leftrightarrow (C \wedge B)$$

LJ (51ms)

$$\frac{\frac{\frac{A, C, A \rightarrow B, B \rightarrow A \vdash A}{A, C, A \rightarrow B, B \rightarrow A \vdash A}^* \quad \frac{\frac{A, B, C, B \rightarrow A \vdash B}{A, B, C, A \rightarrow B \vdash B}^*}{A, C, A \rightarrow B, B \rightarrow A \vdash B} \supset_L \quad \frac{\frac{\frac{B, C, A \rightarrow B, B \rightarrow A \vdash B}{B, C, A \rightarrow B, B \rightarrow A \vdash A}^* \quad \frac{A, B, C, A \rightarrow B \vdash A}{B, C, A \rightarrow B, B \rightarrow A \vdash A}^*}{B, C, A \rightarrow B, B \rightarrow A \vdash A} \supset_L}{A \rightarrow B \wedge B \rightarrow A \vdash A \vdash C \wedge A \rightarrow C \wedge B \wedge C \wedge B \rightarrow C \wedge A}^*$$

MULTIPLICATIVE encoding (98ms)

[illegible]

CALL-BY-NAME encoding (142ms)

[illegible]

CALL-BY-VALUE encoding (227ms)

[illegible]

01-ENC encoding (229ms)

[illegible]

$$(32) \quad (A) \leftrightarrow (B) \vdash (\neg A) \leftrightarrow (\neg B)$$

LJ (74ms)

$$\frac{\overbrace{A \rightarrow B, A \rightarrow C, B \rightarrow C}^{\text{S}} \quad \overbrace{A \rightarrow B, A \rightarrow C, C \rightarrow B}^{\text{S}}}{\overbrace{A \rightarrow B, A \rightarrow C, B \rightarrow C, C \rightarrow B}^{\text{S}}} \text{S}_4 \quad \frac{\overbrace{A \rightarrow B, B \rightarrow A}^{\text{S}} \quad \overbrace{A \rightarrow B, B \rightarrow C, C \rightarrow A}^{\text{S}}}{\overbrace{A \rightarrow B, B \rightarrow A, C \rightarrow A}^{\text{S}}} \text{S}_4 \quad \frac{\overbrace{A \rightarrow B, B \rightarrow C, C \rightarrow A}^{\text{S}} \quad \overbrace{A \rightarrow B, B \rightarrow C, A \rightarrow C}^{\text{S}}}{\overbrace{A \rightarrow B, B \rightarrow C, A \rightarrow C, C \rightarrow A}^{\text{S}}} \text{S}_4$$

MULTIPLICATIVE encoding (97ms)

[illegible]

CALL-BY-NAME encoding (223ms)

CALL-BY-VALUE encoding (304ms)

01-ENC encoding (320ms)

The diagram illustrates the experimental design for two groups: Control and Experimental. Both groups follow a similar sequence of phases: Baseline, Training, Transfer, and Retention. The Control group's performance is shown as a continuous line across all phases. The Experimental group's performance is shown as a line that drops significantly during the Transfer phase, marked with a red 'X' and a red arrow pointing to the Retention phase, indicating a failure to maintain performance during transfer.

$$(33) \cdot \vdash (A \wedge B \wedge C) \leftrightarrow (A \wedge B \wedge C)$$

LJ (21ms)

$$\overline{\cdot \vdash A \wedge B \wedge C \rightarrow A \wedge B \wedge C \wedge A \wedge B \wedge C \rightarrow A \wedge B \wedge C}^*$$

MULTIPLICATIVE encoding (66ms)

$$\frac{\frac{\frac{\vdots}{\vdots} A \vdash A \quad \frac{\vdots}{\vdots} B \vdash B}{\vdots, A, B \vdash A \otimes B} \otimes \quad \frac{\vdots}{\vdots} C \vdash C}{\vdots, A, B, C \vdash A \otimes B \otimes C} \otimes \quad \frac{\frac{\frac{\vdots}{\vdots} A \vdash A \quad \frac{\vdots}{\vdots} B \vdash B}{\vdots, A, B \vdash A \otimes B} \otimes \quad \frac{\vdots}{\vdots} C \vdash C}{\vdots, A, B, C \vdash A \otimes B \otimes C} \otimes}{\vdots, \vdash A \otimes B \otimes C \multimap A \otimes B \otimes C} \star \quad \frac{\frac{\frac{\frac{\vdots}{\vdots} A \vdash A \quad \frac{\vdots}{\vdots} B \vdash B}{\vdots, A, B \vdash A \otimes B} \otimes \quad \frac{\vdots}{\vdots} C \vdash C}{\vdots, A, B, C \vdash A \otimes B \otimes C} \otimes \quad \frac{\vdots}{\vdots} A \otimes B \otimes C \multimap A \otimes B \otimes C}{\vdots, \vdash A \otimes B \otimes C \multimap A \otimes B \otimes C} \star}{\vdots, \vdash A \otimes B \otimes C \multimap A \otimes B \otimes C \multimap A \otimes B \otimes C \multimap A \otimes B \otimes C} \otimes$$

CALL-BY-NAME encoding (34ms)

$$\frac{\overline{A, B, C : \vdash A} \quad \overline{A, B, C : \vdash B} \quad \overline{A, B, C : \vdash C} \quad \overline{A, B, C : \vdash A} \quad \overline{A, B, C : \vdash B} \quad \overline{A, B, C : \vdash C}}{\vdash : \vdash \uparrow (A \& B \& C) \multimap A \& B \& C \& \uparrow (A \& B \& C) \multimap A \& B \& C} *$$

CALL-BY-VALUE encoding (99ms)

[illegible]

01-ENC encoding (124ms)

[illegible]

$$(34) \cdot \vdash (A \wedge B) \leftrightarrow (B \wedge A)$$

LJ (21ms)

$$\overline{\cdot \vdash A \wedge B \rightarrow B \wedge A \wedge B \wedge A \rightarrow A \wedge B} \star$$

MULTIPLICATIVE encoding (73ms)

$$\frac{\frac{\overline{\cdot \vdash B \vdash B} \quad \overline{\cdot \vdash A \vdash A}}{\cdot \vdash A, B \vdash B \otimes A} \otimes \quad \frac{\overline{\cdot \vdash A \vdash A} \quad \overline{\cdot \vdash B \vdash B}}{\cdot \vdash A, B \vdash A \otimes B} \otimes}{\cdot \vdash \cdot \vdash A \otimes B \rightarrow B \otimes A} \star \quad \frac{\cdot \vdash \cdot \vdash B \otimes A \rightarrow A \otimes B}{\cdot \vdash \cdot \vdash A \otimes B \rightarrow B \otimes A \otimes B \otimes A \rightarrow A \otimes B} \star$$

CALL-BY-NAME encoding (35ms)

$$\frac{\overline{A, B : \cdot \vdash B} \quad \overline{A, B : \cdot \vdash A} \quad \overline{A, B : \cdot \vdash A} \quad \overline{A, B : \cdot \vdash B}}{\cdot \vdash \cdot \vdash !(A \& B) \rightarrow B \& A \& !(B \& A) \rightarrow A \& B} \star$$

CALL-BY-VALUE encoding (107ms)

$$\frac{\frac{\overline{A, B : \cdot \vdash B} \quad \overline{A, B : \cdot \vdash A}}{\overline{A, B : \cdot \vdash !(B)} \quad \overline{A, B : \cdot \vdash !(A)}} \quad \frac{\overline{A, B : \cdot \vdash A} \quad \overline{A, B : \cdot \vdash B}}{\overline{A, B : \cdot \vdash !(A)} \quad \overline{A, B : \cdot \vdash !(B)}}}{\frac{\cdot \vdash \cdot \vdash !(A) \otimes !(B) \rightarrow !(B) \otimes !(A)}{\cdot \vdash \cdot \vdash !(!(A) \otimes !(B) \rightarrow !(B) \otimes !(A))} \quad \frac{\cdot \vdash \cdot \vdash !(B) \otimes !(A) \rightarrow !(A) \otimes !(B)}{\cdot \vdash \cdot \vdash !(!(B) \otimes !(A) \rightarrow !(A) \otimes !(B))} \quad \frac{\cdot \vdash \cdot \vdash !(!(A) \otimes !(B) \rightarrow !(B) \otimes !(A)) \otimes !(B) \otimes !(A) \rightarrow !(B) \otimes !(A) \otimes !(B))}{\cdot \vdash \cdot \vdash !(!(A) \otimes !(B) \rightarrow !(B) \otimes !(A)) \otimes !(B) \otimes !(A) \rightarrow !(B) \otimes !(A) \otimes !(B))} \star$$

01-ENC encoding (102ms)

$$\frac{\frac{\overline{A, B : \cdot \vdash B} \quad \overline{A, B : \cdot \vdash A}}{\overline{A, B : \cdot \vdash B \& A} \quad \overline{A, B : \cdot \vdash !(B \& A)}} \star \quad \frac{\overline{A, B : \cdot \vdash A} \quad \overline{A, B : \cdot \vdash B}}{\overline{A, B : \cdot \vdash A \& B} \quad \overline{A, B : \cdot \vdash !(A \& B)}} \star}{\frac{\cdot \vdash \cdot \vdash !(!(A) \& !(B)) \rightarrow !(B \& A)}{\cdot \vdash \cdot \vdash !(!(A) \& !(B)) \rightarrow !(B \& A))} \quad \frac{\cdot \vdash \cdot \vdash !(!(B) \& !(A)) \rightarrow !(A \& B)}{\cdot \vdash \cdot \vdash !(!(B) \& !(A)) \rightarrow !(A \& B))} \quad \frac{\cdot \vdash \cdot \vdash !(!(!(A) \& !(B)) \rightarrow !(B \& A)) \& !(B \& A) \rightarrow !(B \& A) \& !(B \& A))}{\cdot \vdash \cdot \vdash !(!(!(A) \& !(B)) \rightarrow !(B \& A)) \& !(B \& A) \rightarrow !(B \& A) \& !(B \& A))} \star$$

$$(35) \cdot \vdash (A \wedge A) \leftrightarrow (A)$$

LJ (21ms)

$$\overline{\cdot \vdash A \wedge A \rightarrow A \wedge A \rightarrow A \wedge A} \star$$

MULTIPLICATIVE encoding (34ms)

Not provable

CALL-BY-NAME encoding (28ms)

$$\frac{\overline{A : \cdot \vdash A} \quad \overline{A : \cdot \vdash A} \quad \overline{A : \cdot \vdash A}}{\cdot \vdash \cdot \vdash !(A \& A) \rightarrow A \& !(A) \rightarrow A \& A} \star$$

CALL-BY-VALUE encoding (76ms)

$$\frac{\frac{\overline{A : \cdot \vdash A}}{\overline{A : \cdot \vdash !(A)}} \quad \frac{\overline{A : \cdot \vdash A} \quad \overline{A : \cdot \vdash A}}{\overline{A : \cdot \vdash !(A)} \quad \overline{A : \cdot \vdash !(A)}}}{\frac{\cdot \vdash \cdot \vdash !(A) \otimes !(A) \rightarrow !(A)}{\cdot \vdash \cdot \vdash !(!(A) \otimes !(A) \rightarrow !(A))} \quad \frac{\cdot \vdash \cdot \vdash !(A) \otimes !(A) \rightarrow !(A)}{\cdot \vdash \cdot \vdash !(!(A) \otimes !(A) \rightarrow !(A))} \quad \frac{\cdot \vdash \cdot \vdash !(!(A) \otimes !(A) \rightarrow !(A)) \otimes !(A) \rightarrow !(A) \otimes !(A)}{\cdot \vdash \cdot \vdash !(!(A) \otimes !(A) \rightarrow !(A)) \otimes !(A) \rightarrow !(A) \otimes !(A)} \star$$

01-ENC encoding (81ms)

$$\frac{\frac{\overline{A : \cdot \vdash A} \quad \overline{A : \cdot \vdash A}}{\overline{A : \cdot \vdash A \& A} \quad \overline{A : \cdot \vdash !(A \& A)}} \star \quad \frac{\overline{A : \cdot \vdash A}}{\overline{A : \cdot \vdash !(A)}} \star}{\frac{\cdot \vdash \cdot \vdash !(!(A) \& !(A)) \rightarrow A}{\cdot \vdash \cdot \vdash !(!(A) \& !(A)) \rightarrow A)} \quad \frac{\cdot \vdash \cdot \vdash !(A) \rightarrow !(A \& A)}{\cdot \vdash \cdot \vdash !(!(A) \rightarrow !(A \& A))} \quad \frac{\cdot \vdash \cdot \vdash !(!(A) \& !(A)) \rightarrow A \& !(A) \rightarrow !(A \& A)}{\cdot \vdash \cdot \vdash !(!(A) \& !(A)) \rightarrow A \& !(A) \rightarrow !(A \& A))} \star$$

$$(36) A \vdash (A \rightarrow B) \leftrightarrow (B)$$

LJ (37ms)

$$\frac{\frac{\overline{A, A \rightarrow B \vdash A} \star \overline{A, B \vdash B}}{A, A \rightarrow B \vdash B} \supset_L}{A \vdash A \rightarrow B \rightarrow B \wedge B \rightarrow A \rightarrow B} \star$$

MULTIPLICATIVE encoding (42ms)

Not provable

CALL-BY-NAME encoding (73ms)

$$\frac{\frac{\overline{A, ! (A) \multimap B : \cdot \vdash A}}{A, ! (A) \multimap B : \cdot \vdash ! (A)} ! \quad \frac{\overline{A, ! (A) \multimap B : B \vdash B}}{A, ! (A) \multimap B : ! (A) \multimap B \vdash B} \multimap}{\frac{A, ! (A) \multimap B : ! (A) \multimap B \vdash B}{\cdot : ! (A) \vdash ! (! (A) \multimap B) \multimap B \& ! (B) \multimap ! (A) \multimap B} D_C} \star$$

CALL-BY-VALUE encoding (129ms)

$$\frac{\frac{\frac{\overline{A, ! (A) \multimap ! (B) : \cdot \vdash A}}{A, ! (A) \multimap ! (B) : \cdot \vdash ! (A)} ! \quad \frac{\overline{A, B, ! (A) \multimap ! (B) : \cdot \vdash ! (B)} !}{\frac{A, ! (A) \multimap ! (B) : ! (A) \multimap ! (B) \vdash ! (B)}{A, ! (A) \multimap ! (B) : \cdot \vdash ! (B)} D_C} \star \quad \frac{\overline{A, B : \cdot \vdash B}}{A, B : \cdot \vdash ! (A) \multimap ! (B)} !}{\frac{\frac{A, ! (A) \multimap ! (B) : \cdot \vdash ! (B)}{A : \cdot \vdash ! (! (A) \multimap ! (B)) \multimap ! (B)} \star \quad \frac{\overline{A, B : \cdot \vdash ! (A) \multimap ! (B)} !}{A : \cdot \vdash ! (! (! (A) \multimap ! (B)) \multimap ! (B))} !}{\frac{A : \cdot \vdash ! (! (! (A) \multimap ! (B)) \multimap ! (B)) \otimes ! (! (B) \multimap ! (! (A) \multimap ! (B)))}{\cdot : ! (A) \vdash ! (! (! (A) \multimap ! (B)) \multimap ! (B)) \otimes ! (! (B) \multimap ! (! (A) \multimap ! (B)))} \otimes} \star$$

01-ENC encoding (123ms)

$$\frac{\frac{\frac{\overline{A, ! (A) \multimap ! (B) : \cdot \vdash A}}{A, ! (A) \multimap ! (B) : \cdot \vdash ! (A)} ! \quad \frac{\overline{A, B, ! (A) \multimap ! (B) : \cdot \vdash B}}{A, ! (A) \multimap ! (B) : ! (A) \multimap ! (B) \vdash B} \star}{\frac{A, ! (A) \multimap ! (B) : ! (A) \multimap ! (B) \vdash B}{A, ! (A) \multimap ! (B) : \cdot \vdash B} D_C} \star \quad \frac{\overline{A, B : \cdot \vdash B}}{A, B : \cdot \vdash ! (A) \multimap ! (B)} !}{\frac{\frac{A, ! (A) \multimap ! (B) : \cdot \vdash ! (B)}{A : \cdot \vdash ! (! (A) \multimap ! (B)) \multimap ! (B)} \star \quad \frac{\overline{A, B : \cdot \vdash ! (A) \multimap ! (B)} !}{A : \cdot \vdash ! (! (! (A) \multimap ! (B)) \multimap ! (B))} !}{\frac{A : \cdot \vdash ! (! (! (A) \multimap ! (B)) \multimap ! (B)) \otimes ! (! (B) \multimap ! (! (A) \multimap ! (B)))}{\cdot : ! (A) \vdash ! (! (! (A) \multimap ! (B)) \multimap ! (B)) \otimes ! (! (B) \multimap ! (! (A) \multimap ! (B)))} \otimes} \star$$

$$(37) B \vdash (A \rightarrow B) \leftrightarrow (B)$$

LJ (21ms)

$$\overline{B \vdash A \rightarrow B \rightarrow B \wedge B \rightarrow A \rightarrow B} \star$$

MULTIPLICATIVE encoding (40ms)

Not provable

CALL-BY-NAME encoding (39ms)

$$\frac{\overline{B, ! (A) \multimap B : \cdot \vdash B} \quad \overline{A, B : \cdot \vdash B}}{\cdot : ! (B) \vdash ! (! (A) \multimap B) \multimap B \& ! (B) \multimap ! (A) \multimap B} \star$$

CALL-BY-VALUE encoding (152ms)

$$\frac{\frac{\frac{\overline{B, ! (A) \multimap ! (B) : \cdot \vdash B}}{B, ! (A) \multimap ! (B) : \cdot \vdash ! (B)} ! \quad \frac{\overline{A, B : \cdot \vdash B}}{A, B : \cdot \vdash ! (A) \multimap ! (B)} !}{\frac{B : \cdot \vdash ! (! (A) \multimap ! (B)) \multimap ! (B)}{B : \cdot \vdash ! (! (! (A) \multimap ! (B)) \multimap ! (B))} \star \quad \frac{\overline{A, B : \cdot \vdash ! (A) \multimap ! (B)} !}{B : \cdot \vdash ! (! (B) \multimap ! (! (A) \multimap ! (B)))} !}{\frac{B : \cdot \vdash ! (! (! (A) \multimap ! (B)) \multimap ! (B)) \otimes ! (! (B) \multimap ! (! (A) \multimap ! (B)))}{\cdot : ! (B) \vdash ! (! (! (A) \multimap ! (B)) \multimap ! (B)) \otimes ! (! (B) \multimap ! (! (A) \multimap ! (B)))} \otimes} \star$$

01-ENC encoding (88ms)

$$\frac{\frac{\frac{\overline{B, ! (A) \multimap ! (B) : \cdot \vdash B}}{B, ! (A) \multimap ! (B) : \cdot \vdash ! (B)} ! \quad \frac{\overline{A, B : \cdot \vdash B}}{A, B : \cdot \vdash ! (A) \multimap ! (B)} !}{\frac{B : \cdot \vdash ! (! (A) \multimap ! (B)) \multimap ! (B)}{B : \cdot \vdash ! (! (! (A) \multimap ! (B)) \multimap ! (B))} \star \quad \frac{\overline{A, B : \cdot \vdash ! (A) \multimap ! (B)} !}{B : \cdot \vdash ! (! (B) \multimap ! (! (A) \multimap ! (B)))} !}{\frac{B : \cdot \vdash ! (! (! (A) \multimap ! (B)) \multimap ! (B)) \& ! (! (B) \multimap ! (! (A) \multimap ! (B)))}{\cdot : ! (B) \vdash ! (! (! (A) \multimap ! (B)) \multimap ! (B)) \& ! (! (B) \multimap ! (! (A) \multimap ! (B)))} \star}$$

$$(38) \neg A \vdash (A \rightarrow B) \leftrightarrow (\neg A)$$

LJ (52ms)

$$\frac{\frac{\overline{A, A \rightarrow B, A \rightarrow \perp \vdash A}^* \quad \overline{A, \perp, A \rightarrow B \vdash \perp}^*}{A, A \rightarrow B, A \rightarrow \perp \vdash \perp} \supset_L \quad \frac{\overline{A, A \rightarrow \perp \vdash A}^* \quad \overline{A, \perp \vdash B}^*}{A, A \rightarrow \perp \vdash B} \supset_L}{A \rightarrow \perp \vdash A \rightarrow B \rightarrow A \rightarrow \perp \wedge A \rightarrow \perp \rightarrow A \rightarrow B}^*$$

MULTIPLICATIVE encoding (42ms)

Not provable

CALL-BY-NAME encoding (101ms)

[illegible]

CALL-BY-VALUE encoding (201ms)

[illegible]

01-ENC encoding (203ms)

[illegible]

$$(39) \neg B \vdash (A \rightarrow B) \leftrightarrow (\neg A)$$

LJ (61ms)

$$\frac{\frac{\frac{A, A \rightarrow B, B \rightarrow \perp \vdash A}{A, A \rightarrow B, B \rightarrow \perp \vdash \perp} *}{A, A \rightarrow B, B \rightarrow \perp \vdash \perp} *}{B \rightarrow \perp \vdash A \rightarrow B \rightarrow A \rightarrow \perp \wedge A \rightarrow \perp \rightarrow A \rightarrow B} \supset_L \frac{\frac{\frac{A, B, B \rightarrow \perp \vdash B}{A, B, B \rightarrow \perp \vdash \perp} *}{A, B, B \rightarrow \perp \vdash \perp} *}{A, A \rightarrow \perp, B \rightarrow \perp \vdash A} * \supset_L \frac{\frac{A, A \rightarrow \perp, B \rightarrow \perp \vdash A}{A, A \rightarrow \perp, B \rightarrow \perp \vdash \perp} *}{A, A \rightarrow \perp, B \rightarrow \perp \vdash \perp} * \supset_L \frac{\frac{A, A \rightarrow \perp, B \rightarrow \perp \vdash \perp}{B \rightarrow \perp \vdash A \rightarrow B \rightarrow A \rightarrow \perp \wedge A \rightarrow \perp \rightarrow A \rightarrow B} *}{B \rightarrow \perp \vdash A \rightarrow B \rightarrow A \rightarrow \perp \wedge A \rightarrow \perp \rightarrow A \rightarrow B} *$$

MULTIPLICATIVE encoding (43ms)

Not provable

CALL-BY-NAME encoding (185ms)

The diagram illustrates the hierarchical structure of the proof, showing the flow of logical dependencies between various mathematical expressions and theorems. The structure is organized into a series of steps, with each step represented by a box containing a mathematical expression or theorem. The boxes are connected by arrows, indicating the direction of the logical flow. The diagram shows a complex web of dependencies, with many boxes containing the same expression, indicating that the same result is used in multiple places. The overall structure is a series of steps leading from the initial assumptions to the final conclusion.

CALL-BY-VALUE encoding (256ms)

Figure 1 is a schematic representation of the 1000 Genomes Project. It shows a hierarchical structure of genomic data. At the top, 'Genomes' are represented by blue bars. These are grouped into 'Populations' (indicated by brackets). Within each population, 'Individuals' are shown as smaller blue bars. The diagram illustrates the relationship between the total number of genomes, the number of populations, and the number of individuals sampled within each population. The populations are color-coded: African (green), European (blue), East Asian (red), South Asian (orange), and Admixed American (purple).

01-ENC encoding (259ms)

Figure 1 illustrates the hierarchical clustering of 1000 genes into 10 clusters. The tree structure shows the relationships between the genes and the clusters. The genes are listed in a column on the left, and the clusters are listed in a column on the right. The tree structure shows the relationships between the genes and the clusters.

$$(40) \quad B \vdash (A \wedge B) \leftrightarrow (A)$$

LJ (21ms)

$$\overline{B \vdash A \wedge B \rightarrow A \wedge A \rightarrow A \wedge B}^{\star}$$

MULTIPLICATIVE encoding (34ms)

Not provable

CALL-BY-NAME encoding (34ms)

$$\frac{\overline{A, B : \cdot \vdash A} \quad \overline{A, B : \cdot \vdash A} \quad \overline{A, B : \cdot \vdash B}}{\cdot : !(B) \vdash !(A \& B) \multimap A \& !(A) \multimap A \& B} \star$$

CALL-BY-VALUE encoding (88ms)

[illegible]

01-ENC encoding (88ms)

$$\begin{array}{c}
\frac{A, B : \vdash \neg A \quad A, B : \vdash \neg B}{A, B : \vdash \neg (A \& B)} \star \\
\frac{\frac{A, B : \vdash \neg A}{B : \vdash \neg !(\neg(A) \& ! (B)) \rightarrow A} \star \quad \frac{A, B : \vdash \neg ! (A \& B)}{B : \vdash \neg ! (A) \rightarrow ! (A \& B)} \star}{B : \vdash \neg !(\neg(A) \& ! (B)) \rightarrow A} \star \\
\frac{B : \vdash \neg !(\neg(A) \& ! (B)) \rightarrow A \& !(\neg(A) \rightarrow ! (A \& B))}{B : \vdash \neg !(\neg(A) \& ! (B)) \rightarrow A \& ! (A) \rightarrow ! (A \& B))} \star \\
\therefore ! (B) \rightarrow !(\neg(A) \& ! (B)) \rightarrow A \& !(\neg(A) \rightarrow ! (A \& B)) \star
\end{array}$$

$$(41) \neg B \vdash (A \wedge B) \leftrightarrow (B)$$

LJ (37ms)

$$\frac{\frac{\overline{B, B \rightarrow \perp \vdash B}^{\star} \quad \overline{B, \perp \vdash A}^{\star}}{B, B \rightarrow \perp \vdash A} \supset_L}{\overline{B \rightarrow \perp \vdash A \wedge B \rightarrow B \wedge B \rightarrow A \wedge B}^{\star}}$$

MULTIPLICATIVE encoding (41ms)

Not provable

CALL-BY-NAME encoding (78ms)

$$\frac{\frac{\frac{B, \mathfrak{!}(B) \multimap 0 : \vdash B}{B, \mathfrak{!}(B) \multimap 0 : \vdash \mathfrak{!}(B)} \quad \mathfrak{!} \quad \frac{B, \mathfrak{!}(B) \multimap 0 : 0 \vdash A}{B, \mathfrak{!}(B) \multimap 0 : \vdash A} \quad *}{\frac{A, B, \mathfrak{!}(B) \multimap 0 : \vdash B}{B, \mathfrak{!}(B) \multimap 0 : \vdash A} \quad D_C \quad \frac{B, \mathfrak{!}(B) \multimap 0 : \vdash B}{\vdash : \mathfrak{!}(\mathfrak{!}(B) \multimap 0) \vdash \mathfrak{!}(A \& B) \multimap B \& \mathfrak{!}(B) \multimap A \& B} \quad \multimap}$$

CALL-BY-VALUE encoding (172ms)

[illegible]

01-ENC encoding (194ms)

[illegible]

$$(42) \cdot \vdash A \rightarrow \neg\neg A$$

LJ (38ms)

$$\frac{\frac{\frac{A, A \rightarrow \perp \vdash A}{\vdash A}^* \quad \frac{A, \perp \vdash \perp}{\vdash \perp}^*}{A, A \rightarrow \perp \vdash \perp} \quad \supset_L}{\vdash A \rightarrow A \rightarrow \perp \rightarrow \perp}^*$$

MULTIPLICATIVE encoding (41ms)

$$\frac{\frac{\frac{\cdot : A \vdash A}{\cdot : A, A \multimap \perp \vdash \perp} \multimap}{\cdot : \cdot \vdash A \multimap A \multimap \perp \multimap \perp} \star$$

CALL-BY-NAME encoding (66ms)

$$\frac{\frac{\frac{A, !(A) \multimap 0 : \vdash \vdash A}{A, !(A) \multimap 0 : \vdash \vdash !(A)} ! \quad \frac{A, !(A) \multimap 0 : 0 \vdash 0}{A, !(A) \multimap 0 : 0 \vdash 0} *}{\frac{A, !(A) \multimap 0 : !(A) \multimap 0 \vdash 0}{A, !(A) \multimap 0 : \vdash 0} D_C} \multimap$$

CALL-BY-VALUE encoding (84ms)

$$\frac{\frac{A, !(A) \multimap 0 : \vdash A}{A, !(A) \multimap 0 : \vdash ! (A)} ! \quad \frac{A, !(A) \multimap 0 : 0 \vdash 0}{\quad} *}{\frac{A, !(A) \multimap 0 : \vdash 0}{\quad} D_C} \star$$
$$\frac{\frac{A, !(A) \multimap 0 : \vdash 0}{\quad} D_C}{\frac{A : \vdash ! (! (A) \multimap 0) \multimap 0}{\quad} *} !}$$
$$\frac{\frac{A : \vdash ! (! (! (A) \multimap 0) \multimap 0)}{\vdots : \vdash ! (A) \multimap ! (! (! (A) \multimap 0) \multimap 0)} \star}{\vdots : \vdash ! (! (A) \multimap ! (! (! (A) \multimap 0) \multimap 0))} !$$

01-ENC encoding (81ms)

$$\begin{array}{c}
\frac{A, !(\mathbf{A}) \multimap !(\mathbf{0}) : \cdot \vdash A}{A, !(\mathbf{A}) \multimap !(\mathbf{0}) : \cdot \vdash !(\mathbf{A})} ! \quad \frac{A, !(\mathbf{A}) \multimap !(\mathbf{0}) : \cdot \vdash !(\mathbf{0})}{A, !(\mathbf{A}) \multimap !(\mathbf{0}) : \cdot \vdash \mathbf{0}} \star \\
\frac{A, !(\mathbf{A}) \multimap !(\mathbf{0}) : \cdot \vdash !(\mathbf{A}) : !(\mathbf{A}) \multimap !(\mathbf{0}) \vdash \mathbf{0}}{A, !(\mathbf{A}) \multimap !(\mathbf{0}) : \cdot \vdash \mathbf{0}} D_C \\
\frac{A : \cdot \vdash !(!(\mathbf{A}) \multimap !(\mathbf{0})) \multimap \mathbf{0}}{A : \cdot \vdash !(!(!(\mathbf{A}) \multimap !(\mathbf{0})) \multimap \mathbf{0})} \star \\
\frac{A : \cdot \vdash !(!(!(\mathbf{A}) \multimap !(\mathbf{0})) \multimap \mathbf{0})}{\cdot : \cdot \vdash !(!(\mathbf{A}) \multimap !(!(\mathbf{A}) \multimap !(\mathbf{0})) \multimap \mathbf{0})} \star \\
\frac{\cdot : \cdot \vdash !(!(\mathbf{A}) \multimap !(!(\mathbf{A}) \multimap !(\mathbf{0})) \multimap \mathbf{0})}{\cdot : \cdot \vdash !(!(\mathbf{A}) \multimap !(!(\mathbf{A}) \multimap !(\mathbf{0})) \multimap \mathbf{0})} !
\end{array}$$

$$(43) \cdot \vdash (\neg\neg\neg A) \leftrightarrow (\neg A)$$

LJ (81ms)

$$\frac{\frac{A \rightarrow B \rightarrow C \rightarrow D \rightarrow E \rightarrow F \rightarrow G}{A \rightarrow B \rightarrow C \rightarrow D \rightarrow E \rightarrow F} \rightarrow G}{A \rightarrow B \rightarrow C \rightarrow D \rightarrow E \rightarrow F \rightarrow G} \rightarrow G \quad D_4 \quad \frac{\frac{A \rightarrow B \rightarrow C \rightarrow D \rightarrow E \rightarrow F \rightarrow G}{A \rightarrow B \rightarrow C \rightarrow D \rightarrow E \rightarrow F} \rightarrow G}{A \rightarrow B \rightarrow C \rightarrow D \rightarrow E \rightarrow F \rightarrow G} \rightarrow G \quad D_4$$

$$\frac{\frac{\frac{A \rightarrow B \rightarrow C \rightarrow D \rightarrow E \rightarrow F \rightarrow G}{A \rightarrow B \rightarrow C \rightarrow D \rightarrow E \rightarrow F} \rightarrow G}{A \rightarrow B \rightarrow C \rightarrow D \rightarrow E \rightarrow F \rightarrow G} \rightarrow G}{A \rightarrow B \rightarrow C \rightarrow D \rightarrow E \rightarrow F \rightarrow G} \rightarrow G \quad D_4$$

即 $A \rightarrow B \rightarrow C \rightarrow D \rightarrow E \rightarrow F \rightarrow G \rightarrow H \rightarrow I \rightarrow J \rightarrow K \rightarrow L \rightarrow M \rightarrow N \rightarrow O \rightarrow P \rightarrow Q \rightarrow R \rightarrow S \rightarrow T \rightarrow U \rightarrow V \rightarrow W \rightarrow X \rightarrow Y \rightarrow Z$

MULTIPLICATIVE encoding (105ms)

[illegible]

CALL-BY-NAME encoding (244ms)

Figure 1 illustrates the 12-step process for the development of a new product, organized into three phases:

- Phase 1: Initial Planning and Funding (Steps 1-3)**
 - 1. Identify a market need: A person is shown thinking, with a lightbulb icon.
 - 2. Develop a business plan: A person is shown writing on a document, with a document icon.
 - 3. Secure financing: A person is shown shaking hands, with a dollar sign icon.
- Phase 2: Development and Testing (Steps 4-6)**
 - 4. Develop a prototype: A person is shown working on a machine, with a gear icon.
 - 5. Test the prototype: A person is shown holding a device, with a test tube icon.
 - 6. Refine the prototype: A person is shown working on a machine, with a gear icon.
- Phase 3: Marketing and Launch (Steps 7-12)**
 - 7. Develop a marketing plan: A person is shown writing on a document, with a document icon.
 - 8. Launch the product: A person is shown holding a device, with a launch icon.
 - 9. Monitor sales: A person is shown looking at a graph, with a bar chart icon.
 - 10. Evaluate customer feedback: A person is shown talking to a group of people, with a speech bubble icon.
 - 11. Adjust the product: A person is shown working on a machine, with a gear icon.
 - 12. Re-launch the product: A person is shown holding a device, with a launch icon.

CALL-BY-VALUE encoding (295ms)

[illegible]

01-ENC encoding (343ms)

(44) $\vdash \neg A \wedge \neg A$

LJ (38ms)

$$\frac{\frac{\frac{A, A \rightarrow \perp \vdash A}{A, A \rightarrow \perp \vdash \perp} \star}{\cdot \vdash A \wedge A \rightarrow \perp \rightarrow \perp} \star}{\cdot \vdash A \wedge A \rightarrow \perp \rightarrow \perp} \star \supset_L$$

MULTIPLICATIVE encoding (41ms)

$$\frac{\frac{\cdot : A \vdash A \quad \cdot : \perp \vdash \perp}{\cdot : A, A \multimap \perp \vdash \perp} \multimap \quad \cdot : \cdot \vdash A \otimes A \multimap \perp \multimap \perp}{\cdot : \cdot \vdash A \otimes A \multimap \perp \multimap \perp} \star$$

CALL-BY-NAME encoding (65ms)

$$\frac{\frac{\frac{A, !(A) \multimap \mathbf{0} : \vdash \overline{\mathbf{0}}}{A, !(A) \multimap \mathbf{0} : \vdash !(A)} ! \quad \frac{A, !(A) \multimap \mathbf{0} : \mathbf{0} \vdash \mathbf{0}}{} *}{\frac{A, !(A) \multimap \mathbf{0} : !(A) \multimap \mathbf{0} \vdash \mathbf{0}}{A, !(A) \multimap \mathbf{0} : \vdash \mathbf{0}} D_C} \multimap \frac{}{\vdash : \vdash !(A \& !(A) \multimap \mathbf{0}) \multimap \mathbf{0}} *$$

CALL-BY-VALUE encoding (70ms)

$$\frac{\frac{A, ! (A) \multimap \mathbf{0} : \cdot \vdash A}{A, ! (A) \multimap \mathbf{0} : \cdot \vdash ! (A)} ! \quad \frac{A, ! (A) \multimap \mathbf{0} : \mathbf{0} \vdash \mathbf{0}}{} \star}{\frac{A, ! (A) \multimap \mathbf{0} : ! (A) \multimap \mathbf{0} \vdash \mathbf{0}}{A, ! (A) \multimap \mathbf{0} : \cdot \vdash \mathbf{0}} D_C} \star$$

01-ENC encoding (70ms)

$$\frac{\frac{\frac{A, ! (A) \multimap ! (0) : \vdash A}{A, ! (A) \multimap ! (0) : \vdash ! (A)} ! \quad \frac{A, ! (A) \multimap ! (0) : ! (0) \vdash 0}{A, ! (A) \multimap ! (0) : ! (0) \vdash 0} \star}{\frac{A, ! (A) \multimap ! (0) : ! (A) \multimap ! (0) \vdash 0}{A, ! (A) \multimap ! (0) : \vdash 0} D_C} \frac{\vdash : \vdash ! (! (A) \& ! (! (A) \multimap ! (0))) \multimap 0}{\vdash : \vdash ! (! (! (A) \& ! (! (A) \multimap ! (0))) \multimap 0)} \star !$$

$$(45) \cdot \vdash \neg(A) \leftrightarrow (\neg A)$$

LJ (80ms)

$$\frac{\frac{A \rightarrow A \rightarrow \perp \rightarrow \perp \rightarrow \perp \rightarrow \perp}{A \rightarrow A \rightarrow \perp \rightarrow \perp \rightarrow \perp} \supset_2 \quad \frac{\frac{A \rightarrow A \rightarrow \perp \rightarrow \perp \rightarrow \perp \rightarrow \perp}{A \rightarrow A \rightarrow \perp \rightarrow \perp \rightarrow \perp} \supset_2 \quad \frac{A \rightarrow A \rightarrow \perp \rightarrow \perp \rightarrow \perp}{A \rightarrow A \rightarrow \perp \rightarrow \perp \rightarrow \perp} \supset_2}{\frac{A \rightarrow A \rightarrow \perp \rightarrow \perp \rightarrow \perp \rightarrow \perp}{A \rightarrow A \rightarrow \perp \rightarrow \perp \rightarrow \perp} \supset_2} \supset_2$$

MULTIPLICATIVE encoding (41ms)

Not provable

CALL-BY-NAME encoding (231ms)

[illegible]

CALL-BY-VALUE encoding (282ms)

Phylogenetic tree of the 12 studied species, showing relationships and bootstrap values. The tree is rooted at the bottom and branches upwards. Bootstrap values are indicated at the nodes. The species names are listed at the tips of the branches.

01-ENC encoding (300ms)

$$(46) \cdot \vdash \neg\neg\neg\neg A \rightarrow A$$

LJ (66ms)

$$\frac{\frac{\frac{A_1 \rightarrow \perp, A \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \rightarrow A \rightarrow \perp \rightarrow \perp \rightarrow \perp \rightarrow \perp \rightarrow \perp}{A_1, A \rightarrow \perp \rightarrow \perp \rightarrow \perp \rightarrow \perp}^*}{\frac{A_1 \rightarrow \perp, A \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \rightarrow A \rightarrow \perp \rightarrow \perp \rightarrow \perp}{A \rightarrow \perp, A \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \rightarrow A \rightarrow \perp \rightarrow \perp \rightarrow \perp}^*} \supset_k \frac{\frac{A \rightarrow \perp, A \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \rightarrow A \rightarrow \perp \rightarrow \perp \rightarrow \perp}{A \rightarrow \perp \rightarrow \perp \rightarrow A \rightarrow \perp \rightarrow \perp \rightarrow \perp \rightarrow \perp \rightarrow \perp}^*}{\frac{A \rightarrow \perp \rightarrow \perp \rightarrow A \rightarrow \perp \rightarrow \perp \rightarrow \perp \rightarrow \perp \rightarrow \perp}{\perp \rightarrow A \rightarrow \perp \rightarrow \perp \rightarrow A \rightarrow \perp \rightarrow \perp}^*} \supset_k \frac{\frac{A \rightarrow \perp \rightarrow \perp \rightarrow A \rightarrow \perp \rightarrow \perp \rightarrow \perp \rightarrow \perp \rightarrow \perp}{\perp \rightarrow A \rightarrow \perp \rightarrow \perp \rightarrow A \rightarrow \perp \rightarrow \perp}^*}{\perp \rightarrow A \rightarrow \perp \rightarrow \perp \rightarrow A \rightarrow \perp \rightarrow \perp}^* \supset_k$$

MULTIPLICATIVE encoding (41ms)

Not provable

CALL-BY-NAME encoding (198ms)

[illegible]

CALL-BY-VALUE encoding (210ms)

Figure 1 illustrates the hierarchical structure of the 2019-2020 COVID-19 outbreak in Wuhan, China. The diagram is organized into four levels, labeled 1 through 4. Level 1 is the root node, "2019-2020 COVID-19 outbreak in Wuhan, China". Level 2 branches into "Wuhan" and "Non-Wuhan". Level 3 further branches "Wuhan" into "Wuhan City" and "Wuhan City outside Hubei Province", and "Non-Wuhan" into "Non-Wuhan City" and "Non-Wuhan City outside Hubei Province". Level 4 shows specific locations and dates for each branch, such as "Wuhan City" leading to "Wuhan City" and "Wuhan City outside Hubei Province".

01-ENC encoding (217ms)

Figure 1: Schematic representation of the experimental design. The diagram shows a sequence of events for two groups: 'Control' and 'Experimental'. The 'Control' group starts with 'Baseline', followed by 'Training', 'Transfer', and 'Retention'. The 'Experimental' group starts with 'Baseline', followed by 'Training', 'Transfer', and 'Retention'. The 'Transfer' phase for the experimental group is marked with a red 'X' and the text 'No transfer of learning'. The 'Retention' phase for the experimental group is marked with a red 'X' and the text 'No retention of learning'.

$$(47) \cdot \vdash (A \wedge B \wedge \neg B) \leftrightarrow (B \wedge \neg B)$$

LJ (61ms)

$$\frac{\frac{A, B, B \rightarrow \perp \vdash B}{A, B, B \rightarrow \perp \vdash \perp}^* \quad \frac{A, B, \perp \vdash \perp}{\perp}^* \quad \frac{B, B \rightarrow \perp \vdash B \quad B, \perp \vdash A}{B, B \rightarrow \perp \vdash A}^* \quad \frac{B, B \rightarrow \perp \vdash B \quad B, \perp \vdash \perp}{B, B \rightarrow \perp \vdash \perp}^* \quad \frac{B, B \rightarrow \perp \vdash \perp}{\perp}^*}{\vdash A \wedge B \wedge B \rightarrow \perp \rightarrow B \wedge B \rightarrow \perp \wedge B \wedge B \rightarrow \perp \rightarrow A \wedge B \wedge B \rightarrow \perp}^*$$

MULTIPLICATIVE encoding (42ms)

Not provable

CALL-BY-NAME encoding (121ms)

Figure 1 illustrates the construction of a new sequence from an existing one. The top part shows a sequence of elements with various labels above and below them, including 'A', 'B', 'C', 'D', 'E', 'F', 'G', 'H', 'I', 'J', 'K', 'L', 'M', 'N', 'O', 'P', 'Q', 'R', 'S', 'T', 'U', 'V', 'W', 'X', 'Y', 'Z'. The bottom part shows the same sequence with some elements replaced by 'A' and 'B', and the entire sequence is enclosed in a box labeled 'Dn'.

CALL-BY-VALUE encoding (140ms)

[illegible]

01-ENC encoding (132ms)

[illegible]

$$(48) \cdot \vdash A \rightarrow B \rightarrow \neg A \wedge \neg B$$

LJ (49ms)

$$\frac{\frac{\frac{A, A \rightarrow B, B \rightarrow \perp \vdash A}{A, B, B \rightarrow \perp \vdash A}^* \quad \frac{A, B, B \rightarrow \perp \vdash B}{A, B, \perp \vdash \perp}^*}{A, B, B \rightarrow \perp \vdash \perp} \supset_L \quad \frac{A, B, B \rightarrow \perp \vdash \perp}{\vdash A \rightarrow B \rightarrow A \wedge B \rightarrow \perp \vdash \perp}^* \supset_L$$

MULTIPLICATIVE encoding (48ms)

$$\frac{\frac{\frac{\cdot : A \vdash A}{\cdot : A, A \multimap B, B \multimap \perp \vdash \perp} \multimap}{\cdot : B \vdash B \quad \cdot : \perp \vdash \perp} \multimap}{\cdot : \cdot \vdash A \multimap B \multimap A \otimes B \multimap \perp \multimap \perp} \star$$

CALL-BY-NAME encoding (143ms)

[illegible]

CALL-BY-VALUE encoding (167ms)

Figure 1 is a hierarchical tree diagram illustrating the classification of 1000 samples into 10 classes. The tree structure is as follows:

- Root node: 1000
 - Class 1 (100)
 - Sub-class 1.1 (50)
 - Sample 1.1.1 (10)
 - Sample 1.1.2 (10)
 - Sample 1.1.3 (10)
 - Sample 1.1.4 (10)
 - Sub-class 1.2 (50)
 - Sample 1.2.1 (10)
 - Sample 1.2.2 (10)
 - Sample 1.2.3 (10)
 - Sample 1.2.4 (10)
 - Class 2 (100)
 - Sub-class 2.1 (50)
 - Sample 2.1.1 (10)
 - Sample 2.1.2 (10)
 - Sample 2.1.3 (10)
 - Sample 2.1.4 (10)
 - Sub-class 2.2 (50)
 - Sample 2.2.1 (10)
 - Sample 2.2.2 (10)
 - Sample 2.2.3 (10)
 - Sample 2.2.4 (10)
 - Class 3 (100)
 - Sub-class 3.1 (50)
 - Sample 3.1.1 (10)
 - Sample 3.1.2 (10)
 - Sample 3.1.3 (10)
 - Sample 3.1.4 (10)
 - Sub-class 3.2 (50)
 - Sample 3.2.1 (10)
 - Sample 3.2.2 (10)
 - Sample 3.2.3 (10)
 - Sample 3.2.4 (10)
 - Class 4 (100)
 - Sub-class 4.1 (50)
 - Sample 4.1.1 (10)
 - Sample 4.1.2 (10)
 - Sample 4.1.3 (10)
 - Sample 4.1.4 (10)
 - Sub-class 4.2 (50)
 - Sample 4.2.1 (10)
 - Sample 4.2.2 (10)
 - Sample 4.2.3 (10)
 - Sample 4.2.4 (10)
 - Class 5 (100)
 - Sub-class 5.1 (50)
 - Sample 5.1.1 (10)
 - Sample 5.1.2 (10)
 - Sample 5.1.3 (10)
 - Sample 5.1.4 (10)
 - Sub-class 5.2 (50)
 - Sample 5.2.1 (10)
 - Sample 5.2.2 (10)
 - Sample 5.2.3 (10)
 - Sample 5.2.4 (10)
 - Class 6 (100)
 - Sub-class 6.1 (50)
 - Sample 6.1.1 (10)
 - Sample 6.1.2 (10)
 - Sample 6.1.3 (10)
 - Sample 6.1.4 (10)
 - Sub-class 6.2 (50)
 - Sample 6.2.1 (10)
 - Sample 6.2.2 (10)
 - Sample 6.2.3 (10)
 - Sample 6.2.4 (10)
 - Class 7 (100)
 - Sub-class 7.1 (50)
 - Sample 7.1.1 (10)
 - Sample 7.1.2 (10)
 - Sample 7.1.3 (10)
 - Sample 7.1.4 (10)
 - Sub-class 7.2 (50)
 - Sample 7.2.1 (10)
 - Sample 7.2.2 (10)
 - Sample 7.2.3 (10)
 - Sample 7.2.4 (10)
 - Class 8 (100)
 - Sub-class 8.1 (50)
 - Sample 8.1.1 (10)
 - Sample 8.1.2 (10)
 - Sample 8.1.3 (10)
 - Sample 8.1.4 (10)
 - Sub-class 8.2 (50)
 - Sample 8.2.1 (10)
 - Sample 8.2.2 (10)
 - Sample 8.2.3 (10)
 - Sample 8.2.4 (10)
 - Class 9 (100)
 - Sub-class 9.1 (50)
 - Sample 9.1.1 (10)
 - Sample 9.1.2 (10)
 - Sample 9.1.3 (10)
 - Sample 9.1.4 (10)
 - Sub-class 9.2 (50)
 - Sample 9.2.1 (10)
 - Sample 9.2.2 (10)
 - Sample 9.2.3 (10)
 - Sample 9.2.4 (10)
 - Class 10 (100)
 - Sub-class 10.1 (50)
 - Sample 10.1.1 (10)
 - Sample 10.1.2 (10)
 - Sample 10.1.3 (10)
 - Sample 10.1.4 (10)
 - Sub-class 10.2 (50)
 - Sample 10.2.1 (10)
 - Sample 10.2.2 (10)
 - Sample 10.2.3 (10)
 - Sample 10.2.4 (10)

01-ENC encoding (169ms)

[illegible]

(49) $\vdash (A \rightarrow \neg B) \leftrightarrow (\neg A \wedge B)$

LJ (58ms)

[illegible]

MULTIPLICATIVE encoding (88ms)

$$\begin{array}{c}
\frac{\frac{\frac{\vdots}{\vdots} B \vdash B \quad \vdots}{\vdots} \perp \vdash \perp}{\vdots} \quad \frac{\vdots}{\vdots} A \vdash A \quad \frac{\vdots}{\vdots} B \vdash B}{\vdots} \perp \vdash \perp \quad \frac{\vdots}{\vdots} A \vdash A \quad \frac{\vdots}{\vdots} B \vdash B}{\vdots} \perp \vdash \perp \\
\vdots \vdash A \vdash A \quad \vdots \vdash B \vdash B \vdash \perp \vdash \perp \quad \vdots \vdash A \vdash A \quad \vdots \vdash B \vdash A \vdash B \quad \vdots \vdash \perp \vdash \perp \\
\vdots \vdash A, B, A \vdash A \vdash B \vdash \perp \vdash \perp \quad \vdots \vdash A, B, A \vdash B \vdash B \vdash \perp \vdash \perp \\
\vdots \vdash A \vdash A \vdash B \vdash \perp \vdash \perp \vdash A \vdash B \vdash \perp \vdash \perp \quad \vdots \vdash A \vdash A \vdash B \vdash \perp \vdash \perp \vdash A \vdash B \vdash \perp \vdash \perp \\
\vdots \vdash A \vdash A \vdash B \vdash \perp \vdash \perp \vdash A \vdash B \vdash \perp \vdash \perp \quad \vdots \vdash A \vdash A \vdash B \vdash \perp \vdash \perp \vdash A \vdash B \vdash \perp \vdash \perp \\
\vdots \vdash A \vdash A \vdash B \vdash \perp \vdash \perp \vdash A \vdash B \vdash \perp \vdash \perp \quad \vdots \vdash A \vdash A \vdash B \vdash \perp \vdash \perp \vdash A \vdash B \vdash \perp \vdash \perp
\end{array}$$

CALL-BY-NAME encoding (118ms)

[illegible]

CALL-BY-VALUE encoding (281ms)

Figure 1: A complex diagram showing the relationships between various mathematical concepts and their corresponding symbols. The diagram is organized into several horizontal layers. The top layer includes symbols like \mathbb{R} , \mathbb{C} , \mathbb{H} , \mathbb{O} , \mathbb{S} , \mathbb{K} , \mathbb{L} , \mathbb{M} , \mathbb{N} , \mathbb{Z} , \mathbb{Q} , \mathbb{P} , \mathbb{A} , \mathbb{B} , \mathbb{C} , \mathbb{D} , \mathbb{E} , \mathbb{F} , \mathbb{G} , \mathbb{H} , \mathbb{I} , \mathbb{J} , \mathbb{K} , \mathbb{L} , \mathbb{M} , \mathbb{N} , \mathbb{O} , \mathbb{P} , \mathbb{Q} , \mathbb{R} , \mathbb{S} , \mathbb{T} , \mathbb{U} , \mathbb{V} , \mathbb{W} , \mathbb{X} , \mathbb{Y} , \mathbb{Z} . The middle layers contain various mathematical expressions and symbols, including \mathbb{R} , \mathbb{C} , \mathbb{H} , \mathbb{O} , \mathbb{S} , \mathbb{K} , \mathbb{L} , \mathbb{M} , \mathbb{N} , \mathbb{Z} , \mathbb{Q} , \mathbb{P} , \mathbb{A} , \mathbb{B} , \mathbb{C} , \mathbb{D} , \mathbb{E} , \mathbb{F} , \mathbb{G} , \mathbb{H} , \mathbb{I} , \mathbb{J} , \mathbb{K} , \mathbb{L} , \mathbb{M} , \mathbb{N} , \mathbb{O} , \mathbb{P} , \mathbb{Q} , \mathbb{R} , \mathbb{S} , \mathbb{T} , \mathbb{U} , \mathbb{V} , \mathbb{W} , \mathbb{X} , \mathbb{Y} , \mathbb{Z} . The bottom layer contains the text "Figure 1: A complex diagram showing the relationships between various mathematical concepts and their corresponding symbols."

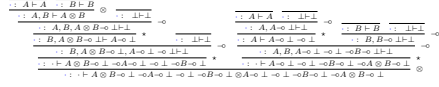
01-ENC encoding (318ms)

$$(50) \cdot \vdash (\neg A \wedge B) \leftrightarrow (\neg \neg A \rightarrow \neg B)$$

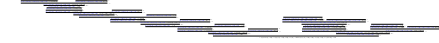
LJ (93ms)



MULTIPLICATIVE encoding (134ms)



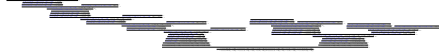
CALL-BY-NAME encoding (333ms)



CALL-BY-VALUE encoding (421ms)

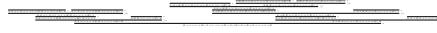


01-ENC encoding (470ms)

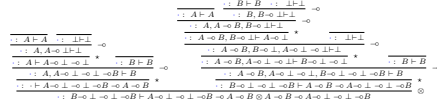


$$(51) \neg \neg B \rightarrow B \vdash (\neg \neg A \rightarrow B) \leftrightarrow (A \rightarrow B)$$

LJ (135ms)



MULTIPLICATIVE encoding (156ms)



CALL-BY-NAME encoding (278ms)



CALL-BY-VALUE encoding (414ms)

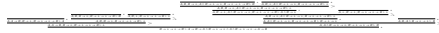


01-ENC encoding (449ms)

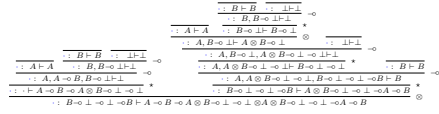


$$(52) \neg\neg B \rightarrow B \vdash (A \rightarrow B) \leftrightarrow (\neg A \wedge \neg B)$$

LJ (94ms)



MULTIPLICATIVE encoding (134ms)



CALL-BY-NAME encoding (296ms)



CALL-BY-VALUE encoding (457ms)

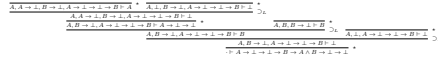


01-ENC encoding (489ms)

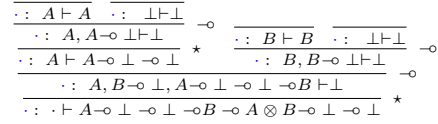


$$(53) \vdash \neg\neg A \rightarrow B \rightarrow \neg A \wedge \neg B$$

LJ (65ms)



MULTIPLICATIVE encoding (75ms)



CALL-BY-NAME encoding (178ms)



CALL-BY-VALUE encoding (204ms)



01-ENC encoding (215ms)



$$(54) \cdot \vdash A \wedge B \rightarrow \neg A \rightarrow \neg B$$

LJ (47ms)

$$\frac{\frac{\frac{A, B, A \rightarrow B \rightarrow \perp \perp A}{A, B, A \rightarrow B \rightarrow \perp \perp \perp} \star \quad \frac{\frac{A, B, B \rightarrow \perp \perp B}{A, B, \perp \perp \perp} \star \quad \frac{A, B, \perp \perp \perp}{A, B, B \rightarrow \perp \perp \perp} \star}{\frac{A, B, A \rightarrow B \rightarrow \perp \perp \perp}{\vdash A \wedge B \rightarrow A \rightarrow B \rightarrow \perp \rightarrow \perp} \star} \supset_L \supset_L$$

MULTIPLICATIVE encoding (48ms)

$$\frac{\frac{\frac{\vdots \vdash A \multimap A}{\vdots \vdash A \multimap A} \quad \frac{\vdots \vdash B \multimap B \quad \vdots \vdash \perp \multimap \perp}{\vdots \vdash B, B \multimap \perp \multimap \perp} \multimap}{\vdots \vdash A, B, A \multimap B \multimap \perp \multimap \perp} \multimap}{\vdots \vdash \cdot \vdash A \otimes B \multimap A \multimap B \multimap \perp \multimap \perp} \star$$

CALL-BY-NAME encoding (77ms)

[illegible]

CALL-BY-VALUE encoding (105ms)

[illegible]

01-ENC encoding (106ms)

[illegible]

$$(55) \cdot \vdash A \wedge \neg B \rightarrow \neg A \rightarrow B$$

LJ (48ms)

$$\frac{\frac{A, A \rightarrow B, B \rightarrow \perp \vdash A}{A, A \rightarrow B, B \rightarrow \perp \vdash \perp} \star \quad \frac{\frac{A, B, B \rightarrow \perp \vdash B}{A, B, B \rightarrow \perp \vdash \perp} \star \quad \frac{A, B, \perp \vdash \perp}{A, B, B \rightarrow \perp \vdash \perp} \star}{A, A \rightarrow B, B \rightarrow \perp \vdash \perp} \supset_L \quad \frac{A, A \rightarrow B, B \rightarrow \perp \vdash \perp}{\vdash A \wedge B \rightarrow \perp \rightarrow A \rightarrow B \rightarrow \perp} \star$$

MULTIPLICATIVE encoding (47ms)

$$\frac{\frac{\frac{\vdots \vdash A \vdash A}{\vdots \vdash A \vdash A} \quad \frac{\frac{\vdots \vdash B \vdash \bar{B}}{\vdots \vdash B, B \multimap \perp \vdash \perp} \quad \frac{\vdots \vdash \perp \vdash \perp}{\vdots \vdash \perp \vdash \perp}}{\vdots \vdash A, A \multimap B, B \multimap \perp \vdash \perp} \multimap \quad \frac{\vdots \vdash A \otimes B \multimap \perp \multimap A \multimap B \multimap \perp}{\vdots \vdash A \otimes B \multimap \perp \multimap A \multimap B \multimap \perp} \star$$

CALL-BY-NAME encoding (144ms)

[illegible]

CALL-BY-VALUE encoding (168ms)

[illegible]

01-ENC encoding (168ms)

[illegible]

$$(56) \cdot \vdash \neg\neg A \wedge B \rightarrow \neg A \rightarrow \neg B$$

LJ (62ms)

$$\frac{\frac{\frac{A, B, A \rightarrow B \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \vdash A}{A, B, A \rightarrow B \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \vdash A}^* \quad \frac{\frac{A, B, B \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \vdash B}{A, B, \perp, A \rightarrow \perp \rightarrow \perp \vdash \perp}^*}{A, B, A \rightarrow B \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \vdash \perp}^* \quad \frac{\frac{A, B, A \rightarrow B \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \vdash \perp}{B, A \rightarrow B \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \vdash \perp}^* \quad \frac{B, \perp, A \rightarrow B \rightarrow \perp \vdash \perp}{B, A \rightarrow B \rightarrow \perp, A \rightarrow \perp \rightarrow \perp \vdash \perp}^*}{\vdash A \rightarrow \perp \rightarrow \perp \wedge B \rightarrow A \rightarrow B \rightarrow \perp \rightarrow \perp}^* \quad \supset_k \quad \supset_k$$

MULTIPLICATIVE encoding (77ms)

$$\begin{array}{c}
\frac{\frac{\frac{\vdots : B \vdash \overline{B}}{\vdots : \perp \vdash \perp}}{\vdots : A \vdash A} \quad \frac{\vdots : \perp \vdash \perp}{\vdots : B, B \multimap \perp \vdash \perp} \multimap}{\vdots : A, B, A \multimap B \multimap \perp \vdash \perp} \multimap \\
\frac{\vdots : B, A \multimap B \multimap \perp \vdash A \multimap \perp \quad \vdots : \perp \vdash \perp}{\vdots : B, A \multimap B \multimap \perp, A \multimap \perp \multimap \perp \vdash \perp} \multimap \\
\frac{\vdots : B, A \multimap B \multimap \perp, A \multimap \perp \multimap \perp \vdash \perp}{\vdots : \vdash A \multimap \perp \multimap \perp \otimes B \multimap A \multimap B \multimap \perp \multimap \perp} \star
\end{array}$$

CALL-BY-NAME encoding (182ms)

[illegible]

CALL-BY-VALUE encoding (212ms)

[illegible]

01-ENC encoding (210ms)

$$(57) \cdot \vdash (\neg\neg A \wedge \neg B) \leftrightarrow (\neg A \rightarrow B)$$

LJ (114ms)

Figure 1. Schematic representation of the experimental design. The figure shows a sequence of events: a stimulus (a word) is presented, followed by a response (a word), and then a feedback (a word). The response is either correct or incorrect. The feedback is either positive (correct) or negative (incorrect). The sequence of events is: Stimulus (word) → Response (word) → Feedback (word). The response is either correct or incorrect. The feedback is either positive (correct) or negative (incorrect). The sequence of events is: Stimulus (word) → Response (word) → Feedback (word).

MULTIPLICATIVE encoding (58ms)

Not provable

CALL-BY-NAME encoding (326ms)

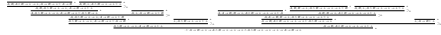
CALL-BY-VALUE encoding (421ms)

Figure 1 is a Gantt chart illustrating the timeline of the study. The chart is divided into two main sections: 'Preparation' and 'Implementation'. The 'Preparation' section includes tasks such as 'Literature review', 'Design of the study', 'Recruitment of participants', 'Development of the intervention', and 'Pilot study'. The 'Implementation' section includes tasks such as 'Data collection', 'Data analysis', 'Reporting', and 'Dissemination'. The timeline spans from 2010 to 2014.

01-ENC encoding (454ms)

$$(58) \cdot \vdash (\neg A \rightarrow B) \leftrightarrow (\neg\neg A \wedge \neg B)$$

LJ (109ms)



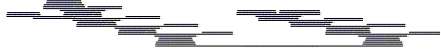
MULTIPLICATIVE encoding (41ms)

Not provable

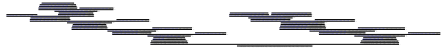
CALL-BY-NAME encoding (597ms)



CALL-BY-VALUE encoding (481ms)

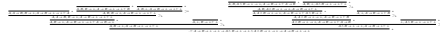


01-ENC encoding (501ms)



$$(59) \cdot \vdash (\neg\neg A \rightarrow B) \leftrightarrow (\neg A \wedge \neg B)$$

LJ (112ms)



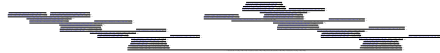
MULTIPLICATIVE encoding (57ms)

Not provable

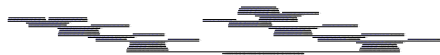
CALL-BY-NAME encoding (594ms)



CALL-BY-VALUE encoding (468ms)



01-ENC encoding (498ms)



$$(60) \cdot \vdash (\neg A \wedge \neg B) \leftrightarrow (A \rightarrow \neg \neg B)$$

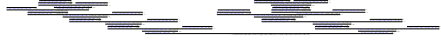
LJ (92ms)



MULTIPLICATIVE encoding (124ms)



CALL-BY-NAME encoding (317ms)



CALL-BY-VALUE encoding (436ms)



01-ENC encoding (468ms)

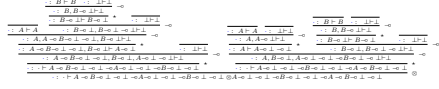


$$(61) \cdot \vdash (A \rightarrow \neg \neg B) \leftrightarrow (\neg \neg A \rightarrow \neg \neg B)$$

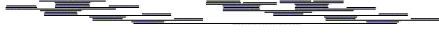
LJ (169ms)



MULTIPLICATIVE encoding (163ms)



CALL-BY-NAME encoding (412ms)



CALL-BY-VALUE encoding (479ms)



01-ENC encoding (552ms)



3 Alternative Translations

$$(10) \cdot : A \multimap \mathbf{0} \vdash A \multimap B$$

encoding (55ms)

$$\frac{\frac{\overline{\cdot : A \vdash A} \quad \overline{\cdot : \mathbf{0} \vdash B}}{\cdot : A, A \multimap \mathbf{0} \vdash B} \multimap}{\cdot : A \multimap \mathbf{0} \vdash A \multimap B} \star$$

$$(11) \cdot : A \vdash A \multimap \mathbf{0} \multimap B$$

encoding (41ms)

$$\frac{\frac{\overline{\cdot : A \vdash A} \quad \overline{\cdot : \mathbf{0} \vdash B}}{\cdot : A, A \multimap \mathbf{0} \vdash B} \multimap}{\cdot : A \vdash A \multimap \mathbf{0} \multimap B} \star$$

$$(12) \cdot : B \vdash !(A) \multimap B$$

encoding (22ms)

$$\frac{\overline{A : B \vdash B}}{\cdot : B \vdash !(A) \multimap B} \star$$

$$(16) \cdot : A \multimap B \otimes !(B \multimap A) \vdash A \multimap B$$

encoding (35ms)

$$\frac{\frac{\overline{B \multimap A : A \vdash A} \quad \overline{B \multimap A : B \vdash B}}{\overline{B \multimap A : A, A \multimap B \vdash B}} \multimap}{\cdot : A \multimap B \otimes !(B \multimap A) \vdash A \multimap B} \star$$

$$(17) \cdot : !(A \multimap B) \otimes B \multimap A \vdash B \multimap A$$

encoding (38ms)

$$\frac{\frac{\overline{A \multimap B : B \vdash B} \quad \overline{A \multimap B : A \vdash A}}{\overline{A \multimap B : B, B \multimap A \vdash A}} \multimap}{\cdot : !(A \multimap B) \otimes B \multimap A \vdash B \multimap A} \star$$

$$(18) \cdot : A, A \multimap B \otimes B \multimap A \vdash B \otimes B \multimap A$$

encoding (56ms)

$$(19) \quad \therefore B, A \multimap B \otimes B \multimap A \vdash A \otimes A \multimap B$$

(26a) $\therefore \cdot \vdash A \& B \multimap \perp \multimap \perp \multimap A \multimap \perp \multimap \perp \& B \multimap \perp \multimap \perp$

$$(26b) \quad \cdot : \cdot \vdash A \multimap \perp \multimap \perp \otimes B \multimap \perp \multimap \perp \multimap A \otimes B \multimap \perp \multimap \perp$$

$$(27a) \quad : \vdash !(A \multimap B) \otimes !(B \multimap A) \multimap \perp \multimap \perp \multimap A \multimap B \multimap \perp \multimap \perp \& B \multimap A \multimap \perp \multimap \perp$$
$$(27b) \vdash A \multimap B \multimap \perp \multimap \perp \otimes B \multimap A \multimap \perp \multimap \perp \multimap A \multimap B \otimes B \multimap A \multimap \perp \multimap \perp$$

(35) $\vdash \vdash ! (A) \otimes ! (A) \multimap ! (A) \otimes ! (A) \multimap ! (A) \otimes ! (A)$

$$(36) \cdot : A \vdash A \multimap B \multimap B \otimes B \multimap !(A) \multimap B$$

$$(37) \quad \vdash : B \vdash !(A \multimap B) \multimap B \otimes B \multimap !(A) \multimap B$$

(38) $\vdash : A \multimap \perp \vdash ! (A \multimap B) \multimap A \multimap \perp \otimes A \multimap \mathbf{0} \multimap A \multimap B$

$$(39) \quad \therefore B \multimap \mathbf{0} \vdash A \multimap B \multimap A \multimap \mathbf{0} \otimes A \multimap \mathbf{0} \multimap A \multimap B$$
$$(40) \cdot : B \vdash A \otimes !(B) \multimap A \otimes A \multimap A \otimes B$$
$$(41) \cdot : B \multimap 0 \vdash !(A) \otimes B \multimap B \otimes B \multimap A \otimes B$$
$$(45) \quad \cdot : \cdot \vdash !(A \multimap A \multimap \perp) \otimes !(A) \multimap \perp \multimap !(A) \multimap \perp$$

$$\begin{array}{c}
\frac{\frac{A, \perp, A \multimap \perp, A \multimap A \multimap \perp : \perp \vdash \perp}{A, \perp, A \multimap \perp, A \multimap A \multimap \perp : \perp \vdash \perp} \quad D_C}{A \multimap A \multimap \perp : \vdash ! (A) \multimap \perp}^* \\
\frac{\frac{A, \perp, A \multimap \perp, A \multimap A \multimap \perp : \perp \vdash \perp}{A \multimap A \multimap \perp : \vdash ! (A) \multimap \perp} \quad D_C}{A \multimap A \multimap \perp : ! (A) \multimap \perp \vdash ! (A) \multimap \perp}^* \\
\frac{A \multimap A \multimap \perp : ! (A) \multimap \perp \vdash ! (A) \multimap \perp}{\vdash \vdash ! (A \multimap A \multimap \perp) \otimes ! (A) \multimap \perp \vdash ! (A) \multimap \perp}^* \quad \neg
\end{array}$$

encoding (138ms)

[illegible]

encoding (83ms)

$$\begin{array}{c}
\frac{\vdash: B \vdash B \quad \vdash: A, 0 \vdash B \otimes B \multimap 0}{\vdash: A, B, B \multimap 0 \vdash B \otimes B \multimap 0} \star \\
\frac{\vdash: A, B, B \multimap 0 \vdash B \otimes B \multimap 0}{\vdash: \vdash A \otimes B \otimes B \multimap 0 \multimap 0 \vdash B \otimes B \multimap 0} \multimap \\
\vdash: \vdash A \otimes B \otimes B \multimap 0 \multimap 0 \vdash B \otimes B \multimap 0 \quad \vdash: \vdash B \otimes B \multimap 0 \multimap 0 \vdash A \otimes B \otimes B \multimap 0 \quad \star \\
\vdash: A \otimes B \otimes B \multimap 0 \multimap 0 \vdash B \otimes B \multimap 0 \quad \vdash: B \otimes B \multimap 0 \multimap 0 \vdash A \otimes B \otimes B \multimap 0 \quad \star \\
\vdash: A \otimes B \otimes B \multimap 0 \multimap 0 \vdash B \otimes B \multimap 0 \quad \vdash: B \otimes B \multimap 0 \multimap 0 \vdash A \otimes B \otimes B \multimap 0 \quad \star
\end{array}$$

encoding (71ms)

$$\frac{\frac{\frac{\vdots : A \vdash A}{\vdots : A \vdash A} \quad \frac{\frac{\vdots : B \vdash B}{\vdots : B, B \multimap \perp \vdash \perp} \quad \frac{\vdots : \perp \vdash \perp}{\vdots : B, B \multimap \perp \vdash \perp} \quad \multimap}{\vdots : A, A \multimap B, B \multimap \perp \vdash \perp} \quad \multimap}{\vdots : A \multimap B, B \multimap \perp \vdash A \multimap \perp} \quad \star \quad \frac{\vdots : \perp \vdash \perp}{\vdots : A \multimap B, B \multimap \perp, A \multimap \perp \multimap \perp \vdash \perp} \quad \multimap}{\vdots : A \multimap \perp, B \multimap \perp \otimes B \multimap \perp \vdash A \multimap B \multimap \perp} \quad \star$$

encoding (92ms)

[illegible]

encoding (162ms)

[illegible]

encoding (69ms)

$$\begin{array}{c}
\frac{\vdots \vdash A \vdash A}{\vdots \vdash A \vdash A} \quad \frac{\vdots \vdash B \vdash B \quad \vdots \vdash \perp \vdash \perp}{\vdots \vdash B, B \multimap \perp \vdash \perp} \quad \multimap \\
\frac{\vdots \vdash A, A \multimap B, B \multimap \perp \vdash \perp}{\vdots \vdash A \multimap B \vdash A \otimes B \multimap \perp \multimap \perp} \quad \multimap \\
\frac{\vdots \vdash A \multimap B \vdash A \otimes B \multimap \perp \multimap \perp \quad \vdots \vdash \perp \vdash \perp}{\vdots \vdash A \multimap B, A \otimes B \multimap \perp \multimap \perp \multimap \perp \multimap \perp \vdash \perp} \quad \star \\
\frac{\vdots \vdash A \otimes B \multimap \perp \multimap \perp \multimap \perp \multimap \perp \multimap A \multimap B \multimap \perp}{\vdots \vdash A \otimes B \multimap \perp \multimap \perp \multimap \perp \multimap A \multimap B \multimap \perp} \quad \star
\end{array}$$

encoding (71ms)

$$\frac{\frac{\frac{\vdots : A \vdash A}{\vdots : A \vdash A} \quad \frac{\vdots : B \vdash B \quad \vdots : \perp \vdash \perp}{\vdots : B, B \multimap \perp \vdash \perp} \multimap}{\vdots : A, A \multimap B, B \multimap \perp \vdash \perp} \multimap \quad \frac{\vdots : A, B \multimap \perp \vdash A \multimap B \multimap \perp \quad \vdots : \perp \vdash \perp}{\vdots : A, B \multimap \perp, A \multimap B \multimap \perp \multimap \perp \vdash \perp} \star}{\vdots : \bullet A \multimap B \multimap \perp \multimap \perp \multimap \perp \multimap A \otimes B \multimap \perp \multimap \perp \vdash \perp} \star$$

encoding (163ms)

[illegible]

encoding (71ms)

$$\frac{\frac{\frac{\vdots : A \vdash A}{\vdots : A \vdash A} \quad \frac{\frac{\vdots : B \vdash B}{\vdots : B, B \multimap \perp \vdash \perp} \quad \frac{\vdots : \perp \vdash \perp}{\vdots : B, B \multimap \perp \vdash \perp} \quad \multimap}{\vdots : A, A \multimap B, B \multimap \perp \vdash \perp} \quad \multimap}{\vdots : A \multimap B, B \multimap \perp \vdash A \multimap \perp} \quad \star \quad \frac{\vdots : \perp \vdash \perp}{\vdots : A \multimap B, B \multimap \perp, A \multimap \perp \multimap \perp \vdash \perp} \quad \multimap}{\vdots : A \multimap \perp, B \multimap \perp \otimes B \multimap \perp \vdash A \multimap B \multimap \perp} \quad \star$$

encoding (92ms)

[illegible]

encoding (162ms)

[illegible]

encoding (69ms)

$$\begin{array}{c}
\frac{\vdots \vdash A \vdash A}{\vdots \vdash A, A \multimap B, B \multimap \perp \vdash \perp} \quad \frac{\vdots \vdash B \vdash B \quad \vdots \vdash \perp \vdash \perp}{\vdots \vdash B, B \multimap \perp \vdash \perp} \quad \multimap \\
\frac{\vdots \vdash A, A \multimap B, B \multimap \perp \vdash \perp}{\vdots \vdash A \multimap B \vdash A \otimes B \multimap \perp \multimap \perp} \quad \frac{\vdots \vdash \perp \vdash \perp}{\vdots \vdash A \multimap B, A \otimes B \multimap \perp \multimap \perp \multimap \perp \multimap \perp \vdash \perp} \quad \star \\
\frac{\vdots \vdash A \multimap B \vdash A \otimes B \multimap \perp \multimap \perp \multimap \perp \multimap \perp \vdash \perp \quad \vdots \vdash A \multimap B \multimap \perp \vdash \perp}{\vdots \vdash A \otimes B \multimap \perp \multimap \perp \multimap \perp \multimap \perp \multimap \perp \vdash \perp} \quad \star
\end{array}$$