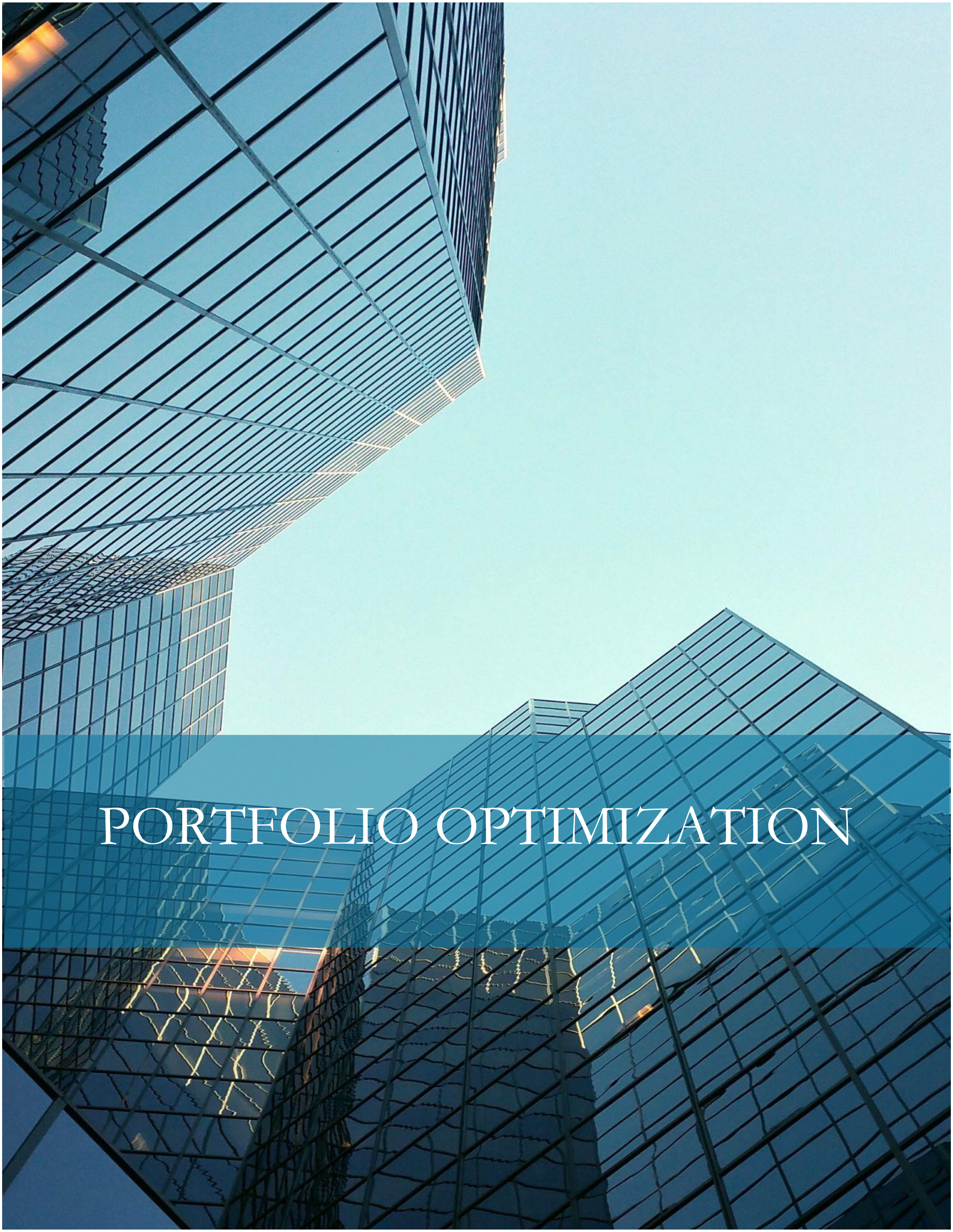

PORTFOLIO OPTIMIZATION

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Executive Summary

This report represents the results of a portfolio optimization process developed for the launch of a new fund, the "Alpha Fund", which aims to invest in a broad selection of equities and risk-free asset in six sectors: Real estate, Pharmaceuticals, Natural resources, Technology, Utilities, and Communication. The primary objective was to identify the optimal asset allocation across these sectors while maximizing returns and minimizing risk. To ensure disciplined risk management, we applied two constraints that prevented us from short selling or investing more than 25% of the capital in a single sector. These measures, combined with the sectoral diversification, will allow us to mitigate idiosyncratic risk.

We utilized Excel-based models to construct this optimal asset allocation for three different time horizons (three, five, and seven years). Using ETFs as proxies for each sector and their expected returns derived from the Capital Asset Pricing Model (CAPM), our portfolio optimization model determined one optimal portfolio for each time horizon. The optimal portfolio maximizes the expected return of our investment while minimizing the risk compared with other portfolios. Based on individual risk preferences, an investment manager can combine this optimal portfolio with a risk-free asset.

Additionally, we assessed the cost and benefit of the constraints; therefore, their impact on our results of portfolio efficiency. We also acknowledged the possible measurement errors and assumptions that helped us undertake our computations. In particular, we highlight the uncertainty increased by the reliance on historical data when making decisions about the future. To address this, we incorporated a modified

variance-covariance matrix and used the CAPM to obtain the ETFs' returns, as these techniques allowed us to reduce our dependency on historical data.

1. Introduction

1.1 Background and Motivation

Portfolio optimization is a cornerstone of modern financial modeling, especially when designing strategies suitable for diverse investor profiles. ETF-based sector investing has gained popularity in recent years for offering a combination of broad diversification and targeted sector exposure.

This report introduces the 'Alpha Fund', a sector-based ETF portfolio designed with a strong emphasis on risk management. The fund imposes two key constraints: (1) short selling is prohibited, and (2) no more than 25% of capital may be allocated to a single sector. These rules aim to prevent overconcentration and speculative risk.

As noted by the Statista Research Department (June 17, 2024), short selling can lead to high potential losses, making it a risky strategy. The Alpha Fund avoids this by focusing on long-only, diversified sector positions.

Sector selection reflects varying sensitivities to economic conditions. For instance, Utilities (XLU) are counter-cyclical and provide stability during downturns, while sectors like Real Estate (VNQ) and Energy (XLE) are pro-cyclical and perform better in expansions. The COVID-19 pandemic illustrated how sector responses to shocks can diverge - Technology (XLK), Communication (XLC), and Biotechnology (XBI) outperformed, while others lagged.

By combining economic reasoning with historical data, the Alpha Fund seeks to deliver well-diversified and resilient returns. Its structure reflects a disciplined strategy that balances opportunity with risk control.

1.2 Objectives

This report aims to construct efficient portfolios by applying the Markowitz optimization model (1952), using six sector-based ETFs across different investment horizons. Specifically, we evaluate 3-year, 5-year, and 7-year time frames to address varying investor preferences.

The 3-year horizon captures short-to-medium term dynamics and is suited for investors seeking faster results. It also provides a more accurate reflection of current market conditions, minimizing the influence of outdated trends.

The 5-year horizon offers a mid-term outlook, balancing long-term structural trends with short-term market movements. Finally, the 7-year horizon represents a medium-to-long-term strategy that is more likely to encompass a complete economic cycle or policy regime and is therefore less impacted by temporary shocks and volatility.
The optimization is performed by combining risky assets with a risk-free asset, resulting in efficient portfolios that lie along the Capital Allocation Line (CAL). The CAL is a straight line that starts at the risk-free rate and is tangent to the minimum variance frontier. The point of tangency, known as the tangency portfolio, maximizes the Sharpe Ratio, or reward-to-volatility ratio, which quantifies the additional expected return gained per unit of risk.

1.3 ETFs Overview

The six ETFs selected for this study represent diverse sectors of the U.S. economy, allowing for balanced exposure to both cyclical and defensive industries. XLK offers concentrated exposure to leading technology companies, a sector characterized by strong growth but also higher volatility. XLC captures firms in media, telecom, and digital communications, reflecting the rapid evolution of the information economy. VNQ provides access to the real estate sector through REITs, which offer high dividend yields but are sensitive to interest rate movements. XBI focuses on biotechnology, a high-risk, high-reward space driven by innovation and FDA approvals. XLE includes energy companies, historically cyclical but essential, especially during inflationary periods. Lastly, XLU offers defensive exposure to regulated utilities, typically less volatile and attractive in downturns due to their stable cash flows.

Ticker	Sector	Full Name	Description / Focus Area
XLK	Technology	Technology Select Sector SPDR Fund	U.S. tech giants in hardware, software, and IT.
XLC	Communication	Communication Services SPDR Fund	Media, telecom, and entertainment companies.
VNQ	Real Estate	Vanguard Real Estate ETF	REITs focused on residential, retail, and office.
XBI	Biotech / Healthcare	SPDR S&P Biotech ETF	Equal-weighted U.S. biotech and pharma firms.
XLE	Energy / Resources	Energy Select Sector SPDR Fund	Oil, gas, and energy equipment & services.
XLU	Utilities	Utilities Select Sector SPDR Fund	Regulated electric, water, and gas utilities.

Figure 1: ETF Overview | Source: www.finance.yahoo.com

Table 2 shows the correlation matrix of the chosen ETFs: low values (near 0) emphasize a low positive correlation between the returns of two ETFs, while high values (near 1) show that two ETFs behave very similarly.

	XLK	XLC	VNQ	XBI	XLE	XLU	SPX
XLK	100%						
XLC	85%	100%					
VNQ	70%	73%	100%				
XBI	61%	60%	58%	100%			
XLE	44%	53%	51%	36%	100%		
XLU	42%	48%	68%	22%	30%	100%	
SPX	60%	56%	54%	55%	44%	38%	100%

Figure 2: Correlation matrix | Source: LSEG Data & Analytics

1.3 Scope and Limitations

While historical average monthly returns are computed, they are not used directly as the primary input for expected returns due to their limited predictive power and sensitivity to recent market anomalies. Instead, we adopt the Capital Asset Pricing Model (CAPM) to generate more forward-looking and theoretically grounded return estimates.

In the Appendices, we include charts illustrating the volatility trends and changes in market prices for all the indices considered in the analysis.

To mitigate the instability of the sample variance-covariance matrix, we apply the shrinkage method as described by Simon Benninga (2014), improving the robustness of our risk estimates and improving the reliability of optimization outcomes.

Our analysis computes the Capital Allocation Line (CAL) to visualize optimal portfolio combinations involving a risk-free asset and the tangency portfolio. We compare portfolio outcomes under various constraints, notably a maximum allocation of 25% per ETF and a ban on short selling.

2. Data Collection

This section outlines the data sources and initial steps used to build the portfolio optimization models. We considered six ETFs, each representing a different sector:

- XLK (Technology)
- XLC (Communication Services)
- VNQ (Real Estate)
- XBI (Pharmaceuticals/Biotech)
- XLE (Natural Resources/Energy)
- XLU (Utilities)

We collected monthly adjusted closing prices for each ETF using the Stock History function in Microsoft Excel, with data sourced from LSEG Data & Analytics and Yahoo Finance. To serve as the market benchmark, we selected the S&P 500 Index, aligning it to the same investment horizons used in the portfolio analysis. The S&P 500 was chosen due to its widespread use as a transparent and representative indicator of U.S. equity market performance.

The risk-free rate was derived from the 10-year U.S. Treasury Bond yield as of April 14, 2025, and subsequently converted into monthly returns. This rate was selected as a credible and widely accepted long-term baseline for comparing returns on risky assets. The market return and risk-free rate inputs were retrieved from reliable financial databases - FRED (Federal Reserve Economic Data) and Bloomberg, respectively. To reflect varying market dynamics and investor preferences, we considered three-time frames: 3 years (01. 04. 2022 – 01. 04. 2025), capturing the post-COVID recovery; 5 years (01. 04. 2020 – 01. 04. 2025), including the COVID shock and its aftermath; and 7 years (01/06/2018 – 01/04/2025), offering a broader balance of pre- and post-COVID data.

Risk-free rate	4.430 %
Monthly risk-free rate	0.362 %
S&P 500 – Market risk (monthly): Seven years data	2.079 %
S&P 500 – Market risk (monthly): Five years data	2.706%
S&P 500 – Market risk (monthly): Three years data	3.117%

Figure 3: Risk-free rate and Market risk premium | Source : Bloomberg, FRED, LSEG Data & Analytics

From the collected monthly prices, we computed monthly returns to better suit medium to long-term investment strategies. While historical average monthly returns were calculated across all horizons, the analysis primarily relies on CAPM-based return estimates due to the potential unreliability of noisy historical returns.

Historical Average Monthly Returns

	XLK	XLC	VNQ	XBI	XLE	XLU
3 Years	0.84 %	0.96 %	-0.46 %	-0.15 %	0.40 %	0.23 %
5 Years	1.70 %	1.37 %	0.49 %	0.29 %	2.07 %	0.68 %
7 Years	1.49 %	0.92 %	0.23 %	0.06 %	0.55 %	0.61 %

Figure 4: Historical Average Monthly Returns | Source: LSEG Data & Analytics

The presence of noise in historical data leads to negative or distorted return estimates, making them less reliable for analytical purposes.

3. Methodology

3.1 Computations in Excel

Firstly, let's define:

- **X** ~ the $T_x N$ matrix
  - T = 82 (7 years of monthly returns)
  - N = 6 assets
- **E(R)** ~ the column matrix $N \times 1$ of the expected returns of the ETFs (XLK, XLC, VNQ, XBI, XLE, XLU in order)
- **S** ~ the sample variance-covariance matrix
- $\mathbf{r_f}$ ~ the risk-free rate
- $\boldsymbol{\sigma_{ii}}$ ~ the standard deviation of the i-th asset
- $\boldsymbol{\sigma_{i,j}}$ ~ the covariance of assets i and j

3. 1. 1 Capital Allocation Line

We calculated the capital allocation line (CAL) for three different time horizons: three, five, and seven years. In each time horizon, we computed two CALs depending on what expected returns of the ETFs we decided to consider-average or CAPM returns. Indeed, using the average monthly returns derived from our data sample presents a lot of drawbacks, such as too significant reliance on the past to predict the future, which empirically can prove to be wrong. This method can also entail negative expected returns, which are non-logical since the market always adapts so that the expected returns are positive, otherwise, nobody would invest. We will then only consider the results obtained using CAPM returns.

3. 1. 2 Capital Asset Pricing Model (CAPM)

Therefore, we also decided to undertake our study by considering expected returns given by the Capital Asset Pricing Model (CAPM), which allows us to provide an estimation of the expected returns of our ETFs based on their historical risk and a market portfolio, the S&P 500. We also, considering each time horizon and each expected return method, computed the CAL with or without the constraints of long positions only and the cap of 25% in the weights of each asset in the efficient portfolios. We partly base our computations on lecture notes from the course Introductory Applied Finance of Evarist Stoja at NHH and Financial Economics of Daniel Schmidt at HEC Paris.

3. 1. 3 Variance-covariance matrix

For our computation, we also needed a variance-covariance matrix of the considered assets. The sample variance-covariance matrix is obtained as follows:

$$S = \frac{\left(X^t - E(R)\right) \cdot (X - E(R)^t)}{T}$$

where $S_{i,j} = \sigma_{i,j}$.

MMULT and TRANSPOSE functions on Excel have been used.

However, S is often too unrealistic and relies too much on historical data to predict future volatility. The disadvantages of the sample variance-covariance matrix are explained in many academic papers (Ledoit and Wolf, 2003) or the book Financial Modeling, Simon Benninga, 4th Edition. In particular, errors can be very significant and eventually lead to unrealistic portfolios with extra-long positions.

To deal with this problem, we resort to the "shrinkage" method as explained by Simon Benninga (2014), which consists of "shrinking" the errors in the matrix and improving how realistic our estimates of the future are. This method assumes that the variance-covariance matrix, which we note V, is a convex combination of the sample one and a target matrix computed as:

$$V = \gamma \cdot A + (1 - \gamma) \cdot S$$

where A is the target matrix and γ is the shrinkage constant.

For our simple computations, we proceeded as Simon Benninga did in *Financial Modeling*, by taking:

$$A_{i,j} = S_{i,j} \text{ when } i = j$$

$$A_{i,j} = 0 \text{ otherwise}$$

$$\gamma = 0.3$$

3. 1. 4 Correlation matrix

The correlation matrix was found by the *Correlation* Analysis Tool in *Data Analysis*. We took as *Input Range* the monthly expected returns of each ETF over the 7-year period.

3. 1. 5 Risk-free rate

The risk-free rate obtained from Bloomberg is an annualized one. We obtain the monthly risk-free rate by using the following formula:

$$(1 + r_{f,\ year}) = (1 + r_{r,\ month})^{12} \Leftrightarrow r_{r,\ month} = (1 + r_{f,\ year})^{\frac{1}{12}} - 1$$

3. 1. 6 Expected returns

3. 1. 6. 1 Average returns

The average returns are different depending on the time horizon. Our latest data are from 01. 04. 2025, so the average monthly return of the asset i over x years is the average of all the monthly returns from 01. 04. 2025 back x years to 01. 04. using the AVERAGE function of Excel.

3. 1. 6. 2 CAPM returns

The CAPM gives an estimate of the expected return of an asset i given its risk, which leads to the formula:

$$E(r_i) = r_f + \beta_i \cdot (r_m - r_f)$$

$$\beta_i = \frac{\sigma_{i,m}}{\sigma_{m,m}}$$

where r_m is the expected return of the market, obtained by computing the average return of the S&P 500 over the time horizon, and β_i is the Beta of the asset i, which measures the systematic risk of the asset, that is, the risk that can't be diversified.

To compute $\beta_,$ we use the COVAR and VAR functions of Excel on our data sample.

The found expected returns are then put into the matrix E(R).

3. 1. 7 Finding the CAL

Having computed the expected returns, the risk-free rate, and the variance-covariance matrix, we define an arbitrary portfolio P, where P is a column matrix $N \times 1$ of the weights of each asset. The variance Var(P) and expected return $E(r_P)$ of portfolio P are obtained as follows:

$$Var(P) = P^t \cdot V \cdot P$$

$$E(r_P) = P^t \cdot E(R)$$

The CAL that we want to find is a straight line since we have to consider N risky assets and a risk-free asset. All efficient portfolios lie on the line given by:

$$E(r_P) = r_f + \gamma_T \cdot \sigma_P$$

Where γ_T is the Sharpe ratio equal to:

$$\frac{E(r_T) - r_f}{\sigma_T} = \max\left\{\frac{\left(E(r_P) - r_f\right)}{\sigma_P}\right\} \quad (1)$$

where P is a portfolio that we change in order to obtain the maximum.

Here, T is the tangency portfolio.

We solve (1) by using Excel's Solver. We put in a cell the formula $\frac{E(r_P) - r_f}{\sigma_P}$, which we maximize with the Solver by changing the weights of each ETF in P. Solver allows us to apply constraint to our maximization, such as the no short-sales constraint, the 25% cap on each weight and the fact that the 6 weights must sum to 1. We finally obtain the Tangency portfolio and the Sharpe Ratio, which allows us to plot the CAL for each situation.

3.2 Computations in Python

While Excel served as the primary tool for portfolio optimization, Python was introduced to extend our analysis; particularly, in assessing the costs of constraints and input robustness (Benninga, 2014). Unlike Excel's Solver, Python offers programmatic control over constraint configurations, return and risk estimators, and simultaneous generation of efficient frontiers (Kolm, 2014). This allowed deeper analysis into how short-selling bans, weight-caps and estimation errors in both expected returns and covariance matrices impact portfolio performance.

3. 2. 1 Measurement Error in Expected Returns (Historical Average VS CAPM)

This section examines how the choice of expected return estimators (Historical Averages VS CAPM) impacts the outcomes of our portfolio optimizations. We conducted our analysis using returns from the 7-year horizon, offering the broadest and most representative time frame across market cycles. This window captures both expansionary and contractionary periods (e.g. COVID-19 shock and recovery), making it suitable for evaluating the long-run return stability and the robustness of optimization outcomes under estimation risk. Four constraint scenarios were analyzed:

- No Short-Selling, 25% Sector Cap
- Short-Selling Allowed, 25% Sector Cap
- No Short-Selling, No Sector Cap
- Short-Selling Allowed, No Sector Cap

Super long and super short positions were excluded to reflect realistic investment constraints, such as leverage limits, regulatory restrictions and standard risk controls (Kolm et al., 2014). This better aligns with the long-only, diversified nature of most ETF-based portfolios. To isolate the effect of return estimation methods, we hold all other inputs constant; the Simon Benninga shrunk covariance matrix is used in both computations. We generated efficient frontiers, summary statistic tables as well as portfolio visualizations, across the 4 scenarios simultaneously.

The summary tables and efficient frontier charts reveal clear disparities in performance. From the minimum variance frontiers plots (Figure 5), portfolios constructed using historical expected returns achieve a higher maximum Sharpe ratio of 21.52% compared to 13.18% under CAPM-based returns. This difference is also reflected in the summary table (Table 5), where portfolios based on historical returns exhibit notably higher in-sample returns and Sharpe ratios.

However, the grouped portfolio weight chart (Figure 5) reveals that these gains are achieved by heavily concentrating on outperforming sectors such as Technology (XLK) and Communications (XLC) while introducing short positions in underperforming sectors like Real Estate (VNQ) and Pharmaceuticals (XBI). This aggressive behaviour reflects overfitting to recent trends; a known risk when using raw historical returns (Fabozzi, 2021).

In contrast, the CAPM-based portfolios maintain more balanced and diversified allocations, assigning moderate weights to all 6 sectors (Figure 5). While the efficient frontiers appear flatter, its allocations reflect systematic risk and offer a more stable and theoretically sound investment basis (Jagannathan & Ma, 2003).

Ultimately, while historical return models may inflate in-sample performance, they carry higher risks of fragile allocations that may not generalize well out-of-sample. CAPM offers a conservative but robust alternative, especially when constraints are applied. These results highlight the trade-off between optimistic but noisy estimations and stable, theoretically grounded ones.

	Return (μ)	Volatility (σ)	Sharpe Ratio
No Short, 25% Cap	0.87%	4.58%	11.14%
Short Allowed, 25% Cap	0.89%	4.65%	11.28%
No Short, No Cap	1.49%	6.22%	18.21%
Short Allowed, No Cap	1.49%	5.26%	21.52%

Summary table for Historical expected returns

	Return (μ)	Volatility (σ)	Sharpe Ratio
No Short, 25% Cap	0.97%	4.62%	13.17%
Short Allowed, 25% Cap	0.97%	4.62%	13.17%
No Short, No Cap	0.98%	4.66%	13.18%
Short Allowed, No Cap	0.98%	4.66%	13.18%

Summary table for CAPM expected returns

Figure 5: Performance Summary for different expected returns | Source: LSEG Data & Analytics

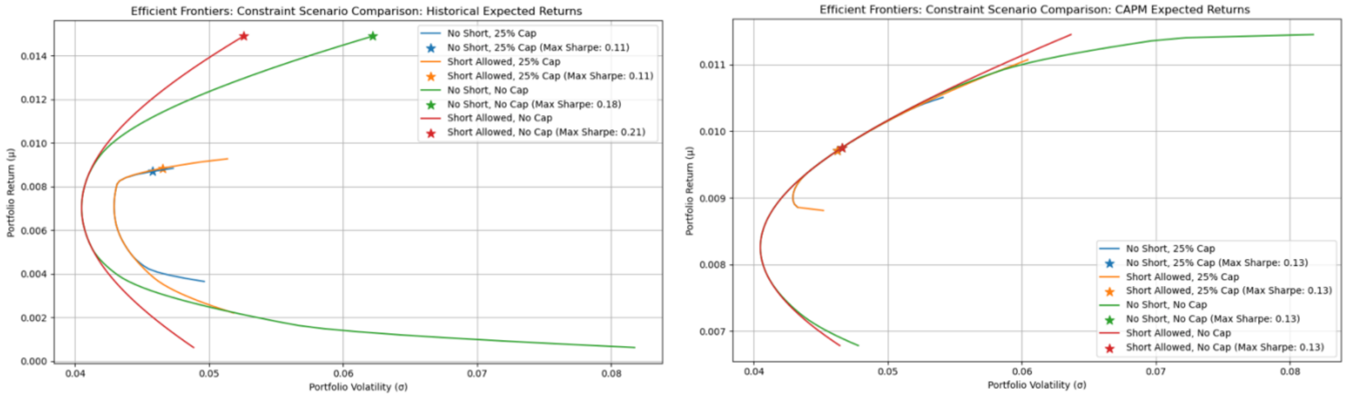

Figure 6: Efficient Frontier using different expected returns | Source: [LSEG Data & Analytics]

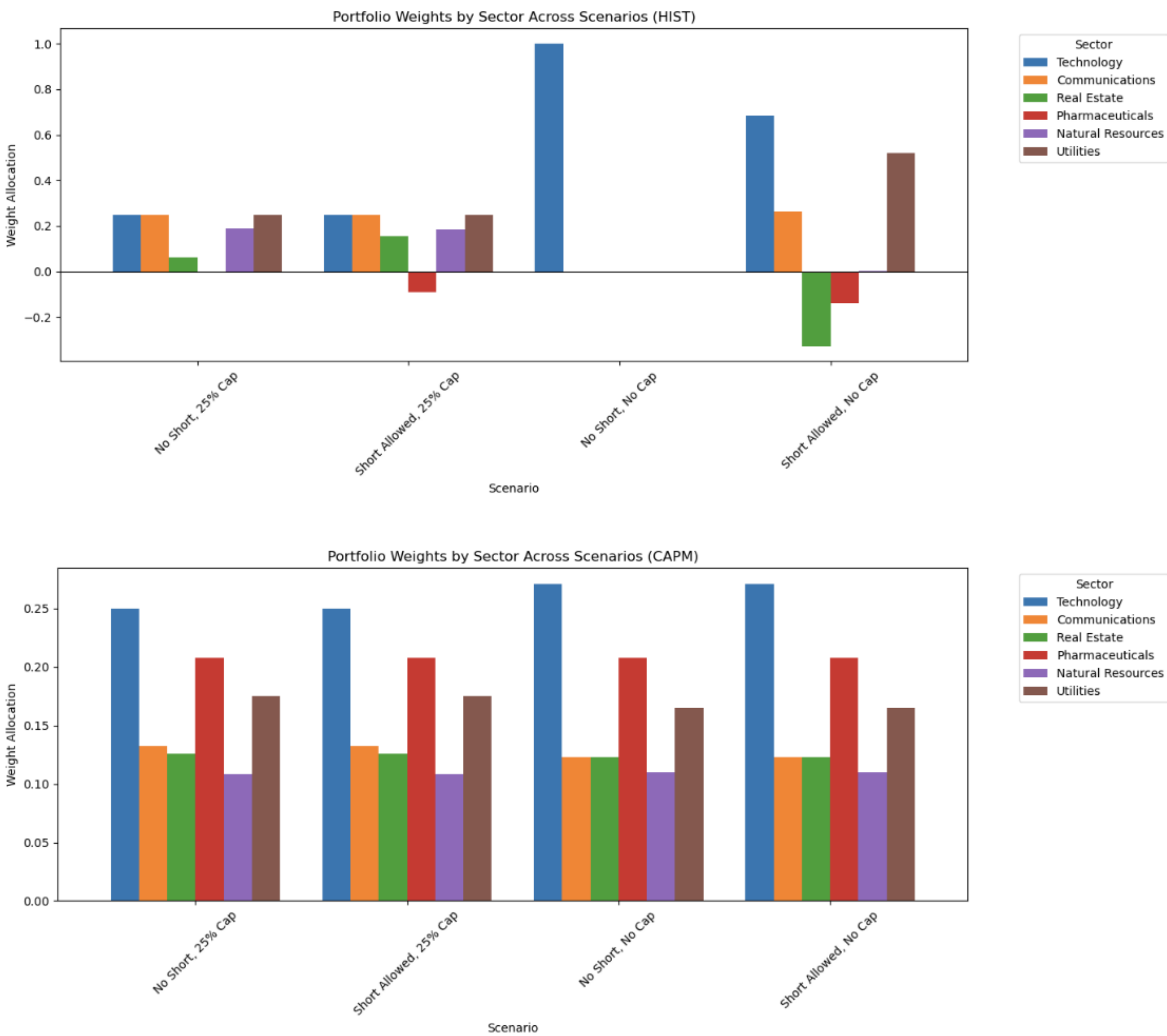

Figure 7: Portfolio Weights by Sector across Constraint Scenarios | Source: [LSEG Data & Analytics]

3. 2. 2 Measurement Error in Covariance Matrix (Sample Covariance VS Diagonal Shrinkage)

To assess the effect of estimation error in risk measurement, we compared the sample variance-covariance matrix with Simon Benninga's shrinkage approach, keeping CAPM-based expected returns constant across both approaches. The shrinkage method combines the noisy sample matrix with a structured diagonal target matrix, reducing the influence of unstable off-diagonal elements (Benninga, 2014). Consistent with Section 3.2.1, we use 7-year historical return data to compute betas for the CAPM, while comparing portfolio performance under the sample covariance matrix with the Simon Benninga shrunk version. We set the shrinkage intensity $\lambda = 0.3$ in line with the example provided by Benninga (2014).

From the minimum variance frontiers plots and Sharpe ratios (Figure 6), we observe that portfolios using the shrunk covariance matrix consistently outperform those built with the raw sample matrix. The maximum Sharpe ratio improves from 11.91% to 13.18%, reflecting reduced estimation error and improved robustness in risk assessment. Additionally, the shrunk frontier appears smoother and more convex-like, avoiding the erratic curvature seen when using sample matrix; this outcome aligns with the existing literature on shrinkage improving portfolio stability (Ledoit & Wolf, 2004).

This improvement is also reflected in our portfolio composition. Referring to Figure 6, portfolios under the raw sample matrix allocate disproportionately to Technology and Utilities while underweighting sectors like Real Estate or Communications. In contrast, optimal portfolios computed using the Benninga-shrunk matrix produced more evenly-spread allocations across all sectors; consistent with the goal of minimizing sampling error and avoiding extreme bets on unstable correlations (Ledoit & Wolf, 2004).

Overall, shrinkage proves to be effective at stabilizing the covariance estimates by reducing the influence of noisy or extreme historical values. This leads to more consistent portfolio performance, especially when data is limited or if correlations between assets are volatile. Although the improvements in Sharpe ratios are modest, they are statistically meaningful and translate to more reliable portfolio decisions (Jagannathan & Ma, 2003).

	Return (μ)	Volatility (σ)	Sharpe Ratio
No Short, 25% Cap	0.97%	5.12%	11.80%
Short Allowed, 25% Cap	0.97%	5.12%	11.80%
No Short, No Cap	0.98%	5.24%	11.88%
Short Allowed, No Cap	0.99%	5.27%	11.91%

Summary table for Sample Covariance Matrix + CAPM expected returns

	Return (μ)	Volatility (σ)	Sharpe Ratio
No Short, 25% Cap	0.97%	4.62%	13.17%
Short Allowed, 25% Cap	0.97%	4.62%	13.17%
No Short, No Cap	0.98%	4.66%	13.18%
Short Allowed, No Cap	0.98%	4.66%	13.18%

Summary table for Shrunk Covariance Matrix + CAPM expected returns

Figure 8: Performance Summary for different expected returns | Source: LSEG Data & Analytics

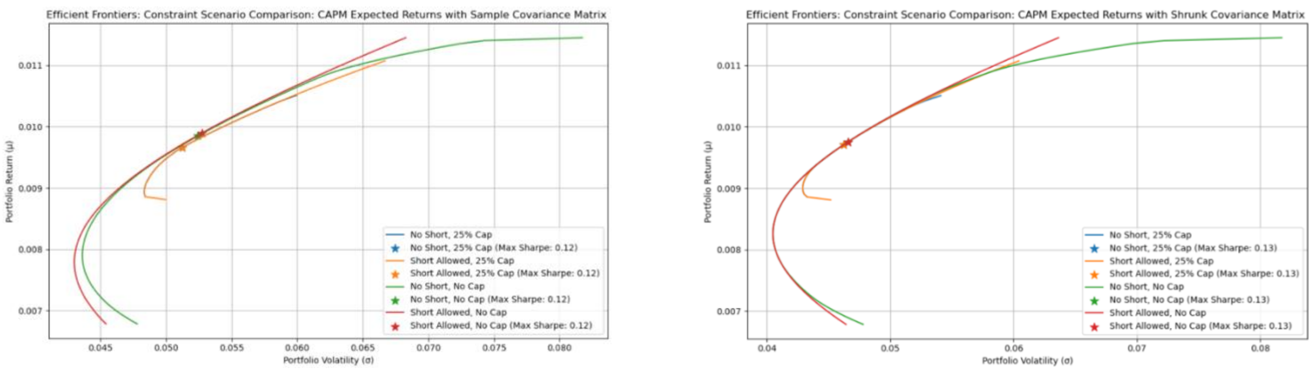

Figure 9: Efficient Frontier using different expected returns | Source: [LSEG Data & Analytics]

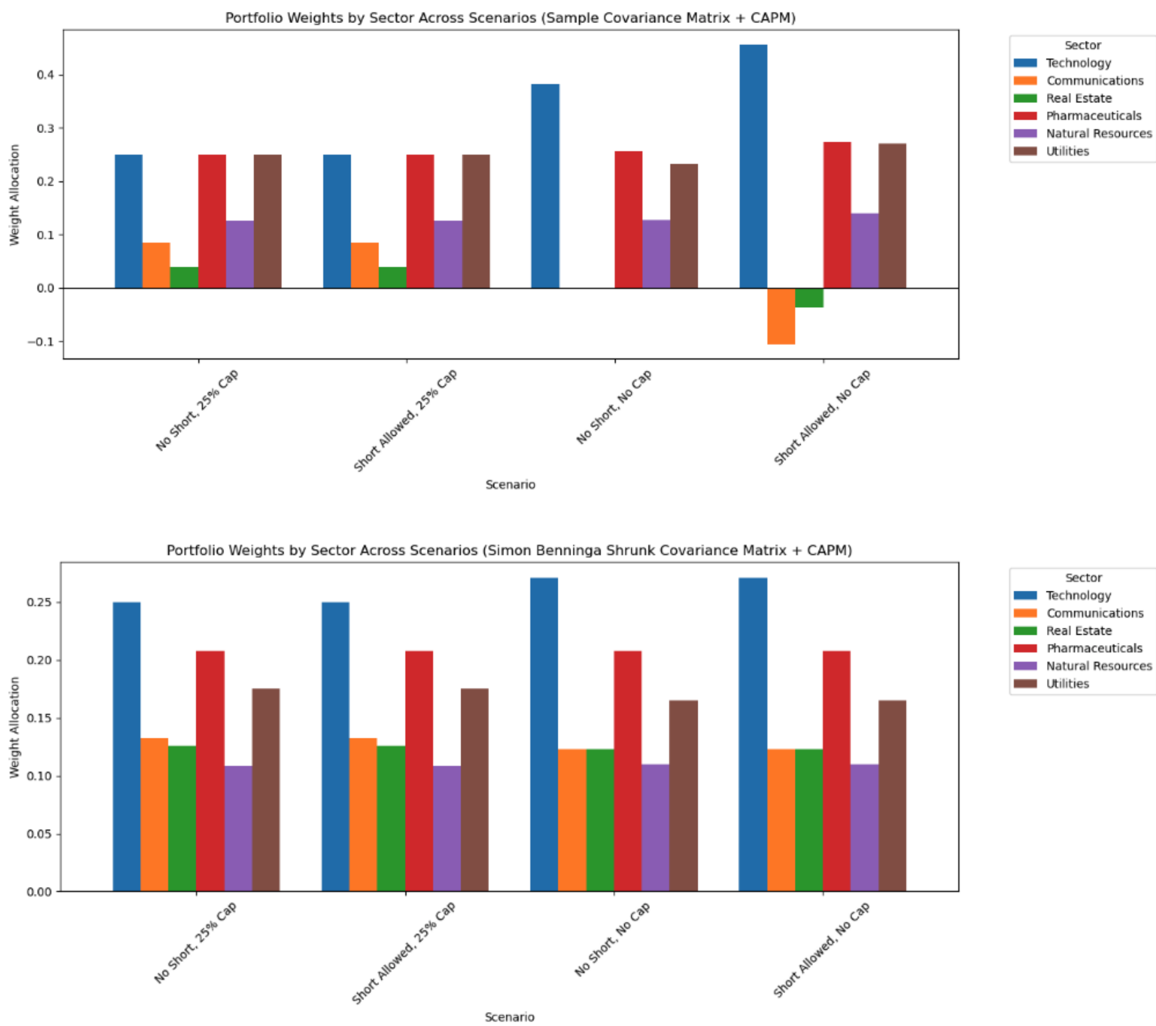

Figure 10: Portfolio Weights by Sector across Constraint Scenarios | Source: [LSEG Data & Analytics]

3. 2. 3 Constraint Scenario Comparison via Efficient Frontiers

From the minimum variance frontiers plots (Figure 6), the 4 curves appear almost indistinguishable, with all scenarios converging to a maximum Sharpe ratio of approximately 13.18%. The performance metrics summary table confirms this negligible difference; portfolios with full flexibility only outperform the most constrained case by 0.01%. This marginal gap suggests that constraints do not materially degrade performance when using robust inputs (Jagannathan & Ma, 2003).

Across all constraint scenarios (Figure 6), the portfolio weights remain well-diversified, with each sector receiving moderate allocations. Technology consistently receives the highest weight, reflecting its strong risk-adjusted returns, but no sector dominates the portfolio excessively. The constraints have little impact on allocation dispersion, reinforcing our earlier observation that with robust inputs, constraint costs are minimal. This consistency in diversification supports the reliability of the shrunk covariance model in guiding stable investment decisions (Benninga, 2014) and aligns with real-world investment principles that limit concentration risk (Fabozzi, 2021).

Ultimately, when using shrunk covariance matrices with CAPM returns, constraints have minimal effect on performance. Sharpe ratios remain nearly identical, frontiers overlap, and allocations are well-distributed. This suggest that constraints do not significantly harm efficiency when estimation error is well-managed (Jagannathan & Ma, 2003).

4. Discussion of Results

4. 1 Sharpe Ratios and Constraints

Expected returns	Sharpe ratio (3 years)	Sharpe ratio (5 years)	Sharpe ratio (7 years)
Constraints	24.46%	21.64%	16.61%
No constraints	24.46%	21.65%	16.62%

Figure 11: Sharpe Ratios | Source: LSEG Data & Analytics

In our analysis, we employed the Markowitz Mean-Variance Optimization (MVO) framework to maximize the Sharpe Ratio of our portfolios. The optimization adjusted asset weights to achieve the steepest possible slope on the Capital Allocation Line, reflecting the highest achievable return per unit of risk, within the bounds of our defined constraints.

As mentioned in the methodology part, our primary goal is to:

$$\max_{x_n} \theta = \frac{\bar{R}_p - r_f}{\sigma_p}, \qquad n = 1, 2, \ldots, 6$$

The Sharpe Ratio measures the slope of the Capital Allocation Line (CAL), indicating the trade-off between risk and return in a portfolio that includes both risky assets and a risk-free asset. A rational investor would prefer higher Sharpe ratios since they reward risk with higher gains in expected returns. Our objective was to create a well-diversified portfolio investing in six strategic sector ETFs, while adding constraints such as a maximum cap of 25% per ETF and prohibiting short selling.

To align our portfolios with realistic investment strategies, we implemented constraints such as no short-selling and sector exposure caps. Interestingly, the cost of these constraints, evaluated by comparing Sharpe Ratios of constrained versus unconstrained portfolios, was negligible across all time horizons, considering the CAPM.

The minimal difference in performance indicates that the constraints did not influence the portfolios' efficiency. On the contrary, they improved portfolio structure by preventing overexposure to individual assets and ensuring diversification. These findings highlight the role of constraints as practical tools for risk management rather than performance limitations.

The following charts display the Capital Allocation Lines (CALs) corresponding to each investment time horizon. Expected returns are expressed on a monthly basis.

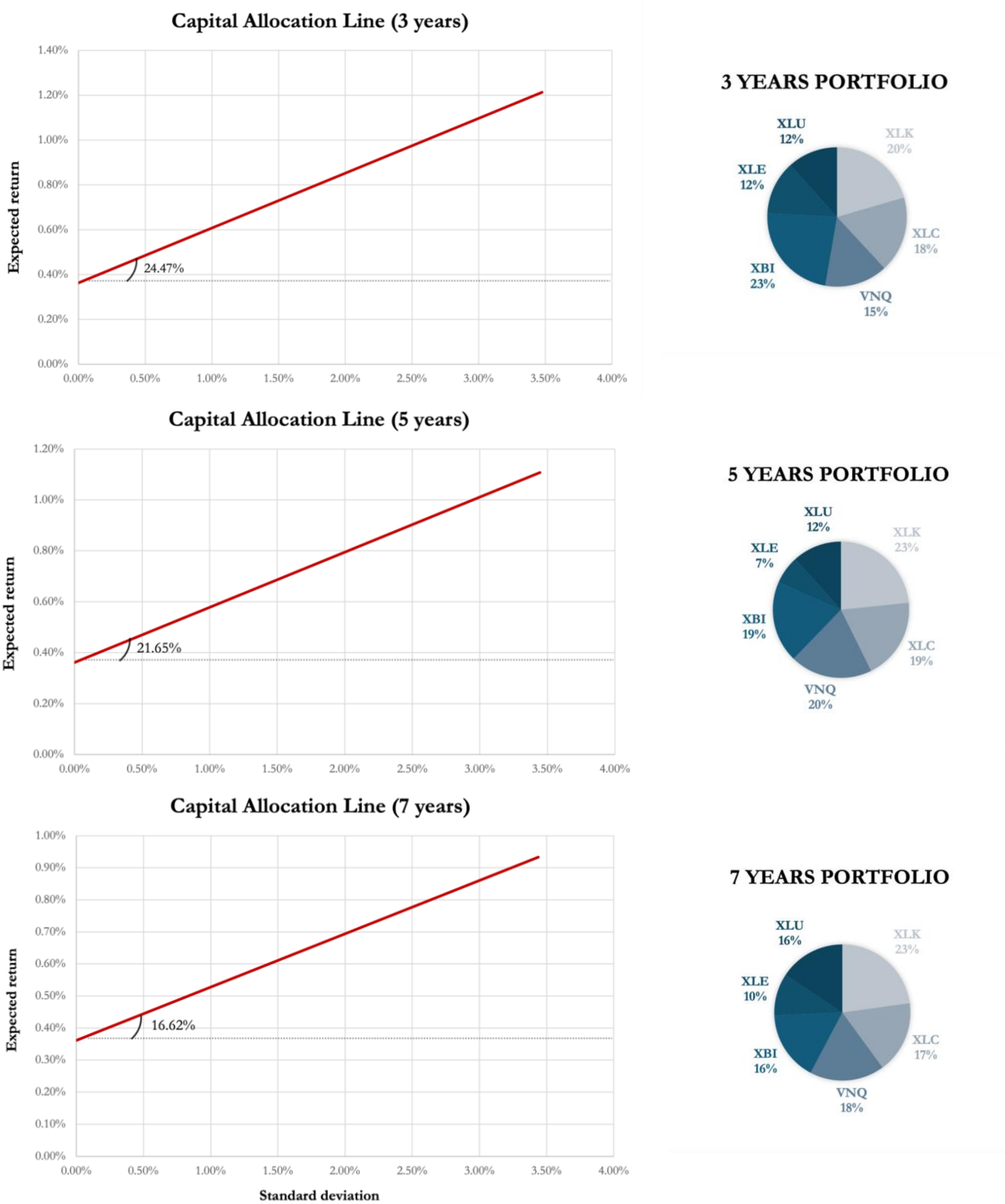

Figure 12: CALs and Tangency Portfolio | Source: [LSEG Data & Analytics]

4. 2 Time Horizon and Asset Allocation Analysis

A key insight from our study relates to the impact of different investment horizons on both portfolio performance and sector allocation. Portfolios were constructed using historical return data over 3-year, 5-year, and 7-year periods. The results revealed that the 3-year horizon delivered the highest Sharpe Ratio at 24.5%, followed by 21.6% for the 5-year horizon and 16.6% for the 7-year horizon.

This outperformance over shorter horizons is largely attributable to the post-COVID economic recovery, a period marked by strong market momentum, aggressive monetary stimulus, and positive investor sentiment. Additionally, geopolitical developments - such as the war in Ukraine - may have further fueled risk-adjusted returns during this timeframe. In contrast, the 7-year window spans the onset of the COVID-19 crisis, introducing substantial volatility and depressed returns during 2020. This weighs down the average performance, resulting in a lower Sharpe Ratio despite broader diversification.

These differences in return also align with shifts in sector allocation across time horizons. The 3-year portfolio leaned heavily into growth-oriented sectors, notably Biotechnology (XBI, 23%), Technology (XLK, 20%), and Communication Services (XLC, 18%). The 5-year portfolio showed more balance, with similar weightings for XLK (23%), XBI (19%), and XLC (19%), while Real Estate (VNQ) increased to 20%.

By the 7-year horizon, the portfolio adopted a noticeably more diversified and risk-mitigated composition. While XLK (23%) and XLC (17%) remained key holdings, exposure to VNQ (18%), XBI (16%), XLU (16%), and Energy (XLE, 10%) increased (compared to the 5 years portfolio).

Overall, this progression demonstrates how shorter time horizons favor concentrated, high-growth strategies, whereas longer horizons emphasize balance, resilience, and sectoral diversification in response to increased uncertainty and volatility over time.

5. Assumptions and Limitations

It is important to acknowledge that our analysis is based on several underlying assumptions, each of which influences the outcomes of the optimization process. These include the use and value of the risk-free rate, the selection of ETFs as proxies for industry performance, the chosen time horizons, and the decision to use the S&P 500 Index as a proxy for the market portfolio. While we have carefully justified each of these choices in alignment with the goals of this study, the reliability of the results is inevitably shaped by these foundational assumptions.

A key limitation of our model lies in its reliance on historical data to make forward-looking investment decisions. Although we applied shrinkage estimation - as proposed by S. Benninga (2014) - to enhance the robustness of the variance-covariance matrix and reduce the sensitivity to sampling noise, the model remains heavily dependent on past market behavior. This raises concerns about its capacity to fully reflect future dynamics, especially in periods of structural change or market disruption.

As discussed earlier, this challenge becomes particularly evident when estimating expected returns. Historical averages, especially over short time frames, are often highly volatile and potentially misleading. For example, during our 3-year sample window, we observed negative average returns for VNQ and XBI, which resulted in unrealistic optimization outcomes, particularly in unconstrained scenarios. To address this issue, we supplemented our analysis with CAPM-based expected returns, offering a more theoretically grounded alternative.

However, the Capital Asset Pricing Model also carries its own set of limitations. While it provides a practical mechanism to estimate expected returns as a function of systematic risk, it relies on strong theoretical assumptions - including market efficiency, the absence of transaction costs and taxes, rational investor behavior, and homogeneous expectations.

Looking forward, the model could be further improved by incorporating additional real-world factors, such as transaction costs, or by adopting alternative frameworks for return estimation. One promising extension is the Black-Litterman model, which not only addresses the shortcomings of traditional mean-variance optimization but also allows investors to incorporate their own market views directly into the return generation process (Benninga, 2014).

6. Conclusion and Recommendations

This report presented a structured portfolio optimization strategy for the Alpha Fund, designed using Markowitz's Mean-Variance Optimization (MVO) model and tailored with realistic investment constraints. Our approach focused on sector diversification through six ETFs, offering a balance between growth potential and downside protection.

Three time horizons were considered, 3, 5, and 7 years, to reflect the preferences of different investor profiles. The 3-year horizon delivered the highest Sharpe Ratios, reflecting the strong post-pandemic recovery. However, these short-term returns were more volatile and concentrated in high-growth sectors like technology and biotech. In contrast, the 7-year horizon provided broader diversification and more stability, capturing a full economic cycle including the COVID-19 shock. The use of investment constraints, specifically a 25% cap per ETF and a prohibition of short selling, proved to be effective. Our results demonstrated that these constraints had minimal impact on performance while improving diversification and reducing portfolio risk.

We also compared different methods for estimating expected returns and risk. Historical averages, while straightforward, often led to unstable and unrealistic weights, especially over short horizons. CAPM-based returns, which we used, on the other hand, offered a more consistent and theoretically sound framework. Similarly, replacing the sample variance-covariance matrix with a shrinkage-adjusted matrix enhanced portfolio robustness, smoothing out unrealistic estimates and improving stability.

For investors, choosing the appropriate time horizon is essential and should reflect both return expectations and risk tolerance. The Capital Allocation Line (CAL) helps visualize this trade-off by showing how different combinations of the tangency portfolio and the risk-free asset yield different levels of return for a given level of risk. A risk-averse investor may choose to stay closer to the risk-free rate, while a more aggressive investor can take on more risk along the CAL to pursue higher returns.

While no optimization model can eliminate uncertainty or predict the future with complete accuracy, this report demonstrates that a disciplined, theory-based approach can offer meaningful insights. The Alpha Fund provides an example of how quantitative techniques, when grounded in economic logic and implemented with care, can support effective and well-diversified investment strategies.

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Appendices

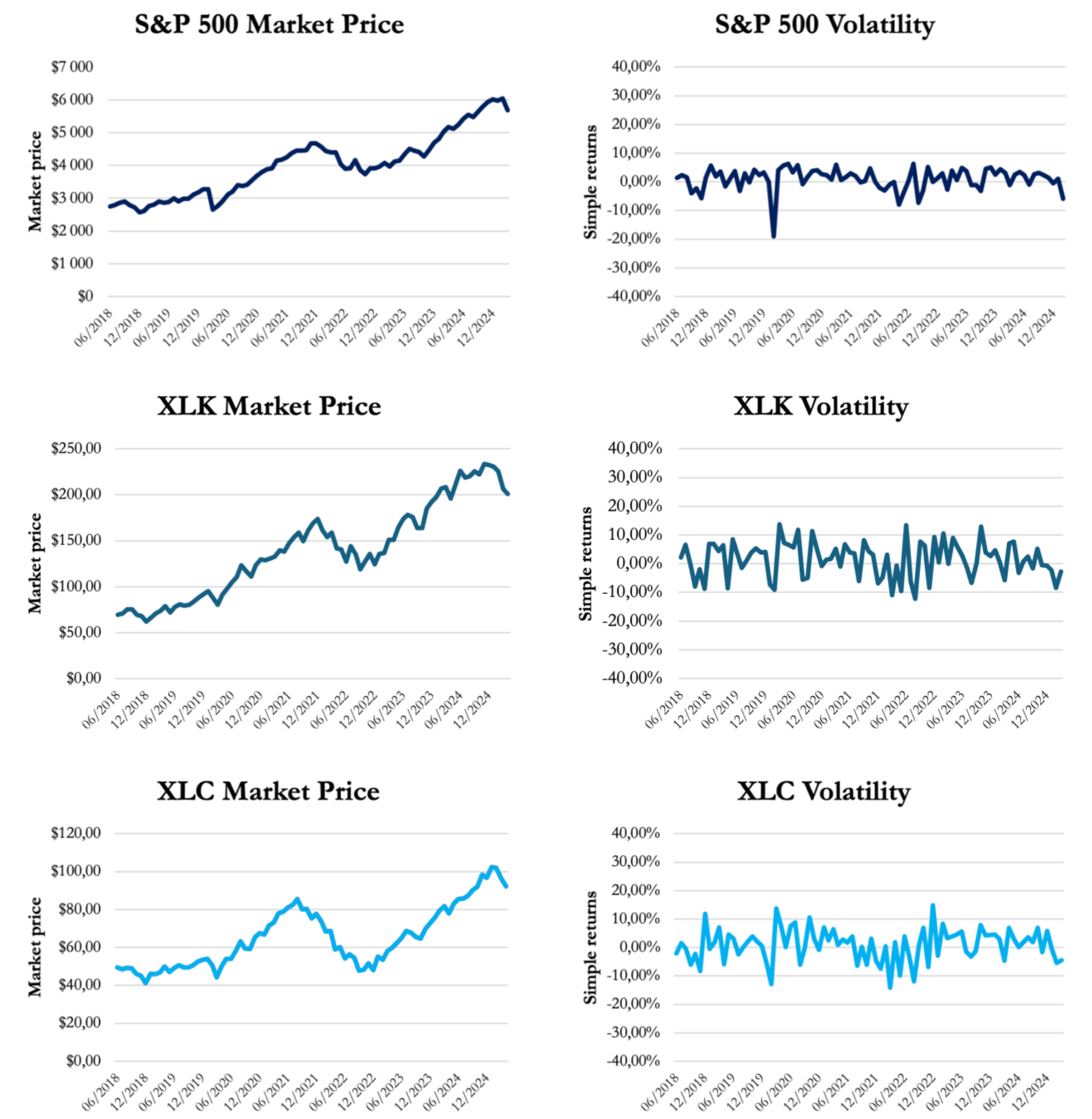

Figure 13: Market Prices and Volatility (Part 1) | Source: LSEG Data & Analytics

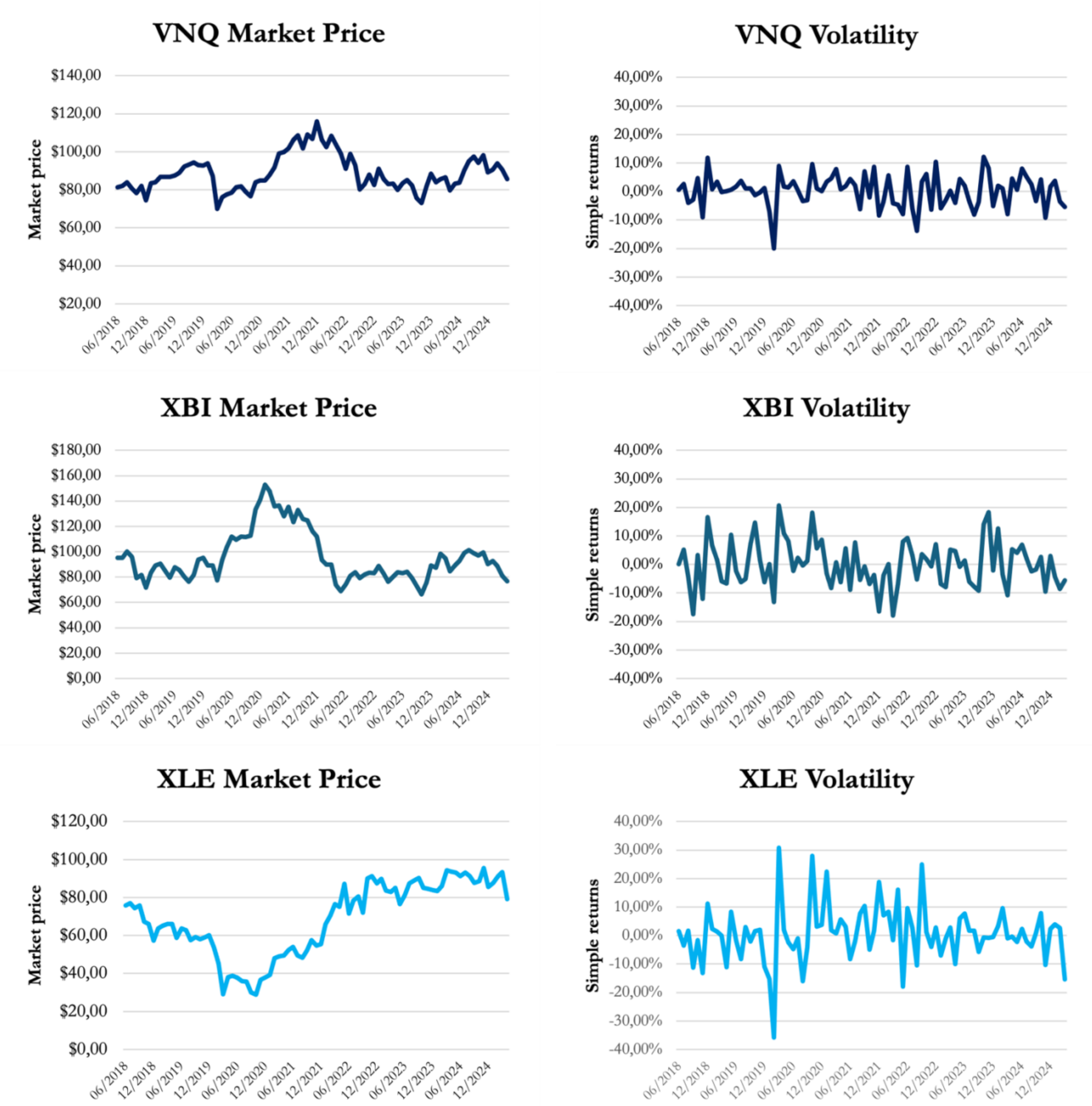

Figure 14: Market Prices and Volatility (Part 2) | Source: LSEG Data & Analytics

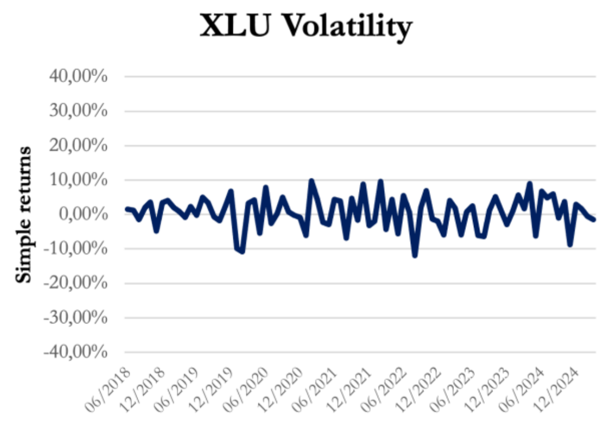

Figure 15: Market Prices and Volatility (Part 3) | Source: LSEG Data & Analytics