1. Box 1 contains 1000 missile parts of which 10 % are defective. Box 2 contains 2000 missile parts of which 5 % are defective. Two parts are picked from a randomly selected box.

a) Find the probability that both parts are defective.

$P(B_1)$ = Prob. of choosing Box 1 = .50
$P(B_2)$ = " " Box 2 = .50
$P(2D)$ = Probability of 2 Defects

$$\therefore \quad P(2D) = P(B_1) \times P(D_1|B_1)\,P(D_2|D_1 \text{ in } B_1) + P(B_2) \times P(D_1|B_2)\,P(D_2|D_1 \text{ in } B_2)$$

$$= .5 \times \left(\frac{100}{1000}\right)\left(\frac{99}{999}\right) + .5 \times \left(\frac{100}{2000}\right)\left(\frac{99}{1999}\right)$$

$$\boxed{P(2D) = .00619307 = 0.619307\,\%}$$

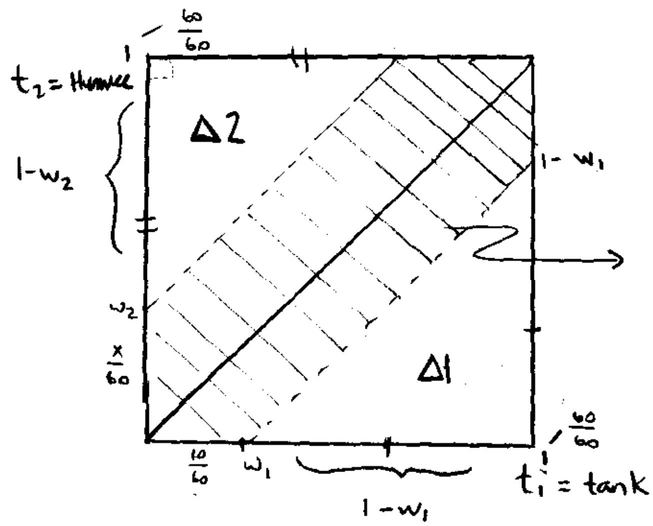
b) Assuming that both are defective, find the probability that they came from Box 1.

RECALL Bayes Rule : $P(A|B) = \dfrac{P(B|A)\,P(A)}{P(B)}$

$$\therefore \quad P(B_1|2D) = \frac{P(2D|B_1) \cdot P(B_1)}{P(2D)} = \frac{\left(\frac{100}{1000} \times \frac{99}{999}\right) \cdot \left(\frac{1}{2}\right)}{0.00619307}$$

$$\approx \quad \boxed{P(B_1|2D) = .80008057 = 80.008057\,\%}$$

(2) A tank and a humvee arrive at a supply depot at random between 9am and 10 am. The tank stops for 10 minutes and the humvee for x minutes. Find x so that the probability that the tank and humvee will meet equals 0.5.



This exact area represents the probability that the two vehicles will meet, since:

$$f_{XY}(x,y) = f_X(x)\, f_Y(y)$$

As graphically shown, the $P(\text{meet}) = 1 - \Delta 1 - \Delta 2$
where $\Delta 1$ = area of $\Delta 1$
$\Delta 2$ = area of $\Delta 2$

$$\therefore \quad P(\text{meet}) = 1 - \Delta 1 - \Delta 2$$

$$= 1 - \tfrac{1}{2}(1-w_1)(1-w_1) - \tfrac{1}{2}(1-w_2)(1-w_2)$$

$$= 1 - \tfrac{1}{2}(1-w_1)^2 - \tfrac{1}{2}(1-w_2)^2$$

$$0.5 = 1 - \tfrac{1}{2}\left(1-\tfrac{10}{60}\right)^2 - \tfrac{1}{2}\left(1-\tfrac{x}{60}\right)^2$$

$$\Rightarrow \boxed{x = 26.83 \text{ minutes}}$$

~ Tank must wait 26.8 minutes to ensure a meet probability of 50 %

3 A register contains 16 random binary digits which are mutually independant. Each digit is a 0 or a 1 with equal probability. Find the probability of the following events:

Note: n = total # of possible combinations = 2^{16}

a) The register contains 1111001100110101 :

Only one unique solution: $\rightarrow$ $\boxed{\frac{1}{2^{16}} = .001526\ \%}$ ↗ .00001526

b) The register contains exactly 4 zeros:

~ All other 12 slots are certain to be 1; therefore $\rightarrow$

# of possible 4 '0' slot combinations: $\binom{16}{4} = 1820$

$\rightarrow$ $\boxed{\text{Probability} = \frac{1820}{2^{16}} = .02777 = 2.777\%}$

c) The first 5 digits are all ones:

1 1 1 1 1 x x x x x x x x x x x $\rightarrow$ x = 0 or 1 (don't care)

$\therefore$ Total # of 2^{11} possible combinations!

$\Rightarrow$ $\boxed{\text{Probability} = \frac{2^{11}}{2^{16}} = .03125 = 3.125\%}$

d) All digits in the register are the same:

~ There are only TWO distinct cases; ALL 0's or ALL 1's

↗ .00003052

$\rightarrow$ $\boxed{\text{Probability} = \frac{2}{2^{16}} = .003052\%}$

4 A random number generator generates integers from 1 – 9 (inclusive). All outcomes are equally likely; each integer is generated independently of any previous integers. Let Σ denote the sum of two consecutively generated integers; that is, $\Sigma = N_1 + N_2$.

~ Create a table of N_1, N_2 Σ combinations...

N_1 \ N_2	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

Remember:

Odd + Odd = Even
Even + Even = Even
Even + Odd = Odd

Total # of combinations = $9 \times 9 = \underline{\underline{81}}$

↳ 40 odd combinations
↳ 41 even combinations

a) Given that Σ is odd, what is the conditional probability that Σ is 7 ?

~ Looking at O's in table: $\boxed{P(\Sigma = 7) = \frac{6}{40} = .15 = 15\%}$

b) Given that $\Sigma > 10$, what is the conditional probability that at least one of the integers is > 7 ?

~ Draw a --- line for $\Sigma > 10$, then find all values of $N_1 / N_2 > 7$. All values in --- area represent the wanted values.

$\boxed{P(\text{1 on integers} > 7 \mid \Sigma > 10) = \frac{26}{36} = \frac{13}{18} = 0.72\bar{2} = 72.22\%}$

c) Given that $N_1 > 8$, what is conditional probability that Σ will be odd ?

~ Representative area where $N_1 > 8$ is below — — —.
Total of 9 possibilities with 4 being odd. ∴

$\boxed{P(\Sigma = \text{odd} \mid N_1 > 8) = \frac{4}{9} = .\bar{4}\bar{4} = 44.44\%}$

5) The time-to-failure in months, x, of computer chips produced at two fabrication plants A and B obey, respectively, the following pdf's:

$$F_X(x|A) = (1 - e^{-x/5})\,u(x) \sim \text{Plant A}$$
$$F_X(x|B) = (1 - e^{-x/2})\,u(x) \sim \text{Plant B}$$

Plant B produces 3x as many chips as plant A. The chips, indistinguishable to the eye, are intermingled and sold. What is the probability that a computer chip purchased at random will work at least: 2 months, 5 months, 7 months.

~ $P(A) = \frac{1}{4}$, $P(B) = \frac{3}{4}$ (given the problem statement)

Since $F_X(x) = \sum_{i=1}^{n} F_X(x|A_i)\,P[A_i]$

$$\therefore\ F_X(x) = F_X(x|A)\,P(A) + F_X(x|B)\,P(B)$$
$$= (1-e^{-x/5})\cdot\tfrac{1}{4} + (1-e^{-x/2})\cdot\tfrac{3}{4}$$

$\Rightarrow F(2) = .55651$ ~ Time to failure in 2 months
$\Rightarrow F(5) = .84647$ ~ Time to failure in 5 months
$\Rightarrow F(7) = .91570$ ~ Time to failure in 7 months

a) P(Chip working atleast 2 months):

$$\boxed{P(\text{atleast 2 months}) = 1 - F(2) = .44349}$$

b) P(Chip working atleast 5 months):

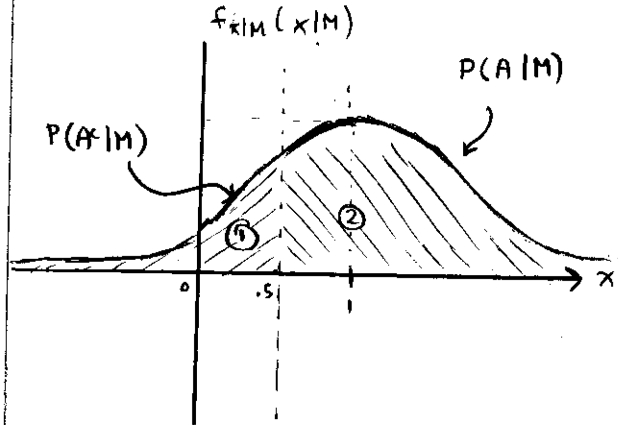
$$\boxed{P(\text{atleast 5 months}) = 1 - F(5) = .15353}$$

c) P(Chip working atleast 7 months):

$$\boxed{P(\text{atleast 7 months}) = 1 - F(7) = .0843}$$

6 A U.S. defense radar scans the skies for UFO's. Let M be the event that a UFO is present and M^c be the event that a UFO is absent. Let A denote the event of an alert, $\{X > x_a\}$. Compute $P(A|M)$, $P(A^c|M)$, $P(A|M^c)$, $P(A^c|M^c)$.

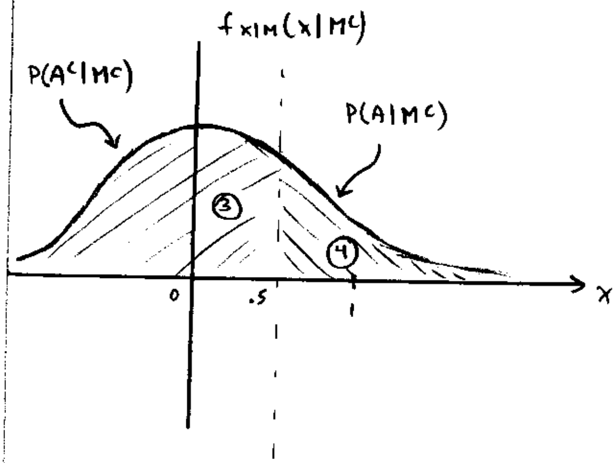
• Graph $f_{x|M}(x|M) = \frac{1}{\sqrt{2\pi}} e^{\left(-\frac{1}{2}[x-1]^2\right)} \sim \begin{matrix}\mu=1\\ \sigma=1\end{matrix}$



① $= \int_{-\infty}^{.5} f_{x|M}(x|M)\,dx = \boxed{.3085 = P(A^c|M)}$

② $= \int_{.5}^{+\infty} f_{x|M}(x|M)\,dx = \boxed{.6915 = P(A|M)}$

• Graph $f_{x|M}(x|M^c) = \frac{1}{\sqrt{2\pi}} e^{\left(-\frac{1}{2}x^2\right)} \sim \begin{matrix}\mu=0\\ \sigma=1\end{matrix}$



③ $= \int_{-\infty}^{.5} f_{x|M}(x|M^c)\,dx = \boxed{.6915 = P(A^c|M^c)}$

④ $= \int_{.5}^{+\infty} f_{x|M}(x|M^c)\,dx = \boxed{.3085 = P(A|M^c)}$

* Graph's confirmed using indefinite integral evaluations of MATLAB and a normal distribution graph as seen at:

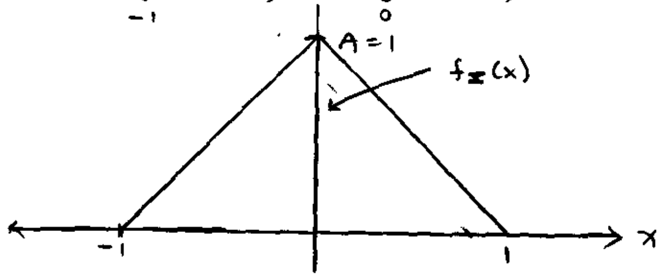
http://www.math.unb.ca/~knight/utility/NormTble.htm

7 Consider the random variable X with pdf $f_X(x)$ given by:

$$f_X(x) = \begin{cases} A(1+x), & -1 < x \le 0 \\ A(1-x); & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

a) Find A and plot pdf $f_X(x)$.

~ $1 = \int_{-1}^{0} A(1+x)\,dx + \int_{0}^{1} A(1-x)\,dx = \frac{1}{2}A + \frac{1}{2}A \Rightarrow A = 1$

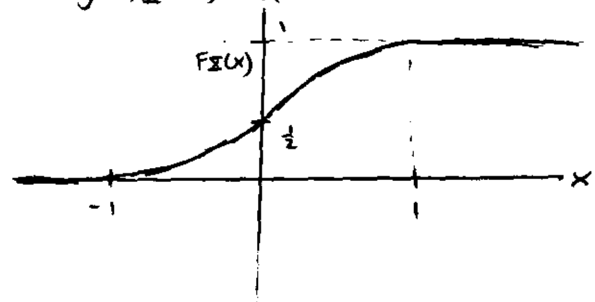


~ PDF of $f_X(x)$.

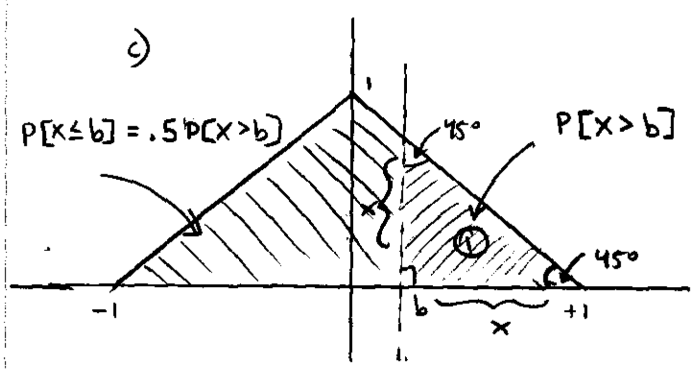
b) Plot cdf of $F_X(x)$.

~ $f_X(x) = \frac{d}{dx} F_X(x)$: $F_X(x) = \int f_X(x)\,dx$

$$F_X(x) = \begin{cases} x + \frac{x^2}{2} + \frac{1}{2}, & -1 < x \le 0 \\ x - \frac{x^2}{2} + \frac{1}{2}; & 0 < x < 1 \\ 0, & \text{elswhere} \end{cases}$$



c)



Find Area of Δ 1. → Given the 45-45-90° triangle:

$A\,\Delta 1 = \frac{1}{2}bh$ → We know that area of 1 must be $\frac{1}{3}$ the total area of the pdf.

$\therefore \quad \frac{1}{3}(1) = \frac{1}{2}x^2 \Rightarrow x = .816497$

$\Rightarrow \boxed{b = 1 - x = .183503}$