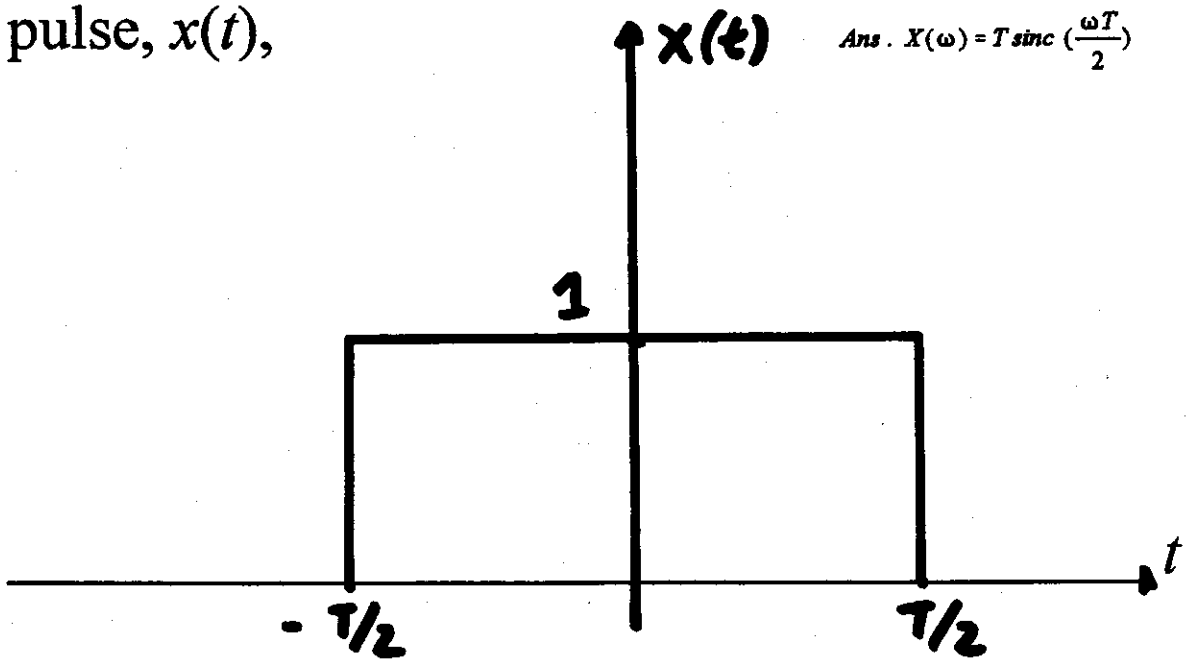
Example

Find the Fourier transform of the rectangular pulse, $x(t)$,



Ans. $X(\omega) = T\,\mathrm{sinc}\left(\frac{\omega T}{2}\right)$

Solution

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt = \int_{-T/2}^{T/2} (1)\, e^{-j\omega t}\, dt$$

$$X(\omega) = -\frac{1}{j\omega}\left[e^{-j\omega t}\right]\Big|_{-T/2}^{T/2} = \frac{-1}{j\omega}\left[e^{-j\omega T/2} - e^{j\omega T/2}\right]$$

$$X(\omega) = \frac{1}{j\omega}\left[e^{j\omega T/2} - e^{-j\omega T/2}\right] \times \frac{2}{2}$$

$$= \frac{2}{\omega}\sin\left(\frac{\omega T}{2}\right) \times \frac{\frac{\omega T}{2}}{\frac{\omega T}{2}}$$

$$X(\omega) = T\,\mathrm{sinc}\left(\frac{\omega T}{2}\right)$$