Near-field and Far-field beam control - ATIRCM

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ELDP 2008 Guided Demo and Homework Assignment

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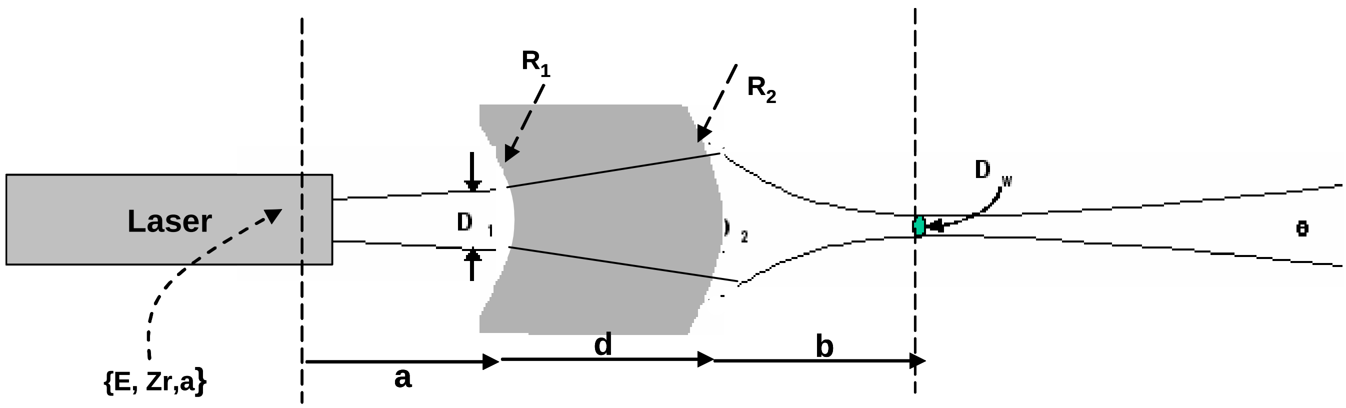
$\text{mrad} := 10^{-3}\text{rad}$

The ELDP single lens design ==



Use CaF2 glass. Go to http://www.luxpop.com/ and find the index of refraction, n, at this wavelength at 25 deg C.

At a wavelength of 4000 nm (0.310 eV) and a temperature of 24 deg C (297.15 deg K), the index of refraction of CaF2 is n = 1.40964 + / - 3e-05. Reference:

Given

Index of Refraction	$N := 1.40964$	Distance from waist to F1	$a := 40\text{mm}$
Laser Wavelength	$\lambda := 4.0\mu\text{m}$	Telescope length	$d := 20\text{mm}$
Beam Quality	$M_{sq} := 2.9$	Near-field beam waist from F2	$b := 500\text{mm}$
Beam Divergence* out of the resonator	$\theta_o := 22\text{mrad}$	Far-field Beam Divergence*	$\theta_{ff} := 2.1\text{mrad}$

Calculate beam parameters {Do, θo, E, Zr}

$$E := \frac{4\lambda}{\pi}\cdot M_{sq} \qquad E = 14.770\,\text{mm}\cdot\text{mrad}$$

$$D_o := \frac{E}{\theta_o} \qquad D_o = 0.671\,\text{mm}$$

$$zr := \frac{D_o}{\theta_o} \qquad zr = 30.516\,\text{mm}$$

Beam in - RP1 is at waist $zo := 0mm$ $q := \begin{pmatrix} \dfrac{0mm + i \cdot zr}{mm} \\ 1 \end{pmatrix}$ $q = \begin{pmatrix} 30.516i \\ 1.000 \end{pmatrix}$ $q_0 = 30.516i$ $q_1 = 1.000$

Paraxial Beam Functions

Parameters of the beam in the region of the RP.

$Zo(q) := Re\left[q_0\, mm \cdot (q_1)^{-1}\right]$ Waist location - distance from waist to RP (Zo > 0, RP is right of waist, or waist if left of RP)

$Zr(q) := Im\left[q_0\, mm \cdot (q_1)^{-1}\right]$ Rayleigh range in this region

$D_{waist}(q) := \sqrt{E \cdot Zr(q)}$ Waist diameter (not beam diameter at RP, unless RP is at the waist.)

$\theta(q) := \sqrt{E \cdot Zr(q)^{-1}}$ Beam Divergence in the far-field in this region

Parameters of the beam at the RP.

$D_{RP}(q) := \sqrt{-E \cdot Im\left[\left[q_0\, mm \cdot (q_1)^{-1}\right]^{-1}\right]^{-1}}$ Diameter of the beam intensity **at the RP** (not at the waist, unless RP is at the waist)

$R(q) := Re\left[\left[q_0\, mm \cdot (q_1)^{-1}\right]^{-1}\right]^{-1}$ Radius of curvature of the beam phase **at the RP.**

Don't confuse Mathcad and yourself.
It is useful to use upper-case for functions, and lower-case for numerical values.

$qo = Q(R_1, R_2)$ $zo = Zo(Q(R_1, R_2))$ $zo = Zo(qo)$

Reserve subscripts for the conponents of a vector or matric
qo,q1,q2.... are a different q's.
q_0, q_1, q_2,...are components of the vector q.
$M2_{1,3}$ is the components of the matrix M2 in the 1st row, and 3rd column.

1) Calculate the ABCD matrix for this new configuration. Check to see if the signs make se nse.

Radii of curvature are numerically negative (center of curvature on left). However, we put them into the equation as unsigned variables, and verify that the solution for each is a negative number as a check.

$$\mathrm{Lens}(R_1,R_2) := \begin{bmatrix} 1 & 0 \\ -\dfrac{(1-N)\cdot mm}{R_2} & 1 \end{bmatrix} \cdot \begin{pmatrix} 1 & \dfrac{d}{N\cdot mm} \\ 0 & 1 \end{pmatrix} \cdot \begin{bmatrix} 1 & 0 \\ -\dfrac{(N-1)\cdot mm}{R_1} & 1 \end{bmatrix}$$

$$M(R_1,R_2) := \begin{pmatrix} 1 & \dfrac{b}{mm} \\ 0 & 1 \end{pmatrix} \cdot \mathrm{Lens}(R_1,R_2) \cdot \begin{pmatrix} 1 & \dfrac{a}{mm} \\ 0 & 1 \end{pmatrix} \qquad |M(-100mm,-100mm)| = 1.000$$

2) Calculate R_1 and R_2 for which the two constraints are satisfied. (Be careful! R1 and R2 are negative or positive. Put them into the matrices as unsigned variables, and let Mathcad calculate their sign.) Verify these R's satisfy the two constraints.

Two degrees of freedom ====> Two restraints
(R1, R2) =====> (1) waist at the correct location, and(2) creates the correct divergence.

Beam out - RP2 at waist, 500 mm from lens

Define function $Q(R_1,R_2) := M(R_1,R_2)\cdot q$

Assume initial guess $R_1 := -100mm$ $\quad R_2 := -100mm$

Given

2 Constraints

$$Zo(Q(R_1,R_2)) = 0mm$$

$$\theta(Q(R_1,R_2)) = 2.26mrad$$

Go Mathcad! $\begin{pmatrix} R_1 \\ R_2 \end{pmatrix} := \mathrm{Find}(R_1,R_2) \qquad \begin{pmatrix} R_1 \\ R_2 \end{pmatrix} = \begin{pmatrix} -1.220 \\ -6.969 \end{pmatrix} mm$

Radii are negative. There center of curvature for each surface is to the left of the surface, as depicted..

Verify results $R_1 = -1.220\ mm \qquad R_2 = -6.969\ mm$

$Zo(Q(R_1,R_2)) = 3.288 \times 10^{-6}\ mm$ finite precision

$\theta(Q(R_1,R_2)) = 2.260000001\ mrad$ as required..

Unique and exact solution