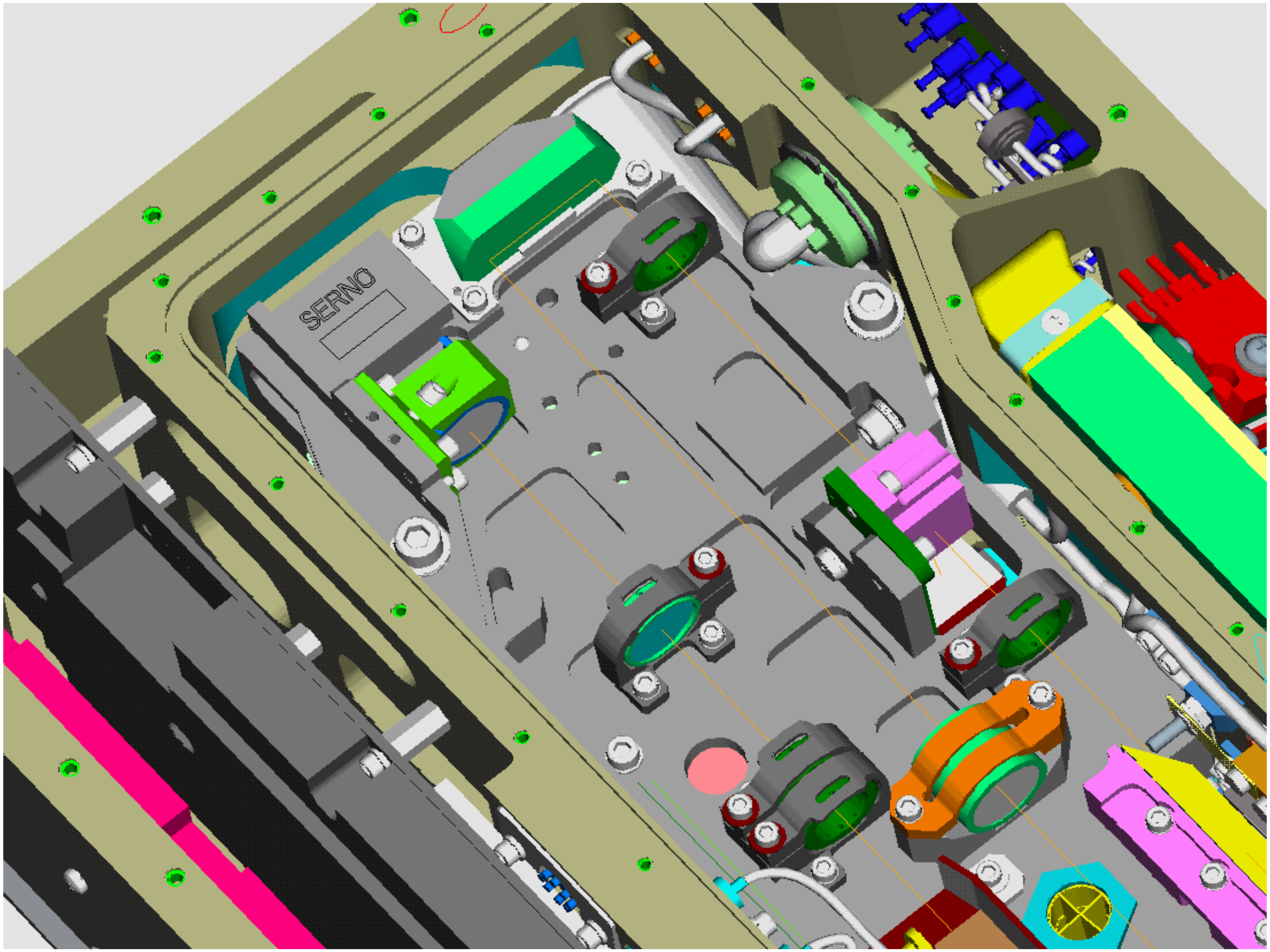
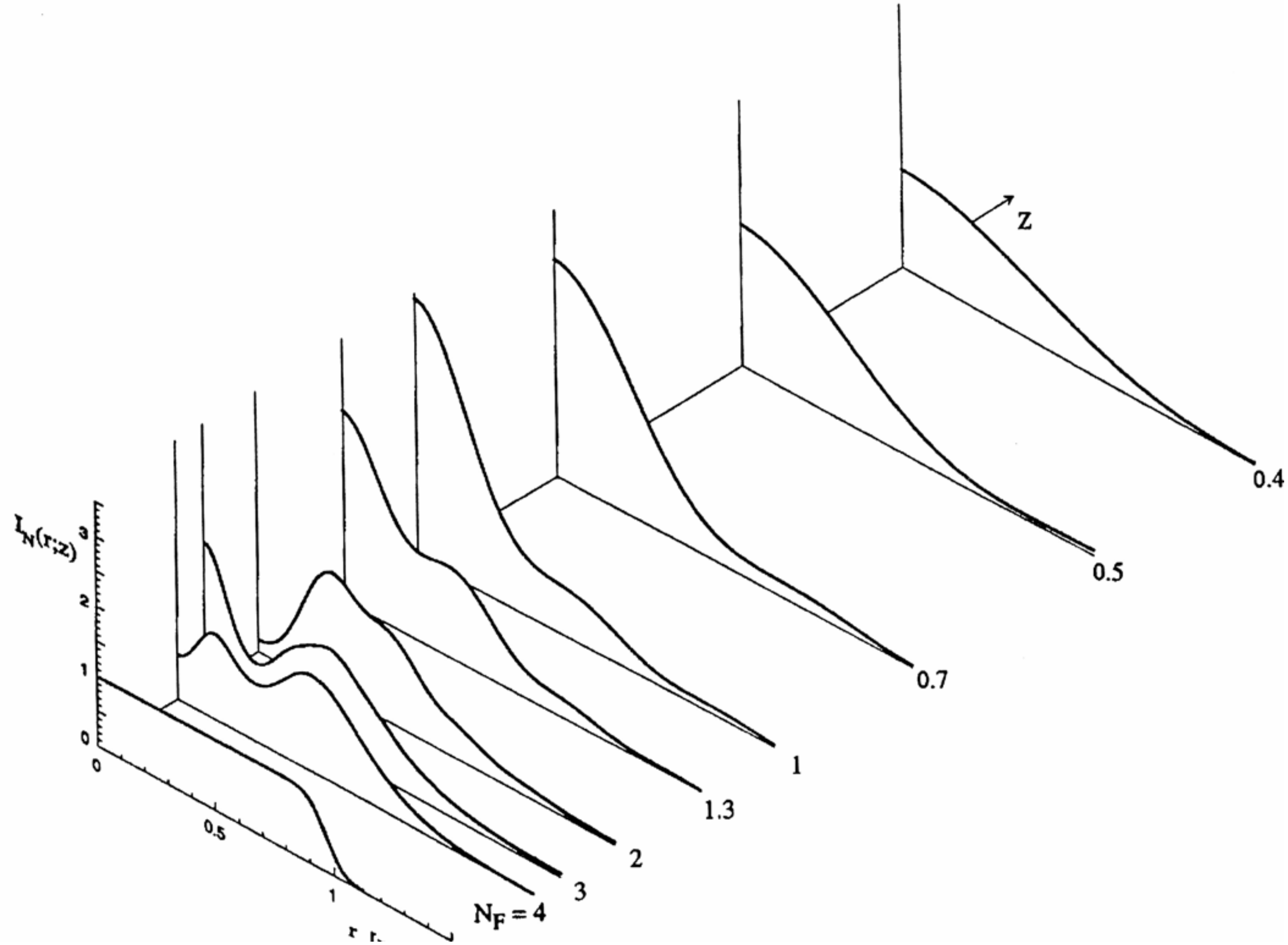
Light propagation through an optical system





The ABCDs of understanding, analysis, and design

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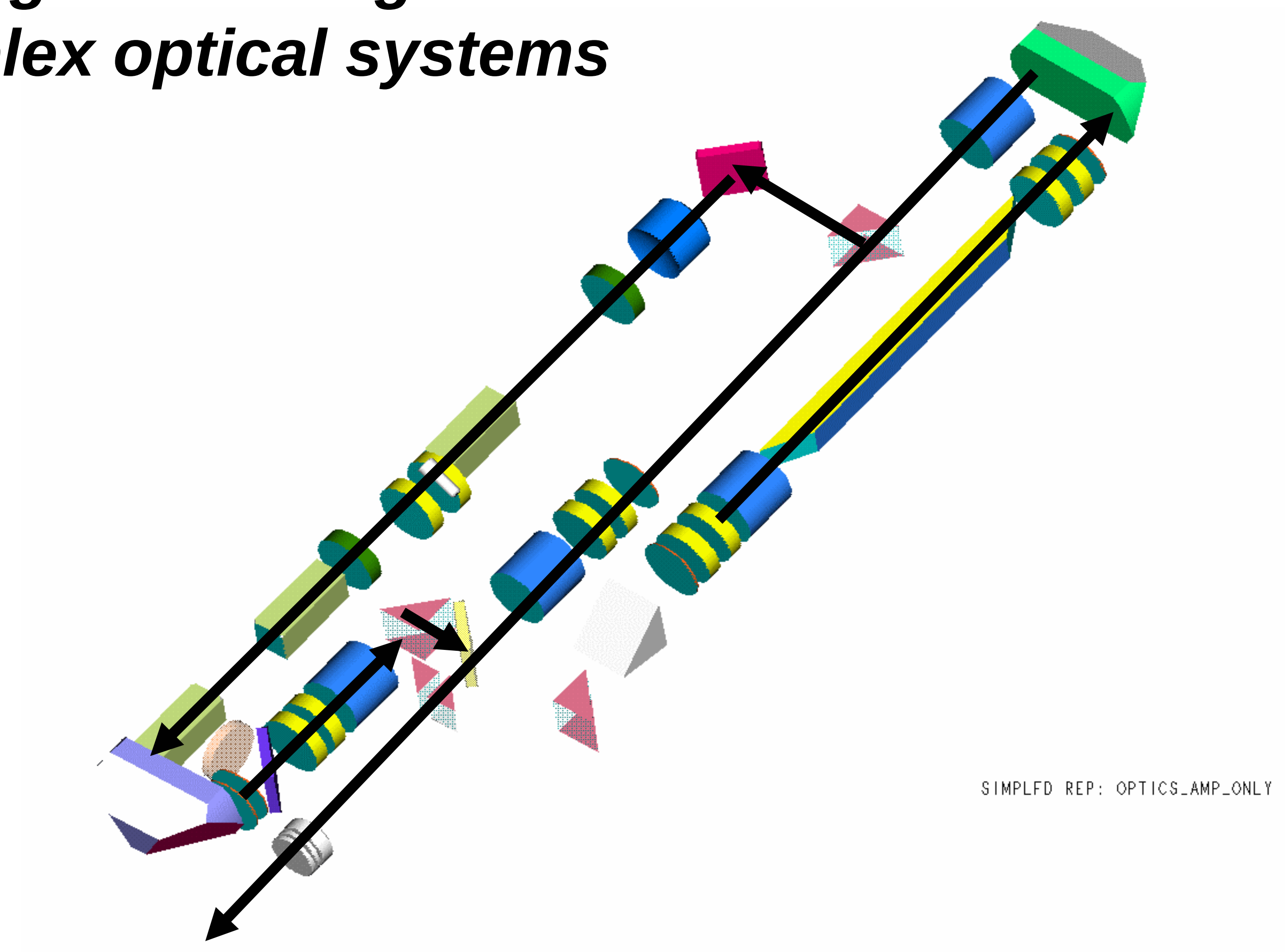
Objectives

- ***Duration:*** ***4 hours***
- ***Intended:***
  - ***For anybody working with optical systems and laser beams, particularly***
    - ***Systems Engineers***
    - ***Non-designers***
  - ***As a refresher for those already familiar with optical systems design***
  - ***As a learning experience for those newly involved in the field***
- ***Objectives:***
  - ***Provide a simple, unified method for analyzing and understanding the propagation of laser beams through complex optical systems***
  - ***Provide systems engineers with the tools to***
    - ***Perform trade studies, explore option spaces, and optimize performance***
    - ***Quantify the salient requirements at the system and component level***
    - ***Develop relevant, meaningful acceptance criteria and validation tests***
- ***Scope:***
  - ***Geometric and diffractive propagation through complex optical systems***
  - ***Tilts, decenters, tolerances and alignment***
  - ***Designing for diffractive beams***
  - ***Characterizing and measuring beams***

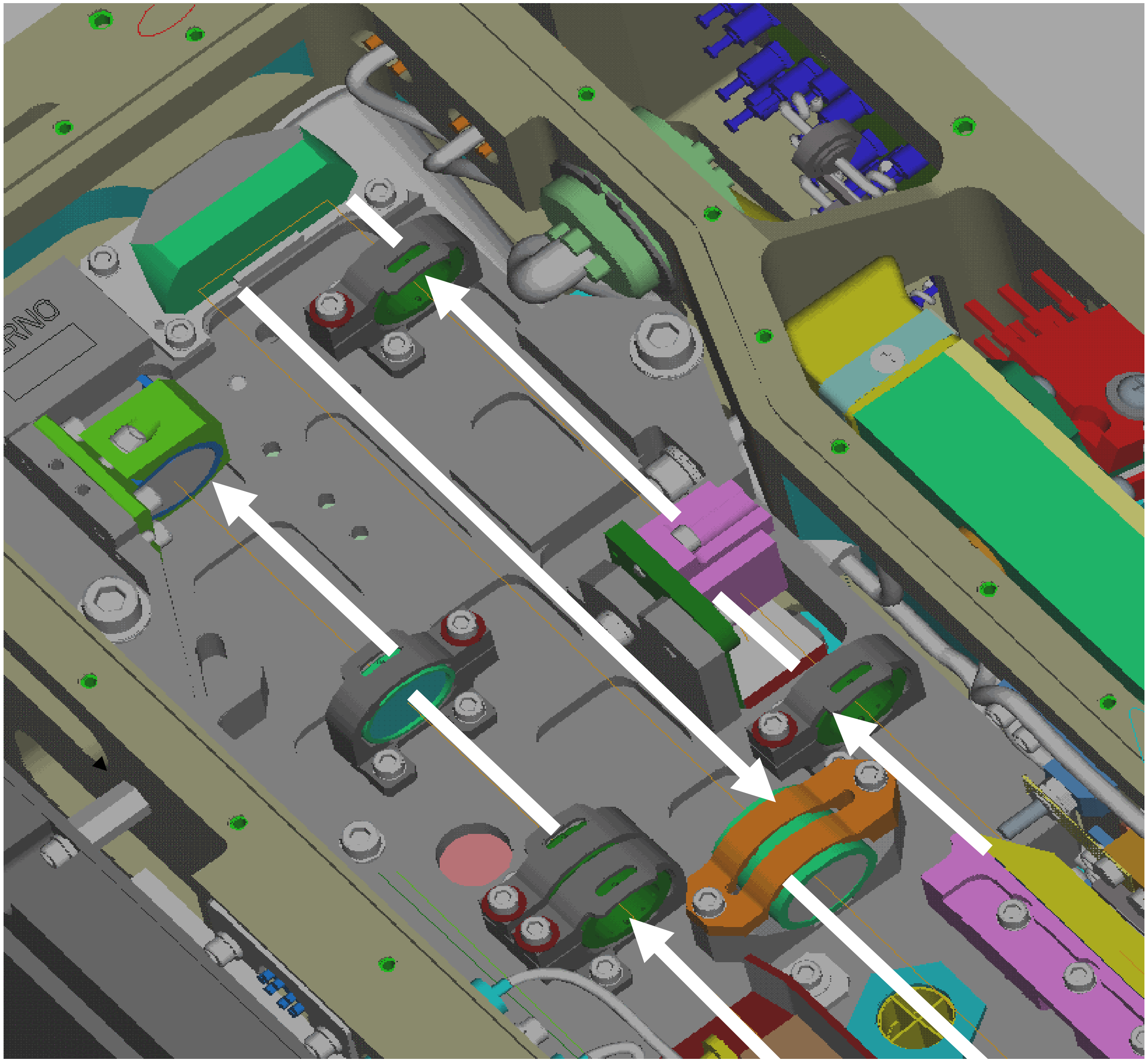
Topics

- ***Basis properties of light rays and beam***
- ***Ray vectors and ABCD matrices***
- ***Effects of propagation and lenses***
- ***Wavefront "radius of curvature" and "curvature"***
- ***Properties of lens and telescope***
- ***Thin lens, thick lens, temperature dependence, wavelength dependence***
- ***Equivalent of complex optical system to a single lens***
- ***Alignment errors - tilts, decenter***
- ***Alignment buildup in complex optical system***
- ***Effects of alignment on telescope***
- ***Modular alignment and degrees of freedom***
- ***Beam parameters***
- ***How all beams propagate - hyperbolic envelope***
- ***Beam waist diameter, waist location, and divergence***
- ***Equation method of beam propagation***
- ***Beam diameter and wavefront radius of curvature***
- ***Etendue and m^2***
- ***Effect of lens on beams***
- ***Refocused beams - location, waist diameter, divergence***
- ***Designing beam optics***
- ***ABCD method of beam propagation***
- ***Advantages and disadvantages of equation and ABCD method***
- ***One equation summary***

Laser beams are generated in and propagated through complex optical systems



How does a laser beam change?



Lenses

Mirrors

Mounts

Spacing

Alignment

Comments

- Optics in collimated beam
  - Do not obey ray (geometrical) optics
  - Requires beam (diffraction, i.e. physical optics)
- Optical Design Codes usually trace rays
  - Code V
  - Zmax
  - Oslo
- Physical Optics codes – 3 types
  - Near- and far-field diffraction
    - FFT on complex (amplitude and phase) data
    - Point Analysis
  - Beam Equations
    - Algebra
  - ABCD Matrices
    - 2x2 matrices
    - Complex numbers

Comments

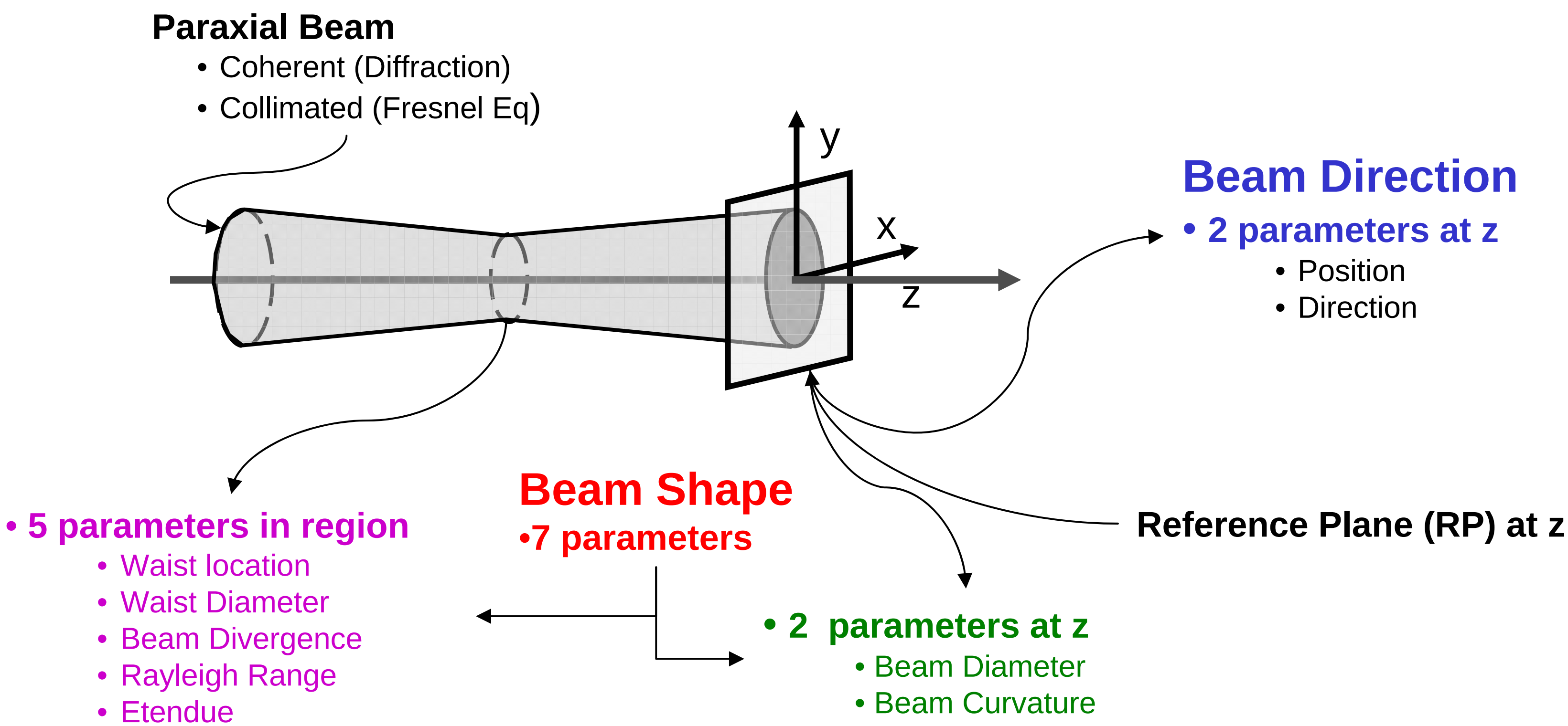
- Designers try to use rays codes to estimate (approximate) results
  - Have and know how to use the tool
  - Can be order of magnitude wrong!
- Optical and System Engineers don't have the tools
  - Complex diffraction simulation codes
  - Need simple analytical tools

- ABCD Matrices
  - Simple, unified
  - Applicable to both rays (geometrical optics) and beams (physical optics)
  - Exactly correct for any paraxial beam
    - Not an approximation
    - Any beam (Gaussian or not)
    - Limited information (9 parameters)
  - Hides all the complexity
  - Easy with MathCad, Mathlab

How is a beam characterized?

- ***On a plane in space – at each point***
  - ***Electric Field amplitude***
  - ***Polarization***
  - ***Phase***
- ***Too much data***
  - ***To know***
  - ***To calculate***
- ***Provides more information than you need***
- ***Pick a minimum set of parameters***
  - ***Meaningful***
  - ***Easy to calculate***
- ***Canonical Set***
  - ***9 parameters in each direction***
  - ***2 directions***

Paraxial beam propagation



- ***Beam characterized by 9 parameters***
  - ***2 direction; 7 beam shape***
  - ***5 in region; 4 on reference plane at any point z***
- ***How do the 9 parameters change?***
  - ***Due to lenses, mirrors, material, propagation, and misalignments***

How is a beam characterized?

- ***Light beam has***

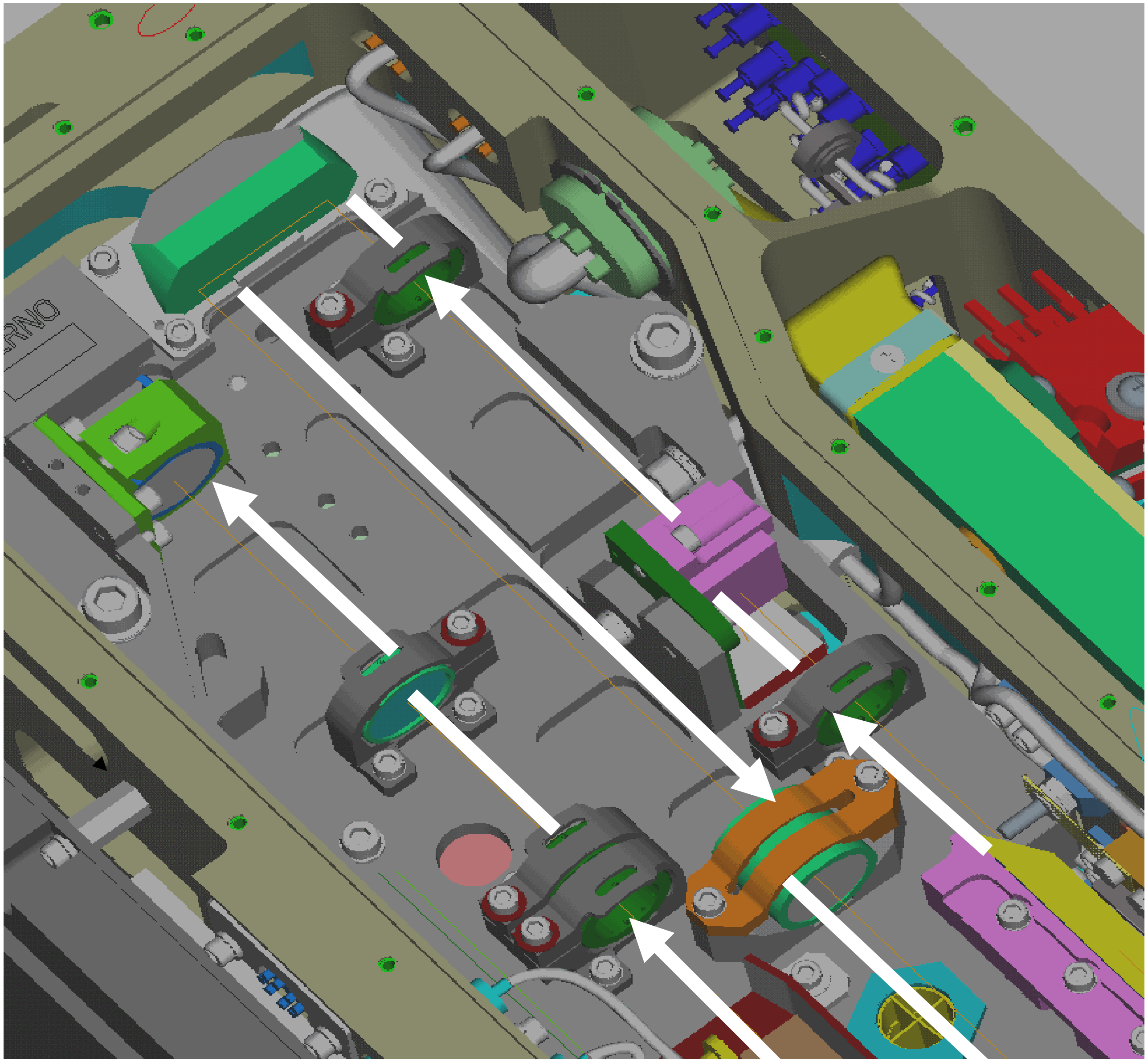
  ✓***Position***
  ✓***Direction***
  ✓***Waist location***
  ✓***Waist diameter***
  ✓***Wavefront diameter***
  ✓***Wavefront radius of curvature***
  ✓***Far-field divergence angle***
  ✓***Rayleigh range***
  ✓***Etendue***
  – ***Intensity distribution (beam shape)***
  – ***Polarization***

ABCD Ray Vector

ABCD Beam Vector

- ***We will analyze the first 9***
  – ***How they are measured***
  – ***How they change***
    - ***the effects of lenses, spacing, wavelength, temperature, tilts, decenters***

How do the 9 parameters change?



Lenses

Mirrors

Mounts

Spacing

Alignment

The only math you need

1) Matrix algebra $y = M \cdot x$

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

where

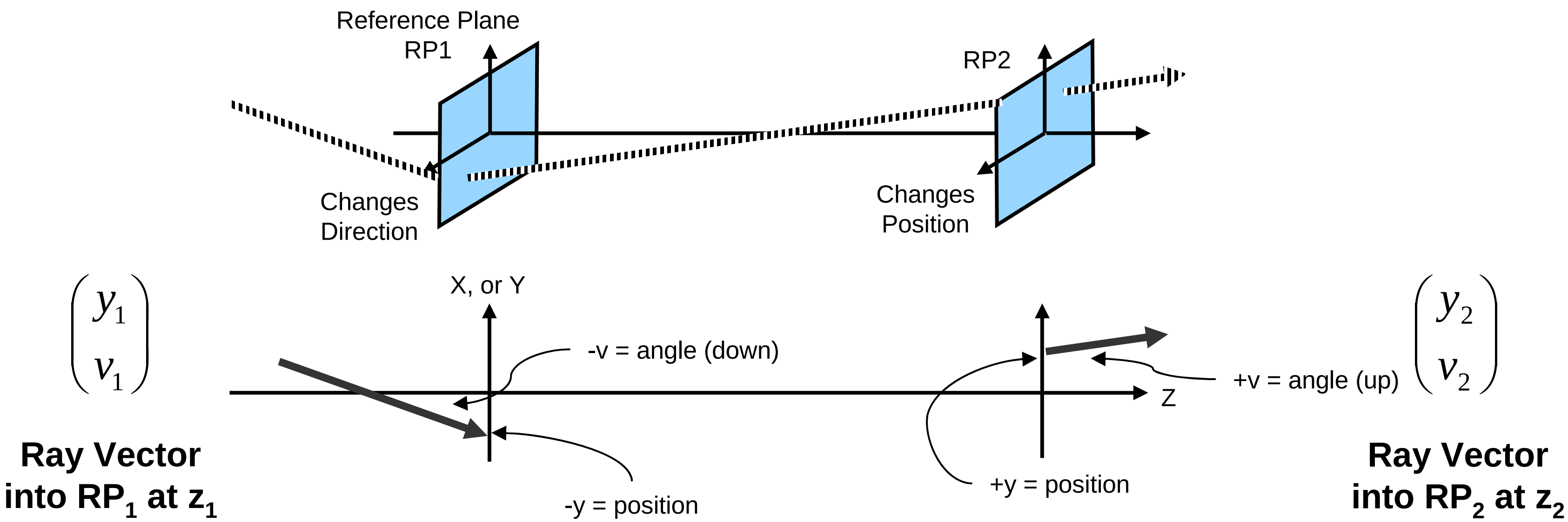
$$y_1 = M_{11}x_1 + M_{12}x_2 + M_{13}x_3$$

etc.

2) Complex numbers **z = x + iy**

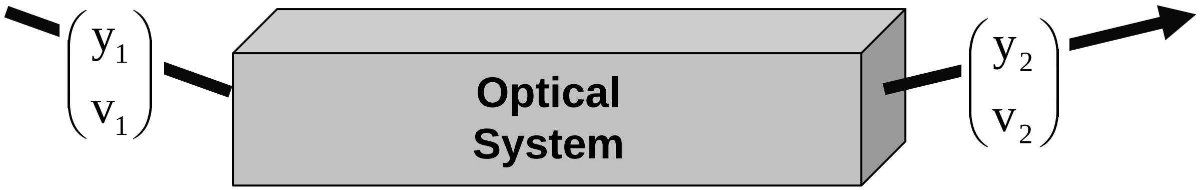
3) Algebra Processor, such as Mathcad, Matlab

Ray vector $\begin{pmatrix} y \\ v \end{pmatrix}$



- ***Two components***
  - ***y, distance from z-axis***
    - ***y > 0 above, y < 0 below***
  - ***v, angle relative to z –axis***
    - ***v > 0 up, v < 0 down***
- ***X-direction and Y-direction***
  - ***treated separately, independently***
- ***Later we will define a “Beam Vector”***

ABCD matrix



$$y_2 = f(y_1, v_1)$$

$$y_2 \approx \frac{\partial f(0,0)}{\partial y_1} y_1 + \frac{\partial f(0,0)}{\partial v_1} v_1 + ... + ...$$

$$y_2 \approx A \cdot y_1 + B \cdot v_1$$

similarly....

$$v_2 \approx C \cdot y_1 + D \cdot v_1$$

$$\begin{pmatrix} y_2 \\ v_2 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} y_1 \\ v_1 \end{pmatrix}$$

- ***Lenses, mirrors***
  - ***f (y,v) continuous function***
  - ***derivatives exist***
- ***Paraxial beams***
  - ***y small***
  - ***v small***
- ***Linear approximation valid***
  - ***Good enough for laser beams!***
  - ***Photon pipes and nozzles – 1st order effects important***
  - ***Usually more uncertainty in the laser beam than error in the ABCD matrix results!***
- ***Ray codes (Code V) good job with optics***
  - ***Not so good with diffraction***
  - ***Use ABCD method to verify ray code design***

Physical units

$$\begin{pmatrix} y_2 \\ v_2 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} y_1 \\ v_1 \end{pmatrix}$$

$$y_2 = A \cdot y_1 + B \cdot v_1$$
$$v_2 = C \cdot y_1 + D \cdot v_1$$

Dimensioned Quantities

y = [mm]
v = [radians] = [1]

A = [mm/mm] = [1]
D = [radians/radians] = [1]
B = [mm/radians] = [mm]
C = [radians/mm] = [mm^{-1}]

Angles (v) must be in radians

Before: change degrees, mrad to radians

- 1 deg = 0.0175 radians
- 1 mrad = 0.001 radians

After: change radians to

- 1 radian = 57.3 deg
- 1 radian = 1000 mrad

ABCD parameters

$$\begin{pmatrix} y_2 \\ v_2 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} y_1 \\ v_1 \end{pmatrix}$$

$$y_2 = A \cdot y_1 + B \cdot v_1$$
$$v_2 = C \cdot y_1 + D \cdot v_1$$

$$A = \frac{dy_2}{dy_1} \equiv M_y$$

M_y = lateral magnification

$$D = \frac{dv_2}{dv_1} \equiv M_v$$

M_v = angular magnification

Cascade

$$\begin{pmatrix} y \\ v \end{pmatrix}_2 = \begin{pmatrix} A & B \\ C & D \end{pmatrix}_{2\leftarrow 1} \begin{pmatrix} y \\ v \end{pmatrix}_1$$

$$\begin{pmatrix} y \\ v \end{pmatrix}_3 = \begin{pmatrix} A & B \\ C & D \end{pmatrix}_{3\leftarrow 2} \begin{pmatrix} y \\ v \end{pmatrix}_2$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}_{3\leftarrow 1} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}_{3\leftarrow 2} \begin{pmatrix} A & B \\ C & D \end{pmatrix}_{2\leftarrow 1}$$

$$\begin{pmatrix} y \\ v \end{pmatrix}_3 = \begin{pmatrix} A & B \\ C & D \end{pmatrix}_{3\leftarrow 2} \begin{pmatrix} A & B \\ C & D \end{pmatrix}_{2\leftarrow 1} \begin{pmatrix} y \\ v \end{pmatrix}_1$$

$$\begin{pmatrix} y \\ v \end{pmatrix}_3 = \begin{pmatrix} A & B \\ C & D \end{pmatrix}_{3\leftarrow 1} \begin{pmatrix} y \\ v \end{pmatrix}_1$$

$In \quad general$

$$\boxed{\begin{pmatrix} A & B \\ C & D \end{pmatrix}_{n\leftarrow m} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}_{n\leftarrow n-1} \cdots\cdot \begin{pmatrix} A & B \\ C & D \end{pmatrix}_{m+1\leftarrow m}}$$

- ***All mathematical complexity is reduced to a 2 x 2 matrix multiplication***
  - *Can use numbers or symbols*
  - *Algebra processors (MatLab, MathCAD) do it for you (no mistakes)*
- ***Calculate only the ray vector you want***
  - *Ray out (n=100) for a ray in (n = 1)*
  - *Ignore internal rays (n=2-99)*
- ***Only 2 types of ABCD matrices***
  - *T-matrix for beam TRANSLATION*
  - *L-matrix for Lenses and Mirrors, Refractive Surfaces, Reflective Surfaces*

T-matrix

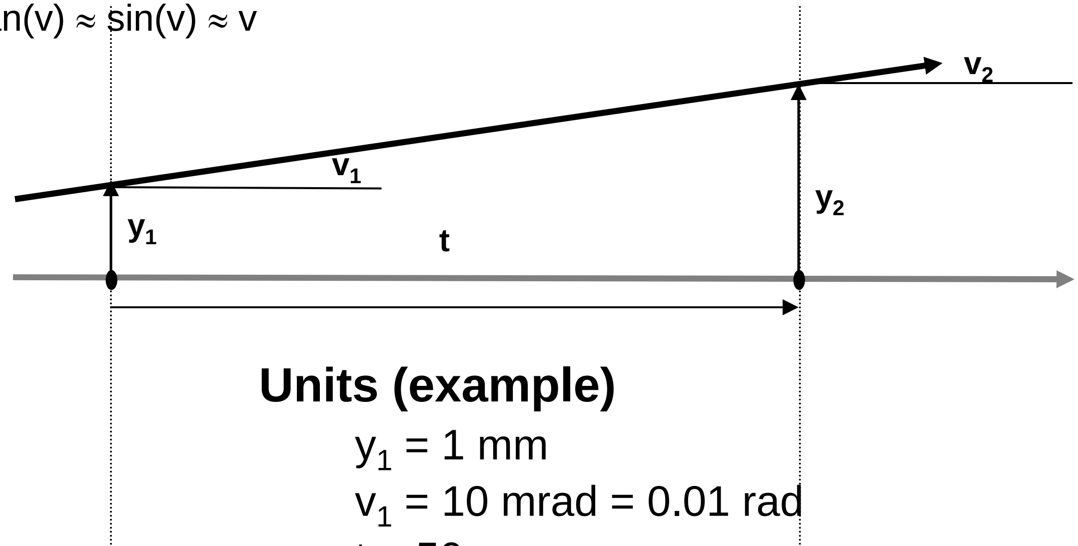
$$y_2 = y_1 + t \cdot \tan(v_1)$$

Paraxial Beams (close to the axis)

- small angles
- tan(v) ≈ sin(v) ≈ v

$$y_2 \cong y_1 + t \cdot v_1$$

$$v_2 = v_1$$



$$\begin{pmatrix} y \\ v \end{pmatrix}_2 = \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} y \\ v \end{pmatrix}_1$$

Units (example)

y_1 = 1 mm
v_1 = 10 mrad = 0.01 rad
t = 50 mm
v_1t = 50 x 0.01 mm rad = 0.5 mm
y_2 = y_1 + v_1t = 1.5 mm (changed)
v_2 = 0.01 rad (unchanged)

$$T = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix}$$

$$|T| = 1 \cdot 1 - t \cdot 0 = 1$$

- ***Unimodular, determinant |T| = 1***

fix

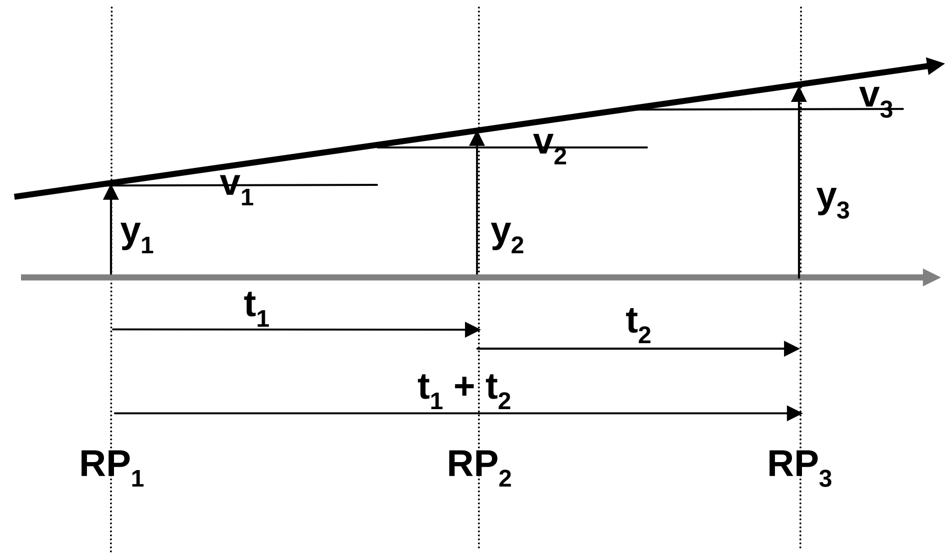
Cascade translations

$$\begin{pmatrix} y \\ v \end{pmatrix}_2 = \begin{pmatrix} 1 & t_1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} y \\ v \end{pmatrix}_1$$

$$\begin{pmatrix} y \\ v \end{pmatrix}_3 = \begin{pmatrix} 1 & t_2 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} y \\ v \end{pmatrix}_2$$

$$\begin{pmatrix} y \\ v \end{pmatrix}_3 = \begin{pmatrix} 1 & t_2 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & t_1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} y \\ v \end{pmatrix}_1$$

$$\begin{pmatrix} 1 & t_2 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & t_1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & (t_1 + t_2) \\ 0 & 1 \end{pmatrix}$$



Cascade

$$\begin{pmatrix} y \\ v \end{pmatrix}_3 = \begin{pmatrix} 1 & (t_1 + t_2) \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} y \\ v \end{pmatrix}_1$$

Makes sense….must be correct!

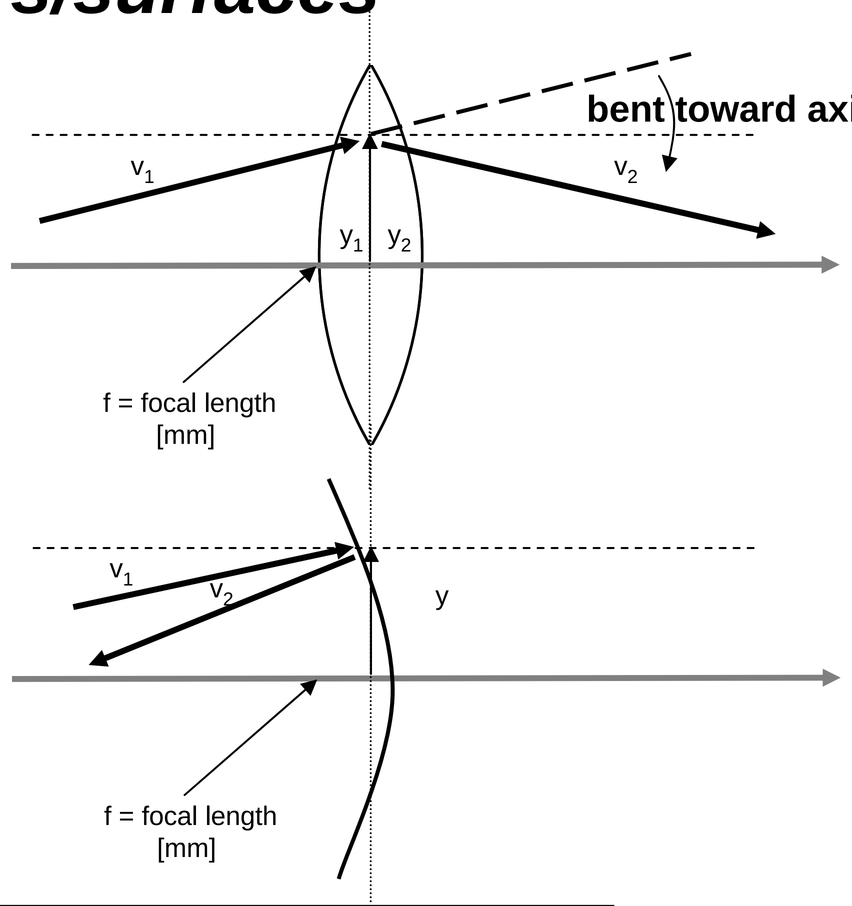
L matrix – lenses/mirrors/surfaces

$$y_2 \cong y_1$$
$$v_2 \cong v_1 - y_1/f$$

$$\begin{pmatrix} y \\ v \end{pmatrix}_2 = \begin{pmatrix} 1 & 0 \\ -\dfrac{1}{f} & 1 \end{pmatrix} \cdot \begin{pmatrix} y \\ v \end{pmatrix}_1$$

$$L = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 \\ -\dfrac{1}{f} & 1 \end{pmatrix}$$

$$|L| = 1$$



- ***Sign Convention***
  - ***f > 0 ➔ toward axis (converging)***
  - ***f < 0 ➔ away from axis (diverging)***
  - ***f = ∞ ➔ no change (L = identity matrix)***
- ***Unimodular, determinant |L| = 1***

Unimodular

$$|T| = 1$$

$$|L| = 1$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = L_n \cdot T_n \cdot .... \cdot L_1 \cdot T_1$$

$$\left| \begin{pmatrix} A & B \\ C & D \end{pmatrix} \right| = |L_n| \cdot |\quad| \cdot |\quad| \cdot |T_1| = 1 \cdot 1 \cdot .... \cdot 1 = 1$$

$$A \cdot B - C \cdot D = 1$$

- ***Product of unmodular matrices is unimodular***
  - ***Easy way to check your math***

On-axis focusing

parallel rays (collimated)

any ray

$$\begin{pmatrix} y \\ v \end{pmatrix}_2 = T \cdot L \begin{pmatrix} y \\ 0 \end{pmatrix}_1$$

focal plane

$$\begin{pmatrix} y \\ v \end{pmatrix}_2 = \begin{pmatrix} 1 & t = f \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -f^{-1} & 1 \end{pmatrix} \begin{pmatrix} y \\ 0 \end{pmatrix}_1$$

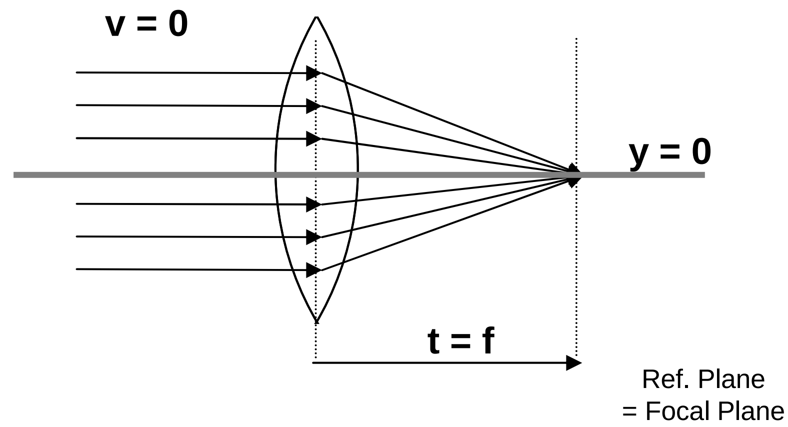
$$\begin{pmatrix} y \\ v \end{pmatrix}_2 = \begin{pmatrix} 0 & f \\ -f^{-1} & 1 \end{pmatrix} \begin{pmatrix} y \\ 0 \end{pmatrix}_1$$

$$y_2 = 0 \cdot y_1 + f \cdot 0$$

$$\boxed{y_2 = 0}$$

independent of y_1

pass though axis at t = f



- ***2 matrices cascaded***
- ***All incoming rays (any y_1) parallel to the axis (all $v_1 = 0$), will pass through the optical axis ($y_2 = 0$) at the focal plane ($t = f$).***
- ***Defines {f, focal length, and focal plane}***

Off-axis focusing

$$\begin{pmatrix} y \\ v \end{pmatrix}_2 = T \cdot L \cdot \begin{pmatrix} y \\ v \end{pmatrix}_1$$

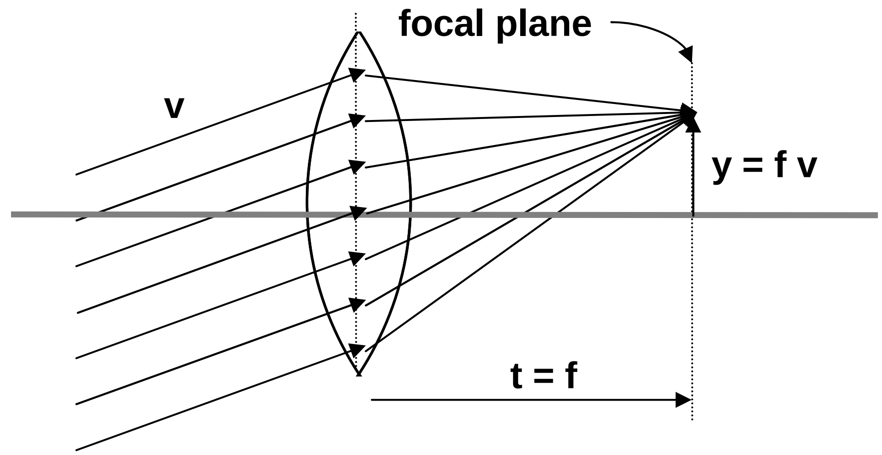
$$\begin{pmatrix} y \\ v \end{pmatrix}_2 = \begin{pmatrix} 1 & f \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -f^{-1} & 1 \end{pmatrix} \begin{pmatrix} y \\ v \end{pmatrix}_1$$

$$\begin{pmatrix} y \\ v \end{pmatrix}_2 = \begin{pmatrix} 0 & f \\ -f^{-1} & 1 \end{pmatrix} \begin{pmatrix} y \\ v \end{pmatrix}_1$$

A = 0

$$y_2 = 0 \cdot y_1 + f \cdot v_1$$

$$\boxed{y_2 = f \cdot v_1}$$



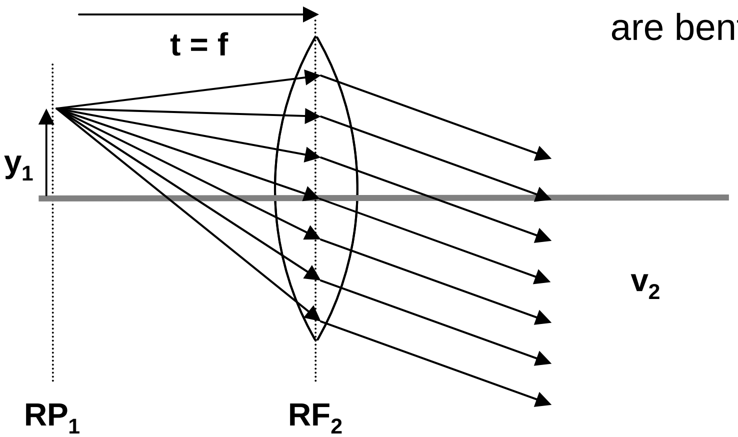
- **Very important for measuring beams in far-field**
  - Angle (v) in far-field ➔ position (y) in near-field (y), on a 2D detector array at the focal plane Z
  - Focal Plane Array **(FPA)**

- ***A = 0 Focusing***
  - ***All incoming collimated rays are mapped to a point on the focal plane.***

- ***Angle ➔ Position Mapping***
  - ***fv ➔y***

Off-axis collimating

Problem: Show that all rays from point y on focal plane are bent into parallel rays with angle v = -y/f.



$$\begin{pmatrix} \mathrm{y} \\ \mathrm{v} \end{pmatrix}_2 = \mathrm{L} \cdot \mathrm{T} \begin{pmatrix} \mathrm{y} \\ \mathrm{v} \end{pmatrix}_1$$

2nd 1st

•Beams "always" go from left to right
•Matrices are ordered right to left
•Deal with it! (be careful)

$$\begin{pmatrix} y \\ v \end{pmatrix}_2 = \begin{pmatrix} 1 & 0 \\ -f^{-1} & 1 \end{pmatrix} \begin{pmatrix} 1 & f \\ 0 & 1 \end{pmatrix} \begin{pmatrix} y \\ v \end{pmatrix}_1$$

$$\begin{pmatrix} y \\ v \end{pmatrix}_2 = \begin{pmatrix} 1 & f \\ -f^{-1} & 0 \end{pmatrix} \begin{pmatrix} y \\ v \end{pmatrix}_1$$

D = 0

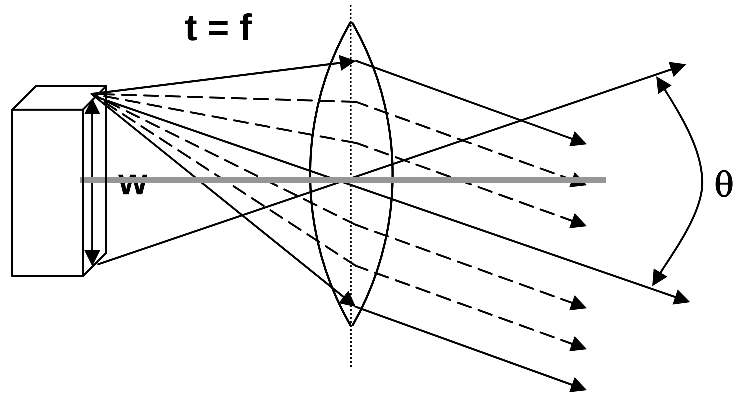
$$y_2 = 1 \cdot y_1 + f \cdot v_1$$

$$\boxed{v_2 = \frac{y_1}{-f}}$$

- ***D = 0 Collimating***
  - ***Rays from a point on focal plane are mapping into a collimated beam***
- ***Position ➔ Angle Mapping***
  - ***Position is transformed into angle v = -y/f***

Focal plane and FOV

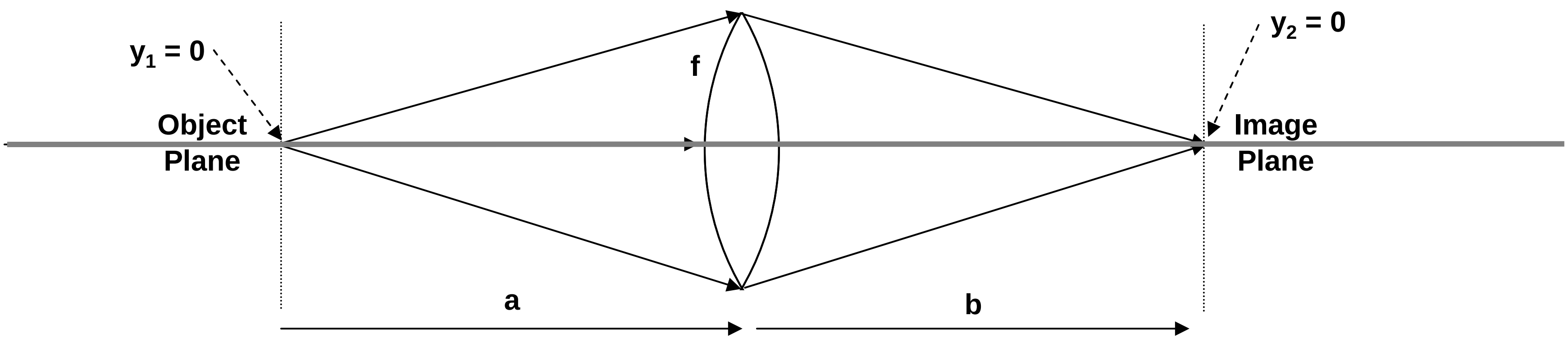
$$y = f \cdot v$$
$$2y = f \cdot 2v$$
$$W = f \cdot \theta$$



Example *We place an FPA (focal plane array) at the focal plane (distance f) of a lens. The size of the array is y1 = ±8mm (16 mm wide), and the field of view is v2 = ±15 degree (30 degrees total). What is the focal length of the lens?*

$$W = 16mm$$
$$\theta = 30° = 0.524 radians$$
$$f = \frac{W}{\theta} = 30.6mm$$

Object and image planes – on axis



Cascade 3 matrices
Watch order

$$\begin{pmatrix} 0 \\ v \end{pmatrix}_2 = T \cdot L \cdot T \cdot \begin{pmatrix} 0 \\ v \end{pmatrix}_1$$

$$\begin{pmatrix} 0 \\ v \end{pmatrix}_2 = \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -f^{-1} & 1 \end{pmatrix} \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix}_1$$

$$\begin{pmatrix} 0 \\ v \end{pmatrix}_2 = \frac{1}{f} \begin{pmatrix} a \cdot f - a \cdot b + b \cdot f \\ f - a \end{pmatrix} v_1$$

$$0 = a \cdot f - a \cdot b + b \cdot f$$

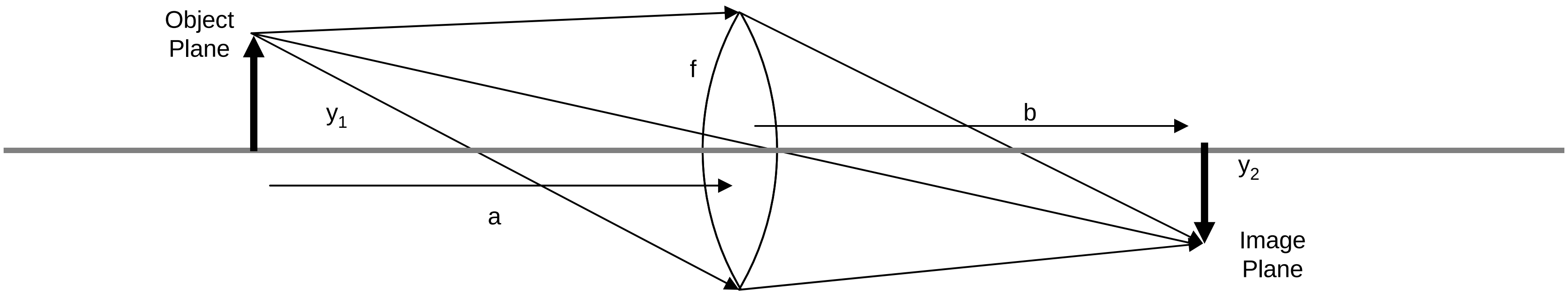
a = object distance
b = image distance

Imaging Law

$$\frac{1}{b} = \frac{1}{f} - \frac{1}{a}$$

$$\frac{1}{f} = \frac{1}{a} + \frac{1}{b}$$

Object/image planes
Method 1 Ray Vectors



$$\begin{pmatrix} y \\ v \end{pmatrix}_2 = \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -f^{-1} & 1 \end{pmatrix} \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \begin{pmatrix} y \\ v \end{pmatrix}_1$$

$$\begin{pmatrix} y \\ v \end{pmatrix}_2 = \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -(a^{-1}+b^{-1}) & 1 \end{pmatrix} \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \begin{pmatrix} y \\ v \end{pmatrix}_1$$

$$\begin{pmatrix} y \\ v \end{pmatrix}_2 = \begin{pmatrix} -b/a & 0 \\ -(a^{-1}+b^{-1}) & -a/b \end{pmatrix} \begin{pmatrix} y \\ v \end{pmatrix}_1$$

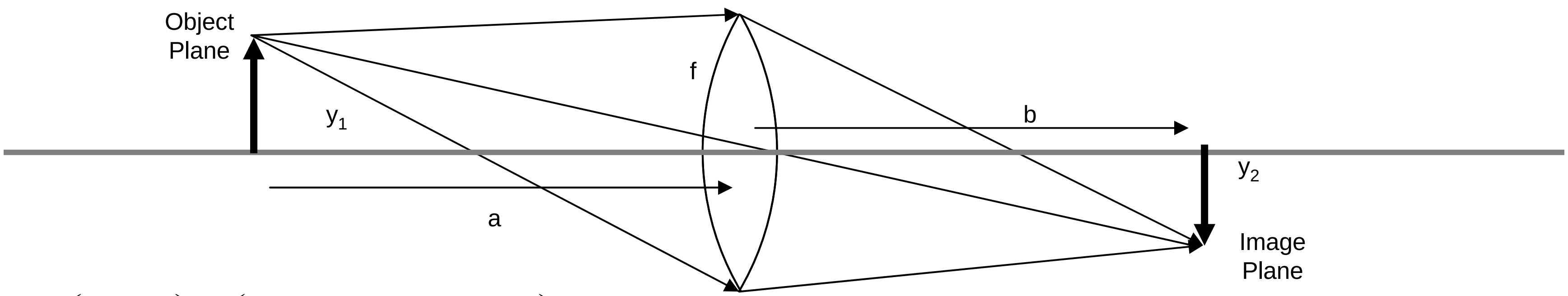
$$y_2 = -\frac{b}{a} y_1$$

$$y_2 = M_y \cdot y_1$$

$$\boxed{M_y = -\frac{b}{a}}$$

- ***y_2 = same for all angles v***
  - ***Point to point mapping between object and image plane***
- ***Scaling factor constant (all y)***
  - ***Image and object planes similar***
  - ***M < 0 inverted, M > 0 non-inverted***
  - ***|M| > 1 enlarged |M| < 1 reduced***

Object/image planes
Method 2 [ABCD]



$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} -b/a & 0 \\ -(a^{-1}+b^{-1}) & -a/b \end{pmatrix}$$

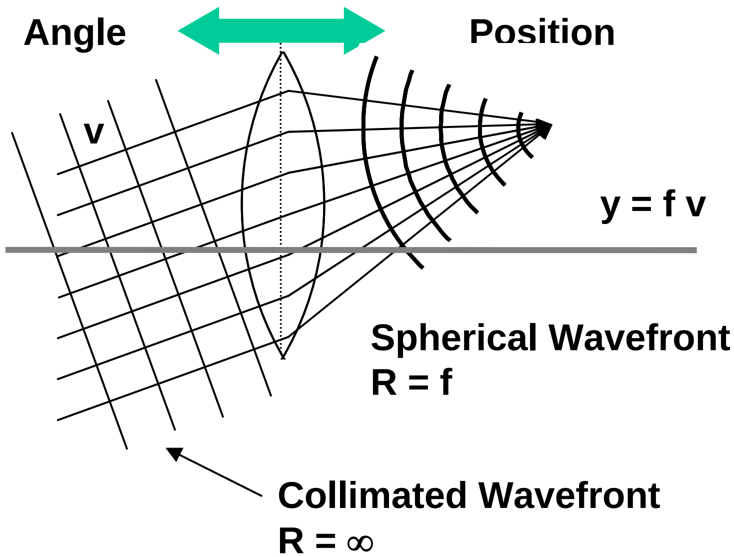
$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} M_y & 0 \\ C & M_v \end{pmatrix}$$

$$M_y = -\frac{b}{a}$$

Problem The distance between object and image is 1000 mm. The object is 3x larger than the inverted image. Calculate the focal length of the lens. Using ABCD matrices show that your calculated values for a, b, and f are correct.

- ***Method 1 – Ray vectors (more intuitive)***
- ***Method 2 – ABCD (less formal)***

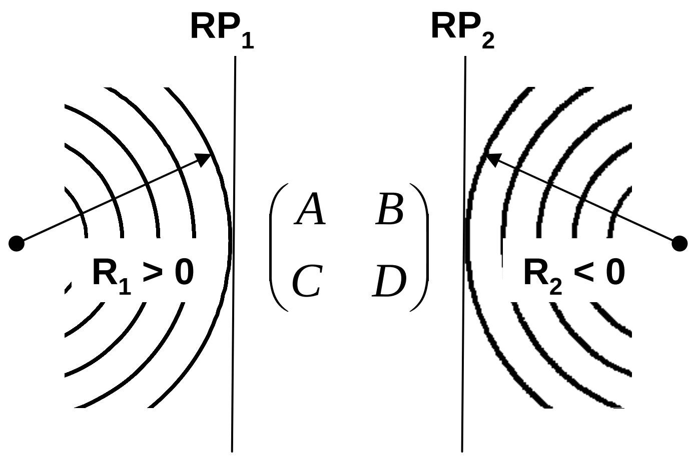
Waterfront radius of curvature



- ***Lens (or mirror) transforms wavefronts (WF)***
  - ***Collimated WF into Spherical WF center on FP***
  - ***Spherical WF center on FP into Collimated WF in the far-field***

- ***Special case***
  - ***Object (or Image) at focal plane***

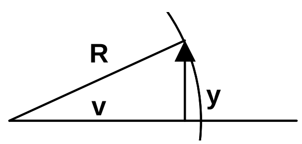
Wavefront optics radius of curvature (R)



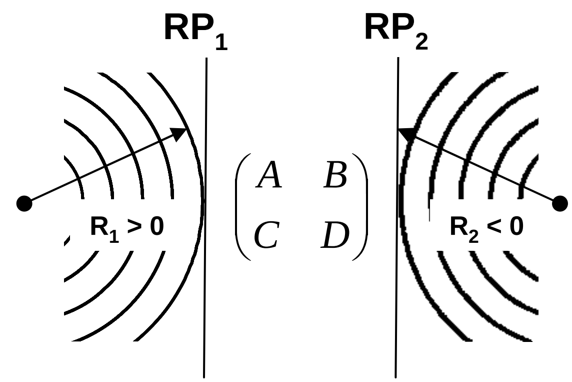
- ***Wavefront Radius of Curvature***
  - ***Not surface of lens or mirror***
- ***Sign convention***
  - ***R > 0, beam is <u>diverging</u>***
  - ***R < 0, beam is <u>converging</u>***
  - ***R = ∞, beam is <u>collimated</u>***
- ***[ABCD] changes R_1 ➔ R_2***

Ray optics: position (y) and angle (v) $\Rightarrow$ wavefront optics: radius of curvature (R)

$$\begin{pmatrix} y \\ v \end{pmatrix}_2 = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \cdot \begin{pmatrix} y \\ v \end{pmatrix}_1$$



$$y = v \cdot R$$



$$\begin{pmatrix} v \cdot R \\ v \end{pmatrix}_2 = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \cdot \begin{pmatrix} v \cdot R \\ v \end{pmatrix}_1$$

$$\frac{v_2 \cdot R_2}{v_2} = \frac{A \cdot v_1 \cdot R_1 + B \cdot v_1}{C \cdot v_1 \cdot R_1 + D \cdot v_1}$$

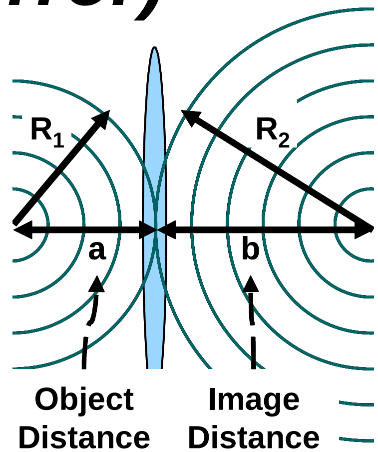
$$\boxed{R_2 = \frac{A \cdot R_1 + B}{C \cdot R_1 + D}}$$

- ***Lens changes R_1 ➔ R_2***
- ***"Bilinear transformation"***
  - ***Beam Optics (Section 4)***
  - ***In EE - Impedances and Admittances***
    - ***"Smith Charts"***
- ***Correct everywhere***
  - ***R (real) ➔ q (complex)***
  - ***Section 4***

Radius of curvature + lens (mirror)

$$\begin{pmatrix} vR \\ v \end{pmatrix}_2 = \begin{pmatrix} 1 & 0 \\ \dfrac{-1}{f} & 1 \end{pmatrix} \begin{pmatrix} vR \\ v \end{pmatrix}_1$$

$$\frac{v_2 R_2}{v_2} = \frac{v_1 R_1}{\dfrac{-v_1 R_1}{f} + v_1}$$



$$\mathrm{R}_1 = +\mathrm{a}$$ + diverging

$$\mathrm{R}_2 = -\mathrm{b}$$ - converging

$$\frac{1}{R_2} = \frac{1}{R_1} - \frac{1}{f}$$

Radius of Curvature Law

$$\frac{1}{\mathrm{a}} + \frac{1}{\mathrm{b}} = \frac{1}{\mathrm{f}}$$

Object-Image Law

- ***Radius of Curvature Law is correct, even near beam waists***
- ***Object-image Law – distances of wavefront centers of curvatures***
  - ***(which are not at the beam waists)***
- ***Karl Friedrich Gauss, 1840.***

Wavefront curvature (K)

$$\frac{1}{R_2} = \frac{1}{R_1} - \frac{1}{f}$$

$$K \equiv \frac{1}{R}$$

Curvature (K) = 1/R **[m^{-1}]**

$$P \equiv \frac{1}{f}$$

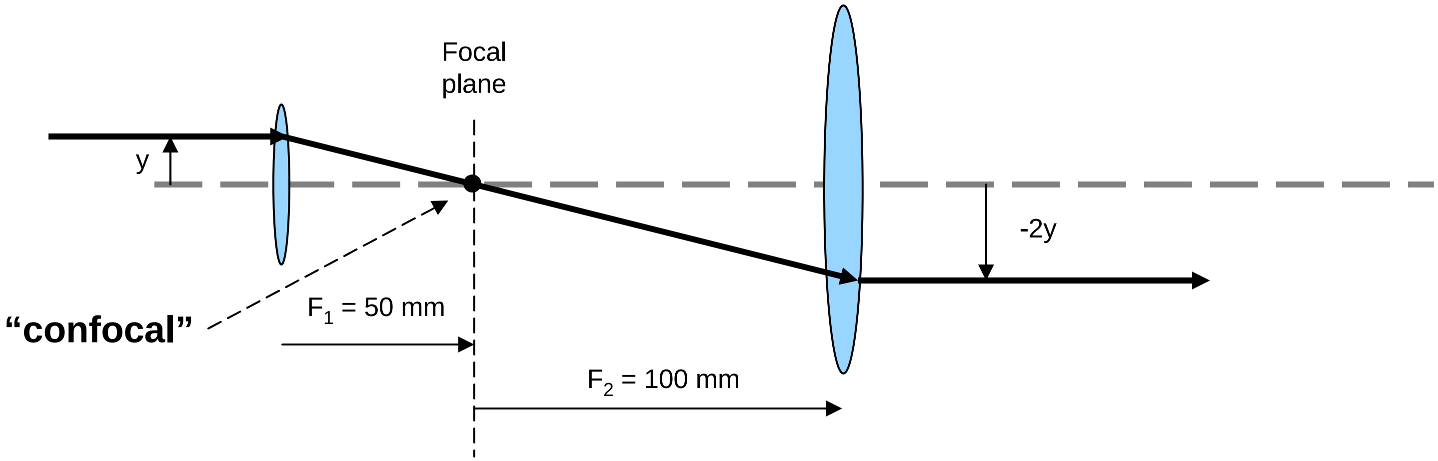
Power (P) = 1/f **[diopter = m-1]**

$$\boxed{K_2 = K_1 - P}$$

A lens changes wavefront <u>curvature</u>

- **Converging lens <u>subtracts</u> from curvature (K)**
- **Diverging lens <u>adds</u> to curvature (K)**

2x confocal telescope (Kipler, 1604) – on-axis



$$\begin{pmatrix} y_2 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{-1}{100} & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 150 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ \frac{-1}{50} & 1 \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ 0 \end{pmatrix}$$

$$\boxed{\begin{pmatrix} y_2 \\ v_2 \end{pmatrix} = \begin{pmatrix} -2 \cdot y_1 \\ 0 \end{pmatrix}}$$

$$M_y = \frac{d}{dy_1} y_2 \qquad M_y = -2$$

Two lenses separated by the sum of their focal lengths

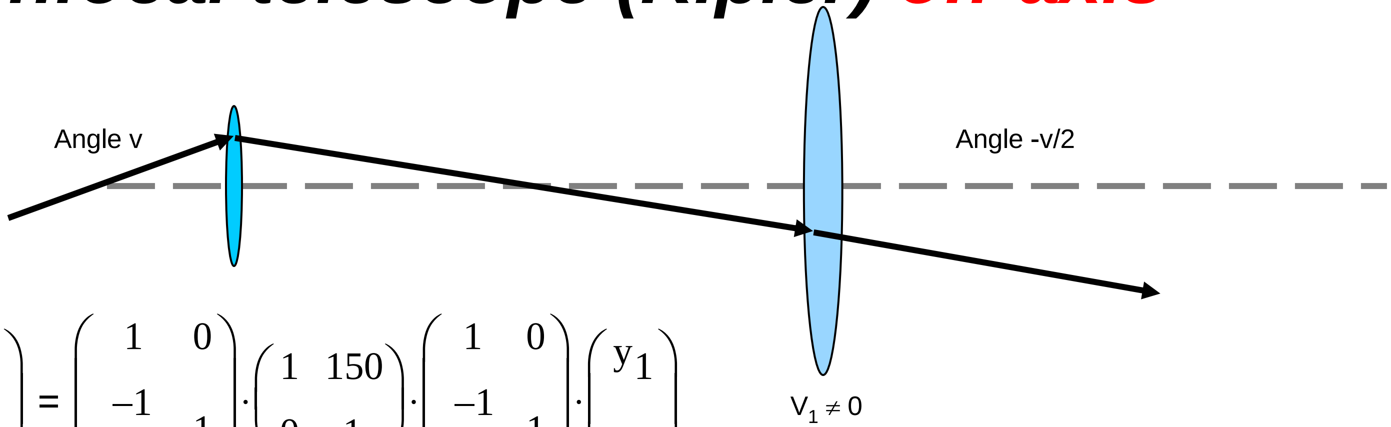
Collimated in = collimated out

2x Beam Expander

Redistributes beam linearly

Mag < 0 Inverting

2x confocal telescope (Kipler) off-axis



$$\begin{pmatrix} y_2 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{-1}{100} & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 150 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ \frac{-1}{50} & 1 \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ v_1 \end{pmatrix}$$

$$\begin{pmatrix} y_2 \\ v_2 \end{pmatrix} = \begin{pmatrix} -2 \cdot y_1 + 150 \cdot v_1 \\ \frac{-1}{2} \cdot v_1 \end{pmatrix}$$

$$M_y = \frac{d}{dy_1} y_2 \qquad M_y = -2$$

$$M_v = \frac{d}{dv_1} v_2 \qquad M_v = -0.5$$

$$M_y \cdot M_v = 1 \qquad M_v = \frac{1}{M_y}$$

Two lenses separated by the sum of their focal lengths

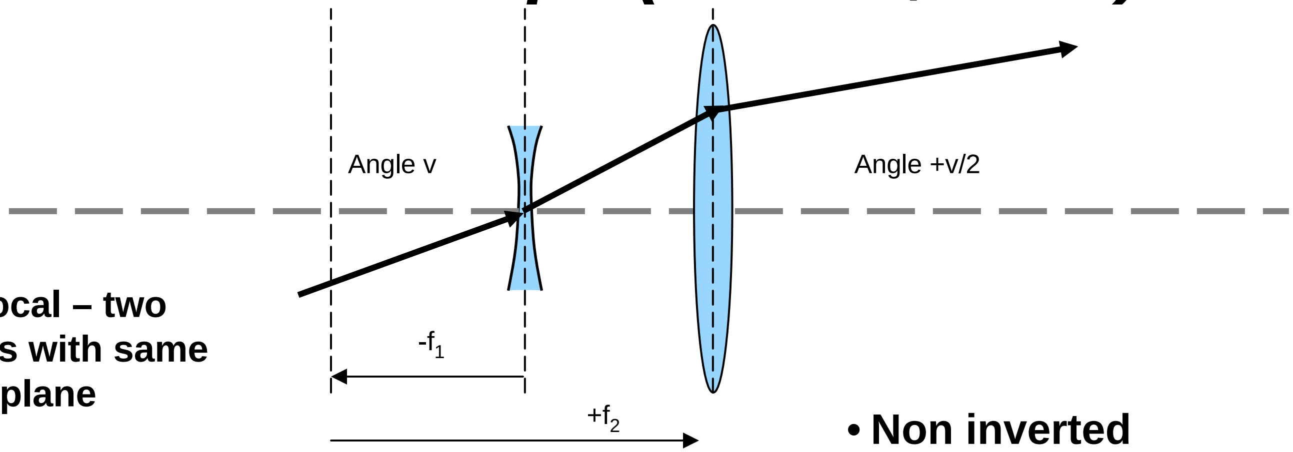
Collimated in = collimated out

2x Beam Expander

Redistributes beam linearly

Mag < 0 Inverting

2x confocal telescope (Galileo, 1609)



Confocal – two lenses with same focal plane

- **Non inverted**
  - **No beam convergence to a point**
- **Shorter, more compact**
  - **L = $f_2 - f_1$**

$$\begin{pmatrix} y_2 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{-1}{f_2} & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & f_2 - f_1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ \frac{1}{f_1} & 1 \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ v_1 \end{pmatrix}$$

$$y_2 = \frac{\left(f_2 \cdot y_1 + f_1 \cdot v_1 \cdot f_2 - f_1^2 \cdot v_1\right)}{f_1} \qquad M_y = \frac{d}{dy_1} y_2 \qquad \boxed{M_y = \frac{f_2}{f_1}}$$

$$v_2 = f_1 \cdot \frac{v_1}{f_2} \qquad M_v = \frac{d}{dv_1} v_2 \qquad \boxed{M_v = \frac{f_1}{f_2}} \qquad M_y \cdot M_v = 1$$

Addition of lens power

Two translations (no lens) $\begin{pmatrix} 1 & t_1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & t_2 \\ 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & t_2 + t_1 \\ 0 & 1 \end{pmatrix}$ $t = t_1 + t_2$

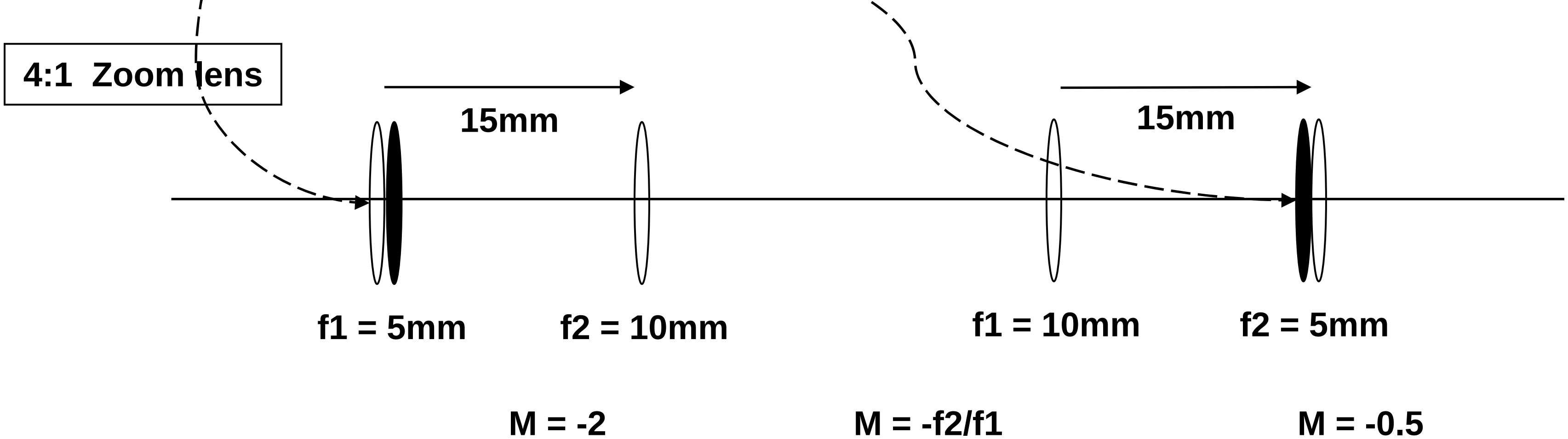
$$K = -\frac{1}{f}$$

Two lenses (no space) $\begin{pmatrix} 1 & 0 \\ K_1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ K_2 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 \\ K_1 + K_2 & 1 \end{pmatrix}$ $K = K_1 + K_2$

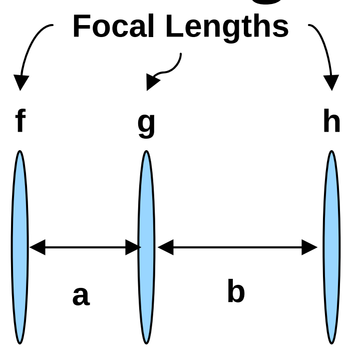
Diopters add

$$\boxed{\frac{1}{10} + \frac{1}{10} \rightarrow \frac{1}{5}}$$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$



Variable magnification telescope

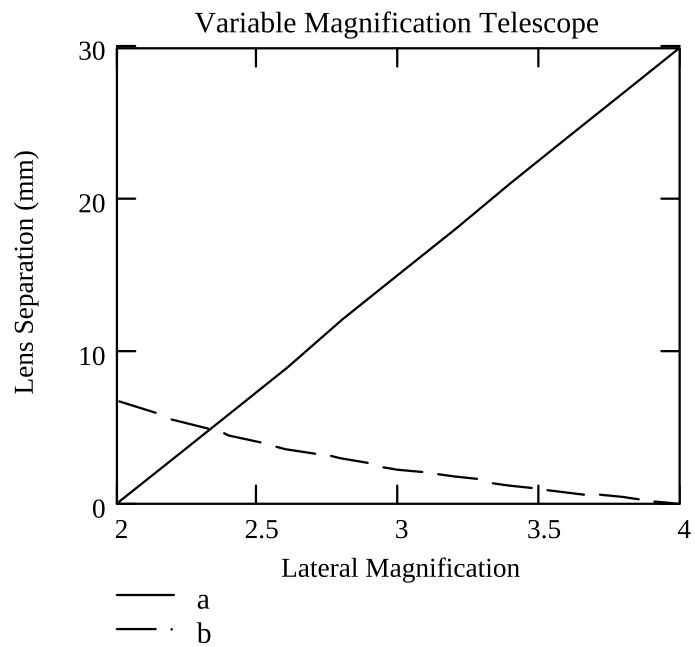


$$M = magnification$$

$$a = f + g + \frac{fg}{h} M$$

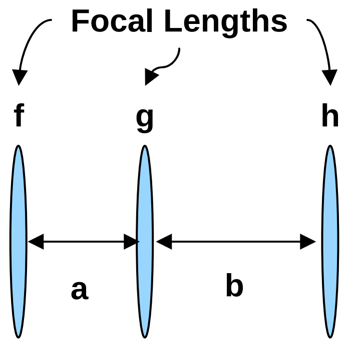
$$b = g + h + \frac{gh}{f} M^{-1}$$

f = - 10mm	g = - 20 mm	h = 13.33mm
M	**a (mm)**	**b (mm)**
2.0	0	6.67
2.2	3	5.45
2.4	6	4.44
2.6	9	3.59
2.8	12	2.86
3.0	15	2.22
3.2	18	1.67
3.4	21	1.18
3.6	24	0.74
3.8	27	0.35
4.0	30	0



- ***Vary magnification 2 ➔ 4 by mechanically changing lens separations***
- ***Sometimes called a Zoom lens!***
- ***Equations were derived from ABCD matrix, C = 0***

Example: variable magnification telescope



$$M = magnification$$

$$a = f + g + \frac{fg}{h} M$$

$$b = g + h + \frac{gh}{f} M^{-1}$$

f = -10mm　　　　g = -20mm　　　　h = +13.333mm

Problem For a = 12mm, b = 2.86 mm, cascade the LTLTL matrices for an on-axis beam. Show that M = 2.8

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} := \overset{\textbf{L}}{\begin{pmatrix} 1 & 0 \\ \frac{-1}{13.333} & 1 \end{pmatrix}} \cdot \overset{\textbf{T}}{\begin{pmatrix} 1 & 2.86 \\ 0 & 1 \end{pmatrix}} \cdot \overset{\textbf{L}}{\begin{pmatrix} 1 & 0 \\ \frac{-1}{-20} & 1 \end{pmatrix}} \cdot \overset{\textbf{T}}{\begin{pmatrix} 1 & 12 \\ 0 & 1 \end{pmatrix}} \cdot \overset{\textbf{L}}{\begin{pmatrix} 1 & 0 \\ \frac{-1}{-10} & 1 \end{pmatrix}}$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 2.801 & 16.576 \\ -5.025 \times 10^{-5} & 0.357 \end{pmatrix}$$

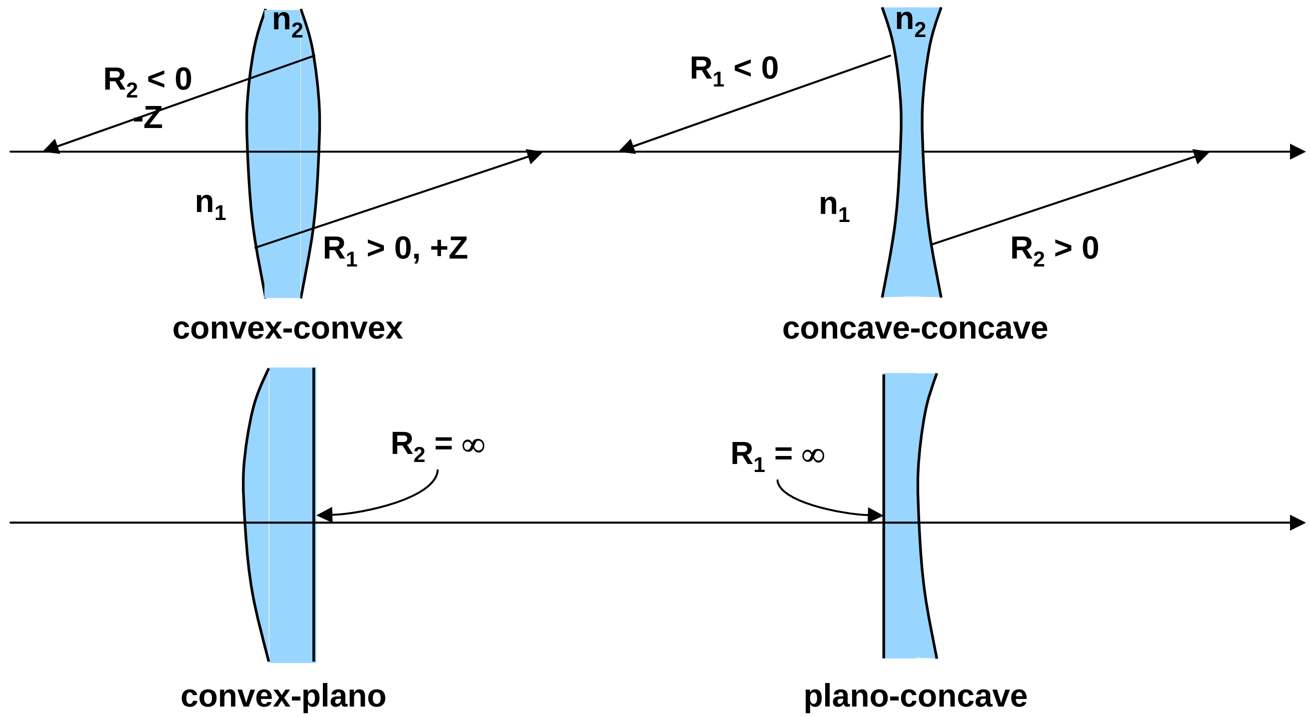
M = 2.8

C ≈ 0, beam expander

That simple!

- **Matrices manage the variables**
- **Mathcad manages the math**

Optical surfaces



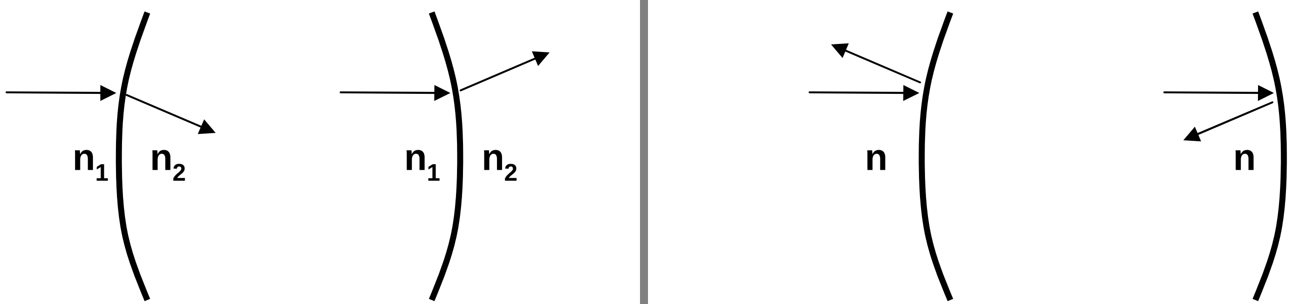
+R, if center of curvature is on the <u>same</u> side as ray <u>out</u>
-R, if center of curvature is on the <u>opposite</u> side as ray <u>out</u>

Be careful: Lens (or mirror) surface radius of curvature IS NOT the same as the wavefront radius of curvature.

L-Matrix at each material interface

$$L = \begin{pmatrix} 1 & 0 \\ -P & 1 \end{pmatrix}$$

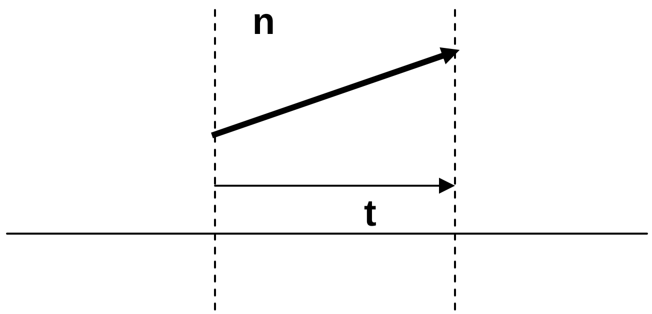
Refraction		Reflection	



Refraction		Reflection	
$P = \dfrac{n_2 - n_1}{+R}$	$P = \dfrac{n_2 - n_1}{-R}$	$P = \dfrac{2n}{-R}$	$P = \dfrac{2n}{+R}$

- **At each material interface, beams are transmitted and reflected.**

T-matrix in materials



$$T = \begin{pmatrix} 1 & \frac{t}{n} \\ 0 & 1 \end{pmatrix}$$

in material, n > 1

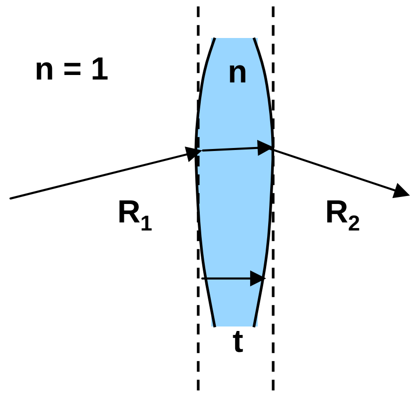
$$T = \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix}$$

in free space, n = 1

Why t/n?

- ***Snell's law*** $n_1 \sin(v_1) = n_2 \sin(v_2)$
  - ***Small angles*** $n_1 v_1 = n_2 v_2$
- ***Redefine*** $(n v = V)$, $(t/n) = T$
  - ***Then*** $y_2 = y_1 + vt = y_1 + VT$
- ***Inside material***
  - ***calculated angle = v***
  - ***physical angle = v/n***

Thin lens t = 0



$$M = L_2 \cdot L_1$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ -\left(\dfrac{1-n}{-R_2}\right) & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ -\left(\dfrac{n-1}{+R_1}\right) & 1 \end{bmatrix}$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\dfrac{1}{f} & 1 \end{pmatrix} \quad \textbf{where} \quad \boxed{\frac{1}{f} = (n-1)\cdot\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$$

- ***Lensmakers Law***
  - ***Use to calculate the effects of temperature and wavelength on the focal length of a lens.***

Temperature and wavelength effects

$$R(\Delta T) = R_0(1 + CTE \cdot \Delta T)$$

$$n(\lambda, \Delta T) = n(\lambda,) + \frac{dn}{dt} \cdot \Delta T$$

BK7 Glass

- **CTE = 7.1 x 10^{-6}/°C**
- **dn/dt = 2.5 x 10^{-6}/°C**

- **λ= 1.06 μm   n = 1.50669**
- **λ = 1.53 μm   n = 1.50094**

Problem At room temperature (20 °C) and at λ= 1.06 μm, the focal length of a lens is 100 mm. Use the Lensmakers Law to calculate its focal length at wavelengths 1.06 μm and 1.53 μm at –40 °C and 50 °C (typical military temperature range).

Solution #1 (Numeric) For each R ➔ R (1 + CTE ΔT), n ➔ n + Δn

Solution #2 (Algebraic) Differentiate the Lensmakers Law and show that

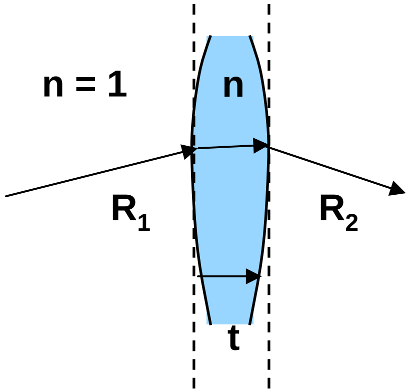
$$\frac{\Delta F}{F} = 1 + \delta_{\Delta T} + \delta_{\Delta\lambda}$$

$$\delta_{\Delta T} = \left[CTE - \frac{1}{n_\lambda - 1}\frac{dn}{dT}\right]\Delta T$$

$$\delta_{\Delta\lambda} = \frac{\Delta n_\lambda}{n_\lambda - 1}$$

- ***For imaging, effects of T, λ may be critical***
- ***For laser beams, probably not!***
- ***Use ABCD to calculate the effects on T, λ on beam performance.***
  - ***Optical System Engineer should establish (and limit) the requirements on the Optical Designer)***
- ***Algebraic form – trade studies***

Thick lens



Problem Use the equivalence theorem to calculate the thin lens equivalent focal length of a thick lens.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = L_2 \cdot T \cdot L_1$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\left(\frac{1-n}{-R_2}\right) & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & \frac{t}{n} \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ -\left(\frac{n-1}{+R_1}\right) & 1 \end{bmatrix}$$

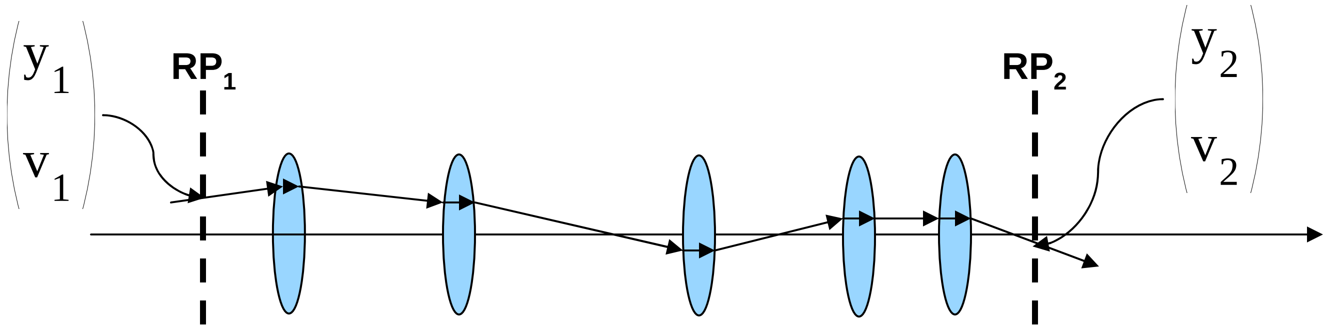
Calculate C. Show that 1/f = -C yields the Lensmakers Law plus a correction term.

$$\boxed{\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - \frac{(n-1)^2}{n}\frac{t}{R_1 R_2}}$$

- *A plano-concave (or plano-convex) lens*
  - *R1 (or R2) = ∞ ➔ focal length is independent of t (no matter how big)*

Break (5 min)

General scheme



21 matrices:

- 10 surfaces
- 6 air translations
- 5 material translations

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = M_{21} \cdot M_{21} \cdot M_{21} \cdot \ldots\ldots\ldots\ldots\ldots M_2 \cdot M_1$$

$$\begin{pmatrix} y \\ v \end{pmatrix}_2 = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} y \\ v \end{pmatrix}_1$$

$check$

$$A \cdot B - C \cdot D = 1$$

- ***No matter how complex, just a matter of L + T + L +***
  - *Focus on the “gozintas” and “gozoutas”*
- ***Matrix algebra keeps the math in order***
  - *Mathcad (Matlab) does the calculation*
- ***Analyze***
  - *Numerically - interested in the performance*
  - *Algebraically - designing, or optimizing*

Equivalence theorem M = TLT

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & h_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\dfrac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} 1 & h_1 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = T \cdot L \cdot T$$

where

$$\mathrm{f} = \frac{-1}{\mathrm{C}}$$

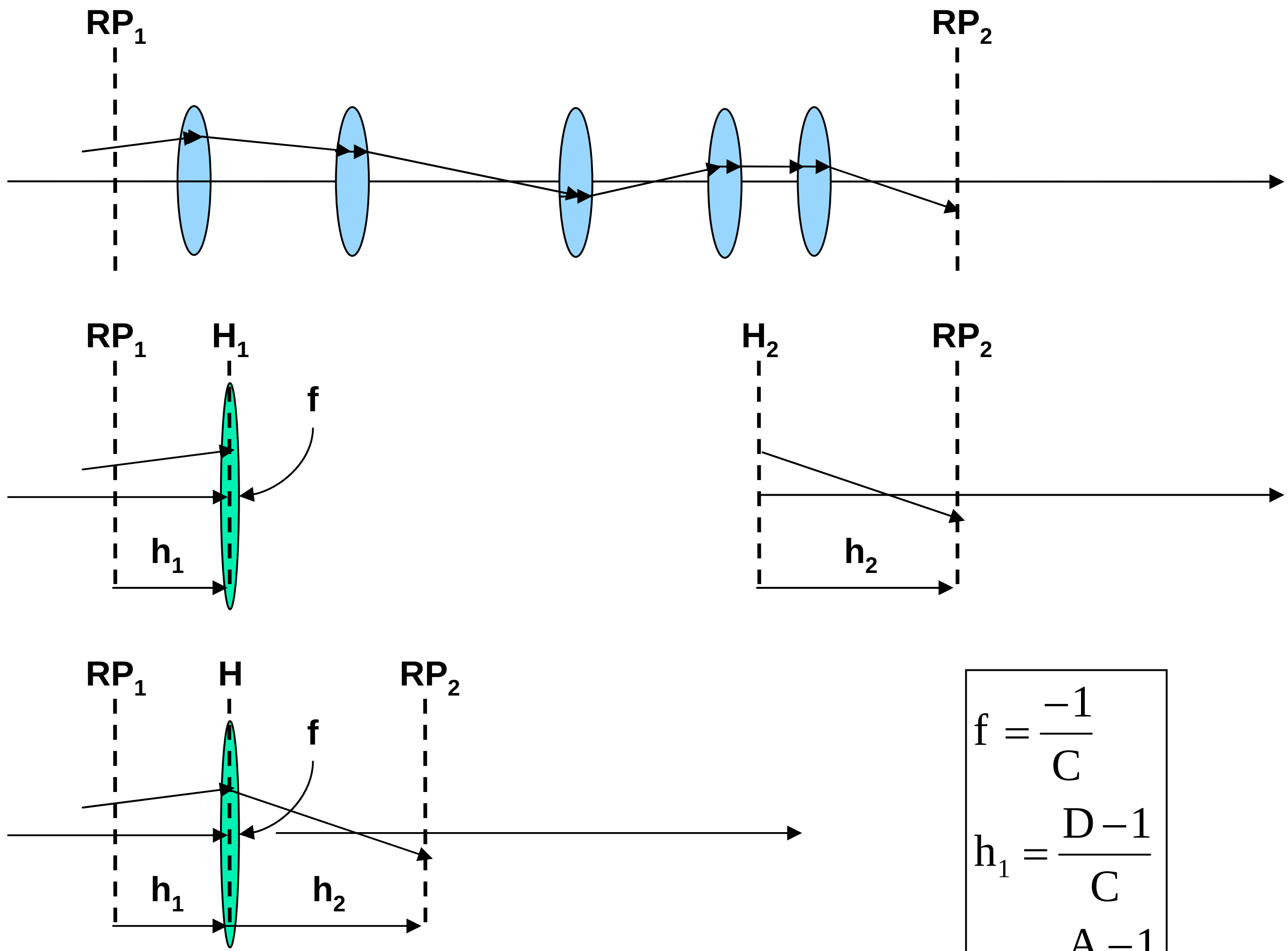
$$\mathrm{h}_1 = \frac{\mathrm{D}-1}{\mathrm{C}}$$

$$\mathrm{h}_2 = \frac{\mathrm{A}-1}{\mathrm{C}}$$

Problem: Substitute for f, h_1, and h_2, and using AD-BC = 1, prove the equality.

- ***If C $\neq$0, then optical system is equivalent to a thin lens at a specific location.***
- ***Any optical system***
  - ***If C =$\neq$0, then the equivalent lens f = -1/C***
  - ***If C = 0, the f, h_1 and h_2 are all infinite (beam expander)***

Equivalence theorem



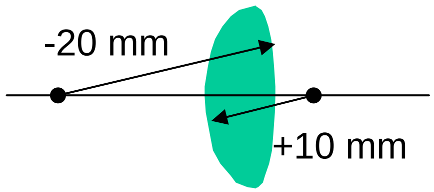
$$f = \frac{-1}{C}$$

$$h_1 = \frac{D-1}{C}$$

$$h_2 = \frac{A-1}{C}$$

Reversing non-symmetric, thick lenses

Forward



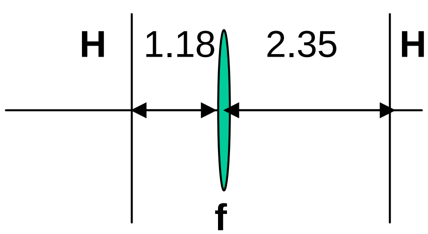
$$M := \begin{bmatrix} 1 & 0 \\ \frac{-(1-n)}{-20} & 1 \end{bmatrix} \cdot \begin{pmatrix} 1 & \frac{5}{n} \\ 0 & 1 \end{pmatrix} \cdot \begin{bmatrix} 1 & 0 \\ \frac{-(n-1)}{10} & 1 \end{bmatrix}$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} := M \qquad M = \begin{pmatrix} 0.833 & 3.333 \\ -0.071 & 0.917 \end{pmatrix}$$

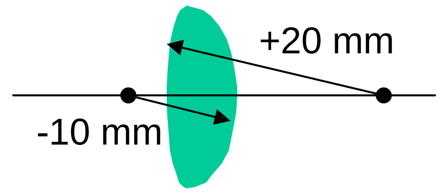
$$h_1 := \frac{D-1}{C} \qquad \boxed{h_1 = 1.176}$$

$$h_2 := \frac{A-1}{C} \qquad \boxed{h_2 = 2.353}$$

$$f := \frac{-1}{C} \qquad \boxed{f = 14.118}$$



Reversed



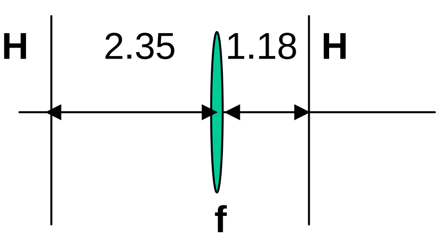
$$M_{rev} := \begin{bmatrix} 1 & 0 \\ \frac{-(1-n)}{-10} & 1 \end{bmatrix} \cdot \begin{pmatrix} 1 & \frac{5}{n} \\ 0 & 1 \end{pmatrix} \cdot \begin{bmatrix} 1 & 0 \\ \frac{-(n-1)}{20} & 1 \end{bmatrix}$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} := M_{rev} \qquad M_{rev} = \begin{pmatrix} 0.917 & 3.333 \\ -0.071 & 0.833 \end{pmatrix}$$

$$h_1 := \frac{D-1}{C} \qquad \boxed{h_1 = 2.353}$$

$$h_2 := \frac{A-1}{C} \qquad \boxed{h_2 = 1.176}$$

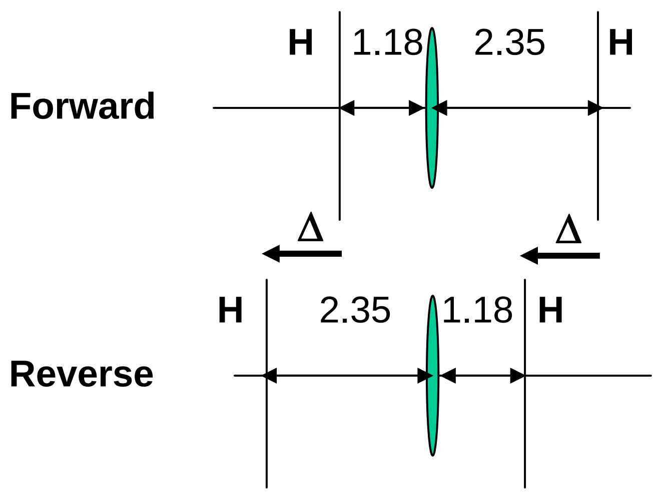
$$f := \frac{-1}{C} \qquad \boxed{f = 14.118}$$



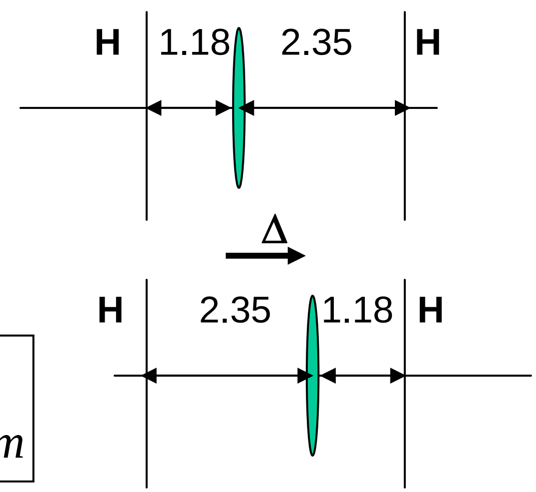
Reversing non-symmetric, thick lenses

Case One
- Place non-symmetric lens REVERSED at the **same** position.

Case Two
- Place non-symmetric lens REVERSED and **shift** position **+**Δ.



$$\boxed{\begin{aligned}\Delta &= h_2 - h_1\\ \Delta &= 1.17 \quad mm\end{aligned}}$$

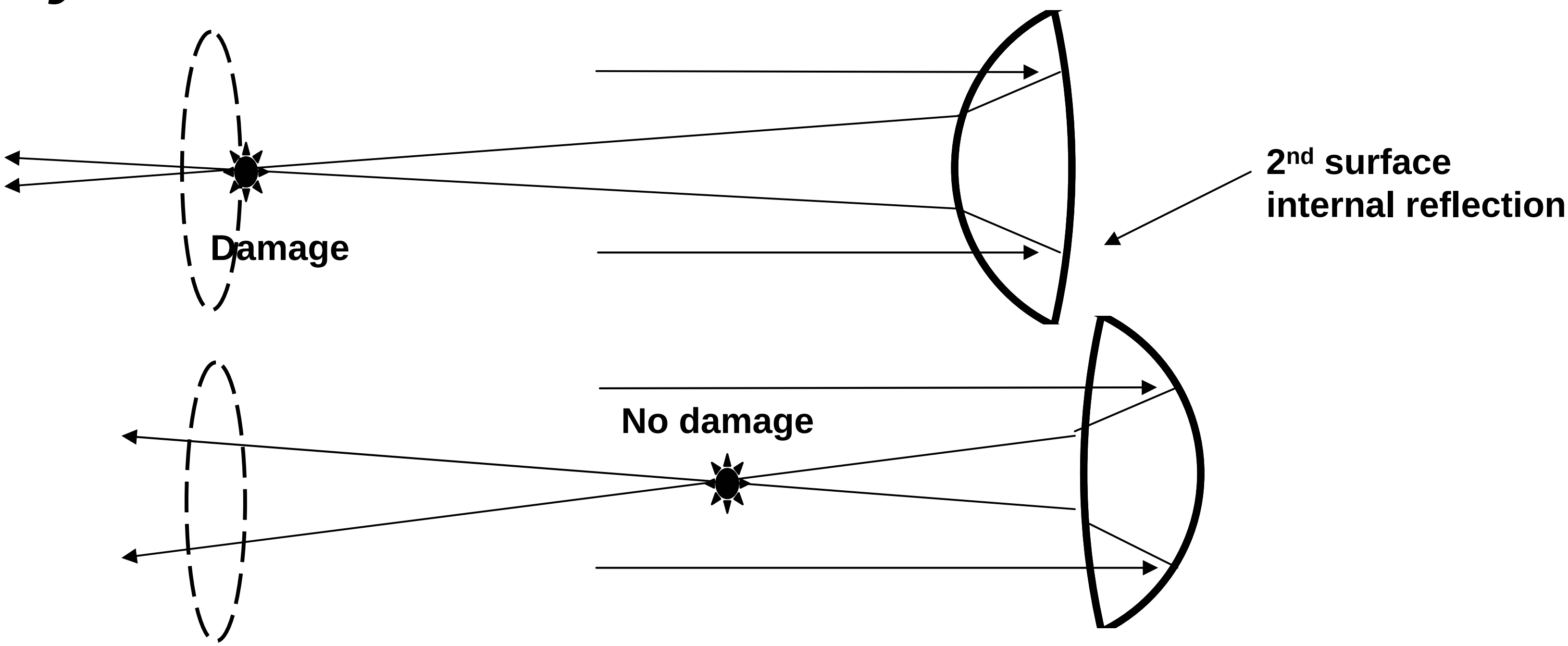


- Principle planes (H) **shift** position **–**Δ
- **Optical path is changed.**

- Principle planes (H) remain at **same** position.
- **Optical path is unchanged.**

- ***Mark non-symmetric lenses to indicate 1st and 2nd surfaces.***

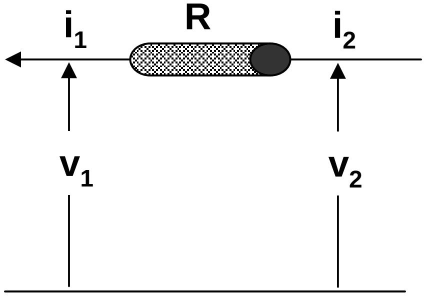
May want to reverse lens



- ***Reflections of first, second surface***
  - ***Either diverge, or will focus beam to spot somewhere!***
  - ***High energy/area cause laser damage***
    - ***1% reflected into a spot 100x smaller than beam***
    - ***100^2 x 0.01 = 100 times higher density***
- ***Keep refocused "ghost" beams away from components.***
  - ***If can, all surfaces with R > 0 – reflected wave diverges.***
  - ***Calculate where the ghost (refocused waists) are located (see Sec. 4)***
  - ***Reverse (and shift) lenses as necessary***

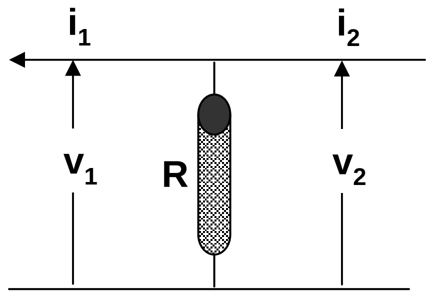
Electrical analog

Series Resistor



$$\begin{pmatrix} V_2 \\ I_2 \end{pmatrix} = \begin{pmatrix} 1 & R \\ 0 & 1 \end{pmatrix} \begin{pmatrix} V_1 \\ I_1 \end{pmatrix}$$

Shunt Resistor



$$\begin{pmatrix} V_2 \\ I_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{1}{R} & 1 \end{pmatrix} \begin{pmatrix} V_1 \\ I_1 \end{pmatrix}$$

Impedance

$$\begin{pmatrix} V_2 \\ I_2 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_1 \\ I_1 \end{pmatrix}$$

$$V = I \cdot Z$$

$$\begin{pmatrix} I_2 Z_2 \\ I_2 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} I_1 Z_1 \\ I_1 \end{pmatrix}$$

$$Z_2 = \frac{A \cdot Z_1 + B}{C \cdot Z_1 + D}$$

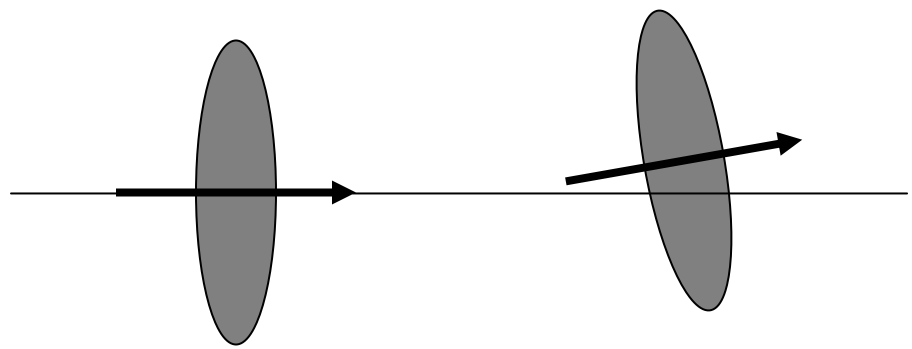
- ***Two-port cascade ABCD matrix methods used widely***
  - ***Electrical circuits, transmission line circuits***
  - ***Ray optics, beam optics***
- ***Optical System can be represented by an electrical circuit of series and shunt resistors.***
  - ***Elegant, but not particularly useful!***
  - ***With enough effort, the entire universe can be made to look like Ohm's Law.***

Effects of Manufacturing Tolerances and Misalignment

Sources of error

- ***Focal Length***
  - ☑***Temperature change***
  - ☑***Wavelength***
  - – *Surface error (radius of curvature)*
- ***Transverse Alignment***
  - ☑***Tilt (2)***
  - ☑***Decenter (2)***
- ***Longitudinal Alignment***
  - – *Rotation (unless axially symmetric)*
  - – *Spacing (1)*
- ***Radiometric***
  - – *Reflection, absorption*
  - – *Polarization*
- ***etc***

Tilts and decenters

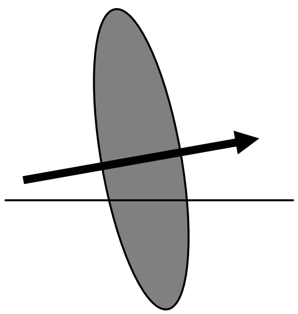


Angular alignment	**tilt**	**a (radians)**
Lateral displacement	**decenter**	**d (mm)**

Sources of component tilt and decenter

- ***Manufacture of lenses (mirrors)***
- ***Manufacture of mounting assemblies***
- ***Installation and alignment***

Errors, tolerances, budgets, and requirements



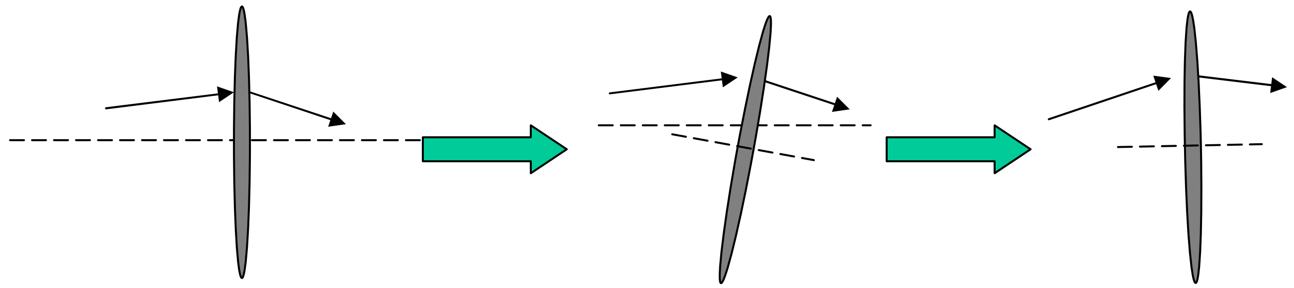
	Decenter	Tilt
Low Cost	0.50 mm	50 mrad
Commercial	0.25 mm	5 mrad
Precision	0.10 mm	2 mrad
Extra Precise	As req'd	As req'd

Poor Engineering	**Good Engineering**
Be Safe	Be Smart
Buy the best	Know what is needed, and not needed
Machine to the most stringent tolerances affordable	**Understand what tolerances are sufficient for the required performance**
Adjust everything during the assembly	Eliminate as many alignments as possible

Know what tolerances are required

- ***What are effects of tilt and decenter on the optical system?***
  - ***Which tilts, decenters affect system performance?***
  - ***Which don't? (You'd be surprised).***
- ***How large can each error be and still meet requirements?***
- ***Tolerance and alignment errors (stack up)***
  - ***Can reduce as well as increase error (partial self-compensation)***
  - ***One or two adjustments can compensate for errors in many components***
- ***What is the minimum number of adjustments needed?***
  - ***Bolt together assembly as much as possible***

ABCDs of tilts and decenters



No tilt, no decenter

Lens tilt and decenter
Rays fixed

Lens fixed
Rays tilt and decenter

$$\begin{pmatrix} y_2 \\ v_2 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} y_1 \\ v_1 \end{pmatrix}$$

$$\begin{pmatrix} y_2 + d \\ v_2 + a \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} y_1 + d \\ v_1 + a \end{pmatrix}$$

Tilt/decenter are in ray vector

- ***In order to cascade the matrices, we must move the tilts/decenter (d,a) into the ABCD matrix.***

TD matrix (tilt/decenter)

2×2

$$\begin{pmatrix} y \\ v \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} y \\ v \end{pmatrix}$$

3×3

$$\begin{pmatrix} y \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} y \\ v \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} y+d \\ v+a \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & d \\ 0 & 1 & a \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} y \\ v \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} y+d \\ v+a \\ 1 \end{pmatrix} = TD_+ \begin{pmatrix} y \\ v \\ 1 \end{pmatrix}$$

where

$$TD_+ \equiv \begin{pmatrix} 1 & 0 & +d \\ 0 & 1 & +a \\ 0 & 0 & 1 \end{pmatrix}$$

inverse

$$TD_- \equiv \begin{pmatrix} 1 & 0 & -d \\ 0 & 1 & -a \\ 0 & 0 & 1 \end{pmatrix}$$

$$TD_+ \cdot TD_- = I$$

$$TD_- \cdot TD_+ = I$$

Misaligned [ABCD]

2 x 2

$$\begin{pmatrix} y_2 + d \\ v_2 + a \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \cdot \begin{pmatrix} y_1 + d \\ v_1 + a \end{pmatrix}$$

3 x 3
TD matrix

$$\begin{pmatrix} 1 & 0 & d \\ 0 & 1 & a \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} y_2 \\ v_2 \\ 1 \end{pmatrix} = \begin{pmatrix} A & B & 0 \\ C & D & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & d \\ 0 & 1 & a \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ v_1 \\ 1 \end{pmatrix}$$

Inverse
TD matrix

$$\begin{pmatrix} 1 & 0 & -d \\ 0 & 1 & -a \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & d \\ 0 & 1 & a \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} y_2 \\ v_2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -d \\ 0 & 1 & -a \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} A & B & 0 \\ C & D & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & d \\ 0 & 1 & a \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ v_1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} y_2 \\ v_2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -d \\ 0 & 1 & -a \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} A & B & 0 \\ C & D & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & d \\ 0 & 1 & a \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ v_1 \\ 1 \end{pmatrix}$$

Misaligned [ABCD]

2 x 2

3 x 3
TD matrix

Inverse
TD matrix

Misaligned [ABCD]

Component Decenter
Component Tilt

$$\begin{pmatrix} y_2 \\ v_2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -d \\ 0 & 1 & -a \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} A & B & 0 \\ C & D & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & d \\ 0 & 1 & a \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ v_1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} y_2 \\ v_2 \\ 1 \end{pmatrix} = \begin{pmatrix} A & B & \Delta Y \\ C & D & \Delta V \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ v_1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} y_2 \\ v_2 \\ 1 \end{pmatrix} = \begin{pmatrix} A & B & 0 \\ C & D & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ v_1 \\ 1 \end{pmatrix} + \begin{pmatrix} \Delta Y \\ \Delta V \\ 0 \end{pmatrix}$$

Beam Decenter
Beam Tilt

$$\begin{pmatrix} y_2 \\ v_2 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ v_1 \end{pmatrix} + \begin{pmatrix} \Delta Y \\ \Delta V \end{pmatrix}$$

Misaligned [ABCD]

$$\begin{pmatrix} A & B & \Delta Y \\ C & D & \Delta V \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -d \\ 0 & 1 & -a \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} A & B & 0 \\ C & D & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & +d \\ 0 & 1 & +a \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} A & B & \Delta Y \\ C & D & \Delta V \\ 0 & 0 & 1 \end{pmatrix} = TD_{-} \cdot \begin{pmatrix} A & B & 0 \\ C & D & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot TD_{+}$$

- ***Misaligned [ABCD] has ΔY, ΔV in 3rd column***
- ***TD matrix pre- and post-multiplies [ABCD]***
  - *Each lens has its own TD {d,a}*
- ***Calculate the effects of the {d,a} misalignments of all the components***
  - *Some may have no effect; some may compensate; some may add*
  - *A decenter {d) of one component may cause a beam tilt (ΔV) at another*
- ***Multiple misalignments are to difficult to manage by hand***
  - *Matrix algebra manages the complexity*

Misaligned [ABCD] Optical Train

$$\begin{pmatrix} A & B & \Delta Y \\ C & D & \Delta V \\ 0 & 0 & 1 \end{pmatrix} = ..etc... \begin{pmatrix} 1 & 0 & -d_2 \\ 0 & 1 & -a_2 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} A_2 & B_2 & 0 \\ C_2 & D_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & +d_2 \\ 0 & 1 & +a_2 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & -d_1 \\ 0 & 1 & -a_1 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} A_2 & B_2 & \Delta Y_2 \\ C_2 & D_2 & \Delta V_2 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & +d_1 \\ 0 & 1 & +a_1 \\ 0 & 0 & 1 \end{pmatrix}$$

Beam decenter and tilt

All component decenters and tilts

- ***Calculate the effects of the {d,a} misalignments of all the components***
  - *Some may have no effect; some may compensate; some may add*
  - *A decenter {d) of one component may cause a beam tilt (ΔV) at another*
- ***Multiple misalignments are to difficult to manage by hand***
  - *Matrix algebra manages the complexity*

Beam Tilt and Decenter {DY, ΔV}

$$\begin{pmatrix} y_2 \\ v_2 \\ 1 \end{pmatrix} = \begin{pmatrix} A & B & \Delta Y \\ C & D & \Delta V \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ v_1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} y_2 \\ v_2 \\ 1 \end{pmatrix} = \begin{pmatrix} A & B & 0 \\ C & D & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ v_1 \\ 1 \end{pmatrix} + \begin{pmatrix} \Delta Y \\ \Delta V \\ 1 \end{pmatrix}$$

Δ<u>Y</u> and Δ<u>V</u> are the tilt and decenter of the BEAM due to the component tilt and decenter {d,a}

Modular Alignment

$$\Delta Y = Ad + Ba - d$$

$$\Delta V = Cd + Da - a$$

BEAM Tilt/Decenter

$$\boxed{\begin{pmatrix} \Delta Y \\ \Delta V \end{pmatrix} = \begin{pmatrix} A-1 & B \\ C & D-1 \end{pmatrix} \cdot \begin{pmatrix} d \\ a \end{pmatrix}}$$

COMPONENT Tilt/Decenter

inverse.......

$$\begin{pmatrix} d \\ a \end{pmatrix} = \begin{pmatrix} A-1 & B \\ C & D-1 \end{pmatrix}^{-1} \cdot \begin{pmatrix} \Delta Y \\ \Delta V \end{pmatrix}$$

COMPONENT Tilt/Decenter

$$\boxed{\begin{pmatrix} d \\ a \end{pmatrix} = \frac{1}{(2-A-D)} \begin{pmatrix} D-1 & -B \\ -C & A-1 \end{pmatrix} \cdot \begin{pmatrix} \Delta Y \\ \Delta V \end{pmatrix}}$$

BEAM Tilt/Decenter

$$\begin{pmatrix} \Delta Y \\ \Delta V \end{pmatrix} \Longleftrightarrow \begin{pmatrix} d \\ a \end{pmatrix}$$

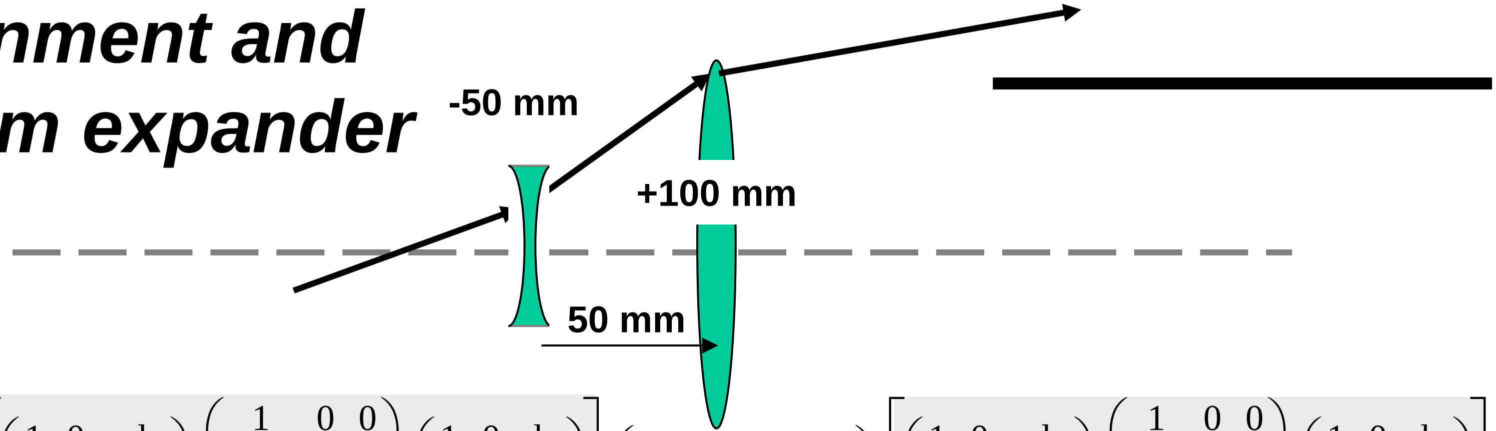
Component Alignment → Modular Alignment

$$\{d_1, a_1, \ldots\ldots\ldots d_n, a_n\} \Rightarrow \begin{pmatrix} \Delta Y_{ass'y} \\ \Delta V_{ass'y} \end{pmatrix}$$

$$\frac{1}{(2-A-D)}\begin{pmatrix} D-1 & -B \\ -C & A-1 \end{pmatrix} \cdot \begin{pmatrix} \Delta Y_{ass'y} \\ \Delta V_{ass'y} \end{pmatrix} = \begin{pmatrix} d_{ass'y} \\ a_{ass'y} \end{pmatrix}$$

Move the entire assembly $\begin{pmatrix} -d_{ass'y} \\ -a_{ass'y} \end{pmatrix} \Rightarrow \begin{pmatrix} \Delta Y \to 0 \\ \Delta V \to 0 \end{pmatrix}$

Misalignment and the beam expander



$$\begin{pmatrix} A & B & \Delta Y \\ C & D & \Delta V \\ 0 & 0 & 1 \end{pmatrix} = \left[\begin{pmatrix} 1 & 0 & -d_2 \\ 0 & 1 & -a_2 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ \frac{-1}{100} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & d_2 \\ 0 & 1 & a_2 \\ 0 & 0 & 1 \end{pmatrix} \right] \cdot \begin{pmatrix} 1 & 100-50 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \left[\begin{pmatrix} 1 & 0 & -d_1 \\ 0 & 1 & -a_1 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{50} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & d_1 \\ 0 & 1 & a_1 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

$$\begin{pmatrix} A & B & \Delta Y \\ C & D & \Delta V \\ 0 & 0 & 1 \end{pmatrix} = \begin{bmatrix} 2 & 50 & (d_1) \\ 0 & (0.5) & \left(\frac{1}{100} \cdot d_1 - \frac{1}{100} \cdot d_2 \right) \\ 0 & 0 & 1 \end{bmatrix}$$

$$M_V = D = 0.5$$

$$\Delta Y = d_1$$

$$\Delta V = \frac{d_1 - d_2}{100mm}$$

- ***Angular Magnification unchanged by lens tilt and decenter***
- ***Beam Tilt and Decenter***
  - ***Insensitive to lens tilt***
  - ***Depends on decenter only***
- ***Beam tilt (ΔV) is caused by lens decenter, only!***

Alignment method #1
adjust two lenses, separately

- ***Step 1 Adjust lens 1 (d_1) ➔ ΔY = 0***
- ***Step 2 Adjust lens 2 (d_2) ➔ ΔV = 0***

$$\Delta Y_{tele} = d_1 \Rightarrow 0mm$$

$$\Delta V_{tele} = \frac{d_1 - d_2}{100mm} = \frac{0 - d_2}{100mm} \Rightarrow 0rad$$

Alignment Method #2
Adjust beam expander (BX)

$$\begin{pmatrix} \Delta Y_{tele} \\ \Delta V_{tele} \end{pmatrix} = \begin{pmatrix} A-1 & B \\ C & D-1 \end{pmatrix} \begin{pmatrix} d_{tele} \\ a_{tele} \end{pmatrix}$$

$$\begin{pmatrix} \Delta Y_{tele} \\ \Delta V_{tele} \end{pmatrix} = \begin{pmatrix} 1 & 50 \\ 0 & -0.5 \end{pmatrix} \begin{pmatrix} d_{tele} \\ a_{tele} \end{pmatrix}$$

- **Step 1 Adjust BX a_{tele} ➔ ΔY = 0**
- **Step 2 Adjust BX d_{tele} ➔ ΔV = 0**

$$\Delta V_{tele} = -0.5 \cdot a_{tele} \Rightarrow 0$$

$$\Delta Y_{tele} = d_{tele} - 50 \cdot a_{tele} = d_{tele} \Rightarrow 0$$

Comments on alignment

- ***Components vs Module***
  - ***2 lens telescope ➔ 2 adjustments in either case***
  - ***6 lens module***
    - ***6 component adjustments***
    - ***2 module adjustments***

- ***ABCD theory of misalignment***
  - ***Quantifies magnitude of effects***
  - ***Provides an optimal sequence of steps***
    - ***Identifies independent adjusts***

Magnitude of module adjustments – statistical

Telescope Decenter and Tilt

$$\Delta Y = d_1$$

$$\Delta V = \frac{d_1 - d_2}{100mm}$$

Statistical Estimation

Component error

$$\Delta d_{1RMS} = \Delta d_{2RMS} = \sigma$$

Telescope Decenter error

$$\Delta Y_{RMS} = \sigma$$

Tilt Error

$$\Delta V_{RMS} = \frac{\sqrt{2}\sigma}{100mm}$$

For example

$$\sigma = 0.5mm$$

$$\Delta Y_{RMS} = 0.5mm$$

$$\Delta V_{RMS} = 0.4\deg$$

$$\begin{pmatrix} d_{tele} \\ a_{tele} \end{pmatrix} = \begin{pmatrix} 1 & 50 \\ 0 & -0.5 \end{pmatrix}^{-1} \begin{pmatrix} \Delta Y_{tele} \\ \Delta V_{tele} \end{pmatrix}$$

$$\begin{pmatrix} d_{tele} \\ a_{tele} \end{pmatrix} = \begin{pmatrix} 1 & 100 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} \Delta Y_{tele} \\ \Delta V_{tele} \end{pmatrix}$$

$$d_{tele} = \Delta Y_{tele} - 100mm \cdot \Delta V_{tele}$$

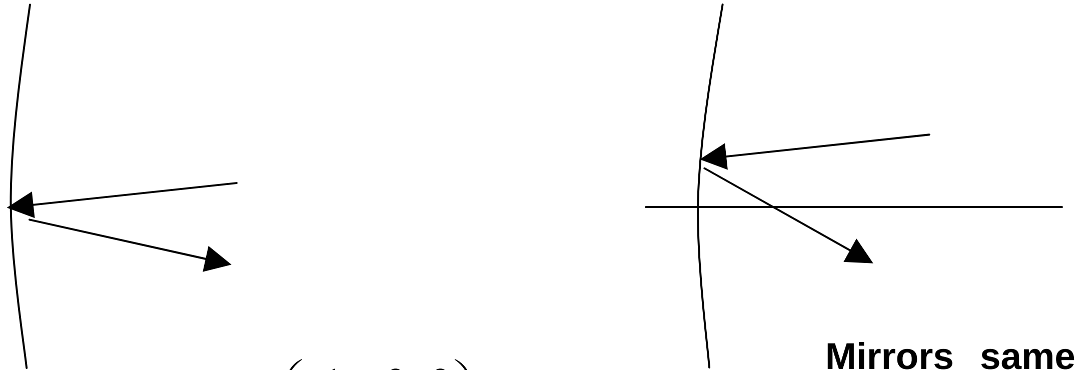
$$a_{tele} = -2 \cdot \Delta V_{tele}$$

$$d_{tele} = 1.20mm$$

$$a_{tele} = -0.8\deg$$

- ***1 RMS ➔ 68% prob; 2 RMS ➔ 95% prob***
  - ***Decenter ± 2.4 mm***
  - ***Tilt ± 1.6 deg***
- ***Can you live with the small angular error?***
  - ***If so, forget tilt adjustment***

Mirror Misalignment is different



Mirrors **<u>same</u> sign**
Lens **<u>opposite</u> sign**

$$\begin{pmatrix} A & B & \Delta Y \\ C & D & \Delta V \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -d \\ 0 & 1 & +a \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ \frac{-1}{f} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & d \\ 0 & 1 & a \\ 0 & 0 & 1 \end{pmatrix}$$

$f = \infty\text{mm}$ **Flat Mirror**

$$\begin{pmatrix} A & B & \Delta Y \\ C & D & \Delta V \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -d \\ 0 & 1 & +a \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & d \\ 0 & 1 & a \\ 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} A & B & \Delta Y \\ C & D & \Delta V \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2\cdot a \\ 0 & 0 & 1 \end{pmatrix}$$

- ***Tilt mirror θ, the beam tilts 2θ***
  - ***Tilt lens θ, the beam tilts = 0***
- ***For flat mirrors ➔ ignore decenter (except for beam walk off)***

Comparing misalignment of lens and mirrors

Lens $$\begin{pmatrix} 1 & 0 & -d \\ 0 & 1 & -a \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ \frac{-1}{f} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & d \\ 0 & 1 & a \\ 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 \\ \frac{-1}{f} & 1 & \frac{-1}{f} \cdot d \\ 0 & 0 & 1 \end{pmatrix} \quad \begin{matrix} \Delta Y \\ \Delta V \end{matrix}$$

Mirror $$\begin{pmatrix} 1 & 0 & -d \\ 0 & 1 & a \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ \frac{-1}{f} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & d \\ 0 & 1 & a \\ 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 \\ \frac{-1}{f} & 1 & \frac{-1}{f} \cdot d + 2 \cdot a \\ 0 & 0 & 1 \end{pmatrix} \quad \begin{matrix} \Delta Y \\ \Delta V \end{matrix}$$

Component Misalignment ➔	Tilt (a)	Decenter (d)
Lens		√
Flat Mirrors	√	
Curved Mirrors	√	√

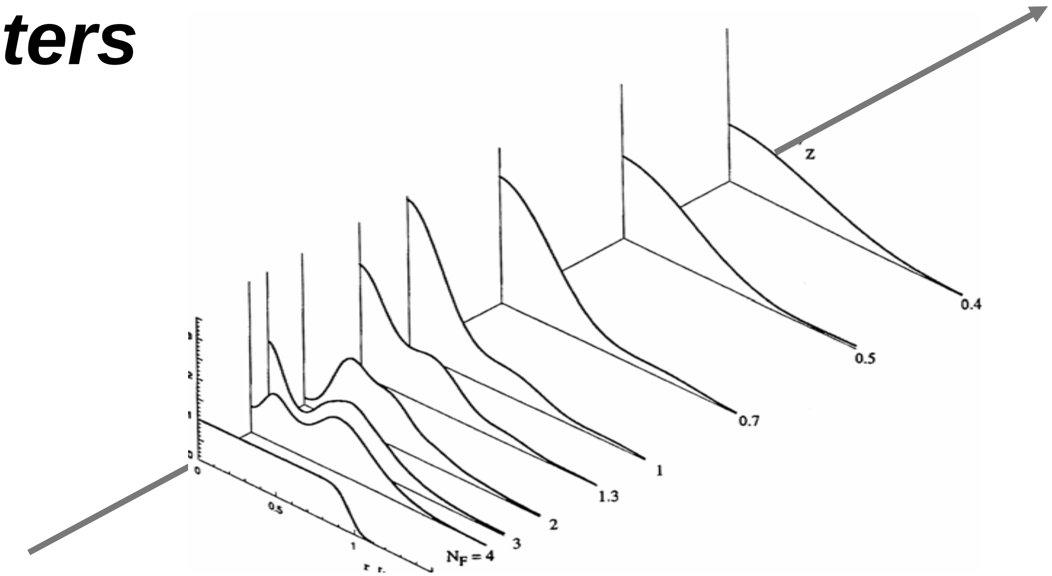
- ***Lens tilt (a) has no effect on beam alignment***
  - *In 1st order*
  - *Good enough for beams; probably not for imaging.*

Final comments

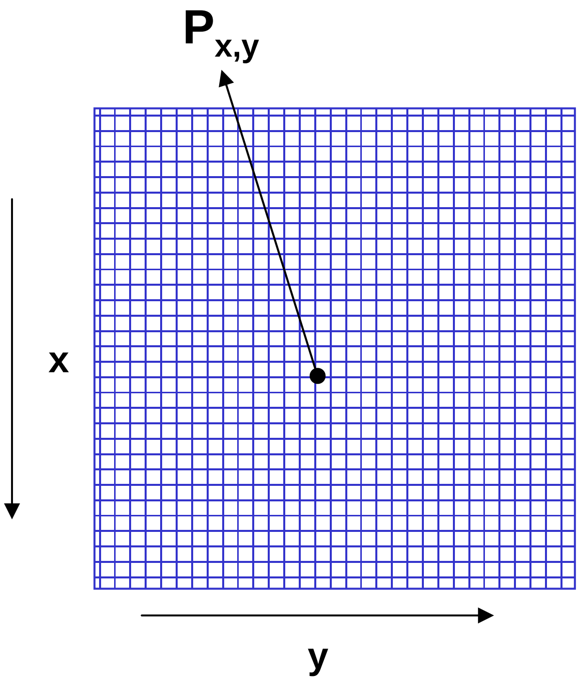
- ***For calculations***
  - ***all lengths in mm {f, t, R}***
  - ***all angles in radians (not degrees, nor mrads)***
- ***A,B,C,D parameters have units***
  - ***A and D are unitless***
  - ***B has units of mm***
  - ***C has units of mm-1***
- ***MathCad does not allow dimensioned components in matrices (radians are fine)***
  - ***Trick - Define mm := 1;***
- ***The Tilt/Decenter transformation for mirrors is different than for lens.***

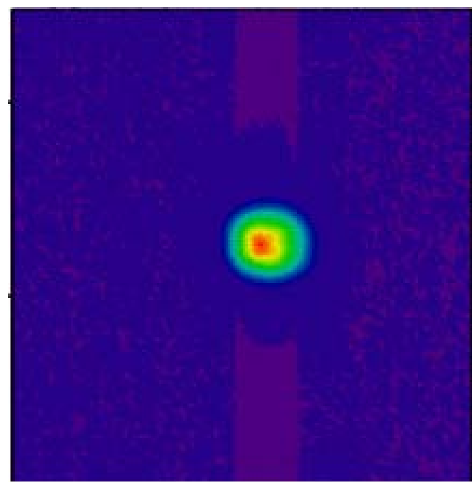
Break (10 min)

Part 3 beam profile – characteristics and parameters

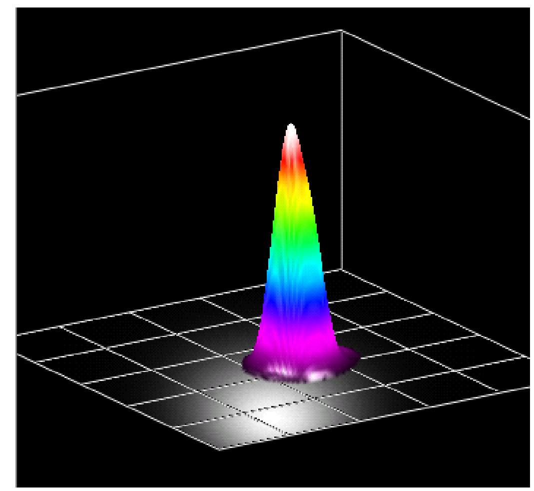


Beam profile - distribution of irradiance





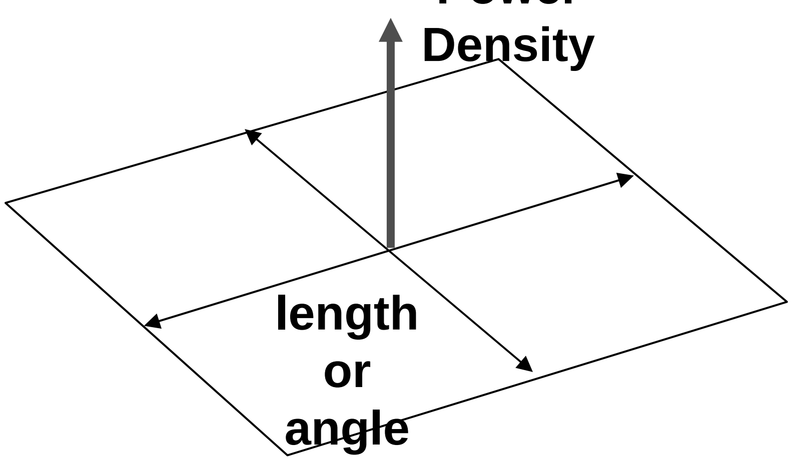
2-D Color

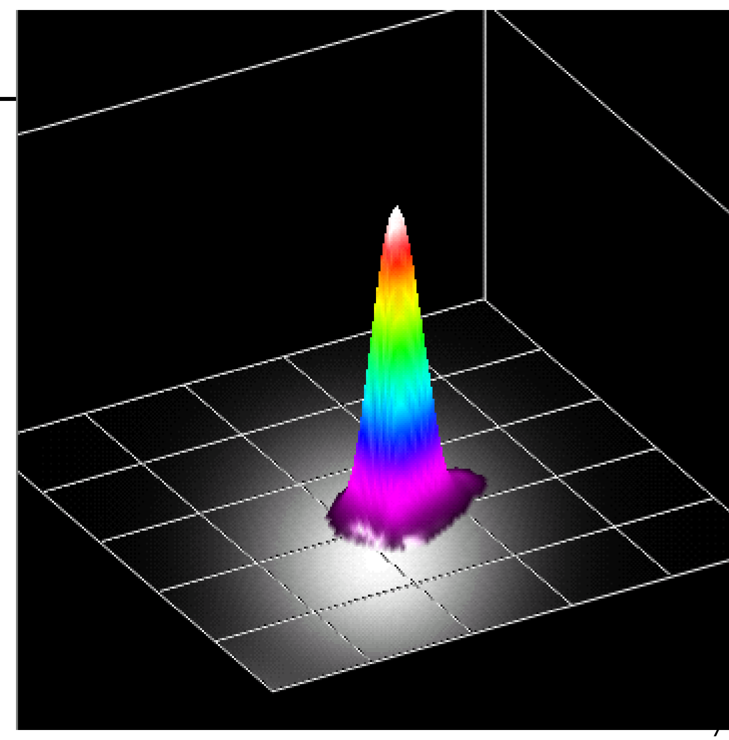


3-D Color

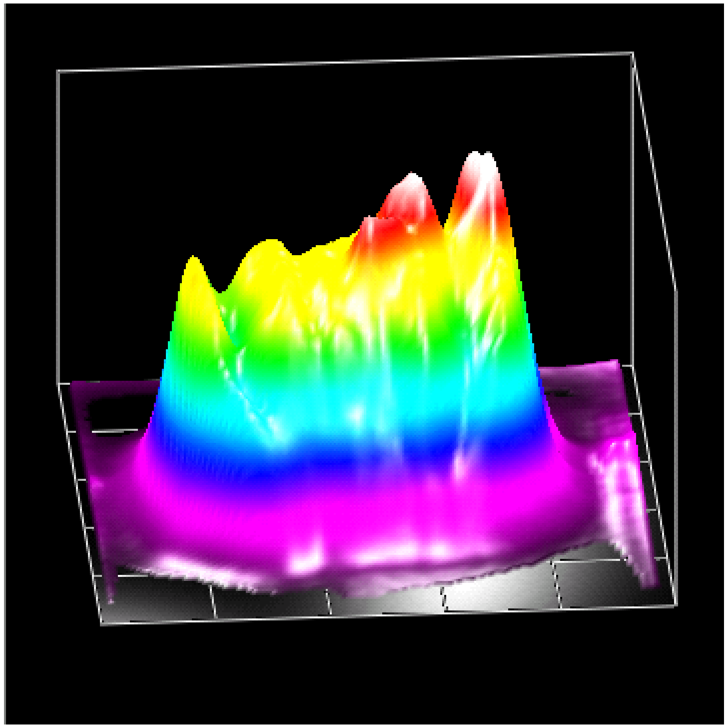
Power Density

- **Power (P) – Total Watts = Joules/sec in a beam passing through a reference plane**
- **Power Density**
  - – **Near-field** **I = P/(unit area)** **[W/mm^2]**
    - **Irradiance**
  - – **Far-field** **J = P/(unit solid angle)** **[W/sr]**
    - **Radiant Intensity**

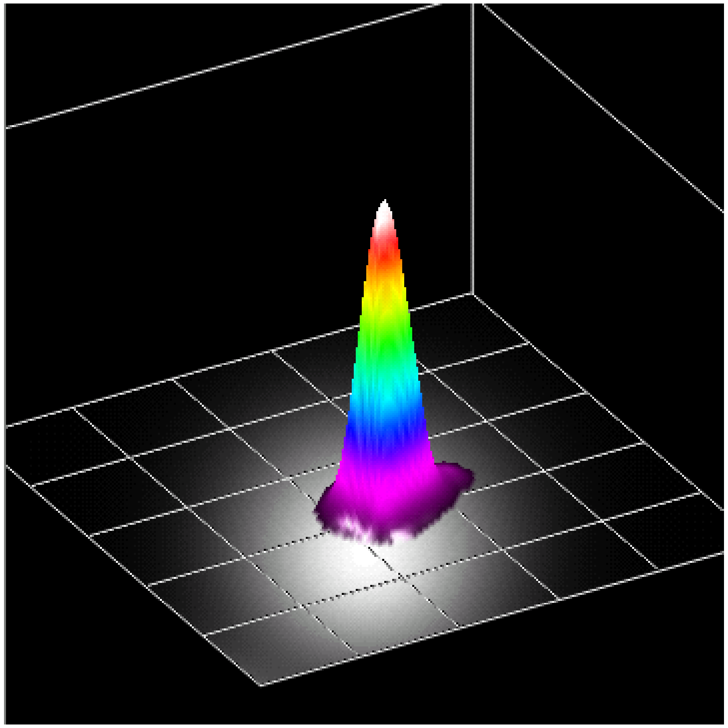




Beam in near- and far-field



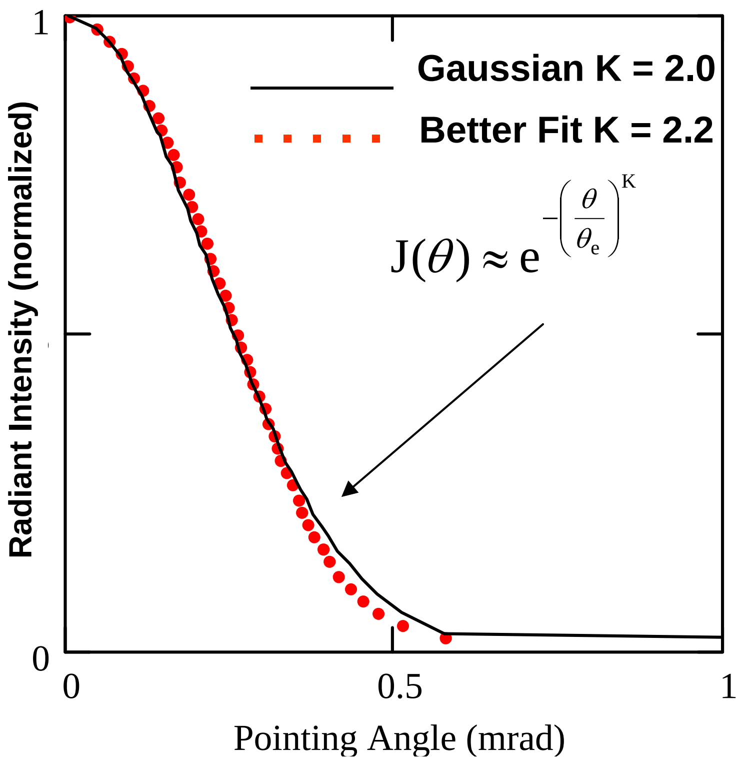
Near-field Power Density
Irradiance (W/mm²)
Spatial Diameter ($4\sigma_I$) = 8.36 mm

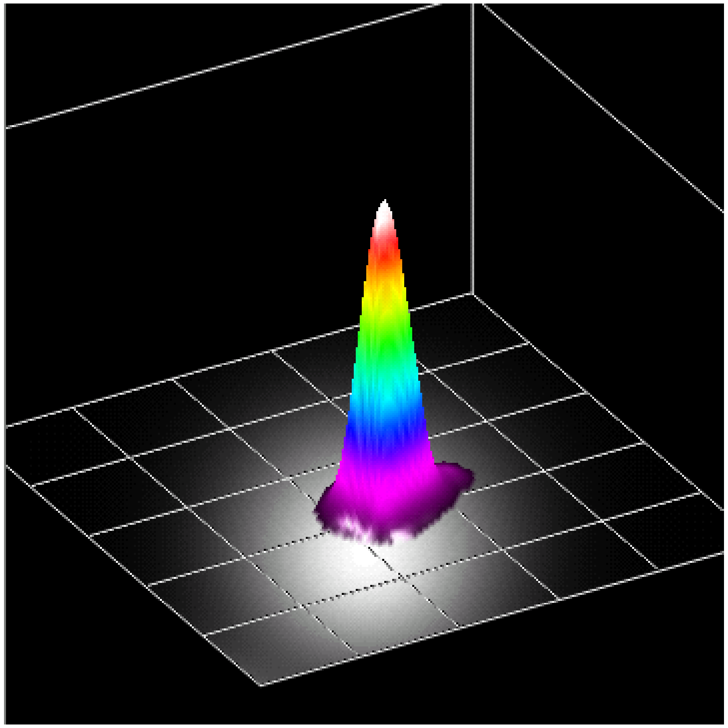


Far-field Power Density
Radiant Intensity (W/sr)
Angular Diameter ($4\sigma_\theta$) = 0.87 mrad

- ***Sharp features in near-field diffract to broad features in far-field.***
- ***Near-field peaks are a problem!***

Far-field gaussian-like





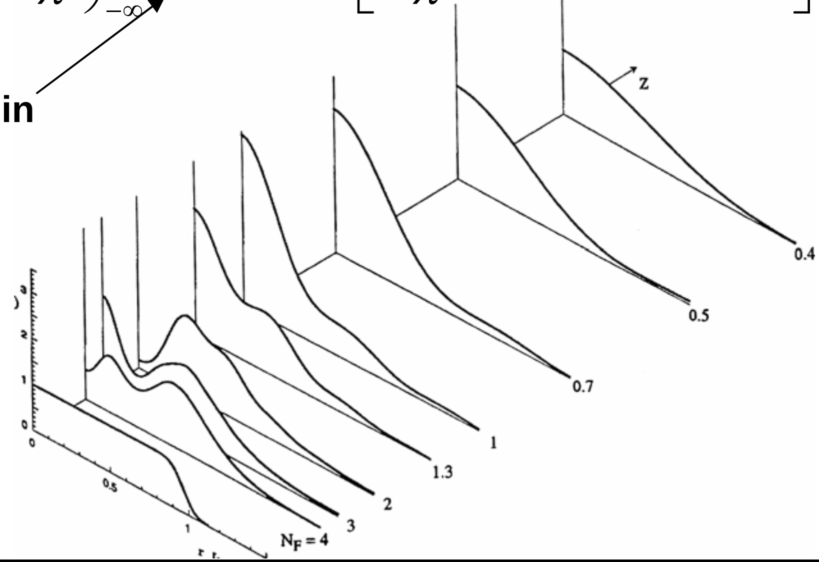
Far-field Beam
Radiant Intensity (W/sr)
Divergence Angle = 0.87 mrad

- ***Divergence (full) angle $\theta_{div} = 4\ \sigma_\theta$, (2nd moment)***
- ***Pointing (half) angle θ_e @ $1/e^2$ of max*** — θ_e ***= 0.435 mrad***
- ***$K \approx 2$ ➔ $\theta_{div} \approx 2\ \theta_e$*** — θ_{div} ***= 0.870 mrad***

Free propagation

$$\sqrt{\frac{-i}{\lambda \cdot z}}\exp\left(i\frac{2\pi}{\lambda}\right)\int_{-\infty}^{\infty} V_0(x')\cdot\exp\left[i\frac{2\pi}{\lambda}\left(x'^2 - 2x'\cdot x + x^2\right)\right]dx' = V_z(x)$$

in **out**

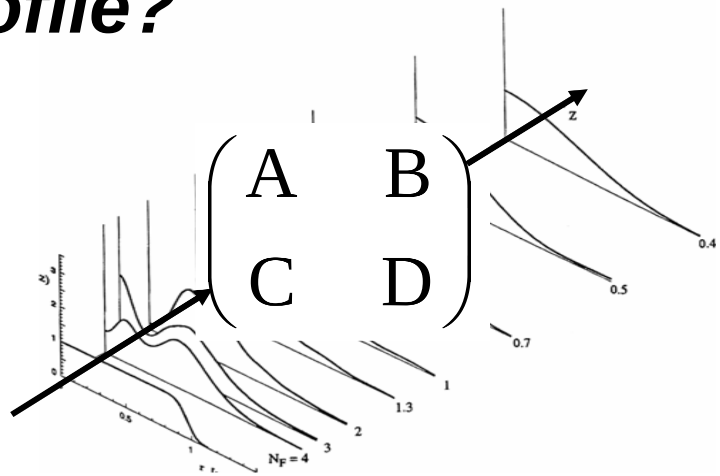


$$P(x) = \left|V_z(x)\right|^2$$

Power Density (W/area)

- **Electromagnetic Fields + Diffraction Integrals**
  - **Arbitrary wavefronts**
  - **Requires at every point on the plane both**
    - **Electric Field [volts/meter]**
    - **Phase angle [radians]**
  - **Diffraction integral ➔ Changes in Field and Phase**
    - **2D – Asymmetric - Fresnel or Fourier transform**
    - **1D – Circularly symmetric – Hankel transform**

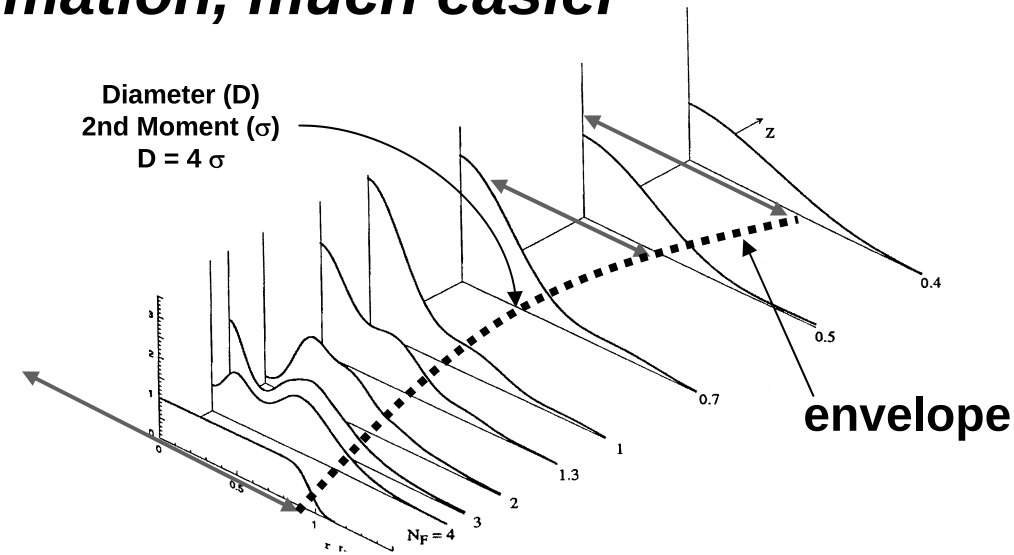
How do optics change the profile?



$$V_z(x) = \sqrt{\frac{-i}{\lambda \cdot B}}\,\exp\left(i\frac{2\pi}{\lambda}\right)\int_{-\infty}^{\infty} V_0(x')\cdot \exp\left[i\frac{2\pi}{\lambda}\left(Ax'^2 - 2x'\cdot x + Dx^2\right)\right]dx'$$

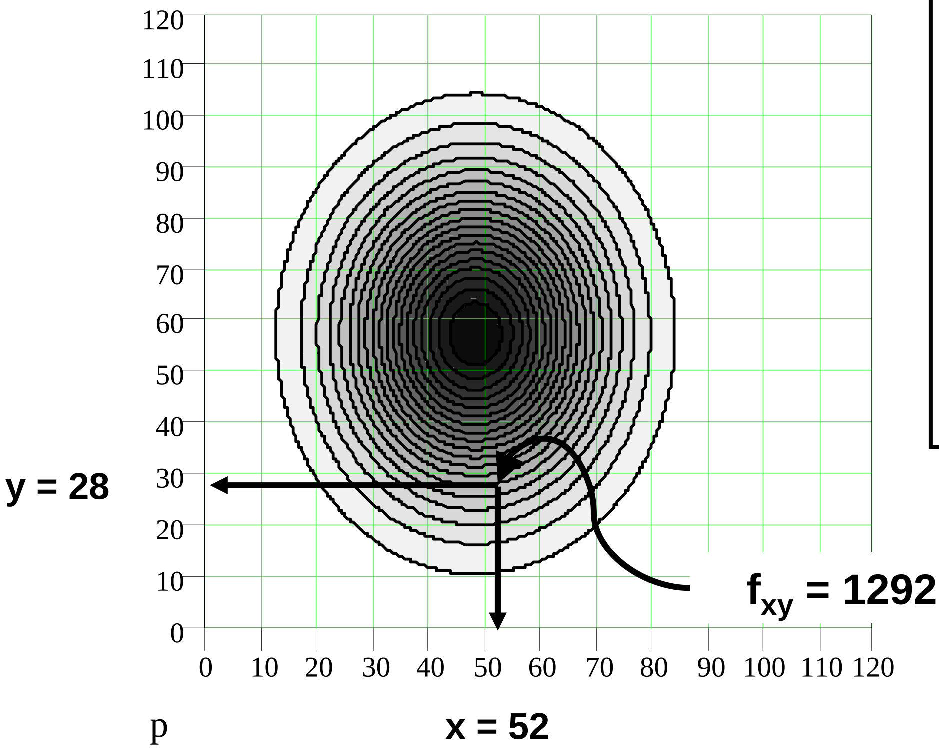
- ***Requires and calculates at every point***
  - ***Electric Field and Phase***

Statistical Parameters
Less information, much easier



- ***Statistical Moments***
  - ***Center (1st moment) and Diameter (2nd moment)***
    - ***plus 7 other derived parameters***
- ***Diffraction Integral ➔ Algebra***
  - ***Changes in 9 parameters ➔ two methods***
    - ***Equation method***
    - ***ABCD method***
  - ***Exact results (Center and Diameter)***
  - ***Any beam profile (that has a 2nd moment)***

Statistical moments



Camera

- ***Amplitude at 120 x 120 points (pixels) on a CCD array***

Statistical Moments

- ***1st Moment – centroid ➔Direction***
- ***2nd Moment – RMS ➔ Diameter***

1^{st} moments – beam axis

Continuous (functions) - integrate

$$M^{0th} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)dxdy$$

normalize

$$F(x,y) = f(x,y)/M^{0th}$$

centroids

$$\hat{x} = M_x^{1st} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x \cdot F(x,y)dxdy$$

$$\hat{y} = M_y^{1st} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} y \cdot F(x,y)dxdy$$

Discrete (camera data) - sum

$$M^{0th} = \sum_{x=0}^{120}\sum_{y=0}^{120} f_{x,y}$$

normalize

$$F_{x,y} = f_{x,y} \big/ M^{0th}$$

centroids

$$\hat{x} = M_x^{1st} = \sum_{x=0}^{120}\sum_{y=0}^{120} x \cdot F_{x,y}$$

$$\hat{y} = M_y^{1st} = \sum_{x=0}^{120}\sum_{y=0}^{120} y \cdot F_{x,y}$$

2^{nd} moments ➔ beam diameter

Continuous (functions) - integrate

$$M_{xx}{}^{2nd} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(x-\hat{x})^2 \cdot F(x,y)dxdy$$

$$M_{yy}{}^{2nd} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(y-\hat{y})^2 \cdot F(x,y)dxdy$$

Discrete (camera data) - sum

$$M_{xx}{}^{2nd} = \sum_{x=0}^{120}\sum_{y=0}^{120}(x-\hat{x})^2 \cdot F_{x,y}$$

$$M_{yy}{}^{2nd} = \sum_{x=0}^{120}\sum_{y=0}^{120}(y-\hat{y})^2 \cdot F_{x,y}$$

$$\sigma_x = \sqrt{M_{xx}{}^{2nd}}$$

$$\sigma_y = \sqrt{M_{yy}{}^{2nd}}$$

In units of pixels

$$\left.\begin{array}{l} D_x \equiv 4 \cdot \sigma_x \\ D_y \equiv 4 \cdot \sigma_y \end{array}\right\} \text{Elliptical Beam, if} \neq$$

$$D \equiv \sqrt{D_x D_y}$$

Diameter of Circle with same area as ellipse

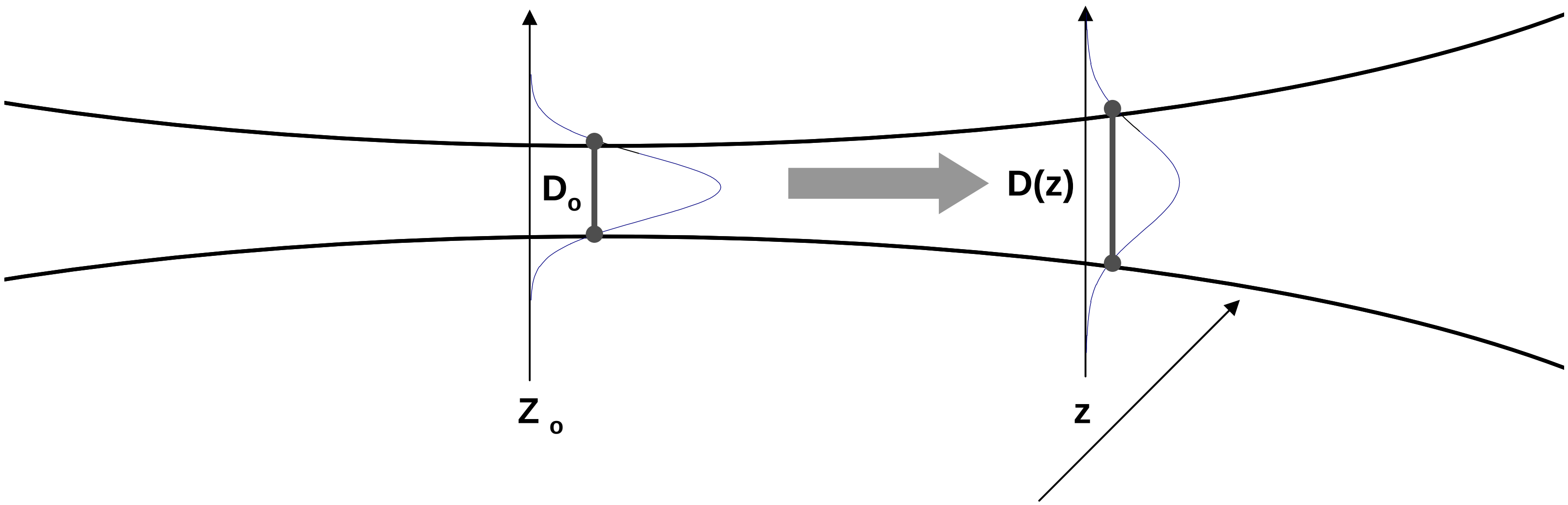
Why 2^{nd} Moment?
The theory is exactly correct for any beam profile, Gaussian or not.

Why Four-Sigma?
If distribution is Gaussian, K = 2, then the irradiance (or radiant intensity) at every point on the circle of diameter is $\mathbf{e^{-2}}$ of the maximum at the center.

D = 57.2 pixels➔L =0.1mm/pixel = 5.72 mm (near-field)

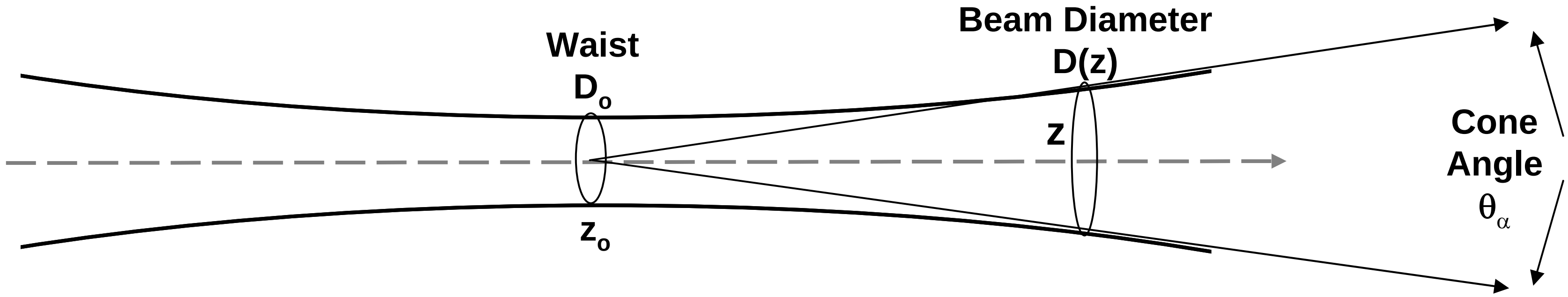
θ = D/F ➔ 5.72mm/2000mm = 2.86 mrad (far-field)

Beam diameter envelope is a hyperbola



- ***The envelope of the beam diameter is a hyperbola***
  - ***True of any (arbitrary) beam (Gaussian, flat top, ugly, donut shaped), as long as diameters D = 4 σ***
  - ***The characteristics of beam propagation are derived from the mathematical form of the hyperbola.***

Beam parameters – free space



Hyperbola $D(z)^2 = A + B(z - z_o)^2$

At waist $D(z_o)^2 = D_0{}^2 = A$

$$D(z) = D_o\sqrt{1 + \left(\frac{z - z_o}{D_o/\theta}\right)^2}$$

Far-field $z >> z_o$

Cone $D(z) \approx \sqrt{B}z$

Divergence angle $\dfrac{D(z)}{z} = \theta = \sqrt{B}$

$$Z_R \equiv \frac{D_o}{\theta}$$

"Rayleigh Range"

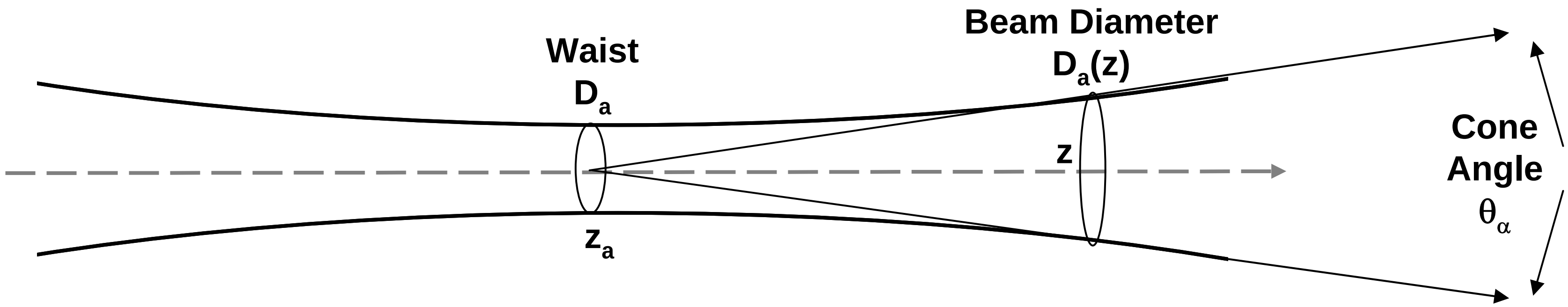
$$\boxed{D(z) = \sqrt{D_o{}^2 + \theta^2(z - z_o)^2}}$$

Form 1

$$\boxed{D(z) = D_o\sqrt{1 + \left(\frac{z - z_o}{Z_R}\right)^2}}$$

Form 2

Notation for beam in region A

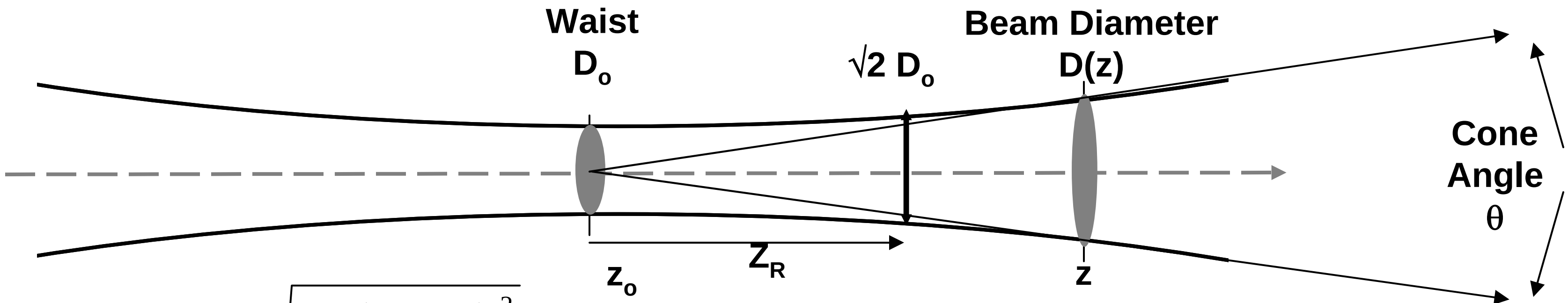


$$D_a(z) = \sqrt{D_a{}^2 + \theta_a{}^2 (z-a)^2}$$

$$Z_a = \frac{D_a}{\theta_a}$$

$$D_a(z) = D_a \sqrt{1 + \left(\frac{z-a}{Z_a}\right)^2}$$

What is the Rayleigh range?



$$D(z) = D_o \sqrt{1 + \left(\frac{z - z_o}{Z_R} \right)^2}$$

- **➔ *Near the waist ("near-field")***
  - ***Beam envelope is cylindrical, collimated***
  - ***Wavefront is flat***

$$(z_0 - Z_R) < z < (z_0 + Z_R)$$
$$D(z) \approx D_o$$

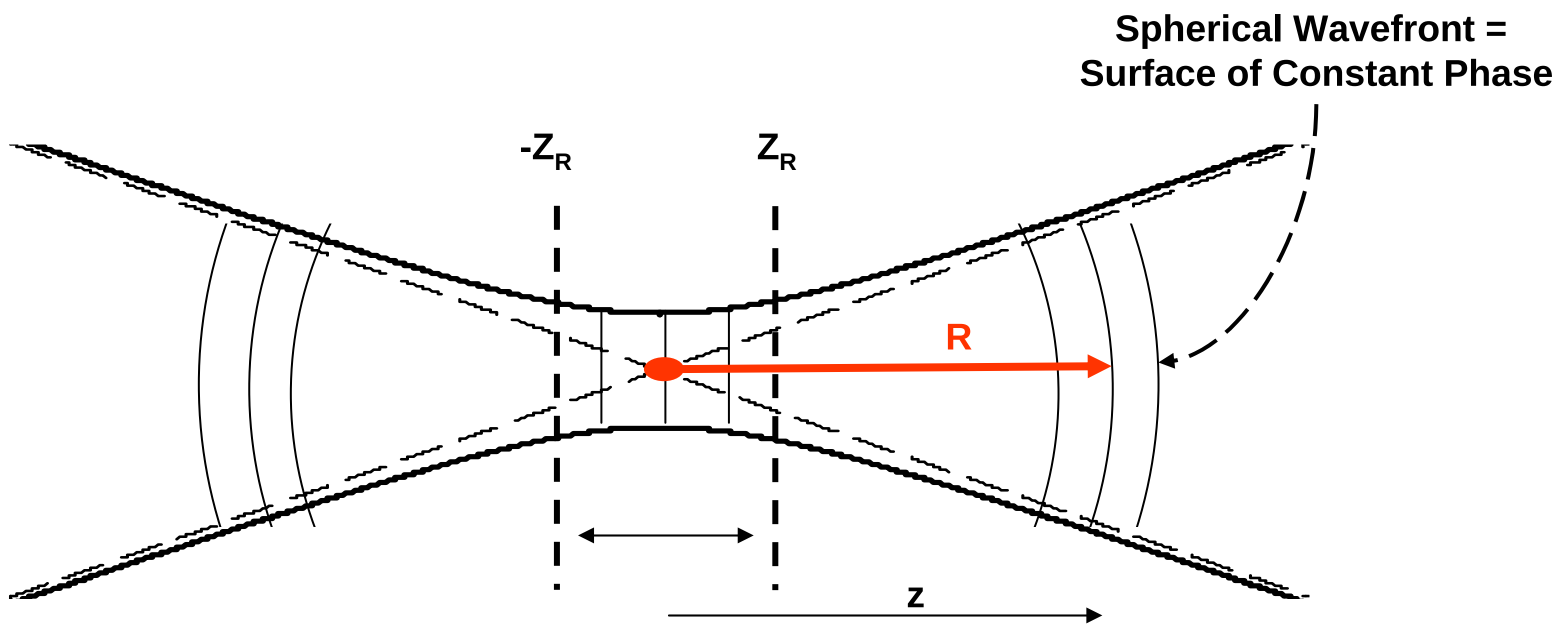
- **➔ At $\pm Z_R$ from waist**
  - **Beam diameter = 1.414 x Waist**

$$z = z_o \pm Z_R$$
$$D(z) = \sqrt{2} \cdot D_o$$

- **➔ Far from the waist ("far-field")**
  - **Beam envelope is conical**
  - **Wavefront is spherical**

$$z >> Z_R$$
$$D(z) \approx \theta \cdot z$$

Wavefront radius of curvature



$R \rightarrow -\infty$

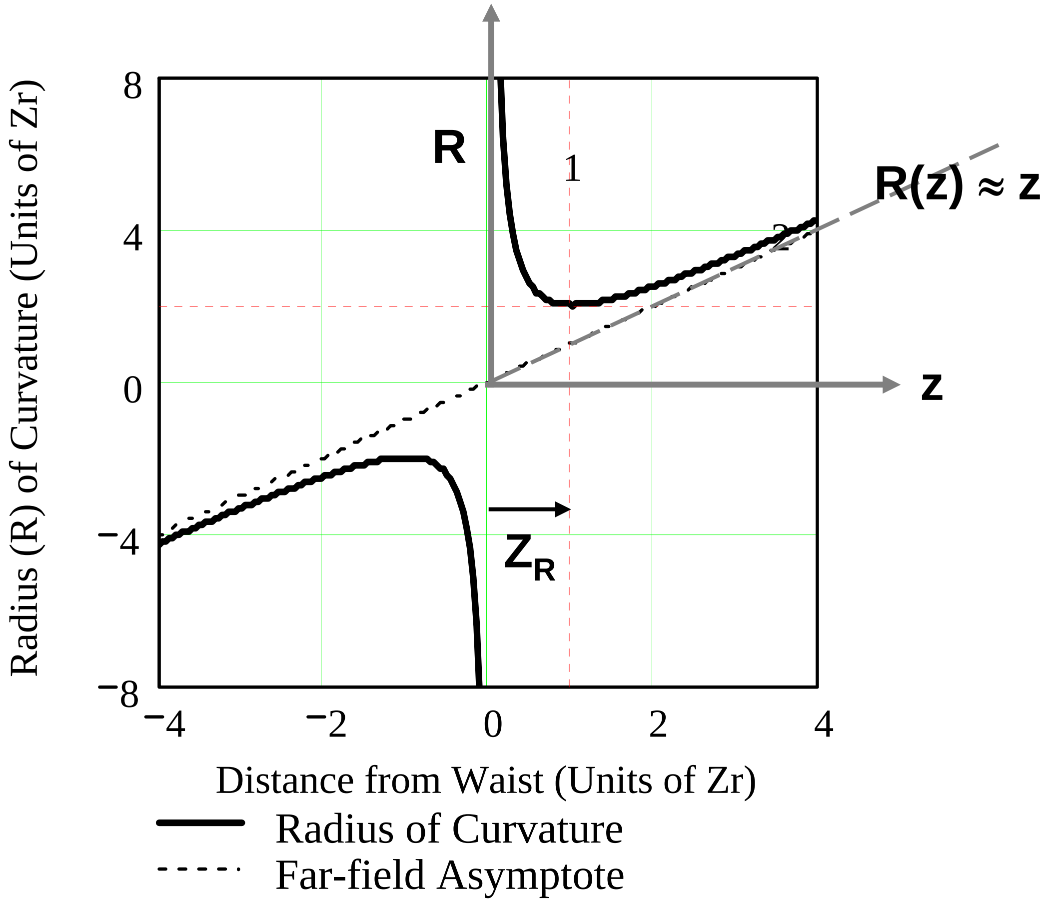
$R \approx -z$
Far-field
Geom (Ray)
Optics

$R \approx \infty$
Near-field
Phys (Diffraction)
Optics

$R \approx z$
Far-field
Geom (Ray)
Optics

$R \rightarrow \infty$

Wavefront radius of curvature



$$R(z) = z + \frac{Z_R^{\ 2}}{z}$$

At waist	$0 = z$	$R \approx \infty$	**Plane** Cylindrical WF
Near-field	$0 \leq z < Z_R$	$R > 2\ Z_R$	**Spherical** Cylindrical WF
At Rayleigh	$z = Z_R$	$R = 2\ Z_R$	Minimum R
Far-field	$Z_R << z$	$R \approx z$	**Spherical** Conical Wavefront
	$z \Rightarrow \infty$	$R \Rightarrow \infty$	**Plane** Conical Wavefront

Etendue

$$\mathrm{E} \equiv \mathrm{D_o} \cdot \theta \qquad\qquad E[mm \cdot mrad] = E[\mu m]$$

E is called "brightness", beam quality, "etendue

Diffraction - Fourier Transform

$$E_{\min} = \left(\frac{4\lambda}{\pi}\right)$$

monochromatic beams, λ=value [μm]
any profile

$$E \geq E_{\min}$$

$$m^2 \equiv \frac{E}{E_{\min}}$$

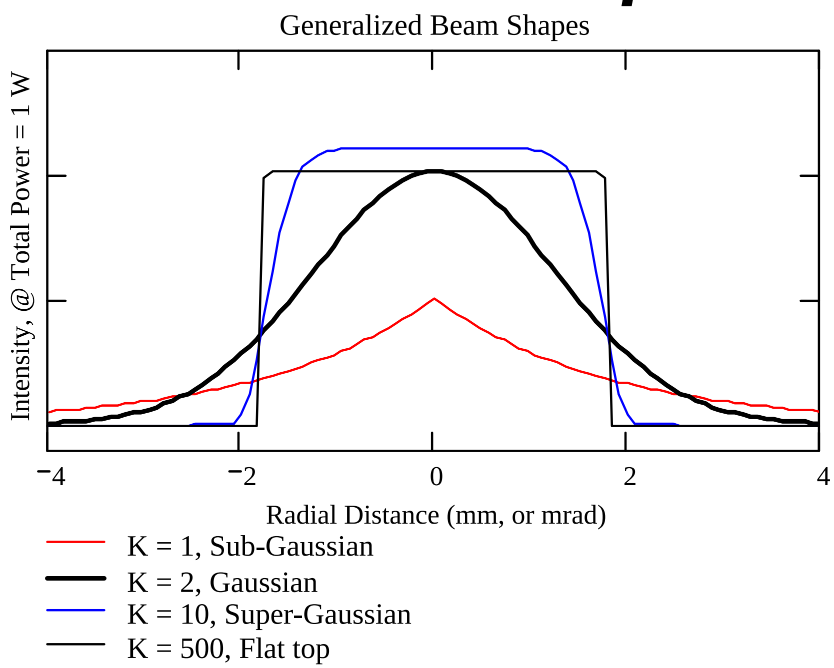
defines m^2

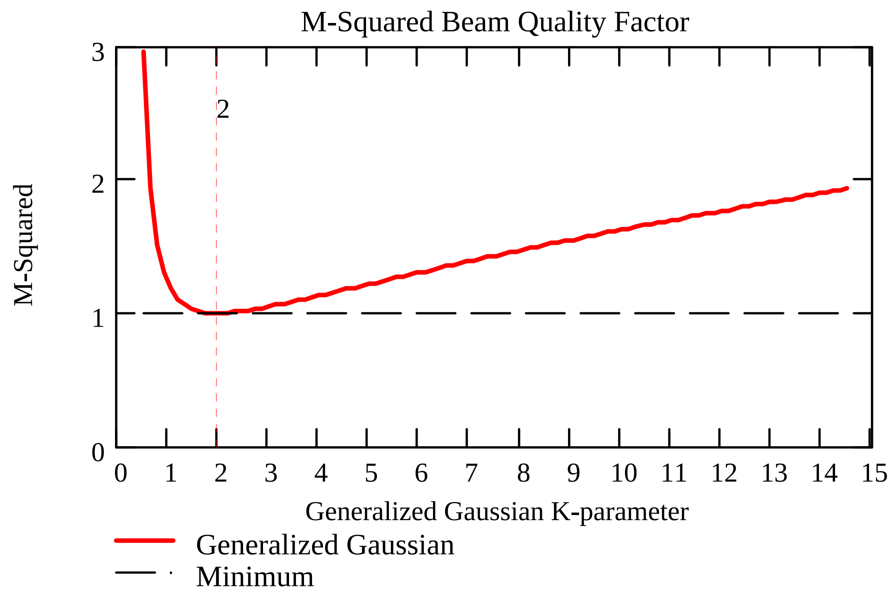
$$m^2 \geq 1$$

m^2 = 1, "diffraction limit", only Ideal Gaussian K = 2

$$E = \left(\frac{4\lambda}{\pi}\right) m^2$$

M^2 and beam shape





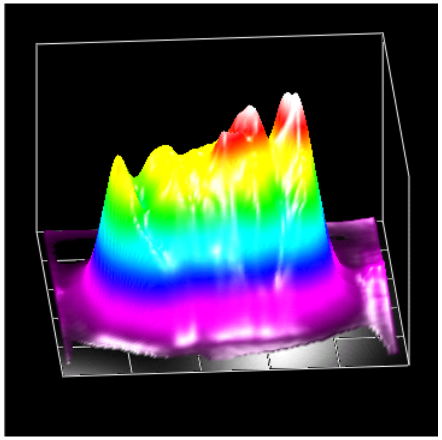
$$F(r) = \frac{1}{\pi \cdot w^2 \Gamma\left(\frac{2}{K}+1\right)} \exp\left(-\left|\frac{r}{w}\right|^K\right)$$

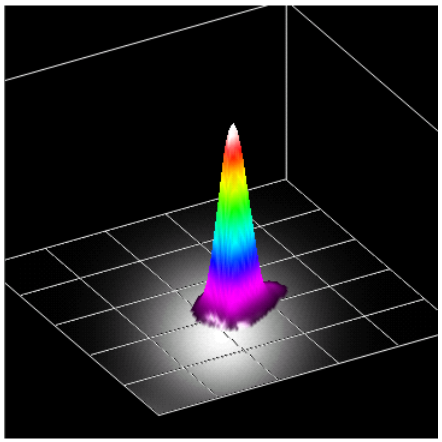
$$M^2 = \frac{\sqrt{\Gamma\left(\frac{4}{K}\right)}}{\left(\frac{2}{K}\right) \cdot \Gamma\left(\frac{2}{K}\right)}$$

- ***To provide minimum intensity over an region (in space or angle), flattened beam requires less power.***
  - ***Flat top***   ***K = ∞,***   ***Power = 1.00 Pmin***
  - ***Flattened***   ***K = 10***   ***Power = 1.55 Pmin***
  - ***Gaussian***   ***K = 2***   ***Power = 2.72 Pmin***
- ***Flattened beams better***
  - ***Near-field – welding, cd writing, scanning***
  - ***Far-field – point applications such as communications, countermeasures***

Beam in near- and far-field

Near-field Beam
Diameter = 8.36 mm





Far-field Beam
Angle = 0.87 mrad

$$D = 8.36mm$$

$$\theta = 0.87mrad$$

$$E = D \cdot \theta = 7.29mm \cdot mrad$$

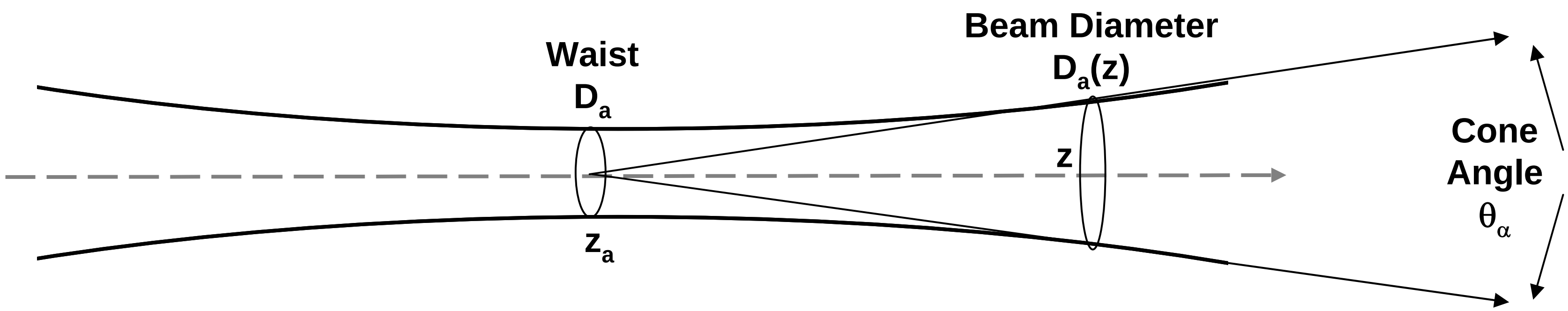
$$\lambda = 1.06\mu m$$

$$E_{\min} = \frac{4\lambda}{\pi} = 1.35mm \cdot mrad$$

$$M^2 = \frac{E}{E_{\min}} = 5.4$$

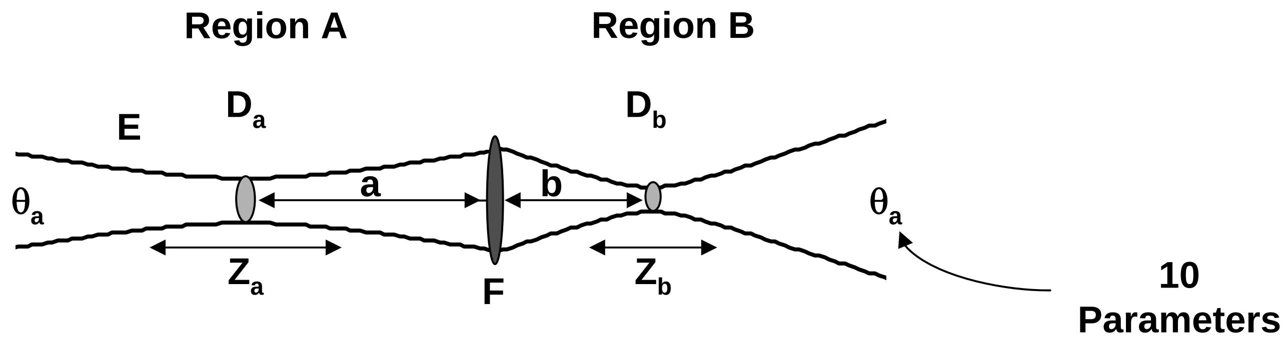
- ***Spectrally broadband beams***
  - ***M^2 is undefined (and meaningless)***
  - ***E is always defined and measurable***
- ***Need only E for beam analysis or design***
  - ***Never need M^2 or λ, except to "guestimate" E, if don't have D, θ***

Too many symbols



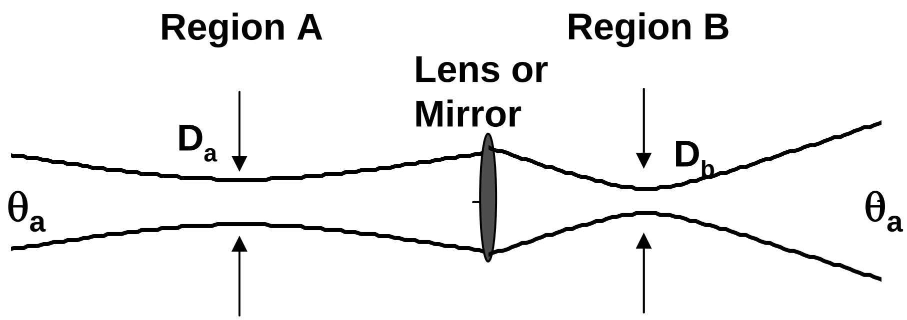
	Beam Parameter		Generic	Region A	Region B
	Position on z-axis		z	z	z
1	Waist position	regional	z_o	a	b
2	Waist Diameter (Near-field)	regional	D_o	D_a	D_b
3	Rayleigh range	regional	Z_R	Z_a	Z_b
4	Divergence Angle (Far-field)	regional	θ	θ_a	θ_b
5	Etendue	regional	E	E	E
6	Beam Diameter at z	local	D(z)	$D_a(z)$	$D_b(z)$
7	Wavefront Radius of Curv	local	R(z)	$R_a(z)$	$R_b(z)$

Refocusing laser beams



- ***Lens (mirror) changes beam envelope***
  - ***Create new waist***
    - ***New position***
    - ***New diameter***
  - ***Change the beam divergence***
- ***How does the lens (F) relate the parameters in Regions A and B***
- ***How many parameters are there? 10***
  - $\{D_a, \theta_a, a, Z_a\} + \{D_b, \theta_b, b, Z_b\} + F + E$
- ***How many parameters are independent? 4***
  - ***Calculate the remaining 6***
- ***Step by step method to follow***

Etendue is invariant



$$E = D_a\theta_a = D_b \cdot \theta_b$$

- ***If lens***
  - ***changes only radius of curvature (no aberrations)***
  - ***does not clip the beam***
- ***then***
  - ***E's in regions A and B are equal***
  - ***Relates parameters in B to those in A***
- ***Called the "Legrangian Invariant"***

Magnification M

Since.....

$$\mathrm{E} = \mathrm{D_a}\theta_\mathrm{a} = \mathrm{D_b}\cdot\theta_\mathrm{b}$$

then.....

$$\frac{\mathrm{D_b}}{\mathrm{D_a}} = \left(\frac{\theta_\mathrm{b}}{\theta_\mathrm{a}}\right)^{-1} \equiv \mathrm{M}$$

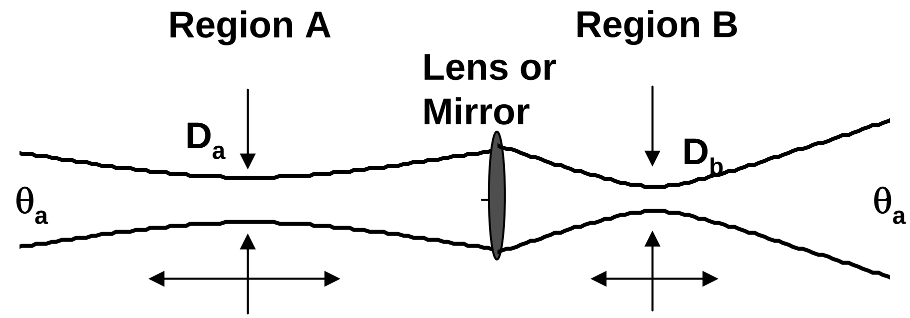
Therefore,

$$\mathrm{D_b} = \mathrm{M}\cdot\mathrm{D_a}$$

$$\mathrm{D_a} = \mathrm{M}^{-1}\cdot\mathrm{D_b}$$

$$\theta_\mathrm{b} = \mathrm{M}^{-1}\cdot\theta_\mathrm{a}$$

$$\theta_\mathrm{a} = \mathrm{M}\cdot\theta_\mathrm{b}$$



$$Z_a = \frac{D_a}{\theta_a} \qquad Z_b = \frac{D_b}{\theta_b}$$

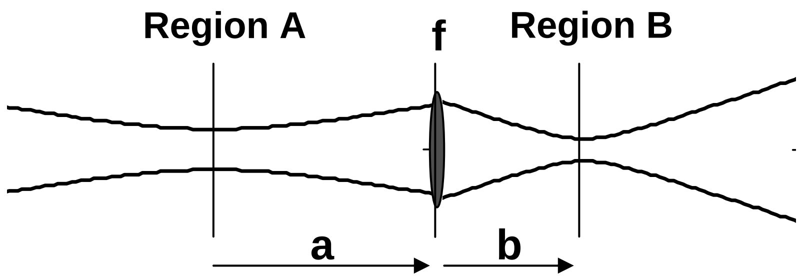
Given 3 of {D_a, θ_a, D_b, θ_b}, calculate:

- ➔ ***$M = D_a/D_b = \theta_b/\theta_a$***
- ➔ ***find the 4th***
- ➔ ***$E = D_a\ \theta_a$***
- ➔ ***$Z_a = D_a/\theta_a$***
- ➔ ***$Z_b = D_b/\theta_b$***

Without knowing the focal length (f)!

(Not to be comfused with M^2.

Magnification and focal length



	$\{a, Z_a\}$	$\{b, Z_b\}$
f→M	$M=\sqrt{\dfrac{f^2}{(a-f)^2+Z_a{}^2}}$	$M^{-1}=\sqrt{\dfrac{f^2}{(b-f)^2+Z_b{}^2}}$
M→f	$f_\pm=\dfrac{a\pm\sqrt{M^{-2}a^2-(1-M^{-2})Z_a{}^2}}{\left(1-M^{-2}\right)}$	$f_\pm=\dfrac{b\pm\sqrt{M^2b^2-(1-M^2)Z_b{}^2}}{\left(1-M^2\right)}$

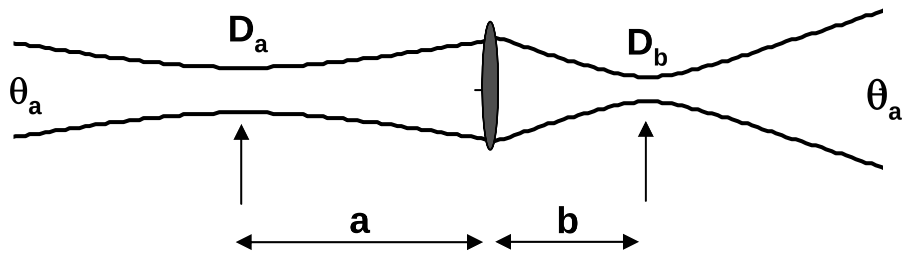
$\{D, \theta\} \Leftrightarrow \{E, Z\}$

Etendue	$E = D_a \cdot \theta_a$		$E = \frac{D_o^{\,2}}{Z_a}$	$E = \theta_a^{\,2} \cdot Z_a$
Rayleigh Range		$Z_a = \frac{D_a}{\theta_a}$	$Z_a = \frac{D_a^{\,2}}{E}$	$Z_a = \frac{E}{\theta_a^{\,2}}$
Waist Diameter	$D_a = \frac{E}{\theta_a}$	$D_a = \theta_a \cdot Z_a$	$D_a = \sqrt{E \cdot Z_a}$	
Beam Divergence	$\theta_a = \frac{E}{D_a}$	$\theta_a = \frac{D_a}{Z_a}$		$\theta_a = \sqrt{\frac{E}{Z_a}}$

Only ones I remember

- ***Given any two, calculate the other two in each region***
  - ***4 parameters = 2 independent + 2 dependent***
  - ***Vertical same equation***

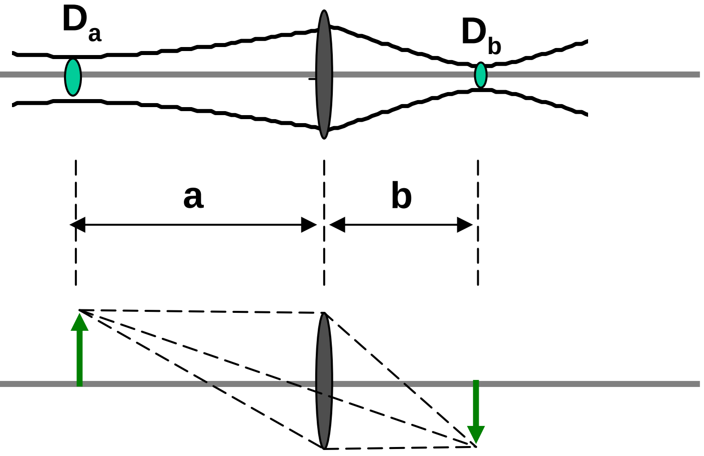
Waist locations and focal length



$\{a, Za, f\} \Rightarrow b$	$\{b, Zb, f\} \Rightarrow a$
$\frac{1}{b}=\frac{1}{f}-\frac{1}{a+\left(\frac{Z_a{}^2}{a-f}\right)}$	$\frac{1}{a}=\frac{1}{f}-\frac{1}{b+\left(\frac{Z_b{}^2}{b-f}\right)}$

Refocusing vs. Imaging

Physical (diffractive) optics



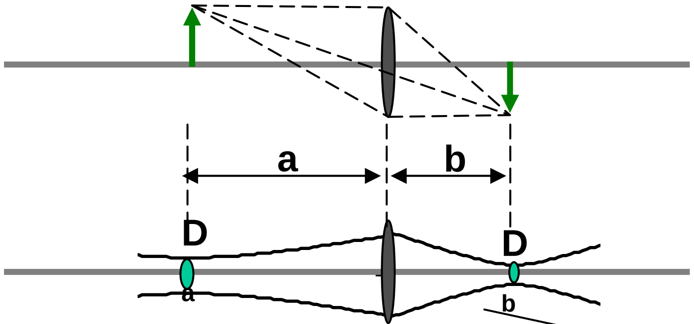
$$\frac{1}{b_{waist}} = \frac{1}{f} - \frac{1}{a + \frac{Z_a^{\ 2}}{(a-f)}}$$

Geometric (Ray) optics

$$\frac{1}{b_{image}} = \frac{1}{f} - \frac{1}{a}$$

- ***Refocusing waists in NOT same as imaging***
  - ***Applied to laser beams, ray optics (geometrical codes) CAN be incorrect***
  - ***Very different behavior***
- ***Diffractive optics is always correct***
  - ***Ray optics is correct only when Z_a << a(a-f)***
    - ***Incoherent light $Z_a \approx 0$***
    - ***Object far from focal plane a > f***
  - ***Geometric optics is the special case of diffractive optics***

Refocused waist position is bounded



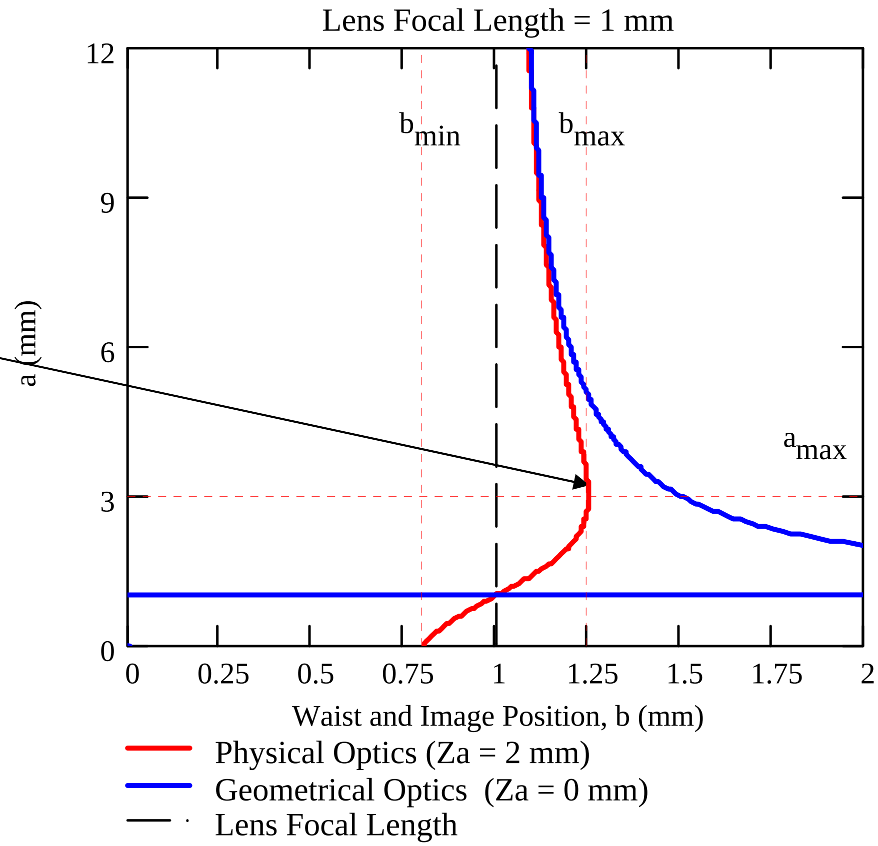
$$f = 1\text{mm}$$

$$Z_a = 2\text{mm}$$

$$a_{max} = f + Z_a = 3\text{mm}$$

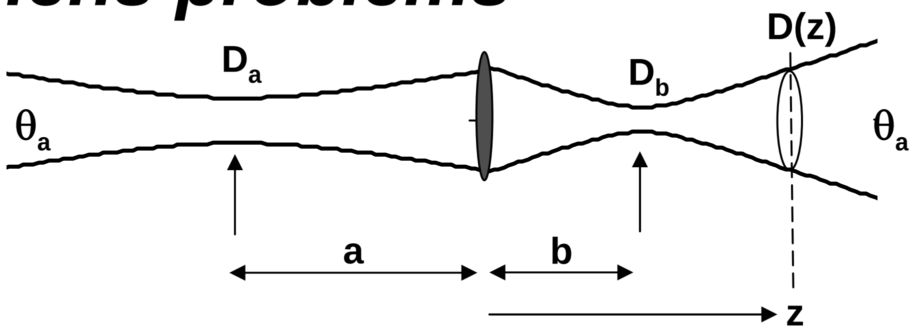
$$b_{max} = f + \frac{f^2}{2Z_a} = 1.25\text{mm}$$

$$b_{min} = f\,\frac{Z_a^2}{f^2 + Z_a^2} = 0.80\text{mm}$$



- ***Refocused waist position is bounded (real, b > 0)***
  - $b_{min} \rightarrow b_{max} \rightarrow f$
- ***Image position not bounded (real, b > 0)***
  - $0 \rightarrow \infty$

All beam + lens problems

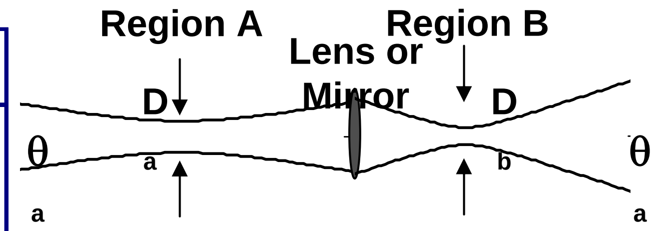


- ***Total number of regional parameters = 10***
  - ***Side A: {a ,D_a, θ_a, Z_a}***
  - ***Side B: {b, D_b, θ_b, Z_b}***
  - ***Focal Length {F}***
  - ***Etendue {E}***
- ***10 = 4 Independent + 6 Dependent***
  - ***Given 4 ➔ Calculate 6***
  - ***Calculate D(z) and R(z) everywhere***
- ***Two types of problems***
  - ***Analysis – given lens (focal length) and its location, calculate the <u>beam</u> parameters***
  - ***Design – given the beam parameters, find the <u>lens</u> (focal length) and its location***

Analysis problems

Table #1

Step	$\text{Given}\{a, D_a, \theta_a\} + f$	$\text{Given}\{b, D_b, \theta_b\} + f$
1	$E = D_a\theta_a$ $Z_a = \dfrac{D_a}{\theta_a}$	$E = D_b\theta_b$ $Z_b = \dfrac{D_b}{\theta_b}$
2	$D_a(z) = D_a\sqrt{1+\left(\dfrac{z-a}{Z_a}\right)^2}$ $R_a(z) = (z-a) + \dfrac{Z_a^{\ 2}}{z-a}$	$D_b(z) = D_b\sqrt{1+\left(\dfrac{z-b}{Z_b}\right)^2}$ $R_b(z) = (z-b) + \dfrac{Z_b^{\ 2}}{(z-b)}$
3	$M = \sqrt{\dfrac{f^2}{(a-f)^2 + Z_a^{\ 2}}}$	$M^{-1} = \sqrt{\dfrac{f^2}{(b-f)^2 + Z_b^{\ 2}}}$
4	$D_b = D_a M$ $\theta_b = \theta_a M^{-1}$	$D_a = D_b M^{-1}$ $\theta_a = \theta_b M$
5	$Z_b = \dfrac{D_b}{\theta_b}$	$Z_a = \dfrac{D_a}{\theta_a}$
6	$\dfrac{1}{b} = \dfrac{1}{f} - \dfrac{1}{a + \dfrac{Z_a^{\ 2}}{(a-f)}}$	$\dfrac{1}{a} = \dfrac{1}{f} - \dfrac{1}{b + \dfrac{Z_b^{\ 2}}{(b-f)}}$
7	$D_b(z) = D_b\sqrt{1+\left(\dfrac{z-b}{Z_b}\right)^2}$ $R_b(z) = (z-b) + \dfrac{Z_b^{\ 2}}{(z-b)}$	$D_a(z) = D_a\sqrt{1+\left(\dfrac{z-a}{Z_a}\right)^2}$ $R_a(z) = (z-a) + \dfrac{Z_a^{\ 2}}{z-a}$

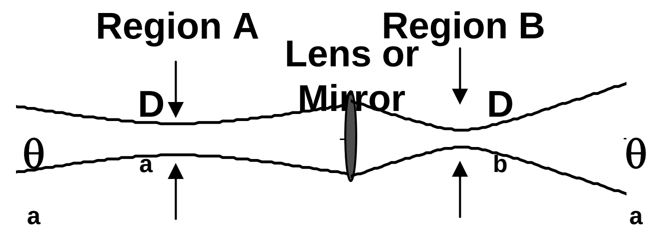


- ***Analysis***
  - *given lens (focal length) and its location*
  - *find all the beam parameters*
- ***Start with either side***
  - *physically and mathematically symmetrical*
- ***Illustrates***
  - *how variables are linked*
  - *flow of calculation*

Design problems

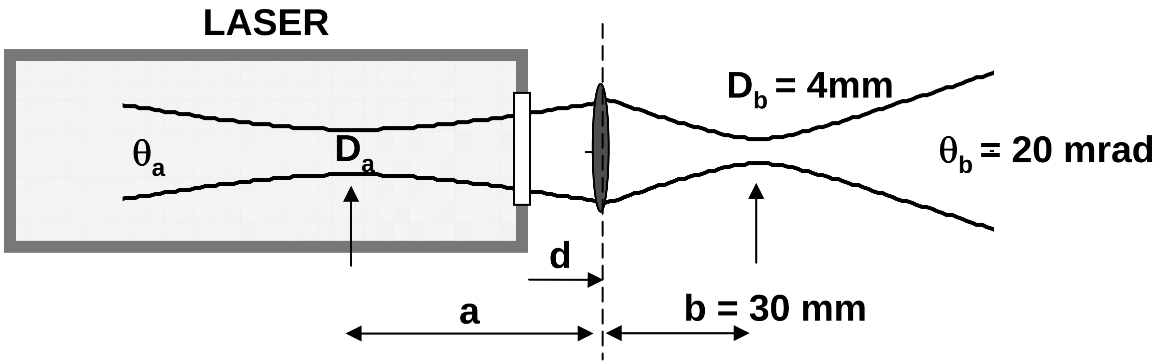
Table #2

Step	$\text{Given}\{a, D_a, \theta_a\} + \{D_b \text{ or } \theta_b\}$	$\text{Given}\{b, D_b, \theta_b\} + \{D_a \text{ or } \theta_a\}$
1	$E = D_a\theta_a$ $Z_a = \dfrac{D_a}{\theta_a}$	$E = D_b\theta_b$ $Z_b = \dfrac{D_b}{\theta_b}$
2	$D_a(z) = D_a\sqrt{1+\left(\dfrac{z-a}{Z_a}\right)^2}$ $R_a(z) = (z-a) + \dfrac{Z_a^{\ 2}}{(z-a)}$	$D_b(z) = D_b\sqrt{1+\left(\dfrac{z-b}{Z_b}\right)^2}$ $R_b(z) = (z-b) + \dfrac{Z_b}{(z-b)}$
3	$M = \dfrac{D_b}{D_a} \text{ or } \dfrac{\theta_a}{\theta_b}$ $f_\pm = \dfrac{a \pm \sqrt{M^{-2}a^2 - (1-M^{-2})Z_a^{\ 2}}}{(1-M^{-2})}$ choose converging (or diverging	$M = \dfrac{D_b}{D_a} \text{ or } \dfrac{\theta_a}{\theta_b}$ $f_\pm = \dfrac{b \pm \sqrt{M^2b^2 - (1-M^2)Z_b^{\ 2}}}{(1-M^2)}$ choose converging (or diverging)
4	$D_b = D_a M$ $\theta_b = \theta_a M^{-1}$	$D_a = D_b M^{-1}$ $\theta_a = \theta_b M$
5	$Z_b = \dfrac{D_b}{\theta_b}$	$Z_a = \dfrac{D_a}{\theta_a}$
6	$\dfrac{1}{b} = \dfrac{1}{f} - \dfrac{1}{a + \dfrac{Z_a^{\ 2}}{(a-f)}}$	$\dfrac{1}{a} = \dfrac{1}{f} - \dfrac{1}{b + \dfrac{Z_b^{\ 2}}{(b-f)}}$
7	$D_b(z) = D_b\sqrt{1+\left(\dfrac{z-b}{Z_b}\right)^2}$ $R_b(z) = (z-b) + \dfrac{Z_b}{(z-b)}$	$D_a(z) = D_a\sqrt{1+\left(\dfrac{z-a}{Z_a}\right)^2}$ $R_a(z) = (z-a) + \dfrac{Z_a^{\ 2}}{(z-a)}$



- **Design**
  - given beam parameters
  - find lens (focal length) and position
- **Start with either side**
  - physically and mathematically symmetrical
- **Illustrates**
  - how variables are linked
  - flow of calculation
- **Analysis and Design flow**
  - same, except at step 3.

Measuring laser beam by refocusing



Problem

The waist of the output beam is usually located inside the laser. In order to characterize the output beam, a lens, f = 100 mm, is placed d = 10 mm from the output window. Careful measurements of the refocused beam yield:

Waist location from lens	b	= 30 mm
Waist diameter	D_b	= 4 mm
Beam divergence	θ_b	= 20 mrad

Calculate {a, D_a, θ_a, Z_a, E} of the laser output beam.

Solution – measure laser beam

Assume refocused beam parameters as "givens"==================================

Refocused waist diameter is 4 mm	$D_b := 4mm$
Refocused beam divergence is 20 mrad	$\theta_b := 0.02rad$
Refoced waist location from lens is 60 mm	$b := 30mm$
Lens focal length is 100 mm	$f := 100mm$
Lens is 10 mm in front of Laser Window	$d := 10mm$

Calculate the input laser beam parameters=====================================

Etendue in B	$E := D_b \cdot \theta_b$	$E = 0.08mm \cdot rad$
Rayleigh range in B	$Z_b := \dfrac{D_b}{\theta_b}$	$Z_b = 200mm$
Magnification	$M := \sqrt{\dfrac{(b-f)^2 + Z_b^2}{f^2}}$	$M = 2.119$
Laser waist diameter	$D_a := \dfrac{D_b}{M}$	$\boxed{D_a = 1.888mm}$
Laser beam divergence	$\theta_a := \theta_b \cdot M$	$\boxed{\theta_a = 0.042rad}$
Laser Rayleigh range	$Z_a := \dfrac{D_a}{\theta_a}$	$\boxed{Z_a = 44.543mm}$
Etendue in A	$E := D_a \cdot \theta_a$	$\boxed{E = 0.08mm \cdot rad}$
Laser waist from Lens	$a := \left[\dfrac{1}{f} - \dfrac{1}{b + \dfrac{Z_b^2}{(b-f)}} \right]^{-1}$	$a = 84.41mm$
Laser waist location inside laser	$L := d - a$	$\boxed{L = -74.41mm}$

Verify results – calculate assumptions

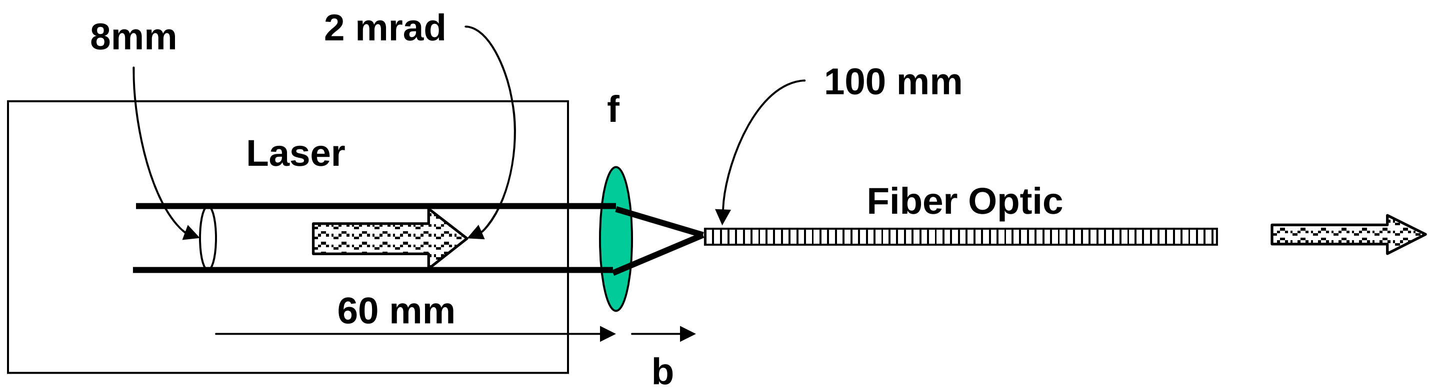
Assume the input beam==

Laser waist diameter	$D_a = 1.888\text{mm}$
Laser divergence (42 mrad)	$\theta_a = 0.042\text{rad}$
Laser waist distance to lens	$a = 84.41\text{mm}$
Lens focal length	$f = 100\text{mm}$

Calculate the refocused beam parameters==================================

Etendue in A (80 mm-mrad)	$E := D_a \cdot \theta_a$	$E = 0.08\text{mm}\cdot\text{rad}$
Rayleigh range in A	$Z_a := \frac{D_a}{\theta_a}$	$Z_a = 44.543\text{mm}$
Magnification	$M := \sqrt{\frac{f^2}{(a-f)^2 + Z_a{}^2}}$	$M = 2.119$
Refocused waist diameter	$D_b := D_a \cdot M$	$\boxed{D_b = 4\text{mm}}$
Refocused beam divergence (20 mrad)	$\theta_b := \frac{\theta_a}{M}$	$\boxed{\theta_b = 0.02\text{rad}}$
Rayleigh Range in B	$Z_b := \frac{D_b}{\theta_b}$	$Z_b = 200\text{mm}$
Etendue in B (80 mm-mrad)	$E := D_b \cdot \theta_b$	$E = 0.08\text{mm}\cdot\text{rad}$
Refocused waist distance from lens	$b := \left[\frac{1}{f} - \frac{1}{a + \frac{Z_a{}^2}{(a-f)}}\right]^{-1}$	$\boxed{b = 30\text{mm}}$

Laser to fiber optic input coupler



Problem We want to couple the output of a laser into a fiber optic cable with a lens. The output beam waist is 60 mm behind the lens. The waist diameter is 8mm. The far-field beam divergence is 2 mrad.

- **What focal length (f) will produce a refocused waist 100 mm, the diameter of the fiber optic core?**
- **How far away from the lens (b) must the fiber optic be located?**
- **How accurately must the fiber optic be positioned (what is the Rayleigh range of the output beam)?**

Solution – input coupler

Assume 4 parameters as "givens"==

Input beam divergence is 2 mrad	$\theta_a := 2 \cdot 10^{-3} \mathrm{rad}$
Input waist diameter is 8 mm	$D_a := 8\mathrm{mm}$
Input waist location is 60 mm	$a := 60\mathrm{mm}$
Output waist diameter is 0.1mm	$D_b := 0.1\mathrm{mm}$

Calculate the other parameters===

Etendue in A	$E := D_a \cdot \theta_a$	$E = 16\mathrm{mm} \cdot \mathrm{mrad}$
Rayleigh range in A	$Z_a := \dfrac{D_a}{\theta_a}$	$Z_a = 4000\mathrm{mm}$
Magnification	$M := \dfrac{D_b}{D_a}$	$M = 0.013$
Refocused beam divergence	$\theta_b := \dfrac{\theta_a}{M}$	$\theta_b = 0.16\mathrm{rad}$
Rayleigh range in B	$Z_b := \dfrac{D_b}{\theta_b}$	$Z_b = 0.625\mathrm{mm}$
Etendue in B	$E := D_b \cdot \theta_b$	$E = 16\mathrm{mm} \cdot \mathrm{mrad}$

Input coupler (cont)

Calculate the mirror focal length and position===================================

$$f := \frac{a + \sqrt{M^{-2} \cdot a^2 - \left(1 - M^{-2}\right) \cdot Z_a^{\,2}}}{\left(1 - M^{-2}\right)} \qquad f = -50.019\,mm$$

Diverging lens; wrong sign; extraneous root; discard

$$f := \frac{a - \sqrt{M^{-2} \cdot a^2 - \left(1 - M^{-2}\right) \cdot Z_a^{\,2}}}{\left(1 - M^{-2}\right)} \qquad \boxed{f = 50\,mm}$$

$$b := \left[\frac{1}{f} - \frac{1}{a + \frac{Z_a^{\,2}}{(a - f)}} \right]^{-1} \qquad \boxed{b = 50.002\,mm}$$

- ***Position a 50.000 mm focal length mirror a distance 60mm from the laser waist.***
- ***Position the fiber optic (100μm core) at a distance 50.002mm from the mirror with a tolerance of +/- 0.625mm (Z_b)***

Verify results – calculate assumptions

Assume four calculated results as "given"==

Focal length $f = 50\,\text{mm}$

Distance to fiber optic $b = 50.002\,\text{mm}$

Refocused beam divergence $\theta_b = 0.16\,\text{rad}$

Rayleigh range in b $Z_b = 0.625\,\text{mm}$

Calculate the original four "given" parameters=================================

Mirror distance to laser waist

$$a := \left[\frac{1}{f} - \frac{1}{b + \frac{Z_b^{\,2}}{(b-f)}}\right]^{-1} \qquad a = 60\,\text{mm}$$

Magnification

$$M := \sqrt{\frac{(b-f)^2 + Z_b^{\,2}}{f^2}} \qquad M = 0.013$$

Refocused waist at F/0

$$D_b := Z_b \cdot \theta_b \qquad D_b = 0.1\,\text{mm}$$

Laser Beam diameter

$$D_a := \frac{D_b}{M} \qquad D_a = 8\,\text{mm}$$

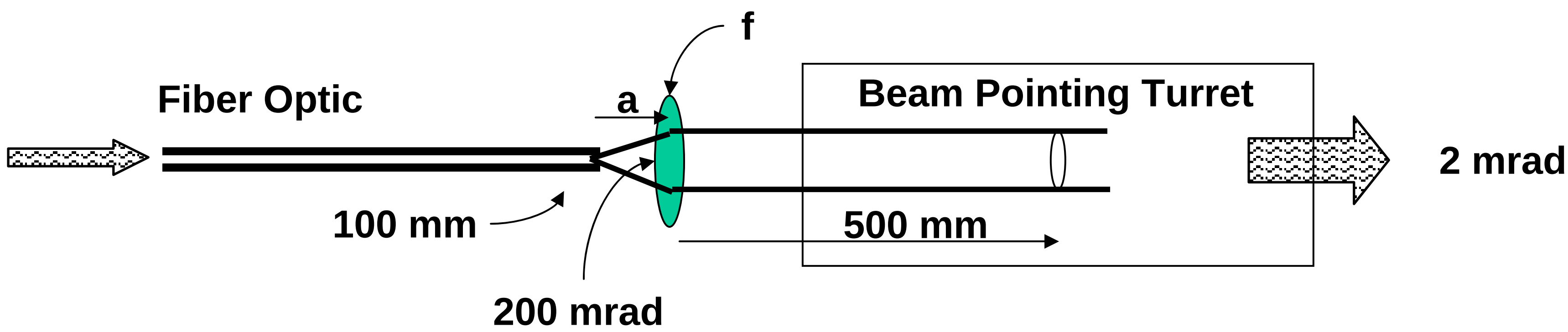
Laser beam divergence

$$\theta_a := \theta_b \cdot M \qquad \theta_a = 2 \times 10^{-3}\,\text{rad}$$

Rayleigh range in A

$$Z_a := \frac{D_a}{\theta_a} \qquad Z_a = 4000\,\text{mm}$$

Fiber optic to far-field output coupler



Problem We want to couple the output of the fiber optic cable (above) into the beam pointing turret. The fiber optic core is 100 mm, and the divergence is 200 mrad. The path length through the turret is 1000 mm long. We want to place a lens at the turret input port, which will produce a beam waist half way through the turret
and a beam divergence of 2 mrad in the far-field. The waist in the middle will keep the beam narrow inside the turret to avoid clipping the beam.

- **What focal length lens (f) placed a distance (a) from the fiber optic will produce the desired beam into the pointing turret?**
- **How accurately does the the fiber optic have to be positioned relative to the lens (what is the Rayleigh range)?**

Solution – output coupler

Assume 4 parameters as "givens"==

F/O beam divergence is 200 mrad	$\theta_a := 0.2\,\text{rad}$
F/Odiameter is 100 μm	$D_a := 0.1\,\text{mm}$
Refocused waist location is 500 mm	$\boxed{b := 500\,\text{mm}}$
Refocused beam divergence is 2 mrad	$\theta_b := 0.002\,\text{rad}$

Calculate the other parameters==

Etendue (before)	$E := D_a \cdot \theta_a$	$E = 20\,\text{mm}\cdot\text{mrad}$
Rayleigh range in A	$Z_a := \dfrac{D_a}{\theta_a}$	$Z_a = 0.5\,\text{mm}$
Magnification	$M := \dfrac{\theta_a}{\theta_b}$	$\boxed{M = 100}$
Refocused waist diameter	$D_b := D_a \cdot M$	$\boxed{D_b = 10\,\text{mm}}$
Rayleigh range in B	$Z_b := \dfrac{D_b}{\theta_b}$	$\boxed{Z_b = 5000\,\text{mm}}$
Etendue (after)	$E := D_b \cdot \theta_b$	$E = 20\,\text{mm}\cdot\text{mrad}$

Output coupler (cont)

Calculate the mirror focal length and position===================================

$$f := \frac{b + \sqrt{M^2 \cdot b^2 - \left(1 - M^2\right) \cdot {Z_b}^2}}{\left(1 - M^2\right)} \qquad f = -50.302\text{mm}$$

Diverging mirror; wrong sign; extraneous root; discard

$$f := \frac{b - \sqrt{M^2 \cdot b^2 - \left(1 - M^2\right) \cdot {Z_b}^2}}{\left(1 - M^2\right)} \qquad \boxed{f = 50.202\text{mm}}$$

$$a := \left[\frac{1}{f} - \frac{1}{b + \frac{{Z_b}^2}{(b - f)}}\right]^{-1} \qquad \boxed{a = 50.247\text{mm}}$$

- **Position a 50.202mm focal length mirror a distance 50.247 from the fiber optic, with a tolerance of +/-0.5mm (Z_a). The beam refocuses 500mm from the mirror to a 10mm waist diameter, and will diverge in the far-field at an angle of 2 mrad - as required.**

Verify results – calculate assumptions

Assume four calculated results as "given"===

Focal length	$f = 50.202\text{mm}$
Distance from fiber optic	$a = 50.247\text{mm}$
Refocused beam diameter	$D_b = 10\text{mm}$
Rayleigh range in A	$Z_a = 0.5\text{mm}$

Calculate the original four "given" parameters==================================

Refocus waist location

$$b := \left[\frac{1}{f} - \frac{1}{a + \frac{Z_a^{\,2}}{(a-f)}}\right]^{-1} \qquad b = 500\text{mm}$$

Magnification

$$M := \sqrt{\frac{f^2}{(a-f)^2 + Z_a^{\,2}}} \qquad M = 100$$

Waist diameter at F/0

$$D_a := \frac{D_b}{M} \qquad D_a = 0.1\text{mm}$$

Beam Divergence at F/0

$$\theta_a := \frac{D_a}{Z_a} \qquad \theta_a = 0.2\text{rad}$$

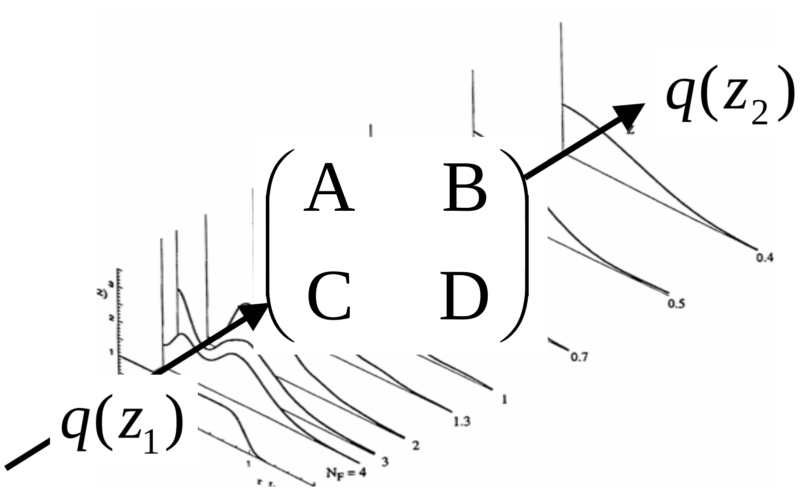
Refocused beam diverence

$$\theta_b := \frac{\theta_a}{M} \qquad \theta_b = 2 \times 10^{-3}\text{rad}$$

Break (5 min)

Part 4 beam parameters – ABCD method

ABCD Method



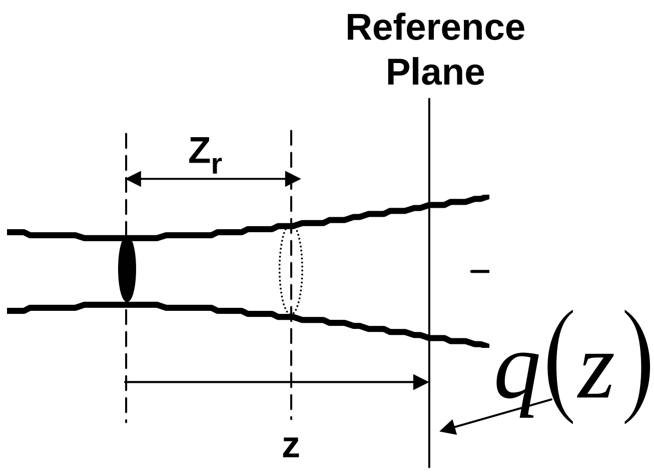
Fact #1 $q(z) = z + i \cdot Z_R$

Fact #2 $\dfrac{1}{q(z)} = \dfrac{1}{R(z)} - i\dfrac{E}{D(z)^2}$

Fact #3 $q(z_2) = \dfrac{A \cdot q(z_1) + B}{C \cdot q(z_2) + D}$

- ***q and 1/q contain 4 beam parameters***
- ***With E, remaining parameters are determined***
- ***[ABCD] changes q, same as wavefront radius of curvature***
- ***q = complex “radius of curvature”***
  - ***1/q = complex “curvature’***

ABCD method for beam parameters



$$q(z) = z + i \cdot Z_R$$

$$z = \mathrm{Re}(q)$$

$$Z_R = \mathrm{Im}(q)$$

$$D_o = \sqrt{E \cdot Z_R}$$

$$\theta = \sqrt{E \cdot Z_R^{-1}}$$

Four parameters:

z <u>waist to reference plane</u>

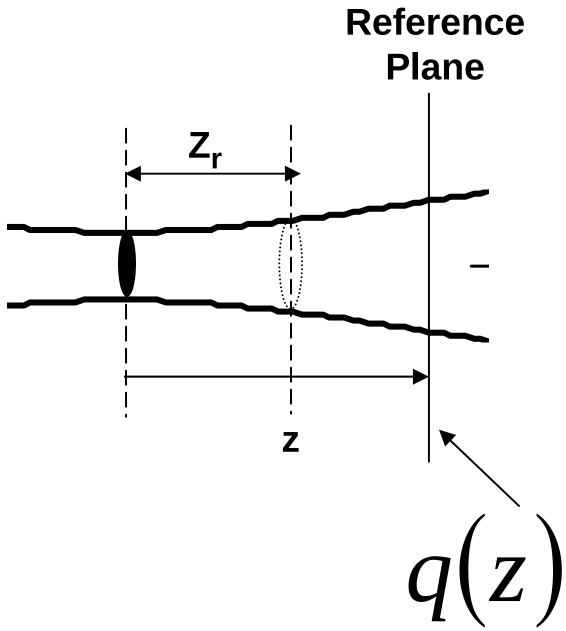
$z > 0$ **Waist left of RP**

$z = 0$ **Waist at RP**

$z < 0$ **Waist right of RP**

Z_R Rayleigh Range
Do = Waist diameter
θ = Divergence

ABCD method for beam parameters



Two parameters:

- **Curvature**
- **Beam diameter**

$$\frac{1}{q(z)} = \frac{1}{R(z)} - i \cdot \frac{E}{D(z)^2}$$

$$R(z) = \left[\mathrm{Re}\left(q(z)^{-1}\right)\right]^{-1}$$

$$D(z) = \sqrt{-\left[\mathrm{Im}\left(q(z)^{-1}\right)\right]^{-1} E}$$

q changes from RP to RP

q at 2 ← q at 1

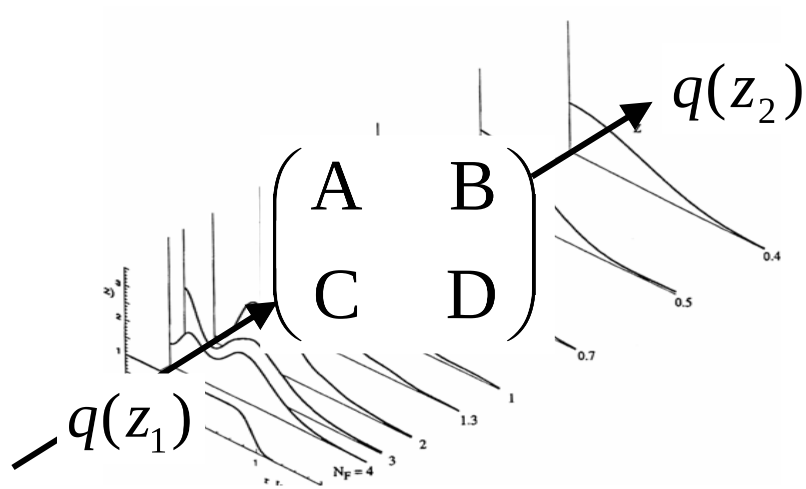
$$\begin{pmatrix} Q_0 \\ Q_1 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \cdot \begin{pmatrix} q_0 \\ 1 \end{pmatrix}$$

$$Q_0 = A \cdot q_1 + B$$

$$Q_1 = C \cdot q_1 + D$$

$$q_2 = \frac{Q_0}{Q_1}$$

$$q_2 = \frac{A \cdot q_1 + B}{C \cdot q_1 + D}$$



Complex q contains all 4 regional parameters

	Complex "Radius"	Regional Parameters
	$q = z + i \cdot Z_R$	
#1	$z = \mathrm{Re}(q)$	**Distance of waist from RF** • z > 0, waist is left • z < 0, waist is right
#2	$Z_R = \mathrm{Im}(q)$	**Rayleigh Range**
#3	$D_0 = \sqrt{E \cdot Z_R}$	**Waist Diameter**
#4	$\theta = \sqrt{E \cdot Z_R^{-1}}$	**Far-field beam divergence**

Complex q^{-1} contains 2 local parameters (at z)

	Complex "Curvature"	Local parameters at position z
	$$\frac{1}{q} = \frac{1}{R(z)} - i\frac{E}{D(z)^2}$$	
#5	$$R(z) = \left[\mathrm{Re}\left(q^{-1}\right)\right]^{-1}$$	**Radius of curvature at z**
#6	$$D(z) = \sqrt{-\left[\mathrm{Im}\left(q^{-1}\right)\right]^{-1} E}$$	**Beam diameter at z (not at waist)**

#7 Etendue

How is a beam characterized?

- ***Light beam has***
  - ✓***Position***
  - ✓***Direction***

    ABCD Ray Vector

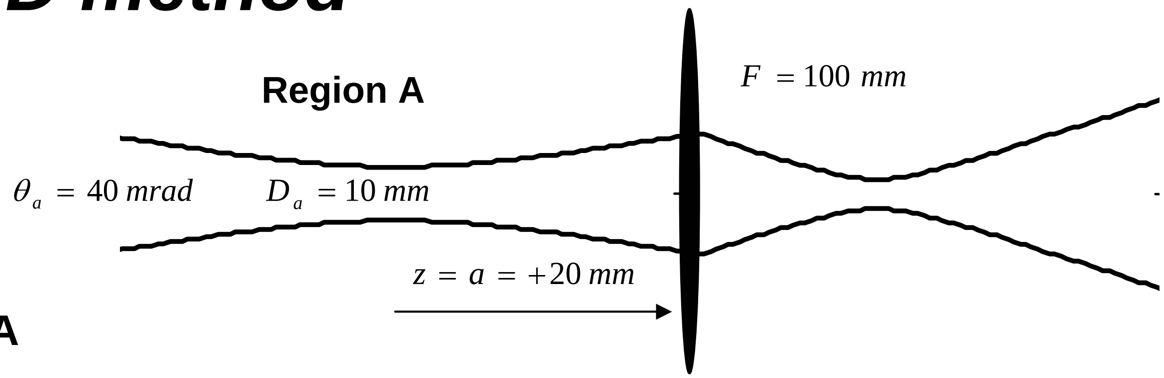
  - ✓***Waist location***
  - ✓***Waist diameter***
  - ✓***Wavefront diameter***
  - ✓***Wavefront radius of curvature*** ***7 parameters***
  - ✓***Far-field divergence angle***
  - ✓***Rayleigh range***
  - ✓***Etendue***

    ABCD Beam Vector

  - ***Intensity distribution (beam shape)***
  - ***Polarization***

- ***We will analyze the first 9***
  - ***How they are measured***
  - ***How they change***
    - ***the effects of lenses, spacing, wavelength, temperature, tilts, decenters***

ABCD method



Region A

q Method

$z = a = 20mm$ (+) waist left of RP (lens)

$D_a = 10mm$

$\theta_a = 40mrad$

$E = D_a \cdot \theta_a = 400mm \cdot mrad$

$Z_a = \dfrac{D_a}{\theta_a} = 250mm$

$q_1 = z + i \cdot Z_R = (20 + i \cdot 250)mm$

$D_a(lens) = \sqrt{-\mathrm{Im}(q_1^{-1}) \cdot E} = 10.032mm$

$R_a(lens) = \mathrm{Re}[q_1^{-1}] = 3145mm$

(-) diverging

Verify

$z = \mathrm{Re}(q_1) = +20mm$

$Z_a = \mathrm{Im}(q_1) = 250mm$

$D_a = \sqrt{E \cdot Z_b} = 10mm$

$\theta_b = \sqrt{E/Z_b} = 40mrad$

Equation Method

$D_a(lens) = D_a\sqrt{1 + (a/Z_a)^2} = 10.032mm$

$R_a(lens) = a + (Z_a{}^2/a) = 3145mm$

ABCD method

Region A --> Region B

$$F = 100mm$$

$$\begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ -F^{-1} & 1 \end{bmatrix} \cdot \begin{pmatrix} q_1 \\ 1 \end{pmatrix}$$

$$q_2 = \frac{Q_1}{Q_2} = (-88.389 + 36.284i)mm$$

$$z_b = \mathrm{Re}(q_2) = -88.39mm$$

(-) waist right of RF (lens)

$$Z_b = \mathrm{Im}(q_2) = 36.28mm$$

$$D_b = \sqrt{E \cdot Z_b} = 3.81mm$$

$$\theta_b = \sqrt{E/Z_b} = 105mrad$$

$$D_b(lens) = \sqrt{\left[-\mathrm{Im}\left(q_2^{-1}\right)\right]^{-1}} = 10.03mm$$

$$R_b(lens) = \left[\mathrm{Re}\left(q_2^{-1}\right)\right]^{-1} = -103.28mm$$

(-) converging

Verify

$$M = \sqrt{\frac{F^2}{(a-F)^2 + Z_a^{\ 2}}} = 0.381$$

$$b = \left[F^{-1} - \left(a + \frac{Z_a^{\ 2}}{(a-F)}\right)^{-1}\right]^{-1} = 88.39mm$$

$$D_b = D_a \cdot M = 3.81mm$$

$$\theta_b = \theta_a \cdot M^{-1} = 105mrad$$

$$Z_b = \frac{D_b}{\theta_b} = 36.28mm$$

$$D_b(lens) = D_b \cdot \sqrt{1 + (b/Z_b)^2} = 10.03mm$$

$$R_b(lens) = -b + \left(Z_b^{\ 2}/-b\right) = -103.28mm$$

ABCD method

Observations/Comments

$$D_a(lens) = D_b(lens) = 10.03mm$$

$$D_a = 10mm \approx D_a(lens) = 10.03mm$$
$$a = 20mm << R_a(lens) = 3145mm$$

$$\frac{1}{R_a} - \frac{1}{F} = -9.682 \cdot 10^{-3}$$
$$\frac{1}{R_b} = -9.682 \cdot 10^{-3}$$
$$\frac{1}{R_b} = \frac{1}{R_a} - \frac{1}{F}$$
$$K_b = K_a - P$$

🡸 ***Thin lens does not change y***

🡸 ***Beam on lens (almost) same size as at waist***
- ***Collimated beam***
- ***Well inside Rayleigh Range***

🡸 ***Gauss's Lens Law (1840)***
- ***Radii of Curvature***
- ***Diffraction Optics***
  - ***Geometrical Optics special case***

ABCD vs Equation-method

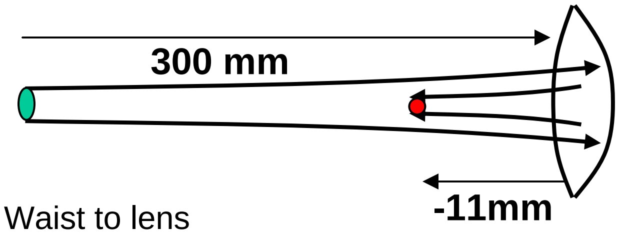
- ***Cascading many lenses***
  - 👍 ***[ABCD] manages complexity, ignores intermediate results***
  - 👎 ***Equation method would be applied successively to each lens***

- ***Design problems (determine lens f and location, given beam on each side)***
  - 👍 ***Equation method - easy; calculate magnification → focal length, etc***
  - 👎 ***Given 1 vector in and 1 vector out, it is impossible to calculate a unique [ABCD} matrix***

- ***Analysis and Trades***
  - 👍 ***Equation method provides analytical expressions which can be differentiated, manipulated algebraically, and analyzed parametrically***
  - 👍 ***[ABCD] provides numerical values directly using the recipe***

- ***Use both***
  - 👍 ***Each method proves a convenient way to check calculations and verify results produced by the other method.***
  - ***The [ABCD] method was used to generate the equations.***

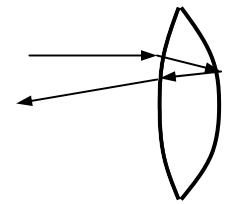
Ghost waists



$n := 1.52$

$L := 300mm$ $\quad M_1 := \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$ $\quad M_1 = \begin{pmatrix} 1 & 300 \\ 0 & 1 \end{pmatrix}$ Waist to lens

$R_1 := 100mm$ $\quad M_2 := \begin{bmatrix} 1 & 0 \\ -\frac{(n-1)}{R_1} & 1 \end{bmatrix}$ $\quad M_2 = \begin{pmatrix} 1 & 0 \\ -5.2 \times 10^{-3} & 1 \end{pmatrix}$ Refraction at 1st surface



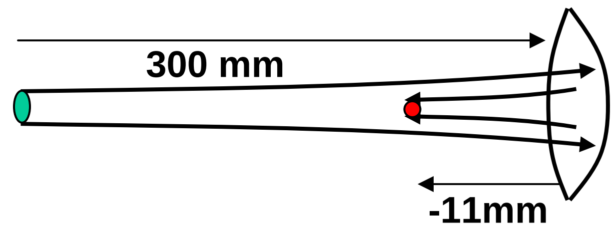
$t := 5mm$ $\quad M_3 := \begin{pmatrix} 1 & \frac{t}{n} \\ 0 & 1 \end{pmatrix}$ $\quad M_3 = \begin{pmatrix} 1 & 3.289 \\ 0 & 1 \end{pmatrix}$ 1st to 2nd surface

$R_2 := 50mm$ $\quad M_4 := \begin{pmatrix} 1 & 0 \\ -\frac{2 \cdot n}{R_2} & 1 \end{pmatrix}$ $\quad M_4 = \begin{pmatrix} 1 & 0 \\ -0.061 & 1 \end{pmatrix}$ Reflection at 2nd surface

$M_5 := \begin{pmatrix} 1 & \frac{t}{n} \\ 0 & 1 \end{pmatrix}$ $\quad M_5 = \begin{pmatrix} 1 & 3.289 \\ 0 & 1 \end{pmatrix}$ 2nd to 1st surface

$M_6 := \begin{bmatrix} 1 & 0 \\ -\frac{(1-n)}{-R_1} & 1 \end{bmatrix}$ $\quad M_6 = \begin{pmatrix} 1 & 0 \\ -5.2 \times 10^{-3} & 1 \end{pmatrix}$ Refraction at 1st surface

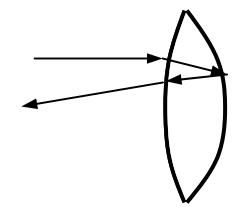
Ghost waists



Given $\quad D := 4\mathrm{mm}$

$\theta := 1.2\mathrm{mrad}$

$E := D \cdot \theta \qquad E = 4.8\mathrm{mm} \cdot \mathrm{mrad}$



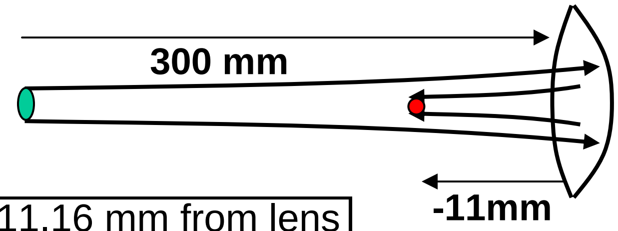
$Z_R := \dfrac{D}{\theta} \qquad Z_R = 3.333 \times 10^3\,\mathrm{mm}$

$q := 0 + i \cdot Z_R \qquad q = 3.333i \times 10^3\,\mathrm{mm}$ **In coming**

$$\begin{pmatrix} \alpha q_2 \\ \alpha_2 \end{pmatrix} := M_6 \cdot M_5 \cdot M_4 \cdot M_3 \cdot M_2 \cdot M_1 \cdot \begin{pmatrix} q \\ 1 \end{pmatrix}$$

$q_2 := \dfrac{\alpha q_2}{\alpha_2} \qquad q_2 = -11.16 + 0.063i$ **Out going**

Ghost waists

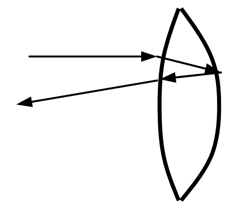


$z_{ghost} := \mathrm{Re}(q_2)$ $\quad z_{ghost} = -11.16\mathrm{mm}$

Ghost waist is 11.16 mm from lens

$Z_R := \mathrm{Im}(q_2)$ $\quad Z_R = 0.063\mathrm{mm}$

New Rayleigh Range



$D_{ghost} := \sqrt{E \cdot Z_R}$ $\quad D_{ghost} = 0.017\mathrm{mm}$

Ghost tight focus

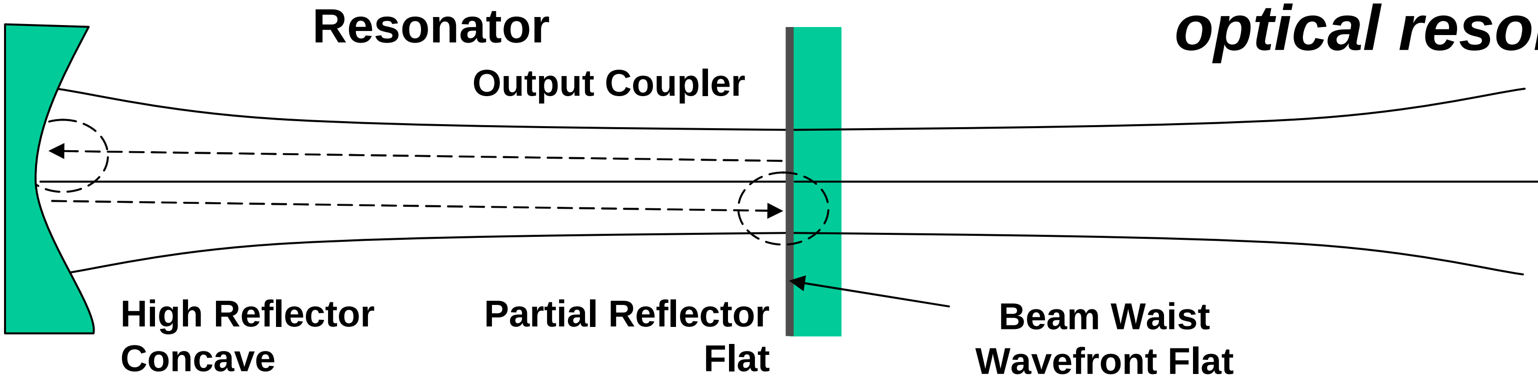
$\mathrm{Reflectivity} := 2\%$

$$\mathrm{PowerDensity} := \left(\frac{D}{D_{ghost}}\right)^2 \cdot \mathrm{Reflectivity}$$

$\mathrm{PowerDensity} = 1065$

- **1% of beam is reflected from 2^{nd} surface**
- **Ghost waist formed 11 mm from lens with vary small diameter**
- **Ghost power density is 1000x**
  - **Damage component there**
  - **Cause air breakdown**

Beams are generated in optical resonators



Inside resonator, beam makes many round trips - (1) Reflects of flat mirror; (2) travels 10 mm to concave mirror; (3) reflects off concave mirror (R = 30 mm); (4) travels 10 mm to flat mirror.

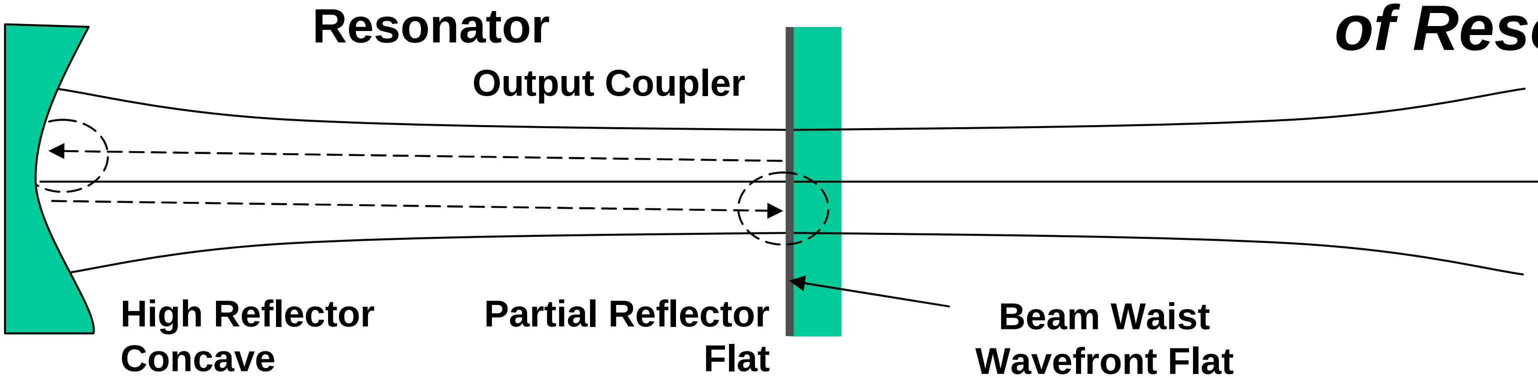
$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} := \overset{(4)}{\begin{pmatrix} 1 & 10 \\ 0 & 1 \end{pmatrix}} \cdot \overset{(3)}{\begin{pmatrix} 1 & 0 \\ \frac{-1}{30} & 1 \end{pmatrix}} \cdot \overset{(2)}{\begin{pmatrix} 1 & 10 \\ 0 & 1 \end{pmatrix}} \cdot \overset{(1)}{\begin{pmatrix} 1 & 0 \\ \frac{-1}{\infty} & 1 \end{pmatrix}}$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 0.667 & 16.667 \\ -0.033 & 0.667 \end{pmatrix}$$

One Cycle – round trip

$$\left| \begin{pmatrix} A & B \\ C & D \end{pmatrix} \right| = 1$$

Beams are Eigenmodes of Resonator



$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \cdot \begin{pmatrix} \alpha q \\ \alpha \end{pmatrix} ===> \begin{pmatrix} \alpha q \\ \alpha \end{pmatrix}$$

Same vector

After several round trips, the beam does not change.

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \cdot \begin{pmatrix} \alpha q \\ \alpha \end{pmatrix} = \varepsilon \cdot \begin{pmatrix} \alpha q \\ \alpha \end{pmatrix}$$

Eigenvalue Equation

$$\begin{pmatrix} \alpha q \\ \alpha \end{pmatrix} := \text{eigenvecs}\left(\begin{pmatrix} A & B \\ C & D \end{pmatrix}\right)^{\langle 1 \rangle}$$

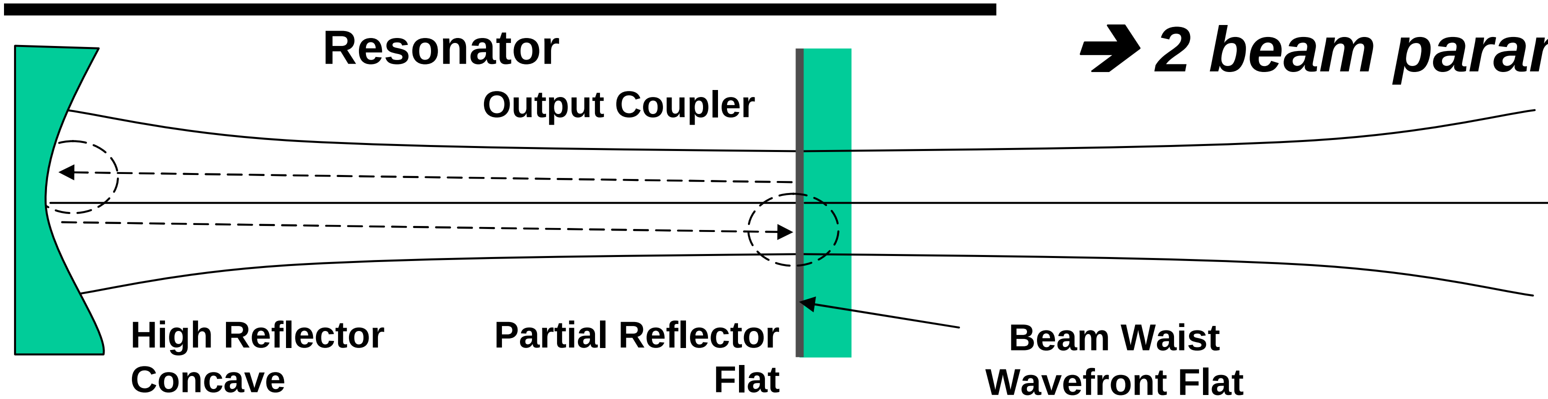
Eigenvector

$$\begin{pmatrix} \alpha q \\ \alpha \end{pmatrix} = \begin{pmatrix} 0.999 \\ -0.045i \end{pmatrix}$$

$$q := \frac{\alpha q}{\alpha} \cdot \text{mm} \qquad q = 22.361i\text{mm}$$

Eigenmode

Resonator optics
➔ 2 beam parameters

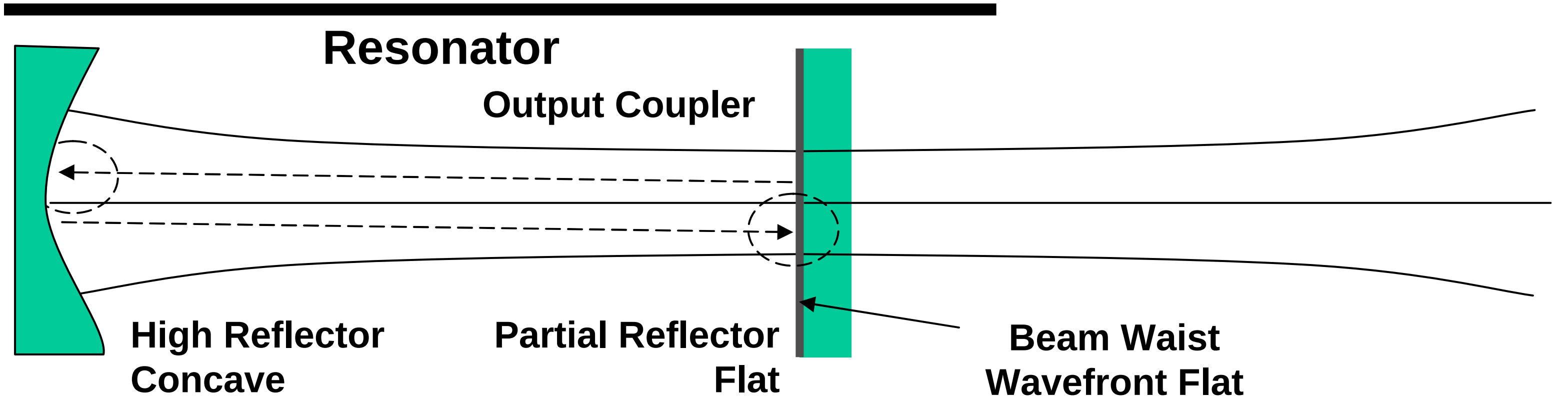


Calculate the beam parameters

Waist Position	Rayleigh Range
$z_o := \mathrm{Re}(q)$	$Z_R := \mathrm{Im}(q)$
$z_o = 0\,\mathrm{mm}$	$Z_R = 22.361\mathrm{mm}$

- ***Reference Plane (RP) at Flat***
  - ***Started and ended at flat***
- ***z_o is the distance of waist from (RP)***
  - ***z_o = 0 mm ➔ beam waist at flat***
  - ***Beam curvature = mirror curvature***
    - ***Mirror flat ➔ beam flat***
    - ***Beam flat ➔ waist***
- ***Rayleigh range and waist location***
  - ***Depend only on focal lengths and distances***
  - ***Independent of wavelength and beam quality***

Near- and Far-field beam



Assume wavelength, and beam quality, calculate waist diameter and divergence angle

$$\lambda := 4\mu m$$

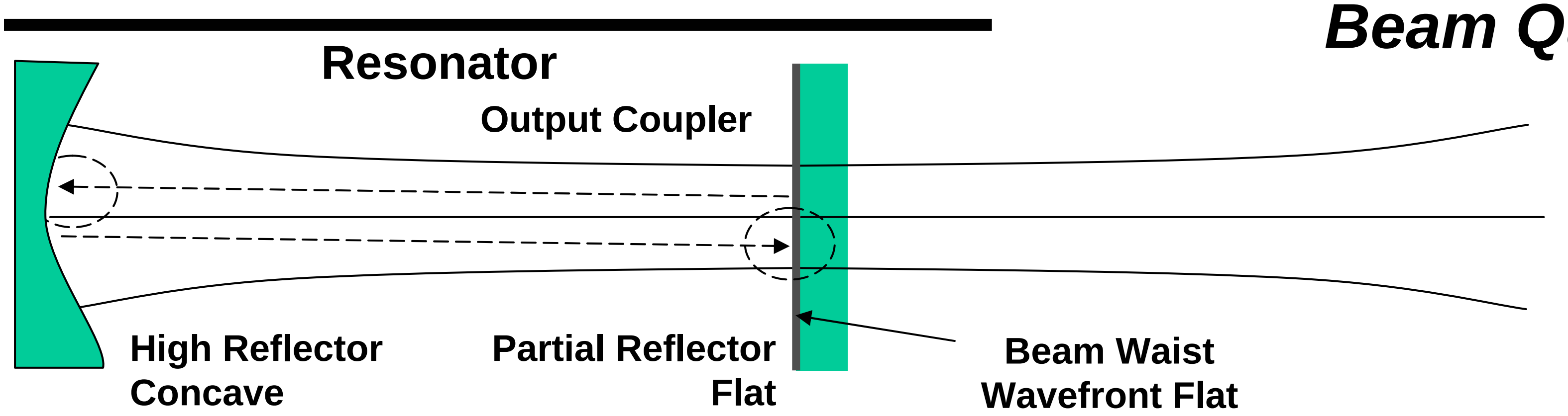
$$E_{min} := \frac{4\cdot\lambda}{\pi} \qquad E_{min} = 5.093\,mm\cdot mrad$$

$$Do_{min} := \sqrt{Z_R \cdot E_{min}} \qquad Do_{min} = 0.337\,mm$$

$$\theta_{min} := \sqrt{\frac{E_{min}}{Z_R}} \qquad \theta_{min} = 15.092\,mrad$$

- ***Minimum ➔ Diffraction limited***
  - ***Gaussian beam shape***
  - ***$M^2 = 1$***
- ***Real beams $M^2 > 1$***
  - ***$Do = Do_{min}\ M$***
  - ***$\theta = \theta_{min}\ M$***
  - ***Non-gaussian beam shape***

Beam Quality



Comparing the actual beam divergence to the minimum, we can calculate beam quality.

$\theta_{min} = 15.092\,\mathrm{mrad}$ $\boxed{\theta_{measured} := 18\,\mathrm{mrad}}$

$M := \dfrac{\theta_{measured}}{\theta_{min}}$ $M^2 = 1.423$

$Do := Do_{min} \cdot M$ $Do = 0.402\,\mathrm{mm}$

$E := E_{min} \cdot M^2$ $E = 7.245\,\mathrm{mm \cdot mrad}$

- ***Results are Inaccurate***
  - ***Neglects the interaction of the beam with the gain medium***
- ***More Accurate***
  - ***Represent gain medium as a lens - thermal lensing***
  - ***No energy loss or gain***
- ***Most accurate***
  - ***Complex matrix elements***
    - ***Energy loss and gain***
  - ***2 x 2 matrices ➔ 3 x 3 matrices***
    - ***Gain medium***

Bibliography

For Every Optical Engineer

- ***A. Gerrard and J.M. Burch, Introduction to Matrix Methods in Optics (Dover, 1975). Clear and methodical with 100s of numerical examples. Must have.***
- ***A.E. Siegman, Lasers (University Science Books, 1986). Excellent and thorough expose of ABCD methods and application to lasers.***
- ***E. Hecht, Optics (Pearson Addison Wesley, 2001). Good basic optics. Superb diagrams.***
- ***H. Kogelnik and T. Li, "Laser Beams and Resonators," Proc. IEEE 54, 1312 – 1329 (1966). One of the earliest, and still one of the best.***

For the Advanced and the Curious

- ***C. Palma and V. Bagini, "Extension of the Fresnel transform to ABCD systems," J. Opt. Soc. Am. A 14, 1774 – 1779 (1997). Shows how to calculate the field and phase of a paraxial beam passing through an optical system described by ABCD.***
- ***A.A. Tovar and L.W. Casperson, "Generalized beam matrices. IV. Optical system design," J. Opt. Soc. Am. A 14. 832 - 893 (1997). Extends beam matrices to 3 x 3 with complex components to account for misalignment and power loss and gain.***

Contact

Dear Student,

Thank you for attending this course. Please assist me in improving these notes for the next students by reporting errors, typos, confusing symbols, or other obstacles to understanding.

Your feedback on what is good and what can be improved is most welcome, as well as suggestions on what might be added or expanded.

Sincerely,

John A. McNeil

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Phone   (603) 673-0753

Summary

All mathematics
- **matrix multiplication**
- **complex numbers**

$$\overset{\textbf{Out}}{\begin{pmatrix} y_2 & Q_0 \\ v_2 & Q_1 \\ 1 & 0 \end{pmatrix}} = \begin{pmatrix} A & B & E \\ C & D & F \\ 0 & 0 & 1 \end{pmatrix} \overset{\textbf{In}}{\begin{pmatrix} y_1 & q_0 \\ v_1 & q_1 \\ 1 & 0 \end{pmatrix}}$$

1. **position**
2. **direction**

3. **waist location**
4. **waist diameter**
5. **far-field divergence**
6. **Rayleigh range**
7. **wavefront radius (and curvature)**
8. **wavefront diameter**
9. **Etendue**

All transverse errors

All lenses and mirrors (f)
All separations (t)
All longitudinal errors