UiT Norges arktiske universitet

DTE-2501 AI Methods and Applications

Boolean logic vs Fuzzy logic

Lecture 1/2 – Probability theory

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Overview

I Repetitive operations and sample spaces

II Events

III Probability

IV Conditional probability

V Example

I. Guttman, S.S. Wilks. *Introductory Engineering Statistics*. John Wiley & Sons, Inc. 1965

I Repetitive operations and sample spaces

Repetitive operation is the notion of being able to repeat an operation over and over again «under essentially the same conditions».

a) Tossing a coin

b) Throwing a die

c) Shuffling a pack of playing cards and then cutting the pack

d) Taking two screws «at random» from a box of 100 screws

Sample space is the set or collection of all possible outcomes of a repetitive operation.

a) Coin: $S = \{H, T\}$

b) Die: $S = \{1, 2, 3, 4, 5, 6\}$

c) Playing cards: 52 outcomes

d) Screws: 4950 possible outcomes

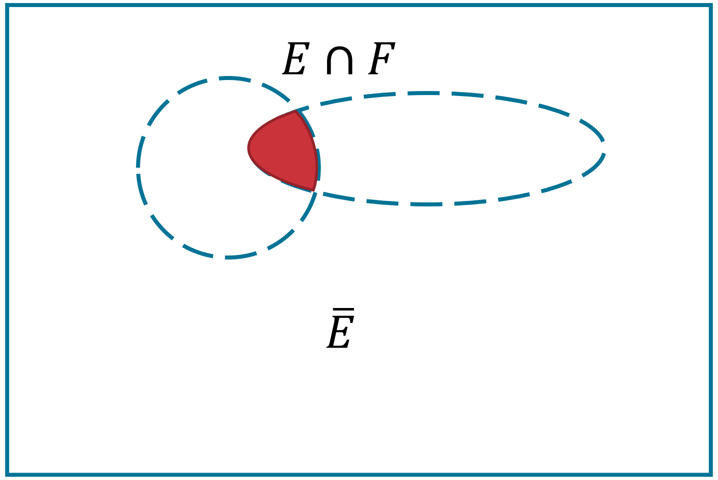
II Events

$S = \{e_1, e_2, \dots, e_m\}$

$E \subset S$

Event is some subset E of the sample space S of particular interest.

S:



III Probability

$S = \{e_1, e_2, \dots, e_m\}$

Example: $S = \{e_1 = H, e_2 = T\}$

Experiment: toss a coin 200 times

Heads: 108

Tails: 92

$f_1 = \dfrac{108}{200}$

$f_2 = \dfrac{92}{200}$

→ repeat this experiment infinitely many times

→ idealized values for the f's

In general, the quantities $p(e_1), \dots, p(e_m)$ are called *probabilities* of occurence of $e_1, \dots, e_m$ respectively and

$$p(e_1) + p(e_2) + \cdots + p(e_m) = 1$$

Suppose E is any event in S, $E = \{e_{i_1}, \dots, e_{i_r}\}$. The probability of the occurrence of E is denoted by $P(E)$, and is defined as follows:

$$P(E) = p(e_{i_1}) + \cdots + p(e_{i_r})$$

The axioms of probability

1. If E is any event in S, then $P(E) \geq 0$
2. $P(S) = 1$
3. If E and F are two disjoint events, then $P(E \cup F) = P(E) + P(F)$

Rule of complementation

If E is an event in a sample space S, then

$$P(\bar{E}) = 1 - P(E)$$

General rule of complementation

If $E_1, \dots, E_k$ are events in a sample space S, then

$$P\left(\cap_{i=1}^{k} \bar{E}_i\right) = 1 - P(\cup_{i=1}^{k} E_i)$$

Rule of addition of probabilities for mutually exclusive events

If $E_1, \dots, E_k$ are *disjoint* events in a sample space S, then

$$P(E_1 \cup \cdots \cup E_k) = P(E_1) + \cdots + P(E_k)$$

Rule for addition of probabilities for two arbitrary events

If E_1 and E_2 are *any* two events in a sample space S, then

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

More generally, for n events $E_1, \dots, E_n$, we have

$$P(E_1 \cup \cdots \cup E_n) = \sum_{i=1}^{n} P(E_i) - \sum_{j>i=1}^{n} P\left(E_i \cap E_j\right) + \sum_{k>j>i=1}^{n} P\left(E_i \cap E_j \cap E_k\right) + \cdots + (-1)^{n-1} P(E_1 \cap \cdots \cap E_n)$$

IV Conditional probability

The probability that an event F occurs if it is known that an event E has occurred is called the *conditional probability of F given E*, and it is denoted by $P(F|E)$

$$P(F|E) = \frac{P(E \cap F)}{P(E)} ,$$

where $P(E) \neq 0$.

Rule of multiplication of probabilities

If E and F are events in a sample space S, such that $P(E) \neq 0$, then

$$P(E \cap F) = P(E)P(F|E)$$

Rule of multiplication of probabilities of two independent events

E and F are independent events if and only if

$$P(E \cap F) = P(E)P(F)$$

V Example

A box of 1000 screws contains:

- 50 screws with type A defects;
- 32 screws with type B defects;
- 18 screws with type C defects;
- 7 screws with type A and type B defects;
- 5 screws with type A and type C defects;
- 4 screws with type B and type C defects;
- 2 screws with all three types of defects.

Event	Number of elements
E_A	50
E_B	32
E_C	18
$E_A \cap E_B$	7
$E_A \cap E_C$	5
$E_B \cap E_C$	4
$E_A \cap E_B \cap E_C$	2

a) The probability that the screw will have a type A *or* type B defect, *or both*, is

$$P(E_A \cup E_B) = P(E_A) + P(E_B) - P(E_A \cap E_B)$$

b) The probability that the screw will have *at least one of three* types of defect is

$$P(E_A \cup E_B \cup E_C) = P(E_A) + P(E_B) + P(E_C) - P(E_A \cap E_B) - P(E_A \cap E_C) - P(E_B \cap E_C) + P(E_A \cap E_B \cap E_C)$$

c) The probability that the screw will be *free of these defects* is

$$P(\bar{E}_A \cap \bar{E}_B \cap \bar{E}_C) = 1 - P(E_A \cup E_B \cup E_C)$$

d) Let the screw drawn from the box has a type A defect. The probability that it *also* have a type B defect is

$$P(E_B|E_A) = \frac{P(E_A \cap E_B)}{P(E_A)}$$