FDTD EN UNA RED NO UNIFORME

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¿Por qué usar una red no uniforme?

- Mejor ajuste a la geometría del sistema.
- Aumento de la resolución espacial.

FDTD en red no uniforme

Definimos distancias entre los puntos de la red:

$$\Delta x_i = x_{i+1} - x_i \;\; ; \; \Delta y_j = y_{j+1} - y_j \;\; ; \; \Delta z_k = z_{k+1} - z_k$$

Posiciones de los centros de la red:

$$x_{i+\frac{1}{2}} = x_i + \frac{\Delta x_i}{2} \; ; \; y_{j+\frac{1}{2}} = y_j + \frac{\Delta y_j}{2} \; ; \; z_{k+\frac{1}{2}} = z_k + \frac{\Delta z_k}{2}$$

Distancias entre los centros de la red:

$$h_i^x = \frac{\Delta x_i + \Delta x_{i-1}}{2} \;\; ; \; h_j^y = \frac{\Delta y_j + \Delta y_{j-1}}{2} \;\; ; \; h_k^z = \frac{\Delta z_k + \Delta z_{k-1}}{2}$$

Condición de estabilidad:

$$\Delta t \leq \frac{1}{c\sqrt{\frac{1}{\left(\Delta x_{i,min}\right)^2} + \frac{1}{\left(\Delta y_{j,min}\right)^2} + \frac{1}{\left(\Delta z_{k,min}\right)^2}}}$$

Solución caso 1D:

$$H_{i+1/2}^{n+1/2} = H_{i+1/2}^{n-1/2} - \frac{\Delta t}{\mu\,\Delta x_i}(E_{i+1}^{n} - E_i^{n})$$

$$E_i^{n+1} = E_i^{n} + \frac{\Delta t}{\epsilon \Delta x_{i+1/2}}(H_{i+1/2}^{n+1/2} - H_{i-1/2}^{n+1/2})$$

FDTD 2D: Grid Uniforme

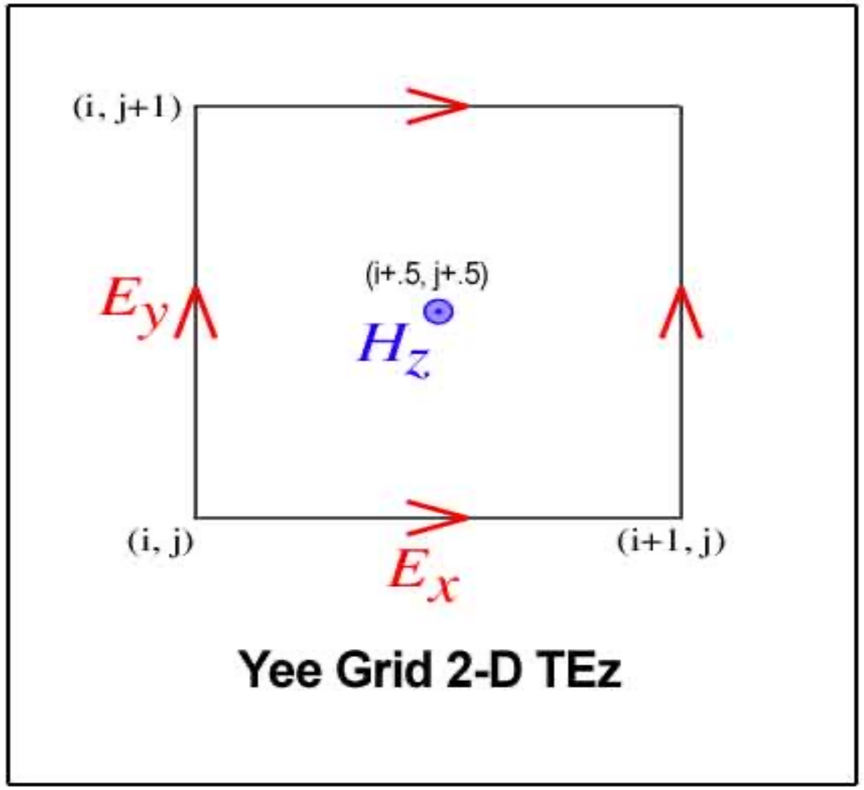
$$\vec{\nabla}\times\vec{E} = -\mu\partial_t\vec{H} \qquad\qquad \vec{\nabla}\times\vec{H} = \varepsilon\partial_t\vec{E}$$

2D: $f(z) = cte \Rightarrow \partial_z f = 0$

Modo TE: $\begin{cases} \partial_x E_y - \partial_y E_x = -\mu\partial_t H_z \\ \partial_y H_z = \varepsilon\partial_t E_x \\ \partial_x H_z = -\varepsilon\partial_t E_y \end{cases}$ Modo TM: $\begin{cases} \partial_x H_y - \partial_y H_x = \varepsilon\partial_t E_z \\ \partial_y E_z = -\mu\partial_t H_x \\ \partial_x E_z = \mu\partial_t H_y \end{cases}$

FDTD 2D: Grid Uniforme

Modo TE: Fijamos H_z en el grid dual $\rightarrow \begin{cases} (x, y) \in \text{ Grid principal} \\ \left(x \pm \frac{\Delta x}{2}, y \pm \frac{\Delta y}{2}\right) \in \text{Grid dual} \end{cases}$



FDTD 2D: Grid Uniforme

Discretizamos las ecuaciones:

$$H_z\left(t+\frac{\Delta t}{2}, x+\frac{\Delta x}{2}, y+\frac{\Delta y}{2}\right)=H_z\left(t-\frac{\Delta t}{2}, x+\frac{\Delta x}{2}, y+\frac{\Delta y}{2}\right)$$
$$-\frac{\Delta t}{\mu}\left[\frac{E_y\left(t, x+\Delta x, y+\frac{\Delta y}{2}\right)-E_y\left(t, x-\Delta x, y+\frac{\Delta y}{2}\right)}{\Delta x}\right.$$
$$\left.-\ \frac{E_x\left(t, x+\frac{\Delta x}{2}, y+\Delta y\right)-E_x\left(t, x+\frac{\Delta x}{2}, y\right)}{\Delta y}\right]$$

$$E_x\left(t+\Delta t, x+\frac{\Delta x}{2}, y\right)=E_x\left(t, x+\frac{\Delta x}{2}, y\right)+\frac{\Delta t}{\varepsilon\Delta y}\left[H_z\left(t+\frac{\Delta t}{2}, x+\frac{\Delta x}{2}, y+\frac{\Delta y}{2}\right)-H_z\left(t+\frac{\Delta t}{2}, x+\frac{\Delta x}{2}, y-\frac{\Delta y}{2}\right)\right]$$

$$E_y\left(t+\Delta t, x, y+\frac{\Delta y}{2}\right)=E_y\left(t, x, y+\frac{\Delta y}{2}\right)-\frac{\Delta t}{\varepsilon\Delta x}\left[H_z\left(t+\frac{\Delta t}{2}, x+\frac{\Delta x}{2}, y+\frac{\Delta y}{2}\right)-H_z\left(t+\frac{\Delta t}{2}, x-\frac{\Delta x}{2}, y+\frac{\Delta y}{2}\right)\right]$$