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Self-fulfilling debt crises: What can monetary policy do?☆

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ABSTRACT

This paper examines the potential for monetary policy to avoid self-fulfilling sovereign debt crises. We combine a version of the slow-moving debt crisis model proposed by Lorenzoni and Werning (2014) with a standard New Keynesian model. Monetary policy could preclude a debt crisis through raising inflation and output and lowering the real interest rate. These reduce the real value of outstanding debt and the cost of new borrowing, and increase tax revenues and seigniorage. We determine the optimal path of inflation required to avoid a self-fulfilling debt crisis. Stronger price rigidity implies more sustained inflation.

1. Introduction

A popular explanation for sovereign debt crises is self-fulfilling sentiments. If market participants believe that sovereign default of a country is more likely, they demand higher spreads, which raises the debt burden and therefore indeed makes eventual default more likely.[1] This view of self-fulfilling beliefs is consistent with the evidence that the surge in sovereign bond spreads in Europe during 2010–2011 was disconnected from debt ratios and other macroeconomic fundamentals (e.g., de Grauwe and Ji, 2013).[2] It has also been suggested as an explanation for the Argentine crisis of 1998–2002 (Ayres et al., 2015). Recently a debate has developed about what role the central bank may play in avoiding such self-fulfilling debt crises. The central bank has additional tools to support the fiscal authority, either in the form of standard inflation policy or by providing liquidity. Some have argued that the US, Japan, UK and others have avoided such crises altogether because they have their own currency and monetary policy.[3]

In this paper we examine what central banks can do to avert self-fulfilling debt crises. We pose this as a general question, without focusing on a specific country or historical episode. For a realistic analysis, we allow for long-term debt, nominal rigidities and dynamics leading to slow-moving debt crises. We do so by combining a standard New Keynesian (NK) model with the “slow moving” debt crisis framework proposed by Lorenzoni and Werning (2014, henceforth LW). The LW model is in the spirit of Calvo (1988), but the latter is

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[1] For discussions of self-fulfilling crises in sovereign debt models see for example Aguiar et al. (2013), Calvo (1988), Camous and Cooper (2014), Cohen and Villemot (2015), Conesa and Kehoe (2015), Corsetti and Dedola (2016), de Grauwe (2012), de Grauwe and Ji (2013), Gros (2012), Jeanne (2012), Jeanne and Wang (2013), Krugman (2013), Lorenzoni and Werning (2014), and Miller and Zhang (2012). Ayres et al. (2015) show that multiple equilibria arise in sovereign debt models when the government chooses current debt, as opposed to debt at maturity. Even if the government chooses debt at maturity, they show that there are still multiple equilibria when lenders move first (choose an interest rate at which they are willing to lend).

[2] This view was held by the ECB President Draghi himself: “…the assessment of the Governing Council is that we are in a situation now where you have large parts of the euro area in what we call a “bad equilibrium”, namely an equilibrium where you may have self-fulfilling expectations that feed upon themselves and generate very adverse scenarios.” (press conference, September 6, 2012).

[3] See for example de Grauwe (2012), de Grauwe and Ji (2013), Jeanne (2012) and Krugman (2013).

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a two-period model with one-period bonds, while the LW model is dynamic and has long-term bonds. As is standard in sovereign debt models with self-fulfilling equilibria, there is a region of debt in which there are multiple equilibria. This exposes the country to self-fulfilling beliefs that lead to lower debt prices, debt accumulation and possible default. We refer to this debt region as the "multiplicity region". The NK block of the model allows us to consider realistic monetary policy. The central bank can reduce the real value of original debt and of distributed coupons through inflation. It can also reduce the real cost of borrowing by reducing the (riskfree) real interest rate. Finally, expansionary monetary policy delivers seigniorage revenue and can raise the primary surplus by raising output. This all helps to slow down the accumulation of debt.

The objective of our investigation is to examine what optimal monetary policy looks like under the constraint that it avoids a self-fulfilling crisis. The central bank is capable of avoiding self-fulfilling default by acting aggressively enough, but it may not always be optimal to do so. There is a tradeoff between the cost of monetary policy, particularly inflation, versus the cost of outright default.[4] A monetary policy aimed at avoiding default is not credible when the cost of the policy is larger than that of outright default. Rather than making judgements about this tradeoff, our aim here is simply to document the cost of the monetary policy that avoids self-fulfilling default. We particularly focus on inflation under such a policy. We examine what the central bank can do when hit by default expectations at an initial time $t = 0$. The factors leading to such a crisis, in particular the initial level of debt, and the future path of fiscal policies, are taken as given by the central bank. In this context our numerical analysis determines the optimal inflation path that is required to avoid a default. This deviation is unexpected, i.e., we determine the required level of surprise inflation.

The required level of surprise inflation depends on the distance between the initial level of debt, B_0, and the maximum level of debt that avoids multiplicity; or equivalently, the minimum debt level in the multiplicity zone, B_{low}. When B_0 is close to B_{low}, the required level of additional inflation is obviously modest. However, we find that the required level of inflation can easily be large. Consider for example our benchmark parameterization that generates a multiplicity region ranging from 80 to 150% of GDP. If we take the debt ratio in the middle of this range, i.e., 115 of GDP, the price level ultimately has to increase by a factor 5 and the additional level of inflation should be higher than 20% for 4 years and over 10% for 8 years.

The potentially high level of inflation needed to avoid self-fulfilling crises has an intuitive explanation. Consider the above example for our benchmark case. If the central bank could raise the price level right away, without delay, the price level would only need to rise by 42% to reduce debt from 115 to 80% of GDP. In reality though, inflation is more gradual, both because of price stickiness and because it is optimal from a welfare point of view to have more gradual inflation.[5] Price indexation contributes to the desire to smooth out inflation as price dispersion increases with changes in the inflation rate. However, raising inflation gradually will ultimately lead to a much larger increase in the price level. The reason is that inflation becomes less effective over time because the interest rate for newly issued debt incorporates inflation expectations. For a realistic maturity of government debt, inflation gradually loses power in reducing debt. Hilscher et al. (2014) take this point seriously in estimating the impact of unexpected inflation in reducing outstanding debt and estimate this impact to be limited.

We devote significant space to the question of how robust this result is. We consider changes to all the parameters of both the LW and NK components of the model and find that the result is quite robust. We also consider results based exclusively on the LW part of the model. This monetary version of the LW model leads to a condition on inflation and real interest rates over time that needs to be satisfied in order to avoid the multiplicity region. It holds independent of how monetary policy affects inflation and real interest rates and is therefore independent of the NK portion of the model. Regarding the LW part of the model, the only key parameter is the maturity of the debt. All the other parameters can have an impact on the multiplicity region, but do not significantly affect the results if we keep B_0/B_{low} fixed.

We consider the standard case where in normal times it is optimal for the central bank to commit to zero inflation. But if initial inflation is already large, the additional surprise inflation needed to avoid a self-fulfilling crisis would be even more costly given the convex cost of inflation. We also abstract from liquidity or rollover crises, such as those considered by Cole and Kehoe (2000). As Bocola and Dovis (2015) point out, we often see a substantial shortening of the maturity structure under steep inflation, which leads to additional problems in terms of exposure to rollover crises that we abstract from.

This is not the first paper to analyze the impact of monetary policy in a self-fulfilling debt crisis environment. The main difference is that previous work focuses on more qualitative questions using more stylized frameworks. It typically does not consider standard interest rate policies and considers mainly two-period models with one period bonds, flexible prices, constant real interest rates and a constant output gap. The role of monetary policy was first analyzed by Calvo (1988), who examined the trade-off between outright default and debt deflation. Corsetti and Dedola (2016) extend the Calvo model to allow for both fundamental and self-fulfilling default. Similarly to our analysis, they show that with optimal monetary policy debt crises can still happen, but for larger levels of debt. They also show that a crisis can be avoided if government debt can be replaced by risk-free central bank debt. Reis (2013) and Jeanne (2012) also develop stylized two-period models with multiple equilibria to illustrate ways in which the central bank can act to avoid the bad equilibrium.

Some papers consider more dynamic models, but still assume flexible prices and one-period bonds.[6] Camous and Cooper (2014) use a dynamic overlapping-generation model with strategic default. They show that the central bank can avoid self-fulfilling default if they commit to a policy where inflation depends on the state (productivity, interest rate, sunspot). Aguiar et al. (2013) consider a dynamic model to analyze the vulnerability to self-fulfilling rollover crises, depending on the aversion of the central bank to inflation. Although a rollover crisis occurs suddenly, it is assumed that there is a grace period to repay the debt, allowing the central bank time to reduce the real value of the debt through inflation. They find that only for intermediate levels of the cost of inflation do debt crises occur under a narrower range of debt values.

The rest of the paper is organized as follows. Section 2 presents the slow-moving debt crisis model based on LW. It starts with a real version of the model and then presents its extension to a

[4] This tradeoff is standard in the literature, e.g. see Aguiar et al. (2013), Camous and Cooper (2014), or Corsetti and Dedola (2016).

[5] Since inflation operates gradually it is not equivalent to outright default. It is true that both inflation and outright default reduce the real value of the debt, but gradual inflation affects differently relative prices and intertemporal decisions by households and firms.

[6] Nuño and Thomas (2015) analyze the role of monetary policy with long-term debt in the context of a dynamic Eaton and Gersovitz (1981) model (with a unique equilibrium) and find that debt deflation is not optimal. There are also recent models that examine the impact of monetary policy in the presence of long-term government bonds, but they do not allow for the possibility of sovereign default. For example, Leeper and Zhou (2013) analyze optimal monetary (and fiscal) policy with flexible prices, while Bhattarai et al. (2013) consider a New Keynesian environment at ZLB. Sheedy (2014) and Gomes et al. (2016) examine monetary policy with long-term private sector bonds.

monetary environment. Subsequently, it analyzes the various channels of monetary policy in this framework. Section 3 describes the New Keynesian part of the model, discusses results under optimal policy and considers sensitivity analysis and extensions. Section 4 provides results that do not rely on the New Keynesian part of the model. Section 5 considers alternative policies. Section 6 concludes. Some of the technical details are left to the Appendix, while additional algebraic details and results can be found in a separate online Appendix.

2. A model of slow-moving self-fulfilling debt crisis

In this section we present a dynamic sovereign debt crisis model based on LW. We first describe the basic structure of the model in a real environment. We then extend the model to a monetary environment and discuss the impact of monetary policy on the existence of self-fulfilling debt crises. We focus on the dynamics of asset prices and debt for given interest rates and goods prices. The latter will be determined in a New Keynesian model that we describe in Section 3.

2.1. A real model

We consider a simplified version of the LW model. As in the applications considered by LW, there is a key date T at which uncertainty about future primary surpluses is resolved and the government makes a decision to default or not.[7] Default occurs at time T if the present value of future primary surpluses is insufficient to repay the debt. We assume that default does not happen prior to date T as there is always a possibility of large primarily surpluses from T onward. In one version of their model LW assume that T is known to all agents, while in another they assume that it is unknown and arrives each period with a certain probability. We adopt the former assumption. In the online Appendix we analyze an extension where T is uncertain. While this significantly complicates the analysis, it does not alter the key findings.[8]

The only simplification we adopt relative to LW concerns the process of the primary surplus. For now we assume that the primary surplus s_t is constant at $\bar{s}$ between periods 0 and $T-1$. LW assume a fiscal rule whereby the surplus is a function of debt. Not surprisingly, they find that the range of debt where a country is vulnerable to self-fulfilling crises narrows if the fiscal surplus is more responsive to debt. Very responsive fiscal policy could in principle eliminate the concern about self-fulfilling debt crises. In this paper, however, we take vulnerability to self-fulfilling debt crises as given in the absence of monetary policy action. We therefore abstract from such strong stabilizing fiscal policy. However, we will consider an extension where the primary surplus depends on output and is pro-cyclical as this provides an additional avenue through which monetary policy can be effective.

A second assumption concerns the primary surplus value starting at date T. Let $\tilde{s}$ denote the maximum potential primary surplus that the government is able to achieve, which becomes known at time T and is constant from thereon. LW assume that it is drawn from a log normal distribution. Instead we assume that it is drawn from a binary distribution, which simplifies the algebra and the presentation. It can take on only two values: s_{low} with probability ψ and s_{high} with probability $1-\psi$. When the present discounted value of $\tilde{s}$ is at least as large as what the government owes on debt, there is no default at time T and the actual surplus is just sufficient to satisfy the budget constraint (generally below $\tilde{s}$). We assume that s_{high} is big enough such that this is always the case when $\tilde{s}=s_{high}$.[9] When $\tilde{s}=s_{low}$ and its present value is insufficient to repay the debt, the government defaults.

A key feature of the model is the presence of long-term debt. As usual in the literature, assume that bonds pay coupons (measured in goods) that depreciate at a rate of $1-\delta$ over time: $\kappa, (1-\delta)\kappa, (1-\delta)^2\kappa$, and so on.[10] A smaller δ therefore implies a longer maturity of debt. This facilitates aggregation as a bond issued at $t-s$ corresponds to $(1-\delta)^s$ bonds issued at time t. We can then define all outstanding bonds in terms of the equivalent of newly issued bonds. We define b_t as debt measured in terms of the equivalent of newly issued bonds at $t-1$ on which the first coupon is due at time t. As in LW, we take δ as given. It is associated with tradeoffs that are not explicitly modeled, and we do not allow the government to change the maturity to avoid default.

Let Q_t be the price of a government bond. At time t the value of government debt is $Q_t b_{t+1}$. In the absence of default the return on the government bond from t to $t+1$ is

$$r_t^g=\frac{(1-\delta)Q_{t+1}+\kappa}{Q_t} \tag{1}$$

If there is default at time T, bond holders are able to recover a proportion $\zeta<1$ of the present discounted value s^{pdv} of the primary surpluses s_{low}.[11] In that case the gross return on the government bond is

$$r_{T-1}^g=\frac{\zeta s^{pdv}}{Q_{T-1}b_T} \tag{2}$$

Government debt evolves according to

$$Q_t b_{t+1}=r_{t-1}^g Q_{t-1}b_t-s_t \tag{3}$$

In the absence of default this may also be written as $Q_t b_{t+1}=((1-\delta)Q_t+\kappa)b_t-s_t$. The initial stock of debt b_0 is given.

We assume that investors also have access to a short-term bond with a gross real interest rate r_t. The only shocks in the model occur at time 0 (self-fulfilling shock to expectations) and time T (value of $\tilde{s}$). In other periods the following risk-free arbitrage condition holds (for $t\geq 0$ and $t\neq T-1$):

$$r_t=\frac{(1-\delta)Q_{t+1}+\kappa}{Q_t} \tag{4}$$

For now we assume, as in LW, a constant interest rate, $r_t=r$. In that case $s^{pdv}=rs_{low}/(r-1)$ is the present discounted value of s_{low}. There is no default at time T if s^{pdv} covers current and future debt service at T, which is $((1-\delta)Q_T+\kappa)b_T$. Since there is no default after time T, Q_T is the risk-free price, equal to the present discounted value of future coupons. For convenience it is assumed that $\kappa=r-1+\delta$, so that Eq. (4) implies that $Q_T=1$. This means that there is no default as long as $s^{pdv}\geq rb_T$, or if

$$b_T\leq\frac{1}{r-1}s_{low}\equiv\tilde{b} \tag{5}$$

[7] One can for example think of countries that have been hit by a shock that adversely affected their primary surpluses, which is followed by a period of uncertainty about whether and how much the government is able to restore primary surpluses through higher taxation or reduced spending.

[8] Notice that uncertainty about T implies uncertainty about the range of the multiplicity zone.

[9] See online Appendix for details.

[10] See for example Hatchondo and Martinez (2009).

[11] One can think of ζ as the outcome of a bargaining process between the government (representing taxpayers) and bondholders. Since governments rarely default on all their debt, we assume $\zeta>0$.

When $b_T > \tilde{b}$, the government partially defaults on debt, with investors seizing a fraction ζ of the present value s^{pdv}.

This framework may lead to multiple equilibria and to a slow-moving debt crisis, as described in LW. The existence of multiple equilibria can be seen graphically from the intersection of two schedules, as illustrated in Fig. 1. The first schedule, labeled "pricing schedule", is a consistency relationship between price and outstanding debt at $T-1$, in view of the default decision that may be taken at T. This is given by

$$Q_{T-1} = 1 \quad \text{if} \quad b_T \leq \tilde{b} \tag{6}$$

$$= \psi \frac{\zeta s^{pdv}}{r b_T} + (1-\psi) \quad \text{if} \quad b_T > \tilde{b} \tag{7}$$

When $b_T \leq \tilde{b}$, the arbitrage condition (4) also applies to $t = T-1$, implying $Q_{T-1} = 1$. When b_T is just above $\tilde{b}$, there is a discrete drop of the price because only a fraction ζ of primary surpluses can be recovered by bond holders in case of default. For larger values of debt, Q_{T-1} will be even lower as the primary surpluses have to be shared among more bonds.

The second schedule is the "debt accumulation schedule," which expresses the amount of debt that accumulates through time $T-1$ as a function of prices between 0 and $T-1$. Every price Q_t between 0 and $T-1$ can be expressed as a function of Q_{T-1} by integrating Eq. (4) backwards from $T-1$ to 0:

$$Q_t - 1 = \left(\frac{1-\delta}{r}\right)^{T-1-t} (Q_{T-1} - 1) \tag{8}$$

Substituting in Eq. (3) and integrating the government budget constraint forward from 0 to $T-1$, we get (see Appendix A):

$$b_T = (1-\delta)^T b_0 + \frac{\chi^\kappa \kappa b_0 - \chi^s \bar{s}}{Q_{T-1}} \tag{9}$$

where

$$\chi^\kappa = r^{T-1} + (1-\delta) r^{T-2} + (1-\delta)^2 r^{T-3} + \ldots + (1-\delta)^{T-1}$$
$$\chi^s = 1 + r + r^2 + \ldots + r^{T-1}$$

The numerator $\chi^\kappa \kappa b_0 - \chi^s \bar{s}$ in Eq. (9) corresponds to the accumulated new borrowing between 0 and T. $\chi^\kappa \kappa b_0$ represents the debt service on the initial debt and $\chi^s \bar{s}$ is the time T value of all surpluses (or deficits) between 0 and T. We assume that $\chi^\kappa \kappa b_0 - \chi^s \bar{s}$ is positive, i.e., the primary surplus is insufficient to pay the coupons on the initial debt and the government needs to issue new debt. In this case a lower price of debt makes financing more difficult and may contribute to a debt crisis. This implies that the debt accumulation schedule (9) gives a negative relationship between b_T and Q_{T-1}. When Q_{T-1} is lower, asset prices from 0 to $T-2$ are also lower. This implies a higher yield on newly issued debt, reflecting a lower price for possible default at time T. Lower issue prices lead to a more rapid accumulation of debt and therefore a higher b_T at $T-1$.

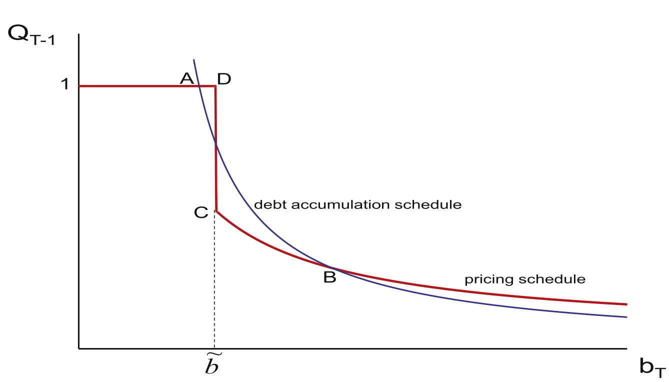

Fig. 1. Multiple equilibria Lorenzoni-Werning model.

Fig. 1 shows these two schedules and illustrates the multiplicity of equilibria. There are two stable equilibria, represented by points A and B. At point A, $Q_{T-1} = 1$. The bond price is then equal to 1 at all times. This is the "good" equilibrium in which there is no default. At point B, $Q_{T-1} < 1$. This is the "bad" equilibrium. Asset prices starting at time 0 are less than 1 in anticipation of possible default at time T. Intuitively, when agents believe that default is likely, they demand lower asset prices, leading to a more rapid accumulation of debt, which in a self-fulfilling way indeed makes default more likely.

In the bad equilibrium there is a slow-moving debt crisis. As can be seen from Eq. (8), using $Q_{T-1} < 1$, the asset price instantaneously drops at time 0 and then continues to drop all the way to $T-1$. Such a slow-moving crisis occurs only for intermediate levels of debt. When b_0 is sufficiently low, the debt accumulation schedule is further to the left, crossing below point C, and only the good equilibrium exists. When b_0 is sufficiently high, the debt accumulation schedule is further to the right, crossing above point D, and only a bad equilibrium exists. In that case default is unavoidable when $\tilde{s} = s_{low}$. There is therefore an intermediate region for b_0 under which there are multiple equilibria, which we refer to as the multiplicity region.

2.2. A monetary model: the impact of monetary policy

In this section we start addressing the central question of this paper, i.e. what the central bank can do to prevent a debt crisis of the type described above. We extend the model to a monetary economy, in which the central bank can set the interest rate and affect the goods price level P_t. For simplicity of exposition, in this section we only consider the case of a cashless economy. In Section 3.5 we will introduce money supply. We will show that the ability of the central bank to collect seigniorage, while being one of the available channels to avoid default, is of limited quantitative importance.

R_t is the gross nominal interest rate and $r_t = R_t P_t / P_{t+1}$ the gross real interest rate in the economy. The coupons on government debt are defined in nominal terms. On the other hand, we assume that primary surpluses before T, and their distribution after T, are known in real terms. The number of bonds outstanding at time $t-1$ is B_t and B_0 is given. We define $b_t = B_t / P_t$. The arbitrage equation with no default remains (Eq. (4)), while the government budget constraint for $t \neq T$ becomes

$$Q_t B_{t+1} = ((1-\delta) Q_t + \kappa) B_t - s_t P_t \tag{10}$$

where s_t is the real primary surplus, $s_t P_t$ the nominal surplus.

At time T the real obligation of the government to bond holders is $[(1-\delta) Q_T + \kappa] b_T$. The no-default condition is $b_T \leq \tilde{b}$, with the latter now defined as

$$\tilde{b} = \frac{s^{pdv}}{(1-\delta) Q_T + \kappa} \tag{11}$$

where

$$s^{pdv} = \left[1 + \frac{1}{r_T} + \frac{1}{r_T r_{T+1}} + \ldots\right] s_{low} \tag{12}$$

and Q_T is equal to the present discounted value of coupons:

$$Q_T = \frac{\kappa}{R_T} + \frac{(1-\delta)\kappa}{R_T R_{T+1}} + \frac{(1-\delta)^2 \kappa}{R_T R_{T+1} R_{T+2}} + \ldots \tag{13}$$

In analogy to the real model, the new pricing schedule becomes

$$Q_{T-1} = \frac{(1-\delta)Q_T + \kappa}{R_{T-1}} \quad \text{if} \quad b_T \le \tilde{b} \tag{14}$$

$$= \psi \frac{\zeta s^{pdv}}{R_{T-1} b_T} + (1-\psi)\frac{(1-\delta)Q_T + \kappa}{R_{T-1}} \quad \text{if} \quad b_T > \tilde{b} \tag{15}$$

The new pricing schedule implies a relationship between Q_{T-1} and b_T that has the same shape as in the real model, but is now impacted by monetary policy through real and nominal interest rates and inflation. In particular, the pricing schedule is affected by inflation and interest rates after T (*ex-post policy*). Inflation after T lowers Q_T. Lower real interest rates after T raise the time-T value of future surpluses, s^{pdv}. Both these effects contribute to raising the default threshold $\tilde{b}$, with the effect of shifting the pricing schedule to the right. Raising s^{pdv} also increases the recovery in case of default, which moves the decreasing branch of the pricing schedule up.

The debt accumulation schedule now becomes (see Appendix A):

$$b_T = (1-\delta)^T \frac{B_0}{P_T} + \frac{P_{T-1}}{P_T} \frac{\chi^\kappa \kappa B_0/P_0 - \chi^s \bar{s}}{Q_{T-1}} \tag{16}$$

where

$$\chi^\kappa = \left[r_{T-2} \ldots r_1 r_0 + (1-\delta) r_{T-2} \ldots r_1 \frac{P_0}{P_1} + (1-\delta)^2 r_{T-2} \ldots r_2 \frac{P_0}{P_2} + \ldots + (1-\delta)^{T-1} \frac{P_0}{P_{T-1}} \right]$$

$$\chi^s = 1 + r_{T-2} + r_{T-2} r_{T-3} + \ldots + r_{T-2} \ldots r_1 r_0$$

As in the real model, the schedule implies a negative relationship between Q_{T-1} and b_T. Monetary policy affects this schedule through its impact on interest rates and inflation before T (*ex-ante policy*). Clearly inflation reduces the real value of the debt outstanding at time 0. This is captured by term $(1-\delta)^T \frac{B_0}{P_T}$ on the RHS of Eq. (16) and by the term proportional to χ^κ. In addition lower real rates can reduce the cost of new borrowing (χ^κ decreases), and decrease the time-T (absolute) value of the surpluses between 0 and T (χ^s decreases), which is beneficial to the government if it is running deficits. All these effects decrease debt accumulation, hence shift the corresponding schedule to the left.

By shifting the two schedules, monetary policy can affect the existence of self-fulfilling debt crises: in terms of Fig. 1, the crisis equilibrium is avoided when the debt accumulation schedule goes through point C or below, so that the two schedules intersect only once, in the good equilibrium. This is the case when

$$\frac{\chi^\kappa \kappa B_0/P_0 - \chi^s \bar{s}}{s^{pdv} - ((1-\delta)Q_T + \kappa)(1-\delta)^T B_0/P_T} r_{T-1} \le 1 - \psi(1-\zeta) \tag{17}$$

Note that point C itself is not on the price schedule as its lower section starts for $b_t > \tilde{b}$. The crisis equilibrium is therefore avoided even if this condition holds as an equality, which corresponds to point C. At point C, $Q_{T-1} < 1$. All prices from 0 to $T-1$ will then be less than one, leading to an accumulation of debt. Inequality (17) gives a condition for what the central bank needs to do to counteract lower prices and avoid default. This condition is key and applies no matter what specific model we assume that relates interest rates, prices and output. We will refer to this as the *default avoidance condition*. Notice that, since the Lagrange multiplier associated with Inequality (17) is positive, the optimal policy that averts default is always such that this condition holds as an equality.

In Section 3.5 we will discuss two more channels, besides inflation and low real rates, through which monetary policy can help. First, through an expansionary monetary policy the central bank can collect extra revenue, seigniorage, that can help avoiding default. The last channel is through output. If we allow the primary surplus to be pro-cyclical, expansionary monetary policy that raises output will raise primary surpluses, which again helps avert default.

3. A new Keynesian model

The default avoidance condition (17) depends on interest rates, prices and output. We now consider a specific New Keynesian model that determines prices and output given interest rates that will be controlled by the central bank. The model is used to examine the policies needed to eliminate the default equilibrium. More precisely, we consider the optimal monetary policy that satisfies both the default avoidance condition and the zero lower bound constraint on nominal interest rates.

3.1. Model description

We consider a standard NK model based on Galí (2008, ch. 3), with three extensions suggested by Woodford (2003): i) habit formation; ii) price indexation; iii) lagged response in price adjustment. These extensions are standard in the monetary DSGE literature and are introduced to generate more realistic responses to monetary shocks. The main effect of these extensions is to generate a delayed impact of a monetary policy shock on output and inflation, leading to the humped-shaped response seen in the data.

3.1.1. Households

With habit formation, households maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t - \eta C_{t-1})^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \right) \tag{18}$$

where total consumption C_t is

$$C_t = \left(\int_0^1 C_t(i)^{1-\frac{1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{19}$$

and N_t is labor. Habit persistence, measured by η, is a common feature in NK models to generate a delayed response of expenditure and output.

The budget constraint is

$$P_t C_t + D_{t+1} + Q_t B_{t+1} = W_t N_t + \Pi_t + R_{t-1} D_t + R^g_{t-1} Q_{t-1} B_t - T_t \tag{20}$$

Here D_{t+1} are holdings of one-period bonds that are in zero net supply. P_t is the standard aggregate price level and W_t is the wage level. Π_t are firms profits distributed to households and T_t are lump-sum taxes. We will abstract from government consumption, so that the primary surplus is $P_t s_t = T_t$.

The first-order conditions with respect to D_{t+1} and B_{t+1} are

$$\tilde{C}_t = \beta E_t R_t \frac{P_t}{P_{t+1}} \tilde{C}_{t+1} \tag{21}$$

$$\tilde{C}_t = \beta E_t R^g_t \frac{P_t}{P_{t+1}} \tilde{C}_{t+1} \tag{22}$$

where

$$\tilde{C}_t \equiv (C_t - \eta C_{t-1})^{-\sigma} - \eta \beta E_t (C_{t+1} - \eta C_t)^{-\sigma}$$

The combination of Eqs. (21) and (22) gives the arbitrage Eqs. (4), (14), and (15). This is because government default, which lowers the return on government bonds, does not affect consumption due to Ricardian equivalence.

3.1.2. Firms

There is a continuum of firms on the interval [0, 1], producing differentiated goods. The production function of firm i is

$$Y_t(i) = AN_t(i)^{1-\alpha} \tag{23}$$

We follow Woodford (2003) by assuming firm-specific labor.

Calvo price setting is assumed, with a fraction $1-\theta$ of firms re-optimizing their price each period. In addition, it is assumed that re-optimization at time t is based on information from date $t-d$. This feature, adopted by Woodford (2003), is in the spirit of the model of information delays of Mankiw and Reis (2002). It has the effect of a delayed impact of a monetary policy shock on inflation, consistent with the data.[12] Analogous to Christiano et al. (2005), Smets and Wouters (2003) and many others, we also adopt an inflation indexation feature in order to generate more persistence of inflation. Firms that do not re-optimize follow the simple indexation rule

$$\ln(P_t(i)) = \ln(P_{t-1}(i)) + \gamma\pi_{t-1} \tag{24}$$

where $\pi_{t-1} = lnP_{t-1} - lnP_{t-2}$ is aggregate inflation one period ago.

3.1.3. Linearized system

Let c_t, y_t and y_t^n denote logs of consumption, output and the natural rate of output. Using $c_t = y_t$, and defining $x_t = y_t - y_t^n$ as the output gap, log-linearization of the Euler equation (Eq. (21)) gives the dynamic IS equation

$$\tilde{x}_t = E_t\tilde{x}_{t+1} - \frac{1-\beta\eta}{\sigma}\left(i_t - E_t\pi_{t+1} - r^n\right) \tag{25}$$

where

$$\tilde{x}_t = x_t - \eta x_{t-1} - \beta\eta E_t\left(x_{t+1} - \eta x_t\right) \tag{26}$$

Here $i_t = ln(R_t)$ will be referred to as the nominal interest rate and $r^n = -\ln(\beta)$ is the natural rate of interest. The latter uses our assumption of constant productivity, which implies a constant natural rate of output.

Leaving the algebra to the online Appendix, we find the following Phillips curve:

$$\pi_t = \gamma\pi_{t-1} + \beta E_{t-d}(\pi_{t+1} - \gamma\pi_t) + E_{t-d}(\omega_1 x_t + \omega_2\tilde{x}_t) \tag{27}$$

where

$$\omega_1 = \frac{1-\theta}{\theta}(1-\theta\beta)\frac{\phi+\alpha}{1-\alpha+(\alpha+\phi)\varepsilon} \tag{28}$$

$$\omega_2 = \frac{1-\theta}{\theta}(1-\theta\beta)\frac{1-\alpha}{1-\alpha+(\alpha+\phi)\varepsilon}\frac{\sigma}{(1-\eta\beta)(1-\eta)} \tag{29}$$

[12] This feature can also be justified in terms of a delay by which newly chosen prices go into effect.

3.1.4. Monetary policy

We follow most of the literature by using a quadratic approximation of utility. Conditional on avoiding the default equilibrium, the central bank then minimizes the following objective function:

$$E_0\sum_{t=0}^{\infty}\beta^t\left\{\mu_x(x_t - \nu x_{t-1})^2 + \mu_\pi(\pi_t - \gamma\pi_{t-1})^2\right\} \tag{30}$$

where ν, μ_x and μ_π a function of model parameters (see Appendix B for the derivation). The central bank chooses the optimal path of nominal interest rates over $H > T$ periods. After that, we assume an interest rate rule as in Clarida et al. (1999):

$$i_t - \bar{i} = \rho(i_{t-1} - \bar{i}) + (1-\rho)\left(\psi_\pi E_t\pi_{t+1} + \psi_y x_t\right) \tag{31}$$

where $\bar{i} = -\ln(\beta)$ is the steady state nominal interest rate. We will choose H to be large. Interest rates between time T and H involve ex-post-policy.[13] Appendix C shows how to solve for the time series of $[\pi_t, x_t, i_t]$ given the policy choice of i_t for $0 \le t < H$ and the adoption of the policy rule (31) for $t \ge H$.

Optimal policy is chosen conditional on two types of constraints. The first is the ZLB constraint that $i_t \ge 0$ for all periods. The second is the default avoidance condition (17) as an equality.[14] Using the NK Phillips curve (Eq. (27)), the dynamic IS equation (Eq. (25)), and the policy rule (Eq. (31)) after time H, we solve for the path of inflation and output gap conditional on the set of H interest rates chosen. We then minimize the welfare cost (30) over the H interest rates subject to $i_t \ge 0$ and the default avoidance condition.

3.2. Calibration

We consider one period to be a quarter and normalize the constant productivity A such that the natural rate of output is equal to 1 annually (0.25 per quarter). The other parameters are listed in Table 1. The left panel shows the parameters from the LW model, while the right panel lists the parameters that pertain to the New Keynesian part of the model.

A key parameter is δ. In the benchmark parameterization we set it equal to 0.05, which implies a government debt duration of 4.2 years. This is typical in the data. For example, OECD estimates of the Macauley duration in 2010 are 4.0 in the US and 4.4 for the average of the five European countries that experienced a sovereign debt crisis (Greece, Italy, Spain, Portugal and Ireland). The coupon is determined such that $\kappa = 1/\beta - 1 + \delta$.

The other LW parameters, β, T and the fiscal surplus parameters, have an impact on the "multiplicity range". The range of B_0 for which there are multiple equilibria under passive monetary policy ($i_t = \bar{i}$) is $[B_{low}, B_{high}]$, where[15]

$$B_{low} = \frac{\beta}{1-\beta}\frac{(\psi\zeta + 1 - \psi)\beta^T s_{low} + (1-\beta^T)\bar{s}}{1-(1-\zeta)(1-\delta)^T\beta^T\psi} \tag{32}$$

$$B_{high} = \frac{\beta}{1-\beta}\left(\beta^T s_{low} + \left(1-\beta^T\right)\bar{s}\right) \tag{33}$$

This range may be wide or narrow, dependent on the chosen parameters. For example, when $\zeta \to 1$, the range narrows to zero.

[13] Since H will be large, the precise policy rule after H does not have much effect on the results.

[14] In the good equilibrium $i_t \ge 0$ is the only constraint and the optimal policy implies $i_t = \bar{i}$ each period, delivering zero inflation and a zero output gap. However, conditional on a sunspot that could trigger a default equilibrium condition (17) becomes an additional constraint.

[15] These values lead to equilibria at points C and D in Fig. 1.

Table 1
Calibration.

Lorenzoni-Werning parameters		New Keynesian parameters	
Parameter	Description	Parameter	Description
$\beta = 0.99$	Discount rate	$\sigma = 1$	Elasticity of intertemporal substitution
$\delta = 0.05$	Coupon depreciation rate	$\phi = 1$	Frisch elasticity
$\kappa = 0.06$	Coupon	$\varepsilon = 6$	Demand elasticity
$T = 20$	Quarters before default decision	$\alpha = 0.33$	Capital share
$\zeta = 0.5$	Recovery rate	$\theta = 0.66$	Calvo pricing parameter
$\psi = 0.95$	Probability low surplus state	$\eta = 0.65$	Habit parameter
$s_{low} = 0.02$	Low state surplus	$\gamma = 1$	Indexation parameter
$\bar{s} = -0.01$	Surplus before T	$d = 2$	Lag in price adjustment
		$\rho = 0.8$	Persistence in interest rate rule
		$\psi_\pi = 1.5$	Inflation parameter in interest rule
		$\psi_y = 0.1$	Output parameter in interest rule

We set $\beta = 0.99$ (4% annual natural real rate of interest), $T = 20$ (uncertainty resolved in 5 years), $\bar{s} = -0.01$ (4% annual primary deficit), $s_{low} = 0.02$, $\zeta = 0.5$ and $\psi = 0.95$. This gives a multiplicity range of $[0.79, 1.46]$, so that a country is subject to multiple equilibria when debt is in the range of 79 to 146% of GDP. Although we are by no means aiming to calibrate to a particular historical episode, we note that during the Eurozone crisis the range of debt of periphery countries varied from 62% in Spain to 148% in Greece.

The New Keynesian parameters are standard in the literature. The first 5 parameters correspond exactly to those in Gali (2008). The habit formation parameter, the indexation parameter and the parameters in the interest rate rule are all the same as in Christiano et al. (2005).[16] We take $d = 2$ from Woodford (2003, pp. 218–219), which also corresponds closely to Rotemberg and Woodford (1997). This set of parameters implies a response to a small monetary policy shock under the Taylor rule that is similar to the empirical VAR results reported by Christiano et al. (2005). The response of output, inflation and interest rates to such a shock are shown in the online Appendix. The levels of output and inflation at their peak are similar to what is found in the data. Both the output and inflation response are humped shaped, although the peak response occurs a bit earlier than in the data.

We also show in some detail what the optimal policy would be if there was no inflation inertia, i.e. $d = 0$. The situation we are considering, with the central bank embarking on a policy of massive inflation to avoid a debt crisis, represents an exceptional occurrence, in which it is not implausible that inflation would react with less inertia than in normal times. Even if this is not the case, showing the comparison between $d = 2$ and $d = 0$ is still useful as it allows us to disentangle the impact of inflation inertia on the optimal policy.

3.3. Results under benchmark parameterization

Fig. 2 shows the dynamics of inflation under optimal policy under the benchmark parameterization for $H = 40$ (and $d = 2$), and compares to the case with $d = 0$. The results are shown for various levels of B_0. The optimal path for inflation is hump shaped in both cases $d = 2$ and $d = 0$. Optimal inflation gradually rises because the welfare cost (30) depends on the change in inflation, and in the case $d = 2$ also due to inflation rigidity. Eventually optimal inflation decreases as it becomes less effective over time when the original debt depreciates and is replaced by new debt that incorporates inflation expectations. For $d = 2$, when $B_0 = B_{middle} = 1.12$, which is in the middle of the range of debt levels giving rise to multiple equilibria, the maximum inflation rate reaches 23.8%. Inflation is over 20% for 4 years, over 10% for 8 years and the price level ultimately increases by a factor 5.3. Inflation needed to avoid default gets much higher for higher debt levels. When B_0 reaches the upper bound B_{high} for multiple equilibria, the maximum inflation rate is close to 47% and ultimately the price level increases by a factor 25! Only when B_0 is very close to the lower bound for multiplicity, as illustrated for $B_0 = 0.8$, is little inflation needed.

By comparison, for $d = 0$, at $B_0 = 1.12$ the maximum inflation rate is 19%, and inflation is over 15% for almost 4 years and above 10% for 7 years. The price level ultimately increases by a factor 4.2. At $B_0 = B_{high}$, inflation peaks at 36% and the final price level increases by a factor 14.

In order to understand why so much inflation is needed, it is useful to first consider a rather extreme experiment where all of the increase in prices happens right away in the first quarter. This cannot happen in the NK model, so assume that prices are perfectly flexible, the real interest rate is constant at $1/\beta$ and the output gap remains zero. When $B_0 = B_{middle} = 1.12$, the price level would need to rise by 42%. This is needed to lower debt so that we are no longer in the multiplicity range.

In reality inflation will be spread out over a period of time, both because sticky prices imply a gradual change in prices and because it is optimal from a welfare perspective not to have the increase in the price level happen all at once. However, such a delay increases the ultimate increase in the price level that is needed. As the time zero debt depreciates, inflation quickly becomes less effective as it only helps to reduce the real value of coupons on the original time zero debt. The interest rate on new debt incorporates the higher inflation expectations. More inflation is therefore needed to avoid the default equilibrium.

3.4. Sensitivity analysis

We now consider changes to both the LW and NK parameters. Changes in the LW parameters change the multiplicity region $[B_{low}, B_{high}]$. As discussed in Section 3.3, the relative distance between the initial debt level B_0 and the lower bound of the multiplicity range B_{low} dramatically affects the inflation path. The objective of this section is instead to examine how the required inflation depends on model parameters, keeping B_0/B_{low} fixed.

Table 2 shows the maximum inflation level and the final price level when changing one parameter at a time, while keeping $B_0/B_{low} = 1.42$ (the same as for the middle of the multiplicity range in the benchmark parameterization). Table 2 also shows how a variation in each LW parameter affects the multiplicity region. Fig. 3 shows how each of the LW affects the inflation path. We see that the only LW parameter that substantially affects the results is δ. A lower debt depreciation δ, which implies a longer maturity of debt, implies lower inflation. The reason is that inflation is effective for a longer period of time as the time 0 debt depreciates more slowly.

[16] Notice that an indexation parameter $\gamma = 1$ implies that it is only the change in inflation that matters in the objective function (30). Full indexation means that large changes in inflation have a strong impact on price dispersion (see Eq. (60) in the Appendix).

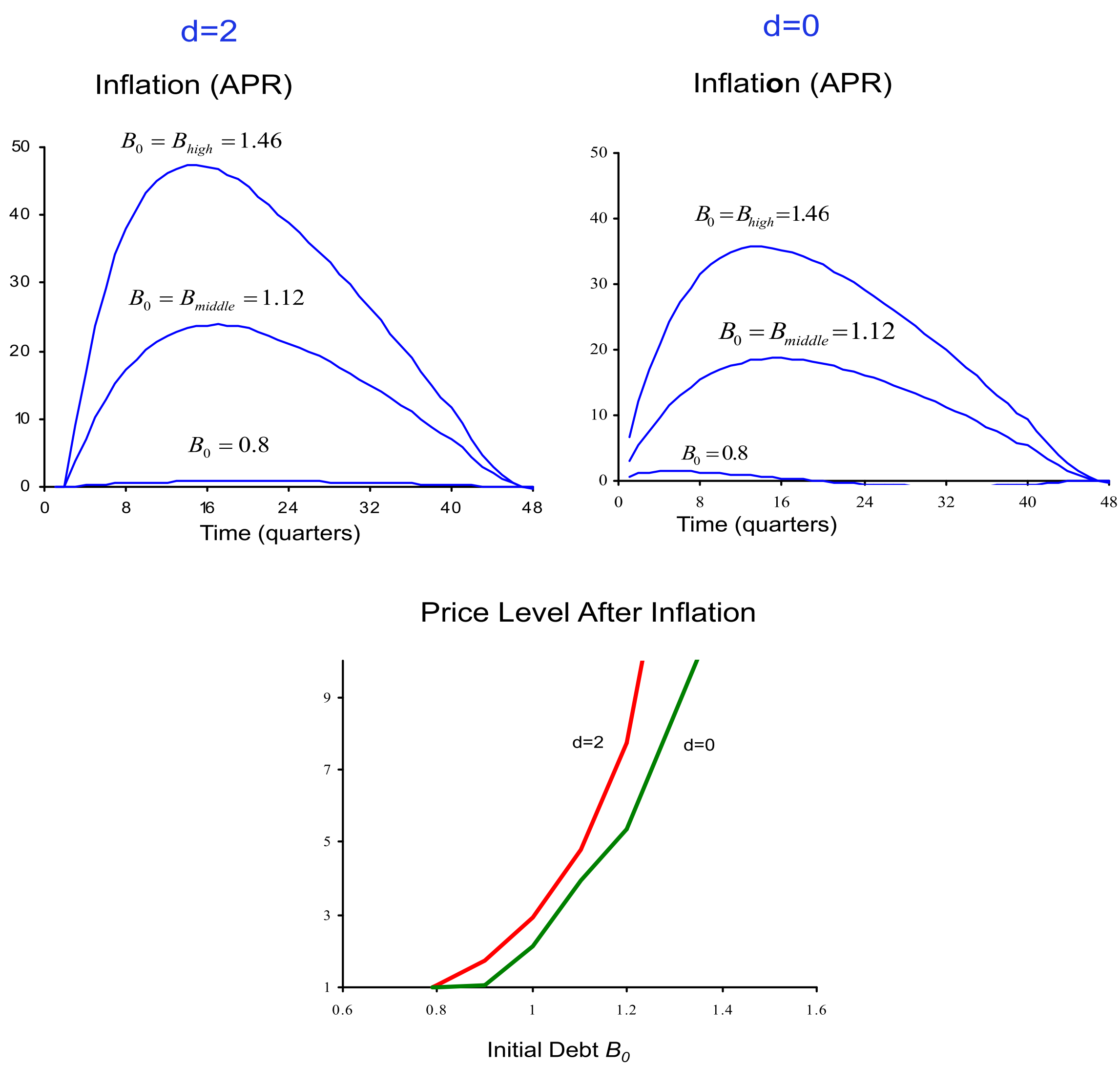

Fig. 2. Benchmark NK model: inflation needed to avoid default.

But even when $\delta = 0.025$, so that the duration is 7.2 years, optimal inflation is still above 10% for 6.5 years and the price level ultimately triples.

Fig. 4 shows analogous results for the NK parameters. Most parameters again have remarkably little effect. There are two cases where parameter values do matter. One is the lag in price adjustment d, which has already been discussed in Fig. 2. The other parameter that significantly affects results is γ. When $\gamma = 0$ (no price indexation), it is optimal to concentrate inflation as early as possible since only the inflation level, and not inflation changes, matters for the central bank objective function (30). In this case the price level ultimately rises by a relatively more modest factor 1.7.

In the last chart we show $\gamma = d = \eta = 0$ as in the Gali (2008) textbook model and compare the results to the benchmark parameterization. This case combines the features of the $d = 0$ and $\gamma = 0$ cases discussed above. All the inflation comes upfront, both because there is no delay in price adjustment and because, with no price indexation, there is no rationale for inflation smoothing. Inflation starts at 23% (APR) in the first quarter, and the ultimate increase in the price level is very similar to the $\gamma = 0$ case seen above, only 66%. However, this case appears to be of little practical relevance. The default avoidance condition is met to a large extent by an unrealistic steep drop in the real interest rate, from 4% (APR) to −19% in the first quarter, while output growth is 25% (APR) in the first quarter.[17] Even for small monetary shocks it is well known that these parameters lead to an unrealistic dynamic response of output and inflation to monetary shocks.

3.5. Extensions

We consider two extensions: seigniorage and pro-cyclical primary surplus.

3.5.1. Seigniorage

We consider a consolidated government whose budget constraint is

$$Q_t B^p_{t+1} = ((1-\delta)Q_t + \kappa)B^p_t - [M_t - M_{t-1}] - s_t P_t \tag{34}$$

where B^p_t is government debt held by the general public. It is $B^p_t = B_t - B^c_t$, where B^c_t is the debt held by the central bank. The consolidated government can reduce the debt of the private sector by issuing

[17] In the case $\gamma = 0$, where all the other parameters are at the benchmark level, including $d = 2$, the output increase in the first quarter is 20%.

Table 2
Sensitivity analysis.

Parameters	B_{low}	B_{high}	Maximum inflation	Price level after inflation
Benchmark	0.79	1.46	23.8	5.3
Lorenzoni-Werning parameters				
$T = 10$	1.15	1.71	25.8	6.3
$T = 30$	0.56	1.22	21.4	4.5
$\delta = 1/40$	0.89	1.46	15.1	3.0
$\delta = 1/10$	0.71	1.46	37.5	12.0
$\bar{s} = -0.02$	0.58	1.28	23.2	5.1
$\bar{s} = 0$	1.00	1.64	23.9	5.3
$s_{low} = 0.01$	0.28	0.63	23.2	5.1
$s_{low} = 0.03$	1.27	2.25	23.7	5.3
$\zeta = 0.3$	0.46	1.46	26.4	6.1
$\zeta = 0.7$	1.08	1.46	21.3	4.5
$\psi = 0.7$	0.98	1.46	22.0	4.7
$\psi = 1$	0.75	1.46	24.0	5.4
$\beta = 0.98$	0.61	1.17	26.7	6.7
$\beta = 0.995$	0.90	1.62	21.8	4.6
New Keynesian parameters				
$\theta = 0.5$	0.79	1.46	23.7	5.0
$\theta = 0.8$	0.79	1.46	22.9	5.5
$\eta = 0$	0.79	1.46	22.3	4.9
$\eta = 0.8$	0.79	1.46	23.8	5.3
$\epsilon = 4$	0.79	1.46	24.1	5.3
$\epsilon = 8$	0.79	1.46	23.2	5.2
$d = 0$	0.79	1.46	19.6	4.2
$d = 4$	0.79	1.46	29.6	7.2
$\sigma = 2$	0.79	1.46	22.7	5.0
$\sigma = 0.5$	0.79	1.46	24.4	5.4
$\phi = 2$	0.79	1.46	23.6	5.3
$\phi = 0.5$	0.79	1.46	23.7	5.2
$\gamma = 0.5$	0.79	1.46	19.5	2.0
$\gamma = 0$	0.79	1.46	26.4	1.7
$\gamma = \eta = d = 0$	0.79	1.46	23.0	1.7

monetary liabilities $M_t - M_{t-1}$. Money demand is generated by a transaction cost $f(M_t, Y_t^n)$, where $Y_t^n = P_t Y_t$ is nominal GDP and $\partial f / \partial M \leq 0$. This transaction cost should be subtracted from the RHS of Eq. (20).

Let $\tilde{m}$ represent accumulated seigniorage between 0 and $T-1$:

$$\tilde{m} = \frac{M_{T-1} - M_{T-2}}{P_{T-1}} + r_{T-2} \frac{M_{T-2} - M_{T-3}}{P_{T-2}} + \ldots + r_0 r_1 \ldots r_{T-2} \frac{M_0 - M_{-1}}{P_0} \tag{35}$$

Similarly, let m^{pdv} denote the present discounted value of seigniorage revenues starting at date T:

$$m^{pdv} = \frac{M_T - M_{T-1}}{P_T} + \frac{1}{r_T} \frac{M_{T+1} - M_T}{P_{T+1}} + \frac{1}{r_T r_{T+1}} \frac{M_{T+2} - M_{T+1}}{P_{T+2}} + \ldots \tag{36}$$

As derived in the online Appendix, if we take seigniorage into account the default avoidance condition becomes

$$\frac{\chi^\kappa \kappa B_0^p / P_0 - \chi^s \bar{s} - \tilde{m}}{s^{pdv} + m^{pdv} - ((1-\delta) Q_T + \kappa)(1-\delta)^T B_0^p / P_T} r_{T-1} \leq \psi \frac{\min\left\{0, \zeta s^{pdv} + m^{pdv}\right\}}{s^{pdv} + m^{pdv}} + 1 - \psi \tag{37}$$

In order to derive a relationship between money supply and interest rates, we use a convenient form of the transaction cost $f(M_t, Y_t^n)$ that gives rise to a standard specification for money demand when $i_t > 0$ $(m_t = ln(M_t))$[18]:

$$m_t = \alpha_m + p_t + y_t - \alpha_i i_t \tag{38}$$

When i_t is close to zero, money demand reaches the satiation level $\alpha_m + p_t + y_t$. We limit ourselves to conventional monetary policy, where the money supply does not go beyond the satiation level. Section 5 makes some comments on unconventional monetary policy.

Seigniorage revenue depends on the semi-elasticity α_i of money demand. Seigniorage is larger for lower values of α_i since that leads to a smaller drop in real money demand when inflation rises. We set $\alpha_i = 40$.[19] The left chart of Fig. 5 compares the optimal inflation path with seigniorage to that in the benchmark without seigniorage, assuming $B_0 = B_{middle}$.[20] The effect of seigniorage is clearly negligible. This result is consistent with Reis (2013) and Hilscher et al. (2014). As Reis (2013) puts it, "In spite of the mystique behind the central bank's balance sheet, its resource constraint bounds the dividends it can distribute by the present value of seigniorage, which is a modest share of GDP".

3.5.2. *Pro-cyclical surplus*

In this case, nominal rigidities give the central bank control over the accumulation of debt through the level of output that affects the primary surplus. From 0 through $T-1$ assume that we have

$$s_t = \bar{s} + \lambda(y_t - \bar{y}) \tag{39}$$

where $\bar{y}$ is steady-state output. We similarly assume that s_{low} is procyclical: $s_{low} = \bar{s}_{low} + \lambda(y_t - \bar{y})$. We set the value of the cyclical parameter of the fiscal surplus to $\lambda = 0.1$, in line with empirical estimates.[21]

With this additional effect from an output increase, the right chart of Fig. 5 shows that the optimal inflation decreases slightly, assuming again $B_0 = B_{middle}$. But the effect is again limited. The maximum inflation rate is reduced from 23.8% in the benchmark to 19.9%. The increase in the price level after inflation is reduced from 5.3 under the benchmark to 4.0, which remains large. Optimal policy now gives more emphasis to raising output, leading to a first quarter output growth rate that is 10% (APR), pushing the boundary of what is plausible.[22]

[18] The transaction cost $f(M_t, Y_t^n) = \alpha_0 + M_t\left(\ln\left(\frac{M_t}{P_t Y_t}\right) - 1 - \alpha_m\right)/\alpha_i$ gives rise to money demand (38). This function applies for values of M_t where $\partial f / \partial M \leq 0$. Once the derivative becomes zero, we reach a satiation level and we assume that the transaction cost remains constant for larger M_t.

[19] Estimates of α_i vary a lot, from as low as 6 in Ireland (2009) to as high as 60 in Bilson (1978). Lucas (2000) finds a value of 28 when translated to a quarterly frequency. Engel and West (2005) review many estimates that also fall in this range.

[20] We calibrate α_m to the U.S., such that the satiation level of money corresponds to the monetary base just prior to its sharp rise in the Fall of 2008 when interest rates approached the ZLB. At that time the velocity of the monetary base was 17. This gives $\alpha_m = -1.45$. The velocity is $4P_t Y_t / M_t$ as output needs to be annualized, which is equal to $4e^{-\alpha_m}$ at the satiation level.

[21] Note that since $\bar{Y} = 0.25$ for quarterly GDP, the specification implies that $\Delta s = 0.4 \Delta Y$. This is consistent for example with estimates by Girouard and André (2005) for the OECD.

[22] It may appear surprising that output increases more than in the benchmark, while inflation increases less. The output increase relative to the benchmark is temporary (5 quarters) and is made possible by an initially more aggressive monetary policy. This increases the surplus and implies that less inflation is needed as debt accumulates more slowly.

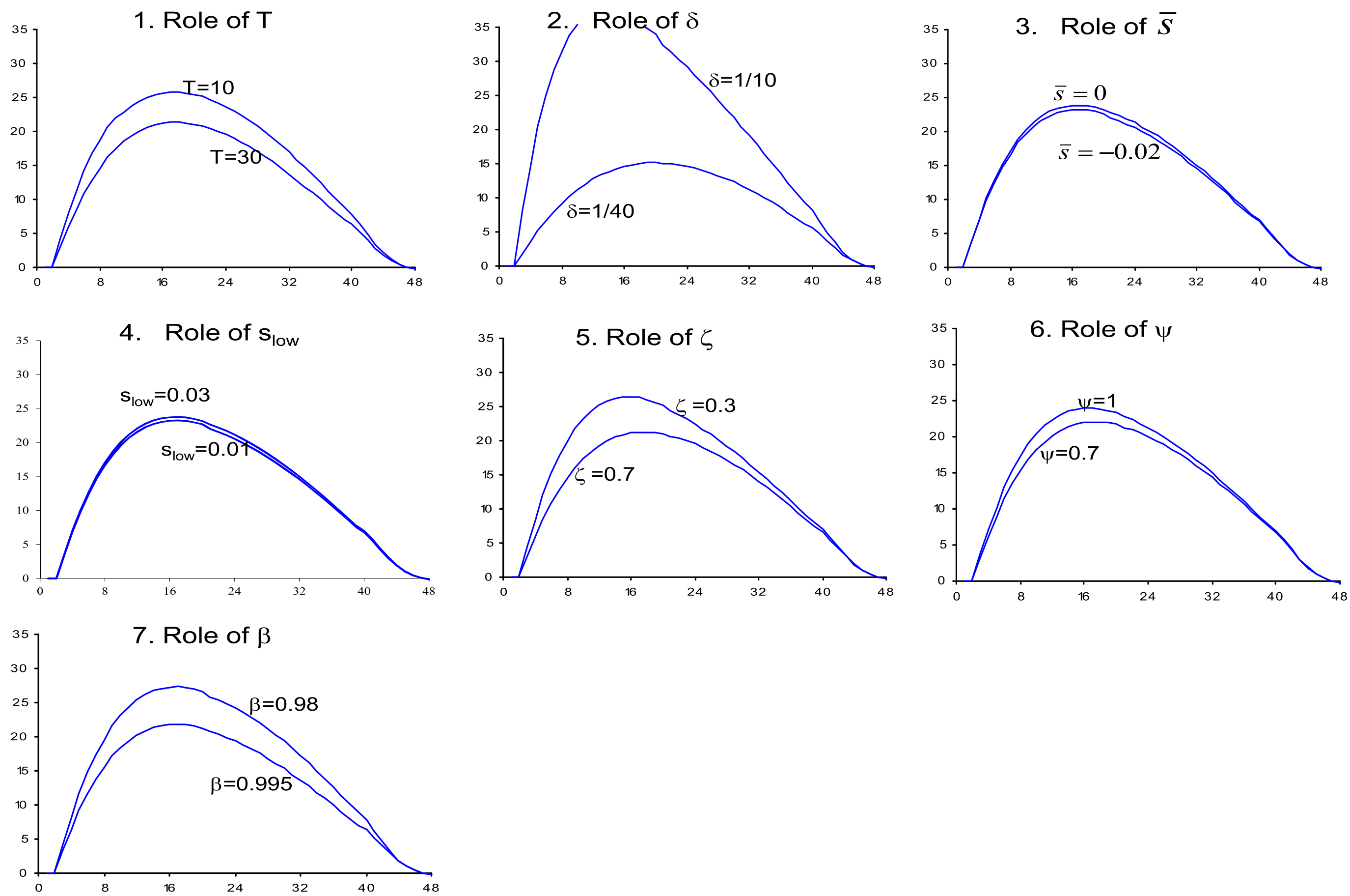

Fig. 3. Sensitivity analysis LW parameters ($B_0/B_{low} = 1.42$).

4. Beyond the NK model

So far we have cast our analysis in the context of a specific NK model, combined with the LW model. While we have done sensitivity analysis with respect to the various NK parameters, we have not considered alternative versions of the NK block of the model. It is not hard to criticize the specific model we have chosen. We have assumed a particular form of price stickiness (Calvo pricing). One could consider alternatives, such as Taylor price setting or menu costs. We have also abstracted from many features that would complicate the structure, but perhaps make it more realistic, such as investment and wage rigidities. Finally, the dynamic IS equation relies on an intertemporal consumption Euler equation that has recently been criticized in the context of the debate about forward guidance.[23]

In this section we therefore take an alternative approach by considering what paths of real interest rates and inflation are consistent with the default avoidance condition. The advantage of this approach is that we do not need to take any stand on the underlying model that maps monetary policy (interest rate decisions) into inflation and real interest rates. Before presenting the results, we first discuss the constraint imposed by the standard consumption Euler equation in the NK model and how it may have affected the results. In particular, we argue that it limits the extent to which the central bank is capable of lowering real interest rates in a sustained way. After that we consider specific paths of real interest rates and inflation that satisfy the default avoidance condition. This relies only on the monetary version of the LW block of the model.

[23] E.g. see Del Negro et al. (2015) or McKay et al. (2015).

4.1. Euler equation

The needed inflation may be smaller when lower real interest rates, by lowering the costs of borrowing, help to avoid the default equilibrium. But the consumption Euler equation may constrain the ability of the central bank to reduce real interest rates in a sustained way. In order to see why, abstract from habit formation for a moment ($\eta = 0$). The dynamic IS equation, which comes from the intertemporal consumption Euler, can then be solved as

$$x_0 = -\frac{1}{\sigma}\sum_{t=0}^{\infty} E_0(r_t - r^n) \tag{40}$$

This precludes a large and sustained drop in the real interest rate as it would imply an enormous and unrealistic immediate change in output at time zero, especially with $\sigma = 1$ as often assumed.

The same point applies when we introduce habit formation, in which case Eq. (40) becomes (see online Appendix)

$$x_0 = -\frac{1}{\sigma}\sum_{t=0}^{\infty} E_0\left(1-(\beta\eta)^{t+1}\right)(r_t - r^n) \tag{41}$$

Removing the expectation operator and the r^n for convenience, for the benchmark parameterization ($\sigma = 1, \eta = 0.65$), we have

$$\begin{aligned} x_0 \;=\; & -0.36r_0 - 0.58r_1 - 0.73r_2 - 0.83r_3 - 0.89r_4 - 0.93r_5 - 0.95r_6 \\ & -0.97r_7 - 0.98r_8 - \ldots \end{aligned} \tag{42}$$

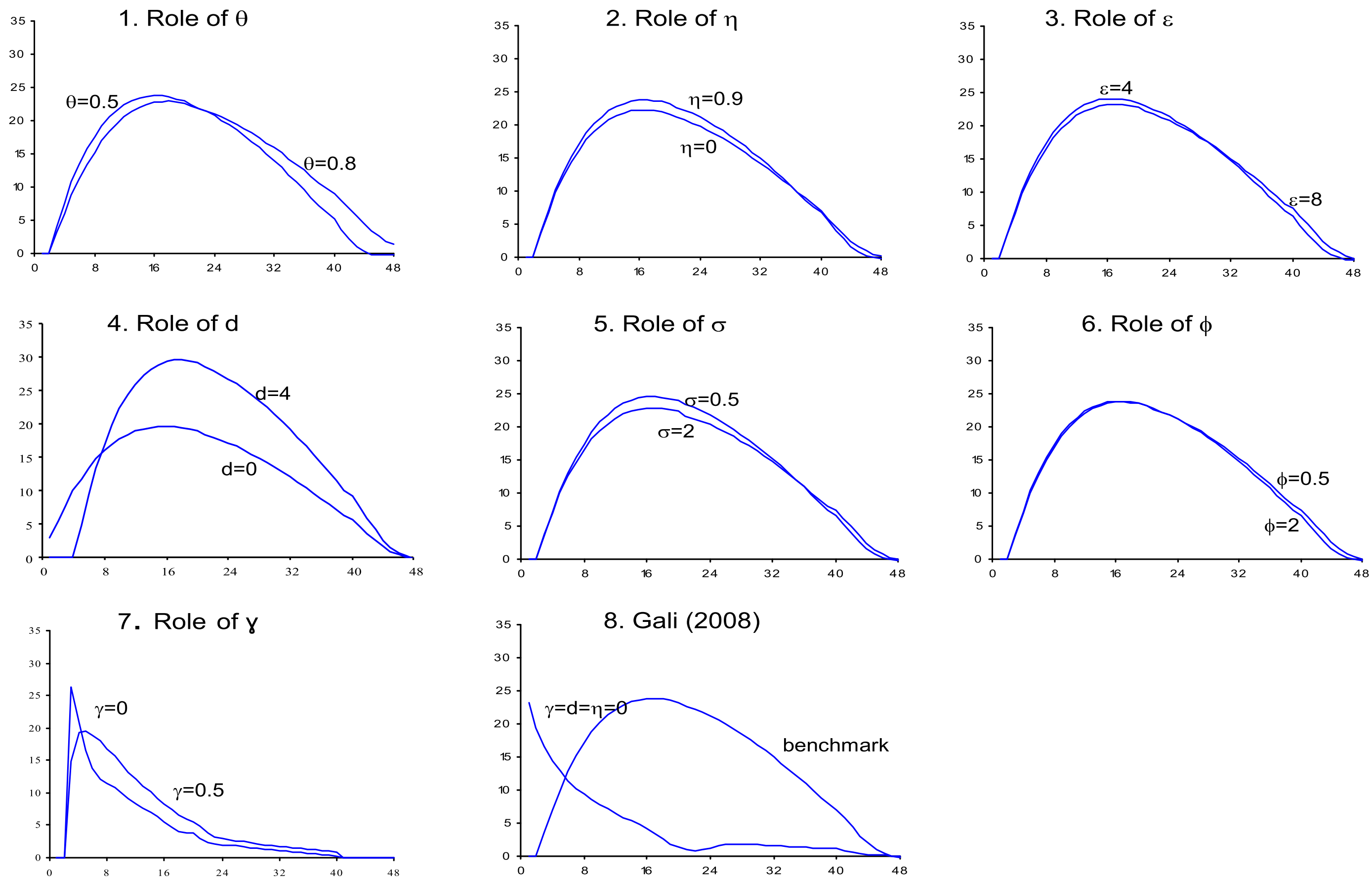

Fig. 4. Sensitivity analysis NK parameters ($B_0/B_{low} = 1.42$).

Subsequent coefficients are very close to −1. For the path of real interest rates under optimal policy this implies $x_0 = 0.0157$. This translates into an immediate output growth of 6.3% on an annualized basis, which is already pushing the boundaries of what is plausible. The real interest rate quickly returns to steady state after dropping from 4% (APR) to 0% for the first two quarters under the benchmark parameterization. This limits the ability of the central bank to satisfy the default avoidance condition.

Related to this, NK models have been found to deliver unrealistically large effects of output and consumption to changes in future interest rates, which Del Negro et al. (2015) have dubbed the forward guidance puzzle. McKay et al. (2015) also argue that it is not realistic that consumption today responds equally to an announced interest rate cut in the far future as to an announced interest rate cut today. Indeed, Eq. (40) shows that real interest rate changes at any future date have the same effect on current output (and consumption) as a current real interest rate change. With habit formation future interest rate changes have an even larger effect than current changes. In order to rectify that problem, models have been proposed leading to a reduced effect of real interest rates on consumption and output today when the expected changes occur further into the future. McKay et al. (2015) do so in the context of a model with idiosyncratic risk and borrowing constraints. Del Negro et al. (2015) do so by introducing finite lives through a positive probability of death. In the context of our model, such alternatives allow for a larger drop in future real interest rates without generating unrealistic

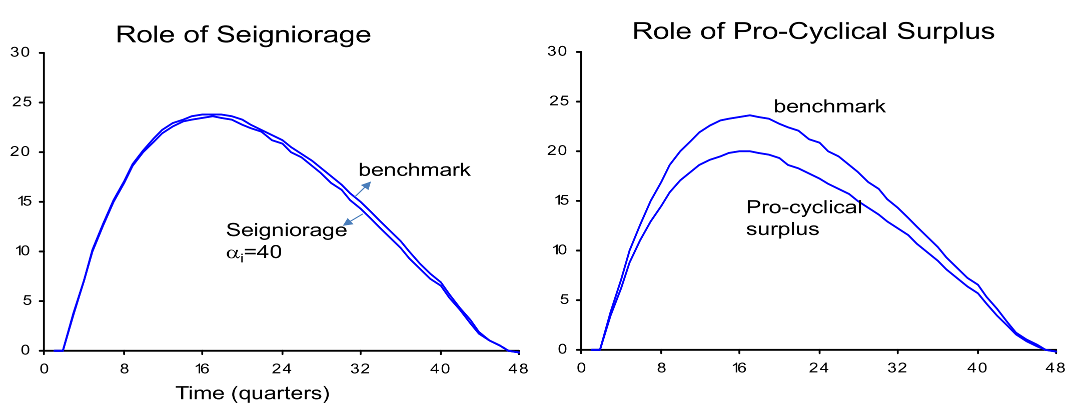

Fig. 5. Role of seigniorage and pro-cyclical surplus*.
**The charts show the inflation rate over time under optimal monetary policy. The left chart compares the benchmark parameterization (cashless economy) to the extension with seigniorage in which α_i=40. The right chart compares the benchmark parameterization to the case where the primary surplus is pro-cyclical with λ=0.1.*

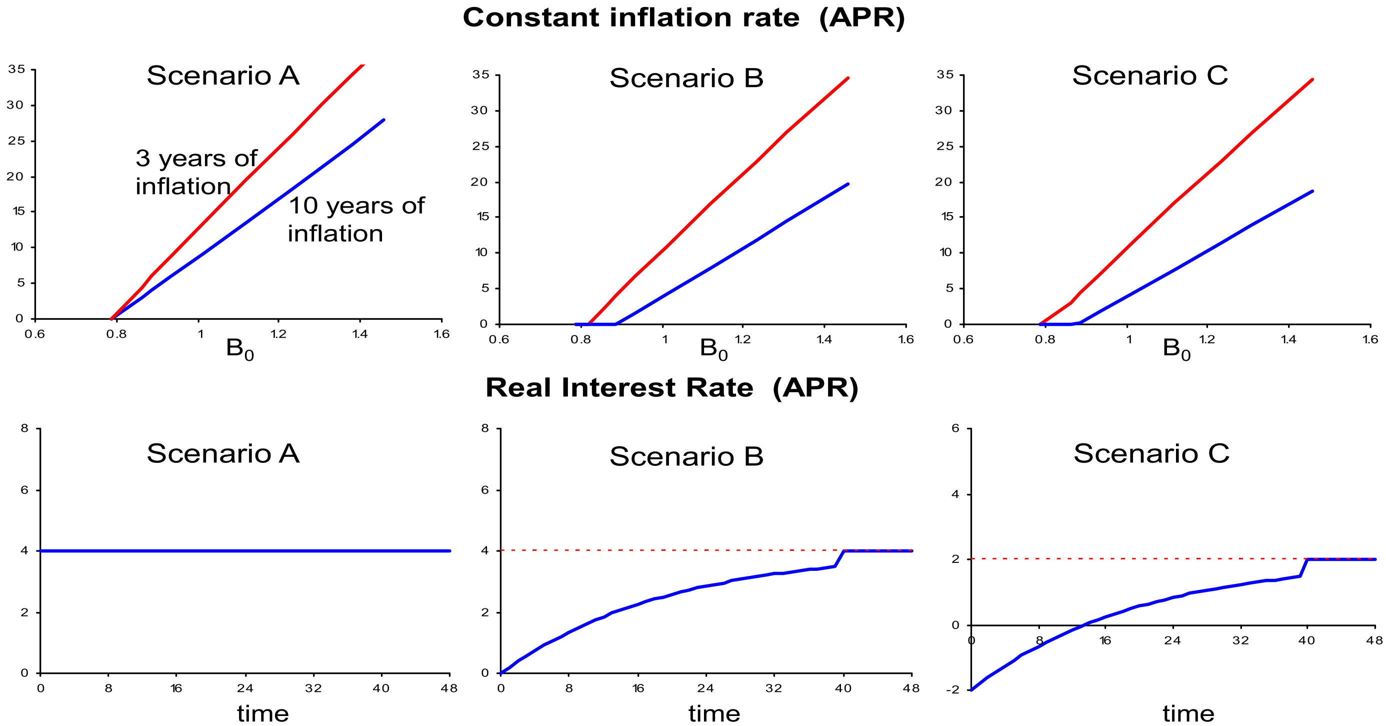

Fig. 6. Constant inflation needed to avoid default*.
Three scenarios for the real interest rate path are shown in the bottom three charts. The corresponding top three charts show the constant inflation that is needed to avoid default. Both a constant inflation rate for 3 years (12 quarters) and a constant inflation rate of 10 years (40 quarters) are reported as a function of the initial debt at time 0.

implications for current output and consumption. We provide an illustration below.

4.2. Some results

Fig. 6 considers three scenarios for the real interest rate path, shown in the bottom charts. The top charts show the corresponding inflation needed to satisfy the default avoidance condition. The results are shown as a function of initial debt. We consider both a constant inflation rate over 10 years and a constant inflation rate over 3 years. The latter is represented by the higher line as a higher inflation rate is needed when inflation is limited to three years. The motivation behind this case is that we have seen that inflation is most effective when it occurs soon after the time zero shock, before most of the original debt has to be rolled over and replaced with new debt that incorporates the higher inflation expectations.

The first scenario for real interest rates (scenario A) assumes that the real interest rate simply remains constant and equal to the natural rate. In this case all the burden is on inflation to reach the default avoidance condition. In scenario B the annualized real interest rate immediately drops in period 0 from 4% (the natural rate) to 0. After that we assume that the gap between real interest rate and the natural rate is multiplied by 0.95 each quarter and we close the remaining gap entirely after 40 quarters. As can be seen, this delivers a very large and persistent drop in the real interest rate. Scenario C is similar to scenario B, but the level of the real interest rate is always 2% (APR) lower than in scenario B. So we start from a 2% real interest rate and it drops right away to −2% and then very gradually, over a period of 10 years, returns to 2%. For this case we set the natural rate to 2% by assuming $\beta = 0.995$, while at the same time we change $\bar{s}$ and s_{low} to keep the multiplicity region unchanged ($\bar{s}$ is lowered from −0.01 to −0.010787 and s_{low} is lowered from 0.02 to 0.00922).

The sharp and persistent drop in real interest rates in scenarios B and C is very large in historical context. Consider for example the 1970s, a decade of significant monetary expansion in many countries leading to steep inflation rates. Italy experienced inflation rates between 10 and 20% for most of the decade. This was the result of extensive money financing of fiscal deficits. The real interest rate in Italy dropped, but no more than in our scenarios B and C.[24]

The drop in real interest rates in scenarios B and C in Fig. 6 would lead to an implausible rise in output by more than 30% within a year under the consumption Euler equation of the model. However, output would respond less under alternative modeling approaches mentioned above where future real interest rate changes have a smaller effect on output today. To illustrate this, we have implemented the dynamic IS equation from Castelnuovo and Nisticò (2010) who, like Del Negro et al. (2015), assume a positive probability of death. They also allow for habit formation as in our model. Their estimated parameters imply

$$x_0 = -0.14 \sum_{t=0}^{\infty} E_0 \Theta^t \left(r_t - r^n\right) \tag{43}$$

with $\Theta = 0.8$. The weight on the real interest rate 2 years from now is then only a fraction 0.16 of the weight on the current real interest rate. Even with the large drop in real interest rates in scenarios B and C, output then rises only a modest 2.4% (APR) during the first quarter.

However, Fig. 6 shows that still large inflation is needed, even with the large sustained drop in real interest rates. Scenarios B and C deliver very similar results for inflation. When inflation is spread

[24] When subtracting expected inflation from nominal interest rates, using either survey data or econometrics, the real interest rate dropped from slightly above zero to about −4% after the first oil price shock, then returned to zero after a couple of years before dropping somewhat again after the second oil price shock (e.g. see Atkinson and Chouraqui, 1985). Moreover, many other factors, specifically the oil price shocks, obviously played a role as well, leading to a decline in real interest rates also in countries with a more modest monetary policy.

over 10 years, annual inflation needs to be anywhere from 0 to 20%, dependent on where we are in the multiplicity region. Correspondingly, the price level needs to increase by a factor between 1 and 6.8. This is a bit less than without the drop in real interest rates, but unless we are near the lower range of debt in the multiplicity region it remains the case that very large and sustained inflation is needed to avoid default.

The lowest ultimate increase in the price level is achieved when the real interest rate drops as in scenarios B and C, while at the same time inflation is limited to the first 3 years, when it is most effective. Annual inflation then varies between 0 to 34% per year. Correspondingly, the price level rises by a factor between 1 and 2.7. Even in this case, unless debt is near the lower end of the multiplicity range, the inflation cost of such policies is generally very substantial.

Of course these results are not entirely model free as we still rely on the LW part of the model. But the LW model matters mainly in generating a certain multiplicity range and in the duration of the government debt. For a realistic assumption about duration and a broad range of multiplicity for debt it is generally difficult to avoid the default equilibrium through monetary policy other than by generating very steep inflation. It is hard to see how this result would change by changing aspects of the LW model as the intuition behind our findings (Section 3.3) does not depend on details of the LW framework.

5. Discussion of alternative policies

In the NK model, we have examined the role of optimal interest rate policies. But other policies are often mentioned in the context of sovereign debt crises. In this section, we examine three of these policies: i) an interest rate ceiling; ii) quantitative easing at the ZLB; iii) sterilized purchase of debt.

5.1. Interest rate ceiling

Some, including Calvo (1988), have argued that the bad equilibrium may be avoided if the government commits to an interest rate ceiling. LW counter that if the government refuses to pay more than a certain (real) interest rate, and the market is unwilling to lend at that interest rate, the government would be forced to significantly cut spending or raise taxes without delay. They consider this not to be credible. LW argue that in reality the government will have to do another auction at a price that the market is willing to pay (the bad equilibrium). While they consider a real model without a central bank, alternatively one could imagine the central bank committing to buy government debt at a low (default free) real interest rate. But as we have already seen, this will make little difference. Seigniorage revenue is very small in reality. Ultimately the private sector will need to absorb new debt unless the central bank is able to slow down debt accumulation through the inflationary policies that we have already considered.

5.2. Quantitative easing

When considering the role of seigniorage in Section 3.5.1, we assumed that monetary expansions do not go beyond the satiation level. But one can consider much larger monetary expansions that go well beyond the satiation level, where we reach the ZLB. We examine such policies in an earlier draft of this paper, Bacchetta et al. (2015). Such a large monetary expansion can for example result from the central bank buying back a lot of government debt or providing liquidity support to the government that obviates the need for new government borrowing. In both cases government debt to the private sector is reduced and replaced by monetary liabilities. However, the large monetary expansion will be unwound when the economy exits the ZLB. Therefore, the present discounted value of seigniorage does not change and large asset purchases have no impact.[25]

5.3. Sterilized debt purchases

The central bank could potentially sterilize the purchase of government debt by the sale of other securities. There are two ways in which one can imagine the sterilized purchase of government debt by the central bank, which does not increase monetary liabilities and is therefore not inflationary. One case is where the central bank has assets other than government debt on its balance sheet, such as for example gold or foreign exchange reserves. The central bank can purchase government debt, sell gold or reserves, and keep the money supply unchanged. This typically is not a solution though, as central banks' other assets are small compared to government debt.[26] More generally, this illustrates that we should look at government debt as a net concept, subtracting any assets (other than government bonds) that either the central bank or the government itself has on its balance sheet. When this net government debt is in the multiplicity region, the economy becomes subject to self-fulfilling debt crises.

The second case of sterilization applies to a monetary union, where the central bank intervenes in a crisis in its periphery. In the context of the Eurozone it would involve the ECB intervening in a self-fulfilling crisis in its periphery. In that case the central bank can be quite effective. For example, the ECB could buy periphery debt at a low interest rate and sell German debt, without a change in monetary liabilities. The threat alone of doing so is sufficient, which is, in our view, what happened under the OMT policy in the summer of 2012 and the famous Draghi statement "to do whatever it takes". This threat was credible as such an intervention would not overwhelm the ECB.[27] This explains why sovereign spreads quickly fell due to the change in policy. But such a policy applies to a periphery and would not work if the ECB aimed to avoid a self-fulfilling sovereign debt crisis across the entire Eurozone.

6. Conclusion

Several recent contributions have derived analytical conditions under which the central bank can avoid a self-fulfilling sovereign debt crisis. Extreme central bank intervention, generating extraordinary inflation, would surely avoid a sovereign debt crisis. In order to have a better quantitative assessment of what a successful intervention would require, we have adopted a dynamic model with many realistic elements. We introduced a New Keynesian model with nominal rigidities in which monetary policy has realistic effects on output and inflation. We introduced long-term bonds in a slow-moving debt crises model and calibrated the debt maturity to what is observed in many industrialized countries. Overall our conclusion is that, unless debt is close to the bottom of the interval where multiple equilibria occur, monetary policy leads to very high inflation for a sustained period of time.

To determine the desirability of asking the central bank to deflate the government debt, a more public finance approach would be needed as typically done in the literature. The cost of inflation should be weighted against the cost of outright default or the cost of fiscal austerity. Such an analysis goes beyond the scope of this paper. Nevertheless, the high unanticipated inflation rates required when

[25] The only case we find where there could be an impact is when the natural rate of interest stays at zero or negative for a sustained period of time.

[26] There may however be special cases. For example, Switzerland has FX reserves that are larger than government debt.

[27] For example, in 2010 the sum of all the periphery country government deficits together (Greece, Ireland, Portugal, Spain, Italy) amounted to 13% of the ECB balance sheet. And a self-fulfilling default can be avoided even if only a portion of these financing needs are covered by the ECB.

initial debt level is in the middle or high range of the multiplicity region appear highly implausible in developed economies. First, there is currently a strong preference for low inflation rates. Second, even in previous decades with higher inflation rates, the use of inflation to reduce the debt burden has been limited. Indeed, Reinhart and Sbrancia (2015) find that in the post-WWII era, particularly 1945–1980, public debt reductions in industrialized countries have been achieved mostly through financial repression as opposed to inflation surprises. To the extent that debt reduction has been partly achieved through inflation surprises, this often has gone hand in hand with financial repression. The extensive use of financial repression tools is a reflection of the difficulty of achieving debt reduction through inflation surprises alone.

Several extensions are worthwhile considering for future work. We have focused on a closed economy. In an open economy monetary policy also affects the exchange rate, which affects relative prices and output. Related to that, one can also consider the case where a large share of the debt is held by foreigners or is denominated in a foreign currency. Both might make inflationary central bank policy an even less attractive option. Default is more appealing to the extent that the debt is held by foreigners, while in the case of foreign currency denominated debt a depreciation increases the value of debt denominated in domestic currency. Finally, we have only considered one type of self-fulfilling debt crises, associated with the interaction between sovereign spreads and debt. It would be of interest to also consider rollover crises or even a combination of both types of crises. This also provides an opportunity to consider the endogenous maturity of sovereign debt. As pointed out in the introduction, high inflation can reduce the maturity of government debt, which can lead to exposure to rollover crises.

Appendix A. Derivation of the debt accumulation schedule

We derive the debt accumulation schedule in the monetary cashless economy in Section 2.2. We first derive a relationship between Q_0 and Q_{T-1}. Integrating forward the one-period arbitrage Eq. (4) from $t = 1$ to $t = T - 1$, we have

$$Q_0 = A^{\kappa}\kappa + A^{Q}Q_{T-1} \tag{44}$$

where

$$A^{\kappa} = \frac{1}{R_0} + \frac{1-\delta}{R_0R_1} + \frac{(1-\delta)^2}{R_0R_1R_2} + \ldots + \frac{(1-\delta)^{T-2}}{R_0R_1R_2\ldots R_{T-2}} \tag{45}$$

$$A^{Q} = \frac{(1-\delta)^{T-1}}{R_0R_1R_2\ldots R_{T-2}} \tag{46}$$

Next consider the government budget constraint (34):

$$Q_tB_{t+1} = ((1-\delta)Q_t + \kappa)B_t - s_tP_t \tag{47}$$

At $t = 0$ this is

$$\frac{Q_0B_1}{P_0} = ((1-\delta)Q_0 + \kappa)\, b_0 - \bar{s} \tag{48}$$

For $1 < t < T$

$$\frac{Q_tB_{t+1}}{P_t} = r_{t-1}\frac{Q_{t-1}B_t}{P_{t-1}} - \bar{s} \tag{49}$$

Using Eqs. (49) and (48) and integrating forward, we obtain

$$\begin{aligned}\frac{Q_{T-1}B_T}{P_{T-1}} &= r_{T-2}\ldots r_1r_0\frac{Q_0B_1}{P_0}\\ &\quad -\bar{s}(1 + r_{T-2} + r_{T-2}r_{T-1} + \ldots + r_{T-2}r_{T-1}\ldots r_1)\end{aligned} \tag{50}$$

Combining Eq. (50) with Eqs. (48) and (44), we obtain

$$\begin{aligned}\frac{Q_{T-1}B_T}{P_{T-1}} &= r_{T-2}\ldots r_1r_0(1-\delta)b_0Q_0 + r_{T-2}\ldots r_1r_0\kappa b_0\\ &\quad -\bar{s}(1 + r_{T-2} + r_{T-2}r_{T-3} + \ldots + r_{T-2}\ldots r_1r_0)\end{aligned} \tag{51}$$

Using Eqs. (44)–(46), we can rewrite Eq. (51) as

$$\begin{aligned}\frac{Q_{T-1}B_T}{P_{T-1}} &= \frac{P_0}{P_{T-1}}(1-\delta)^Tb_0Q_{T-1}\\ &\quad + r_{T-2}\ldots r_1r_0\left[1 + \frac{1-\delta}{R_0} + \frac{(1-\delta)^2}{R_0R_1} + \ldots + \frac{(1-\delta)^{T-1}}{R_0R_1R_2\ldots R_{T-2}}\right]\kappa b_0\\ &\quad -\bar{s}(1 + r_{T-2} + r_{T-2}r_{T-3} + \ldots + r_{T-2}\ldots r_1r_0)\end{aligned} \tag{52}$$

This yields Eq. (16).

Appendix B. Objective function of the Central Bank

The Central Bank wants to rule out the bad equilibrium while minimizing welfare losses. Welfare is equal to

$$E_0\sum_{t=0}^{\infty}\beta^tU_t \tag{53}$$

With the specific functional form that we have used, a second order approximation of U_t is

$$U_t = -0.5(cU_c)\left\{\mu_x(x_t - \nu x_{t-1})^2 + \left[\frac{1}{\varepsilon} + \frac{\alpha+\phi}{1-\alpha}\right](1-\beta\eta)var_i\hat{y}_t(i)\right\} \tag{54}$$

(see Woodford, 2003), where

$$\mu_x = \frac{1-\beta\eta}{1+\beta\nu^2}\left[\frac{\alpha+\phi}{1-\alpha} + \frac{\sigma}{(1-\beta\eta)(1-\eta)}\left(1+\beta\eta^2\right)\right] \tag{55}$$

and ν is defined as the smaller root of the quadratic equation

$$\eta\frac{\sigma}{(1-\beta\eta)(1-\eta)}\left(1+\beta\nu^2\right) = \left[\frac{\alpha+\phi}{1-\alpha} + \frac{\sigma}{(1-\beta\eta)(1-\eta)}\left(1+\beta\eta^2\right)\right]\nu \tag{56}$$

Welfare expression (54) assumes that there is a subsidy to firms to offset the inefficiency caused by monopolistic competition.

We now derive an expression for $var_i\hat{y}_t(i)$, the cross sectional variance of output across firms. The demand for the goods of firm i is

$$\ln(y_t(i)) = \ln(Y_t) - \varepsilon(p_t(i) - p_t) \tag{57}$$

where $p_t(i)$ is the log price of the goods produced by firm i. Hence it is

$$var_i\hat{y}_t(i) = \varepsilon^2 var_ip_t(i) \tag{58}$$

Defining

$$\Delta_t = var_i p_t(i) \tag{59}$$

It can be shown (see Woodford, 2003) that

$$\Delta_t = \theta\Delta_{t-1} + \frac{\theta}{1-\theta}(\pi_t - \gamma\pi_{t-1})^2 \tag{60}$$

which implies

$$\sum_{t=0}^{\infty}\beta^t\Delta_t = \frac{\theta}{(1-\theta)(1-\theta\beta)}\sum_{t=0}^{\infty}\beta^t(\pi_t - \gamma\pi_{t-1})^2 \tag{61}$$

Using Eqs. (57)–(61) welfare becomes

$$-0.5(cU_c)\sum_{t=0}^{\infty}\beta^t\left\{\mu_x(x_t - \nu x_{t-1})^2 + \mu_\pi(\pi_t - \gamma\pi_{t-1})^2\right\} \tag{62}$$

with

$$\mu_\pi = \left[\frac{1}{\varepsilon} + \frac{\alpha+\phi}{1-\alpha}\right]\frac{\theta\varepsilon^2(1-\beta\eta)}{(1-\theta)(1-\theta\beta)} \tag{63}$$

Starting from a steady state, a marginal change in consumption by dc that lasts 4 quarters raises welfare by

$$\left(1 + (1-\eta)\beta + (1-\eta)\beta^2 + (1-\eta)\beta^3 - \eta\beta^4\right)(cU_c)(dc/c) \tag{64}$$

We can then express the welfare loss in terms of one year's consumption, or output, as

$$\begin{aligned}\frac{dc}{c} = &-\frac{0.5}{1 + (1-\eta)\beta + (1-\eta)\beta^2 + (1-\eta)\beta^3 - \eta\beta^4} \\ &\times\sum_{t=0}^{\infty}\beta^t\left\{\mu_x(x_t - \nu x_{t-1})^2 + \mu_\pi(\pi_t - \gamma\pi_{t-1})^2\right\}\end{aligned} \tag{65}$$

Appendix C. Solution method for the optimal policy

We outline the solution method for the optimal policy. The central bank chooses the interest rate i_t for $t = [0, H-1]$. At time H the Central Bank resumes the Taylor rule (31). The first step is to compute the sequences of output gap x_t, inflation π_t and interest rate i_t implied by the choice of $\{i_1, i_2, \ldots, i_{H\text{-}1}\}$. Once we have these sequences, we can compute the objective function (30). The optimal policy is the choice of $\{i_1, i_2, \ldots, i_{H\text{-}1}\}$ that maximizes the objective function.

From time H on, the economy is described by the system of 3 equations

$$\tilde{x}_t = \tilde{x}_{t+1} - \frac{1-\beta\eta}{\sigma}\left(i_t - \pi_{t+1} - r^n\right) \tag{66}$$

$$\pi_t = \gamma\pi_{t-1} + \beta\pi_{t+1} - \beta\gamma\pi_t + (\omega_1 x_t + \omega_2\tilde{x}_t) \tag{67}$$

$$i_t - \bar{\imath} = \rho(i_{t-1} - \bar{\imath}) + (1-\rho)\left(\psi_\pi\pi_{t+1} + \psi_y x_t\right) \tag{68}$$

We omitted all expectations as the only shock in the economy after time 0 is the shock to the primary surplus at time T, which does not affect output, inflation or interest rates. $\tilde{x}_t$ is defined in Eq. (26) and ω_1 and ω_2 are defined in Eqs. (28) and (29). We can write this system as

$$M_1 v_{t+1} + M_2 v_t = 0 \tag{69}$$

where

$$v_t = \begin{pmatrix} i_{t-1} \\ \pi_t \\ \pi_{t-1} \\ x_{t+1} \\ x_t \\ x_{t-1} \end{pmatrix} \tag{70}$$

$$M_1 = \begin{pmatrix} -1 & (1-\rho)\psi_\pi & 0 & 0 & 0 & 0 \\ -\frac{1-\beta\eta}{\sigma} & \frac{1-\beta\eta}{\sigma} & 0 & -\beta\eta & \beta\eta + 1 + \beta\eta^2 & 0 \\ 0 & \beta & -1-\gamma\beta & 0 & -\beta\eta\omega_2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \tag{71}$$

and

$$M_2 = \begin{pmatrix} \rho & 0 & 0 & 0 & (1-\rho)\psi_y & 0 \\ 0 & 0 & 0 & 0 & -\eta - 1 - \beta\eta^2 & \eta \\ 0 & 0 & \gamma & 0 & \omega_1 + \omega_2 + \omega_2\beta\eta^2 & -\omega_2\eta \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{72}$$

Eq. (69) implies

$$v_{t+1} = Mv_t \tag{73}$$

with $M = -M_1^{-1}M_2$. It can be verified that the matrix M has 3 non-explosive eigenvalues, equal to the number of pre-determined variables in v_t. v_t at time H needs to be a linear combination of the 3 eigenvectors corresponding to the non-explosive eigenvalues, which implies a relationship between the non-pre-determined and the pre-determined variables at time H.

$$\begin{pmatrix} \pi_H \\ x_{H+1} \\ x_H \end{pmatrix} = G\begin{pmatrix} i_{H-1} \\ \pi_{H-1} \\ x_{H-1} \end{pmatrix} \tag{74}$$

Between time 0 and time $H-1$, call $\left\{\bar{\imath}_0, \ldots, \bar{\imath}_{H-1}\right\}$ the sequence of interest rates chosen by the central bank. The economy is described by the following system of $2H$ equations:

$$\tilde{x}_t = \tilde{x}_{t+1} - \frac{1-\beta\eta}{\sigma}\left(\bar{\imath}_t - \pi_{t+1} - r^n\right) \tag{75}$$

$$\pi_t = 0 \tag{76}$$

for $t < d$ and

$$\tilde{x}_t = \tilde{x}_{t+1} - \frac{1-\beta\eta}{\sigma}\left(\bar{\imath}_t - \pi_{t+1} - r^n\right) \tag{77}$$

$$\pi_t = \gamma\pi_{t-1} + \beta\pi_{t+1} - \beta\gamma\pi_t + (\omega_1 x_t + \omega_2\tilde{x}_t) \tag{78}$$

for $d \le t \le H-1$. After substituting Eq. (74) into Eqs. (77)–(78), Eqs. (75)–(78) become a linear system of 2H equations in 2H variables. After obtaining $[\pi_t, x_t]$ for $0 \le t \le H-1$, we can obtain $[\pi_t, x_t, i_t]$ for $t \ge H$ by using Eq. (73).

Appendix D. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.jinteco.2017.11.004.

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