Sectoral Productivity Losses from Sovereign Default Events: a simulation-based approach

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Abstract

Sovereign defaults are traumatic events with economically sizable losses in output. Their effects, however, have an asymmetric impact on economic sectors. Data on sovereign debt crises from the last 40 years suggest the tradable GDP is affected more severely than the nontradable during these events. In that sense, this report assesses the hypothesis of asymmetric sovereign default costs between sectors. Taking the typical case of Argentina, I provide evidence of the differential effect of default crises in some macroeconomic aggregates. Then, I estimate the productivity loss functions for tradable and nontradable GDP through the lens of a small open economy model with two sectors and sovereign default via the simulated method of moments and the indirect inference approach. Results show that productivity losses in the tradable sector are between 2.5 to 6 times higher than in the nontradable one. I also provide evidence in favor of models with heterogeneous sovereign default costs in terms of a better match with parameters from auxiliary models and observed moments than models with a symmetric cost function.

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1 Introduction

Sovereign defaults are costly events. They hurt the economy severely by their effects on trade, financial markets, the banking sector, and the real economy. Because of that, debt crisis is a persistent topic of interest in academia and policy-making worlds, and its transmission mechanisms and quantitative consequences are continuously studied. Some estimates point out the economically significant and persistent adverse effects of default episodes. For instance, Esteves, Kenny, and Lennard (2021) find that GDP falls up to 3.3% in the aftermath of a sovereign crisis (two years after), while Kuvshinov and Zimmermann (2019) estimate a reduction in output of 3.7% after five years.[1] However, sovereign defaults have an asymmetric impact on different economic sectors. Empirical evidence highlights a disproportionate GDP decline in the more export-oriented industries, a severe effect in sectors with a high degree of financial dependence, and a strong decline in trade and investment components.[2] Similar patterns are found in data on sovereign debt crises from the last 40 years. In figure 1, I show the cyclical behavior of Manufacturing (tradable) and Services (nontradable) value-added during a debt crisis for a sample of 12 defaulters.[3] Data reveals that the tradable sector is more sensitive than nontradable to sovereign default events in the majority of cases. The large negative correlation between tradable GDP and default is remarkable in the cases of Argentina in 2002, or Ecuador in 1999, but also in European countries, like Greece in 2012, Türkiye in 2001, and Ukraine in 2015. In all of them, the downfall in tradable GDP is about double of the nontradable sector. Given these findings, questions about the quantitative differences in output losses between sectors arise naturally.

In this report, I test the hypothesis of heterogeneous default costs in sectoral-level GDP for the case of Argentina.[4] To do so, I proceed in two steps. I first obtain some estimates on the impact of sovereign default crises in Argentina's macroeconomic aggregates. I follow a simple methodology, exposed in Gourinchas and Obstfeld (2012), to examine the behavior of tradable and nontradable GDP around debt crises. I find that output in tradable and nontradable sectors are negatively associated with default episodes, with an average drop of about 13% and 7.7% relative to "tranquil times", respectively. In a second step, I estimate the parameters of a nonlinear default cost function[5] through the lens of a dynamic stochastic small open economy model with incomplete markets, two productive sectors, and sovereign default. In the model, default costs are interpreted as productivity losses and are asymmetric between sectors. The estimation is made by two similar approaches: the simulated method of moments (SMM) and the indirect inference method (IIM). I find that parameters governing productivity losses are numerically different between sectors, with a higher punishment in the tradable sector. I also provide some evidence in favor of models with heterogeneous default costs. The rest of this report is organized as follows. In section 2, I provide a brief description of related literature. Section 3 describes some empirical findings, while the quantitative model and the estimation are presented in section 4. Finally, section 5 concludes. Figures and tables are shown at the end of this document.

2 Literature review

In this section, I provide a brief description of some papers on sovereign default. I focus on two topics of this literature. On the one hand, I revise quantitative models with two (or more) sectors. On the other hand, I inspect the methodology to calibrate/estimate the main parameters from these models. Two conclusions emerge from this

[1] Furthermore, Furceri and Zdzienicka (2012) obtains an estimate of an output downfall by about 10% after eight years of a sovereign default episode.

[2] See Borensztein and Panizza (2010), Zymek (2012), and Asonuma, Chamon, and Sasahara (2016) respectively.

[3] Variables are in annual frequency and are log-detrended using the Hodrick-Prescott filter with a smoothing parameter of 100. Dates are obtained from Gourinchas and Obstfeld (2012).

[4] I chose Argentina as the country case due to the ease of obtaining calibrated parameters and comparing results from other similar studies.

[5] As in Arellano (2008).

review: (i) the majority of studies do not model asymmetric default costs between sectors, and (ii) most of them use calibration to match some targets from data.

Sovereign default models with more than one sector are mainly employed to study the impact of this event on real exchange rate and the reallocation of factors. For instance, Asonuma (2016) build a model with two countries and two sectors to study the link between real exchange rate and default probability. The default cost is symmetric and modeled as in Arellano (2008). Moreover, the output loss is calibrated to 2%. The link between debt and real exchange rate is as follows: negative tradable endowment shocks cause the accumulation of more debt, a relative larger supply of nontraded goods and a real depreciation. In turn, the interaction with sovereign's large share of foreign currency debt trigger defaults. Post-default, output costs and loss of market access lead to further real exchange rate depreciation since the sovereign prefers to consume more traded goods. On the other hand, Arellano, Bai, and Mihalache (2018) build a two-sector model of sovereign default with capital accumulation to replicate the persistent declines in economic activity caused by this event. In this case, the output loss is modeled as in Chatterjee and Eyigungor (2012), by a quadratic function, and is symmetric between sectors. Its values are calibrated to match the mean of spread, the volatility of spread, the relative consumption volatility and the relative investment volatility. In the model, adverse domestic shocks increase the default risk, which is the main mechanism for explaining the enduring deterioration: it limits capital inflows and restricts the ability of the economy to exploit investment opportunities. Furthermore, Na, Schmitt-Grohé, Uribe, and Yue (2018) present a model with sovereign default, nominal wage rigidities and two sectors to study the Twin Ds phenomenon (joint occurrence of default and a large nominal exchange rate depreciation). The model has the property that periods in which it is optimal to default are also periods in which it is optimal to bring about a large change in relative prices (real depreciation). The interaction between the decision to default and the downward wage rigidity is key to explain this behavior. They also use a quadratic output loss function and assume symmetry between sectors. The calibration matches the average debt to traded GDP ratio in periods of good financial standing, the frequency of default, and the average output loss from Argentina's data.

In terms of methodology, calibration is the main strategy in this literature. Parameters of the default cost function (probability of reentry and output loss) and the discount factor are usually chosen to match targets like the default probability, trade balance volatility and debt service to GDP, like Arellano (2008), or the average external debt-to-output ratio and the average default spread and its standard deviation, as in Chatterjee and Eyigungor (2012). In the same vein, Gordon and Guerron-Quintana (2018) calibrate six parameters (including the default cost parameters) to match six empirical moments: the debt-output ratio, the average spread, the spread volatility, the volatility of investment, the volatility of output, and the ratio of the volatilities of consumption and output. Probably the closest work to the methodology exposes in this report is Arellano, Mateos-Planas, and Ríos-Rull (2019) where the authors develop a quantitative model with partial default. They obtain the parameters that characterize the default cost function and the discount rate, among others, using a minimum distance estimator that targets the empirical distribution of partial default and the behavior of interest rate spreads, debt, and output in emerging markets. However, they do not provide standard errors or statistical tests on their estimates. Other interesting papers that bridge theoretical models with data, but are beyond the scope of this report, are Arellano, Bai, and Bocola (2017) and Moretti (2020).

3 Empirical evidence

To identify empirical regularities during sovereign default events, I follow the methodology exposed in Gourinchas and Obstfeld (2012). These authors examine the behavior of key economic variables around crisis episodes by the estimation of the variable's conditional expectation as a function of the temporal distance from various types of crisis, relative to a common "tranquil times" baseline. In contrast to them, my estimation just focuses on Argentina.

The econometric specification is:

$$y_t = \gamma + \delta_{d,s}\mathcal{I}_{d,s} + \delta_{c,s}\mathcal{I}_{c,s} + \delta_{b,s}\mathcal{I}_{b,s} + \delta_{i,s}\mathcal{I}_{i,s} + \zeta_t \quad (1)$$

where y_t is the variable of interest, $\mathcal{I}_{j,s}$ is a dummy variable equal to 1 when Argentina is s periods away from a crisis of type j in period t, and ζ_t is the error term. The index j denotes, respectively, sovereign default crisis (d), currency crisis (c), systemic banking crisis (b), and the 2009 international financial crisis (i). Moreover, the event window around crisis episodes is set to 9 years (four years before, four years after). The parameter of interest is $\delta_{d,s}$. It measures the conditional effect of a sovereign default crisis on variable y over the event window $-4 \leq s \leq 4$ relative to "tranquil times", which corresponds to the years when Argentina does not fall into any crisis-event window. It is important to highlight these estimates do not have a causal interpretation, but just reflect the evolution of macroeconomic aggregates over different crisis episodes. The model is estimated by OLS.

Data Variables of interest are the sectoral level GDP (tradable and nontradable). I extract this information from the World Bank's World Development Indicators in annual frequency for the period 1965-2019. I define tradable GDP as the Manufacturing value-added, whereas the nontradable GDP is the Services value-added.[6] Both variables are measured in $US\$$ at constant 2015 prices. Additionally, I add 6 variables: total GDP, Consumption, Investment, Net Exports, the bilateral Real Exchange Rate (RER), and the Total Factor Productivity (TFP). The latter variable is obtained from Feenstra, Inklaar, and Timmer (2015), while the others are extracted from the same World Bank database.[7] Variables are log-detrended using the Hodrick-Prescott filter with a smoothing parameter of 100. Exception is Net Exports which is defined as a percentage of GDP. In the case of the indicator variables, crises dates are extracted from Gourinchas and Obstfeld (2012) for the period 1973-2010. For the remainder years (1965-1972 and 2011-2019), I fill the information in using the Global Crisis dataset[8] and the updated version of Asonuma and Trebesch (2016) dataset.

Results They are shown graphically in figure 2 for the case of the cyclical components of sectoral-level GDP (tradable and nontradable). Table 1 presents the whole estimation for all the 8 dependent variables. It shows that a sovereign default episode in Argentina is associated with a significant decline in tradable and nontradable GDP relative to "tranquil times". Moreover, tradable GDP is more sensitive than its nontradable counterpart. Quantitatively, a sovereign default event is correlated with a fall of about 13% of the tradable GDP, while the nontradable GDP falls 7.7% at the year of the event relative to "tranquil times". Furthermore, it is interesting to see economically large drops in tradable GDP one year before and after the materialization of the default episode. This is not statistically observed in the case of nontradable GDP. Additionally, notice the total GDP, consumption and productivity fall almost 10%, 11% and 7%, respectively, relative to "tranquil times" at $s = 0$. On the other hand, investment reaction to a default event represented the strongest decline (-31.4%) at the year of the default event. Finally, real exchange rate and net exports are associated with jumps (34.8% and 7.9%, respectively), at the year of the default episode. Large real depreciations and trade surpluses around sovereign default events are patterns found in

[6]Manufacturing refers to industries belonging to ISIC divisions 15-37, while Services correspond to ISIC divisions 45-99. Both sectors represent about 3/4 of Argentina's GDP in average. Although the standard definition of tradable GDP includes primary sectors, this information is not reliably available in the World Bank database.

[7]I constructed the Real Exchange Rate with information of Consumer Price Indices from Argentina and the USA, and the end-of-period nominal exchange rate between the Argentinean currency and US dollars.

[8]It is available here.

Asonuma (2016)[9] and Kuvshinov and Zimmermann (2019).[10]

4 The model

I build a quantitative dynamic stochastic small open economy model with two sectors. Each sector hires labor to produce tradable and nontradable goods. Moreover, there is a benevolent government that trades one-period bonds with international lenders. Notice the international debt is unenforceable so the government can default on it. This model follows Uribe and Schmitt-Grohé (2017)'s TNT model closely and differs from the standard quantitative model of sovereign default, like Arellano (2008), in three aspects: (i) it contains two productive sectors that demand labor each one, (ii) the "primary punishment" of default events is modeled as productivity (not total output) losses and is heterogeneous between sectors,[11] and (iii) it is calibrated for annual data instead of a quarterly calibration.[12] In the following lines, I detail the main features of the model. This description follows Arellano, Bai, and Mihalache (2018).

Firms There are two types of firms producing tradable $(y_{T,t})$ and nontradable $(y_{N,t})$ goods by the following technologies:

$$y_{T,t} = z_t h_{T,t}^{\alpha_T} \tag{2}$$

$$y_{N,t} = z_t h_{N,t}^{\alpha_N} \tag{3}$$

where z_t is a common productivity shock which follows a first-order Markov process. Moreover, $h_{T,t}$ and $h_{N,t}$ are the hours worked in tradable and nontradable sectors respectively, while α_T and α_N are the labor shares.

Households Their preferences are defined as follows:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_{T,t}, h_{N,t}) \tag{4}$$

where β is the discount factor. In my model c_t is a CES bundle of tradable $(c_{T,t})$ and nontradable $(c_{N,t})$ goods:

$$c_t = \left[(1-\theta) c_{T,t}^{\frac{\eta-1}{\eta}} + \theta c_{N,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{5}$$

where θ is the share of nontradables in the consumption basket and η is the elasticity of substitution between both types of goods.

[9]The author suggests a double causality explanation: on the one hand, the real exchange rate depreciation increases the burden of debt service payments for a debtor sovereign and helps trigger default. On the other hand, the country's announcement of default or restructuring leads to further real exchange rate depreciation.

[10]According to these authors, defaulters tend to increase net exports by around 3% of GDP in the medium term via a rapid and sharp reduction in imports.

[11]In quantitative models, the default cost is controlled by two parameters: the probability of reentry and the endowment loss. As Aguiar and Amador (2021) explain, the former *"proxies for the time it takes for governments to reach a settlement with their creditors, while the latter is a catch-all that proxies for disruption of trade, financial markets, etc"*. Although both are critical to influence government's optimal behavior, the endowment loss is the "primary punishment".

[12]However, I do not include capital in the model. It is well known that adding state variables increases strongly the computational cost of solving sovereign default models. Moreover, as Gordon and Guerron-Quintana (2018) point out, capital addition generates complex relationships between investment, debt, and default, and makes it necessary to include long-term debt for matching debt and spread statistics simultaneously for conventional parameter values. Although adding capital is a natural extension to the standard model of sovereign default, I leave this task for future work given the time constraints, computational requirements, and the scope of this report.

Government It trades international bonds denominated in tradable goods (b_t) with risk-neutral lenders. The discount rate of investors is R which is the international interest rate. If the government decides to payback its debt, then it can borrow b_{t+1} at price q_t. Otherwise, it is out of the international markets and, consequently, enters the autarky state and the economy suffers heterogeneous productivity losses between sectors. In this model, traded goods and new borrowing are used for consumption and for paying back the debt, while nontraded goods are used exclusively for consumption:

$$c_{T,t} = y_{T,t} + q_t b_{t+1} - b_t \tag{6}$$

$$c_{N,t} = y_{N,t} \tag{7}$$

4.1 Recursive formulation

There are three state variables in this model: the productivity shock (z), the stock of debt (b) and a dummy variable (x) which records of whether a country is in default ($x = 1$) or in normal market access ($x = 0$). Let $\mathcal{S}$ be the state of the economy given by (x, s), with $s = (b, z)$, and $V(s)$ be the value of the benevolent government in the "normal" regime with state s. The government chooses d_t to maximize its value:

$$V(s) = \max_{d=\{0,1\}} \left\{ dV^n(s) + (1-d)V^d(z) \right\} \tag{8}$$

where $V^n(s)$ is the value of repayment and $V^d(z)$ is the value of default. Note that V^d depends only on z because government has not access to the debt market in that state. If the government defaults, then $x' = 1$. Conditional on repaying its debt, the benevolent government chooses the allocation of labor between tradable and nontradable sectors, consumption of both types of goods, and new borrowing b' to maximize its value:

$$V^n(s) = \max_{\{h_T, h_N, c_T, c_N, b'\}} u(c_t, h_{T,t}, h_{N,t}) + \beta \mathbb{E}_{z'|z} V(s') \tag{9}$$

subject to equations 5, 6 and 7. However, if the government decides to default, it is excluded from international financial markets and the economy suffers productivity losses. In that case, the government faces a probability of reentry equal to λ. When it reenters financial markets, productivity losses vanish and the government starts with zero debt. In the default state, the government chooses optimally the allocation of labor between sectors and consumption of tradable and nontradable goods to maximize its value:

$$V^d(z) = \max_{\{h_T, h_N, c_T, c_N\}} u(c_t, h_{T,t}, h_{N,t}) + \beta \mathbb{E}_{z'|z} \left[\lambda V(0, z') + (1-\lambda) V^d(z') \right] \tag{10}$$

subject to equation 5 and the following resource constraints:

$$c_{T,t} = z_t^{T,d}(z) h_{T,t}^{\alpha_T} \tag{11}$$

$$c_{N,t} = z_t^{N,d}(z) h_{N,t}^{\alpha_N} \tag{12}$$

where $z_t^{i,d}(z) \leq z$ indicates the productivity cost of default per sector $i \in \{T, N\}$. Finally, there is a large number of identical international lenders that are risk neutral and discount at the rate R. They lend to the government in a perfectly competitive market. Their profits are:

$$\Pi(b', z) = -q(b', z) \times b' + \frac{1}{R} \mathbb{E}_{z'|z} d(b', z) \times 0 + \frac{1}{R} \mathbb{E}_{z'|z} \left[1 - d(b', z) \right] \times b' \tag{13}$$

where $\mathbb{E}_{z'|z}d(b',z)$ is the default probability. In this context, the bond price schedule $q(b',z)$ compensates international lenders for default risk. The break-even (zero profits) bond price satisfies:

$$q(b',z) = \frac{1}{R}\mathbb{E}_{z'|z}\left[1 - d(b',z)\right] \tag{14}$$

4.2 Competitive equilibrium

Given the state $\mathcal{S} = (x,s)$, the recursive Markov equilibrium consists of policy functions for default $d(s)$, the allocation of labor $\{h_T(\mathcal{S}), h_N(\mathcal{S})\}$, tradable and nontradable consumption $\{c_T(\mathcal{S}), c_N(\mathcal{S})\}$, and borrowing $b'(s)$; value functions $\{V(s), V^n(s), V^d(z)\}$, and the bond price function $q(s)$ such that:

- the policy functions for sectoral consumption and labor solve households' optimization problem,
- the policy and value functions for the government satisfy its optimization problem,
- international lenders obtain zero expected profits for the equilibrium bond price (it satisfies equation 14).

4.3 Quantitative analysis

Functional forms I define the functional forms of utility, the default cost and the productivity process. Following Uribe and Schmitt-Grohé (2017), households' preferences follow a GHH form:

$$u(c_t, h_{T,t}, h_{N,t}) = \frac{\left(c_t - \frac{h_{T,t}^{\omega_T}}{\omega_T} - \frac{h_{N,t}^{\omega_N}}{\omega_N}\right)^{1-\sigma} - 1}{1-\sigma} \tag{15}$$

where β is the discount factor, σ is the risk aversion (intertemporal elasticity of substitution), and ω_T and ω_N determine the wage elasticities of labor supply in each sector.[13] Notice this functional form implies households' labor supply is not affected by wealth effect. In the case of the default cost, I assume heterogeneity between the two sectors, so the declines in tradable and nontradable sectoral GDP due to a sovereign default event are not necessarily equal. In other words, productivity losses due to a default episode are different between sectors. I consider the nonlinear function from Arellano (2008), such that for each $i \in \{T, N\}$ the default cost is given by:

$$z_t^{i,d}(z) = \min\left(z_t, (1-\chi^i)\mathbb{E}_t(z_t)\right) \tag{16}$$

Finally, the productivity shock is assumed to follow an AR(1) process:

$$\ln(z_t) = (1-\rho_z)\ln(\bar{z}) + \rho_z \ln(z_{t-1}) + \epsilon_t \tag{17}$$

where $\bar{z}$ is its steady state value, $\mathbb{E}(\epsilon) = 0$ and $\mathbb{E}(\epsilon^2) = \sigma_z^2$.

Calibration It is based on the literature about the Argentinean case. Calibration is in annual frequency and it is shown in table 2. I set the risk aversion parameter σ to 2, which is the standard value in the literature, and the annual net risk-free rate $R-1$ to 4%. Moreover, the share of nontradable goods in the consumption basket is $\theta = 0.68$, which is obtained from the data.[14] I follow Uribe and Schmitt-Grohé (2017) for calibrating the elasticity

[13] These elasticities are defined as $\frac{1}{\omega_i - 1}$ for all $i \in \{T, N\}$.

[14] I made a simple calculation to obtain this value. Using data extracted from the World Bank's World Development Indicators, θ is equal to the sample average (1965-2019) of the nontradable (Services) relative weight for Argentina: $\theta = \frac{Services}{Services+Manufacturing}$. In the sample, Services and Manufacturing sectors represent about 52.0% and 25.0% of the total GDP, respectively.

of substitution between tradable and nontradable goods to $\eta = 0.5$, as well as both values of ω_T and ω_N to 1.455 (they imply a wage elasticity of 2.2% in both sectors). Nontradable and tradable labor shares are set to $\alpha_N = 0.34$ and $\alpha_T = 0.43$ from Arellano, Bai, and Mihalache (2018). The discount factor is set to $\beta = 0.825$ which is the yearly equivalence of the quarterly calibration from Arellano (2008).[15] In the case of the probability of reentry, I set a value of λ to 0.25 so the defaulting countries are excluded from international markets for 4 years, which is in line with Gelos, Sahay, and Sandleris (2011).[16] Finally, values of parameters that govern the productivity process z_t are obtained from an OLS regression of Argentina's TFP (in logs) for the period 1965-2019.[17]

Simulation and estimation I discretize the productivity process into a 21-state Markov chain using the Tauchen's algorithm while bonds are discretized by a grid of 151 points equally spaced. The model is solved by value function iteration, following the computational algorithm described in Appendix 2 from Arellano (2008). On the other hand, parameters of the productivity loss functions are estimated using both, the simulated method of moments (SMM), and the indirect inference method (IIM). As it is explained in chapter 4 from Adda and Cooper (2003), IIM requires a simple auxiliary model which is estimated both on the observed data and on simulated data. In this case, the auxiliary model is provided in section 3. I define $\hat{\boldsymbol{\delta}}_{d,0} = [\hat{\delta}^T_{d,0}, \hat{\delta}^N_{d,0}]$ as the vector of estimated parameters that reflect the reaction of tradable and nontradable GDP at the year of the debt crisis (see table 1). Moreover, $\boldsymbol{\chi} = [\chi^T, \chi^N]$ is the vector of parameters that govern the nonlinear productivity loss functions. Note that the system is exactly identified. The indirect inference estimator $\hat{\boldsymbol{\chi}}^{(\mathcal{IN})}_{IIM}$ is the solution to:

$$\hat{\boldsymbol{\chi}}^{(\mathcal{IN})}_{IIM} = \arg\min_{\chi} \left[\hat{\boldsymbol{\delta}}_{d,0} - \hat{\boldsymbol{\delta}}^{(\mathcal{IN})}_{d,0}(\boldsymbol{\chi})\right]' \Omega_{\mathcal{N}} \left[\hat{\boldsymbol{\delta}}_{d,0} - \hat{\boldsymbol{\delta}}^{(\mathcal{IN})}_{d,0}(\boldsymbol{\chi})\right] \tag{18}$$

where $\Omega_{\mathcal{N}}$ is the weight (identity) matrix and $\hat{\boldsymbol{\delta}}^{(\mathcal{IN})}_{d,s}$ is the average value of the auxiliary parameters, over all simulations:[18]

$$\hat{\boldsymbol{\delta}}^{(\mathcal{IN})}_{d,s} = \frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} \hat{\boldsymbol{\delta}}^{(i\mathcal{N})}_{d,s}(\boldsymbol{\chi}) \tag{19}$$

On the other hand, SMM minimizes the distance between moments from the observed and simulated data. I define $\boldsymbol{\mu}(\boldsymbol{x}_t) = \left[\frac{\sigma^T}{\sigma^N}, Mean(Spread)\right]$ as the vector of functions of the observed data, $\boldsymbol{x}_t$, where $\frac{\sigma^T}{\sigma^N}$ is the volatility of tradable relative to nontradable GDP (cycle) and $Mean(Spread)$ is the sample average of the Argentina's bond spread. The latter moment is key to discipline models with sovereign default and is usually consider in the majority of calibration exercises from the literature. The former moment is an obvious choice since it is expected the volatility of tradable sector is bigger than the nontradable during a sovereign default episode (see figure 1, for instance). The estimator for the SMM is defined as

$$\hat{\boldsymbol{\chi}}^{(\mathcal{IN})}_{SMM} = \arg\min_{\chi} \left[\boldsymbol{\mu}(\boldsymbol{x}_t) - \boldsymbol{\mu}^{(\mathcal{IN})}(\boldsymbol{x}(\boldsymbol{\chi}))\right]' \Omega_{\mathcal{N}} \left[\boldsymbol{\mu}(\boldsymbol{x}_t) - \boldsymbol{\mu}^{(\mathcal{IN})}(\boldsymbol{x}(\boldsymbol{\chi}))\right] \tag{20}$$

where $\boldsymbol{\mu}^{(\mathcal{IN})}(\boldsymbol{x}(\boldsymbol{\chi}))$ is the average value of the moments, over all simulations:

$$\boldsymbol{\mu}^{(\mathcal{IN})}(\boldsymbol{x}(\boldsymbol{\chi})) = \frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} \boldsymbol{\mu}^{(i\mathcal{N})}(\boldsymbol{x}(\boldsymbol{\chi})) \tag{21}$$

[15] The author sets a value of $\beta^q = 0.953$. Since the quarterly gross discount rate is $1+\rho = \frac{1}{\beta^q}$, then the yearly gross discount rate is $(1+\rho)^4 \approx 1.21$ and the annual discount factor is $\beta = \frac{1}{1.21} \approx 0.825$.

[16] The median of the years out of the international markets for countries that regained access after the default is 4 for the period 1980-2000. See table 4.

[17] The data is extracted from the Penn World Table 10.0, see Feenstra, Inklaar, and Timmer (2015).

[18] Parameters estimated in the simulation correspond to the regression of y^T_t (or y^N_t) on $d(b', z)$.

I obtain the optimal values by a grid search algorithm over plausible values of $\boldsymbol{\chi}$.[19] Grids for χ^T and χ^N have 35 points each one, which makes a total of 1225 combinations to assess. In each round, I run $\mathcal{I} = 100$ simulations of $\mathcal{N} = 55$ periods each one for the variables of interest,[20] in order to replicate the sample length. All the simulated data is treated as their empirical counterpart: they are log-detrended using the Hodrick-Prescott filter with a smoothing parameter of 100. Moreover, I also estimate the model under the assumption of $\chi^T = \chi^N$ as a benchmark.

4.4 Results

Table 3 contains the results from the optimization algorithm. The value of $\hat{\chi}_T$ is 2.5 (6.0) times larger than $\hat{\chi}_N$ when they are obtained by SMM (IIM). These findings imply that a sovereign default event punishes sharply tradable than the nontradable sector. In the case of SMM, $\hat{\chi}^T$ is 7.8% and $\hat{\chi}^N$ is 3.0%. Both values are statistically significant. These numbers are interpreted as no productivity losses if z is lower than 92.2% and 97% of its mean, respectively. In the case of the IMM estimates, the difference between both parameters is bigger: $\hat{\chi}^T$ is 6.9% and $\hat{\chi}^N$ is 1.1%, although both are also statistically significant. Then, the economy has no productivity losses if z is lower than 93.1% and 98.9% of its mean, respectively. In that sense, the probability of a productivity downfall is bigger in the tradable than the nontradable sector amid a sovereign default event.

In panel (a) from figures 3 and 4, I show graphical representations of the productivity loss function. Notice that for sufficiently large values of z, the tradable sector faces stronger losses than the nontradable once a default occurs. I also show the variation in productivity because of the default episode in panel (b). Again, tradable sector productivity faces a severe decline over a large interval of z values. For instance, if $z = 0.8$ (just 10% higher than the average), then a sovereign default causes a collapse in the tradable productivity of about 15%, while the nontradable sector productivity declines about 9%. I also show results under the assumption of $\chi^T = \chi^N$ (symmetric default cost) in table 3. Some aspects are relevant: (i) the parameter estimated by SMM lies below the values obtained under the asymmetric assumption, which is a strange result, (ii) also this parameter is not statistically significant,[21] and (iii) the magnitude of the objective criterion is bigger under the assumption of symmetric default cost. The latter indicates the assumption of heterogeneous productivity losses achieves a better match with moments and parameter estimates chosen for SMM and IIM methodologies. I also ran Wald tests (not reported) where I assess the joint hypothesis that both parameters (χ^T, χ^N) are equal to the value obtained under the symmetric assumption.[22] The null hypothesis was rejected in both cases (SMM and IIM). Another way to assess the model's fit is provided in table 4, where I compare observed and simulated moments (for SMM) and parameter estimates (for IIM) from models with symmetric and heterogeneous productivity losses. In the case of the SMM estimates, the model with asymmetric productivity losses works well to match the relative volatility of sectoral-level GDP, but not the sample average of the spread. The distribution of both moments is also shown in figure 5. Instead, parameter estimates by IIM show a remarkable match when productivity losses are different between sectors (see also figure 6).

Discussion Results show productivity losses due to a debt crisis are larger in tradable than nontradable sector. This is in line with evidence on the asymmetric impact of default episodes. However, it is important to highlight this model is blinded to the mechanisms behind these results since default cost is modeled exogenously. In the model, a sequence of negative productivity shocks makes production and labor in both sectors decline, which incentives the

[19]The choice of the interval was guided by the previous literature. For instance, Arellano (2008) sets a default cost parameter of 0.0310. Other authors define this value between 1% to 2%. Consequently, I run a grid search over the interval $[0.005, 0.1]$ for both parameters and obtain a set of optimal values. Then I run Matlab's solver `fminsearch` taking the previous results as initial points.

[20]Actually, I run 85 periods per simulation but discard the first 30 periods in order to avoid initial condition problems.

[21]This could be explained by the choice of the identity matrix as $\Omega_{\mathcal{N}}$. When $\chi^T = \chi^N$ the model is overidentified and the optimal weighting matrix can minimize the asymptotic variance of the estimator.

[22]For instance, in the case of the IMM: $H_0 : \chi^T = \chi^N = 0.0478$. For SMM we have $H_0 : \chi^T = \chi^N = 0.0187$.

government to issue more debt. Eventually, this triggers the default status which, in turn, provokes a severe decline in tradable sector productivity in relative terms. The magnitude of this decline is managed by χ^T. Default cost causes a direct negative effect over GDP through a lower productivity, but also an indirect effect through the labor market,[23] so a higher χ^T implies a stronger negative reaction of the correspondent sectoral GDP to a default event. An investigation of what triggers this reaction is beyond the scope of this report and, probably, require a richer model. A potential extension, of course, is to endogenize the default cost function. An interesting paper on this line is Mendoza and Yue (2012), where the default cost is associated with an efficiency loss since imported inputs are replaced by imperfect substitutes, because both firms and the government are excluded from credit markets.[24]

There are two important caveats to these results. The first one corresponds to the difficulty to find a global minimum since the quadratic objective function is not smooth, but has sparse local minimums (see figure 7). This is specially true in the case of the indirect inference approach. I address this issue by an exhaustive, computationally demanding, search of potential combinations of χ^T and χ^N, and by discarding values without economic intuition (my search interval starts at 0.005, not at zero). As a robustness check (not reported), I used two additional Matlab solvers (`fmincon` and `patternsearch`) and find no significant changes in the estimated parameters. The second caveat corresponds to the calibration of α^T and α^N. Na, Schmitt-Grohé, Uribe, and Yue (2018) sets a value of $\alpha^N = 0.75$ for the Argentinean case, however, it does not work in this model. A high labor share in the nontradable sector implies a stronger sensitivity of labor supply to the productivity losses caused by the default. As a consequence, nontradable GDP reacts severely, regardless of the value of χ^N. The algorithm takes this fact and sets an optimal value of χ^N close to zero. This is the reason for calibrating values for α^T and α^N from Arellano, Bai, and Mihalache (2018) which are taken from the emerging markets business cycle literature. However, a potential solution to this issue is the joint estimation of the default cost and sectoral labor share parameters.

5 Conclusions

Sovereign defaults are costly events and affect the economy asymmetrically. A simple analysis suggests the tradable sector is more affected by debt crises than the nontradable one. In this report, I tested this hypothesis by the estimation of some parameters via simulation based-methods. To do so, I built a small open economy model with two productive sectors and sovereign default. In the model, the default cost is interpreted as a productivity loss and is asymmetric between sectors, as a way of capturing the differential impact of debt crises. Parameters from the productivity loss functions were estimated by the simulated method of moments and the indirect inference approach.

Results showed the tradable sector is severely punished by a default episode regarding the nontradable one, with estimated parameters of productivity loss between 2.5 to 6 times higher in the former sector. Note, however, that the model is blinded to the mechanisms behind these results since productivity losses are exogenous. Results also pointed out the better performance of the model with heterogeneous default costs regarding the assumption of symmetry between sectors. Although the estimation algorithm performs well, especially in the case of the indirect inference method, some caveats are made in terms of the probability of finding local minimums and the joint estimation of a bigger set of parameters.

[23] Note that $w_t^T = \alpha_T \frac{z_t}{h_{T,t}^{1-\alpha_T}}$, where w_t^T are wages in tradable sector. If z_t falls stronger than $h_{T,t}$, which is the case usually, then w_t also falls and households face lower incentives to work.

[24] Moreover, this report does not say anything about the real exchange rate, even though two-sector models are usually employed to describe its behavior. Formally, it requires to solve the model via the decentralized economy. However, it is possible to infer its path during a sovereign crisis. In the model, default causes the government enters the autarky state. Given the values of $\hat{\chi}^T$ and $\hat{\chi}^N$, there is a relatively low supply of traded goods with respect to nontraded goods during default which, in turn, causes lower nontraded (relative) prices and, consequently, lower consumer prices too. Finally, the latter causes a real exchange rate depreciation. As it is described in Asonuma (2016), this is a pattern found during sovereign default crisis and, according to the empirical section of this report, is particularly severe in Argentina.

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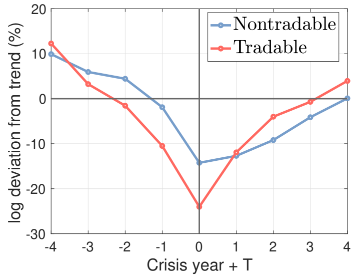
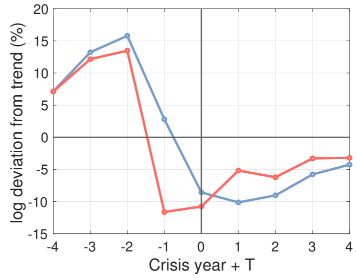
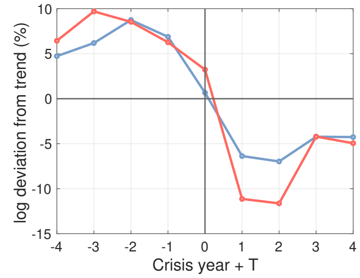
Figure 1: Sectoral level GDP during a sovereign default event



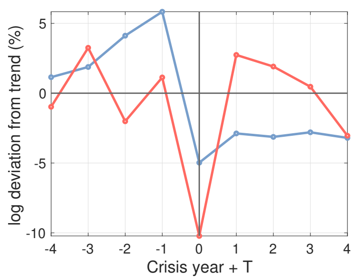
(a) Argentina, 2002



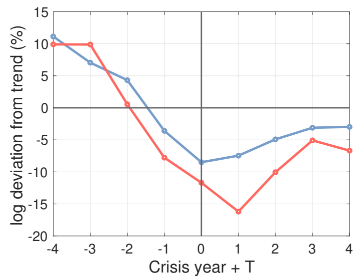
(b) Chile, 1983



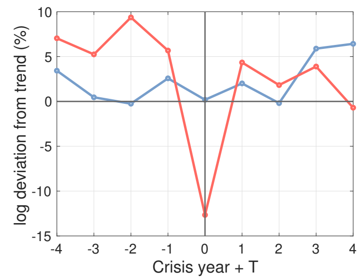
(c) Costa Rica, 1981



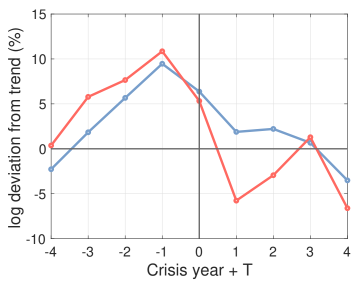
(d) Ecuador, 1999



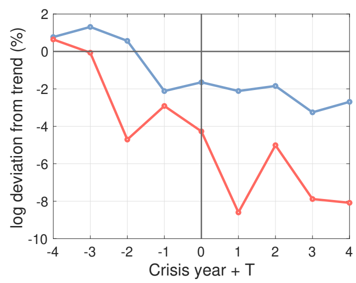
(e) Greece, 2012



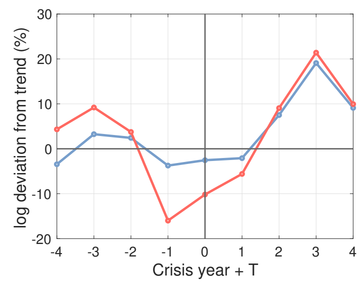
(f) Grenada, 2004



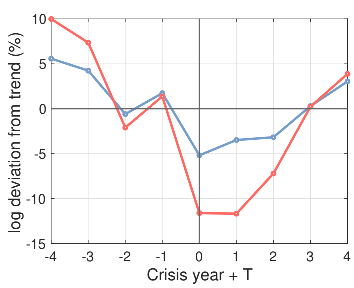
(g) Mexico, 1982



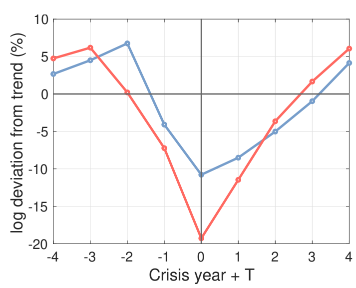
(h) Pakistan, 1999



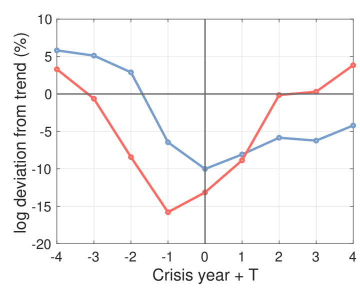
(i) Peru, 1984



(j) Türkiye, 2001



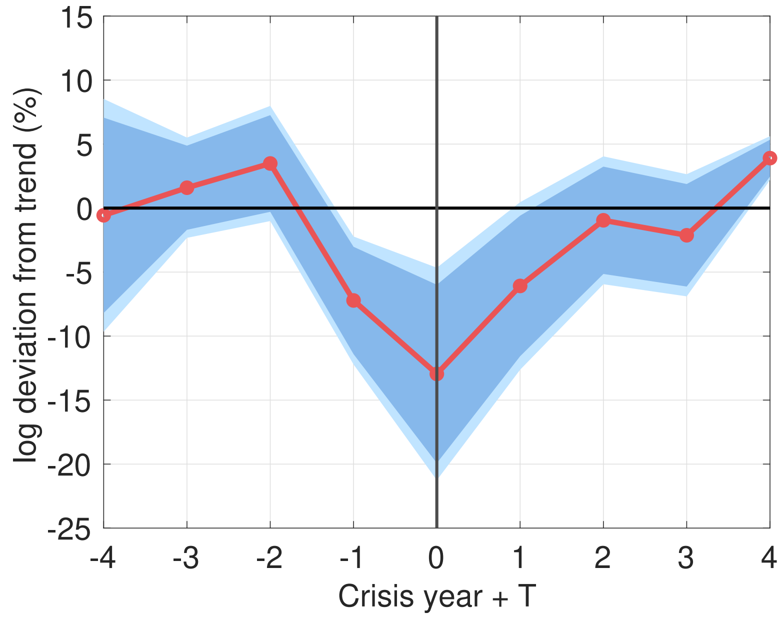
(k) Ukraine, 2015



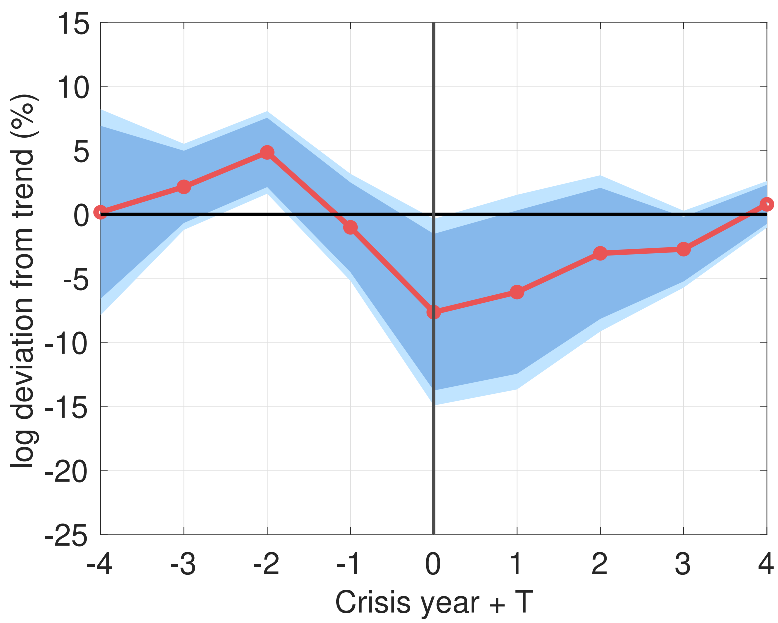
(l) Uruguay, 2003

***Note**: Red (blue) lines corresponds to Manufacturing (Services) value-added. This information is extracted from World Bank's World Development Indicators. Variables are in annual frequency and are log-detrended using the Hodrick-Prescott filter with a smoothing parameter of 100. Dates are obtained from Gourinchas and Obstfeld (2012).*

Figure 2: Tradable and nontradable GDP during sovereign default crises: Argentina



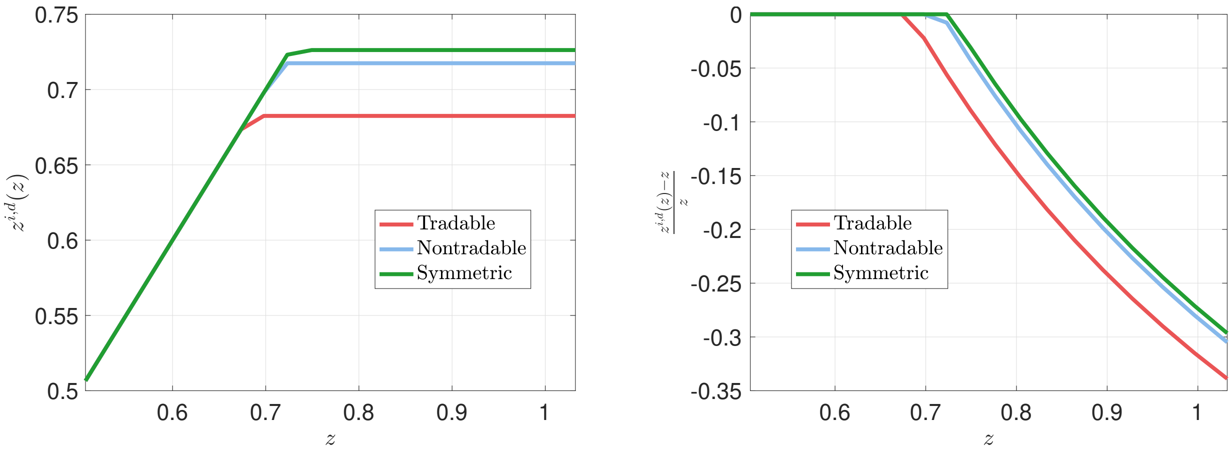
(a) Tradable GDP



(b) Nontradable GDP

Note 1: *Light (dark) blue areas are 95% (90%) confidence bounds based on robust heteroskedasticity standard errors.*
Note 2: *Red lines correspond to estimates of $\delta_{d,s} \times 100$ for an event window $-4 \leq s \leq 4$. It corresponds to the conditional effect of a sovereign default crisis on variable y over the event window relative to "tranquil times". See Gourinchas and Obstfeld (2012).*

Figure 3: Default cost and productivity losses: simulated method of moments

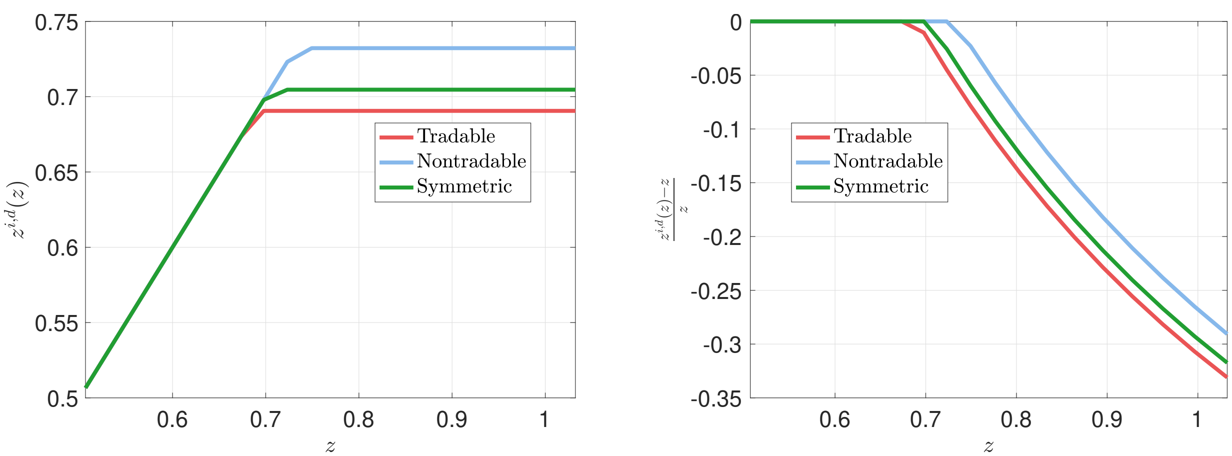


(a) Productivity loss function

(b) Productivity losses

Note: *These functions were constructed with values of* $\chi^T = 0.0776$ *and* $\chi^N = 0.0301$. *In the symmetric case* $\chi^T = \chi^N = 0.0187$

Figure 4: Default cost and productivity losses: indirect inference method

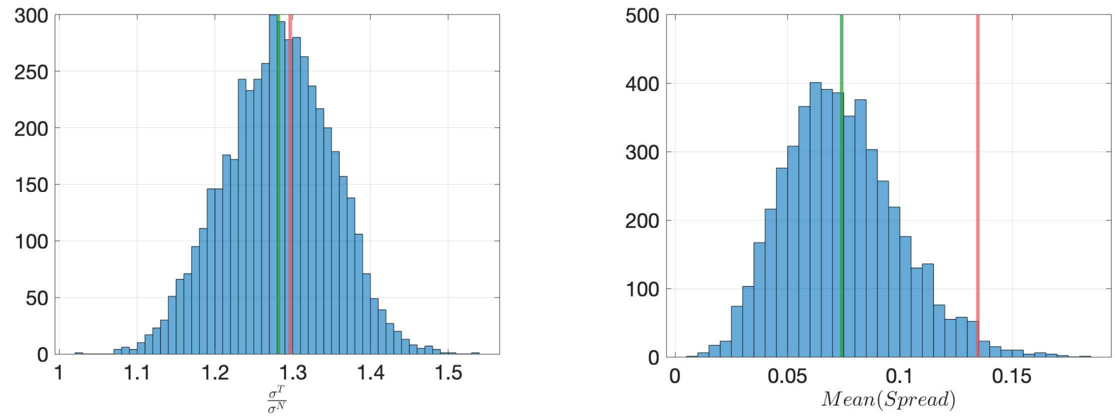


(a) Productivity loss function

(b) Productivity losses

Note: *These functions were constructed with values of* $\chi^T = 0.0669$ *and* $\chi^N = 0.0106$. *In the symmetric case* $\chi^T = \chi^N = 0.0478$

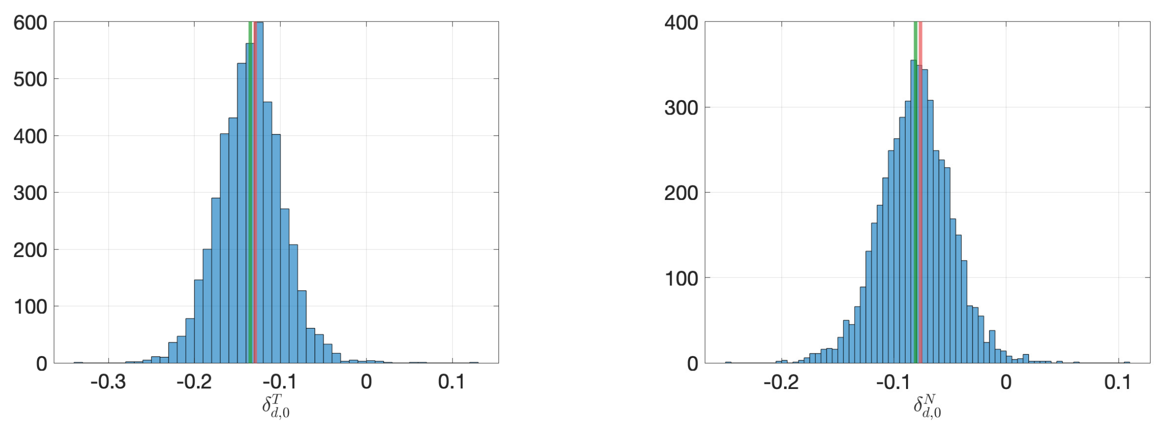
Figure 5: Distribution of simulated moments with optimal values from SMM



(a) Volatility of tradable relative to nontradable GDP (b) Average of spread

***Note**: The red (green) lines correspond to moments obtained from (simulated) observed data. Distributions were obtained after 5000 simulations of the model using the optimal values for χ^T and χ^N. SMM: simulated method of moments.*

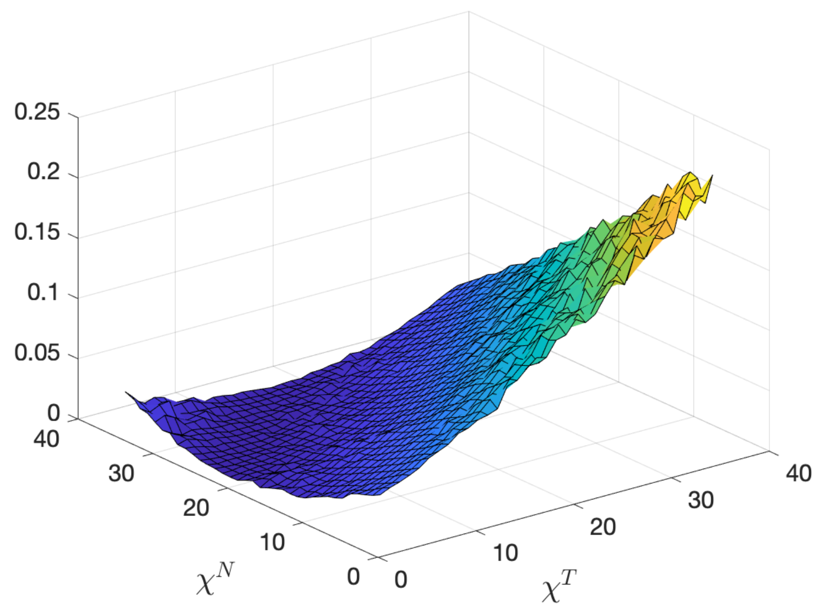
Figure 6: Distribution of "auxiliary" parameters with optimal values from IIM



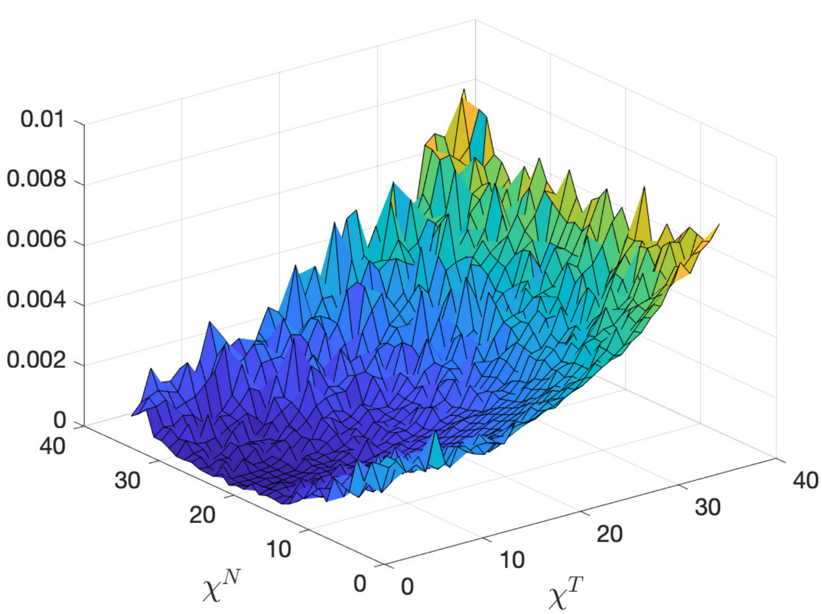
(a) Reaction of tradable GDP at $s = 0$ (b) Reaction of nontradable GDP at $s = 0$

***Note**: The red (green) lines correspond to auxiliary parameter estimates obtained from (simulated) observed data. Distributions were obtained after 5000 simulations of the model using the optimal values for χ^T and χ^N. IIM: indirect inference method.*

Figure 7: Criterion function for values of χ^T and χ^N



(a) Simulated methods of moments



(b) Indirect inference method

Table 1: Empirical regularities during a sovereign default crisis: Argentina

Variable (y)	Event window (s)								
	-4	-3	-2	-1	0	1	2	3	4
Tradable GDP	-0.56	1.59	3.49	-7.22**	-12.96**	-6.08*	-0.95	-2.12	3.90**
Nontradable GDP	0.15	2.14	4.83**	-1.03	-7.65**	-6.08	-3.06	-2.73	0.77
RER	6.85	-5.39	-15.82	-14.22	34.83**	7.34	-2.74	17.76	12.61
Consumption	-1.10	3.31*	5.25**	-2.40	-11.21**	-7.83**	-1.88	-2.22	2.24
Investment	3.43	0.90	11.11*	-2.16	-31.37**	-20.55**	-7.94*	-4.48	3.41
Net exports (% GDP)	0.42	-1.58	-3.37*	0.39	7.89**	4.54	1.89	1.12	-0.54
Total GDP	0.60	2.61	3.33**	-3.17**	-9.66**	-6.09*	-2.63	-2.62	1.94**
Productivity	0.16	0.44	1.13	-1.96**	-6.96*	-3.79	-1.79	-2.96*	-0.26

Note 1: *Superscripts $\star$ and $\star\star$ indicate significance at the 90% and 95% level, respectively.*
Note 2: *Values correspond to $\delta_{d,s} \times 100$ for an event window $-4 \leq s \leq 4$. They are estimates of the conditional effect of a sovereign default crisis on variable y over the event window relative to "tranquil times". See Gourinchas and Obstfeld (2012).*

Table 2: Parameter values

Description	Parameter	Value
Risk aversion	σ	2.0
Gross risk-free rate	R	1.04
NT share	θ	0.68
Elasticity of substitution	η	0.50
Labor share in NT	α_N	0.34
Labor share in T	α_T	0.43
Wage elasticity for T labor	ω_T	1.455
Wage elasticity for NT labor	ω_N	1.455
Discount factor	β	0.825
Probability of reentry	λ	0.25
AR(1) - productivity	ρ_z	0.906
Steady state - productivity	$\bar{z}$	0.723
Standard deviation - productivity	σ_z	0.05
Productivity range	$[\underline{z}, \bar{z}]$	$[0.5066, 1.0323]$
Debt range	$[\underline{b}, \bar{b}]$	$[0, 0.4]$

Note: *Calibration is in annual frequency. It is based on the literature about the Argentinean case, like Arellano (2008), Gelos, Sahay, and Sandleris (2011), Uribe and Schmitt-Grohé (2017), Gordon and Guerron-Quintana (2018), and Na, Schmitt-Grohé, Uribe, and Yue (2018).*

Table 3: Parameter estimates

SMM			IIM		
$\chi^T = \chi^N$	χ^T	χ^N	$\chi^T = \chi^N$	χ^T	χ^N
0.0187	0.0776	0.0301	0.0478	0.0669	0.0106
(0.0372)	(0.0024)	(0.0015)	(0.0032)	(0.0012)	(0.0094)
[0.0452]	[0.0030]		$[1.05e-5]$	$[4.94e-7]$	

Note 1: *This table shows the estimated parameters of the productivity loss functions in tradable and nontradable sector obtained by the simulated method of moments (SMM) and the indirect inference approach (IIM). Values in parenthesis are the standard errors. Numbers in brackets correspond to the minimized value of the objective criterion.*
Note 2: *When $\chi^T = \chi^N$ parameters are estimated under the assumption that default cost is symmetric between sectors.*

Table 4: Moments and parameter estimates from observed and simulated data

	SMM		IIM	
	$\frac{\sigma^T}{\sigma^N}$	$Mean(Spread)$	$\hat{\delta}^T_{d,0}$	$\hat{\delta}^N_{d,0}$
Target	1.2967	0.1349	-0.1296	-0.0765
Asymmetric	1.2817	0.0740	-0.1350	-0.0808
Symmetric	1.0878	0.0893	-0.1156	-0.1063

***Note**: This table shows the simulated moments for the case of the simulated methods of moments (SMM), and the simulated estimates for the case of the indirect inference method (IMM). They are obtained after 5000 simulations of the model using the optimal values for χ^T and χ^N found in each method.* Target *refers to moments and parameter estimates from the observed data.*