Advanced Kernel Methods for Multi-Task Learning

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Acknowledgements 1

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Outline

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- Machine Learning tries to automatize the learning process
- In the supervised setting we have:
  - An input space $\mathcal{X}$,
  - an output space $\mathcal{Y}$,
  - and the (unknown) probability $P(x, y)$ over $\mathcal{X} \times \mathcal{Y}$
- Given a function $f : \mathcal{X} \to \mathcal{Y}$, we define the loss function:

$$\begin{aligned} \ell : \mathcal{Y} \times \mathcal{Y} \quad &\to \quad [0, \infty) \\ (y, f(x)) \quad &\to \quad \ell(y, f(x)), \end{aligned}$$

such that $\ell(y, y) = 0$ for all $y \in \mathcal{Y}$

Loss Functions

- In classification, with the class labels $y_i \in \{-1, 1\}$, we can use:

$$\ell(y, f(x)) = [1 - yf(x)]_+ = \begin{cases} 0, & yf(x) \geq 1, \\ 1 - yf(x), & yf(x) < 1. \end{cases} \tag{1}$$

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Expected Risk

- Given a space of hypothesis $\mathcal{H} = \{h(\cdot, \alpha), \alpha \in A\}$
- Definition: Expected Risk

$$R_P(\alpha) = \int_{\mathcal{X}\times\mathcal{Y}} \ell(y, h(x, \alpha))dP(x, y)$$

- Our goal is to find

$$\alpha^* = \arg\min_{\alpha\in A} \left\{ R_P(\alpha) = \int_{\mathcal{X}\times\mathcal{Y}} \ell(y, h(x, \alpha))dP(x, y) \right\},$$

however the distribution $P(x, y)$ is unknown

Empirical Risk

- Instead, we have a set of n instances sampled from $P(x, y)$:

$$D_n = \{(x_i, y_i),\ i = 1, \ldots, n\},$$

- Definition: Empirical Risk

$$\hat{R}_{D_n}(\alpha) = \frac{1}{n}\sum_{i=1}^{n} \ell(y_i, h(x_i, \alpha))$$

- Instead of the Expected Risk, we minimize this empirical risk:

$$\arg\min_{\alpha \in A} \left\{ \hat{R}_D(\alpha) = \frac{1}{n}\sum_{i=1}^{n} \ell(y_i, h(x_i, \alpha)) \right\}$$

Multi-Task Learning

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Good Luck!

- Enough for an introduction! You should know enough by now

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Thank you for listening!

Any questions?