Reset Adaptive Observer for a Class of Nonlinear Systems

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Abstract

This paper proposes a novel kind of estimator called reset adaptive observer (ReO). A ReO is an adaptive observer consisting of an integrator and a reset law that resets the output of the integrator depending on a predefined condition. The main contribution of this paper is the generalization of the reset element theory to the nonlinear adaptive observer framework. The introduction of the reset element in the adaptive law can decrease the overshooting and settling time of the estimation process without sacrificing the rising time. The stability and convergence LMI-based analysis of the proposed ReO is also addressed. Additionally, an easily computable method to determine the $\mathcal{L}_2$ gain of the ReO dealing with noise-corrupted systems is presented. Simulation examples show the potential benefit of the proposed ReO.

Chapter 1

Introduction

An adaptive observer is a recursive algorithm for joint state and parameter estimation in dynamic systems. This kind of algorithm play a key role in many applications such as failure detection and recovery, monitoring and maintenance, fault forecasting and adaptive and fault tolerant control.

The research on adaptive observers started in the 1970s. Initially, the research was focused on linear time invariant systems [1], [2], and afterwards on nonlinear systems [3], [4], [5], [6]. All those works were characterized by having only a proportional feedback term of the output observation error in both state observer and parameter adaptation law. This proportional approach ensures a bounded estimation of the state and the unknown parameter, assuming a persistent excitation condition as well as the lack of disturbances. The performance of proportional adaptive observers was improved by adding an integral term to the adaptive laws, [7], [8], [9]. This additional term can increase the steady state accuracy and improve the robustness against modeling errors and disturbances.

However, since the adaptive laws are still linear, they have the inherent limitations of linear feedback control. Namely, they cannot decrease the settling time and the overshoot of the estimation process simultaneously. Therefore, a trade-off between both requirements is needed. Nevertheless, this limitation can be solved by adding a reset element. A reset element consists of an integrator and a reset law that resets the output of the integrator as long as the reset condition holds. Reset elements were introduced by Clegg in 1958 [10], who proposed an integrator which was reset to zero when its input is zero. In 1974, Horowitz generalized that initial work substituting the Clegg integrator by a more general structure called the first order reset element (FORE), [11]. During the last years, the research on the stability analysis and switching stabilization for reset systems is attracting the attention of many academics and engineers. The main difference between the state-of-art reset control works is how to address the stability analysis. Originally, stability analysis was only dependent on the reset law design rather than the reset intervals. A general analysis for such time independent reset control systems can be found in [12]. There, the authors modified the reset condition in such a manner that the system is reset when its input and output have different sign, rather than as long as its input is equal to zero. This is the main difference of [12], compared with other relevant time independent reset control works [13], [14]. Indeed, this approach addresses and solves the lack of robustness of the original formulation, which cannot be implemented in simulation packages (e.g. Simulink), since the integrator state is never reset due to the time discretization performed by the simulator.

Recently, some authors have proposed to include the reset intervals in the stability analysis. These proposals are based on checking the stability of the induced discrete time system, which is obtained representing the dynamics of the reset system between two consecutive after-reset instants. Therefore, they claim to guarantee the stability of the original reset control system checking only the stability of the induced discrete system, thus it is expected to obtain less conservative stability results. Relevant works related to this approach are [15], who addresses the stability at fixed reset time instants, and [16], who provides a method to determine the minimal reset time intervals which guarantee the stability of the induced discrete time system.

Although the research on reset elements is still an open and challenging topic, this research has been mainly focused on control issues. The first application of the reset elements to the adaptive observer framework is [17]. There, the authors proposed a new sort of adaptive observer called reset adaptive observer (ReO). A ReO is an adaptive observer whose integral term has been substituted for a reset element. The reset condition of the ReO is based on the approach proposed by [12], that is, its integral term is reset as long as the estimation error and the integrated estimation error have opposite sign. The introduction of the reset element in the adaptive laws can improve the performance of the observer, as it is possible to decrease the overshoot and settling time of the estimation process simultaneously.

This paper is an extended version of the work [17], which now considers nonlinear formulation as well as joint state and uncertain parameter estimation. In Section 1.1, the ReO formulation for a class of nonlinear systems is presented. In Section 1.2, a LMI-based stability condition which guarantees the convergence and stability of the estimation process is developed. Besides, an easily computable method to obtain the $\mathcal{L}_2$ gain of the ReO dealing with noise-corrupted system is presented. Simulation examples are presented in Section 1.3 in order to test the performance of our proposed ReO compared with traditional PIAO. Finally, concluding remarks are given in Section 1.4.

Notation: In the following, we use the notation $(x, y) = \left[x^T \; y^T\right]^T$. Given a state variable x of a hybrid system with switches, we will denote its time derivative with respect to the time by $\dot{x}$. Furthermore, we will denote the value of the state variable after the switch by x^+. Finally, we omit its time argument and we write $x(t)$ as x.

1.1 Reset Adaptive Observer Formulation

In this paper we address the problem of joint state and unknown parameter estimation for uncertain nonlinear systems which can be described by

$$\begin{aligned} \dot{x} &= Ax + Bu + \Delta\phi\theta + B_w w \\ y &= Cx \end{aligned} \tag{1.1}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^l$ is the input vector, $y \in \mathbb{R}^m$ is the output vector, $\theta \in \mathbb{R}^p$ is the unknown constant parameter vector which can be used to represent modeling uncertainties, $w \in \mathbb{R}^w$ is the disturbance vector, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times l}$, $C \in \mathbb{R}^{m\times n}$, $\Delta \in \mathbb{R}^{n\times m}$, and $B_w \in \mathbb{R}^{n\times w}$ are known constant matrices. The nonlinearity $\phi \in \mathbb{R}^{m\times p}$ is a time-varying matrix which depends on the input u and/or the output y. In addition, u and ϕ are assumed persistently exciting, and the pair (A, C) is assumed completely observable. As it is shown in [3], general nonlinear systems can be formulated as a system described by (1.1) through a change of coordinates. Here, we consider single-input single-output (SISO) systems, since a suitable formulation of reset elements for multiple-input multiple-output (MIMO) systems is still an open research topic.

The structure of our proposed ReO applied to a nonlinear system (1.1) is given in Fig. 1.1. The ReO dynamics are described as follows:

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + Bu + \Delta\phi\hat{\theta} + K_I\zeta + K_P\tilde{y} \\ \dot{\hat{\theta}} &= \Gamma\phi^T\tilde{y} \\ \hat{y} &= C\hat{x} \end{aligned} \tag{1.2}$$

where $\hat{x}$ is the estimated state, K_I and K_P represent the integral and proportional gain respectively, $\tilde{y} = C\tilde{x} = C(x - \hat{x})$ is the output estimation error, and $\Gamma \in \mathbb{R}^{p\times p}$ is a positive definite matrix. In addition, ζ is the reset integral term which can be computed as

$$\begin{aligned} \dot{\zeta} &= A_\zeta\zeta + B_\zeta\tilde{y} & \tilde{y}\cdot\zeta \geq 0 \\ \zeta^+ &= A_r\zeta & \tilde{y}\cdot\zeta \leq 0 \end{aligned} \tag{1.3}$$

where $A_\zeta \in \mathbb{R}$ and $B_\zeta \in \mathbb{R}$ are two tuning scalars which regulate the transient response of ζ, and A_r is the reset matrix. Specifically, we define $A_r = 0$, since the reset integral term ζ is reset to zero when $\tilde{y}\cdot\zeta \leq 0$.

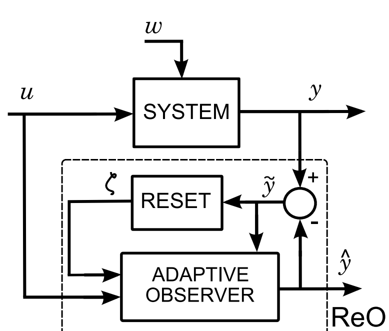

Figure 1.1: Reset adaptive observer applied to a dynamic system.

The reset observer can be regarded as a hybrid system with a *flow set* $\mathcal{F}$ and a *jump or reset set* $\mathcal{J}$. Regarding (1.3), the two conditions at the right side are the *flow* and the *jump* condition respectively. On one hand, as long as $(\tilde{y}, \zeta) \in \mathcal{F}$ the observer behaves as a proportional integral observer. On the other hand, if the pair $(\tilde{y}, \zeta)$ satisfies the *jump* condition, the integral term is reset according to the reset map A_r.

Thus, the observer flows whenever $\tilde{y}\cdot\zeta \geq 0$, that is, if $\tilde{y}$ and ζ have the same sign, whereas the observer jumps whenever $\tilde{y}\cdot\zeta \leq 0$, that is, if $\tilde{y}$ and ζ have different sign. According to this statement and since $\tilde{y} = C\tilde{x}$, the definition of both sets can be formalized by using the following representation:

$$
\begin{aligned}
\mathcal{F} &:= \left\{ (\tilde{x}, \zeta) : \begin{bmatrix} \tilde{x} \\ \zeta \end{bmatrix}^T M \begin{bmatrix} \tilde{x} \\ \zeta \end{bmatrix} \geq 0 \right\}, \\
\mathcal{J} &:= \left\{ (\tilde{x}, \zeta) : \begin{bmatrix} \tilde{x} \\ \zeta \end{bmatrix}^T M \begin{bmatrix} \tilde{x} \\ \zeta \end{bmatrix} \leq 0 \right\},
\end{aligned} \tag{1.4}
$$

where M is defined as

$$
M = \begin{bmatrix} 0 & C^T \\ C & 0 \end{bmatrix}. \tag{1.5}
$$

1.2 Stability and Convergence Analysis

In this section we analyze the stability and convergence of the ReO defined by (1.2) and (1.3) applied to nonlinear systems described by (1.1). Firstly, the different error dynamics which are involved in the estimation process are shown. Secondly, a computable method to determine the stability of the ReO assuming absence of disturbances is given. Thirdly, stability results are extended to noise-corrupted systems. For this reason, a method to compute the $\mathcal{L}_2$ gain minimizing the effect of the disturbances on the output estimation error is also provided. Finally, guidelines for tuning the parameters involved in our ReO are given.

1.2.1 Observer error dynamics

Let us begin analyzing the error system dynamics which can be obtained subtracting (1.2) from (1.1). Then, the state error dynamic $\tilde{x} = x - \hat{x}$ is defined by:

$$
\dot{\tilde{x}} \quad = \quad (A - K_P C)\tilde{x} + \Delta\phi\tilde{\theta} - K_I \zeta + B_w w \tag{1.6}
$$

while the parameter error dynamic $\tilde{\theta} = \theta - \hat{\theta}$ is described by:

$$
\dot{\tilde{\theta}} = \quad -\Gamma \phi^T \tilde{y} \tag{1.7}
$$

The state error dynamic can be augmented by connecting (1.6) to (1.3) as follows:

$$
\begin{array}{lll}
\dot{\eta} & = A_\eta \, \eta + B_\Delta \, \phi\tilde{\theta} + B_\eta \, w & \eta \in \mathcal{F} \\
\eta^+ & = A_R \, \eta & \eta \in \mathcal{J} \\
\xi & = C_\eta \, \eta &
\end{array} \tag{1.8}
$$

where $\eta = [\tilde{x} \;\; \zeta]^T$,

$$
\begin{aligned}
A_\eta = \begin{bmatrix} A - K_P C & -K_I \\ B_\zeta C & A_\zeta \end{bmatrix}, B_\Delta = \begin{bmatrix} \Delta \\ 0 \end{bmatrix}, B_\eta = \begin{bmatrix} B_w \\ 0 \end{bmatrix}, \\
C_\eta = \begin{bmatrix} C & 0 \end{bmatrix}, A_R = \begin{bmatrix} I & 0 \\ 0 & A_r \end{bmatrix}.
\end{aligned} \tag{1.9}
$$

Notice that the parameter error dynamic does not change after resets, that is, $\tilde{\theta}^+ = \tilde{\theta}$. Only the reset term ζ of the augmented state error dynamic η is modified through A_R after resets, since $\eta^+ = A_R\,\eta$. It is also worth pointing out that the output of the augmented error dynamic (1.8) is equal to the output of the ReO observer (1.2), that is, $\tilde{y} = C\tilde{x} = C_\eta\,\eta = \xi$.

Additionally, we assume in the following that the reset observer (1.2)-(1.3) holds the following assumptions:

Assumption 1. *The reset observer described by (1.2)-(1.3) is such that* $\eta \in \mathcal{J} \;\Rightarrow\; A_R\,\eta \in \mathcal{F}$.

This condition guarantees that after each reset, the solution will be mapped to the flow set $\mathcal{F}$ and, as a consequence, it is possible to flow after resets.

Assumption 2. *The reset observer described by (1.2)-(1.3) is such that the reset times* $t_{i+1} \;- t_i \geq\; \rho\; \forall i \in \mathbb{N}$, $\rho \in \mathbb{R} > 0$.

This assumption ensures that the reset observer uses time regularization to avoid Zeno solutions. It guarantees that the time interval between any two consecutive resets is not smaller than ρ which is a positive constant called the dwell time.

It is important to note that both assumptions are quite natural to assume for hybrid system [18], and consequently, these conditions are commonly used in most of current reset system formulations available in literature [12], [14].

1.2.2 State stability analysis

Here, we state a sufficient condition for the existence of a quadratically stable ReO assuming absence of disturbances, that is, $w = 0$. This analysis is based on a LMI approach.

Theorem 1. *For given* A_η, B_η, C_η, B_Δ *and* A_R *the augmented error dynamic shown in (1.8) and (1.7) with* $w = 0$ *is quadratically stable, if there exist a matrix* $P = P^T > 0$ *and scalars* $\tau_F \geq 0$ *and* $\tau_J \geq 0$ *subject to*

$$\begin{aligned} A_\eta^T P + P A_\eta + \tau_F M &< 0, \\ A_R^T P A_R - P - \tau_J M &\leq 0, \\ P B_\Delta &= C_\eta^T, \end{aligned} \tag{1.10}$$

which is a linear matrix inequality problem in the variables P, τ_F *and* τ_J.

Proof. Let us begin considering the following quadratic Lyapunov function for the error dynamics described by (1.8) and (1.7):

$$V(\eta, \tilde{\theta}) = \eta^T\, P\, \eta + \tilde{\theta}^T\, \Gamma^{-1}\, \tilde{\theta} \tag{1.11}$$

where $P = P^T > 0$ and $\Gamma = \Gamma^T > 0$.

To prove the quadratically stability of our proposed reset adaptive observer, we have to check that:

$$\begin{aligned} &\dot{V}(\eta, \tilde{\theta}) < 0 && \eta \in \mathcal{F} \\ &V(\eta^+, \tilde{\theta}^+) \leq V(\eta, \tilde{\theta}) && \eta \in \mathcal{J} \end{aligned} \tag{1.12}$$

Since $\mathcal{F} := \left\{\eta : \eta^T\, M\, \eta \geq 0\right\}$, employing the S-procedure [19], the first term of (1.12) is equivalent to the existence of $\tau_F \geq 0$ such that

$$\dot{V}(\eta, \tilde{\theta}) < -\eta^T \tau_F M \eta \tag{1.13}$$

Then, let us take derivative of (1.11) to obtain

$$\begin{aligned}
\dot{V}(\eta, \tilde{\theta}) &= \dot{\eta}^T P \eta + \eta^T P \dot{\eta} + \dot{\tilde{\theta}}^T\, \Gamma^{-1}\, \tilde{\theta} + \tilde{\theta}^T\, \Gamma^{-1}\, \dot{\tilde{\theta}} \\
&= \eta^T (A_\eta^T P + P A_\eta) \eta + \eta^T P B_\Delta \phi \tilde{\theta} + \tilde{\theta}^T \phi^T B_\Delta^T P \eta \\
&- \eta^T C_\eta^T \phi \tilde{\theta} - \tilde{\theta}^T \phi^T C_\eta \eta
\end{aligned} \tag{1.14}$$

Rearranging terms of equations (1.13) and (1.14), and by using $P B_\Delta = C_\eta^T$, the first term of (1.12) holds if the following inequality is satisfied

$$\eta^T (A_\eta^T P + P A_\eta) \eta + \eta^T \tau_F M \eta < 0, \tag{1.15}$$

which can be rearranged as an equivalent LMI problem in the variables $P > 0$ and $\tau_F \geq 0$

$$A_\eta^T P + P A_\eta + \tau_F M < 0, \tag{1.16}$$

which is the first term of (1.10) and consequently, proves the first equation of (1.12).

Similarly, employing again the S-procedure, the second term of (1.12) holds if there exits $\tau_J \geq 0$ such that

$$V(\eta^+, \tilde{\theta}^+) \leq V(\eta, \tilde{\theta}) + \eta^T \tau_J M \eta, \tag{1.17}$$

which is equivalent to

$$\eta^T A_R^T P A_R \eta - \eta^T P \eta - \eta^T \tau_J M \eta \leq 0. \tag{1.18}$$

Rearranging terms, (1.17) can be also rewritten as an equivalent LMI problem in the variables $P > 0$ and $\tau_J \geq 0$ as follows

$$A_R^T P A_R - P - \tau_J M \leq 0, \tag{1.19}$$

which is the second term of (1.10) and proves the second equation of (1.12) and, as a consequence, completes the proof of the theorem. □

1.2.3 Input-output stability analysis

Now, we present our results on the input-output properties of the ReO. The aim is to develop a ReO such as the effect of the disturbance w on the output estimation error ξ is minimized. For this reason, let us define the $\mathcal{L}_2$ or Root Mean Square (RMS) gain of the system (1.8) as the following quantity

$$\mathcal{L}_2 = \sup_{\|w\|_2 \neq 0} \frac{\|\xi\|_2}{\|w\|_2} \tag{1.20}$$

where the $\mathcal{L}_2$ norm $\|u\|_2^2$ of a signal u is defined as follows

$$\|u\|_2^2 = \int_0^\infty u^T u \, dt \tag{1.21}$$

and sup means *supremum* which is taken over all non-zero trajectories of (1.8).

Additionally, we present the following lemma that will be used in the sequel [19].

Lemma 1. *The $\mathcal{L}_2$ gain of a LTI system with an input signal u and an output signal y is less than γ, if there exists a quadratic function $V(x) = x^T Q x$, $Q = Q^T > 0$ and $\gamma > 0$ such that*

$$\dot{V}(x) < \gamma^2 u^T u - y^T y \tag{1.22}$$

Now, we apply this lemma to the augmented error dynamics (1.8) and (1.7) to obtain the following theorem.

Theorem 2. *For given A_η, B_η, C_η, B_Δ and A_R the augmented error dynamic shown in (1.8) and (1.7) is quadratically stable and have a $\mathcal{L}_2$ gain from w to ξ which is smaller than γ, if there exist a matrix $P = P^T > 0$ and scalars $\tau_F \geq 0$, $\tau_J \geq 0$ and $\gamma > 0$ subject to*

$$\begin{aligned} \begin{bmatrix} A_\eta^T P + P A_\eta + C_\eta^T C_\eta + \tau_F M & P B_\eta \\ B_\eta^T P & -\gamma^2 I \end{bmatrix} &< 0, \\ A_R^T P A_R - P - \tau_J M &\leq 0, \\ P B_\Delta &= C_\eta^T, \end{aligned} \tag{1.23}$$

which is a linear matrix inequality problem in the variables P, τ_F, τ_J and γ.

Proof. To prove the stability of our proposed reset adaptive observer and that the $\mathcal{L}_2$ gain from w to ξ is smaller than γ, we have to check that:

$$\begin{array}{ll} \dot{V}(\eta, \tilde{\theta}) < \gamma^2 w^T w - \xi^T \xi & \eta \in \mathcal{F} \\ V(\eta^+, \tilde{\theta}^+) \leq V(\eta, \tilde{\theta}) & \eta \in \mathcal{J} \end{array} \tag{1.24}$$

The first equation of (1.24) relays on (1.22) and the second equation of (1.24) is equal to the second equation of (1.12) which has been already proved. Then, let us concentrate on the first equation of (1.24). Again, since $\mathcal{F} := \left\{\eta : \eta^T M \eta \geq 0\right\}$ and employing the S-procedure, the first term of (1.24) is equivalent to the existence of $\tau_F \geq 0$ such that

$$\dot{V}(\eta, \tilde{\theta}) < \gamma^2 w^T w - \xi^T \xi - \eta^T \tau_F M \eta \tag{1.25}$$

In this case, the time derivative of (1.12) is

$$\begin{aligned} \dot{V}(\eta, \tilde{\theta}) &= \dot{\eta}^T P \eta + \eta^T P \dot{\eta} + \dot{\tilde{\theta}}^T \Gamma^{-1} \tilde{\theta} + \tilde{\theta}^T \Gamma^{-1} \dot{\tilde{\theta}} \\ &= \eta^T (A_\eta^T P + P A_\eta) \eta + w^T B_\eta^T P \eta + \eta^T P B_\eta w \\ &\quad + \eta^T P B_\Delta \phi \tilde{\theta} + \tilde{\theta}^T \phi^T B_\Delta^T P \eta - \eta^T C_\eta^T \phi \tilde{\theta} - \tilde{\theta}^T \phi^T C_\eta \eta \end{aligned} \tag{1.26}$$

Rearranging terms of equations (1.25) and (1.26), and by using $P B_\Delta = C_\eta^T$, the first term of (1.24) holds if the following inequality is satisfied

$$\eta^T (A_\eta^T P + P A_\eta) \eta + w^T B_\eta^T P \eta + \eta^T P B_\eta w + \xi^T \xi + \eta^T \tau_F M \eta - \gamma^2 w^T w < 0 \tag{1.27}$$

Since $\xi^T\xi = \eta^T C_\eta^T C_\eta \eta$, (1.27) can also be rearranged as an equivalent LMI problem in the variables $P > 0$ and $\tau_F \geq 0$ as follows

$$\begin{bmatrix} A_\eta^T P + P A_\eta + C_\eta^T C_\eta + \tau_F M & P B_\eta \\ B_\eta^T P & -\gamma^2 I \end{bmatrix} < 0, \tag{1.28}$$

which is the first inequality of (1.23) and proves the first equation of (1.24) and, as a consequence, completes the proof of the theorem. □

1.2.4 Reset observer gains. Tuning and design

According to (1.2) and (1.3), there are several tuning gains which play a key role in the performance of our proposed reset adaptive observer. For this reason, guidelines about how to tune these parameters are given. Specifically, there are five tuning gains: the proportional gain K_P and the integral gain K_I, which modify the convergence speed of the state estimation error, the parameter gain Γ, which balances the convergence speed of the parameter estimation error, and the reset term gains A_ζ and B_ζ, which regulate the transient response of the reset term. Although there are available results of optimal gain design for PIAO [8], these results cannot directly be used for reset adaptive observers. Therefore, before tuning the algorithm on a real system, it is strongly recommended to first carry out some numerical simulations, and tune the observer gains following the next guidelines.

Firstly, the gains of the reset term A_ζ and B_ζ have to be chosen. Analyzing (1.3), it is evident that the reset term ζ can be regarded as a low-pass filter whose cutoff frequency relays on A_ζ and whose gain depends on B_ζ. Typically, it is selected $B_\zeta = 1$, because the effect of the integral term can be increased by tuning the integral gain K_I, therefore the transient response of the integral term only relays on A_ζ. To guarantee a proper integration of the error dynamic, A_ζ should be chosen to be Hurwitch with $|A_\zeta|$ at least 10 times lower than the minimum absolute value of the eigenvalues of A.

The second step is to find suitable K_P and K_I in such a manner that the response of the state estimation error is fast enough but without overshooting. Since the pair (A_η, C_η) is constant, it can be done solving the appropriate Riccati equation or by using any pole placement method. Once both gains have been computed, it is time to exploit the potential benefit of the reset element. The aim is to increase the integral gain in order to obtain a quicker and oscillating response due to the fact that most of the overshoots will be removed by resetting the integral term. Consequently, we will obtain a state estimation error response quicker than before but without overshooting. This fact underlines the benefit of the reset adaptive observers, which are mainly nonlinear and, as a consequence, it can achieve some specifications that cannot be achieved by pure linear observers.

Once the state estimation error has the desired dynamic, we can focus on designing an appropriate parameter gain Γ. Typically, Γ is chosen to be a positive diagonal matrix in such a manner that the convergence speeds of each estimated parameter can be tuned separately. Although some authors have proposed to use time varying $\Gamma(t)$ matrix [20], [21], we consider only constant parameter gain Γ due to the fact that a constant Γ highly simplifies the Lyapunov stability analysis.

1.3 Simulation Results

In this section, two examples are presented in order to show the effectiveness of our proposed ReO. In the first example, the performance of the ReO applied to an uncertain high-order nonlinear plant is shown. It can achieve a zero steady-state estimation error for all the state variables as well as for the uncertain parameter. After that, the results obtained by the ReO are compared with a PIAO with the same tuning parameters than the ReO, which is denoted by Std-PIAO, and with an optimal PIAO designed according to [8], which is denoted by J-PIAO. On the one hand, comparing the ReO with the Std-PIAO we show the potential benefit of including a reset element, which can be used to minimize the rise time without overshooting. On the other hand, comparing the ReO with the J-PIAO we show the real effectiveness of our proposal, due to the fact that we are comparing two adaptive observers that have been tuned to maximize their performances. In the second example, to highlight the performance of the ReO, we have applied it to the example proposed in [8], and we have compared the results obtained by both observers. Since in that example the system is only affected by unknown disturbances, it represents a challenging benchmark for the ReO. Finally, notice that all these simulation results have been obtained by using Matlab-Simulink with the ode45 solver.

1.3.1 Example 1

Let us begin considering the following third-order noise-corrupted nonlinear system:

$$\begin{aligned} \dot{x}_1 &= -2x_1 - x_2 - 2x_3 + u + 0.2w \\ \dot{x}_2 &= -x_2 - 2x_3 - 0.5u + 0.2w \\ \dot{x}_3 &= x_2 - x_3 + 0.2y^3\theta + 0.5u + 0.2w \\ y &= x_3 \end{aligned} \tag{1.29}$$

with $x(t=0) = [1.5; 0.5; 1]^T$, $u(t) = sin(6t)$, $w(t) = sin(23t)$ and an uncertain parameter $\theta = 1$. The aim is to develop an adaptive observer for the system described by (1.29) which estimates all the state variables as well as the uncertain parameter without overshooting as fast as possible. According to (1.1), the nonlinear system (1.29) has the following parameters:

$$A = \begin{bmatrix} -2 & -1 & -2 \\ 0 & -1 & -2 \\ 0 & 1 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ -0.5 \\ 0.5 \end{bmatrix}, \Delta = \begin{bmatrix} 0 \\ 0 \\ 0.2 \end{bmatrix}, \phi = y^3,$$

$$B_w = \begin{bmatrix} 0.2 \\ 0.2 \\ 0.2 \end{bmatrix}, C = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}^T.$$

Following the tuning guidelines given previously, we have designed a ReO for the nonlinear system (1.29) with the following parameters: $\hat{x}(t=0) = [0; 0; 0]^T$, $\zeta(t=0) = 0$, $A_\zeta = -0.1$, $B_\zeta = 1$, $K_P = [70; 20; 50]^T$, $K_I = [600; 200; 400]^T$, $A_r = 0$, and $\Gamma = 65$.

Additionally, we can compute the bound of the $\mathcal{L}_2$ gain of our ReO minimizing the value of γ^2 according to Theorem 2. Specifically, our ReO obtains a

$\mathcal{L}_2 = 0.1585$ with the following matrices:

$$P = \begin{bmatrix} 0.0001 & 0 & 0 & 0 \\ 0 & 0.023 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 2475 \end{bmatrix}, \tau_F = 0.5055, \tau_J = 0.2873.$$

Here, we compare the previous results obtained by the ReO with two different PIAO observers denoted by Std-PIAO and J-PIAO. The former is a PIAO with the same tuning parameters than the ReO and the latter is an optimal PIAO designed according to the tuning rules proposed by [8]. Generally, PIAO for nonlinear systems are described by:

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + Bu + K_I z + K_P \tilde{y} \\ \hat{y} &= C\hat{x} \\ \dot{z} &= A_z z + B_z \tilde{y} \end{aligned} \tag{1.30}$$

where $A_z \in \mathbb{R}$ and $B_z \in \mathbb{R}$ are two tuning scalars which regulate the transient response of the integral term z.

Specifically, the parameters of the Std-PIAO are $\hat{x}(t=0) = [0;0;0]^T$, $z(t=0) = 0$, $A_z = -0.1$, $B_z = 1$, $K_P = [70;20;50]^T$, $K_I = [600;200;400]^T$, and $\Gamma = 65$, whereas the parameters of the J-PIAO are obtained by solving the minimization problem that appears in [8]. Specifically, the parameter of the J-PIAO are: $\hat{x}(t=0) = [0;0;0]^T$, $z(t=0) = 0$, $A_z = -0.1$, $B_z = 1$, $K_P = [400;856;200]^T$, $K_I = [-0.0007;0.0006;1.14]^T$, and $\Gamma = 525$. The proportional gain K_P and the integral gain K_I are calculated as $K_P = P^{-1}S^T$ and $K_I = P^{-1}Q$ where the matrices P, Q and S are the solution of the minimization problem. Namely, the optimal solutions are:

$$P = \begin{bmatrix} 0.008 & 0 & 0 \\ 0 & 0.011 & 0 \\ 0 & 0 & 5 \end{bmatrix}, Q = \begin{bmatrix} -5.6 \cdot 10^{-6} \\ 6.6 \cdot 10^{-6} \\ 5.7 \end{bmatrix}, S = \begin{bmatrix} 3.21 & 9.38 & 1000 \end{bmatrix}.$$

The state estimation error $\tilde{x}(t) = x(t) - \hat{x}(t) = [\tilde{x}_1(t); \tilde{x}_2(t); \tilde{x}_3(t)]^T$ of all adaptive observers is shown in Fig. 1.2. It is evident that the ReO has a much better performance compared with the Std-PIAO, since it has a response as quick as Std-PIAO but without overshooting. The integral gain is too high for the Std-PIAO and, as consequence, it causes an oscillating estimation process. If we decrease the integral gain of the Std-PIAO to avoid overshooting it will give a slower response. However, the overshoots associated with the high integral gain are almost removed reseting the integral term of the ReO. That result underlines the potential benefit of the reset element, due to the fact that we can decrease the settling time as long as we increase the integral gain, while we can remove the overshoots resetting the integral term. On the other hand, comparing the results of the ReO with the J-PIAO, it is easy to see that both observers achieve a fast estimation of the measured variable x_3. Nevertheless, there are significant differences in how the observers estimate the non-accessible variables x_1, x_2. As before, the ReO exploits the reset element properties to estimate x_1, x_2 as fast as the J-PIAO but without overshooting.

Finally, we also show in Fig. 1.2 the parameter estimation error of each observer. Fig. 1.2 points out that the three observers could achieve zero steady-state error once they have been properly tuned. Notice that the Std-PIAO

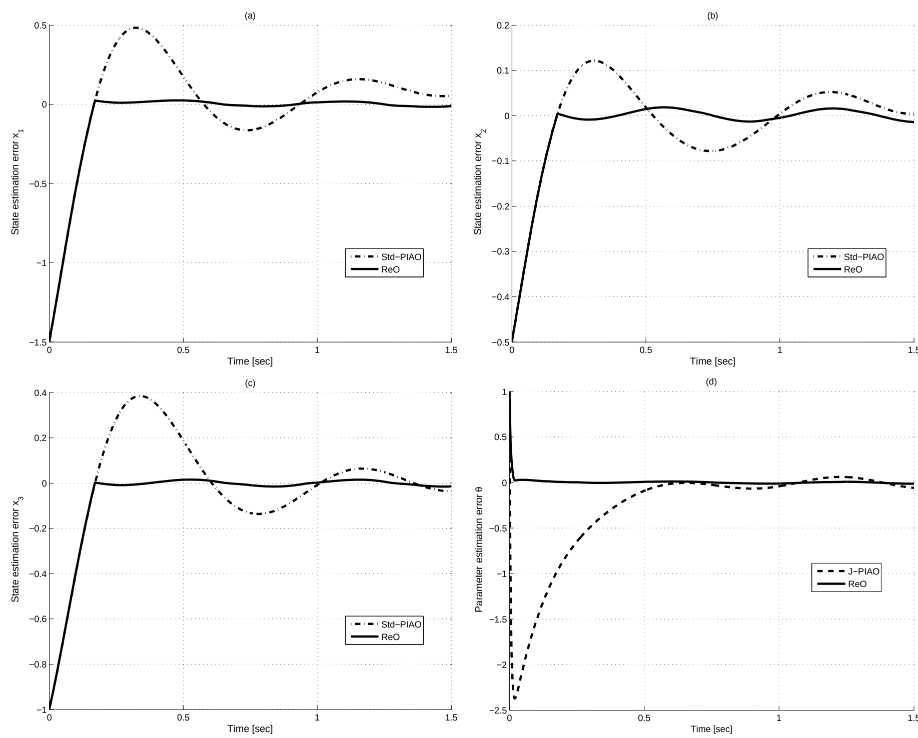

Figure 1.2: Estimation results of Example 1. (a), (b), and (c) show the state estimation error $\tilde{x}_1$, $\tilde{x}_2$, and $\tilde{x}_3$ respectively. (d) shows the parameter estimation error $\tilde{\theta}$. Dashed lines have been obtained by using the J-PIAO. Dash-dot lines have been obtained by using the Std-PIAO. Solid lines have been obtained by using the ReO.

has a poor performance because of it has the same parameter gain Γ than the ReO. This result confirms the fact that traditional PIAOs require different adjustments than our proposed ReO to achieve a reasonable performance.

1.3.2 Example 2

This example is taken from [8], and it represents a useful example to check the behavior of our proposed ReO dealing with systems affected only by unknown disturbances. For this reason, let us define the scalar system

$$\begin{aligned} \dot{x}_1(t) &= -x_2(t) + y^3(t)\theta + 0.1w(t) \\ \dot{x}_2(t) &= -x_1(t) - 2x_2(t) + 0.1w(t) \\ y(t) &= x_1(t) \end{aligned} \tag{1.31}$$

with $x(t=0) = [-0.5, 0.2]$, $\theta = 1.1$, $u(t) = 0$ and $w(t) = sin(0.5t)$.

In this case, the optimal parameters of the J-PIAO are set as in [8] too. Specifically, the tuning parameters of the J-PIAO are $\hat{x}(t=0) = [0; 0]^T$, $z(t = 0) = 0$, $A_z = -10$, $B_z = 1$, $K_P = [3.18; 5]^T$, $K_I = [0.93; 0]^T$, and $\Gamma = 50$.

On the other hand, the tuning parameter of the ReO have been obtained following the tuning guidelines given previously. Specifically, we have designed a ReO for the nonlinear system (1.31) with the following parameters: $\hat{x}(t = 0) = [0; 0]^T$, $\zeta(t = 0) = 0$, $A_\zeta = -0.1$, $B_\zeta = 1$, $K_P = [3.18; 5]^T$, $K_I = [25; 15]^T$, $A_r = 0$, and $\Gamma = 50$.

Fig. 1.3 shows the true state variables and the respective estimates of both adaptive observers. It is evident that our proposed ReO achieves a better performance than the J-PIAO as long as we compare the estimation process of both observers, due to the fact that the ReO has a lower steady state error as well as a faster estimation process. The true uncertain parameter and its parameter estimate of both adaptive observers are shown in Fig. 1.3. In this case, the behavior of both observers is similar. The two observers are not able to estimate the unknown parameter because the system (1.31) is not persistently excited since $u(t) = 0$.

1.4 Conclusion

This paper addresses the application of the reset adaptive observers to the nonlinear framework. The proposed algorithm can jointly estimate the unknown states and the uncertain parameters of a dynamic system. The stability and convergence analysis of this novel proposal has been proved by using quadratic Lyapunov functions. Moreover, a method to determine the $\mathcal{L}_2$ gain of the proposed reset adaptive observer has also been developed. This method is based on a linear matrix approach which is easily computable. Additionally, tuning guidelines have been given to facilitate the design process.

Simulation results have been given to highlight the potential benefit of including a reset element in the adaptive laws. Since the reset adaptive observer is mainly nonlinear it can meet requirements that cannot be satisfied by pure linear observers. Namely, the reset element can decrease the overshoot and settling time of the estimation process without sacrificing the rise time.

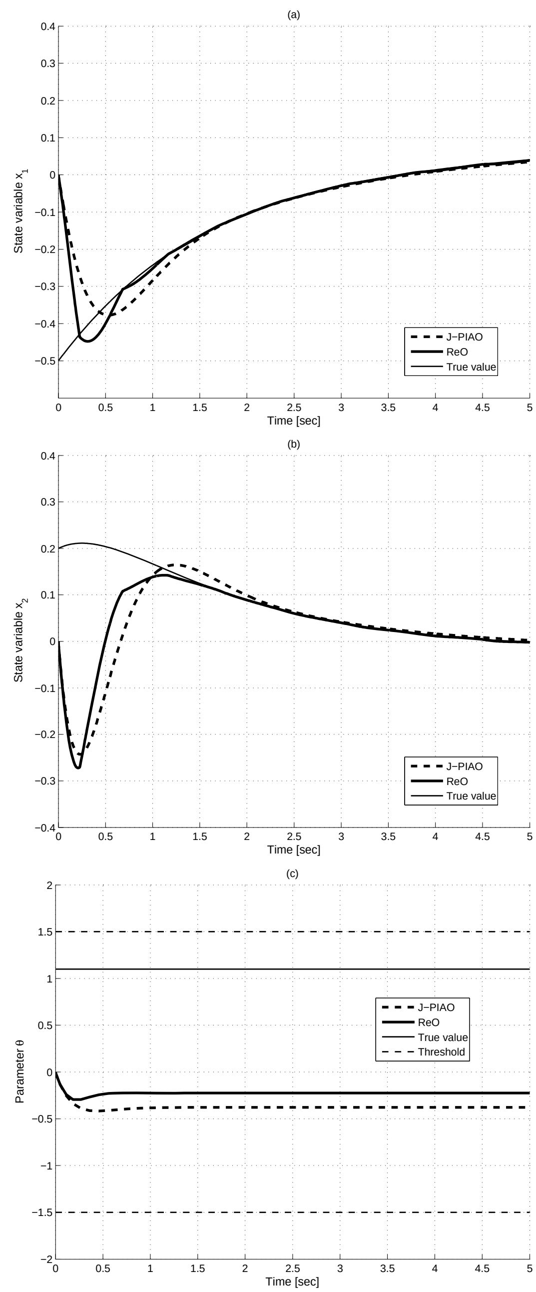

Figure 1.3: Estimation results of Example 2. (a) and (b) show the state estimation error $\tilde{x}_1$ and $\tilde{x}_2$ respectively. (c) shows the parameter estimation error $\tilde{\theta}$. Dashed lines represent the estimate obtained by using the J-PIAO. Thick solid lines represent the estimate obtained by using the ReO. Thin solid lines represent the true values of the states and parameter.

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