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Structuring bilateral energy contract portfolios in competitive markets

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Summary. A multistage complete recourse model for structuring energy contract portfolios in competitive markets is presented for price taker operators. The main uncertain parameters are spot price, exogenous water inflow to the hydro system and fuel-oil and gas cost. A mean-risk objective function is considered as a composite function of the expected trading profit and the weighted probability of reaching a given profit target. The expected profit is given by the bilateral contract profit and the spot market trading profit along the time horizon over the scenarios. The uncertainty is represented by a set of scenarios. The problem is formulated as a mixed 0-1 Deterministic Equivalent Model. Only 0-1 variables have nonzero coefficients in the first-stage constraint system, such that the continuous variables only show up in the formulation of the later stages. A problem solving approach based on a splitting variable mathematical representation of the scenario clusters is considered. The approach uses the Twin Node Family concept within the algorithmic framework presented in the chapter. The Kyoto protocol-based regulations for the pollutant emission are considered.

Key words: energy trading contracts portfolio, stochastic programming, mean-risk, mixed 0-1 models, splitting variable, branch-and-fix coordination.

1.1 Introduction

Given a power generation system and a set of committed as well as candidate energy trading contracts, the *Energy Contract Portfolio Problem* (*ECP2*) is concerned with selecting the bilateral contracts for energy purchasing and selling to be delivered along a given time horizon. It is one of the main problems faced today by the power generation companies and the energy service providers. The main uncertain parameters are spot market price, exogenous water inflow to the hydro system, and fuel-oil and gas cost. The maximization of the weighted *Reaching Probability (RP)* is included as a risk measure in the objective function, together with the expected energy trading profit along the time horizon over the scenarios, and the economic value of satisfying the Kyoto protocol-based regulations for the pollutant emissions by the thermal generators. The constraint system comprises the power generation constraints for the thermal and hydro generators, the logistic constraints related to the energy contracts and the power load already committed to purchasing and selling. See [TLL02] for different types of financial hedging contracts. See also [FWZ01] for an approach to hedge electricity portfolios via stochastic optimization. An approach in [PPN05] uses dynamic programming for problem solving. [SPLF05] presents a medium term hydropower planning with bilateral contracts. [SYG06] integrates the unit commitment model with financial decision making and spot market considerations in a non hydro-power generation environment. [FK07] presents a model for determining optimal bidding strategies taking uncertainty into account by a price take hydropower producer.

Nowadays, most of the traded energy (around 75 per cent) is done via bilateral contracts (e.g., the daily market APX in the Netherlands has only between 10 and 15 percent of the total energy of the country, the daily market EEX in Germany sells something less than 15 percent of the total energy, etc.). Moreover, there are some other countries (Spain is a good example, see [Per05]), with a very small bilateral contracts development due, mainly, to disruption of market regulations.

The deterministic version of the problem can be represented as a mixed 0-1 model. Given today's state-of-the-art optimization tools, no major difficulties should arise for problem solving of moderate size instances, at least. However, given the uncertainty of the main parameters, the modelling and algorithmic approach to the problem makes *ECP2* an interesting application case of Stochastic Integer Programming.

The aim of the model that we present in the paper consists of helping a better decision making on the energy selling / purchasing (so-called first stage) policy. For this purpose the future uncertainties in the spot market and generation availability are considered. (Since the model assumes price taking decision maker, no restrictions are considered for satisfying the uncertain demand).

Some of the stochastic approaches related to power generation only consider continuous variables. Moreover, there are different schemes to address the energy power generation planning under uncertainty, by modelling it with 0-1 and continuous variables, see [CCCR96, CS98, CS99, DGMRS97, DR98, GMNRS99, GKNRW01, HS01, KV01, NLR04, NR02, SNW05, TB00, TBG05], among others. Most of these approaches only consider mean (expected) objective functions and Lagrangian Decomposition schemes, some of which propose bidding strategies for 24 hour horizons.

Alternatively, very few approaches in general deal with the mean-risk measures by considering semi-deviations [OR99], excess probabilities [AEP09, ST04], Value-at-Risk (VaR) and conditional VaR [RU00, ST06], see also [Der98, LS04, Sch03, Val02]. These approaches are more amenable for large-scale problem solving than the classical mean-variance schemes, particularly, in the presence of 0-1 variables.

In this chapter we present a mixed 0-1 *Deterministic Equivalent Model* (*DEM*) for the multistage stochastic *ECP2* with complete recourse, where the parameters' uncertainty is represented by a set of scenarios. Only 0-1 variables (that represent strategic trading decisions) have nonzero coefficients in the first-stage constraint system, such that the continuous variables (that represent operational decisions) only show up in the formulation of the later stages. The values of the variables do consider all scenarios without being subordinated to any of them. The *Reaching Probability (RP)* is considered as a risk measure in contrast with approaches where only mean functions appear in the objective function to optimize.

The model has not direct relationship with the enterprise-wide risk management function, or the bidding to each submarket function. However, the max mean-risk function to consider assumes this interrelation with the enterprise-wide risk management function. On the other hand, the elements of the spot market in the model assume the interrelation with the bidding function in an aggregated way. After structuring the portfolio, the model can be run again, in a rolling horizon approach, once new opportunities for energy selling / purchasing, restructuring and cancellation appear in the horizon.

One of the main contributions of the paper consists of considering a coupling of a generic electricity generation planning problem and the contract management problem, since in our opinion the structuring of the bilateral contract portfolio must take into account the spot market availability and the energy generation restrictions albeit in an aggregate way.

We present a *Branch-and-Fix Coordination (BFC)* scheme to exploit the structure of the *splitting variable* representation of the mixed 0-1 *DEM* and, specifically, the structure of the *nonanticipativity* constraints for the 0-1 variables. The algorithm makes use of the *Twin Node Family (TNF)* concept introduced in [AEO03]. It is specially designed for coordinating and reinforcing the branching node and the branching variable selections at each *Branch-and-Fix (BF)* tree. The trees result from the relaxation of the *nonanticipativity* constraints in the *splitting variable* representation of the *DEM* for the scenario *clusters*.

Additionally, the proposed approach considers the *DEM* at each *TNF integer set*, i.e., each set of integer nodes, one for each *BF* tree, where the *nonanticipativity* constraints for the 0-1 variables are satisfied. By fixing those variables to the nodes' values, the *DEM* has only continuous variables. The approach ensures the satisfaction of the *nonanticipativity* constraints for the continuous variables associated with each scenario group in the scenario tree and so, provides the *LP* optimal solution for the given *TNF integer set*.

The remainder of the chapter is organized as follows. Section 2 states the *ECP2* and introduces the mixed 0-1 *DEM* for the multistage stochastic model, compact representation. Section 3 introduces the mean-risk function and the splitting variable representation of the *DEM*. Section 4 presents our approach for problem solving. Section 5 concludes. The Appendix gives more algorithmic details of the approach presented in this chapter.

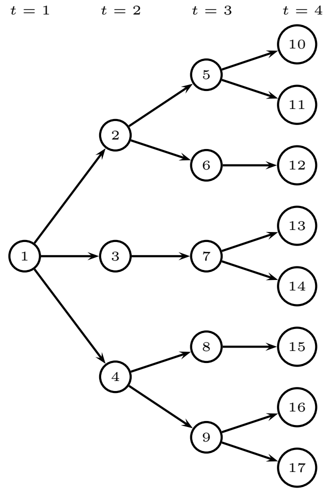
1.2 Problem description

1.2.1 Problem statement

Consider a set of thermal-power generators, and a set of hydro-power generators distributed along river basins whose reservoirs can store water from one period to the next one in a given time horizon. Consider also a set of scenarios for the uncertain parameters, mainly, spot market price, exogenous water inflow to the hydro system and fuel-oil and gas cost. Finally, let a set of candidate energy bilateral trading (purchasing/selling) contracts be considered. Each contract is characterized by the energy price depending on the scenario to occur and the lower and upper bounds of the purchasing/selling load to deliver in each time period of the time horizon. The load in the selling contracts will be assumed to be dependent on the scenario to occur. It is assumed that the energy generator / trading company is a price taker when operating in the spot market. Note that the contract portfolio can combine contracts in different ways to reduce risk and profit less or, viceversa, increase profit and increase risk.

The problem consists of deciding which candidate purchasing/selling contracts to accept, such that some logistic constraints related to the combination of the contracts are satisfied, the power to generate satisfies the thermal and hydro generation constraints, the balance equations of the water flow along the time periods in the river basins are satisfied, the Kyoto protocol-based pollutant emission constraints are satisfied by the thermal units, and the purchasing/selling loads already committed are also satisfied. See also [CP01, Esc01]. The objective function to maximize is a composite function of the estimated profit along the time horizon and the weighted *RP* of having a profit greater than or equal to a given threshold over the scenarios.

We present below a mixed 0-1 model for structuring the energy contract portfolio, where the uncertainty is treated such that the occurrence of the events is represented by a multistage scenario tree.



$\Omega = \Omega^1 = \{10, 11, \ldots, 17\}$; $\Omega^2 = \{10, 11, 12\}$

$\mathcal{G}^2 = \{2, 3, 4\}$; $t(5) = 3$

$\mathcal{N}^9 = \{1, 4, 9\}$;$\phi(9) = 4$

Fig. 1.1. Scenario tree

1.2.2 Scenario tree

Let the tree shown in Figure 1.1 represent the stochasticity of the problem to be dealt with. Each node in the figure is associated with a point in time where a decision can be made. Once a decision is made, some contingencies can happen (in this example, the number of contingencies is three for time period $t = 2$), and information related to them is available at the beginning of the period. Each root-to-leaf path in the tree represents one specific scenario and corresponds to one realization of the whole set of the uncertain parameters along a time horizon. Each node in the tree can be associated with a scenario group, such that two scenarios belong to the same group for a given time period, provided that they have the same realizations of the uncertain parameters up to that period. The *nonanticipativity* principle [BL97, RW91] requires that both scenarios have the same values for the related variables up to the given period.

The following notation related to the scenario tree will be used throughout the chapter:

$\mathcal{T}$, set of stages (time periods, in our case). $\mathcal{T}^- = \mathcal{T} - \{|\mathcal{T}|\}$.
Ω, set of (consecutively numbered) scenarios.
Γ, set of multiperiods for pollutant emission regulation. Note: The regulation is performed at the multiperiod level (e.g., quarter, year), instead of the period level (e.g., week).
$\mathcal{M}_\gamma \subseteq \mathcal{T}$, set of (consecutive) time periods for multiperiod γ, for $\gamma \in \Gamma$.
τ_γ, last time period for multiperiod γ, for $\gamma \in \Gamma$.
$\mathcal{G}$, set of (consecutively numbered) scenario groups.

$\mathcal{G}^t$, set of scenario groups in time period t, for $t \in \mathcal{T}$, such that the scenarios that belong to the same group are identical in all realizations of the uncertain parameters up to period t ($\mathcal{G}^t \subseteq \mathcal{G}$).

$t(g)$, time period for scenario group g, for $g \in \mathcal{G}$. Note: $g \in \mathcal{G}^{t(g)}$.

Ω^g, set of scenarios in group g, for $g \in \mathcal{G}$ ($\Omega^g \subseteq \Omega$).

$\mathcal{N}^g$, set of scenario groups $\{k\}$ such that $\Omega^g \subseteq \Omega^k$, for $g \in \mathcal{G}$ ($\mathcal{N}^g \subseteq \mathcal{G}$). That is, set of ancestor nodes of node g.

$\phi(g)$, immediate ancestor node of node g, for $g \in \mathcal{G}$. Note: $\phi(g) \in \mathcal{N}^g$.

1.2.3 Mixed 0-1 *DEM*. Mean function maximization

The following is additional notation used in *ECP2*.

Sets:

$\mathcal{D}$, set of thermal-power generator types (coal, fuel-oil, gas, combined cycle, etc.).

$\mathcal{I}$, set of thermal-power generators in the problem.

$d(i)$, type of generator i, such that $d(i) \in \mathcal{D}$, for $i \in \mathcal{I}$. Note : Two generators that belong to the same type share the allowed pollutant emission regulations.

$\mathcal{J}$, set of hydro generation units in the river basins under consideration.

$\mathcal{U}_j$, set of immediate up-stream hydro units to hydro unit j to represent the topology of the river basins, for $j \in \mathcal{J}$.

$\mathcal{F}^{cu}$, current energy bilateral selling contracts.

$\mathcal{K}^{cu}$, current energy bilateral purchasing contracts.

$\mathcal{F}^{ca}$, candidate energy bilateral selling contracts.

$\mathcal{K}^{ca}$, candidate energy bilateral purchasing contracts.

$\mathcal{F} = \mathcal{F}^{cu} \bigcup \mathcal{F}^{ca}$ and $\mathcal{K} = \mathcal{K}^{cu} \bigcup \mathcal{K}^{ca}$.

Deterministic function and parameters:

$p_j\left(x_j^g, \hat{z}_j^g\right)$, power output of hydro unit j for a given water release, say, x_j^g and storage level, say, z_j^g at time period $t(g)$ under scenario group g, for $g \in \mathcal{G}$, $j \in \mathcal{J}$. Although the formulation in terms of the variables z_j would provide a better representation of the influence of the water related variables in the hydro power generation function, it would result in a significant increase in problem complexity. As a compromise, a representative value for the stored water, say $\hat{z}_j^g$, is used in the formulation to linearize the resulting expression. It is represented by a concave piecewise function on x_j^g.

$\overline{z}_j$, upper bound of the stored water in the reservoir associated with hydro unit j at any time period, for $j \in \mathcal{J}$.

$\overline{xy}_j$, upper bound of the (released and spilled) water flow down stream from the reservoir associated with hydro unit j at any time period, for $j \in \mathcal{J}$, $t \in \mathcal{T}$.

$\overline{p}_j$, upper bound of the power to generate by hydro unit j at any time period, for $j \in \mathcal{J}$.

$\overline{p}_i$, upper bound of the power to generate by thermal unit i, for for $i \in \mathcal{I}$.

$\underline{\theta}_{kt}, \overline{\theta}_{kt}$, lower and upper bounds for energy bilateral purchasing contract k during time period t, respectively, for $k \in \mathcal{K}, t \in \mathcal{T}$. This instrument is evidently a physical instrument, but it can be used as a financial one and, in particular, can be used as a load factor contract, see its definition in [MGG01]

v_d, unit pollutant emission from thermal generator type d, for $d \in \mathcal{D}$.

$\hat{\varrho}_d$, threshold for the non-penalized pollutant emission for the Kyoto protocol regulation in the thermal generation type d, for $d \in \mathcal{D}$.

ζ, unit penalty (res., reward) for Kyoto protocol-based pollutant emission excess (res., deficit) from the allowed threshold. It is constant for the time horizon, at least.

a, unit trading fee for energy purchasing/selling from the spot market.

Feasibility spaces:

Φ_i, operating feasible region for the production of thermal generator i at any time period, for $i \in \mathcal{I}$. There is ample bibliography in the open literature describing the operating constraint system to define the feasible space, see [AC00, DGMRS97, CNA02, DR98, GKNRW01, HROCh01, JC97, KV01, SNW05, NR02, WW96], among many others. However since the problem treated in this chapter considers a long planning horizon, the set of operating constraints has not been considered.

Δ, feasible region for the set of energy bilateral selling contracts to be chosen, such as maximum percentage for each contract, regional considerations, exclusivity constraints mainly in the financial contracts, etc.

Ψ, feasible region for the set of energy bilateral purchasing contracts to be chosen, such as maximum allowed budget, maximum percentage for each contract, regional considerations, exclusivity constraints mainly in the financial contracts, etc.

Stochastic function and parameters for time period $t(g)$ under scenario group g, for $g \in \mathcal{G}$:

$c_i^g(p_i^g)$, production cost for power output, say, p_i^g of thermal generator i, for $i \in \mathcal{I}$. It can be represented by a convex piecewise linear function, where the stochasticity is mainly due to the cost of the fuel-oil and gas along the time horizon.

$\underline{z}_j^g$, lower bound of the stored water in reservoir j, for $j \in \mathcal{J}$. Special attention should be given to the lower bounds of the stored water during the last period of the time horizon.

$\underline{xy}_j^g$, lower bound of the (released and spilled) water flow downstream from reservoir j, for $j \in \mathcal{J}$.

e_j^g, net exogenous water inflow to reservoir j, for $j \in \mathcal{J}$.

ι_f^g, energy requirement from bilateral selling contract f, for $f \in \mathcal{F}$.

μ_f^g, unit payment received from bilateral selling contract f, for $f \in \mathcal{F}$.

λ_k^g, unit payment due for bilateral purchasing contract k, for $k \in \mathcal{K}$.

η^g, unit energy price in the spot market.

Let ω^g denote the weight assigned to scenario group g, for $g \in \mathcal{G}$.

Generation and trading variables for time period $t(g)$ ***under scenario group*** g***, for*** $g \in \mathcal{G}$***:***

θ_k^g, energy requirement from bilateral purchasing contract k, for $k \in \mathcal{K}$.

p_i^g, power output of thermal generator i, for $i \in \mathcal{I}$.

${\varrho_d^+}^g, {\varrho_d^-}^g$, excess and deficit pollutant emissions by the thermal generators of type d from the allowed target for satisfying the Kyoto protocol-based regulations, respectively, for $d \in \mathcal{D}, g \in \mathcal{G}^{\tau_\gamma}, \gamma \in \Gamma$.

x_j^g, released water from reservoir j, for $j \in \mathcal{J}$.

y_j^g, spilled water from reservoir j, for $j \in \mathcal{J}$.

z_j^g, stored water at (the end of) time period $t(g)$ in reservoir j, for $j \in \mathcal{J}$.

s^g, power output to be sold to the spot market by the generation/energy service company.

b^g, power to be purchased from the spot market by the generation/energy service company.

First stage variables. Candidate contracts:

δ_f, 0-1 variable having the value 1 if the candidate energy bilateral selling contract f is chosen and, otherwise, its value is 0, for $f \in \mathcal{F}^{ca}$.

β_k, 0-1 variable having the value 1 if the candidate energy bilateral purchasing contract k is chosen and, otherwise, its value is 0, for $k \in \mathcal{K}^{ca}$.

The following is a so-called *compact* representation of the mixed 0-1 *DEM* for the *multistage* stochastic problem with complete recourse.

Objective: Determine the portfolio of the energy bilateral trading contracts to maximize the mean (expected) profit from the contract portfolio exploitation and spot market trading along the time horizon over the scenarios, subject to the constraints (1.2)-(1.14).

$$Q_E = \sum_{g \in \mathcal{G}} \sum_{f \in \mathcal{F}^{cu}} w^g \mu_f^g \iota_f^g$$

$$+\max \sum_{g\in\mathcal{G}} w^g \Big[\sum_{f\in\mathcal{F}^{ca}} \mu_f^g \iota_f^g \delta_f + (\eta^g - a)s^g -$$

$$\Big(\sum_{k\in\mathcal{K}} \lambda_k^g \theta_k^g + \sum_{i\in\mathcal{I}} c_i^g(p_i^g) + (\eta^g + a)b^g + \sum_{d\in\mathcal{D}|g\in\mathcal{G}^{\tau_\gamma},\gamma\in\Gamma} \zeta({\varrho_d^+}^g - {\varrho_d^-}^g) \Big) \Big]. \quad (1.1)$$

Constraints:

$$\sum_{i\in\mathcal{I}} p_i^g + \sum_{j\in\mathcal{J}} p_j(x_j^g, \hat{z}_j^g) + \sum_{k\in\mathcal{K}} \theta_k^g + b^g = \quad (1.2)$$

$$\sum_{f\in\mathcal{F}^{cu}} \iota_f^g + \sum_{f\in\mathcal{F}^{ca}} \iota_f^g \delta_f + s^g \quad \forall g \in \mathcal{G}$$

$$0 \le p_i^g \le \overline{p}_i \quad \forall i \in \mathcal{I}, g \in \mathcal{G} \quad (1.3)$$

$$v_d \sum_{k\in\mathcal{N}^g|t(k)\in\mathcal{M}_\gamma} \sum_{i\in\mathcal{I}|d(i)=d,} p_i^k + {\varrho_d^-}^g = \hat{\varrho}_d + {\varrho_d^+}^g \quad d \in \mathcal{D}, g \in \mathcal{G}^{\tau_\gamma}, \gamma \in \Gamma \quad (1.4)$$

$$p_j(x_j^g, \hat{z}_j^g) \le \overline{p}_j \quad \forall j \in \mathcal{J}, g \in \mathcal{G} \quad (1.5)$$

$$-z_j^{\phi(g)} - \sum_{u\in\mathcal{U}_j} (x_u^g + y_u^g) + x_j^g + y_j^g + z_j^g = e_j^g \quad \forall j \in \mathcal{J}, g \in \mathcal{G} \quad (1.6)$$

$$\underline{z}_j^g \le z_j^g \le \overline{z}_j \quad \forall j \in \mathcal{J}, g \in \mathcal{G} \quad (1.7)$$

$$\underline{xy}_j^g \le x_j^g + y_j^g \le \overline{xy}_j \quad \forall j \in \mathcal{J}, g \in \mathcal{G} \quad (1.8)$$

$$\underline{\theta}_{k,t(g)} \le \theta_k^g \le \overline{\theta}_{k,t(g)} \quad \forall k \in \mathcal{K}^{cu}, g \in \mathcal{G} \quad (1.9)$$

$$\underline{\theta}_{k,t(g)}\beta_k \le \theta_k^g \le \overline{\theta}_{k,t(g)}\beta_k \quad \forall k \in \mathcal{K}^{ca}, g \in \mathcal{G} \quad (1.10)$$

$$\delta_f \in \Delta \quad \forall f \in \mathcal{F} \quad (1.11)$$

$$\beta_k \in \Psi \quad \forall k \in \mathcal{K} \quad (1.12)$$

$$x_j^g, y_j^g, s^g, b^g \ge 0 \quad \forall i \in \mathcal{I}, j \in \mathcal{J}, g \in \mathcal{G} \quad (1.13)$$

$${\varrho_d^+}^g, {\varrho_d^-}^g \ge 0 \quad \forall d \in \mathcal{D}, g \in \mathcal{G}^{\tau_\gamma}, \gamma \in \Gamma. \quad (1.14)$$

The model determines the selling/purchasing contract (so-called *first stage*) policy to be followed when restructuring the energy bilateral contracts portfolio. The restructuring aims to maximize the expected profit given by the exploitation of the portfolio as well as the spot market trading as represented in (1.1). The balance equations (1.2) equate the power availability (i.e., thermal and hydro generation plus purchased power already committed plus candidate power to purchase in bilateral trading contracts and power to purchase on the spot market) on one side and the power needs (i.e., sold power already committed plus candidate power to sell in bilateral trading contracts and power to sell on the spot market) on the other side. The bounds (1.3) ensure the feasibility of the thermal power to generate. The equations (1.4) define the positive and negative output from the pollutant thermal system to satisfy the regulation based on the Kyoto protocol. The constraints (1.5) force upper bounds on the hydro-power generation. The balance equations (1.6) ensure the water availability along the time horizon, given the topology of the river basins under the control of the generation company. (See in [CChW06] an approach for optimizing large scale water reservoir network by stochastic dynamic programming).

The bounds (1.7)-(1.8) ensure the bounding of the stored water in the reservoirs and the lower and upper bounds of the water flow (i.e., released and spilled water) in the segments of the river basins. The bounds (1.9) ensure that the generation company / energy service provider will purchase the required energy from the current bilateral contracts satisfying the requirements stated in the contracts. The constraints (1.10) ensure that the delivery of the energy from the candidate bilateral purchasing contracts will satisfy the requirements of the contracts, if any. The memberships (1.11)-(1.12) ensure that the logistic constraints are satisfied for the set of bilateral energy selling and purchasing contracts to be chosen.

It is worth pointing out the different mechanism of the energy bilateral selling and purchasing contracts. The model decides the energy to purchase, within a given set of lower and upper bounds. On the other hand, the decisions on the energy to be sold in a given time period are exogenous to the model (i.e., they are made by the buyer within committed lower and upper bounds); they are represented by uncertain energy requirements which are modelled under the scenario groups for both types of contracts, current and candidate.

Notice that the decision on the acceptance of the contracts considers the scenario-related system, but it is not subordinated to any scenario.

Remark: The bilateral contracts enter the constraint system in the power balance equation restriction (1.2), but they also appear in the constraints that define the feasible spaces (1.11)-(1.12) besides the objective function (1.1) (and the function (1.15)). It is enough for deciding the selling / purchasing (so-called first stage) policy. Notice also that the physical contracts are the main contracts in the model, however the financial ones are also allowed. The differences between the two types of contracts are considered in the spaces defined by (1.11)-(1.12). It is up to the contract manager to selling or purchasing a financial contract but (s)he should take into account according to the model that there is a cost for energy purchasing (including fees) and an income for energy selling (subtracting the related fees), such that the solution must also satisfied the constraints (1.2), (1.11) and (1.12).

1.3 Mean-Risk function maximization and *VaR*

As an alternative to maximizing the mean function (1.1), we propose to consider a mean-risk objective function, defined as a composite function of the expected trading profit and RP, the probability of reaching a given profit threshold, say, τ. It can be formulated as follows,

$$\max Q_E + \rho Q_P, \tag{1.15}$$

where

$$Q_E = \sum_{\omega \in \Omega} w_\omega \mathcal{O}^\omega, \tag{1.16}$$

w_ω gives the weight assigned to scenario ω, $\mathcal{O}^\omega$ gives the profit to obtain under scenario ω for $\omega \in \Omega$, ρ is a nonnegative weighting parameter and Q_P is the *Reaching Probability, RP*. The profit $\mathcal{O}^\omega$ can be expressed

$$\begin{aligned}\mathcal{O}^\omega = &\sum_{g \in \mathcal{N}^n} \Big[\sum_{f \in \mathcal{F}^{cu}} \mu_f^g \iota_f^g + \sum_{f \in \mathcal{F}^{ca}} \mu_f^g \iota_f^g \delta_f + (\eta^g - a) s^g \Big] \\ - &\sum_{g \in \mathcal{N}^n} \Big[\sum_{k \in \mathcal{K}} \lambda_k^g \theta_k^g + \sum_{i \in \mathcal{I}} c_i^g (p_i^g) + (\eta^g + a) b^g + \zeta \sum_{d \in \mathcal{D} | g \in \mathcal{G}^{\tau_\gamma}, \gamma \in \Gamma} (\varrho_d^{+g} - \varrho_d^{-g}) \Big],\end{aligned} \tag{1.17}$$

where $n \in \mathcal{G}^{|\mathcal{T}|}$ such that $\omega \in \Omega^n$, and RP can be expressed

$$Q_P = P\big(\omega \in \Omega : \mathcal{O}^\omega \geq \tau\big). \tag{1.18}$$

Based on an approach for modelling RP given in [ST04], a more amenable expression of (1.15) for computational purposes, at least, is given by

$$\begin{aligned} &\max \sum_{\omega \in \Omega} w_\omega \big(\mathcal{O}^\omega + \rho \nu^\omega\big) \\ &\text{s.t. } \mathcal{O}^\omega + M(1 - \nu^\omega) \geq \tau \ \forall \omega \in \Omega \\ &\qquad \nu^\omega \in \{0, 1\} \qquad\qquad \forall \omega \in \Omega, \end{aligned} \tag{1.19}$$

where M is a parameter whose value is chosen as the smallest value which does not eliminate any feasible solution under any scenario, and ν^ω is a 0-1 variable such that

$$\nu^\omega = \begin{cases} 1, & \text{if the trading profit } \mathcal{O}^\omega \text{ under scenario } \omega \\ & \text{is greater or equal than threshold } \tau \\ 0, & \text{otherwise} \end{cases} \qquad \forall \omega \in \Omega.$$

Note: The term $M(1 - \nu^\omega)$ in (1.19) allows the benefit $\mathcal{O}^\omega$ to be negative for some scenario $\omega \in \Omega$.

As alternatives to the mean-risk optimization, the *Value-at-Risk (VaR)* and *Conditional VaR* functionals to maximize for a given α-risk, where $0 \leq \alpha < 1$, can be expressed as

$$\begin{aligned} &\max VaR \\ &\text{s.t. } \mathcal{O}^\omega + M(1 - \vartheta^\omega) \geq VaR \ \forall \omega \in \Omega \\ &\qquad \sum_{\omega \in \Omega} w_\omega \vartheta^\omega \geq 1 - \alpha \\ &\qquad \vartheta^\omega \in \{0, 1\} \qquad\qquad \forall \omega \in \Omega, \end{aligned} \tag{1.20}$$

for the *Value-at-Risk (VaR)*, where ϑ^ω is a 0-1 variable, and

$$\begin{aligned} &\max CVaR \\ &\text{s.t. } \kappa + \frac{1}{1-\alpha} \sum_{\omega \in \Omega} \Big[w_\omega \mathcal{O}^\omega - \kappa \Big]^+ \geq CVaR, \end{aligned} \tag{1.21}$$

for the *Conditional VaR*, where $[x]^+$ gives the positive value of x and κ gives the *VaR* value. Note: The Kyoto-based pollutant emission regulation has not been considered.

Notice that the replacement of the expected trading profit (1.1) by the mean-risk system (1.19) with (1.17) does not change the structure of the model. On the contrary, the maximization of *VaR* (1.20) and the maximization of *Conditional VAR* (1.21) destroy the structure of the model, since the related constraints are non-separable.

The instances of the mixed 0-1 *DEM* (1.1)-(1.14) can have such large dimensions that the plain use of state-of-the-art optimization engines becomes unaffordable.

Additionally, a *splitting variable* representation can be introduced by replacing the variables with their siblings, such that θ_k^g, p_i^g, x_j^g, y_j^g, z_j^g, s^g and b^g are replaced with θ_{kt}^ω, p_{it}^ω, x_{jt}^ω, y_{jt}^ω, z_{jt}^ω, s_t^ω and b_t^ω, respectively, for $t = t(g)$, $\omega \in \Omega^g$, $g \in \mathcal{G}$; ϱ_d^{+g} and ϱ_d^{-g} are replaced with $\varrho_{dt}^{+\omega}$ and $\varrho_{dt}^{-\omega}$, respectively, for $\omega \in \Omega^g$, $g \in \mathcal{G}^t$, $t = \tau_\gamma, \gamma \in \Gamma$; and δ_f and β_k are replaced with δ_f^ω and β_k^ω, respectively, for $\omega \in \Omega$. Additionally, the *nonanticipativity* constraints (1.22)-(1.32) are appended to the model, $\forall \omega \in \Omega^g$, $g \in \mathcal{G}^t$, $t \in \mathcal{T}^-$, where from now on $\omega + 1$ is assumed to be any scenario in set Ω^g, but the last one, whenever ω is the last scenario in the set.

$$\theta_{kt}^\omega - \theta_{kt}^{\omega+1} = 0 \qquad \forall k \in \mathcal{K} \tag{1.22}$$

$$p_{it}^\omega - p_{it}^{\omega+1} = 0 \qquad \forall i \in \mathcal{I} \tag{1.23}$$

$$\varrho_{dt}^{+\omega} - \varrho_{dt}^{+\omega+1} = 0 \qquad \forall d \in \mathcal{D}, \gamma \in \Gamma, \text{ where } t = \tau_\gamma \tag{1.24}$$

$$\varrho_{dt}^{-\omega} - \varrho_{dt}^{-\omega+1} = 0 \qquad \forall d \in \mathcal{D}, \gamma \in \Gamma, \text{ where } t = \tau_\gamma \tag{1.25}$$

$$x_{jt}^\omega - x_{jt}^{\omega+1} = 0 \qquad \forall j \in \mathcal{J} \tag{1.26}$$

$$y_{jt}^\omega - y_{jt}^{\omega+1} = 0 \qquad \forall j \in \mathcal{J} \tag{1.27}$$

$$z_{jt}^\omega - z_{jt}^{\omega+1} = 0 \qquad \forall j \in \mathcal{J} \tag{1.28}$$

$$s_t^\omega - s_t^{\omega+1} = 0 \tag{1.29}$$

$$b_t^\omega - b_t^{\omega+1} = 0 \tag{1.30}$$

$$\delta_f^\omega - \delta_f^{\omega+1} = 0 \qquad \forall f \in \mathcal{F}^{ca} \tag{1.31}$$

$$\beta_k^\omega - \beta_k^{\omega+1} = 0 \qquad \forall k \in \mathcal{K}^{ca}. \tag{1.32}$$

By considering the *nonanticipativity* constraints (1.22)-(1.32) as the 'difficult' ones, Benders [Ben62] Decomposition schemes can be used, see [ALMP06, BL97, EGMP07, LL93], among others. Alternatively, by dualizing these constraints at each so-called supernode of a branch-and-bound phase, Lagrangian Decomposition schemes can be used for solving the model given by the function (1.19) with (1.17) and the constraint system (1.2)-(1.14), see [CS99, HS01, KV99, KV01, NR02, RS01, Sch03, SNW05, ST04, TB00], among others. However, some heuristics could be needed for obtaining solutions to the problem, such that the *nonanticipativity* constraints are also satisfied. Independently of the constraints' dualization, a *Branch-and-Fix Coordination* approach can be used, see below.

1.4 A Branch-and-Fix Coordination approach

1.4.1 BFC methodology

By slightly abusing the notation, consider the following *splitting variable* representation of the *DEM*: maximize (1.19) with (1.17) subject to (1.2)-(1.14) and (1.22)-(1.32).

$$
\begin{aligned}
Z^{IP} = \max & \sum_{\omega \in \Omega} w_\omega \big(\sum_{r \in \mathcal{R}} e_r^\omega \gamma_r^\omega + \sum_{t \in \mathcal{T}} c_t^\omega v_t^\omega + \rho \nu^\omega \big) \\
\text{s.t.} & \sum_{r \in \mathcal{R}} e_r^\omega \gamma_r^\omega + \sum_{t \in \mathcal{T}} c_t^\omega v_t^\omega + M(1-\nu^\omega) \geq \tau \ \forall \omega \in \Omega \\
& A_t^\omega \gamma^\omega + B_t'^\omega v_{t-1}^\omega + B_t^\omega v_t^\omega = b_t^\omega && \forall \omega \in \Omega, t \in \mathcal{T} \\
& \gamma_r^\omega - \gamma_r^{\omega+1} = 0 && \forall \omega \in \Omega, r \in \mathcal{R} \\
& v_{ht}^\omega - v_{ht}^{\omega+1} = 0 && \forall \omega \in \Omega^g, g \in \mathcal{G}^t, t \in \mathcal{T}^-, h \in \mathcal{H} \\
& v_{ht}^\omega \geq 0 && \forall \omega \in \Omega, t \in \mathcal{T}, h \in \mathcal{H} \\
& \gamma_r^\omega, \nu^\omega \in \{0,1\} && \forall \omega \in \Omega, r \in \mathcal{R},
\end{aligned}
\tag{1.33}
$$

where v_t^ω denotes the vector of the continuous variables $\{v_{ht}^\omega,\ \forall h \in \mathcal{H}\}$ for $\omega \in \Omega, t \in \mathcal{T}$, representing the variables θ_{kt}^ω, p_{it}^ω, ${\varrho_{dt}^+}^\omega$, ${\varrho_{dt}^-}^\omega$, x_{jt}^ω, y_{jt}^ω, z_{jt}^ω, s_t^ω and b_t^ω in *ECP2*; $\mathcal{H}$ is the set of indices of the variables in v_t^ω; γ^ω denotes the vector of the 0-1 variables $\{\gamma_r^\omega,\ \forall r \in \mathcal{R}\}$ for $\omega \in \Omega$ representing the variables δ_f^ω $\forall f \in \mathcal{F}$ and β_k^ω $\forall k \in \mathcal{K}$; $\mathcal{R}$ is the set of indices of the variables in γ^ω; c_t^ω is the row vector of the objective function coefficients for the vector v_t^ω and e_r^ω is the objective function coefficient for the variable γ_r^ω; A_t^ω, $B_t'^\omega$ and B_t^ω are the constraint matrices for the vectors γ^ω, v_{t-1}^ω and v_t^ω, respectively; b_t^ω is from now on the right-hand-side (*rhs*) vector for $\omega \in \Omega$, $t \in \mathcal{T}$; and τ and ν are as defined above, all of them with conformable dimensions.

Notice that the relaxation of the *nonanticipativity* constraints

$$\gamma_r^\omega - \gamma_r^{\omega+1} = 0 \quad \forall \omega \in \Omega, r \in \mathcal{R} \tag{1.34}$$

$$v_{ht}^\omega - v_{ht}^{\omega+1} = 0 \quad \forall \omega \in \Omega^g, g \in \mathcal{G}^t, t \in \mathcal{T}^-, h \in \mathcal{H} \tag{1.35}$$

in model (1.33) results in a set of $|\Omega|$ independent mixed 0-1 models, where (1.36) is the model for scenario $\omega \in \Omega$.

$$\begin{aligned}
\max & \sum_{r\in\mathcal{R}} e_r^\omega \gamma_r^\omega + \sum_{t\in\mathcal{T}} c_t^\omega v_t^\omega + \rho\nu^\omega \\
\text{s.t.} & \sum_{r\in\mathcal{R}} e_r^\omega \gamma_r^\omega + \sum_{t\in\mathcal{T}} c_t^\omega v_t^\omega + M(1-\nu^\omega) \geq \tau \\
& A_t^\omega \gamma^\omega + B_t'^\omega v_{t-1}^\omega + B_t^\omega v_t^\omega = b_t^\omega \qquad \forall t \in \mathcal{T} \\
& v_{ht}^\omega \geq 0 \qquad \forall t \in \mathcal{T}, h \in \mathcal{H} \\
& \gamma_r^\omega, \nu^\omega \in \{0,1\} \qquad \forall r \in \mathcal{R}.
\end{aligned} \tag{1.36}$$

To improve computational efficiency, it is not necessary to relax all constraints of the form (1.34)-(1.35). The number of constraints to relax in a given model will depend on the dimensions of the scenario related model (1.36) (i.e., the parameters $|\mathcal{T}|$, $|\mathcal{I}|$, $|\mathcal{J}|$, $|\mathcal{D}|$, $|\mathcal{F}^{ca}|$ and $|\mathcal{K}|$ in model (1.1)-(1.14)). Let q denote the number of scenario *clusters* that results after taking into account the size considerations mentioned above. Let Ω_p denote the set of scenarios that belong to *cluster* p. The criterion for the choice of scenario *clusters* for the sets $\Omega_1, \ldots, \Omega_q$, such that $\Omega_p \cap \Omega_{p'} = \emptyset$, $p, p\prime = 1, \ldots, q : p \neq p\prime$ and $\Omega = \cup_{p=1}^q \Omega_p$ could alternatively be based on the smallest internal deviation of the uncertain parameters, the greatest deviation, etc. We favor the assignation of a higher priority to include in the same *cluster* those scenarios with greater number of common ancestor nodes in the associated scenario tree.

By slightly continuing to abuse the notation, the model to consider for scenario *cluster* $p = 1, \ldots, q$ can be expressed by the *compact* representation (1.37), where ω for $n \in G^{|\mathcal{T}|}$ is such that ω is the unique scenario in Ω^n and, on the other hand, $G_p = \{g \in \mathcal{G} : \Omega^g \bigcap \Omega_p \neq \emptyset\}$.

$$Z_p^{IP} = \max \sum_{n\in\mathcal{G}^{|\mathcal{T}|}\bigcap\mathcal{G}_p} w_\omega \big[\sum_{g\in\mathcal{N}^n} \big(e^g\gamma^p + c^g v^g\big) + \rho\nu^\omega \big]$$

subject to

$$\begin{aligned}
& \sum_{g\in\mathcal{N}^n} \big(e^g\gamma^p + c^g v^g\big) + M(1-\nu^\omega) \geq \tau \ \forall n \in \mathcal{G}^{|\mathcal{T}|} \textstyle\bigcap \mathcal{G}_p \\
& A^g\gamma^p + B'^g v^{\phi(g)} + B^g v^g = b^g \qquad \forall g \in \mathcal{G}_p \\
& v_h^g \geq 0 \qquad \forall g \in \mathcal{G}_p, h \in \mathcal{H} \\
& \gamma_r^p, \nu^\omega \in \{0,1\} \qquad \forall \omega \in \Omega_p, r \in \mathcal{R},
\end{aligned} \tag{1.37}$$

where v^g is the vector of the continuous variables for the constraint system related to time period $t(g)$ under scenario group g, γ^p is the vector of the 0-1 variables, e^g and c^g are the row vectors of the objective function coefficients for the γ^p and v^g vectors, respectively, A^g, B'^g and B^g are the constraint matrices, b^g is the *rhs* under scenario group g, and the other parameters are as above, all with conformable dimensions. Notice that the model (1.37) for $q = 1$ is the *compact* representation equivalent to the *splitting variable* representation (1.33), and it is the model (1.36) for $q = |\Omega|$.

The q models (1.37) are linked by the *nonanticipativity* constraints:

$$\gamma^p - \gamma^{p\prime} = 0 \tag{1.38}$$

$$v^{g^p} - v^{g^{p\prime}} = 0 \tag{1.39}$$

$\forall g \in \mathcal{G}_p \bigcap \mathcal{G}_{p\prime}, p, p' = 1, \ldots, q : p \neq p'$, and γ^p and $\gamma^{p\prime}$ are the γ vectors and v^{g^p} and $v^{g^{p\prime}}$ are the v vectors for the submodels (1.37) related to the *clusters* p and p', respectively.

We could apply a Branch-and-Bound procedure to ensure the integrality condition in the models (1.37), for $p = 1, \ldots, q$. However, instead of obtaining independently the optimal solution for each one, elsewhere [AEO03, AEG03] we propose an approach so-called *Branch-and-Fix Coordination* (*BFC*). Our specialization in this chapter is designed to coordinate the selection of the branching node and branching variable for each scenario *cluster*-related so-called *Branch-and-Fix (BF)* tree, such that the relaxed constraints (1.38) are satisfied when fixing the appropriate γ variables to either one or zero. The proposed approach also coordinates and reinforces the scenario *cluster*-related *BF* node pruning, the variable fixing and the objective function bounding for the submodels attached to the nodes. See also [AEP09, Esc09, EGMP07, EGMP09a, EGMP09b, EGMP10]

Let $\mathcal{C}_p$ denote the *BF* tree associated with scenario *cluster* p and $\mathcal{A}_p$ the set of active nodes in $\mathcal{C}_p$ for $p = 1, \ldots, q$. Any two nodes, say, $a \in \mathcal{A}_p$ and $a' \in \mathcal{A}_{p'}$ are said to be *twin* nodes if either they are the *root* nodes or the paths from their *root* nodes to each of them in their own *BF* trees $\mathcal{C}_p$ and $\mathcal{C}_{p'}$, respectively, have branched on / fixed at the same values of some or all their (*first stage*) variables γ_r^p and $\gamma_r^{p'}$, for $p, p' = 1, \ldots, q : p \neq p'$. Notice that in order to satisfy the *nonanticipativity* constraints (1.38), the γ variables must be branched on / fixed at the same 0-1 value for the *twin* nodes. A *Twin Node Family* (*TNF*), say, $\mathcal{L}_n$ is a set of active nodes, such that any one is a *twin* node to all the other members of the family, for $n \in \Upsilon$, where Υ is the set of *TNFs*.

1.4.2 On obtaining *LP* optimal solutions for *TNF integer sets*

Recall that a *TNF integer set* is each set of integer nodes, one for each *BF* tree, where the *nonanticipativity* constraints (1.38) for the 0-1 variables are satisfied.

Consider the following *LP* model, obtained after fixing in model (1.33) the γ and ν variables to their 0-1 values, say, $\hat{\gamma}_r \; \forall r \in \mathcal{R}$ and $\hat{\nu}^\omega \; \forall \omega \in \Omega$ for a given *TNF integer set*, respectively,

$$Z_{TNF}^{LP} = \sum_{\omega \in \Omega} w_\omega \big(e^\omega \hat{\gamma} + \rho \hat{\nu^\omega} \big) + \max \sum_{\omega \in \Omega} \sum_{t \in \mathcal{T}} w_\omega c_t^\omega v_t^\omega$$

subject to

$$\begin{aligned}
&\sum_{t \in \mathcal{T}} c_t^\omega v_t^\omega \geq \tau - e^\omega \hat{\gamma} - M(1 - \hat{\nu}^\omega) \; \forall \omega \in \Omega \\
&B_t'^\omega v_{t-1}^\omega + B_t^\omega v_t^\omega = b_t^\omega - A_t^\omega \hat{\gamma} && \forall \omega \in \Omega, t \in \mathcal{T} \\
&v_{ht}^\omega - v_{ht}^{\omega+1} = 0 && \forall \omega \in \Omega^g, g \in \mathcal{G}^t, t \in \mathcal{T}^-, h \in \mathcal{H} \\
&v_{ht}^\omega \geq 0 && \forall \omega \in \Omega, t \in \mathcal{T}, h \in \mathcal{H}.
\end{aligned} \tag{1.40}$$

Notice that the *RP* related constraint for scenario ω is redundant for $\hat{\nu^\omega} = 0$, $\omega \in \Omega$.

1.4.3 On *TNF integer set* bounding

Let $\mathcal{R}f$ denote the set of non-yet branched on / fixed at γ variables in a given *TNF integer set*, such that $\mathcal{R}01 = \mathcal{R} - \mathcal{R}f$. Let also $A_t^\omega = (Af_t^\omega, A01_t^\omega)$ $\forall \omega \in \Omega$, where Af_t^ω and $A01_t^\omega$ are the constraint submatrices related to the sets $\mathcal{R}f$ and $\mathcal{R}01$, respectively, for $t \in \mathcal{T}, \omega \in \Omega$. Similarly, let $\Omega = \Omega f \bigcup \Omega 01$, such that Ωf and $\Omega 01$ are the set of scenarios where the ν variables have not yet been branched on / fixed at 0-1 and the set where these variables have already been branched on / fixed at integer 0-1 values, respectively.

The model (1.41) enforces the satisfaction of the *nonanticipativity* constraints (1.34)-(1.35) but, on the other hand, it allows the γ and ν variables from the sets $\mathcal{R}f$ and Ωf, respectively, to take continuous values for the given *TNF integer set*.

$$\begin{aligned}
Z_{CON}^{LP} = &\sum_{\omega \in \Omega} \sum_{r \in \mathcal{R}01} w_\omega e^r \gamma^r + \rho \sum_{\omega \in \Omega 01} w_\omega \nu^\omega \\
&+ \max \sum_{\omega \in \Omega} \sum_{r \in \mathcal{R}f} w_\omega e^r \gamma^r + \sum_{\omega \in \Omega} w_\omega c^\omega v^\omega + \rho \sum_{\omega \in \Omega f} w_\omega \nu^\omega
\end{aligned}$$

subject to

$$
\begin{array}{ll}
e^{\omega f}\gamma^{\omega f}+c^{\omega}v^{\omega}+M^{\omega f}(1-\nu^{\omega})\geq & \\
\tau-e^{\omega 01}\gamma^{\omega 01}-M^{\omega 01}(1-\nu^{\omega}) & \forall\omega\in\Omega \\
Af_t^{\omega}\gamma^{\omega f}+B_t'^{\omega}v_{t-1}^{\omega}+B_t^{\omega}v_t^{\omega}=b_t^{\omega}-A01_t^{\omega}\gamma^{\omega 01} & \forall\omega\in\Omega,t\in\mathcal{T} \\
\gamma^{\omega f}-\gamma^{\omega f+1}=0 & \forall\omega\in\Omega \\
v_{ht}^{\omega}-v_{ht}^{\omega+1}=0 & \forall\omega\in\Omega^g,g\in\mathcal{G}^t,t\in\mathcal{T}^-,h\in\mathcal{H} \\
v_{ht}^{\omega}\geq 0 & \forall\omega\in\Omega^g,g\in\mathcal{G}^t,t\in\mathcal{T},h\in\mathcal{H} \\
0\leq\gamma^{\omega f}\leq 1 & \forall\omega\in\Omega \\
0\leq\nu^{\omega}\leq 1, & \forall\omega\in\Omega f,
\end{array}
\tag{1.41}
$$

where $\gamma^{\omega 01}$ and $\gamma^{\omega f}$ give the subvectors of the vector γ^{ω} for the sets $\mathcal{R}01$ and $\mathcal{R}f$, respectively, similarly for the subvectors $e^{\omega 01}$ and $e^{\omega f}$, and the subvectors $M^{f\omega}=M$ and $M^{01\omega}=M$ for $\omega\in\Omega f$ and $\omega\in\Omega 01$, respectively and, otherwise, they are zero.

1.4.4 BFC algorithm

Consider the following algorithm for solving model (1.33) by using the scenario *cluster*-related submodels (1.37).

Step 1: Solve the *LP* relaxations of the q models (1.37). Each model is attached to the *root* node in the trees $\mathcal{C}_p\ \forall p=1,\dots,q$. If the integrality constraints and the constraints (1.38)-(1.39) are satisfied then stop, the optimal solution to the original mixed 0-1 model (1.33) has been obtained.

Step 2: The following parameters are saved in a centralized device (*CD*): the values of the variables and the solution value (i.e., the optimal objective function value) of the *LP* models attached to the nodes in $\mathcal{A}_p\ \forall p=1,\dots,q$, as well as the appropriate information for branching on the 0-1 γ and ν variables in the *TNFs* $\mathcal{L}_n\,\forall n\in\Upsilon$. A decision is made in *CD* for the selection of the branching *TNF* and the branching variable. The decision is made available for the execution of each scenario *cluster*-related *BF* phase.

Step 3: Optimization of the *LP* models attached to the newly created *TNF* after branching on the chosen 0-1 variable. Prune the *TNF* if its related *LP* model is infeasible or its solution value (i.e., the weighted sum of the solution values of its node members) is not greater than the value of the incumbent solution and, then, go to Step 8.

Step 4: If the optimal solution that has been obtained in Step 3 has 0-1 values for all the γ and ν variables and satisfies the constraints (1.38) (i.e., it is a *TNF integer set*), either of the two following situations has happened:

(a) The *nonanticipativity* constraints (1.39) have been satisfied and, then, a new solution has been found for the original mixed 0-1 model (1.33).

The related *incumbent* solution can be updated and, additionally, the updating of the sets $\mathcal{A}_p$ at the trees $\mathcal{C}_p$ $\forall p = 1, \ldots, q$ can also be performed. In any case, the *TNF* is pruned. Goto Step 8.

(b) The *nonanticipativity* constraints (1.39) have not been satisfied. Goto Step 5.

Otherwise, go to Step 2.

Step 5: Optimizing the *LP* model (1.40) that results from fixing the γ and ν variables in model (1.33) to the 0-1 values given in the *TNF integer set* whose associated models (1.37) have been optimized in Step 3. See section 1.4.2. If the *LP* model is feasible (and, thus, its optimal solution has been obtained), then the updating of the *incumbent* solution and active node sets can be performed.

Step 6: Solve the *LP* model (1.41) that results from fixing the branched on / fixed at γ and ν variables in model (1.33) to their integer values and allowing the other 0-1 variables to take continuous values. See section 1.4.3.

Step 7: If the solution value of the model (1.41) for the given *TNF* is not greater than the *incumbent* solution value, or the *LP* models to be optimized in Steps 5 and 6 yield the same solution value, then the family is pruned.

Step 8: If the sets of active nodes are empty then stop, since the optimality of the *incumbent* solution has been proved, if any. Otherwise, go to Step 2.

1.5 Conclusions

In this chapter we have introduced a multistage stochastic mixed 0-1 model with complete recourse for structuring portfolios of bilateral energy trading contracts in competitive electricity markets. The treatment of the uncertainty plays a central role in the approach that we have presented. Scenario treatment has proved to be a useful mechanism to represent the uncertainty. The risk-measure given by the composite function of the expected trading profit and the *Reaching Probability* functional is considered, instead of optimizing mean values alone. The modelling approach to select candidate purchasing and selling contracts has been proved to be successful for maximizing the mean-risk function and, thus, balancing the net revenue from the contracts exploitation and the trading in the spot market under each scenario group along the time horizon. It is worth noting that the approach can be used for portfolio structuring by energy service providers in addition to energy generation companies. It can also be easily adapted to the existence of fixed costs associated with the contracts, and to the consideration of contract cancellations. The model considers the influence of the constraints imposed by the Kyoto protocol-based pollutant emission regulations. A specialization of the *Branch-and-Fix Coordination* algorithm has been proposed for optimizing multistage stochastic problems with 0-1 variables in the first stage and continuous variables in the other stages.

Appendix

Different types of implementations can be considered within the algorithmic framework presented in Section 1.4. This Appendix presents the version that we favor.

We use the *depth first* strategy for selecting the branching *TNF*. The branching criterion followed for the energy selling contracts is to perform first a "branching on the ones" and, afterwards, a "branching on the zeros". However, the branching criterion for the energy purchasing contracts performs first a "branching on the zeros" and, afterwards, a "branching on the ones".

Given the significance of the ν variables, they have the highest priority for branching purposes. The priority is given according to the non-increasing scenario weight criterion. It is branched first on the ones and then on the zeros.

Another topic of interest is the branching ordering for the energy trading contracts. We have considered a *static* ordering as follows: The highest priority is given to the selling contracts over the purchasing contracts. Within each category, the priority is given according to the non-increasing expected income criterion for the selling contracts

$$\sum_{g \in \mathcal{G}} w^g \mu_f^g \iota_f^g \quad \forall f \in \mathcal{F}^{ca},$$

and the non-decreasing expected cost criterion for the purchasing contracts,

$$\sum_{g \in \mathcal{G}} w^g \lambda_k^g \frac{1}{2} \big(\overline{\theta}_{k,t(g)} + \underline{\theta}_{k,t(g)} \big) \quad \forall k \in \mathcal{K}^{ca}.$$

Notice that a *TNF* can be pruned for any of the following reasons: (a) the *LP* relaxation of the scenario *cluster* model (1.37) attached to any node member is infeasible, (b) there is no guarantee that a better solution than the *incumbent* one can be obtained from the best descendant *TNF integer set* (currently, it is based on the *LP* objective function value), (c) the *LP* model (1.40) attached to the *TNF integer set* is infeasible or its solution value is not better than the value of the *incumbent* solution, in case that all γ and ν variables have already been branched on / fixed at for the family, and (d) see below the case where some integer valued γ or ν variable have not yet been branched on / fixed at.

Once a *TNF* has been pruned, the same branching criterion allows us to perform either a "branching on the zeros (resp., "on the ones") in the case that the *TNF* has already been "branched on the ones" for the energy selling contracts (resp., "on the zeros" for the energy purchasing contracts), or a *backtracking* to the previous branched *TNF* if it has already been branched "on the ones" and "on the zeros".

It is worth noting, see also [EGMP07], that if Z^{LP}_{TNF} (1.40) $= Z^{LP}_{CON}$ (1.41), then the related *TNF* can be pruned. Notice that the solution space defined by model (1.40) is included in the space defined by model (1.41) and, so, in this case, there is not a greater solution value than Z^{LP}_{TNF} from the descendant nodes to be obtained by branching on the non-yet branched on / fixed at γ and ν variables. For the same reason, the family is also pruned if Z^{LP}_{CON} is not greater than the *incumbent* solution value.

To gain computational efficiency, the optimization of model (1.41) should not be performed for a small number of the γ and ν variables already branched on / fixed at in the given *TNF*. Let λ denote the minimum fraction of the branched on / fixed at variables that is required for optimizing the model, where $0 \leq \lambda \leq 1$.

For presenting the detailed *BFC* algorithm to solve model (1.33), let the following additional notation.

LP_p, *LP* relaxation of the scenario *cluster*-related model (1.37) attached to a node member from the *BF* tree $\mathcal{C}_p$ in the given *TNF*, for $p = 1, \ldots, q$.

Z^{LP}_p, solution value of the *LP* model LP_p, for $p = 1, \ldots, q$.

$\overline{Z}^{IP}$, upper bound of the solution value of the original model (1.33) to be obtained from the best descendant *TNF integer set* for a given family. It is computed as $\overline{Z}^{IP} = \sum_{p=1,\ldots,q} Z^{LP}_p$.

$\underline{Z}^{IP}$, lower bound of the solution value of the original model (1.33). It is the *incumbent* solution value, and it ends up with the value of the optimal solution.

By convention, $Z^{LP}_* = -\infty$ for any infeasible LP_* model.

Procedure

Step 0: Initialize $\underline{Z}^{IP} := -\infty$.

Assign $\sigma_r := 1$ where r represents a scenario or an energy selling contract, and $\sigma_r := 0$ if it is an energy purchasing contract, for $r \in \Omega \bigcup \mathcal{R}$.

Step 1: Solve the q independent *LP* models LP_p, $\forall p = 1, \ldots, q$, and compute $\overline{Z}^{IP}$. If the γ and ν variables take integer values and the *nonanticipativity* constraints (1.38) and (1.39) are satisfied, then the optimal solution to the original model (1.33) has been found and, so, $\underline{Z}^{IP} := \overline{Z}^{IP}$ and stop.

Step 2: If there is any γ or ν variable that takes continuous values or there is a γ variable that takes different values for some of the q scenario *clusters* then go to Step 3.

Solve the *LP* model (1.40) to satisfy the constraints (1.35) for the v variables in the given *TNF integer set* (in this case, the set of *root* nodes in the *BF* trees). Notice that the solution value is denoted by Z^{LP}_{TNF}.

Update $\underline{Z}^{IP} := Z^{LP}_{TNF}$. If $\underline{Z}^{IP} = \overline{Z}^{IP}$ then the optimal solution to the original model has been found and stop.

Step 3: Initialize $r := 1$ and go to Step 5.

Step 4: Reset $r := r + 1$.

If $r = |\Omega \bigcup \mathcal{R}| + 1$ then go to Step 9.

Step 5: Branch $\gamma^p_r := \sigma_r$, $\forall p = 1, \ldots, q$.

Step 6: Solve the linear models LP_p, $\forall p = 1, \ldots, q$ and compute $\overline{Z}^{IP}$.

If $\overline{Z}^{IP} \leq \underline{Z}^{IP}$ then go to Step 8.

If there is any γ or ν variable that takes continuous values or there is a γ variable that takes different values for some of the q scenario *clusters* then go to Step 4.

If all the v variables take the same value for all scenario *clusters* $p = 1, \ldots, q$ then update $\underline{Z}^{IP} := \overline{Z}^{IP}$ and go to Step 8.

Solve the *LP* model (1.40) to satisfy the constraints (1.35) for the v variables in the given *TNF integer set*.

Update $\underline{Z}^{IP} := \max\{Z^{LP}_{TNF}, \underline{Z}^{IP}\}$. If $r = |\Omega \bigcup \mathcal{R}|$ then go to Step 8.

If $r < \lambda|\Omega \bigcup \mathcal{R}|$ then go to Step 4.

Step 7: Solve the *LP* model (1.41), where the 0-1 continuous variables are the non-yet been branched on / fixed at γ and ν variables in the solution of the current *TNF*. Notice that the solution value is denoted by Z^{LP}_{CON}. If $Z^{LP}_{TNF} < Z^{LP}_{CON}$ and $Z^{LP}_{CON} > \underline{Z}^{IP}$ then go to Step 4.

Step 8: Prune the branch.

If $\gamma^p_r = \sigma_r$, $\forall p = 1, \ldots, q$ then go to Step 11.

Step 9: Reset $r := r - 1$.

If $r = 0$ then stop, since the optimal solution $\underline{Z}^{IP}$ has been found, if any.

Step 10: If $\gamma_r = 1 - \sigma_r$, $\forall p = 1, \ldots, q$ then go to Step 9.

Step 11: Branch $\gamma_r := 1 - \sigma_r$ $\forall p = 1, \ldots, q$.

Go to Step 6.

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