A mixed 0–1 nonlinear approach for the collision avoidance in ATM: Velocity changes through a time horizon

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***Abstract*—A mixed 0–1 nonlinear model for the Collision Avoidance in Air Traffic Management (ATM) problem is presented. The aim of the problem consist of deciding the best strategy for an arbitrary aircraft configuration such that all conflicts in the airspace are avoided where a conflict is the loss of the minimum safety distance that two aircraft have to keep in their flight plans. The optimization model is based on geometric constructions. It requires to know the initial flight plan (coordinates, angles and velocities in each time period). The objective is the minimization of the acceleration variations where aircraft are forced to return to the original flight plan when no aircraft are in conflict. A linear approximation by using iteratively Taylor polynomials is developed to solve the problem in mixed 0–1 linear terms.**

***Index Terms*—Air Traffic Management (ATM), collision avoidance, mixed 0–1 nonlinear optimization, Taylor approximations.**

I. INTRODUCTION AND PROBLEM DESCRIPTION

Collision avoidance within the Air Traffic Management (ATM) is an interesting problem nowadays due to the increasing demand in the aerial field. On the other hand, the interest for the denominated “free flight” is another incentive for studying carefully this problem. There is a recent paper by EUROCONTROL [7], aimed to specify the required capabilities of Medium-Term Conflict Detection (MTCD) for Air Traffic Management Systems. The MTCD system is required to detect and notify the controller about the probable loss of the required separation between two aircrafts, an aircraft penetrating restricted airspace, or an aircraft blocking airspace that might have been used by some other ones. That paper considers that, although flight data and trajectories are provided to the MTCD, some uncertainty is likely to be on the trajectories. It distinguishes too between tactical and planned trajectories.

The previous works tackling the problem have been developed from different fields in mathematics and engineering. Kuchar and Yang (2000) [8] reference several works devoted to Collision Avoidance classified under their own criteria. Martín-Campo (2010) [10] presents an extended state-of-the-art about the problem. In our paper, we provide an interesting point of view from the optimization field, obtaining good results in a difficult problem. Early works based on mathematical optimization give weak approximations to the real problem like Richards and How (2002) [12] who consider uninhabited aerial vehicle. Pallottino et al. (2002) [11] propose two independent models based on mixed integer linear optimization in order to solve the problem by considering velocity changes (VC) and heading angle changes (HAC), whose geometric construction is very useful for this work. Dell’Olmo and Lulli (2003) [6] combine exact optimization software with an heuristic approach for large problems. Christodoulou and Costoulakis (2004) [5] propose a mixed integer nonlinear optimization model based on the geometric construction proposed in [11] considering the three dimensions in the airspace, and assuming the safety area for each aircraft like a sphere, when the safety areas are cylinders. In Alonso-Ayuso et al. (2010) [2] we use the geometric idea proposed in [11] and include altitude changes as a new maneuver in order to avoid the infeasible situations in the velocity changes model, and, then, resulting the so-called Velocity and Altitude Changes model (VAC). In Alonso-Ayuso et al. (2011) [4] two different integer linear optimization models are proposed where they propose velocity and altitude changes as maneuvers to avoid conflicts in a medium-term environment.

In this work we present a strong tightening of the VC model in order to include continuity in the velocity changes and to consider possible nonlinear trajectories for each aircraft. Including continuity in the speed maneuvers is not an easy goal due to the geometric construction that causes nonlinear trigonometric constraints (where some tangents are involved in) that have been reduced in a difficult fraction. For this purpose, a mixed 0–1 nonlinear optimization (MINLO) is proposed in this work.

This paper is organized as follows. In Section II, the general features of the problem required to build the MINLO model are described. In Section III a formulation of the VCTH model is presented. Section IV introduces the algorithmic approach proposed for problem solving. Section V reports the main computational results and, finally, Section VI draws the main conclusions and outlines the main lines of future research.

II. Problem statement

Aerial sectors, a given number F of aircraft that fly in a certain sector and T the number of time periods that discretize the time horizon are considered. In the following, all elements related to the VCTH model are detailed.

Sets

- $\mathcal{F}$, set of aircraft in the airspace.
- $\mathcal{T}$, set of time periods.

Parameters

- s, safety distance between aircraft, usually, 2.5 nautical miles.
- e, distance bound to consider a pair of aircraft.
- w_1, w_2, weight (between 0 and 1) for each objective function term.
- div, integer parameter greater than 1 to be considered for the bounds of some variables.

For all $t \in \mathcal{T}$:

- I_t, length of the time period between times instants $t-1$ and t.

For all $f \in \mathcal{F}$ and $t \in \mathcal{T}$:

- x_f^{*t}, y_f^{*t}, initial configuration of position i.e. abscissa and ordinate, for aircraft f in time period t, respectively.
- d_f^{*t}, covered distance for aircraft f during time period t in the initial configuration.
- v_f^{*t}, initial velocity configuration for aircraft f in time period t.
- a_f^{*t}, initial acceleration configuration for aircraft f in time period t.
- r_f^t, safety radius for each aircraft f in time period t, usually 2.5 nautical miles (nm).
- $\underline{v}_f^t, \overline{v}_f^t$, minimum and maximum velocities allowed for aircraft f in time period t, respectively.
- $\underline{a}_f^t, \overline{a}_f^t$, minimum and accelerations allowed for aircraft f in time period t, respectively.
- m_f^{*t}, direction of motion in $(-\pi, \pi]$ for aircraft f in time period t.
- $\hat{x}_f^t, \hat{y}_f^t$, position parameters to be updated in the Taylor approximation for aircraft f in time period t.
- $\hat{d}_f^t$, distance parameter to be updated in the Taylor approximation for aircraft f in time period t.
- $\hat{v}_f^t$, velocity parameter to be updated in the Taylor approximation for aircraft f in time period t.
- $c_{ft}^{a^+}, c_{ft}^{a^-}$, costs for positive and negative acceleration changes for aircraft f in time period t, respectively.
- c_{ft}^v, costs for the difference between the initial and optimal velocity configuration for aircraft f in time period t.
- c_{ft}^d, costs for the difference between the initial and optimal covered distance for aircraft f in time period t.

For all $f \in \mathcal{F}$:

- $x_f^{*t_f^d}, y_f^{*t_f^d}$, departure positions (abscissa and ordinate) for aircraft f.
- $x_f^{*t_f^r}, y_f^{*t_f^r}$, arrival positions (abscissa and ordinate) for aircraft f.
- d_f^{tot}, total length of the polygonal of the trajectory for aircraft f.
- t_f^d, t_f^r, scheduled departure and arrival times for flight f.

Data preprocessing

For all $f \in \mathcal{F}$ and $t \in \mathcal{T}$:

- $\overline{x}_f^t, \overline{y}_f^t$, upper bounds for variables x and y, respectively.
- $\underline{x}_f^t, \underline{y}_f^t$, lower bounds for variables x and y, respectively.

For all $i, j \in \mathcal{F} : i < j$, for all $t \in \mathcal{T}$:
$t = \left\{ \max\{t_i^d, t_j^d\} + 1, \ldots, \min\{t_i^r, t_j^r\} - 1 \right\}$:

- fc_{ij}^t, 0–1 parameter that detects if there is a “false conflict” between aircraft i and j in time period t being a false conflict the situation such that the geometric construction gives a conflict, but the two aircraft are not in conflict.
- p_{ij}^t, 0–1 parameter that will be 1 if the pair of aircraft i and j will not be taken into account in time period t for conflict resolution. This parameter depends on the criterion decided by the ATC. Notice that this parameter will be 1 if $fc_{ij}^t = 1$.
- ip_{ijt}, intersection point between the straight line trajectories of the corresponding polygonal segment for time period t for aircraft i and j, provided that the trajectories are not parallel o coincident.
- d_{ijt}^1, distance between the i aircraft position and the intersection point ip_{ijt} in time period t.

d^2_{ijt}, distance between points $\big(x^t_i + \cos(m^{*t}_i), y^t_i + \sin(m^{*t}_i)\big)$ and ip_{ijt} for aircraft i and j in time period t.

Variables

For all $f \in \mathcal{F}$ and for all $t \in \mathcal{T}$:

x^t_f, y^t_f, the position (i.e., abscissa and ordinate) of aircraft f in time period t, for $f \in \mathcal{F}$, $t \in \mathcal{T}$, respectively.

a^t_f, acceleration variation for aircraft f in time period t, for $f \in \mathcal{F}$, $t \in \mathcal{T}$. This variable is real and can be divided in two positive variables, say, a^{t+}_f and a^{t-}_f, such that $a^t_f = a^{t+}_f - a^{t-}_f$ as a traditional way in LP.

a^{t+}_f, positive acceleration variation for aircraft f in time period t, for $f \in \mathcal{F}$, $t \in \mathcal{T}$.

a^{t-}_f, negative acceleration variation for aircraft f in time period t, for $f \in \mathcal{F}$, $t \in \mathcal{T}$.

v^t_f, velocity for aircraft f in time period t, for $f \in \mathcal{F}$, $t \in \mathcal{T}$.

d^t_f, covered distance for aircraft f during time period t, for $f \in \mathcal{F}$, $t \in \mathcal{T}$.

γ^t_f, auxiliary 0–1 variable to model the case of early or delay for aircraft f in time period t, for $f \in \mathcal{F}$, $t \in \mathcal{T}$.

For all $i, j \in \mathcal{F} : i < j$, for all $t \in \mathcal{T}$: $t = \big\{ \max\{t^d_i, t^d_j\} + 1, \ldots, \min\{t^r_i, t^r_j\} - 1\big\}$ and $n = 1, \ldots, 8$:

δ^n_{ijt}, auxiliary 0–1 variables for modeling the or constraints for $n = 1, \ldots, 4$ and for $i < j \in \mathcal{F}$, $t \in \mathcal{T}$.

III. The partial formulation of the VCTH model

Now, a partial formulation for the VCTP model is presented below. The full formulation of the model is given in the full version [3] of the paper. However, the formulation below presents the most important constraints of the problem.

$$\min w_1 \sum_{f\in\mathcal{F}} \sum_{t\in\mathcal{T}} \left(\frac{c^{a^+}_{ft} a^{t+}_f}{\overline{a}^t_f - \underline{a}^t_f} + \frac{c^{a^-}_{ft} a^{t-}_f}{\overline{a}^t_f - \underline{a}^t_f} \right) + w_2 \sum_{f\in\mathcal{F}} \sum_{t\in\mathcal{T}} c^d_{ft} \beta^t_f \tag{1}$$

subject to $\forall f \in \mathcal{F}, \forall t \in \mathcal{T} : t = \{t^d_f + 1, \ldots, t^r_f\}$

$$\underline{v}^t_f \leqslant v^{t-1}_f + a^t_f I_t \leqslant \overline{v}^t_f \tag{2a}$$

$$\underline{a}^t_f \leqslant a^t_f \leqslant \overline{a}^t_f \tag{2b}$$

$\forall f \in \mathcal{F}, \forall t \in \mathcal{T} : t = \{t^d_f + 1, \ldots, t^r_f\}$

$$d^t_f = v^{t-1}_f I_t + \frac{1}{2}(a^{t+}_f - a^{t-}_f) I^2_t \tag{3}$$

$\forall i, j \in \mathcal{F} : i < j \wedge p_{ij} = 0, \forall t \in \mathcal{T} : t = \big\{ \max\{t^d_i, t^d_j\} + 1, \ldots, \min\{t^r_i, t^r_j\} - 1\big\}$

$$v^t_i\big(\cos(m^{*t}_i)(1 - pc^t_{ij}) - \sin(m^{*t}_i) pc^t_{ij}\big) - v^t_j\big(\cos(m^{*t}_j)(1 - pc^t_{ij}) - \sin(m^{*t}_j) pc^t_{ij}\big) \leqslant (\overline{v}^t_i + \overline{v}^t_j)(1 - \delta^1_{ijt}) \tag{4a}$$

$$- v^t_i\big(h^t_i(1 - pc^t_{ij}) + h'^t_i pc^t_{ij}\big) + v^t_j\big(h^t_j(1 - pc^t_{ij}) + h'^t_j pc^t_{ij}\big) \leqslant \Big((\overline{v}^t_i |h^t_i| + \overline{v}^t_j |h^t_j|)(1 - pc^t_{ij}) + (\overline{v}^t_i |h'^t_i| + \overline{v}^t_j |h'^t_j|) pc^t_{ij} \Big)(1 - \delta^1_{ijt}) \tag{4b}$$

$$\delta^1_{ijt} + \delta^2_{ijt} + \delta^3_{ijt} + \delta^4_{ijt} = 1 \tag{4c}$$

$\forall f \in \mathcal{F}, \forall t \in \mathcal{T} : t = \{t^d_f + 1, \ldots, t^r_f\}$

$$\sum_{\ell=1}^{t} d^\ell_f - \sum_{\ell=1}^{t} d^{*\ell}_f \leqslant \frac{d^{*t}_f}{div} \gamma^t_f \tag{5a}$$

$$\sum_{\ell=1}^{t} d^\ell_f - \sum_{\ell=1}^{t} d^{*\ell}_f - \varepsilon \geqslant (-\frac{d^{*t}_f}{div} - \varepsilon)(1 - \gamma^t_f) \tag{5b}$$

$$x^t_f - x^{*t}_f - \Big(\sum_{\ell=1}^{t} d^\ell_f - \sum_{\ell=1}^{t} d^{*\ell}_f \Big) \cos(m^t_f) \leqslant (\overline{x}^t_f - x^{*t}_f + \frac{d^{*t}_f}{div})(1 - \gamma^t_f) \tag{5c}$$

$$x^t_f - x^{*t}_f - \Big(\sum_{\ell=1}^{t} d^\ell_f - \sum_{\ell=1}^{t} d^{*\ell}_f \Big) \cos(m^{*t}_f) \geqslant (\underline{x}^t_f - x^{*t}_f - \frac{d^{*t}_f}{div})(1 - \gamma^t_f) \tag{5d}$$

$$y^t_f - y^{*t}_f - \Big(\sum_{\ell=1}^{t} d^\ell_f - \sum_{\ell=1}^{t} d^{*\ell}_f \Big) \sin(m^{*t}_f) \leqslant (\overline{y}^t_f - y^{*t}_f + \frac{d^{*t}_f}{div})(1 - \gamma^t_f) \tag{5e}$$

$$y^t_f - y^{*t}_f - \Big(\sum_{\ell=1}^{t} d^\ell_f - \sum_{\ell=1}^{t} d^{*\ell}_f \Big) \sin(m^{*t}_f) \geqslant (\underline{y}^t_f - y^{*t}_f - \frac{d^{*t}_f}{div})(1 - \gamma^t_f) \tag{5f}$$

$$\sum_{\ell=1}^{t} d^\ell_f - \sum_{\ell=1}^{t} d^{*\ell}_f \leqslant \beta^t_f \tag{6a}$$

$$\sum_{\ell=1}^{t} d^{*\ell}_f - \sum_{\ell=1}^{t} d^\ell_f \leqslant \beta^t_f \tag{6b}$$

$\forall f \in \mathcal{F}, \forall t \in \mathcal{T} : t = \{t^d_f + 1, \ldots, t^r_f - 1\}$

$$\underline{d}^t_f = d^{*t}_f - \frac{d^{*t}_f}{div} \leqslant d^t_f \leqslant d^{*t}_f + \frac{d^{*t}_f}{div} = \overline{d}^t_f \tag{7a}$$

$$\underline{x}^t_f = x^{*t}_f - \frac{d^{*t}_f}{div} \cos(m^{*t-1}_f) \leqslant x^t_f \leqslant x^{*t}_f + \frac{d^{*t}_f}{div} \cos(m^{*t}_f) = \overline{x}^t_f \tag{7b}$$

$$\underline{y}^t_f = y^{*t}_f - \frac{d^{*t}_f}{div} \sin(m^{*t-1}_f) \leqslant y^t_f \leqslant y^{*t}_f + \frac{d^{*t}_f}{div} \sin(m^{*t}_f) = \overline{y}^t_f \tag{7c}$$

$\forall f \in \mathcal{F}, \forall t \in \mathcal{T} : t = \{t^d_f, \ldots, t^r_f\}$

$$x^t_f, y^t_f, a^t_f \in \mathbb{R} \tag{8a}$$

$$v^t_f, a^{t+}_f, a^{t-}_f, d^t_f, \beta^t_f \in \mathbb{R}^+ \tag{8b}$$

$\forall i, j \in \mathcal{F} : i < j \wedge p_{ij} = 0, \forall t \in \mathcal{T} : t = \big\{ \max\{t^d_i, t^d_j\} + 1, \ldots, \min\{t^r_i, t^r_j\} - 1\big\}$

$$\delta^1_{ijt}, \delta^2_{ijt}, \delta^3_{ijt}, \delta^4_{ijt} \in \{0, 1\} \tag{8c}$$

$\forall f \in \mathcal{F}, \forall t \in \mathcal{T} : t = \{t^d_f, \ldots, t^r_f\}$:

$$\gamma^t_f \in \{0, 1\} \tag{8d}$$

The objective function (1) gives the optimization criterion for the model. It has two terms, one for minimizing the sum of the absolute values of the accelerations and the other one for forcing aircraft to return to the initial configuration, where the parameters w_1 and w_2

emphasize one term over the other. If the second term of the objective function is contemplated, it must be accompanied by constraints (6). Constraints (2) avoid the velocity and the acceleration to be bigger or smaller than the upper or lower bound, respectively. Constraints (3) update the covered distance by an aircraft after the changes in its configuration in time period $t \in \mathcal{T}$. Constraints (4) detect and solve the conflicts in the airspace. Notice that only the first block of equations is presented where the next terms are nonlinear ones (there exist another block analogue to this one):

$$h_i^t = \frac{(x_i^t - x_j^t)s + (y_i^t - y_j^t)\sqrt{(x_i^t - x_j^t)^2 + (y_i^t - y_j^t)^2 - s^2}}{(x_i^t - x_j^t)\sqrt{(x_i^t - x_j^t)^2 + (y_i^t - y_j^t)^2 - s^2} - (y_i^t - y_j^t)s}\cos(m_i^{*t}) - \sin(m_i^{*t}) \tag{9a}$$

$$h_j^t = \frac{(x_i^t - x_j^t)s + (y_i^t - y_j^t)\sqrt{(x_i^t - x_j^t)^2 + (y_i^t - y_j^t)^2 - s^2}}{(x_i^t - x_j^t)\sqrt{(x_i^t - x_j^t)^2 + (y_i^t - y_j^t)^2 - s^2} - (y_i^t - y_j^t)s}\cos(m_j^{*t}) - \sin(m_j^{*t}) \tag{9b}$$

Constraints (5) update the positions in each time period $t \in \mathcal{T}$, according to the previous changes in velocity made until the current time period. Notice that only the case of an earlier arrival to the predicted destination point is considered. Constraints (6) transform the second term of the objective function in a linear function, since it minimizes an absolute value between the covered distances in the initial flight plan and the covered distances in the resolution. Constraints (7) force some variables to be into fixed bounds, in order to calculate the big M in the position projection constraints. Note: It is well known that the smaller the “big M” is the tighter the model is and, then, hopefully smaller computing time is required.

IV. ALGORITHMIC APPROACH

For solving iteratively the linearized model, the algorithmic approach described in Almiñana et al. (2007) [1] in a different context inspires the work presented in this paper. It is based on a iterative optimization by starting with the initial configuration and updating the input parameters where the derivatives are centered until a stop criterion is satisfied.

First, the nonlinear constraints have to be linearized by using Taylor polynomials, so the new mathematical expression for each inequality ($n = 1, \ldots, 4$) can be expressed as follows,

$$\left\{ C_n + \left(\frac{\partial C_n}{\partial(v^t_{i \text{ and } j}, x^t_{i \text{ and } j}, y^t_{i \text{ and } j})} \right) \right\}_{|(\hat{v}^t_{i \text{ and } j}, \hat{x}^t_{i \text{ and } j}, \hat{y}^t_{i \text{ and } j})} \begin{pmatrix} v_{i \text{ and } j} - \hat{v}^t_{i \text{ and } j} \\ x_{i \text{ and } j} - \hat{x}^t_{i \text{ and } j} \\ y_{i \text{ and } j} - \hat{y}^t_{i \text{ and } j} \end{pmatrix} \le M(1 - \delta^n_{ijt}).$$

The algorithm for the resolution is presented below:

Step 1. Computing the values of the nonlinear constraints $\forall i, j \in \mathcal{F} : i < j$ and $\forall f \in \mathcal{F}$, such that $v_i^t, v_j^t, x_i^t, x_j^t, y_i^t, y_j^t$ are replaced with $\hat{v}_i^t, \hat{v}_j^t, \hat{x}_i^t, \hat{x}_j^t, \hat{y}_i^t, \hat{y}_j^t$, respectively. In the first iteration, $\hat{v}_i^t = v_i^{*t}, \hat{v}_j^t = v_j^{*t}, \hat{x}_i^t = x_i^{*t}, \hat{x}_j^t = x_j^{*t}, \hat{y}_i^t = y_i^{*t}, \hat{y}_j^t = y_j^{*t}, \hat{d}_f^t$.

Step 2. Solving the mixed zero-one model, where the nonlinear constraints are substituted by their linear approximations. Let d_f^t be the optimal values of the respective variables for $f \in \mathcal{F}$ and $t \in \mathcal{T}$.

Step 3. Optimality test. If the following condition is satisfied then stop, the quasi-optimal solution has been obtained. Otherwise, go to Step 4.

$$\sum_{f \in \mathcal{F}} \sum_{t \in \mathcal{T}} (d_f^t - \hat{d}_f^t)^2 \le \varepsilon$$

where ε is a positive tolerance.

Step 4. Updating the covered distance, the acceleration, the velocity and the positions and go to Step 1.

Now, the linearized model can be successively solved by using the optimization engine of choice for mixed 0–1 linear models.

V. CASE STUDIES

Table I shows the most important results (averages), after solving a huge variety of simulated cases with different number of aircraft flying in the same aerial sector. An extensive computational experience is reported in the full paper. The headings are as follows:

- *Case*: Case configuration: CAAA denotes number of aircraft (AAA).
- *nit*: Number of iterations in the Taylor approximation.
- z_{lp}: Value of the objective function in the continuous linear relaxation.
- z_s: Value of the bound after performing the CPLEX cut identification and appending at node 0
- z_{ip}: Value of the objective function for the optimal solution of the problem.
- GAP_{lp}: $\frac{z_{ip} - z_{lp}}{z_{ip}}$ (optimality gap of the LP solution)
- GAP_s: $\frac{z_{ip} - z_s}{z_{ip}}$ (optimality gap of the tightened LP solution)
- nb: Number of times that there is branching
- nn: Number of CPLEX branch-and-cut nodes
- t_{lp}: Time (secs.) to obtain the z_{lp} value
- t_s: Time (secs.) to obtain the z_s value.
- t_{ip}: Time (secs.) to obtain the z_{ip} value.
- nc: Total number of cuts performed by CPLEX.

Notice that the minimum, average and maximum GAPs are reported in Table I.

In the biggest cases of study, the number of constraints is about 11000 whereas the number of variables is about 30000.

VI. Conclusions

A mixed 0–1 nonlinear optimization model has been presented in order to solve the collision avoidance problem for ATM. Due to four hard nonlinear constraint types, the model has been successively linearized by using iteratively Taylor polynomials approximations since no optimization engine to solve mixed integer nonlinear models could solve the large-scale instances of the problem, as we know. Currently, we are working on a metaheuristic scheme so called Variable Neighbourhood Decomposition Search (VNDS) [9] to hopefully produce good solutions of the problem in very short computing time, although the optimality cannot be guaranteed at this point in time.

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TABLE I
COMPUTATIONAL RESULTS

Case	**nit**	z_{lp}	z_s	z_{ip}	GAP_{lp}	GAP_s	**nb**	**nn**	t_{lp}	t_s	t_{ip}	**nc**
C020	3.4	0.0013	0.0184	0.0186	0.6848 0.9291 1.0000	0.0000 0.0101 0.0957	51	29.76	0.76	4.60	9.83	101.8
C025	5.0	0.0010	0.0225	0.0227	0.7999 0.9572 1.0000	0.0000 0.0097 0.0864	111	28.14	1.37	9.31	19.95	185.4
C030	4.3	0.0011	0.0264	0.0268	0.8630 0.9599 1.0000	0.0000 0.0139 0.0609	94	34.94	1.52	9.34	24.25	221.0
C035	5.6	0.0017	0.0312	0.0315	0.8781 0.9463 1.0000	0.0000 0.0114 0.0508	129	42.95	2.42	17.52	54.54	351.8
C040	6.6	0.0036	0.0369	0.0374	0.6406 0.9038 1.0000	0.0000 0.0117 0.0584	99	36.96	3.23	20.13	54.41	286.6
C045	7.2	0.0045	0.0458	0.0457	0.6084 0.9011 0.9969	0.0000 0.0009 0.0449	157	45.18	4.70	31.43	116.55	505.0
C050	5.7	0.0034	0.0486	0.0496	0.8043 0.9321 1.0000	0.0000 0.0189 0.0893	90	47.58	4.03	27.55	81.05	510.9