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***Abstract*—The usual Muti-objective Evolutionary Algorithms (MOEAs) are designed to provide acceptable results in short-term executions. The principal reason is that most MOEAs converge to sub-optimal regions in a fast way, as result the search process could stagnate. Therefore, in long-term executions some Evolutionary Algorithms (EAs) are not capable to provide quality solutions. Despite the fact that in literature has been showed that algorithms based in Differential Evolution (DE) perform significally better than Genetics Algorithms (GA) in short-term executions, in long term executions is unkown their behavior. It is well known the stability issues of parameter-configuration of DE and the accelerated convergence that in general results in poor quality solutions, therefore GA operators can be ideal with this setting. In this paper we propose several improvements to the Simulated Binary Crossover (SBX) where are considered the criteria stop as part of the search process. Experimental validation with DTLZ, WFG and UFs problems shows the benefits of the proposal.**

The abstract goes here.

I. Introduction

Evolutionary Algorithms (EA) have become a promising alternative in several practical problems where is not suitable a deterministic approach. Multi-objective Optimization Problems (MOP) involves the simultaneous optimization of two or more objective functions that are usually in conflict. A continuous minimization multi-objective problem can be defined as follows:

$$\begin{array}{ll} minimize & F(x) = (f_1(x), f_2(x), ..., f_m(x)) \\ subject \quad to & x \in \Omega \end{array} \tag{1}$$

where $x = (x_1, ..., x_n) \in R^n$ indicate a decision variable vector, n correspond to the number of decision variables, Ω is the feasible space, $F : \Omega \rightarrow R^m$ which consist of m objective functions and R^m is known as the *objective space*.

Particularly, a MOP which consists in minimization of the m objective functions and given two decision variable vectors $x, y \in \Omega$, x dominates y denoted by $x \prec y$, iff $f_i(x) \leq f_i(y)$ for all objectives $\{1, ..., m\}$, and $f_i(x)$ is better than $f_i(y)$ in at least one objective function $F(x) \neq F(y)$. Accordingly this, the solution x is not worse than y in any of the objectives and x is strictly better than y in at least one objective. The Pareto dominance is defined as the set of the best solutions that are not dominated by any feasible solution. A decision variable vector $x^* \in \Omega$ is known as the Pareto optimal solution if does not exist any solution $x \in \Omega$ that dominates x^*. The Pareto set correspond to the set of all Pareto optimal decision vectors and the Pareto front are the images of the Pareto set. Principally, the goal in multi-objective optimization is to obtain an approximation of the Pareto front. Therefore is required to obtain diverse and converged solutions among the Pareto front.

In the last decade several paradigms of MOEAs have been arised REF, among them, the Non-Dominated Sorting Genetic Algortihm II (NSGA-II) ref, the MOEA based on decomposition (MOEA/D) ref, the S-metric Selection Evolutionary Multi-objective Optimization Algorithm (SMS-EMOA) ref are three representative methods that can be considered as state of the art.

Differential Evolution (DE) is popular algorithm which outperform the Genetic Algorithms (GAs) ref. However, it is well known that DE suffers of diversity issues and can present important drawbacks in long-term executions. Based on the acelerated convergence it can locate individuals in a sub-optimal region at the first stages of the execution and the rest of the function evaluations could be wasted. To deal with this issues, it is evident a bias of design in the recent algorithms, most of them are designed with strategies that considers the criteria stop as part of the search process, such as adaptive parametes ref, population reduction ref and differents mutation strategies ref. A substantial weakness in DE is the high paramete dependence, also there exist a different behabior between single-objective and multi-objective problems which are related with the decision variable space diversity(referencia GDE3).

On the other hand GAs imply a less agressive behavior, this can be a reason of the preference of DE than GA in short-term executions both in continuous and discrete domains ref, ref. However, considering a long-term executions GAs has better properties, likely for the flexibility of the operators involved, as the selection, crossover and mutation.

In this paper the principal issue addresed is to provide a crossover operator which considers the criteria stop as part of the search process, guiding to a gradual change between exploration and intensification. The Simulated Binary

Crossover (SBX) is analyzed and modified to offer a suiteable perfomance in long-term executions. Specifically, in this paper a variant of the SBX is proposed with the aim of better exploring different regions of the search space.

The rest of this paper is organized as follows. In Section II provides a detailled review of the literature related with the SBX operator, also a brief review of the state of the art of MOEAs is showed. Section III describe the key components of the SBX, therefore is showed a proposal. The experimental validation of the proposal is shown in Section IV. Finally, conslusions and some lines of future work are given in Section V.

II. Literature Review

This section is devoted to review the particularities of the Simulated Binary Crossover (SBX) discussed in this work. Thereafter, are briefly explained some popular algorithms, one of them widely used in mono-objective problems, and the rest designed for multi-objective problems.

A. Multi-objective Evolutionary Algorithms

Currently, there exist a large number of MOEAs that follows different design principles. In orde to better classfy them, several taxonomies have been propose ref21. Attending to the principles of design, MOEAs can be based on Pareto dominance, indicators and/or decomoposition ref4. Currently, none of these methods have reported a clear advantage over the other ones. Particuarly, the experimental validation has been carry out by including the Non-Dominated Sorting Genetic Algorithm (NSGA-II) ref, the MOEA based on Decomposition ref, and the S-Metric Selection Evolutionary Multi-objective Optimization Algorithm (SMS-EMOA) ref. Therefore, they are repsentative methos of the domination-based, decomposition-based and indicator based paradigms, respectively.

B. Domination-Based MOEAs - NSGAII

One of the most reconized paradigms are the domination based algorithms, particularly this family are based on the application of the dominance relation to design different components of the EAs. Since the dominance relation does not inheritly promotes diversidy in the objective space, auxiliary techniques such as niching crowding and/or clustering are usually integrated with the aim of obtain an aceptable spread and diversity in the solution space. One of the most problematic issues of implement the dominance relation is related with the number of objectives, therefore if the number of function objective increase the selection pressure is decremented considerabily, based in this some strategies have been developed to deal with the many-objective problems.

Probably, the most popular technique of this group is the NSGA-II. This algorithm ref2 implement a special parent selection operator. This operator is based on two mechanism: fast-non-dominated-sort and crowding. The first one tend to provide a convergence and the second one promotes the presetvation of diversity in the objective space. Last versions is the NSGA-II and is designed to deal with many-objective problems.

C. Decomposition-Based MOEAs - MOEA/D

Decomposition-based MOEAs ref transform a MOP in a set of single-objective optimization problems that are considered simultaneously ref3. This tranformation can be achieved throguh several approaches. The most popular of them is based in a weighted Tchebycheff function, therefore it is necessary provide weigth vectors well distribuited with the aim of obtain well-spread solutions. However, this is a difficulty of these kinds of approaches because the quality of the aproximation is related with the weight vectors. MOEA/D ref25 is a recently designed decomposition-base MOEA. Its main principles include problem decomposition, weighted aggregation of objectives and mating restrictions throguh the use of neighborhoods. Particularly, the neighborhoods are considered in the variation operators. A very used variant of the MOEA/D is the MOEA/D-DE, which use the DE operators (lampinen ref) and polynomial mutation operator for producing new solutions, also it has two extra measures for maintaining the population diversity (refs polynomial mutation). However these two extra mechanism are not enough to deal with long-term executions.

D. Indicator-Based MOEAs - SMS-EMOA

In multi-objective optimization several quality indicators have been developed to compare the performance of MOEAs. Since these indicators measure the quality of the approximations attained by MOEAs, a paradigm based on the application of these indicators was proposed. Particularly, instead of dominance concept, the indicators are used in the MOEAs to guide the optimization process. Among the different indicators, hypervolume is a widely accepted Pareto-compliance quality indicator. The principal advantage of this algorithms is that the indicator usually takes into account both the quality and diversity of the solutions.

A popular and extensively used indicator-based algorithm is the SMS-EMOA ref. This algorithm might be considered as hybrid, since it involves an indicator and dominance concepts. Scencially it integrates the non-dominated sorting with the use of the hypervolume metric. Thus, SMS-EMOA uses the hypervolume as a density estimator which results in a computationally extensive task. Taking into account the promising behavior of SMS-EMOA, it has been used in our experimental validation.

E. Analysis of the Simulated Binary Crossover SBX

The principal components involved in the search process of a GA are the reproduction operators. Specifically, the crossover and mutation operators are highly related with quality and diversity issues ref. Particularly, in this paper is addresed the crossover operator.

The Simulated Binary Crossover (SBX) ref is popularly implemented in GAs ref. The SBX is classified as Parent-Centric, meaning that two offspring values (c_1 and c_2) are created around the parent values (p_1 and p_2). Also the process of generate the offspring values is based in a probability distribution. This distribution is defined by a non-dimensional variable, better known as the spread factor $\beta = |c_1 - c_2|/|p_1 - p_2|$, indicating the ratio of the spread children values to the parent values.

Additionally, this density function uses a distribution index η (user-defined control parameter) that alters the exploration capability of the operator. Specifically, a small index induce a larger probability of building offsprings values more dissimilar than parents values, whereas with a high index the probabilities of generating offspring solutions that are similar to the parents increse.

Principally, the SBX has non-zero probability of creating any number in the search space by recombining any two parent values from the search space. The probability distribution to create an offspring value is defined as a function of a non-dimensionalized parameter $\beta \in [0, \infty]$ as follows:

$$P(\beta) = \begin{cases} 0.5(\eta_c + 1)\beta^{\eta_c}, & \text{if} \quad \beta \leq 1 \\ 0.5(\eta_c + 1)\frac{1}{\beta^{\eta_c+2}}, & \text{otherwise} \end{cases} \tag{2}$$

Based in the mean-preserving property of offspring values and parent values, the distribution probability preserve the following properties:

- Both offspring values are equi-distant from parent values.
- There exist a non-zero probability to create offspring solutions in the entire space from any two parent values.
- The overall probability of creating a pair offspring values within the range of parent values is identical to the overall probability of creating two offspring values outside the range of parent values.

Therefore, considering two participating parent values (p_1 and p_2), two offspring values (c_1 and c_2) can be created as linear combination of parent values with a random number $u \in [0, 1]$, as follows:

$$\begin{aligned} c_1 &= 0.5(1 + \beta(u))p_1 + 0.5(1 - \beta(u))p_2 \\ c_2 &= 0.5(1 - \beta(u))p_1 + 0.5(1 + \beta(u))p_2 \end{aligned} \tag{3}$$

The parameter $\beta(u)$ depends on the random number u, as follows:

$$\beta(u) = \begin{cases} (2u)^{\frac{1}{\eta_c+1}}, & \text{if} \quad u \leq 0.5, \\ (\frac{1}{2(1-u)})^{\frac{1}{\eta+1}}, & \text{otherwise} \end{cases} \tag{4}$$

The above equation only considers a optimization problem having no variable bounds. In most practical problems, each variable is bounded within a lower and upper bound. Thus, Deb and Beyer in 1999 proposed a modification of the probability distribution as showed in the equation (**??**).

$$\beta(u, a) = \begin{cases} (2u(1 - \gamma_a))^{\frac{1}{\eta_c+1}}, & \text{if} \quad u \leq 0.5/(1 - \gamma_a), \\ (\frac{1}{2(1-u(1-\gamma_a))})^{\frac{1}{\eta+1}}, & \text{otherwise} \end{cases} \tag{5}$$

$$c_1 = 0.5(1 + \beta(u, a))p_1 + 0.5(1 - \beta(u, a))p_2 \tag{6}$$

where the offspring c_1 near to p_1 is calculated as indicate the equation (**??**). Therefore, for $p_1 < p_2$, lower bound a is closer to p_1 than to p_2, thus $\gamma_a = 1/(\beta_a^{\eta_c+1})$, where $\beta_a = 1 + (p_1 - a)/(p_2 - p_1)$. Similarly, γ_b is computed in a similar way except β_a must be replaced by $\beta_b = 1 + (b - p_2)/(p_2 - p_1)$ and $\beta(u, b)$ is computed replacing γ_a with γ_b. Then, the second offspring is computed as indicates the equation (**??**).

$$c_2 = 0.5(1 + \beta(u, b))p_1 + 0.5(1 - \beta(u, b))p_2 \tag{7}$$

In the literature [**?**] is not enterely studied the SBX extension to multi-variables problems, in fact the authors use a mechanism similar to uniform crossover for multiple variables (Syswerda 1989) for choosing which variables to cross. However those authors reconized the important implications with the linkage issues, therefore that mechanism does not alliviate the linkage problem.

F. Implementation and analyses of the SBX operator

Particularly, the behavior of the operator is directly affected by three key components. Firstly, it applies a probability of altering each variable that is fixed to 0.5, therefore in average the half of each parent is modified to generate each offspring. Increasing this probability value (depending in the index distribution) could generate more dissimilar offsprings, since more variables can be modified. An appropiate setting of this probability is related with the MOP, therefore a high probability value is better with objective functions with higth dependece-parameters and provide a rotationally invariant behavior (ref lampinen). Otherwise, a low probability value is suiteable for objective functions that are separable, due that few decision variables are changed by crossover operation (ref MOEA decision).

The second relevant component indicates if two childs are interchanged with a probability of 0.5, in some enviroments this probability is known as “Variable uniform crossover probability” ref or “discrete recombination” ref. Although that in single-objective this action provides auto-adaptive behavior, in multi-objective optimization could not be a desirable effect at first stages, the principal reason is that it could be a disruptive actios, due that depending of the dimensions swaping increases exponentially the number of reflections.

Finally, the index distribution plays an important role, since a low index results in a greater exploration and a high index implies a intensification.

In the algorithm **??** is showed the crossover process to generate two offsprings given two parents. The usual SBX operator is setted with $\delta = 0.5$ and $\eta_c = 20$. It is important take into account that this setting does not consider the dimension of the decision variables space.

III. Proposal

Based in the previous analysis, to achieve the effect of balance between exploration at first stages and intensification at the end of the executions the following modifications are proposed. First, the probability of modify a variable (P_v) change among the execution, thus at first generations almost all variables are modified or sampled by the distribution and in the last stages less variables are sampled. This change is

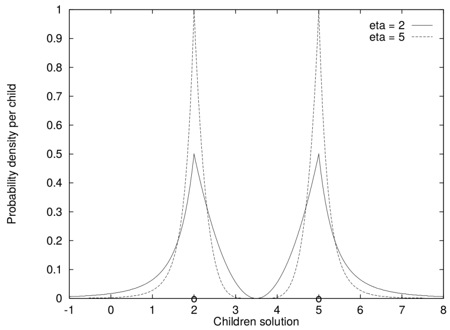

Fig. 1. Probability density function SBX with indexes of distribution 2 and 5.

Algorithm 1 Simulated Binary Crossover (SBX)

Input: Parents (P_1, P_2), Index distribution (η_c), Probability distribution (P_c).
Output: Offsprings (C_1, C_2).
if $U[0,1] \leq P_c$ **then**
  for each variable d **do**
    if $U[0,1] \leq \delta$ **then**
      Generate $C_{1,d}$ with equations (**??**) and (**??**).
      Generate $C_{2,d}$ with equations (**??**) and (**??**).
      if $U[0,1] \geq \delta$ **then**
        Swap $C_{1,d}$ with $C_{2,d}$.
    else
      $C_{1,d} = P_{1,d}$.
      $C_{2,d} = P_{2,d}$.
else
  $C_{1,d} = P_{1,d}$.
  $C_{2,d} = P_{2,d}$.

based in a linear decrement model, where initially is 100% and at the end it is 50%.

In a similar way the second change is related with the "variable uniform crossover probability" which is incremented to the 50%.

$$\delta = max\left(0.5, 1.0 - \frac{T_{Elapsed}}{T_{End}}\right) \tag{8}$$

Finally, the index distribution change among the execution, where at the first stages it is low inducing a high degree of exploration and is incremented to the last stages as indicate the equation (**??**).

$$\eta_c = 2 + 20 \times \left(\frac{T_{Elapsed}}{T_{End}}\right) \tag{9}$$

IV. Experimental validation

This section is devoted to validate our proposal. The nine WFG tests proposed in ref 10 have been used for out purpose. Our experimental validation includ Given thath all of them are stochastic algorithms, each execution was repeated 35 times with different seeds.

Our experimental analysis has been performed in base of attainment surfaces, hypervolume and IGD+. In order to comparate the hypervolume results, a similar guideline than the one proposed in ref27 was used. First a Shapiro-Wilk test was performed to check wahtever or not the values of the results followed a Gaussian distribution. If, so, the Levene test was used to check for the homogeneity of the variances. If samples had equal variance, an ANOVA test was done; if not, a

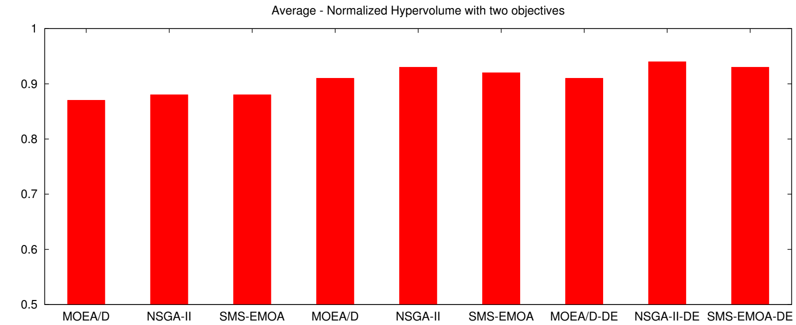

Fig. 2. Average of normalized hypervolume considering all the instances and two objective.

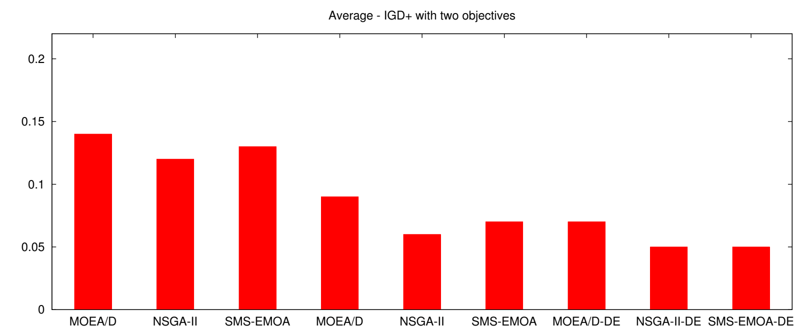

Fig. 3. Average of Inverted Genralized Distance Plus (IGD+) considering all the instances and two objective.

Welch test was performed. For non-Gaussian distributions, the non-parametric Kruskal-Wallis test was used to test whether samples are drawn from the same distribution. An algoritm X is said to win an algorithm Y when the differences between them are statiscally significant, and the mean and median and mean obtained by X are higher than the mean and median achieved by Y.

La validación experimental se implementa considerando tres sub-secciones: 1) Gráfica de barras considerando dos y tres objetivos, que corresponden a la media del hipervolumen y el IGD+ de todas las instancias. 2) Pruebas estadísticas comparación entre la forma estándar y cada forma individual. — la propuesta y cada forma individual + DE. 3) Tabla de resultados estadísticos de la media, con 23 filas y cada forma con cada forma con cada algoritmo.

V. Conclusions

[**?**]

TABLE I

MEAN OF THE NORMALIZED HYPERVOLUME WITH TWO OBJECTIVES

	Case 1			Case 2			Case 3			Case4			Case 5			DE		
	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
DTLZ1	1.00	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.00	1.00
DTLZ2	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.98	0.99	0.99
DTLZ3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.99	0.98
DTLZ4	0.84	0.78	0.76	0.72	0.78	0.74	0.82	0.78	0.68	0.92	0.86	0.74	0.96	0.84	0.76	0.98	0.99	0.99
DTLZ5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
DTLZ6	0.71	0.71	0.72	0.81	0.95	0.85	0.72	0.71	0.71	0.95	0.96	0.95	0.97	0.97	0.97	0.99	1.00	0.99
DTLZ7	0.99	0.99	1.00	0.99	0.99	1.00	0.92	0.99	0.71	0.99	0.99	1.00	0.80	0.99	1.00	0.96	0.99	1.00
UF1	0.90	0.97	0.93	0.91	0.95	0.93	0.92	0.98	0.93	0.94	0.98	0.94	0.97	1.00	0.99	0.97	0.97	0.97
UF2	0.94	0.98	0.97	0.96	0.97	0.97	0.95	0.99	0.98	0.97	0.99	0.98	0.99	0.99	0.99	0.99	0.99	0.99
UF3	0.70	0.79	0.72	0.69	0.79	0.71	0.79	0.92	0.90	0.70	0.81	0.72	0.93	0.96	0.95	0.83	0.84	0.87
UF4	0.97	0.96	0.97	0.96	0.96	0.97	0.97	0.96	0.97	0.97	0.97	0.97	0.97	0.96	0.97	0.97	0.97	0.97
UF5	0.38	0.49	0.51	0.37	0.51	0.51	0.44	0.57	0.54	0.45	0.64	0.62	0.46	0.57	0.53	0.45	0.81	0.80
UF6	0.71	0.78	0.77	0.70	0.79	0.76	0.70	0.81	0.75	0.69	0.78	0.77	0.64	0.79	0.75	0.70	0.85	0.83
UF7	0.74	0.88	0.79	0.76	0.85	0.81	0.69	0.83	0.75	0.78	0.88	0.88	0.72	0.96	0.85	0.86	0.98	0.96
WFG1	0.81	0.86	0.75	0.93	0.88	0.87	0.84	0.86	0.75	0.99	0.98	0.98	0.96	0.98	0.95	0.96	0.91	0.86
WFG2	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.96	0.97	0.97	0.97	0.97	0.98	0.98	1.00	1.00
WFG3	0.99	0.98	0.99	0.99	0.98	0.99	0.99	0.98	0.99	0.99	0.98	0.99	0.99	0.98	0.99	0.99	0.99	0.99
WFG4	0.98	0.98	0.99	0.98	0.98	0.99	0.98	0.98	0.99	0.98	0.98	0.99	0.98	0.98	0.99	0.98	0.98	0.99
WFG5	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85	0.85
WFG6	0.87	0.86	0.87	0.87	0.87	0.88	0.88	0.88	0.88	0.89	0.88	0.89	0.92	0.92	0.92	0.90	0.91	0.92
WFG7	0.98	0.98	0.99	0.98	0.98	0.99	0.98	0.98	0.99	0.98	0.98	0.99	0.98	0.98	0.99	0.98	0.98	0.99
WFG8	0.78	0.75	0.78	0.78	0.75	0.78	0.80	0.78	0.81	0.78	0.75	0.78	0.95	0.78	0.81	0.79	0.78	0.80
WFG9	0.89	0.79	0.89	0.89	0.81	0.89	0.91	0.89	0.93	0.89	0.80	0.87	0.96	0.96	0.97	0.76	0.73	0.74
Average	0.87	0.88	0.88	0.87	0.90	0.89	0.87	0.90	0.87	0.90	0.91	0.91	0.91	0.93	0.92	0.91	0.94	0.93

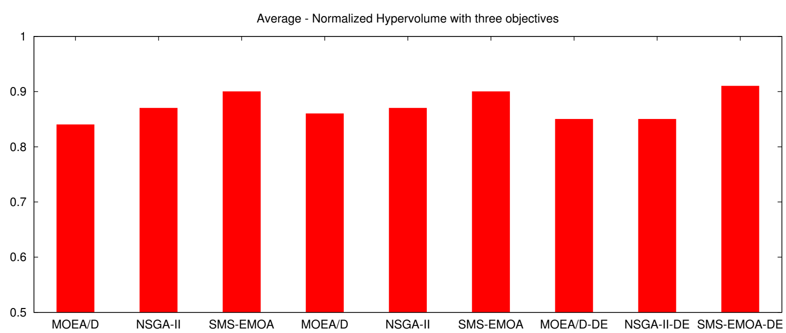

Fig. 4. Average of normalized hypervolume considering all the instances and three objective.

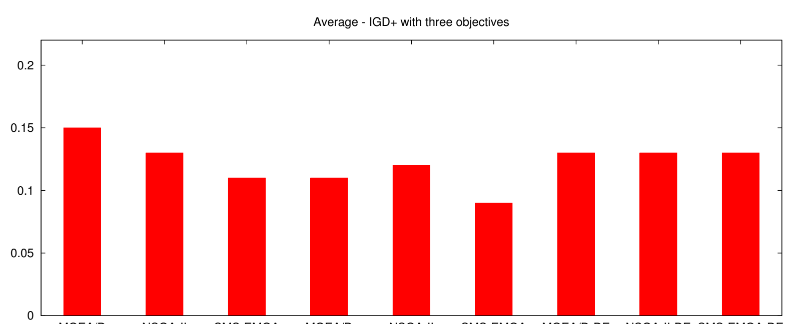

Fig. 5. Average of Inverted Genralized Distance Plus (IGD+) considering all the instances and three objective.

TABLE II

MEAN OF THE IGD+ WITH TWO OBJECTIVES

	Case 1			Case 2			Case 3			Case4			Case 5			DE		
	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
DTLZ1	0.001	0.002	0.001	0.001	0.002	0.001	0.001	0.002	0.001	0.001	0.002	0.001	0.001	0.002	0.001	0.001	0.002	0.001
DTLZ2	0.002	0.003	0.002	0.002	0.003	0.002	0.002	0.003	0.002	0.002	0.003	0.002	0.002	0.003	0.002	0.002	0.003	0.002
DTLZ3	0.002	0.003	0.002	0.002	0.003	0.002	0.002	0.003	0.002	0.002	0.003	0.002	0.002	0.003	0.002	0.002	0.031	0.059
DTLZ4	0.074	0.106	0.115	0.136	0.106	0.126	0.085	0.106	0.157	0.033	0.064	0.126	0.013	0.075	0.115	0.002	0.003	0.002
DTLZ5	0.002	0.003	0.002	0.002	0.003	0.002	0.002	0.003	0.002	0.002	0.003	0.002	0.002	0.003	0.002	0.002	0.003	0.002
DTLZ6	0.975	0.985	0.958	0.686	0.237	0.580	0.958	0.993	0.997	0.224	0.203	0.219	0.142	0.122	0.145	0.021	0.017	0.024
DTLZ7	0.003	0.003	0.002	0.003	0.003	0.002	0.054	0.003	0.197	0.003	0.003	0.002	0.136	0.003	0.002	0.024	0.003	0.002
UF1	0.058	0.028	0.063	0.044	0.042	0.060	0.043	0.022	0.057	0.022	0.016	0.038	0.014	0.007	0.017	0.021	0.027	0.031
UF2	0.037	0.016	0.017	0.026	0.018	0.021	0.034	0.013	0.016	0.018	0.014	0.013	0.006	0.014	0.010	0.005	0.015	0.011
UF3	0.210	0.126	0.181	0.214	0.127	0.193	0.167	0.066	0.075	0.209	0.105	0.179	0.065	0.060	0.060	0.095	0.188	0.126
UF4	0.036	0.044	0.035	0.040	0.045	0.038	0.039	0.045	0.038	0.033	0.043	0.033	0.039	0.045	0.038	0.035	0.035	0.035
UF5	0.724	0.583	0.532	0.740	0.546	0.526	0.643	0.485	0.513	0.628	0.393	0.392	0.634	0.488	0.523	0.618	0.225	0.223
UF6	0.309	0.233	0.233	0.309	0.208	0.246	0.340	0.214	0.267	0.321	0.230	0.234	0.450	0.257	0.312	0.308	0.153	0.174
UF7	0.234	0.104	0.190	0.218	0.138	0.171	0.279	0.146	0.238	0.205	0.108	0.115	0.269	0.039	0.142	0.131	0.026	0.050
WFG1	0.207	0.148	0.287	0.076	0.126	0.145	0.180	0.146	0.281	0.014	0.016	0.016	0.038	0.020	0.050	0.038	0.092	0.154
WFG2	0.055	0.052	0.057	0.055	0.053	0.053	0.060	0.054	0.059	0.072	0.053	0.052	0.055	0.053	0.048	0.042	0.003	0.002
WFG3	0.008	0.012	0.007	0.008	0.014	0.007	0.008	0.012	0.007	0.008	0.012	0.007	0.008	0.012	0.007	0.008	0.011	0.007
WFG4	0.007	0.008	0.005	0.007	0.010	0.005	0.007	0.008	0.005	0.007	0.008	0.005	0.007	0.008	0.005	0.007	0.008	0.005
WFG5	0.068	0.066	0.065	0.068	0.067	0.067	0.068	0.066	0.065	0.068	0.066	0.065	0.069	0.066	0.066	0.068	0.066	0.066
WFG6	0.063	0.059	0.055	0.065	0.057	0.054	0.055	0.053	0.051	0.049	0.049	0.049	0.034	0.035	0.034	0.040	0.036	0.034
WFG7	0.007	0.009	0.005	0.007	0.009	0.005	0.007	0.008	0.005	0.007	0.008	0.005	0.007	0.008	0.005	0.007	0.007	0.005
WFG8	0.112	0.124	0.112	0.111	0.124	0.113	0.097	0.107	0.095	0.113	0.124	0.113	0.026	0.101	0.089	0.102	0.107	0.099
WFG9	0.049	0.099	0.050	0.047	0.087	0.051	0.038	0.051	0.033	0.049	0.095	0.061	0.016	0.015	0.011	0.111	0.125	0.124
Average	0.14	0.12	0.13	0.12	0.09	0.11	0.14	0.11	0.14	0.09	0.07	0.08	0.09	0.06	0.07	0.07	0.05	0.05

TABLE III

MEAN OF THE NORMALIZED HYPERVOLUME WITH THREE OBJECTIVES

	Case 1			Case 2			Case 3			Case4			Case 5			DE		
	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
DTLZ1	0.99	0.99	1.00	0.98	0.99	1.00	0.99	0.99	1.00	0.99	0.99	1.00	0.99	0.99	1.00	0.98	0.99	1.00
DTLZ2	0.89	0.89	0.95	0.89	0.88	0.95	0.89	0.88	0.95	0.89	0.89	0.95	0.89	0.88	0.95	0.88	0.90	0.95
DTLZ3	0.99	1.00	1.00	0.99	1.00	1.00	0.99	1.00	1.00	0.99	1.00	1.00	0.99	1.00	1.00	0.99	0.99	0.93
DTLZ4	0.73	0.89	0.71	0.63	0.88	0.72	0.77	0.87	0.76	0.87	0.90	0.83	0.82	0.89	0.82	0.88	0.90	0.95
DTLZ5	0.99	1.00	1.00	0.99	1.00	1.00	0.99	1.00	1.00	0.99	1.00	1.00	0.99	1.00	1.00	0.99	1.00	1.00
DTLZ6	0.87	0.95	0.92	0.92	1.00	0.95	0.88	0.95	0.92	0.96	0.97	0.97	0.97	0.98	0.98	0.99	1.00	0.99
DTLZ7	0.89	0.91	0.87	0.90	0.89	0.88	0.78	0.85	0.61	0.90	0.91	0.89	0.89	0.91	0.90	0.85	0.93	0.90
UF10	0.52	0.72	0.68	0.62	0.70	0.65	0.52	0.71	0.62	0.55	0.75	0.71	0.49	0.63	0.51	0.63	0.66	0.82
UF8	0.90	0.85	0.91	0.87	0.81	0.86	0.83	0.91	0.91	0.91	0.84	0.89	0.82	0.87	0.85	0.93	0.78	0.84
UF9	0.90	0.94	0.99	0.89	0.96	0.99	0.94	0.99	0.99	0.92	0.93	0.99	0.94	0.95	0.97	0.96	0.88	1.01
WFG1	0.93	0.93	0.93	0.81	0.45	0.93	0.93	0.93	0.93	0.95	0.93	0.99	0.94	0.93	0.98	0.71	0.95	0.91
WFG2	0.86	0.91	0.90	0.85	0.91	0.90	0.87	0.91	0.91	0.88	0.96	0.98	0.92	0.96	0.99	0.95	0.97	0.99
WFG3	0.99	0.98	0.99	0.98	0.97	0.99	0.99	0.98	0.99	0.99	0.97	1.00	0.99	0.98	1.00	0.99	0.97	1.00
WFG4	0.81	0.83	0.92	0.81	0.81	0.92	0.81	0.82	0.92	0.81	0.83	0.92	0.81	0.83	0.92	0.80	0.75	0.92
WFG5	0.73	0.76	0.84	0.73	0.75	0.84	0.73	0.76	0.84	0.73	0.77	0.84	0.73	0.76	0.84	0.73	0.76	0.84
WFG6	0.75	0.75	0.85	0.74	0.74	0.85	0.75	0.76	0.86	0.75	0.76	0.86	0.76	0.79	0.88	0.71	0.76	0.88
WFG7	0.82	0.83	0.92	0.82	0.83	0.92	0.82	0.83	0.92	0.82	0.83	0.92	0.82	0.83	0.92	0.82	0.78	0.92
WFG8	0.70	0.65	0.79	0.70	0.64	0.79	0.72	0.63	0.80	0.70	0.66	0.79	0.80	0.65	0.80	0.71	0.60	0.79
WFG9	0.74	0.68	0.84	0.76	0.72	0.86	0.75	0.67	0.88	0.75	0.68	0.84	0.79	0.66	0.87	0.67	0.66	0.74
Average	0.84	0.87	0.90	0.84	0.84	0.89	0.84	0.87	0.88	0.86	0.87	0.91	0.86	0.87	0.90	0.85	0.85	0.91

TABLE IV
MEAN OF THE IGD+ WITH THREE OBJECTIVES

	Case 1			Case 2			Case 3			Case4			Case 5			DE		
	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
DTLZ1	0.014	0.020	0.014	0.014	0.022	0.013	0.014	0.020	0.014	0.014	0.020	0.014	0.014	0.020	0.013	0.014	0.018	0.014
DTLZ2	0.028	0.032	0.019	0.028	0.033	0.019	0.028	0.032	0.019	0.028	0.032	0.019	0.028	0.033	0.019	0.028	0.028	0.019
DTLZ3	0.028	0.031	0.019	0.028	0.041	0.019	0.028	0.031	0.019	0.028	0.031	0.019	0.028	0.031	0.019	0.028	0.056	1.065
DTLZ4	0.140	0.030	0.156	0.201	0.032	0.166	0.108	0.046	0.129	0.032	0.030	0.100	0.060	0.030	0.090	0.028	0.028	0.019
DTLZ5	0.003	0.003	0.002	0.003	0.003	0.002	0.003	0.003	0.002	0.003	0.003	0.002	0.003	0.003	0.002	0.003	0.003	0.002
DTLZ6	0.674	0.273	0.511	0.445	0.008	0.321	0.627	0.272	0.506	0.210	0.190	0.226	0.137	0.103	0.152	0.060	0.009	0.056
DTLZ7	0.055	0.041	0.078	0.044	0.045	0.083	0.189	0.098	0.438	0.044	0.039	0.057	0.050	0.040	0.055	0.101	0.035	0.056
UF10	0.421	0.295	0.284	0.323	0.319	0.308	0.377	0.278	0.297	0.366	0.262	0.251	0.382	0.321	0.358	0.312	0.333	0.189
UF8	0.085	0.145	0.085	0.114	0.188	0.124	0.134	0.099	0.088	0.077	0.151	0.101	0.144	0.135	0.136	0.062	0.207	0.146
UF9	0.148	0.193	0.105	0.133	0.142	0.104	0.129	0.139	0.094	0.129	0.197	0.095	0.149	0.192	0.137	0.127	0.315	0.076
WFG1	0.124	0.148	0.127	0.307	0.987	0.134	0.118	0.146	0.120	0.090	0.136	0.046	0.101	0.148	0.048	0.465	0.100	0.159
WFG2	0.093	0.140	0.065	0.097	0.138	0.064	0.089	0.141	0.061	0.085	0.158	0.036	0.067	0.147	0.031	0.059	0.092	0.030
WFG3	0.025	0.048	0.012	0.025	0.052	0.012	0.025	0.046	0.012	0.024	0.055	0.012	0.024	0.047	0.012	0.024	0.074	0.012
WFG4	0.123	0.122	0.069	0.125	0.129	0.070	0.123	0.124	0.070	0.123	0.119	0.069	0.123	0.122	0.069	0.132	0.178	0.070
WFG5	0.184	0.167	0.128	0.184	0.169	0.128	0.184	0.167	0.128	0.184	0.165	0.128	0.184	0.168	0.128	0.185	0.164	0.129
WFG6	0.175	0.174	0.117	0.174	0.177	0.118	0.176	0.170	0.115	0.173	0.167	0.112	0.162	0.143	0.097	0.206	0.175	0.100
WFG7	0.126	0.117	0.069	0.124	0.118	0.069	0.126	0.117	0.070	0.126	0.116	0.069	0.125	0.116	0.069	0.127	0.157	0.070
WFG8	0.195	0.238	0.149	0.197	0.246	0.149	0.181	0.249	0.144	0.195	0.233	0.149	0.137	0.234	0.142	0.190	0.284	0.146
WFG9	0.172	0.219	0.116	0.156	0.190	0.097	0.165	0.223	0.087	0.161	0.214	0.110	0.137	0.232	0.082	0.231	0.234	0.192
Average	0.15	0.13	0.11	0.14	0.16	0.11	0.15	0.13	0.13	0.11	0.12	0.09	0.11	0.12	0.09	0.13	0.13	0.13