OPERATIONS RESEARCH PROJECT REPORT

<u>AUTO ASSEMBLY</u>

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DESCRIPTION OF THE PROBLEM

William Smith, manager of one of the plants for a large automobile manufacturer, needs our help determining the production schedule for the next month. William naturally wants us to determine the way in which he can maximize profits for his company. He tells us that his plant is in charge of producing two different models of midsize luxury cars:

The Family Adventurer:

- Each model generates a profit of $3,700 for the company.
- Each model takes 6 labor hours to be assembled.
- This is a very demanded model: there is no demand limit within the capacity limits of the production plan (i.e. we must not worry about recommending a larger number of Adventurers to be produced).
- 4-door car.

The Classic Transporter:

- Each model generates a profit of $5,300 for the company.
- Each model takes 10.5 labor hours to be assembled.
- Reports indicate that the maximum demand for this model is 3,500 cars (that is, we must not recommend a larger number of Transporters, as it would exceed the demand and not generate any profit).
- 2-door car.

William also tells us about some of the plant capacities:

- The plant receives doors from a door manufacturer. Next month the plant will only receive a total of 20,000 doors:
  - 10,000 of each side
  - Both cars use these same doors
- The maximum monthly labor-hours is 48,500

Now we have all the necessary information to develop our model. We will write from the perspective of a team of Operations Analysts whose job is to advise William so that he can present the CEO with an optimal course of action.

PROBLEM MODELING

Description of the chosen model: the following image showcases the problem formulation, written in LaTeX using www.overleaf.com

1 Problem formulation

DECISION VARIABLES:
Let x_1 be the number of Family Adventurer models
Let x_2 be the number of Classic Transporter models

OBJECTIVE FUNCTION:
$max(3700x_1 + 5300x_2)$

CONSTRAINTS:
$6x_1 + 10.5x_2 \leq 48500$
$2x_1 + x_2 \leq 10000$
$x_2 \leq 3500$
$x_1, x_2 \geq 0$

Justification for the chosen model: we decided to go with a linear programming model, simple yet effective. Still, one must always explore and discuss how well the conditions of the problem adapt to the 4 assumptions of a linear programming model:

1. **Proportionality:** the information we receive from the company suggests that the amount of hours that it takes to assemble each model as well as the profit margin that each car yields, they both stay consistent regardless of how many units are produced. Since we are not taking into account the monetary cost of the physical inputs and the labor (we can actually disregard it, since we are calculating based on the profit margin which, unlike revenue, takes these costs into consideration), we can consider the necessary hours of labor as the 'cost' of production. This cost stays constant, so this property is satisfied.

2. **Additivity:** we are considering two different products with different features, costs and profit margins, and looking at the mathematical expressions that have been determined to set the constraints, there is not any factor that could simultaneously contribute to both products. Note that the manufacturer will produce cars of one of the two models, one at a time, and will sell to consumers again a particular model. Formally, the algebraic sum of the individual weighted effects is a good representation for the combined effect of the products, so this property is also satisfied.

3. **Divisibility:** if we knew that this plant only intends to produce for one more month, then close having attained maximum profits, we would not want to have any decimal numbers in the optimal solution: that would mean one car for the corresponding model would be

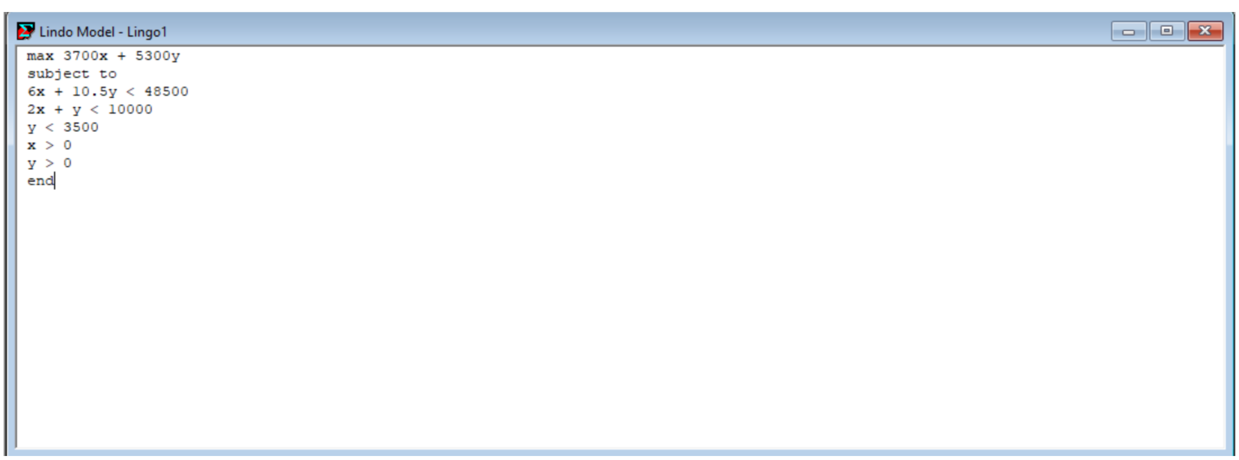
left undone (assemblement would start but would need more resources such as labor time or doors to be finished), and it would be useless. A solution for this would be to use an integer programming model. However, we are developing a model for a big automobile manufacturer, so we can assume that having a big presence in the market, it will operate long term, and we can afford to have 'unfinished' units that will be finished as soon as next month's production starts. Under this assumption, divisibility is satisfied.

4. **Certainty:** we are provided with data from the company that is definitely clear and precise, from the number of components available, to the profit margin (which in itself needs certainty from the sales revenue of cars, the cost of the components and the labor), to the number of labor hours. All of these can be measured in detail by the company analysts so we should be able to assume certainty. Although the future demand of one of the car models is taken into account and it is impossible to predict with 100% certainty, we can assume that the company has enough data from previous months on how this product has perform in terms of demand, and we can also assume that they have been conservative in order not to derive risks from the results of our research.

SOLUTION OF THE MODEL

The problem can be solved using Lingo software. The following images illustrate:

- (1) How the model was formulated using the Lingo syntax: the only change is now the decision variables are declared as x (Family Adventurer) and y (Classic Transporter).
- (2) The solution provided after running the program with Lingo. Interpretation of this solution is provided in the next page of this report.
- (3) The range of solutions. This is the sensitivity analysis, which tells us how much the variables and constraints could be altered without changing the optimal solution that has been determined for the current values.



Solution Report - Lingo1

LINGO/WIN64 19.0.40 (26 Apr 2021), LINDO API 13.0.4099.270

Licensee info: Eval Use Only
License expires: 16 JUL 2022

Global optimal solution found.	
Objective value:	0.2701000E+08
Infeasibilities:	0.000000
Total solver iterations:	2
Elapsed runtime seconds:	0.06
Model Class:	LP
Total variables:	2
Nonlinear variables:	0
Integer variables:	0
Total constraints:	6
Nonlinear constraints:	0
Total nonzeros:	9
Nonlinear nonzeros:	0

Variable	Value	Reduced Cost
X	3766.667	0.000000
Y	2466.667	0.000000

Row	Slack or Surplus	Dual Price
1	0.2701000E+08	1.000000
2	0.000000	460.0000
3	0.000000	470.0000
4	1033.333	0.000000
5	3766.667	0.000000
6	2466.667	0.000000

Range Report - Lingo1

Ranges in which the basis is unchanged:

Objective Coefficient Ranges:

Variable	Current Coefficient	Allowable Increase	Allowable Decrease
X	3700.000	6900.000	671.4286
Y	5300.000	1175.000	3450.000

Righthand Side Ranges:

Row	Current RHS	Allowable Increase	Allowable Decrease
2	48500.00	7750.000	18500.00
3	10000.00	6166.667	2583.333
4	3500.000	INFINITY	1033.333
5	0.000000	3766.667	INFINITY
6	0.000000	2466.667	INFINITY

INTERPRETATION OF THE SOLUTION

Translating the numerical results obtained into understandable guidance:

- The optimal combination consists of producing 3,766 whole units of Family Adventurer and 2,466 whole units of Classic Transporter. Whole is a keyword, since following the production scheme that allows to maximize profits will result in two thirds of a Family Adventurer and another two thirds of a Classic Transporter for which assembly will start but will be left unfinished.
- These numbers give us an output of 27,010,000, which means that the company would obtain $27.01 million in profit. This result has to be minorly tweaked, as the previously mentioned fractional units will not be sold and therefore not contribute to the profits. The real profit for the number of whole units that can actually be sold is $27.004 million.
- Slack or surplus variables in rows 2 and 3 come from constraints 1 and 2. Since they equal 0, this indicates that:
    - Mathematically, we reach the equality part of the constraint rather than staying in the inequality.
    - In words, we don't waste any available resources out of those considered for these constraints: the company uses all possible labor hours and all available doors in order to produce these quantities.
- Slack variables in rows 4 and 6 both are directly related to y (the amount of Classic Transporters). Note that when added, they are equal to the maximum demand of this car model that we have assumed for the market. Particularly, row 6 = 2,466.667 is, as previously mentioned, nothing but the optimal amount of Classic Transporter Units, and row 4 = 1,033.333 indicates how much more units we could assemble without surpassing the market demand.
- Similarly, row 5 = 3,766.667 is nothing but the optimal x value (number of Family Adventurer units).

RECOMMENDATIONS

As operations analysts, we are interested in studying how certain actions that the company could potentially take would either affect its results in a negative or positive way. After considering a series of alternatives, we expand on each of them and elaborate on our recommendations.

1. A **$500,000 marketing campaign** that would raise Classic Transporter demand by 20% is proposed. We recommend **not to implement this**. The reason is that, when constructing a new formulation where 500,000 is taken into account as a negative summand in the objective function, and the new demand constraint is determined by 4,200 rather than 3,500, we actually obtain a lower optimal value (= profit) than we had before. This comes from the fact that under our initial model we don't reach the maximum assumed demand for Classic Transporter, therefore we should not care about raising it since all that will do is leave the optimal amounts the same, yielding the same profit margin, only this time with $500,000 deducted. In other words, **this would make the company waste $500,000.**

2. We are presented with the possibility to **increase the plant's labor hour capacity by 25%**, making it 60,625. The more capacity, the more cars will be assembled, and the higher the profits, so we recommend doing so and we recommend the following way: **assemble a total of 3,250 Family Adventurers and 3,500 Classic Transporters**. These amounts will lead to an optimal profit of 30,575,000. Side note: **this means a 25% increase in labor hour capacity turns into a 13.2% increase in profits.** Anything over 0% is recommended, but this is especially good considering we are only changing one of the factors.

3. Now we face reality and realize that i**ncreasing labor hour capacity implicates making workers do extra time, for which the hourly rate is higher**. Since the profits were augmented by **$3,565,000**, William can distribute any amount of money lower (or equal) to that between all extra hours worked for this proposal to still manifest positive results. A total of 0.25*48500 = 12,125 extra hours of labor are added, and dividing we obtain that the maximum William should be willing to raise overtime wage is by no more than $294.02/hour. We do not know about the wages of the plant employees but this looks like an extremely high number, so **William could definitely pay a reasonable amount for extra labor hours and still be better off.**

4. If William decided to move forward with both the marketing campaign and the extra labor hours, the optimal combination we recommend is **2,958 (and a third) units of Family Adventurer and 4,083 (and a third) units of Classic Transporter**, which yield a (real) profit of $32,584,500. Here we are again assuming that extra labor hours are not compensated in a higher manner and also considering no cost from the marketing campaign (whatever that is it will be a constant negative summand so it will not change the optimal quantities). This is enough to determine the optimal combination for such

demand and labor increases, but before recommending we need consider their corresponding added costs, which we do next up:

5. If the marketing campaign costs $500,000 and the overtime labor hours cost an extra $1,600,000, the company is left with **$30,484,500 in net profit. We recommend moving on with this strategy**, as even after being realistic and subtracting the extra cost, the net profit is higher than the initial one (the profit with no further actions was $27,004,000).

6. Repeating all these previous scenarios for **a new profit margin on $2,800 on the Family Adventurer** (from now on keeping the assumptions of $500,000 on the marketing campaign and $1,600,000 on the extra labor hours, and referring to Family Adventurer as AF and to Classic Transporter as CT):

   - No marketing and no extra labor: 1,958 AC; 3,500 CT; $24,032,400 profit
   - Only marketing: 733 AC; 4,200 CT; $23,812,400 profit
   - Only extra labor: 3,250 AC; 3,500 CT; $26,050,000 profit
   - Both marketing and extra labor: 2,754 AC; 4,200 CT; $27,871,200 profit
   - **With these new conditions, we still recommend both the marketing campaign and the extra labor hours**

7. Due to recurrent defects in the **FA, its assembly time increases by 1.5 hours**. The new recommendations considering this change (and no marketing nor extra labor hours, still $3,700 for the FA) are: assemble **1,566 (whole) FA; 3,500 CT; the profit will be $24,344,200 profit.**

8. The board wishes to meet the full demand for CT (3,500 units). Formulating a new LP model in which y is fixed at 3,500; we can produce a total of 1,958 (and a third) FA. The corresponding profit is $25,794,600. This number is **$1,209,400 lower** than the original (the optimal without having to meet the CT demand). Since the decrease in profit is less than $2,000,000; **William should meet the full demand.**

9. As explained in detail above, regardless of whether the profit margin of the FA decreases to $2,800 or stays at $3,700; **it is the smartest decision to combine both the marketing campaign and the extra labor hours**, as long as these added hours don't induce a cost of over 1,600,000. That will lead the company to:
   - A profit of **$27,871,200 if the FA profit = $2,800**
     - Units assembled: 2,754 AC; 4,200 CT
   - A profit of **$30,484,500 if the FA profit = $3,700**
     - Units assembled: 2,958 AC; 4,083 CT

Note: all the optimal profits are calculated based on the WHOLE number of units produced (the real optimal value of the objective function would take as inputs fractional optimal values of the car quantities, which we ignore in order to provide assistance in a realistic way.