On Mistaking a Coordinate System for a Concept

Carlos Tomas Grahm

Response to "No, S^2 Does Not Live in Two Dimensions"

I appreciate Dr. Matheson's enthusiasm in defending the orthodoxy, but his post unintentionally proves the very point of my note. The reflexive leap to geometric language—embedding, tangent space, curvature—illustrates how thoroughly geometry has colonized logic within mathematics.

My argument was never that S^2 "lives" in two Euclidean dimensions; it was that the *logical minimal space capable of containing* S^2 has the same abstract capacity as $\mathbb{R}^2$. The distinction is simple but profound: geometry presupposes a metric, logic does not. Once we strip away the metric, the extra coordinate of $\mathbb{R}^3$ ceases to serve a definitional purpose. It exists only for visualization.

To invoke stereographic projection as if it neutralizes this claim is to miss it entirely. The projection assumes $\mathbb{R}^3$ as its host; the minimal space argument removes that assumption. It does not say "we can map the sphere to the plane." It says "the plane already has the logical resources to host the sphere's topology without importing a third coordinate." The difference is between expressing a property within a model and choosing the model itself.

That geometry has long conflated these layers is not a sin—it is a historical artifact. Gauss and Riemann worked before the formal separation of logic, topology, and set theory. To appeal to their era as final authority is to forget how much the language of mathematics has evolved since.

The collaboration that produced this idea was between a human and an AI system, but the logic is entirely classical. What is new is not the bijection; it is the refusal to confuse representational excess with necessity. If mathematics cannot occasionally revisit its own assumptions, it becomes liturgy, not science.

In short: the paper does not mistake geometry for philosophy; it reveals that geometry has mistaken philosophy for proof. The coordinate system is not a convenience—it is a constraint we have mistaken for truth.

Carlos Tomas Grahm
`carlostomas@research.mail`
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