A Strategy to Solve the Multistage Transmission Expansion Planning Problem

Guillermo Vinasco, Marcos J. Rider, *Member, IEEE*, and Ruben Romero, *Senior Member, IEEE*

***Abstract*—In this letter, a heuristic to reduce the combinatorial search space (CSS) of the multistage transmission expansion planning (MTEP) problem is presented. The aim is to solve the MTEP modeled like a mixed binary linear programming (MBLP) problem using a commercial solver with a low computational time. The heuristic uses the solution of several static transmission expansion planning problems to obtain the reduced CSS. Results using some test and real systems show that the use of the reduced CSS solves the MTEP problem with better solutions compared to other strategies in the literature.**

***Index Terms*—Combinatorial optimization, mixed binary linear programming problem, multistage transmission expansion planning.**

I. Introduction

The objective of the power system transmission expansion planning (TEP) problem is to determine *where*, *how many*, and *when* new devices must be added to a network in order to make its operation viable for a predefined horizon of planning at a minimum cost. To solve the TEP problem, relaxed mathematical models using only the active part (real power and voltage angles) are commonly used, as the DC model. According to the model, one can have a static TEP in which only one stage (planning horizon) is considered, or a multistage (dynamic) problem in which several planning horizons are considered in the same TEP. The mathematical models used for the static and multistage TEP problems are presented in [1], [2] and [3], [4], respectively.

Static and multistage TEP can be modeled as an MBLP problem as shown in [3]. About MTEP, [3] says, "*The extension to multiple stages increases the number of the continuous and the binary variables, as well as the number of network constraints. As a consequence, the planning problem rapidly becomes intractable by integer programming techniques*." Metaheuristics, integer programming techniques (IPTs), and heuristic algorithms (HA) have been used to solve the MTEP problem. Although metaheuristics are easy and simple methods that provide good results, they present various problems such as high processing demand and their incapacity to identify the optimum solution. HAs are robust, easily applied, and normally converge to a local optimum solution. Even though IPTs guarantee the optimum solution of an MBLP problem, the search space size of the problem leads to a substantial computational effort.

An approximate method is to solve the MTEP like a sequence of static plannings (pseudo-dynamic or forward planning). The lines of the solution of the static planning of a state will be considered as part of the base topology for the solution of the static planning of the following stage. Another approach is the backward heuristic [3]: First solve the TEP problem for the last stage of the planning horizon (the "target solution"). Then, go backwards in time and solve the TEP problem in each stage considering as candidates only the reinforcements previously made for the last stage (in other words, anticipating only the investment decisions made for the horizon year is allowed).

The forward planning does not see future benefits from present reinforcements; e.g., 500-kV circuits (more expensive) could have future benefits and be a better choice for now than 220-kV circuits (cheaper), phenomena economies of scale. The backward heuristic takes into account an approximation of these future benefits [3].

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G. Vinasco is with the Programa Ingeniería Eléctrica, University of Antioquia and Interconexión Eléctrica S.A.-ISA, Medellin, Colombia (e-mail: gevinasco@isa.com.co).

M. J. Rider and R. Romero are with the Faculdade de Engenharia de Ilha Solteira, UNESP, Departamento de Engenharia Elétrica, Ilha Solteira-SP, Brazil (e-mail: mjrider@dee.feis.unesp.br; ruben@dee.feis.unesp.br).

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A. Multistage Transmission Expansion Planning Model

The MTEP is modeled like a MBLP problem as shown in (1)–(11), assuming that the electricity market is centrally operated and that generators do not have the ability to exercise local market power:

$$\min v = \sum_{t\in T}\alpha_t \sum_{km\in\Omega} c_{km} \sum_{y\in Y}(x_{km,y,t} - x_{km,y,t-1}) \quad (1)$$

$$\text{s.t. } g_{k,t} + \sum_{mk\in\Omega}\left(f^0_{mk,t} + \sum_{y\in Y} f_{mk,y,t}\right) - \sum_{km\in\Omega}\left(f^0_{km,t} + \sum_{y\in Y} f_{km,y,t}\right) = d_{k,t} \quad \forall k\in B, \forall t\in T \quad (2)$$

$$f^0_{km,t} = -b_{km} n^0_{km}(\delta_{k,t} - \delta_{m,t}) \quad \forall km\in\Omega, \forall t\in T \quad (3)$$

$$- n^0_{km}\overline{f}_{km} \le f^0_{km,t} \le n^0_{km}\overline{f}_{km} \quad \forall km\in\Omega, \forall t\in T \quad (4)$$

$$- \bar{\delta} \le \delta_{k,t} \le \bar{\delta} \quad \forall k\in B, \forall t\in T \quad (5)$$

$$- 2\bar{\delta}(1 - x_{km,y,t}) \le \frac{f_{km,y,t}}{b_{km}} + (\delta_{k,t} - \delta_{m,t}) \le 2\bar{\delta}(1 - x_{km,y,t}) \quad \forall km\in\Omega, \forall y\in Y, \forall t\in T \quad (6)$$

$$- \overline{f}_{km} x_{km,y,t} \le f_{km,y,t} \le \overline{f}_{km} x_{km,y,t} \quad \forall km\in\Omega, \forall y\in Y, \forall t\in T \quad (7)$$

$$x_{km,y,t} \le x_{km,y-1,t} \quad \forall km\in\Omega, \forall y\in Y/y>1, \forall t\in T \quad (8)$$

$$x_{km,y,t-1} \le x_{km,y,t} \quad \forall km\in\Omega, \forall y\in Y, \forall t\in T/t>1 \quad (9)$$

$$\sum_{y\in Y} x_{km,y,t} \le \bar{n}_{km} \quad \forall km\in\Omega, \forall t\in T \quad (10)$$

$$x_{km,y,t} \text{ binary} \quad \forall km\in\Omega, \forall y\in Y, \forall t\in T \quad (11)$$

where Ω, B, Y, and T are the sets containing right-of-ways, buses, circuits by right-of-ways, and stages, respectively. α_t is the discount factor to find the net present value for the transmission investment. So for any right-of-way, km: c_{km}, b_{km}, n^0_{km}, $\bar{n}_{km}$ and $\overline{f}_{km}$ represent the cost of a circuit, the susceptance for the transmission circuit, the number of circuits in the base case, the maximum number of circuits that can be added, and the maximum power flow, respectively. Thus $f^0_{km,t}$ and $f_{km,y,t}$ represent the power flow in the base case and in the circuit y of stage t. $\bar{\delta}$ is the maximum voltage angle. $\delta_{k,t}$, $g_{k,t}$, and $d_{k,t}$ represent the voltage angle, the generation, and the demand in bus k at stage t. $x_{km,y,t}$ is the investment binary variable for circuit y at stage t, where $x_{km,y,_0} = 0, \forall\, km \in \Omega$, $\forall\, y \in Y$. Equation (1) is investment in transmission lines; (2) represents Kirchhoff's first law. The real power flows are calculated in (3) and (6). In (6), $2\bar{\delta}$ plays exactly the role of the "Big M" factor, as shown in [3], in that it limits the voltage angle difference between two nodes that are not connected. In other words, given the maximum voltage angle, $2\bar{\delta}$ provides a sufficient degree of freedom to the voltage angle difference between every unconnected node of the network. Nevertheless, it could happen that the "Big M" factors (M^*) computed in [3] are smaller than $2\bar{\delta}$. In such a case, it would be a better idea to use M^*. Thus, the proper definition of the "Big M" factors for the MTEP problem is $\min\{2\bar{\delta}, M^*\}$. In (6) is used just $2\bar{\delta}$, instead of $\min\{2\bar{\delta}, M^*\}$, to avoid the computation of M^* (solving the longest and shortest path problems) prescribed in [3]. Equations (4) and (7) limit the real power flows. Equation (5) is the maximum phase angle allowed and represents a stability constraint. Equation (8) guarantees a sequential installation in the transmission line for each corridor, while (9) guarantees that a line installed in a stage must be present in later stages. The maximum number of transmission lines is represented by (10). In fact, the literature includes other TEP approaches dealing with decentralized strategic oligopolistic generation firms [5].

II. Strategy to Solve the MTEP Problem

The strategy to solve the MTEP uses the following steps:

1) solve the static TEP problem independently for each stage in the planning horizon;
2) use the static TEP solution obtained in 1), for each stage, like a base case and carry out the forward planning from this stage until the last planning stage; a set of reinforcement circuits is thus obtained;
3) at the end of step 2), SN sets of reinforcement circuits are obtained, where SN is the stage number of the horizon planning. Merge all these sets to obtain a new set, which is the reduced CSS for the MTEP problem;
4) solve the MTEP problem (1)–(11) considering as expansion candidates only the reinforcement circuits included in the reduced CSS obtained in step 3) (in other words, construction of only the reinforcement circuits in the reduced CSS is allowed).

In steps 1) and 2), the static TEP can be obtained solving the model (1)–(11) considering $T = \{1\}$. The reduced CSS is obtained using the solution of several static TEP problems. Note that in step 2), the forward solution is obtained for the MTEP problem (set of reinforcement circuits for the first stage), and the "target solution" used in the backward heuristic (set of reinforcement circuits for the last stage) is also obtained; both sets are in the reduced CSS. In step 3), there is no guarantee that the reduced CSS contains the optimum solution for the MTEP problem, but it does contain both solutions for forward and backward planning, and guarantees that the solution for the MTEP problem using the reduced CSS is better than or equal to the best solution of the forward and backward planning. The reduced CSS contains few of the reinforcement circuits that are used to solve the MTEP problem. Step 4) takes into consideration the MTEP model, including the links between decisions taken stage to stage; in consequence, phenomena like economies of scale are taken into account. Furthermore, the proposed methodology is not useful when the electricity market is decentralized and/or when generators have the ability to exercise local market power because, in those cases, the strategy has the problem of "dynamic inconsistency".

TABLE I
Results Summary for MTEP Problem

Systems	Binary variables	Best known solution [MUS$]	Forward planning [MUS$]	Backward planning [MUS$]	Proposed strategy [MUS$]
IEEE24	615	220.29(80 sec)	232.62 (3 sec)	220.29 (5 sec)	220.29 (9 sec)
Colombian	930	492.17 (10 hr)	518.26 (3 min)	497.16 (7 min)	492.17(12 min)
Bolivian	764	71.78 (7 hr)	78.08 (2 min)	71.93 (4 min)	71.78 (6 min)

III. Tests and Results

All the tests were made on a SunFire X2200 with two dual core AMD processor at 3 GHz and 32 Gb of RAM memory. The problem (1)–(11) was written in AMPL [6] and solved with the CPLEX 12.1 [7] (called with default options). The proposed strategy was used to solve three test systems. The economic evaluations for all the systems use the discount factors shown in [4]. For all tests, the maximum phase angle is 90 degrees. The data of the systems can be obtained upon request from the authors.

A result summary is shown in Table I, including the number of binary investment variables and the approximate computational times (in parentheses) for all approaches. The best solution for all test systems was obtained by directly solving (1)–(11). Note that the difference in the computational effort required to solve the MTEP problem using the proposed strategy instead of the original one is due to the reduced CSS size.

1) IEEE24 System: Plan P1 (164 MUS$): $n_{6-10} = 1$, $n_{7-8} = 2$, $n_{10-12} = 1$, $n_{11-13} = 1$; Plan P2 (30 MUS$):$n_{20-23} = 1$; Plan P3 (72 MUS$): $n_{1-5} = 1$, $n_{3-24} = 1$. From Table I, the results using the proposed strategy, the backward planning, and the best solution are the same and are better than the forward planning. The reduced CSS contains approximately 5% of the reinforcement circuits used to solve the MTEP problem.

2) Colombian System [4]: Plan P1 (338.74 MUS$): $n_{57-81} = 2$, $n_{55-57} = 1$, $n_{55-62} = 1$, $n_{45-81} = 1$, $n_{82-85} = 1$. Plan P2 (104.75 MUS$): $n_{27-29} = 1$, $n_{62-73} = 1$, $n_{72-73} = 1$, $n_{19-82} = 1$. Plan P3 (161.21 MUS$): $n_{43-88} = 2$, $n_{15-18} = 1$, $n_{30-65} = 1$, $n_{30-72} = 1$, $n_{55-84} = 1$, $n_{27-64} = 1$, $n_{19-82} = 1$, $n_{68-86} = 1$. From Table I, the results using the proposed strategy and the best solution are the same and better than the other methods. The reduced CSS contains approximately 9% of the reinforcement circuits used to solve the MTEP problem.

3) Bolivian System: This system has 57 buses, 92 branches, and four stages (2010, 2014, 2017, and 2021); therefore, $\alpha_4 = 0.349$ is used for the fourth stage. This system includes: 1) change in generation location; 2) location/removal of the mining load; and 3) interconnection with Peru in 2021 (exporting 300 MW). Plan P1 (1.54 MUS$): $n_{36-39} = 1$, $n_{13-14} = 1$, $n_{21-39} = 1$; Plan P2 (20.32 MUS$): $n_{27-50} = 1$, $n_{21-39} = 1$; Plan P3 (18.96 MUS$): $n_{43-51} = 2$; Plan P4 (132.84 MUS$): $n_{43-53} = 2$, $n_{53-51} = 1$, $n_{51-54} = 1$, $n_{21-55} = 2$, $n_{55-32} = 3$, $n_{52-53} = 2$, $n_{55-20} = 2$, $n_{55-35} = 2$, $n_{24-25} = 1$, $n_{41-45} = 1$. From Table I, the results using the proposed strategy were equal to the best known solution and better than the other methods. The reduced CSS contains approximately 19% of the reinforcement circuits used to solve the MTEP problem.

IV. Conclusion

A simple and quick heuristic to reduce the CSS was presented in order to use IPTs to solve the MTEP problem using less computational time. The heuristic uses the solution of several static TEP problems. In the TEP model, if the maximum voltage angle of the system is known, it can be shown that the double of the maximum voltage angle is a "Big M" factor large enough to formulate the MTEP problem correctly. Test and real systems were used to demonstrate the usefulness of this heuristic. The reduced CSS contains few of the reinforcement circuits that are used to solve the MTEP problem. The backward planning found good solutions, while the forward planning found the worst solutions for the MTEP problem. However, the proposed strategy is faster than the direct approach and finds better solutions than previous heuristic approaches for all systems.

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