Crescimento de seres vivos, indivíduo ou população. Introdução a modelagem estatística.

NÚCLEO DE ESTUDOS EM REGRESSÃO NÃO LINEAR APLICADA

NLIN – Núcleo de estudos em regressão não linear aplicada

Universidade Federal do Oeste do Pará (UFOPA)
Curso de Engenharia de Aquicultura Campus de Monte Alegre

UNIVERSIDADE FEDERAL DE LAVRAS

Universidade Federal de Lavras (UFLA)
Programa pós graduação em Estatística e Experimentação agropecuária

113 anos UFLA
Faz Extensão

Palestra: Carlos Antônio Zarzar
E-mail: carloszarzar_@hotmail.com
Coordenador grupo: Tales Jesus Fernandes
Data: 03/09/2021 às 15:00
Extensão UFLA

carloszarzar_@hotmail.com

- O que é crescimento?
- Como ocorre o fenômeno do crescimento?
- Por que é importante estudar o crescimento dos organismos?
- Como mensurar (quantificar, medir) o crescimento dos animais ou plantas?
- Como amostramos o crescimento de um lote?
- Crescimento observacional e experimental.
- Quais os fatores que influenciam o crescimento?
- Autocorrelação do crescimento.
- Quais os modelos matemáticos / estatísticos de crescimento? (Modelos não lineares sigmoides)
- Ganho de peso e o crescimento dos indivíduos?
- Modelagem do ganho de peso através das derivadas.
- Ganho de peso máximo e sua relação econômica (custo e receita).
- Famílias de modelos não lineares.
- Parametrizações de modelos não lineares.

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- O que é crescimento?

https://www.suno.com.br/artigos/analisando-o-potencial-de-crescimento-de-uma-empresa/

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- O que é crescimento?

https://www.drogariasultrapopular.com.br

O crescimento corresponde às **alterações físicas nas dimensões** do corpo como um **todo**, ou de **partes** específicas, em relação ao fator **tempo** (Karlberg e Taranger, 1976).

YouTube

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- O que é crescimento?

https://www.drogariasultrapopular.com.br

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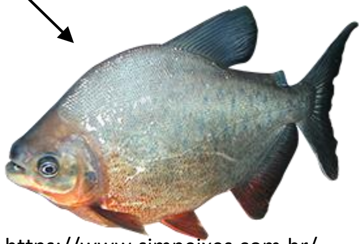

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https://agroara.com.br/

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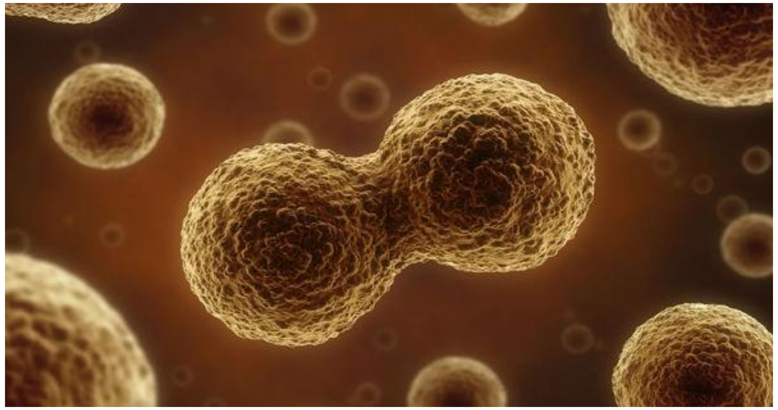
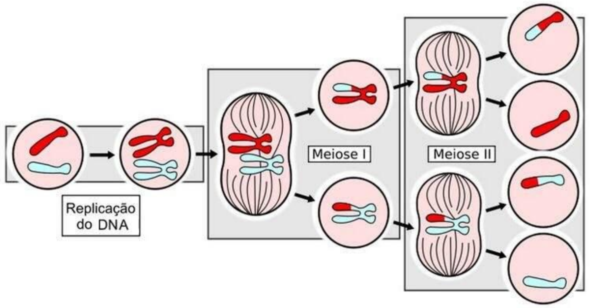
- Como ocorre o fenômeno do crescimento dos organismos?

https://www.drogariasultrapopular.com.br



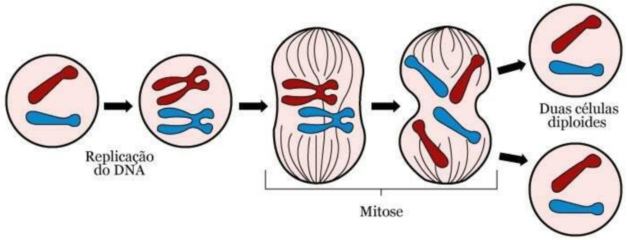
https://trabalhosparaescola.com.br/divisao-celular/

Meiose



https://www.educamaisbrasil.com.br/enem/biologia/meiose

Mitose



https://www.educamaisbrasil.com.br/enem/biologia/mitose

O crescimento somático envolve o aumento do número de células (hiperplasia), um aumento no seu tamanho (hipertrofia), e um incremento no conteúdo extracelular (Malina e BoucharcL, 1991; Fischbein, 1977).

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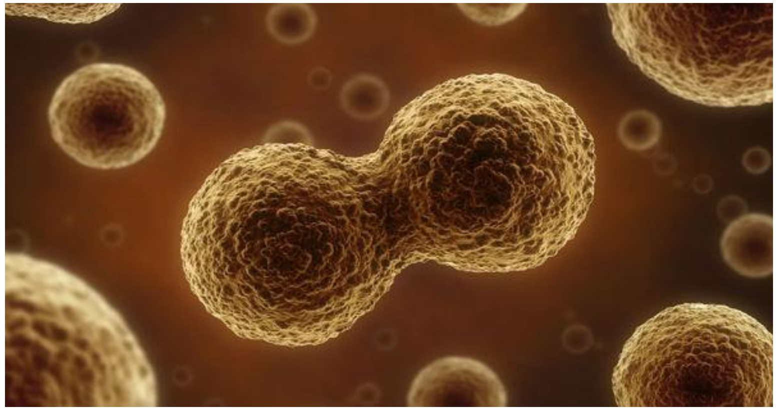
- Como ocorre o fenômeno do crescimento dos organismos?

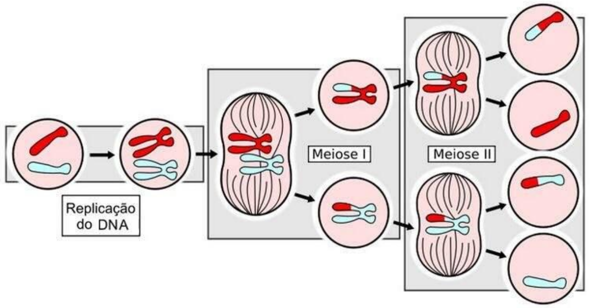
https://www.drogariasultrapopular.com.br

https://www.voninmaquinas.com.br/dicas-crescimento-do-seu-negocio/



https://trabalhosparaescola.com.br/divisao-celular/

Meiose



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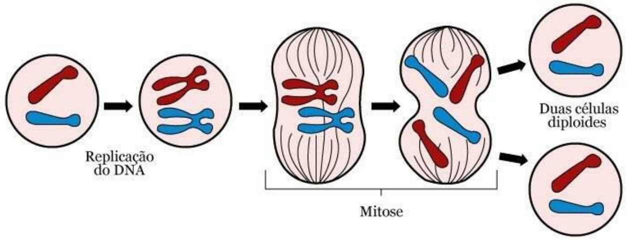
Mitose



https://www.educamaisbrasil.com.br/enem/biologia/mitose

- Por que é importante estudar o crescimento dos organismos?

https://earthtechling.com/2012/08/farmed-fish-float-almost-free-in-clever-geodesic-domes/

http://www2.senar.com.br/

https://www.voninmaquinas.com.br/dicas-crescimento-do-seu-negocio/

https://br.freepik.com/

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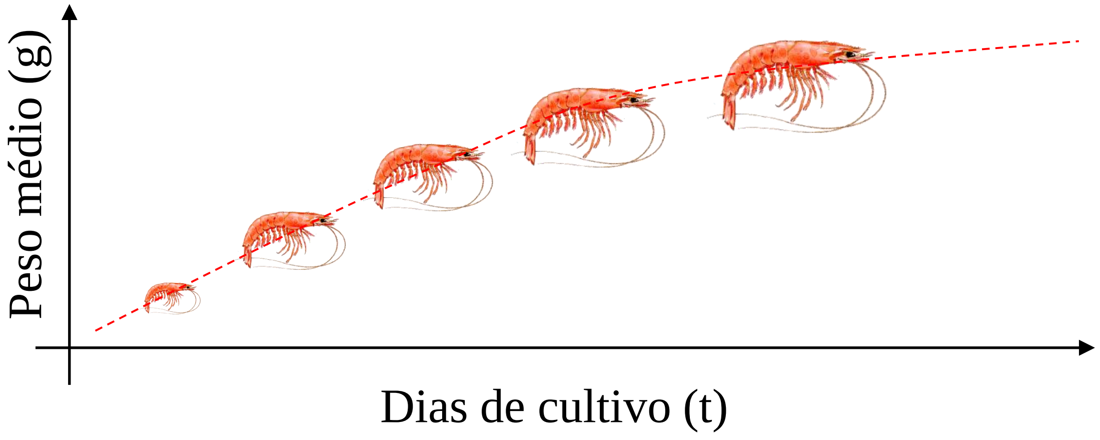
- Como mensurar (quantificar, medir) o crescimento dos animais ou plantas?

Crescimento em função do tempo

$$f(t) = \cdots$$



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- Como mensurar (quantificar, medir) o crescimento dos animais ou plantas?

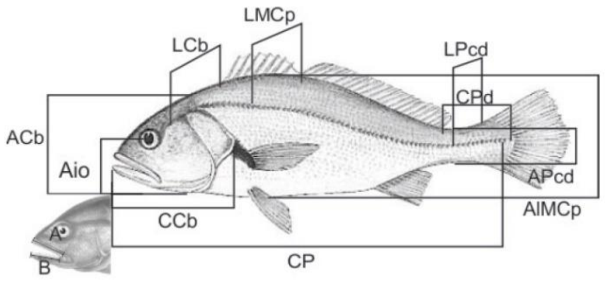
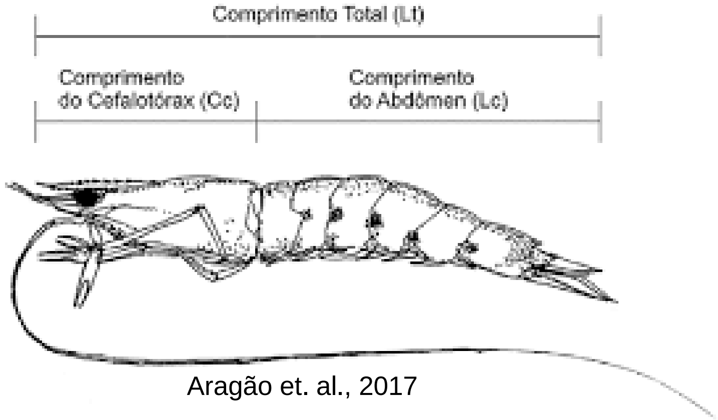


Fig. 2. Medidas morfométricas utilizadas para a pescada *Plagioscion squamosissimus* Heckel, 1840.

Filho et. al., 2014



Aragão et. al., 2017

Crescimento em função do tempo

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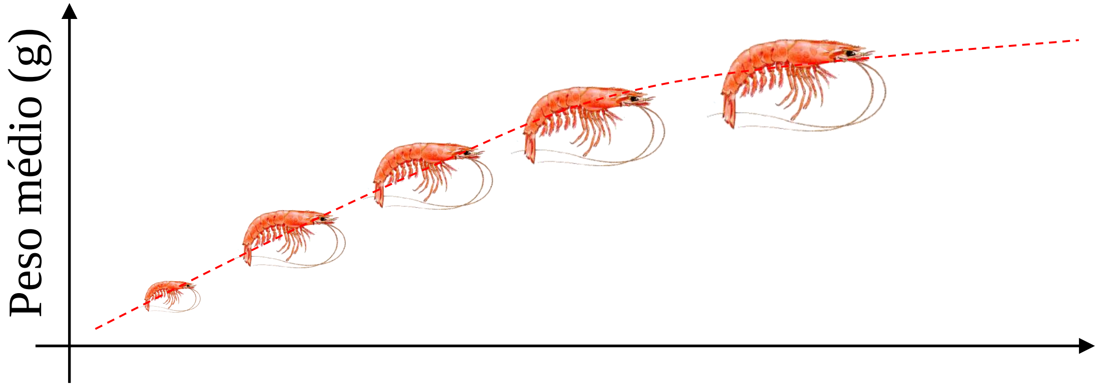

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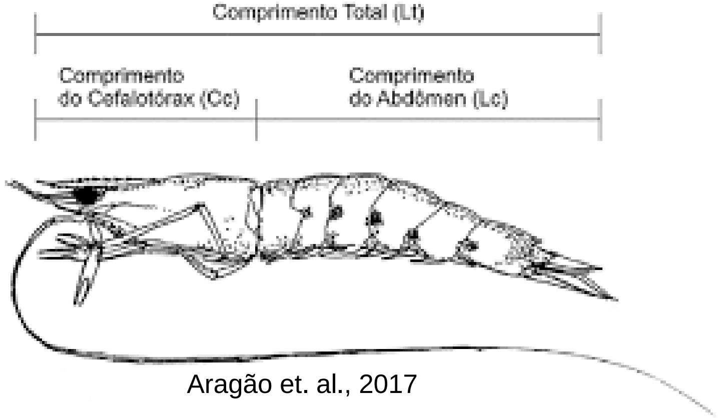
- Como mensurar (quantificar, medir) o crescimento dos animais ou plantas?

➢ São várias notações:

- *Peso - Weight: $W(t)$;*
- *Sobrevivência: $S(t)$;*
- *Número de células / bactérias ou tamanho da população: $N(t)$;*
- *Densidade de células ou microrganismos: $D(t)$;*
- *Concentração de organismos: $C(t)$;*
- *Volume: $V(t)$;*
- *Massa corporal: $M(t)$;*
- *Comprimento - Length: $L(t)$;*



Aragão et. al., 2017

➢ Variáveis dependentes podem ser declaradas como valores relativos:

- $W(t)/A$, onde A é a assíntota superior (α);
- $W(t)/W_0$, onde W_0 é valor inicial;

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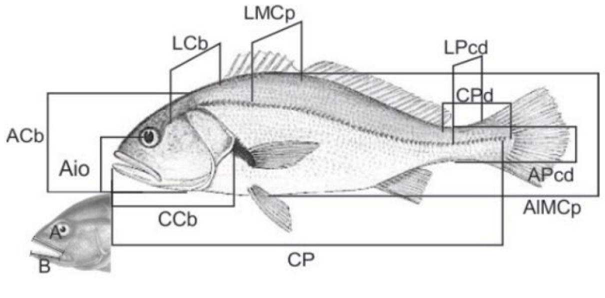
- Como amostramos o crescimento de um lote?



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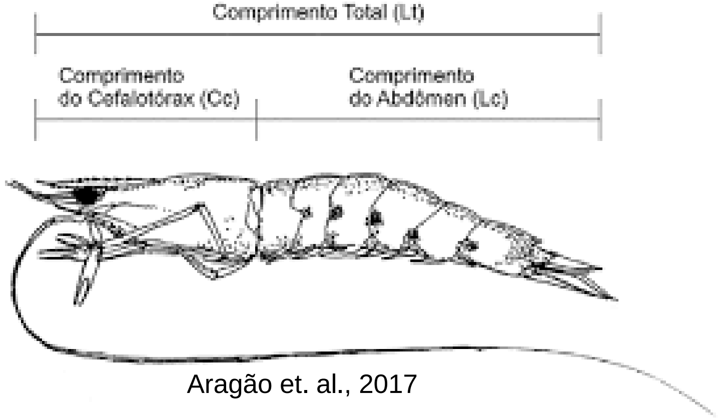
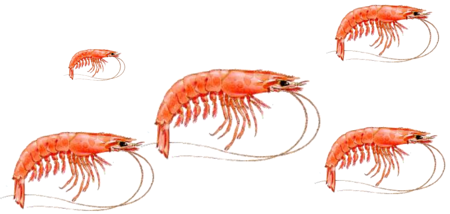


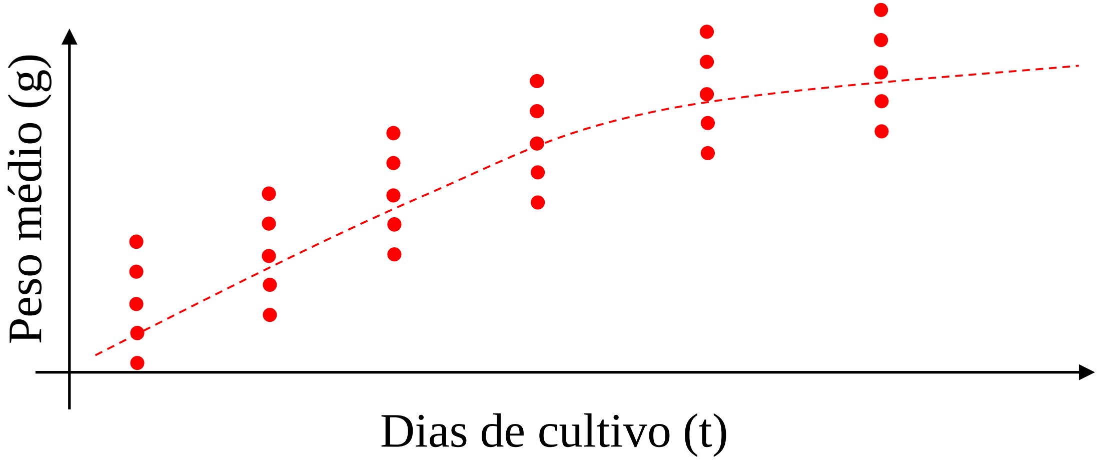
Aragão et. al., 2017

Crescimento em função do tempo

$$f(t) = \cdots$$



n=5 amostras
Peso (g)
Comprimento (cm)



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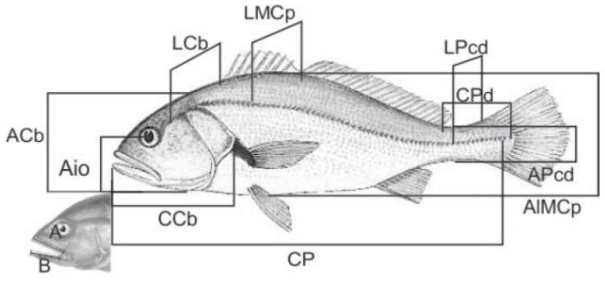
- Como amostramos o crescimento de um lote?



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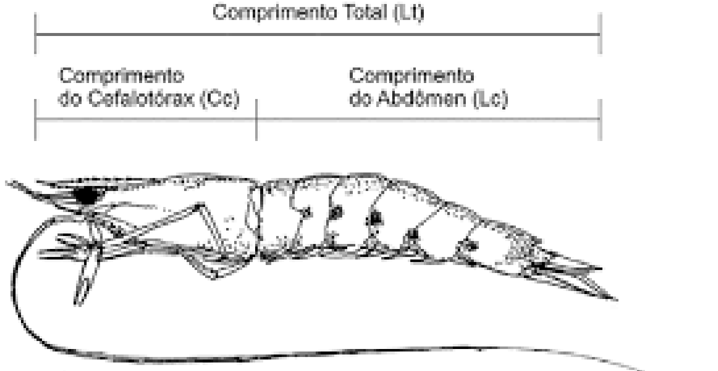
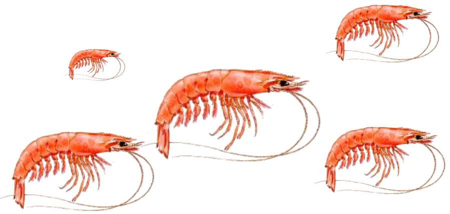
Filho et. al., 2014

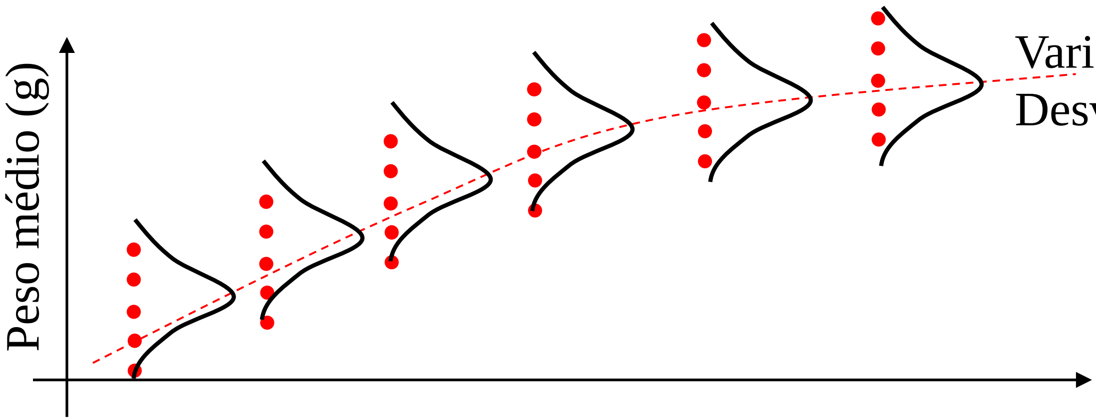


Aragão et. al., 2017

Crescimento em função do tempo
$f(t) = \cdots$



n=5 amostras
Peso (g)
Comprimento (cm)



Variância (σ^2)
Desvio padrão (σ)

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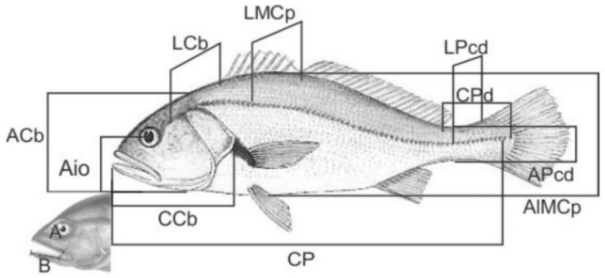
- Como amostramos o crescimento de um lote?



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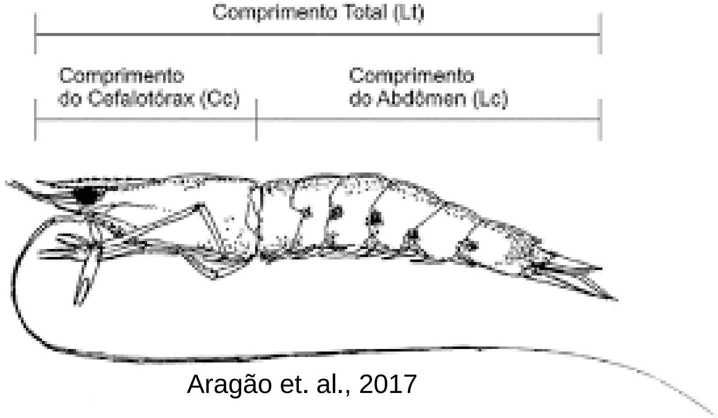

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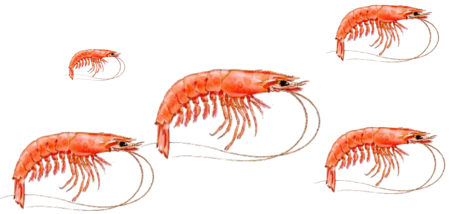

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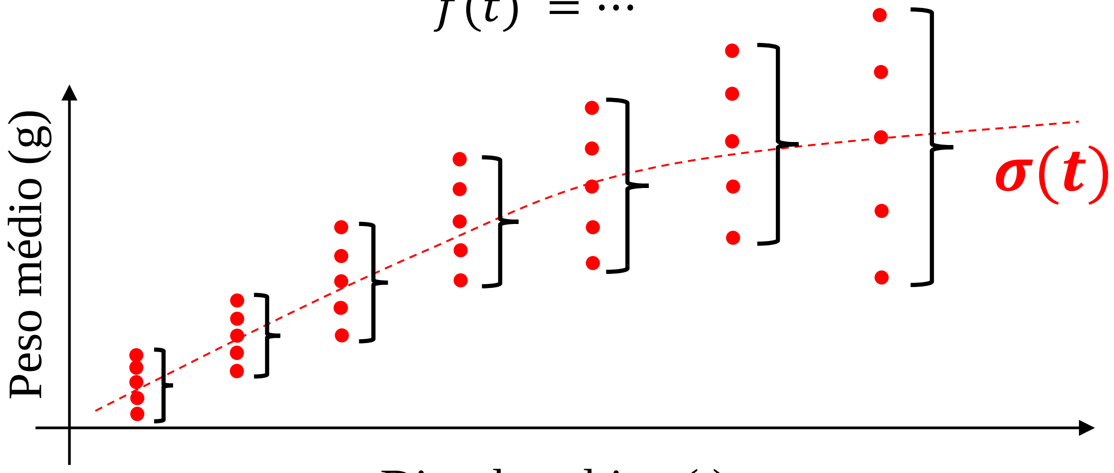

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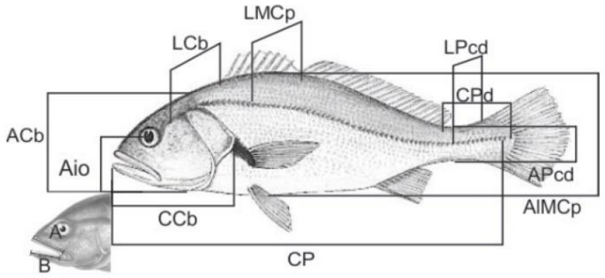
- Como amostramos o crescimento de um lote?



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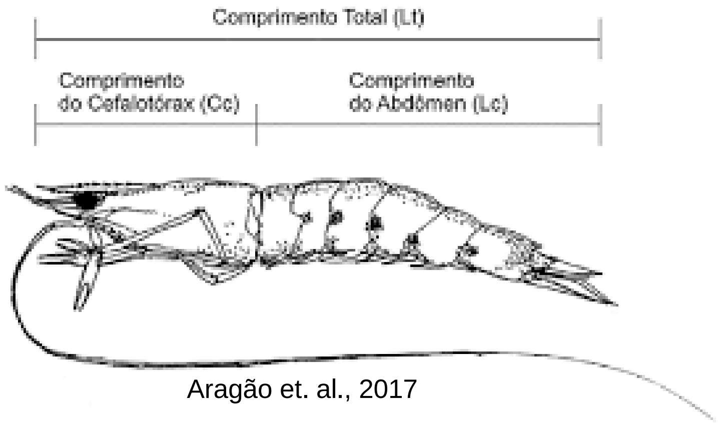
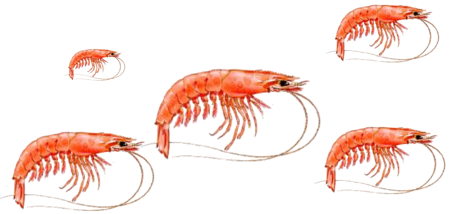


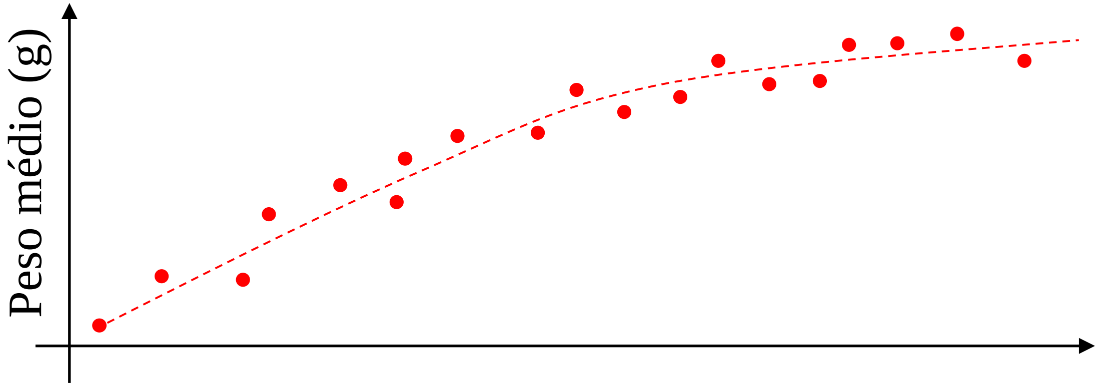
Aragão et. al., 2017

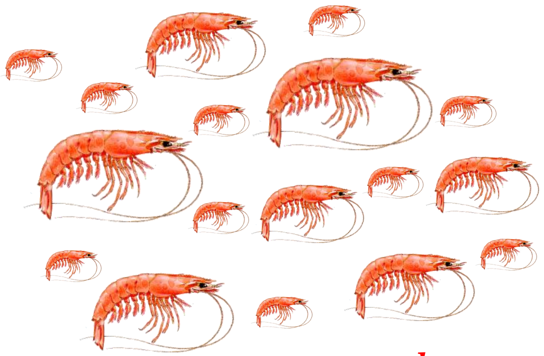
Crescimento em função do tempo

$$f(t) = \cdots$$



***n*=5** amostras
Peso (g)
Comprimento (cm)





$$\tilde{x} = \frac{peso\ total}{n\ indivíduo}$$

Média Peso (g)
Média Comprimento (cm)

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- Como amostramos o crescimento de um lote?

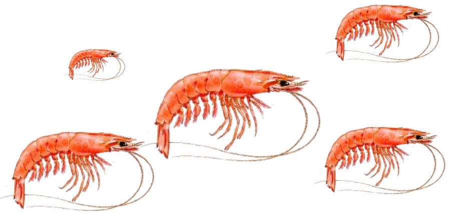
Erro aleatório:

$\epsilon \sim N(\mu, \sigma)$

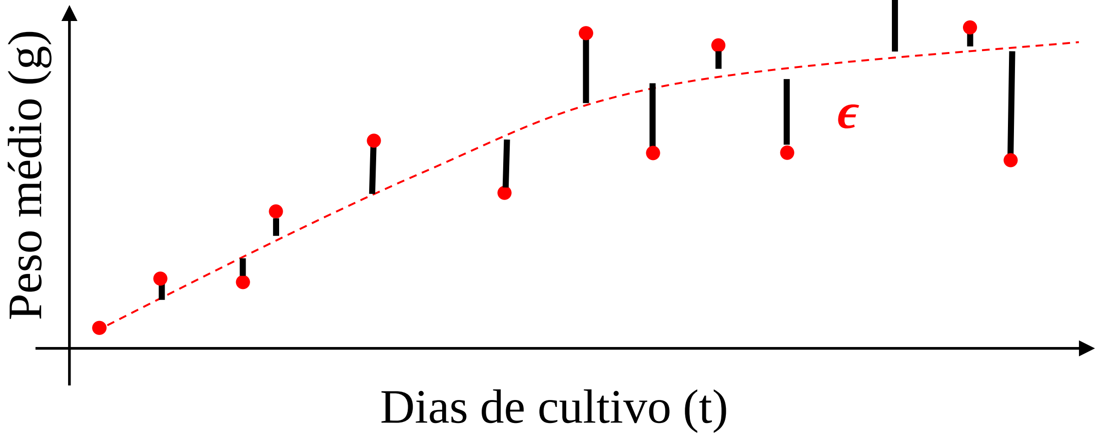
$\mu = 0$

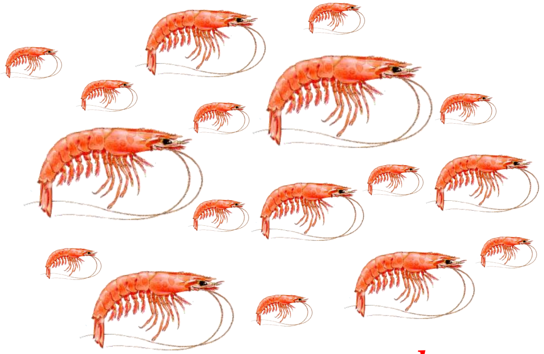
Crescimento em função do tempo

$f(t) = \cdots$



***n*=5** amostras
Peso (g)
Comprimento (cm)





$$\tilde{x} = \frac{peso\ total}{n\ indivíduo}$$

Média Peso (g)
Média Comprimento (cm)

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- Como amostramos o crescimento de um lote?

Erro aleatório:

$\epsilon \sim N(\mu, \sigma)$

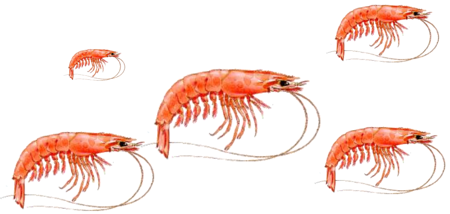
$\mu = 0$

$\sigma = {\color{red}\tau} \cdot f(t)$

$\sigma = {\color{red}\tau} \cdot peso$

Crescimento em função do tempo

$f(t) = \cdots$



***n*=5** amostras
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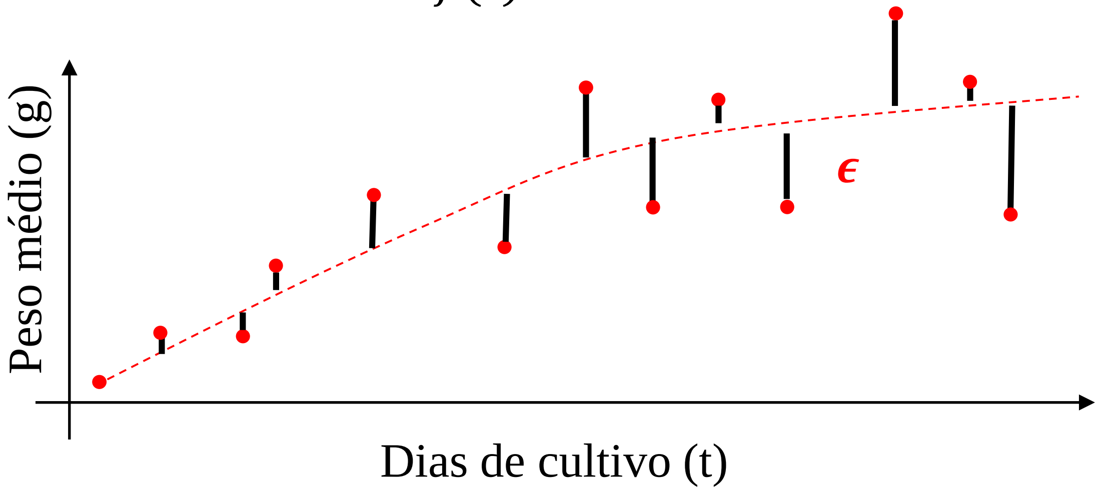
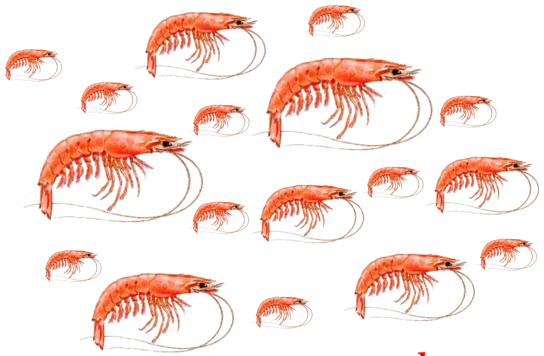
Comprimento (cm)





$\tilde{x} = \frac{peso\ total}{n\ indivíduo}$

Média Peso (g)
Média Comprimento (cm)

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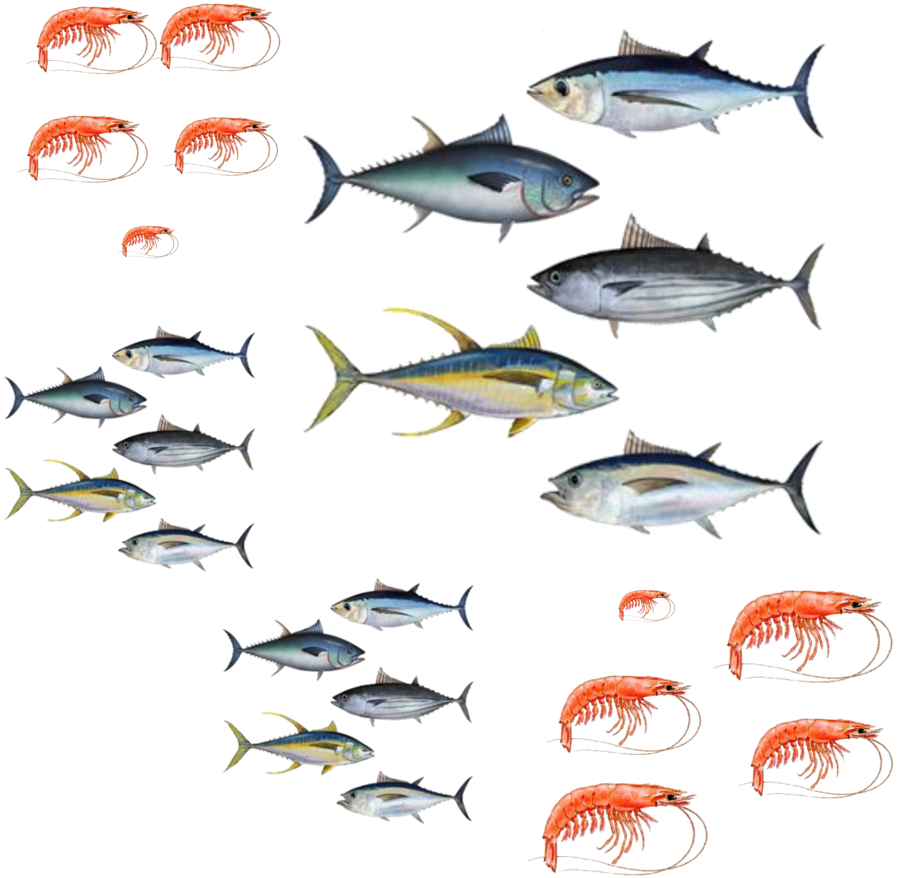
Estudo observacional

Ambientes heterogêneos



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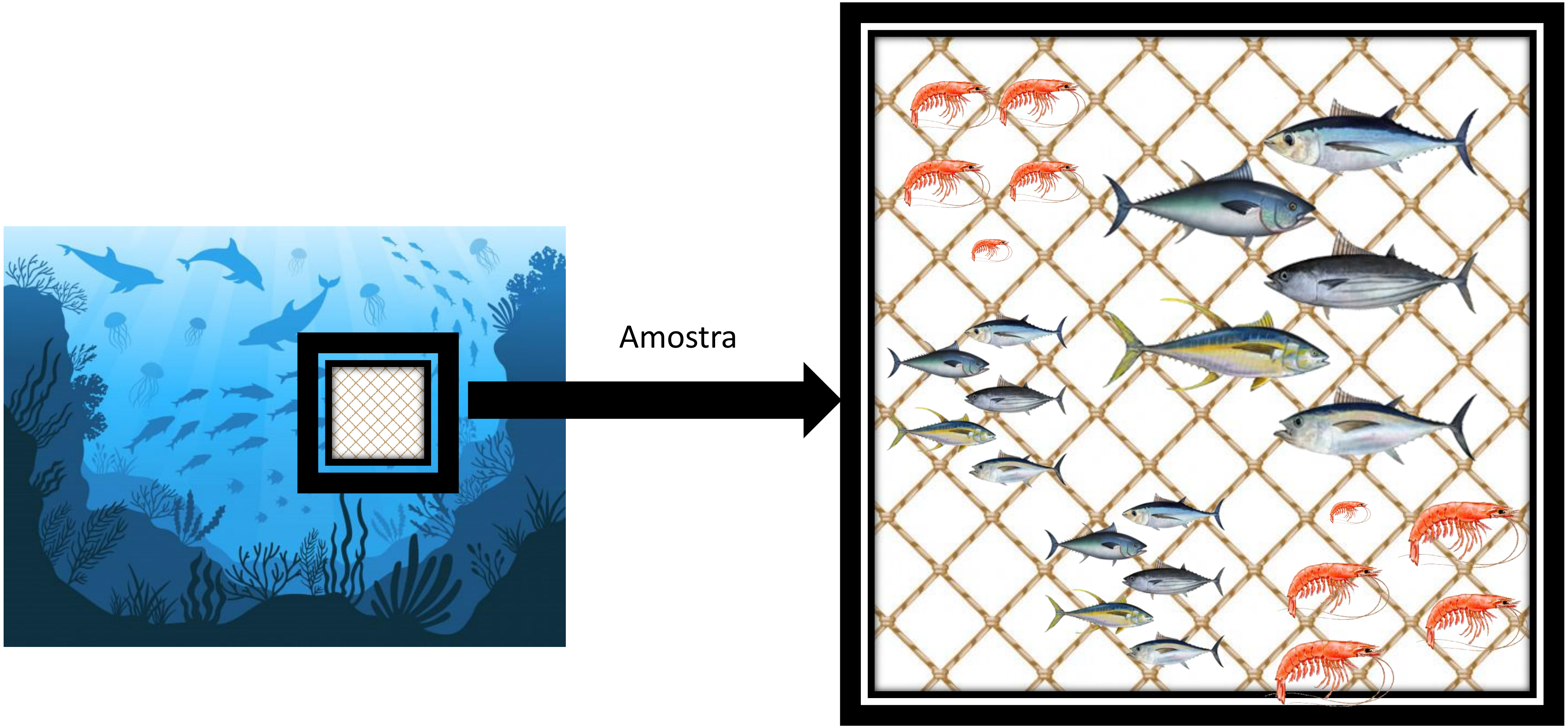
Estudo observacional



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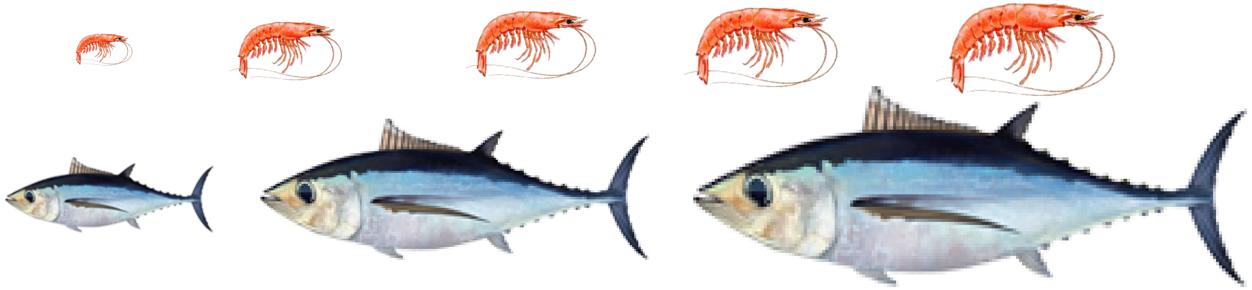
Estudo observacional



Faixa etária

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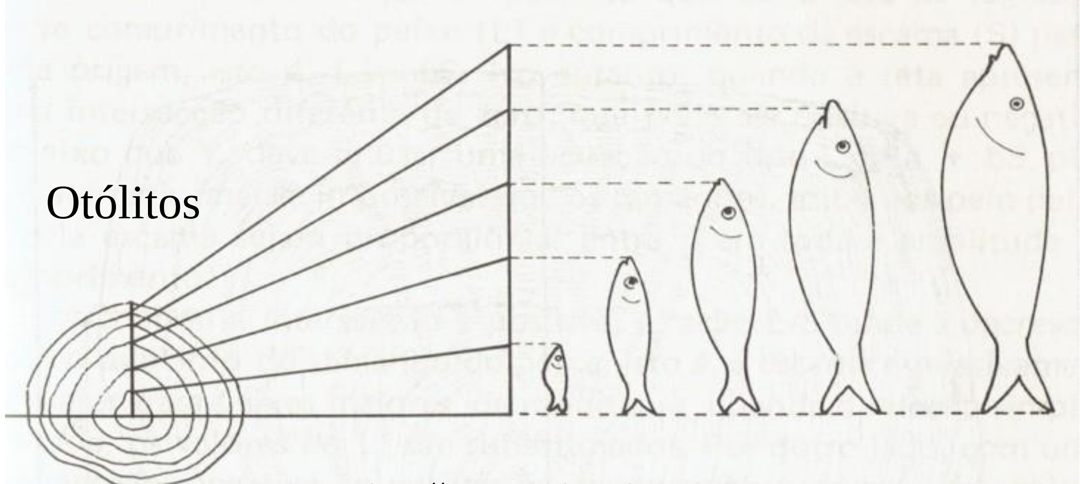
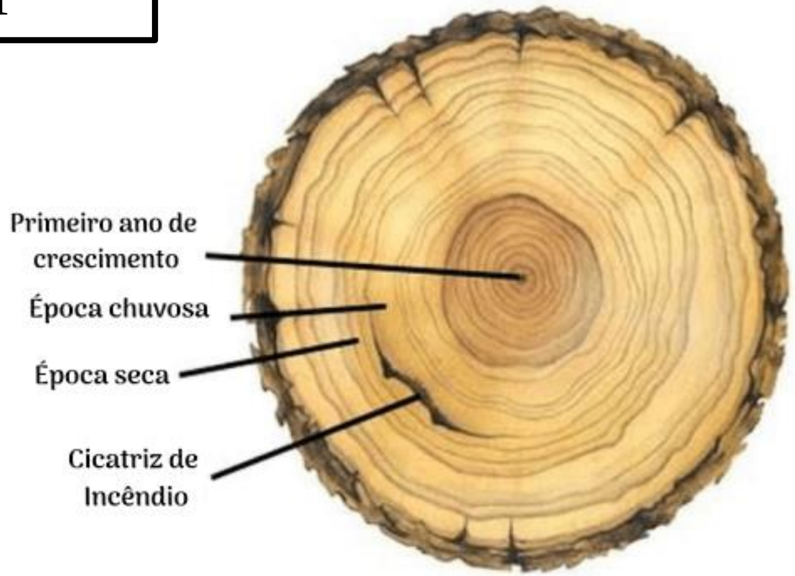
Estudo observacional

Otólitos



https://www.jnippak.com.b



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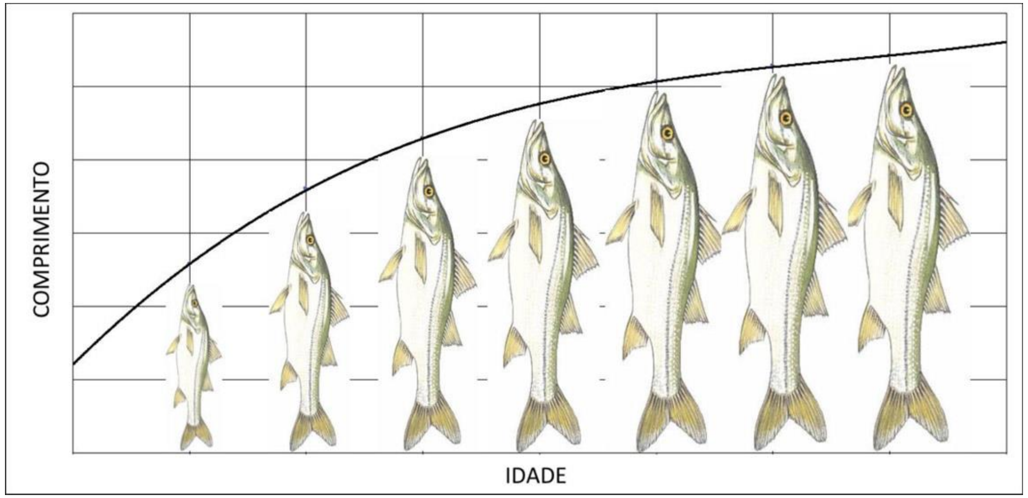

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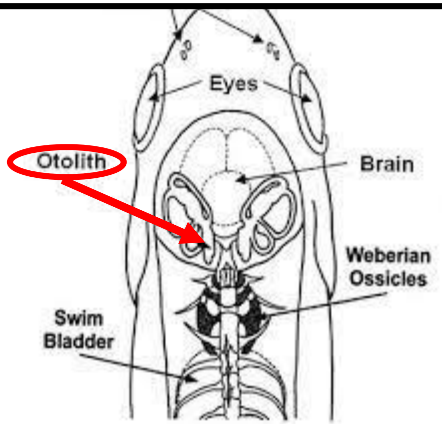
Estudo observacional



https://pt.wikipedia.org/

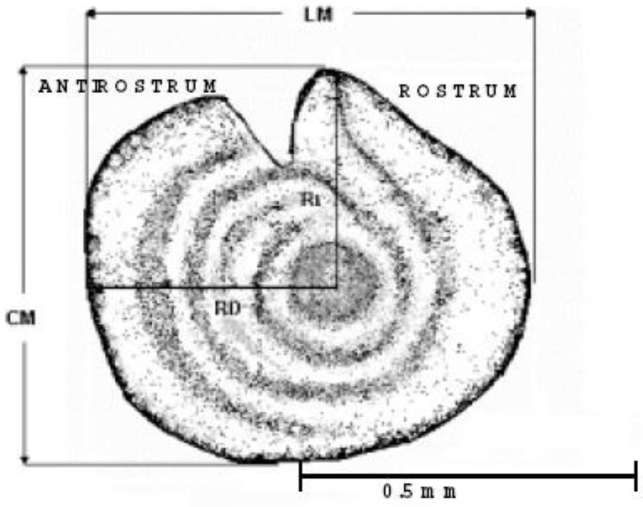
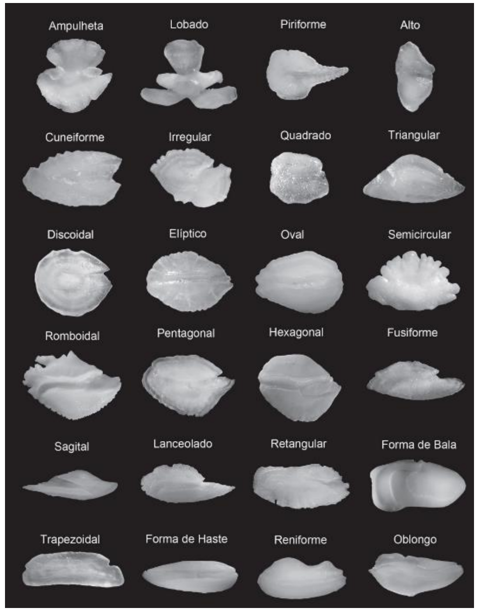


Figura 2. Esquema da face externa do otólito *asteriscus*, indicando as medidas obtidas nesta estrutura, (LM: largura máxima; CM: comprimento máximo; RD: raio dorsal; Rt: raio total).

Pérez e Fabré 2003



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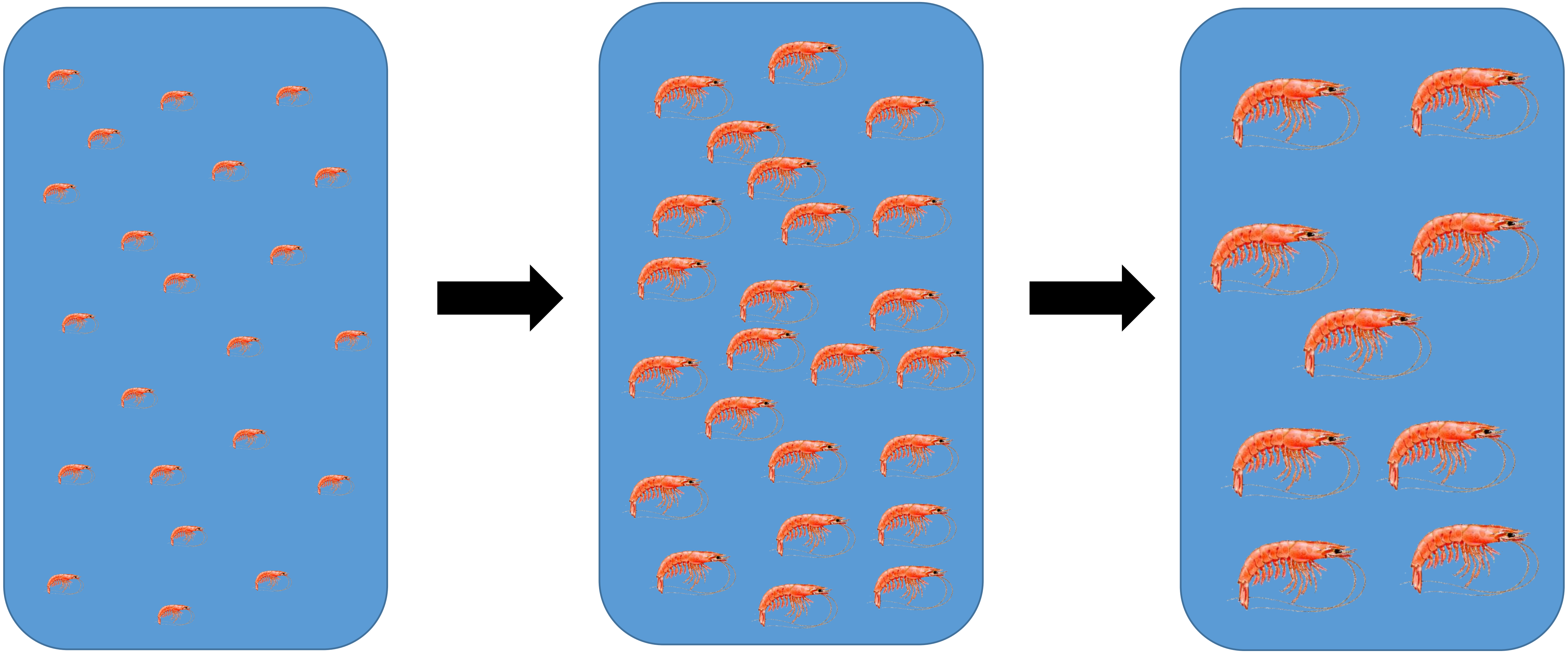
Estudo experimental

1 dia

7 dias

15 dias



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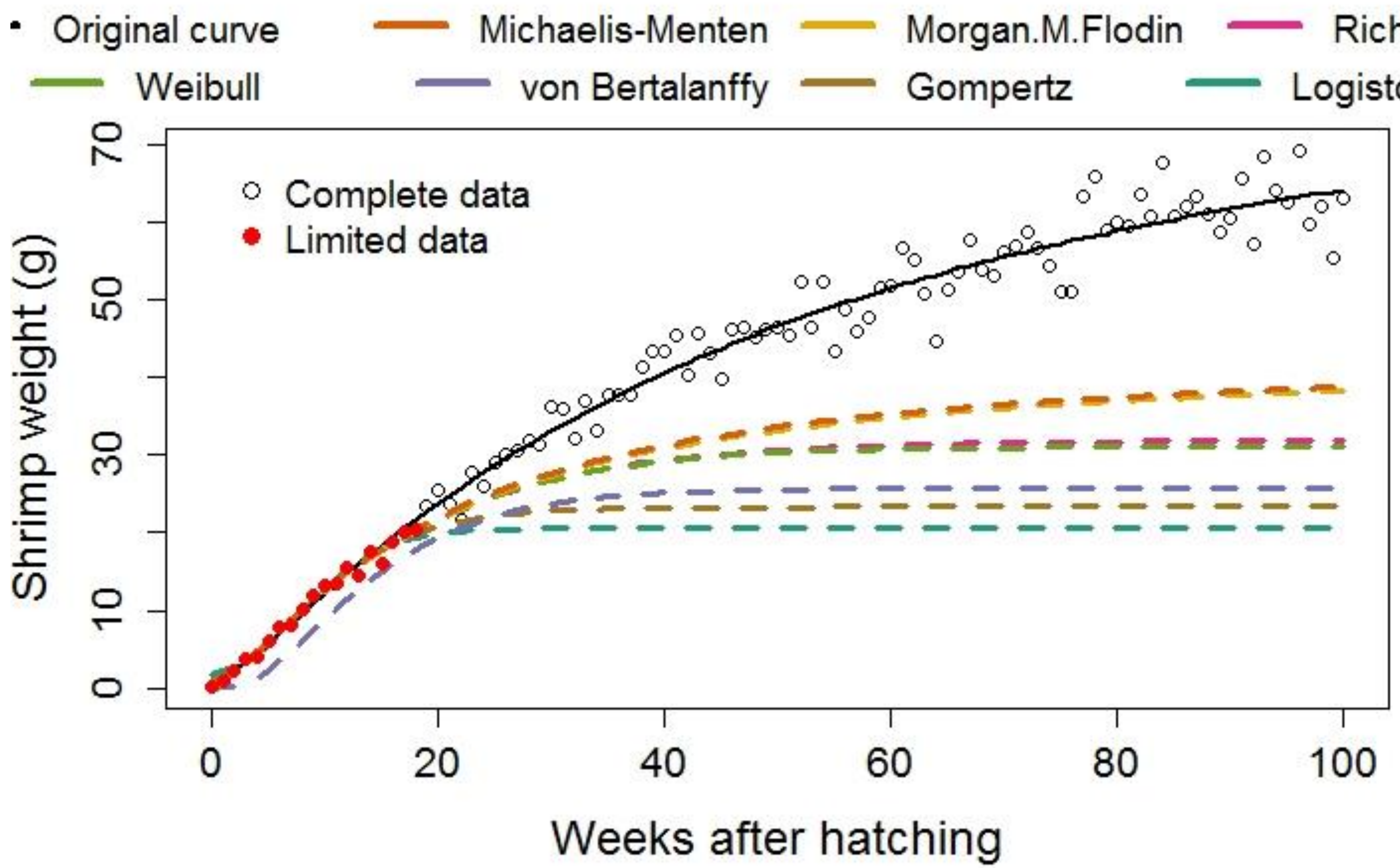
Estudo experimental



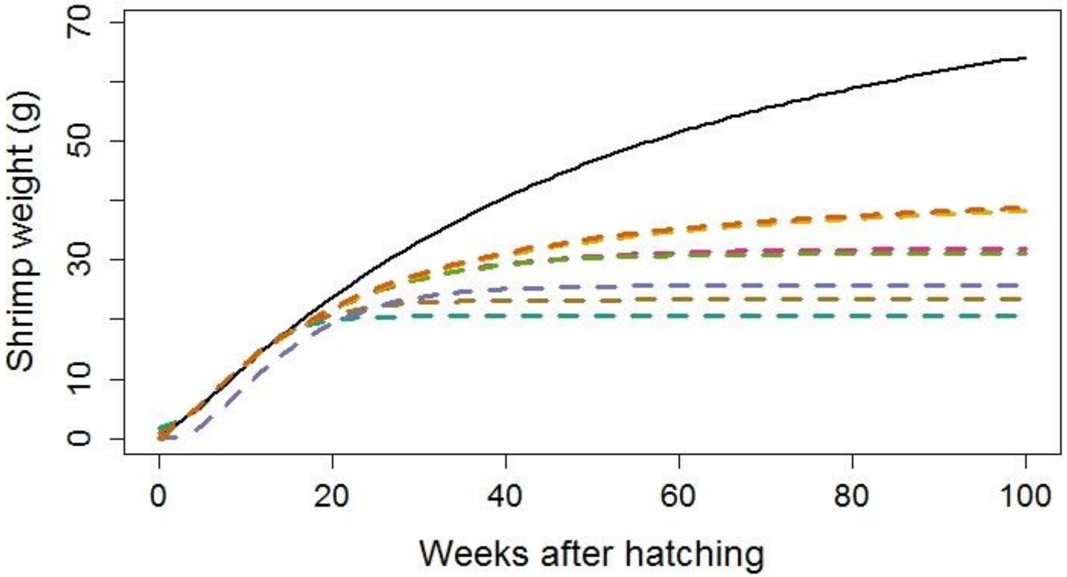
- Quais os fatores que influenciam o crescimento?

O crescimento varia com a idade, sendo governada por fatores genéticos, regulados por mecanismos hormonais complexos e atualizados pela natureza sempre variável do ambiente (Malina & BoucharcL, 1991; Fischbein, 1977).

Crescimento em função do tempo

$$f(t) = \cdots$$

Fatores intrínsecos



Fatores extrínsecos

carloszarzar_@hotmail.com

Autocorrelação

https://pt.vecteezy.com/

https://www.comprerural.com/

https://pt.aliexpress.com/

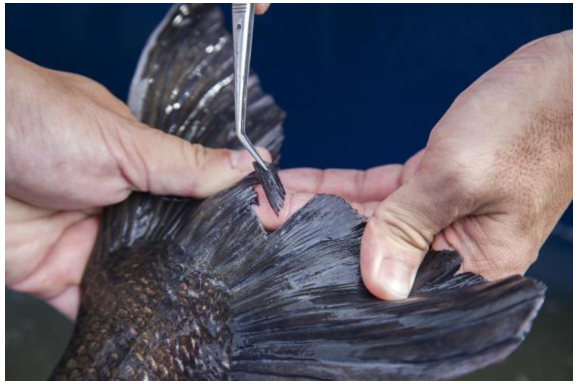
carloszarzar_@hotmail.com

Autocorrelação

https://pt.vecteezy.com/

https://www.comprerural.com/

https://pt.aliexpress.com/



https://www.embrapa.br/

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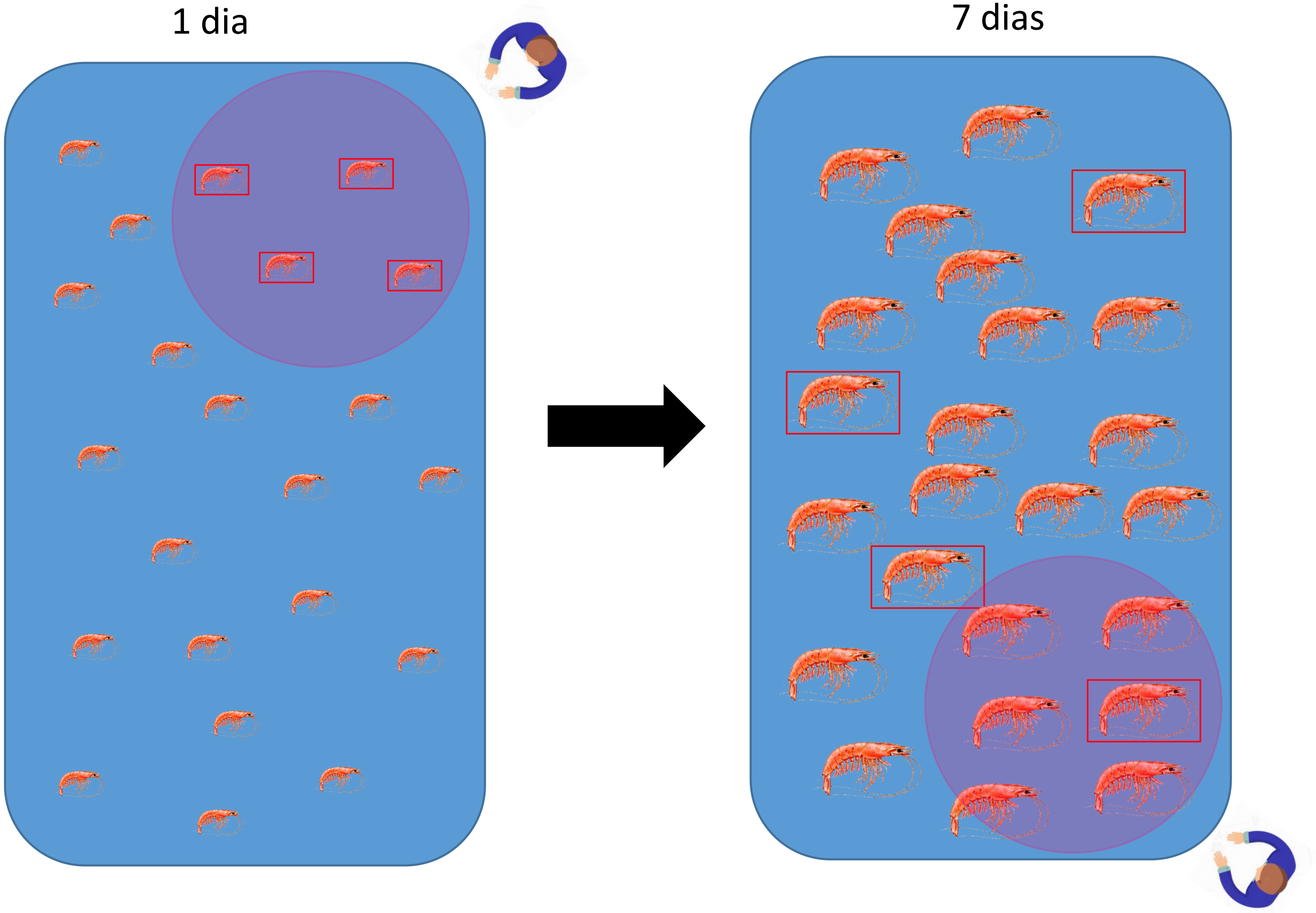

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Predicting shrimp growth: Artificial neural network versus nonlinear regression models

Run Yu [a,b], PingSun Leung [a,*], Paul Bienfang [c]

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[b] Department of Economics, University of Hawaii at Manoa, Honolulu, Hawaii 96822, USA
[c] Department of Oceanography, University of Hawaii at Manoa, Honolulu, Hawaii 96822, USA

Abstract

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Keywords: Shrimp...

1. Introductio...

A reliable p... determining h...

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E-mail address... psleung@hawaii.ed... bienfang@soest.hav...

Aquaculture 306 (2010) 205–210
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A Bayesian hierarchical model for modeling white shrimp (*Litopenaeus vannamei*) growth in a commercial shrimp farm

Run Yu *, PingSun Leung

Department of Natural Resources and Environmental Management, University of Hawaii at Manoa, 3050 Maile Way Gilmore 111 Honolulu, HI 96822, United States

ARTICLE INFO

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Keywords:
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ABSTRACT

The paper explored the Bayesian hierarchical model as a possible way to incorporate growth variability in estimating shrimp growth function to enhance forecasting accuracy, using data from 16 growout ponds of a commercial shrimp farm in Hawaii. Based on a dataset of 571 weekly growth observations, the Bayesian hierarchical model is found to fit the data better than the simple nonlinear model that neglects growth variability, with respect to the deviance information criterion, root mean squared error and mean absolute percentage error. The Bayesian hierarchical model therefore could be a promising alternative for forecasting shrimp growth in commercial aquaculture practice.

1. Introduction

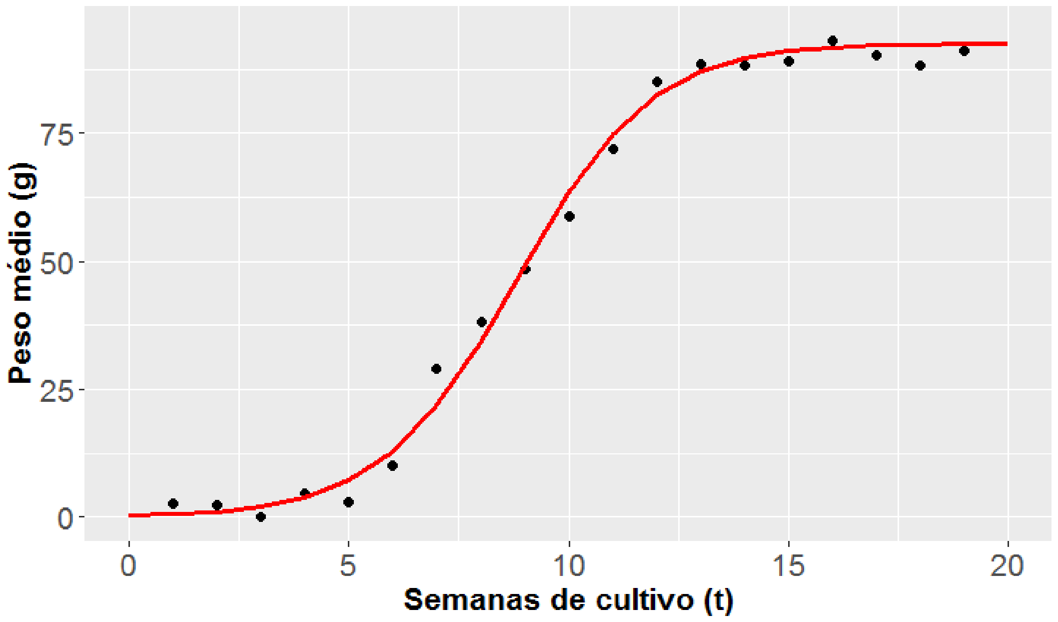
Forecasting growth of aquatic organisms bears great significance for any aquaculture enterprise. Many statistical models have been explored for modeling the growth of aquatic animals and plants. These models vary in the cultured species, explanatory factors, statistical methods employed, and the issues of interests. Shrimp is one of the most studied crustaceans (Wyban et al., 1995; Araneda et al., 2008). As for the statistical methods employed, various regression models have been widely applied in modeling shrimp growth (Tian et al., 1993). Yu et al. (2006) applied the neural network ponds, pens, or raceways) usually vary and thus pose a particular challenge for shrimp growth modeling. For example, the shrimp farm in this study operated 40 growout ponds year round. While these ponds were constructed with the same physical characteristics such as depth and surface area, historical records indicated that growth performances were different across ponds even under similar cultivation conditions such as water temperature, stocking density, and feeding rate. Ideally, it is best to trace the growth curve for each pond individually. Unfortunately, sampling data from individual ponds were generally not sufficient to accomplish this task. The conventional solution is to fit a single growth curve with pooled data,

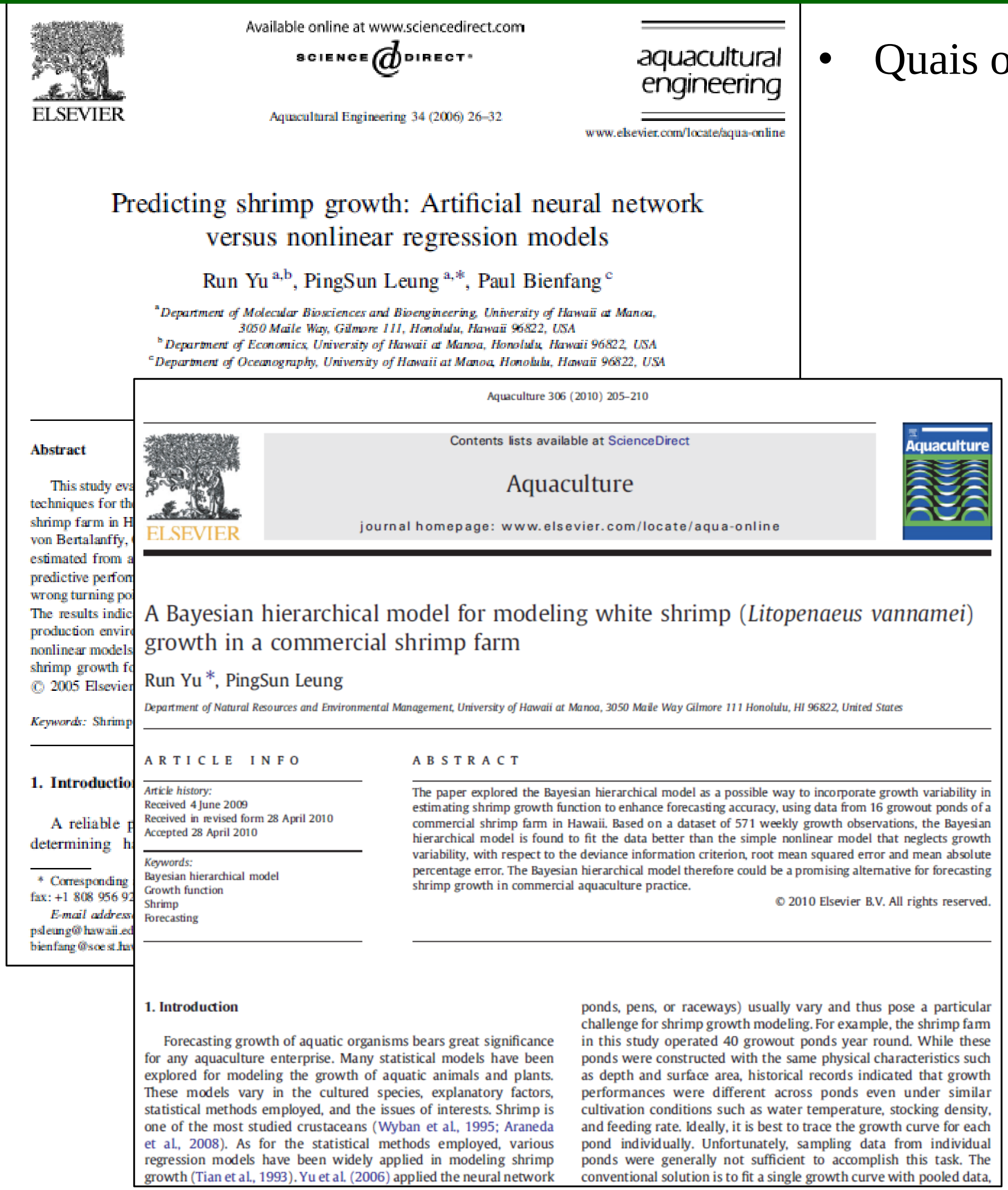
- Quais os modelos matemáticos / estatísticos de crescimento?

$$f_{(t)} = \frac{\alpha}{1 + e^{k(\beta - t)}}$$

Modelos não lineares para crescimento (sigmoidal)

Modelo Logístico





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1. Introduction

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Modelos não lineares para crescimento (sigmoidal)

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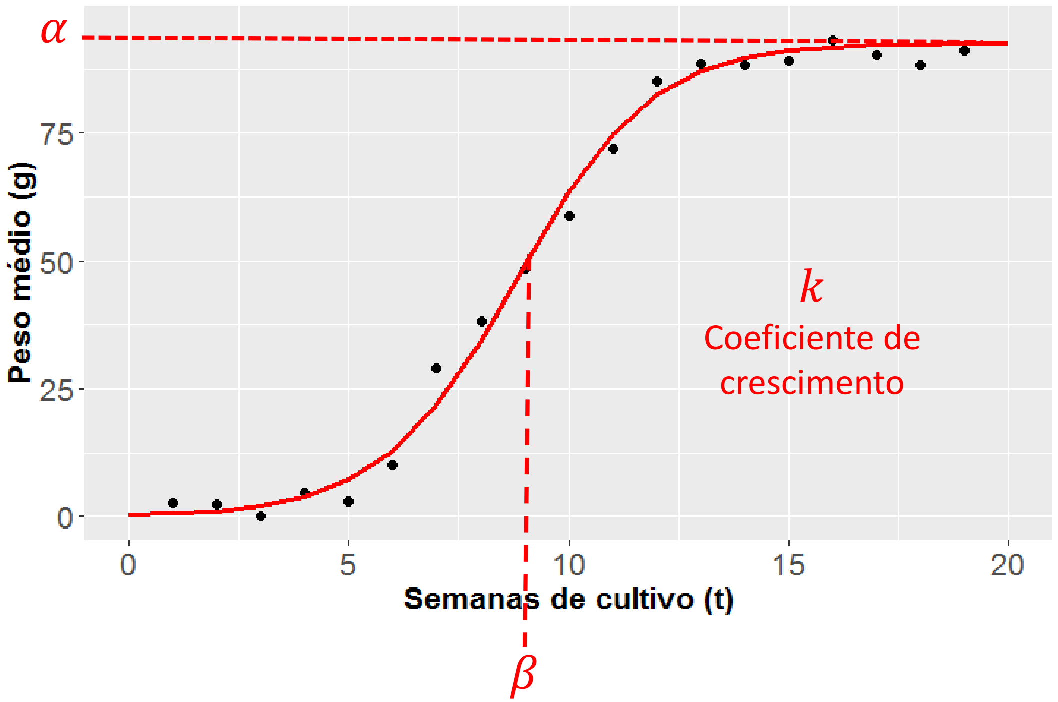

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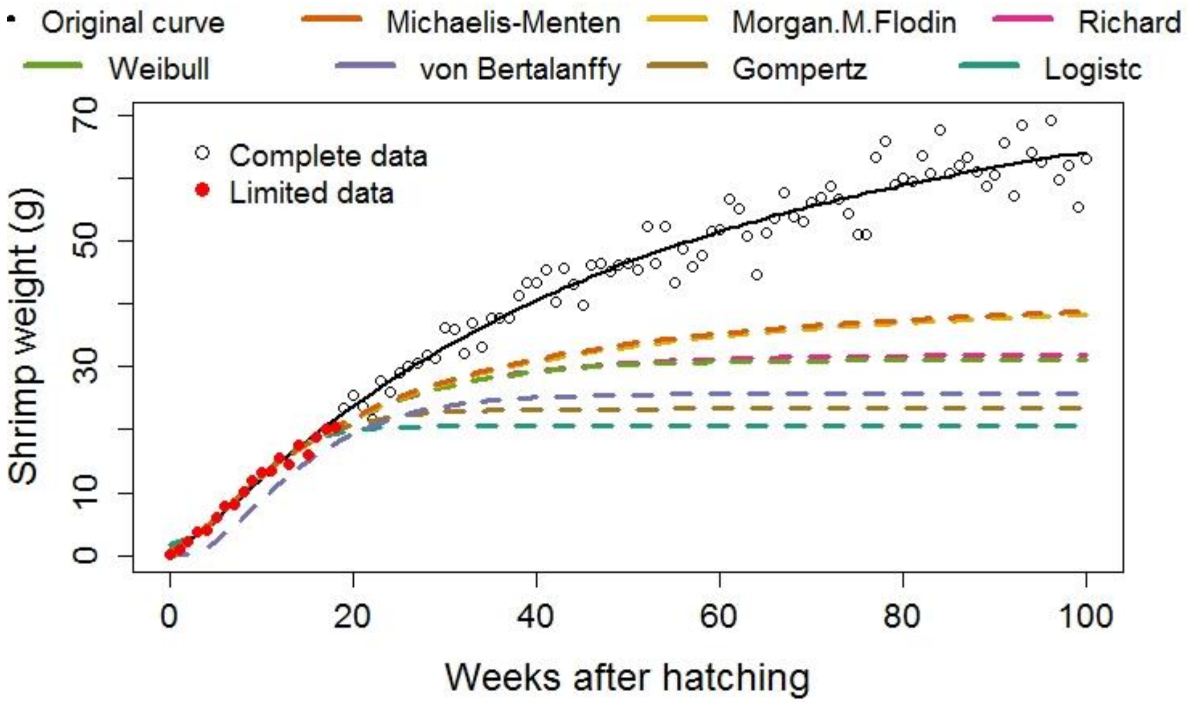
- Quais os modelos matemáticos / estatísticos de crescimento?



Modelos não lineares para crescimento (sigmoidal)

Function name	Mathematical expression	Parameters restrictions
Michaelis-Menten Generalized	$f(t) = \frac{w_0\ \beta^\kappa + \alpha\ t^\kappa}{\beta^\kappa + t^\kappa}$	$\alpha > 0;\ \beta > 0;$ $\kappa > 0$ and $w_0 \geq 0$
Gompertz function	$f(t) = \alpha\ exp[-exp(\kappa\ (\beta - t))]$	$\alpha > 0;\ \beta \in \mathbb{R};$ and $\kappa > 0$
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von Bertalanffy	$f(t) = \alpha\ (1 - exp(-\kappa\ (t + \beta)))^3$	$\alpha > 0;\ \beta \in \mathbb{R}$ and $\kappa > 0$
Richards curve	$f(t) = \frac{\alpha}{[1 + exp(-\kappa\ \delta\ (t - \beta))]^{1/\delta}}$	$\alpha > 0;\ \beta \in \mathbb{R};$ $\kappa > 0$ and $\delta > 0$
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Morgan-Mercer-Flodin (MMF)	$f(t) = \alpha - \frac{\alpha - w_0}{1 + (\kappa\ t)^\delta}$	$\alpha > 0;\ w_0 \geq 0;$ $\kappa > 0$ and $\delta \in \mathbb{R}_+^* \mid \delta \neq 1$

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Crescimento: Peso x Ganho de Peso

Crescimento

$\downarrow$

Ganho de Peso $\longrightarrow$ $\Delta w \sim \underbrace{w_1 - w_0}_{\text{Peso}}$

- Ganho de peso e o crescimento dos indivíduos?
- Modelagem do ganho de peso através das derivadas.

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Crescimento: Peso x Ganho de Peso

Crescimento

Ganho de Peso → $\Delta w \sim \underline{w_1 - w_0}$

Peso

- Ganho de peso e o crescimento dos indivíduos?
- Modelagem do ganho de peso através das derivadas.

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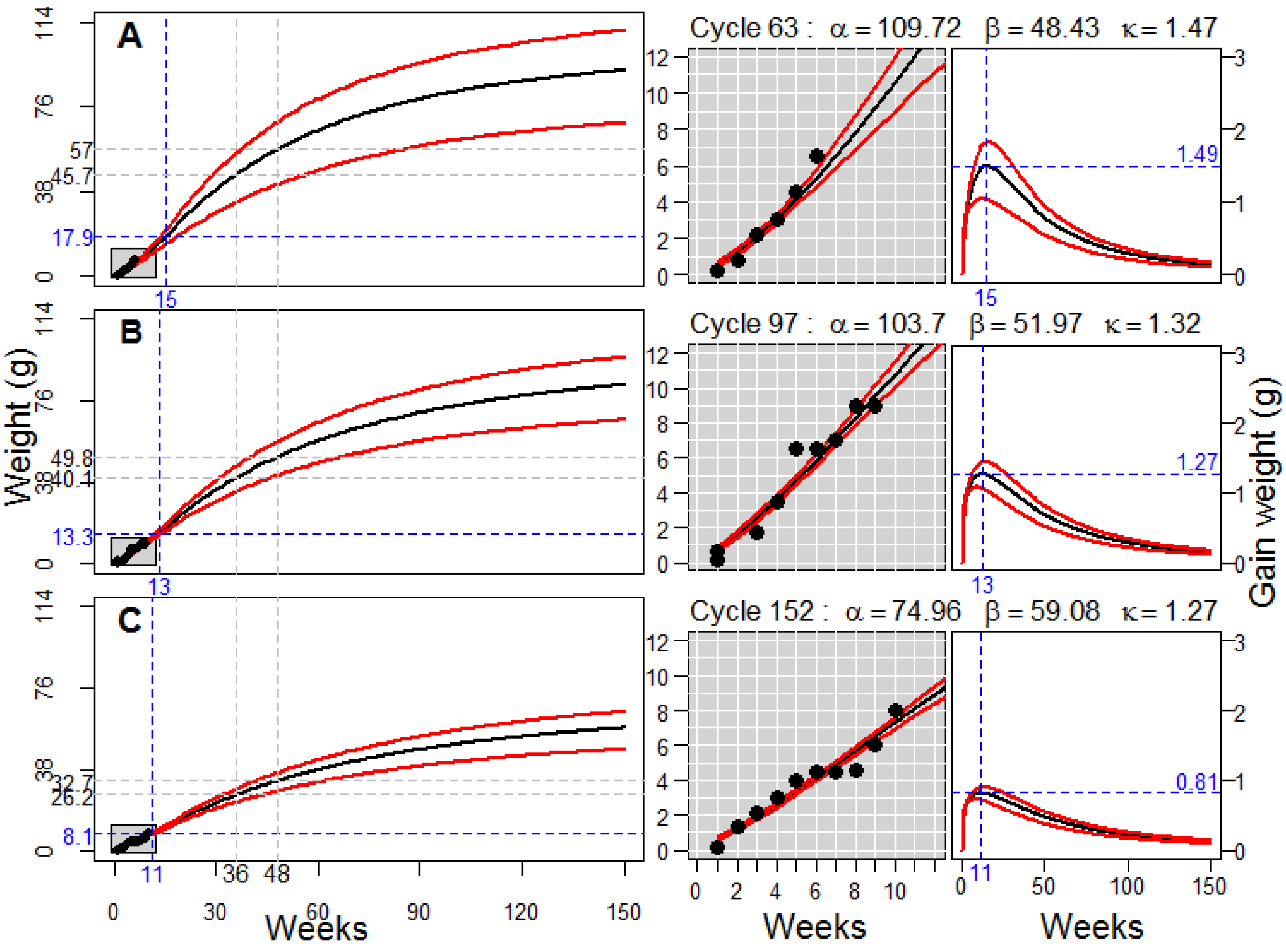
Crescimento: Peso x Ganho de Peso

Michaelis Menten Generalized

$$f(t) = \frac{w_0\, \beta^k + \alpha\, t^k}{\beta^k + t^k}$$



Derivada Michaelis Menten Generalized

$$f'(t) = \frac{\alpha\, \beta^k \cdot k t^{k-1} - \beta^k \cdot k\, w_0\, t^{k-1}}{(\beta^k)^2 + 2\beta^k\, t^k + (t^k)^2}$$

- Ganho de peso e o crescimento dos indivíduos?
- Modelagem do ganho de peso através das derivadas.

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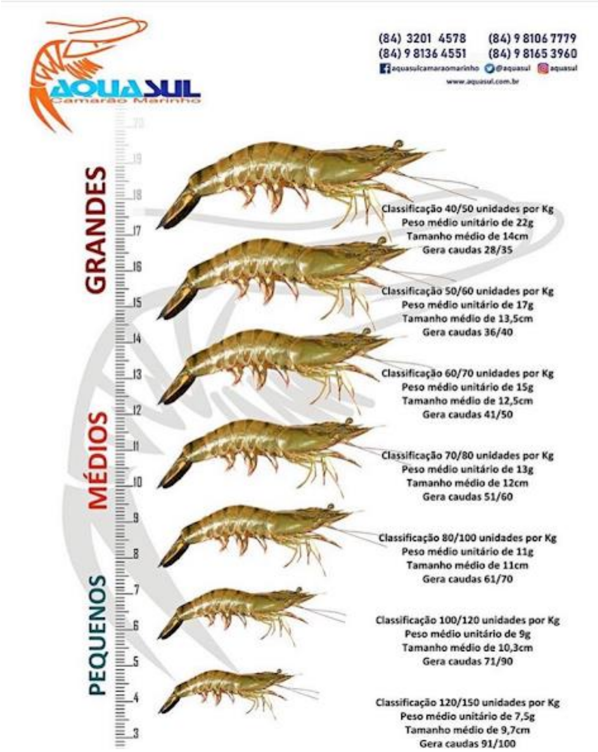
Crescimento: Peso x Ganho de Peso



https://aquasul-camarao-marinho.business.site/

Receita
Custo
Lucro

https://abccam.com.br/

Valor camarão descascado

R$ 62,90 R$ 47,90 R$ 45,90

	Tabela de Equivalência p/ Camarão Inteiro (p/ kg)							
Classificação	40/50	50/60	60/70	70/80	80/100	100/120	120/150	150 up
Peso Unitário	22 g	18 g	15 g	13 g	11 g	9 g	7,8 g	6,5 g (-)

https://villapescados.com.br/

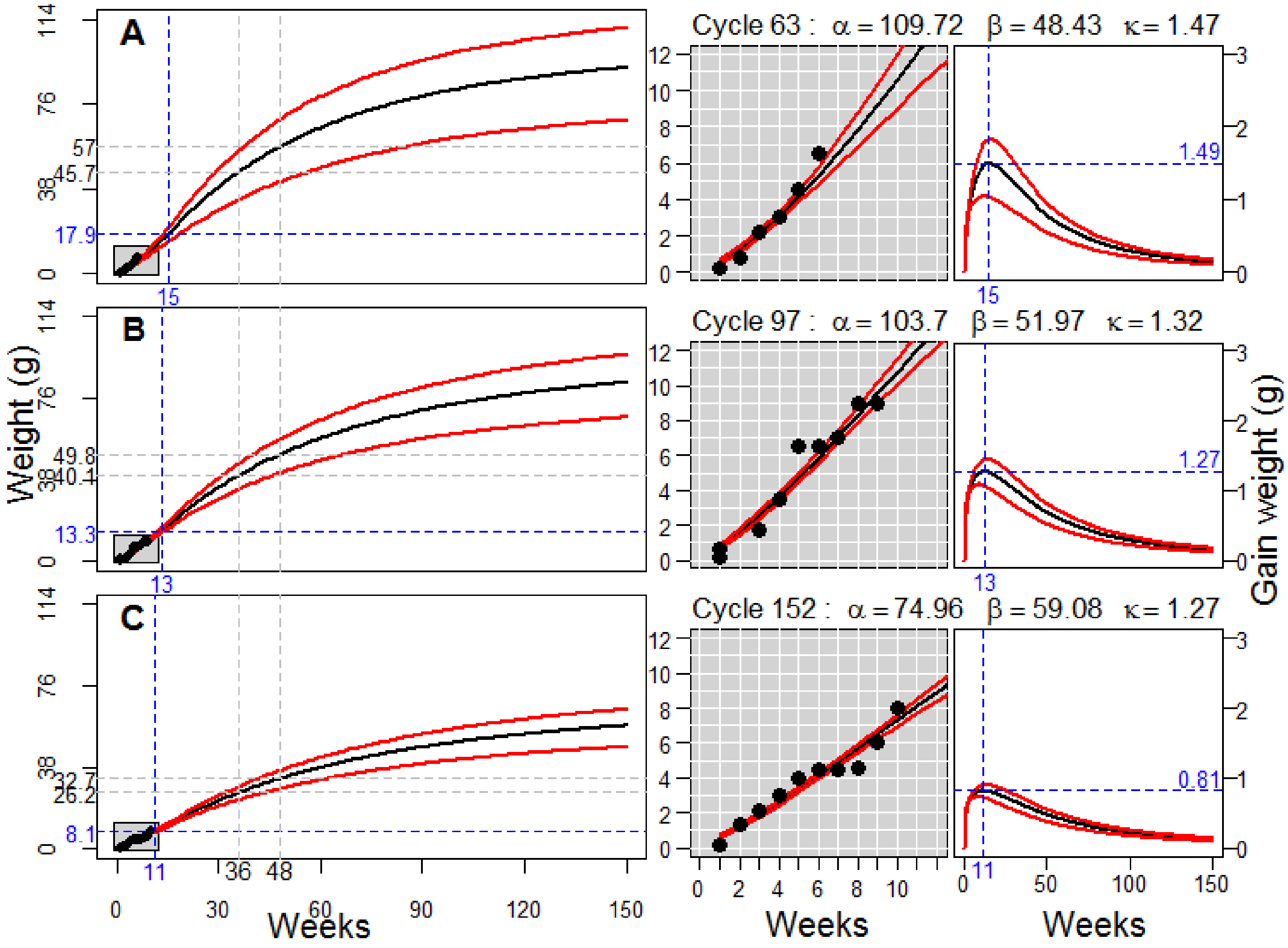
carloszarzar_@hotmail.com

Crescimento: Peso x Ganho de Peso

Lucro = Receita - Custo

Michaelis Menten Generalized

$$f(t) = \frac{w_0\, \beta^k + \alpha\, t^k}{\beta^k + t^k}$$



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- Ganho de peso máximo e sua relação econômica.

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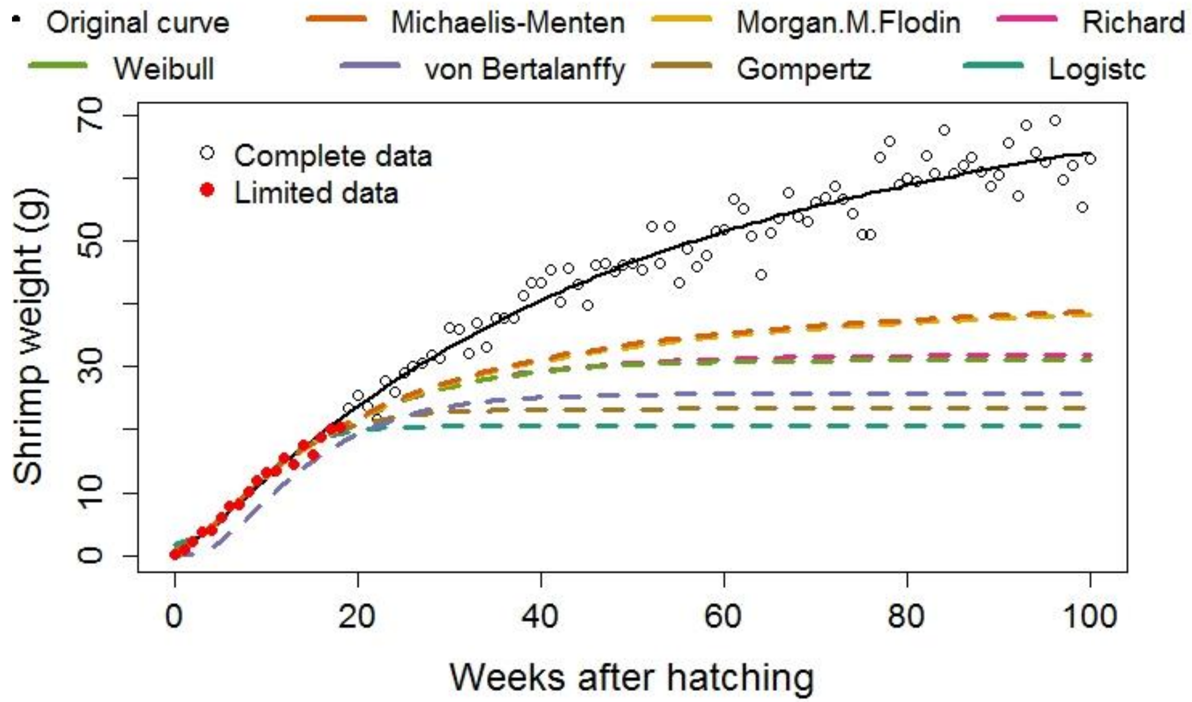
- Famílias de modelos não lineares.



Modelos não lineares para crescimento (sigmoidal)

Function name	Mathematical expression	Parameters restrictions
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Unified-Richards (U-Richards).

$$f(t) = \alpha \left[1 + (\delta - 1) \cdot \exp\left(\frac{-k(t-\beta)}{\delta^{\delta/(1-\delta)}}\right)\right]^{1/(1-\delta)}$$

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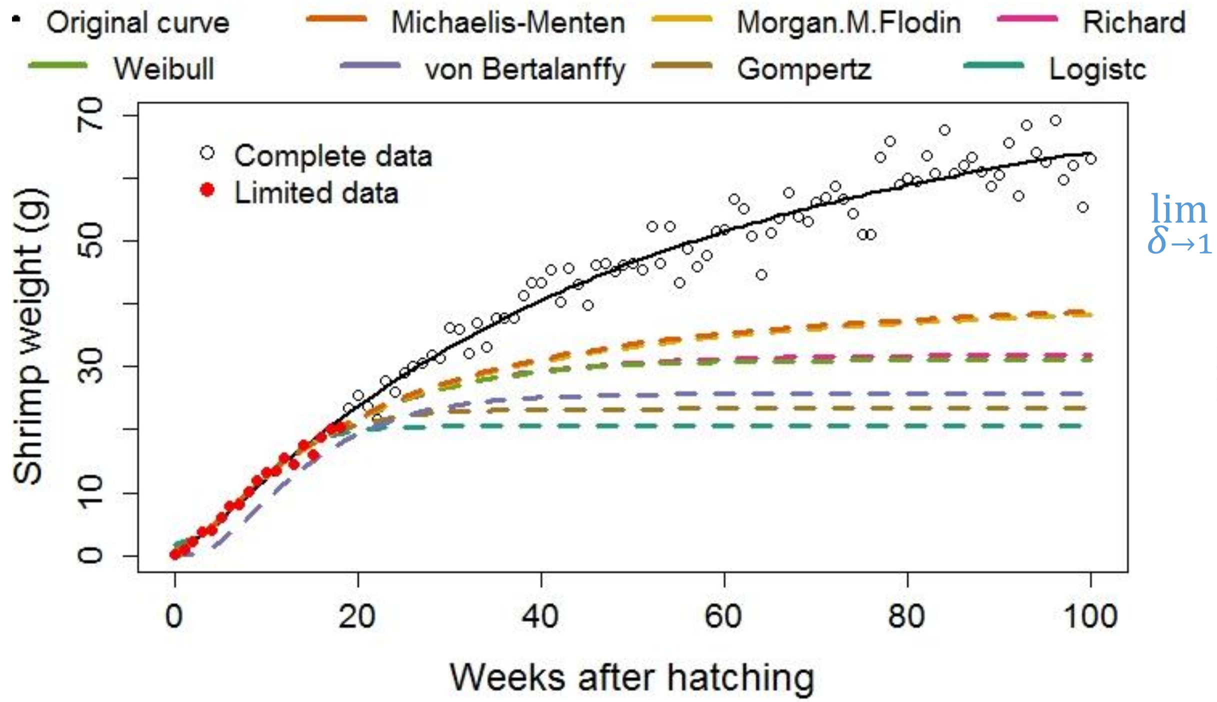
- Famílias de modelos não lineares.



Modelos não lineares para crescimento (sigmoidal)

$\lim_{\delta \to 1} [f(t)]$

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Logistic function δ=2	$f(t) = \frac{\alpha}{1 + exp[\kappa\, (\beta - t)]}$	$\alpha > 0;\ \beta \in \mathbb{R};$ and $\kappa > 0$
von Bertalanffy δ=2/3	$f(t) = \alpha\, (1 - exp(-\kappa\, (t + \beta)))^3$	$\alpha > 0;\ \beta \in \mathbb{R}$ and $\kappa > 0$
Richards curve	$f(t) = \frac{\alpha}{[1 + exp(-\kappa\, \delta\, (t - \beta))]^{1/\delta}}$	$\alpha > 0;\ \beta \in \mathbb{R};$ $\kappa > 0$ and $\delta > 0$
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THEORY OF GROWTH 371

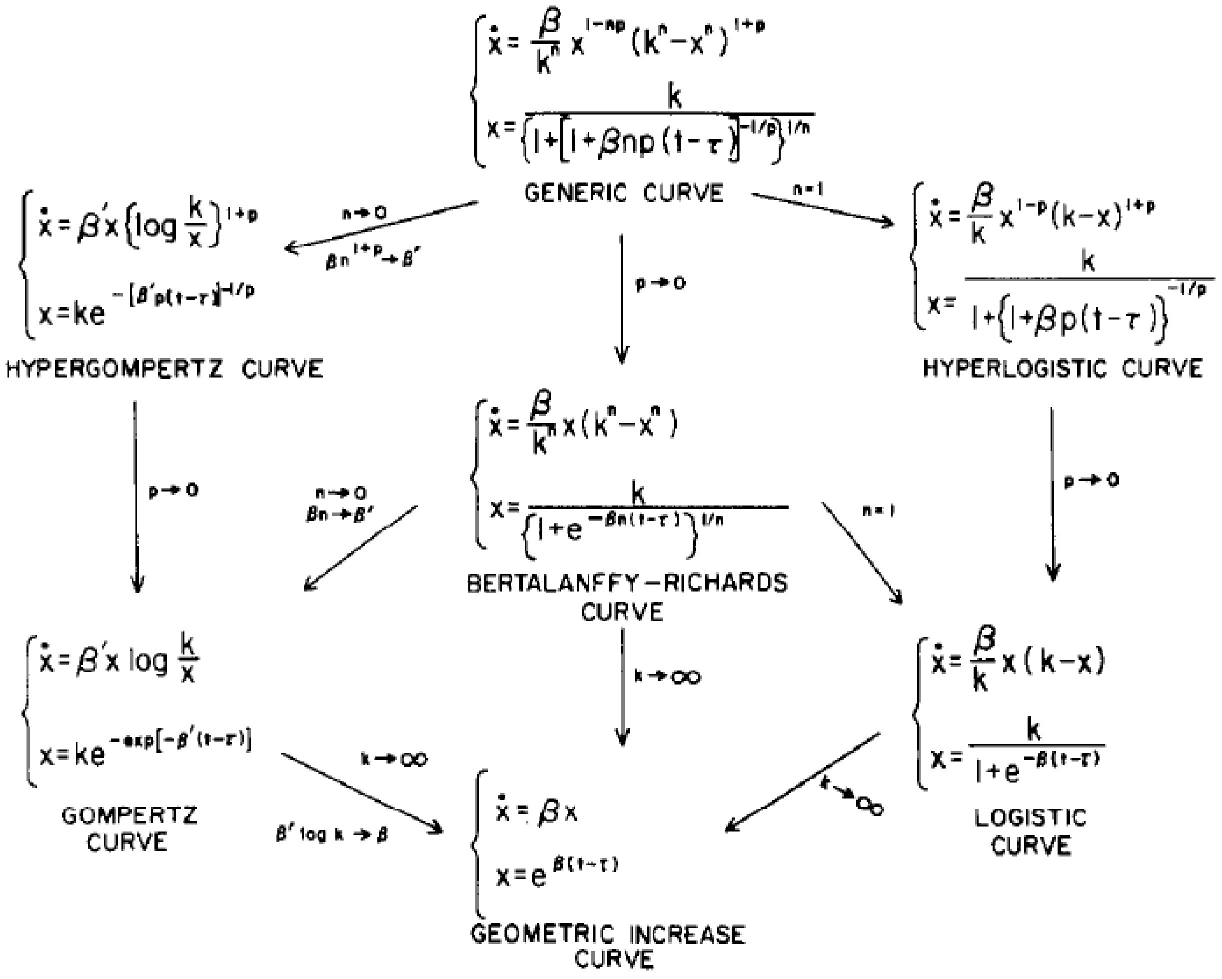

FIG. 1. Interrelations of growth-family members.

A Theory of Growth

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Department of Biostatistics, University of Alabama in Birmingham, Birmingham, Alabama 25294
AND
KENNETH M. PRUITT
Laboratory of Molecular Biology, University of Alabama in Birmingham, Birmingham, Alabama 25294

Communicated by K. E. F. Watt

ABSTRACT

A generalized theory of growth is presented based upon three postulates. The first asserts that the rate of growth is jointly proportional to a monotonic function of the generalized distance from the origin to present size ("reproductive capability"), and to a monotonic function of the generalized distance from present size to ultimate size ("limiting factor"). The second postulate restricts the monotonic function to power (or "mass action") functions. The third postulate constrains the model to a mathematically tractable set which never-the-less is sufficiently general to include the cases of Malthusian, Gompertz, logistic and Bertalanffy-Richards Growth. The most general case is termed the "generic growth model". Other special cases are termed hyperGompertzian and hyperlogistic growth. Generic growth is illustrated by rat weight data from the literature.

INTRODUCTION

There have been many contributors to kinetic theories of growth, following the pioneering work of Quetelet [1], Verhulst [2], Pearl and Reed [3], and Lotka [4]. Glass [5] reviews the early history of the subject. Later writers include Medawar [6], Bertalanffy [7, 8], Richards [9], Nelder [10], and Turner et al. [11].

In the spirit of this tradition, we offer in the present paper a somewhat abstract but elementary frame for a kinetic theory of growth. On the basis of three postulates we obtain a generic growth function which has as special or limiting cases several of the well-known growth curves such as the Verhulst logistic curve, the Gompertz curve, and the generalized growth curve of Bertalanffy [8] and Richards [9]. In addition, we obtain several new forms.

MATHEMATICAL BIOSCIENCES 29, 367–373 (1976) 367

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Turner Jr, M. E., Bradley Jr, E. L., Kirk, K. A., & Pruitt, K. M. (1976). A theory of growth. *Mathematical Biosciences*, *29*(3-4), 367-373.

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Tjørve, K. M., & Tjørve, E. (2017). The use of Gompertz models in growth analyses, and new Gompertz-model approach: An addition to the Unified-Richards family. *PloS one*, *12*(6), e0178691.

PLOS | ONE

RESEARCH ARTICLE

The use of Gompertz models in growth analyses, and new Gompertz-model approach: An addition to the Unified-Richards family

Kathleen M. C. Tjørve*, Even Tjørve

Inland Norway University of Applied Sciences, Elverum, Norway

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OPEN ACCESS

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Competing interests: The authors have declared that no competing interests exist.

Abstract

The Gompertz model is well known and widely used in many aspects of biology. It has been frequently used to describe the growth of animals and plants, as well as the number or volume of bacteria and cancer cells. Numerous parametrisations and re-parametrisations of varying usefulness are found in the literature, whereof the Gompertz-Laird is one of the more commonly used. Here, we review, present, and discuss the many re-parametrisations and some parameterisations of the Gompertz model, which we divide into T_i (type I)- and W_0 (type II)-forms. In the W_0-form a starting-point parameter, meaning birth or hatching value (W_0), replaces the inflection-time parameter (T_i). We also propose new "unified" versions (U-versions) of both the traditional T_i-form and a simplified W_0-form. In these, the growth-rate constant represents the relative growth rate instead of merely an unspecified growth coefficient. We also present U-versions where the growth-rate parameters return absolute growth rate (instead of relative). The new U-Gompertz models are special cases of the Unified-Richards (U-Richards) model and thus belong to the Richards family of U-models. As U-models, they have a set of parameters, which are comparable across models in the family, without conversion equations. The improvements are simple, and may seem trivial, but are of great importance to those who study organismal growth, as the two new U-Gompertz forms give easy and fast access to all shape parameters needed for describing most types of growth following the shape of the Gompertz model.

Introduction

The Gompertz model [1] is one of the most frequently used sigmoid models fitted to growth data and other data, perhaps only second to the logistic model (also called the Verhulst model) [2]. Researchers have fitted the Gompertz model to everything from plant growth, bird growth, fish growth, and growth of other animals, to tumour growth and bacterial growth [3–12], and the literature is enormous. The Gompertz is a special case of the four parameter Richards model, and thus belongs to the Richards family of three-parameter sigmoidal growth models,

Modelo Gompertz

Normalmente Parâmetros

2 – Forma;

1 – Localização;

Classificou os modelos Gompertz 2 tipos

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Modelo Gompertz

Tipo I

- **Um único parâmetro (normalmente ponto de inflexão T_i) está condicionado ao tempo;**
- **No momento em que esse ponto ocorre não é afetado pelos outros parâmetros (embora todos os outros pontos ao longo da curva sejam);**
- **Esse ponto representa uma proporção fixa da assíntota superior (inflexão 36,8% para o Gompertz);**

$$w(t) = A\, e^{-e^{-K\,(t\,-\,T_i)}}$$

Tipo II

- **Um único parâmetro controla o valor inicial da curva W_0;**
- **Nessas parametrizações, os outros parâmetros não afetam o ponto de partida;**
- **Todos modelos W_0 são do tipo II.**

$$w(t) = W_0\, e^{m(1-e^{-K\cdot t})}$$

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Modelo Gompertz

Tipo I

Tipo II

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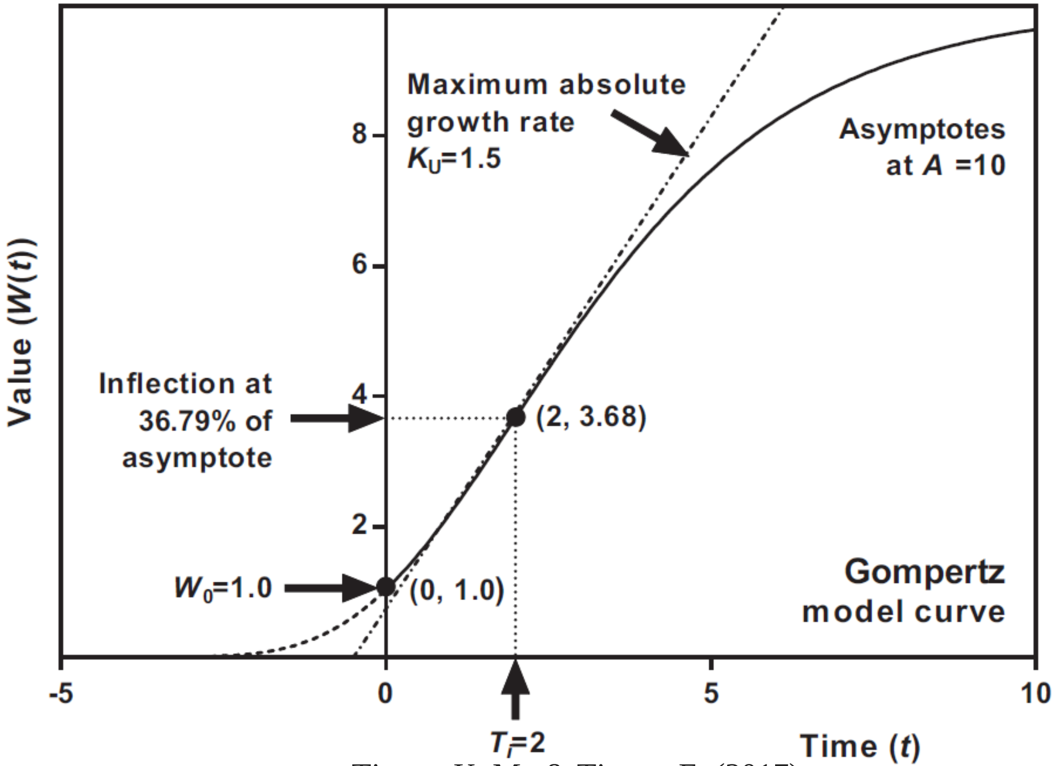
- **Esse ponto representa uma proporção fixa da assíntota superior (inflexão 36,79% para o Gompertz);**

$$w(t) = A\, e^{-e^{-K\,(t\,-\,T_i)}}$$



Tjørve, K. M., & Tjørve, E. (2017)

Modelo Gompertz

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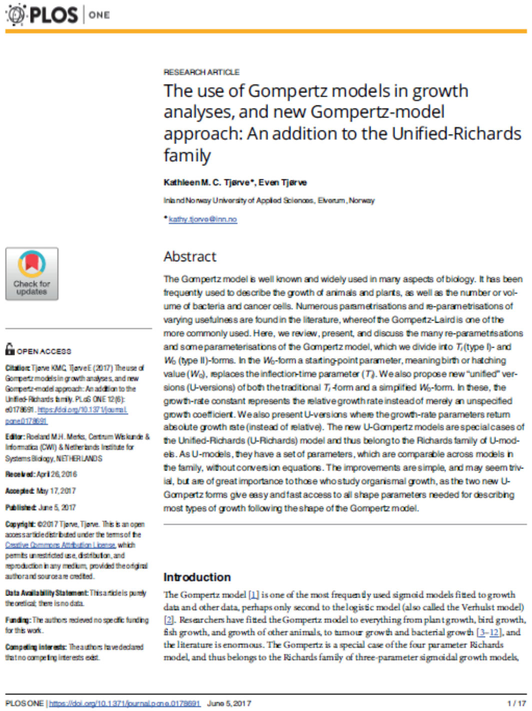

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RESEARCH ARTICLE

The use of Gompertz models in growth analyses, and new Gompertz-model approach: An addition to the Unified-Richards family

Kathleen M. C. Tjørve*, Even Tjørve
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Abstract

The Gompertz model is well known and widely used in many aspects of biology. It has been frequently used to describe the growth of animals and plants, as well as the number or volume of bacteria and cancer cells. Numerous parametrisations and re-parametrisations of varying usefulness are found in the literature, whereof the Gompertz-Laird is one of the more commonly used. Here, we review, present, and discuss the many re-parametrisations and some parameterisations of the Gompertz model, which we divide into T_i (type I)- and W_0 (type II)-forms. In the W_0-form a starting-point parameter, meaning birth or hatching value (W_0), replaces the inflection-time parameter (T_i). We also propose new "unified" versions (U-versions) of both the traditional T_i-form and a simplified W_0-form. In these, the growth-rate constant represents the relative growth rate instead of merely an unspecified growth coefficient. We also present U-versions where the growth-rate parameters return absolute growth rate (instead of relative). The new U-Gompertz models are special cases of the Unified-Richards (U-Richards) model and thus belong to the Richards family of U-models. As U-models, they have a set of parameters, which are comparable across models in the family, without conversion equations. The improvements are simple, and may seem trivial, but are of great importance to those who study organismal growth, as the two new U-Gompertz forms give easy and fast access to all shape parameters needed for describing most types of growth following the shape of the Gompertz model.

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Competing interests: The authors have declared that no competing interests exist.

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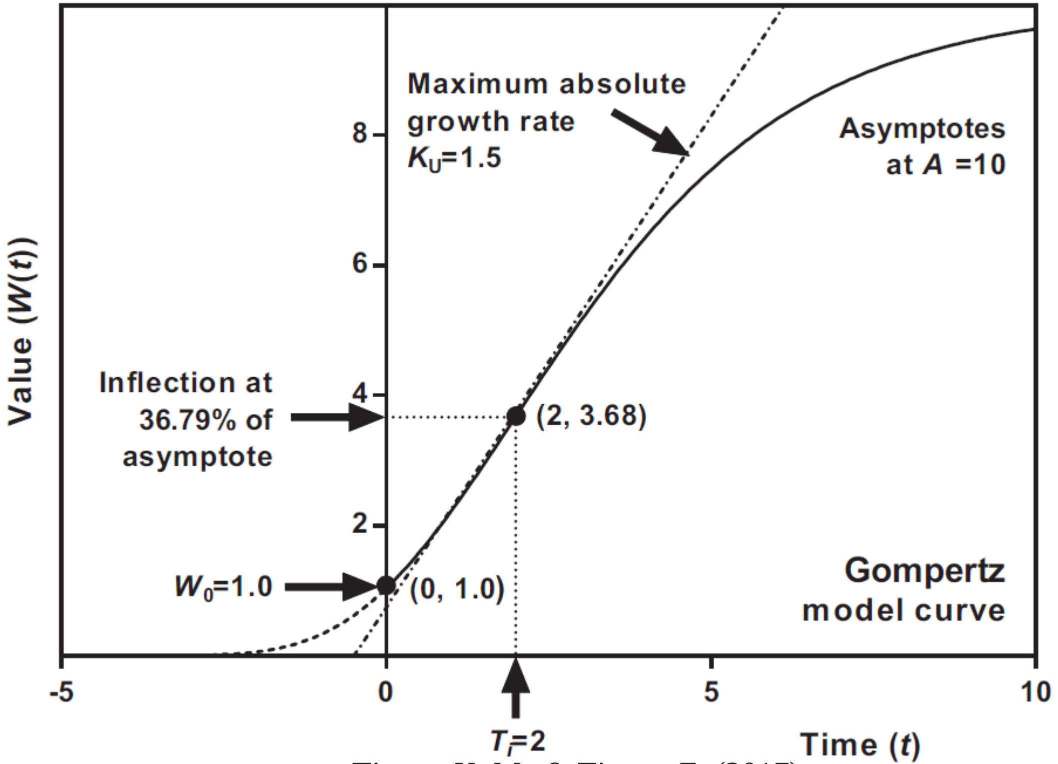
The Gompertz model [1] is one of the most frequently used sigmoid models fitted to growth data and other data, perhaps only second to the logistic model (also called the Verhulst model) [2]. Researchers have fitted the Gompertz model to everything from plant growth, bird growth, fish growth, and growth of other animals, to tumour growth and bacterial growth [3–12], and the literature is enormous. The Gompertz is a special case of the four parameter Richards model, and thus belongs to the Richards family of three-parameter sigmoidal growth models,

K - controla a inclinação na inflexão (taxa de crescimento máxima)



Tjørve, K. M., & Tjørve, E. (2017)

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Gompertz Clássico

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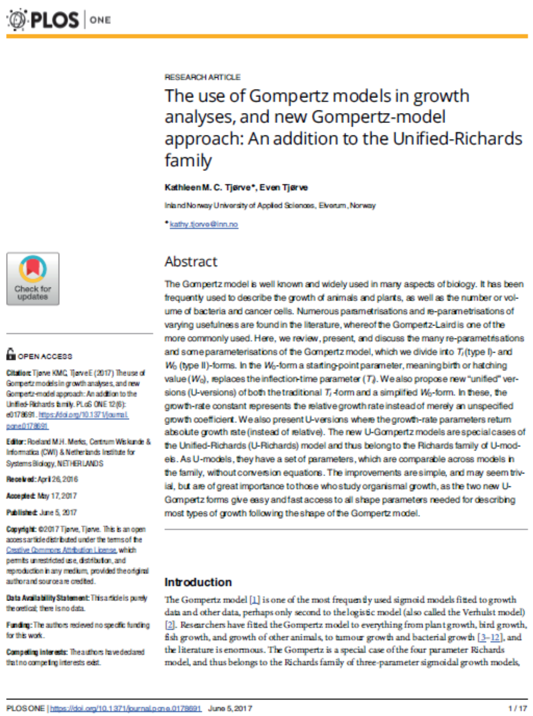

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K - controla a inclinação na inflexão (taxa de crescimento máxima)

K - coeficiente de taxa de crescimento;
K - coeficiente de crescimento;
K - constante de crescimento;
K - taxa de crescimento relativo a assíntota superior (A);
K - taxa de crescimento relativo na inflexão (incorreto, se somente se: $\frac{K}{e}$);
Consequentemente a taxa de crescimento absoluto será $\frac{K}{e} \cdot A$
K não é comparável a outros modelos sigmoides:
(e.g. Logístico, Gompertz, von Bertalanffy e Richards).

$$w(t) = A\, e^{-e^{-K\,(t\,-\,T_i)}}$$

Gompertz Clássico

$$w(t) = W_0\, e^{m(1-e^{-K\cdot t})}$$

A reparametrização de Zweifel e Lasker
Citado no livro de Ricker
Também conhecido como Gompertz modificado

Modelo Gompertz

Tipo I

Tipo II

a)

b)

- **Um único parâmetro (normalmente ponto de inflexão T_i) está condicionado ao tempo;**
- **No momento em que esse ponto ocorre não é afetado pelos outros parâmetros (embora todos os outros pontos ao longo da curva sejam);**
- **Esse ponto representa uma proporção fixa da assíntota superior (inflexão 36,8% para o Gompertz);**

$$w(t) = A\, e^{-e^{-K\,(t\,-\,T_i)}}$$

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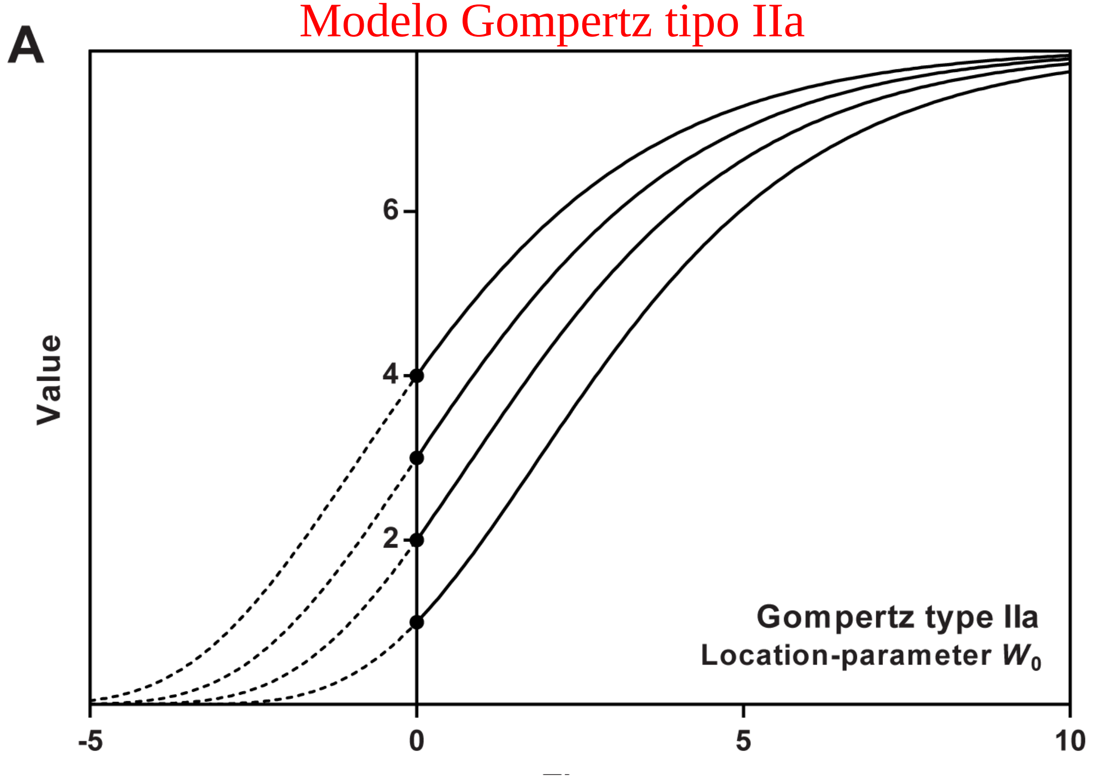
Modelo Gompertz tipo II
W_0 - controla o valor inicial

Modelo Gompertz tipo IIa



Dois tipos do parâmetro W_0

a) W_0 atua como um parâmetro de localização que desloca a curva horizontalmente sem alterar sua forma.

$$w(t) = W_0\, e^{m(1-e^{-K\cdot t})}$$

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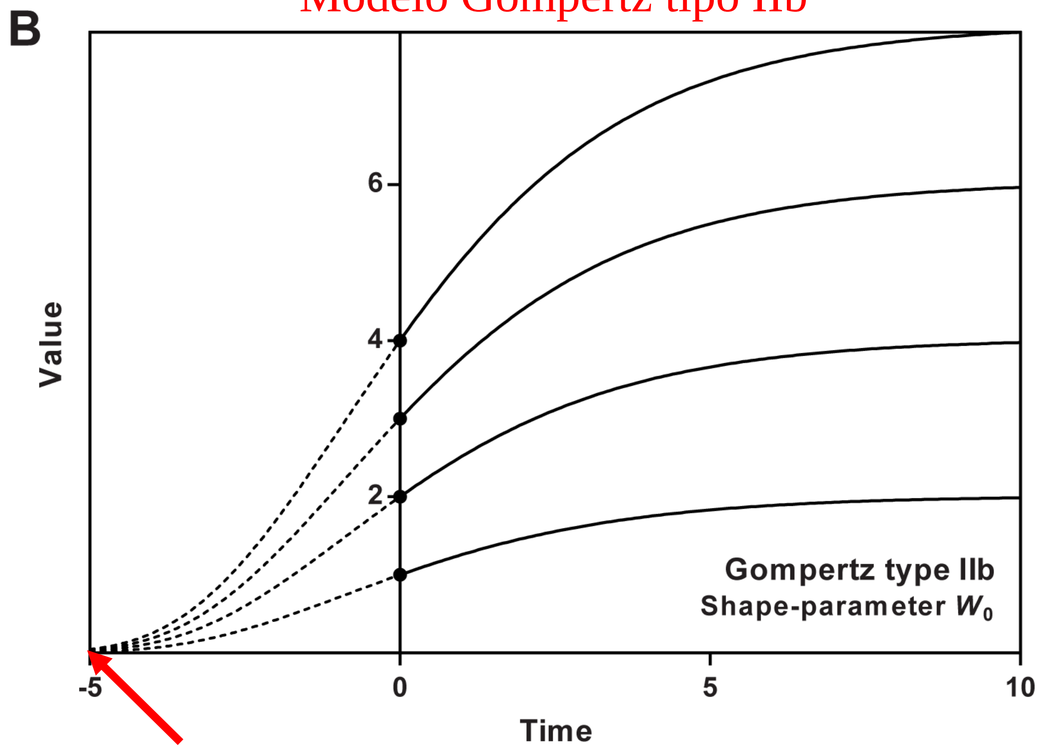
Modelo Gompertz tipo II
W_0 - controla o valor inicial

Modelo Gompertz tipo IIb



Fixando um ponto de partida

Dois tipos do parâmetro W_0

b) W_0 atua como um parâmetro de forma que dimensiona toda a curva verticalmente, afetando a assíntota superior.

$$w(t) = W_0 \, e^{m\left(1-e^{-K\cdot t}\right)}$$

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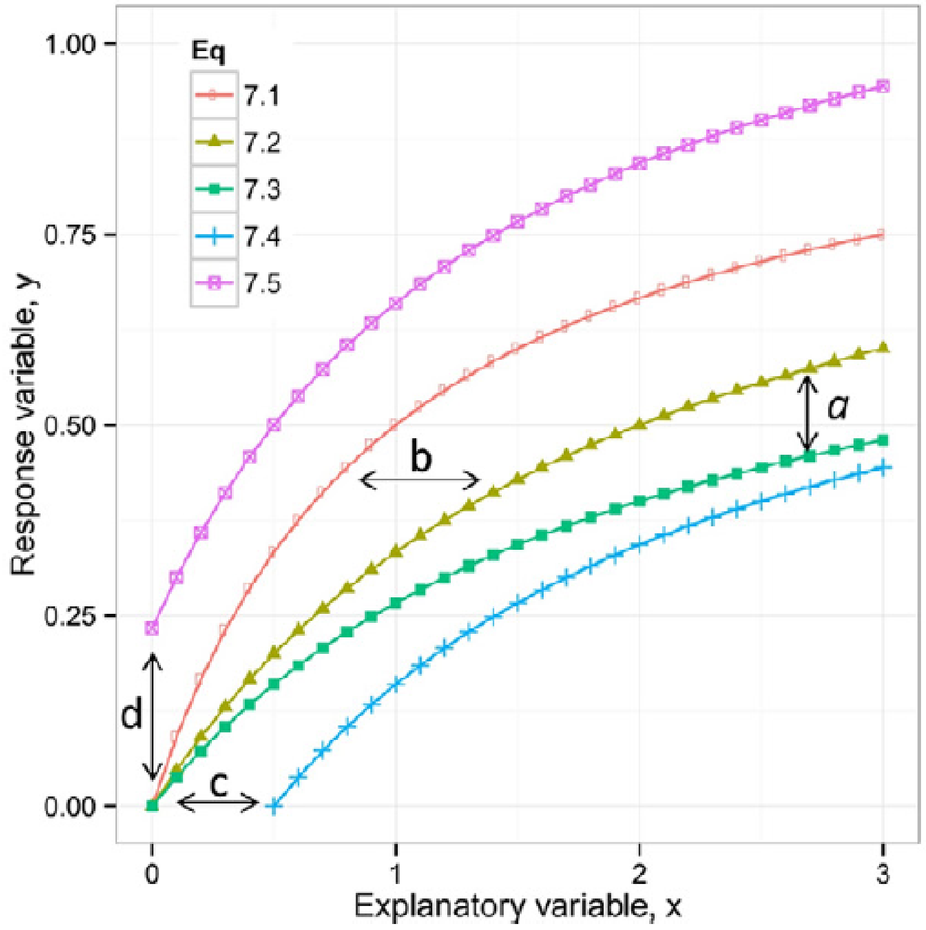
Parametrizações de modelos não lineares



(#)	Equation	Parameters
7.1	$Y = \frac{x}{1+x}$	-
7.2	$Y = \frac{bx}{1+bx}$	b=0.5
7.3	$Y = a\frac{bx}{1+bx}$	b=0.5, *a*=0.8
7.4	$Y = a\frac{b(x-c)}{1+b(x-c)}$	b=0.5, *a*=0.8, c=0.5
7.5	$Y = a\frac{b(x-c)}{1+b(x-c)} + \text{d}$	b=0.5, *a*=0.8, c=0.5, d=0.5

Fig. 3. Example of a nonlinear model modification. Starting with Eq. [7.1], the parameters ***a***, ***b***, ***c***, and ***d*** were added step by step to Eq. [7.1], resulting in four new equations: Eq. [7.2–7.5]. Horizontal or vertical arrows in the figure panel indicate how the additional parameters affected the model.

Archontoulis, S. V., & Miguez, F. E. (2015). Nonlinear regression models and applications in agricultural research. Agronomy Journal, 107(2), 786-798.

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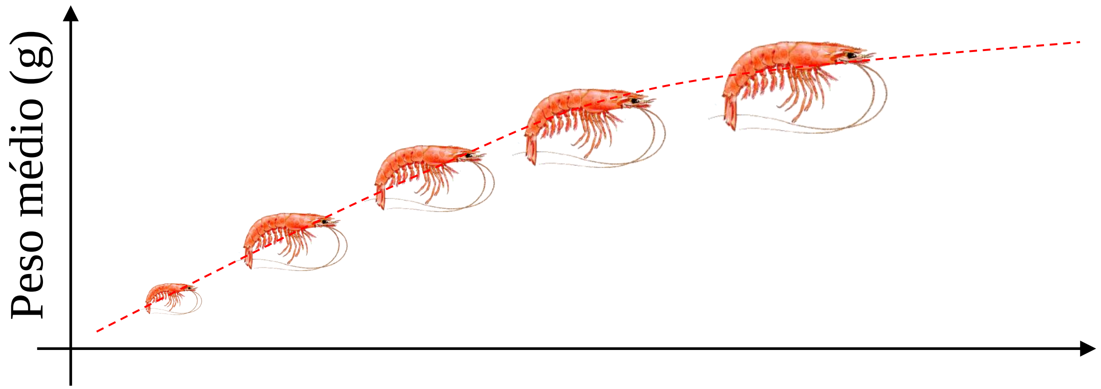
Crescimento em função do tempo



Dias de cultivo (t)

- Existem várias curvas lineares e não lineares;
- Parametrizações diferentes;
- Existe a relação de autodependência série temporal;
- Diferentes métodos de amostragem e modelagem;
- Que devem ser levados em consideração.

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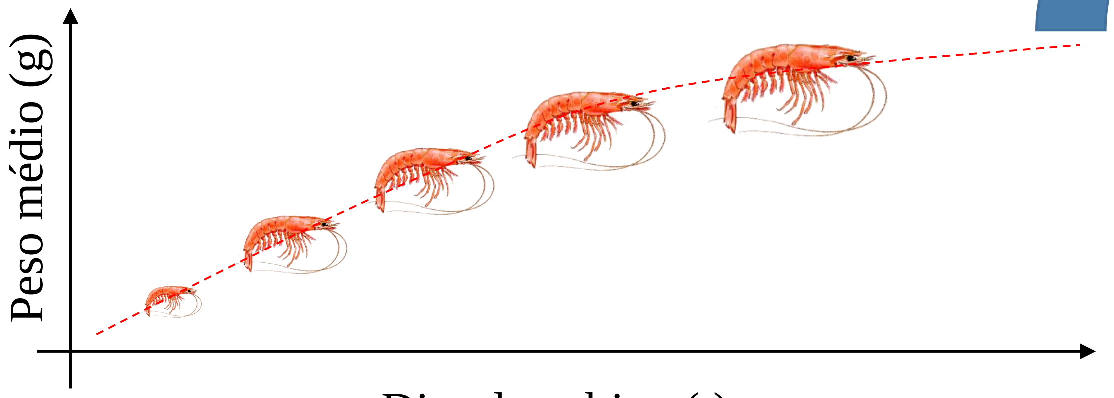
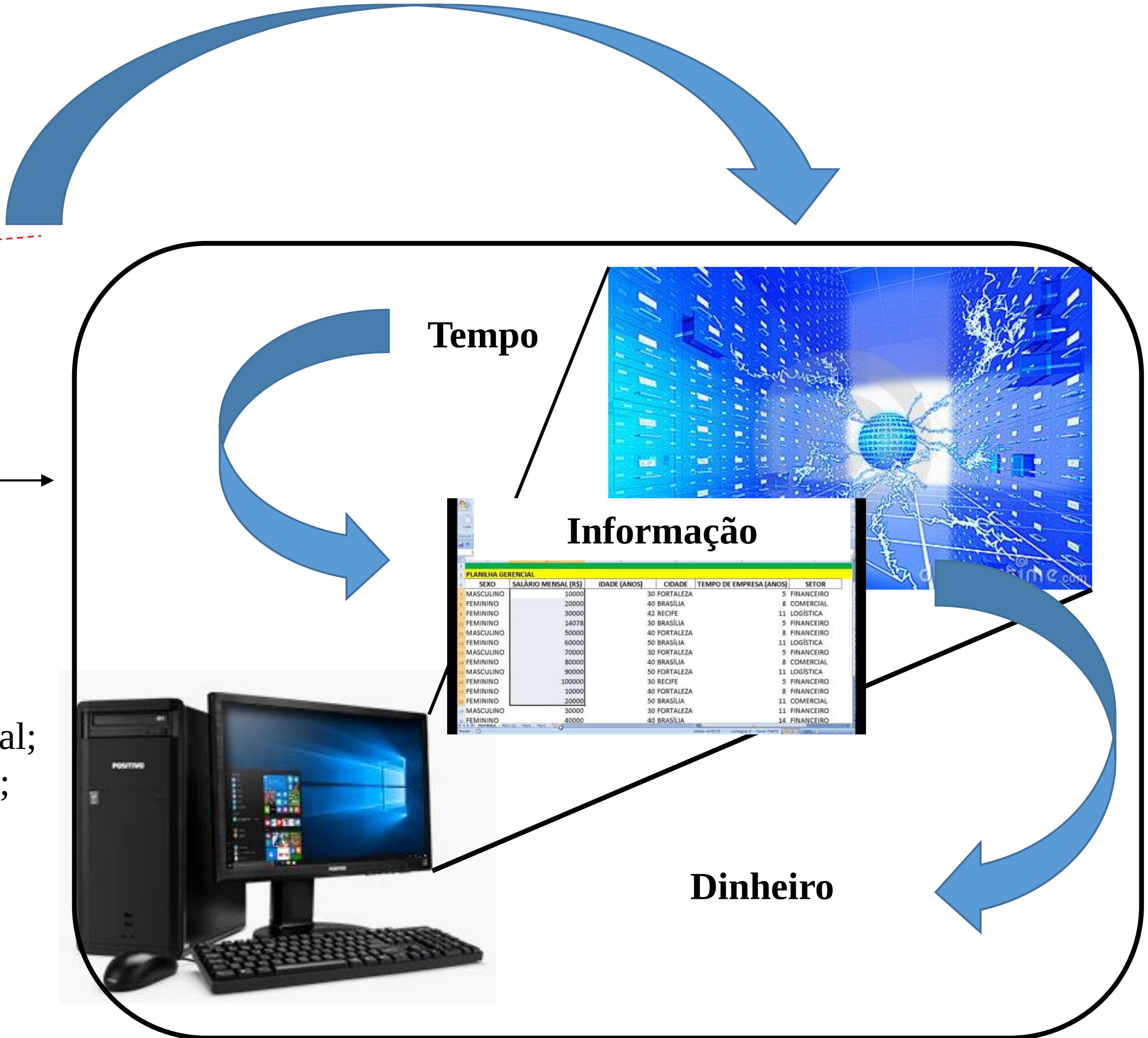
Crescimento em função do tempo





SEXO	SALÁRIO MENSAL (R$)	IDADE (ANOS)	CIDADE	TEMPO DE EMPRESA (ANOS)	SETOR
MASCULINO	10000	30	FORTALEZA	5	FINANCEIRO
FEMININO	20000	40	BRASÍLIA	8	COMERCIAL
FEMININO	30000	42	RECIFE	11	LOGÍSTICA
FEMININO	14078	30	BRASÍLIA	5	FINANCEIRO
MASCULINO	50000	40	FORTALEZA	8	FINANCEIRO
FEMININO	60000	50	BRASÍLIA	11	LOGÍSTICA
MASCULINO	70000	30	FORTALEZA	5	FINANCEIRO
FEMININO	80000	40	BRASÍLIA	8	COMERCIAL
MASCULINO	90000	50	FORTALEZA	11	LOGÍSTICA
FEMININO	100000	30	RECIFE	5	FINANCEIRO
FEMININO	10000	40	FORTALEZA	8	FINANCEIRO
FEMININO	20000	50	BRASÍLIA	11	COMERCIAL
MASCULINO	30000	30	FORTALEZA	11	FINANCEIRO
FEMININO	40000	40	BRASÍLIA	14	FINANCEIRO

- Existem várias curvas lineares e não lineares;
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Modelagem do crescimento de indivíduos, dos organismos ou populações

NÚCLEO DE ESTUDOS EM REGRESSÃO NÃO LINEAR APLICADA

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Programa pós graduação em Estatística e Experimentação agropecuária

Palestra: Carlos Antônio Zarzar
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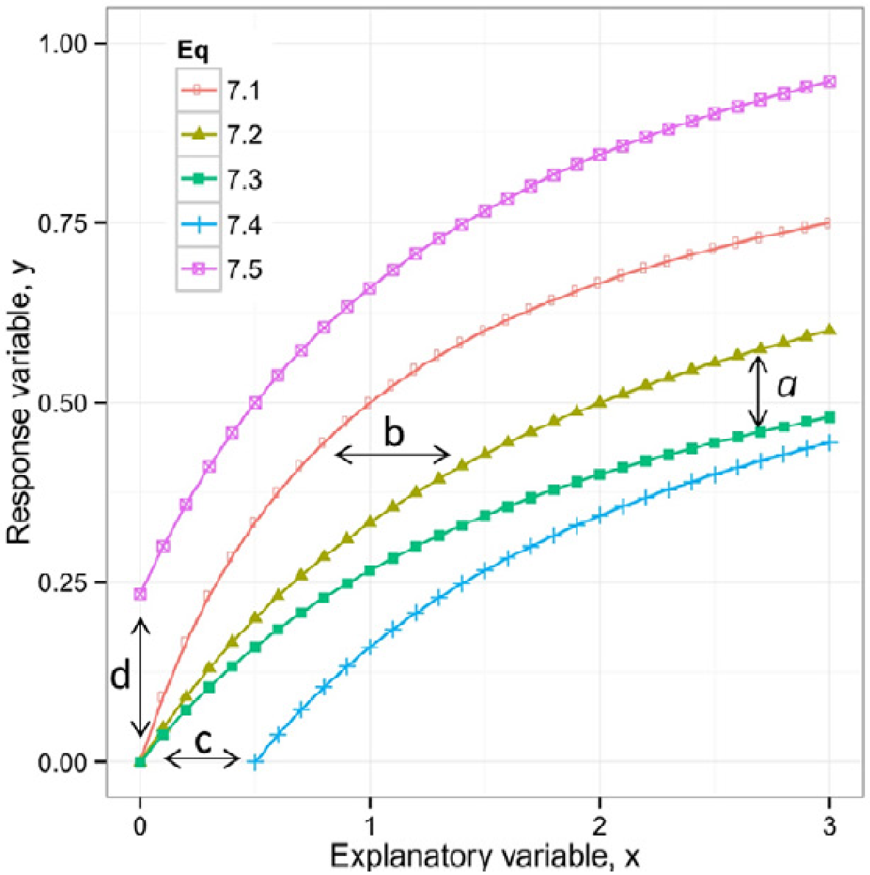

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