London Business School

Data Science

Lecture 4: Classification with Logistic Regression

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Welcome to Hybrid Teaching

- **Class starts 4pm. Thank you for being early!**
- Please log in on Zoom even if you are a roomie
  - If you are a **roomie, do not connect audio**
  - Please turn on your **camera**
- For your zoom username use
  - Your full (real) name in Latin characters
  - **Add R or Z** in front of your name if you are Rommie or a Zommie
  - **Add your study group** number after your name
  - For example: **Z - Nicos Savva - 20**
- Engage, ask & answer questions, as usual
  - Raise your digital hands on zoom!
  - The session will be recorded
  - **Let's make this interactive and fun ☺**

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Lectures

Course contents (first part of the course – Nicos)

- Session 1: The Art & Science of Regression Models For Prediction
- Session 2: More on Using Linear Regression For Prediction
  - Quiz 1, due 3 days after session 2
- Session 3: Workshop I – Engineer an algorithm that sets interest rates for new Lending Club loans
  - Group assignment 1, due 6 days after the workshop
- Session 4: Classification using Logistic Regression
  - Quiz 2, due 3 das after session 4
- Session 5: Workshop – Invest in a portfolio of Lending Club loans
  - Individual project 1, due 6 days after the end of the workshop

Course contents (second part of the course – Tolga)

- See canvas syllabus

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The classification problem

- Recap from regression
  - Regression analysis is the ***PROCESSES*** of finding the linear relationship that links a ***continuous*** variable Y to a set of features (or explanatory variables) X

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots \beta_n X_n + \epsilon$$

  - Features can be continuous or binary dummies (e.g., own house or rent)
- Suppose now the variable Y is a binary variable
  - *Y is called a binary response variable*
  - *Example: Loan paid back or not (charged-off* $Y = 1$*, paid back* $Y = 0$*)*

annual_inc	dti	loan_status
22000	14.29	0
40000	2.55	0
150000	0	0
95000	3.83	1
120000	2.29	0
85000	0.31	0
200000	3.72	1

- Classification problem: Given a set of explanatory variables, is it more likely that an observation will belong to “category” 1 or 0?

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Modelling classification

- What is this useful for?
  - **Explanatory**: Quantify the impact of each variable X on the probability that variable Y is equal to 1
  - **Predictive**: Predict the chance that variable Y is equal to 1 using information from the X variables

- Examples
  - Credit card fraud
    - Explanatory: Are jewellery-shop transactions more likely to be fraudulent than average?
    - Predictive: Is a particular transaction fraudulent?
  - Student admissions at LBS
    - Explanatory: Does the GMAT score affect a candidate's probability of being admitted?
    - Predictive: Will I be admitted to LBS?
  - Medicine
    - Explanatory: Does the presence of a specific gene make someone more likely to develop cancer?
    - Predictive: Is John Doe infected by Covid 19?

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Today's lecture

- Goals
  - Introduction to the classification problem and logistic regression
  - Discuss how to interpret the results and use them to
    - Reach conclusions regarding explanatory factors
    - Calculate risk scores and make predictions
  - Choose between multiple models
  - Validation using out-of-sample data
  - Give a "cookbook" to follow when you need to do classification using logistic regression
  - Allow you to critically assess the classification work of others

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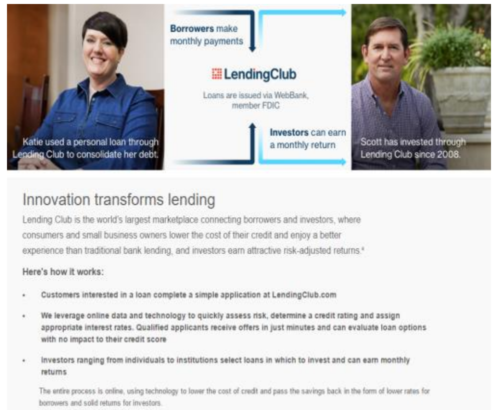
Predicting defaults at the lending club

- **Why is this useful?**

- **How would you start?**



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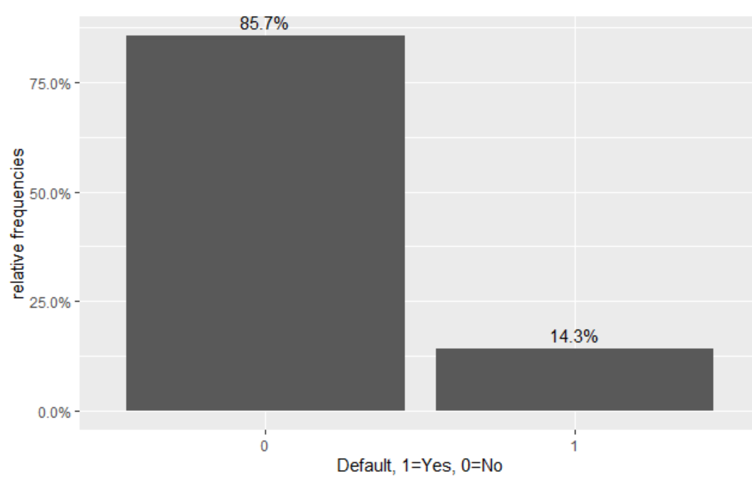
ICE the data!



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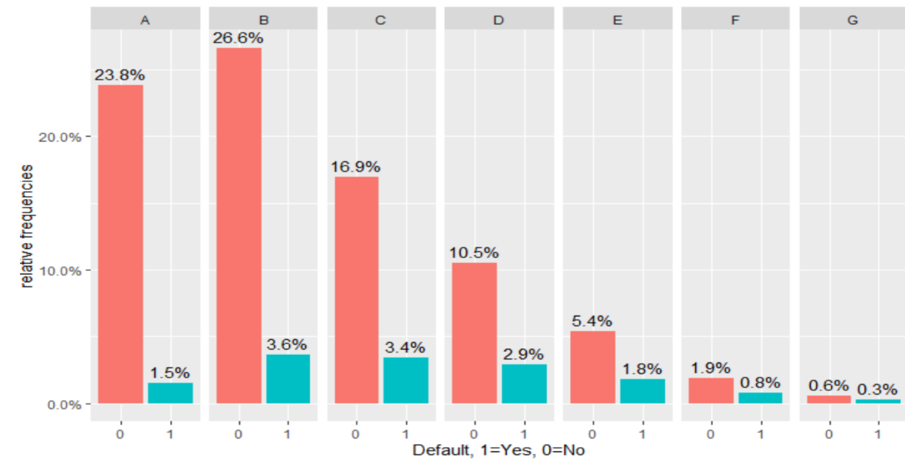
More ICE



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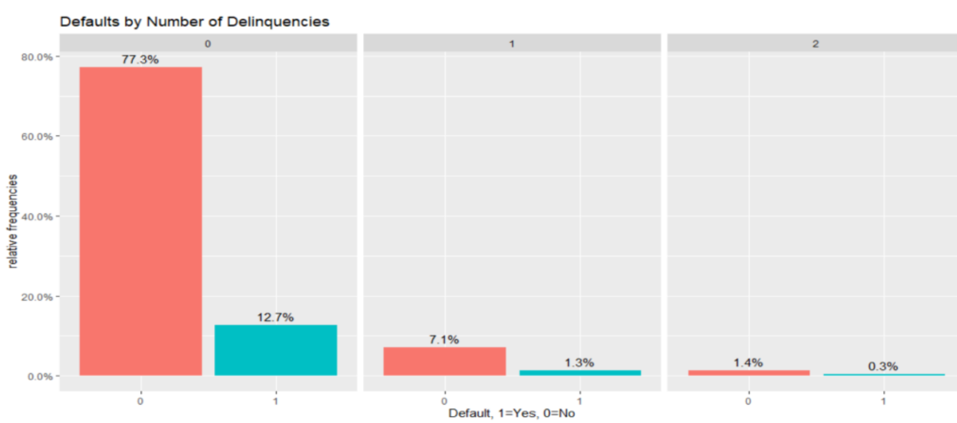
More ICE



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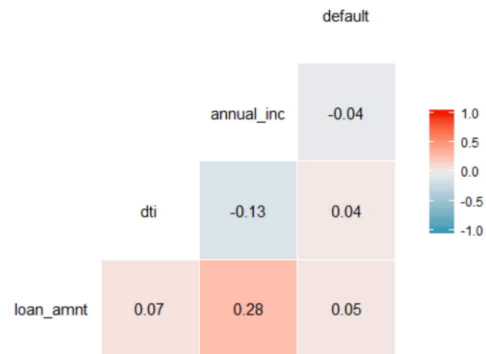
Lending Club and loan defaults

- Let's set $Y = 1$ if charged off and $Y = 0$ otherwise
- Correlation analysis
  - How do you interpret the correlation coefficient?
    - Those have higher income are more likely to pay back
    - Those who borrow more are more likely to default
- Do these relationships sound plausible?



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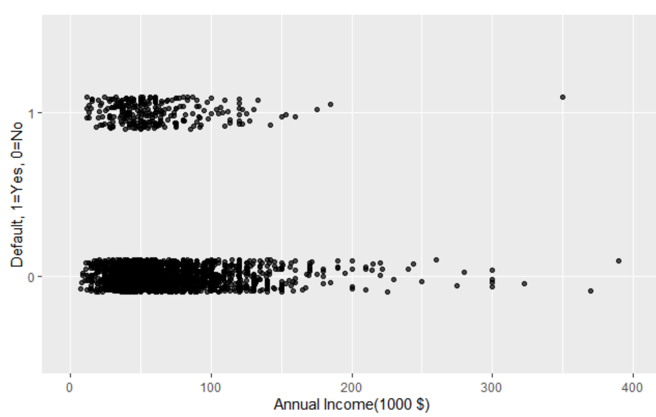
Scatter plot



- Can we run a linear regression between the two variables?

$$Default = \beta_0 + \beta_1(Income) + \epsilon$$

- How would you interpret the coefficients?
- Can you see any problem(s) with this regression?

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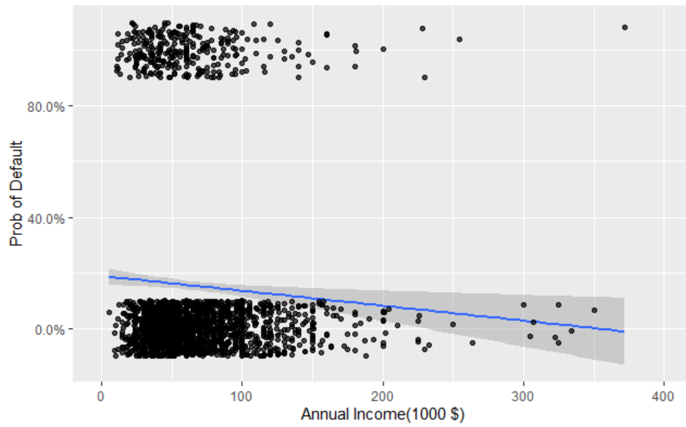
Linear regression plot



- Can we run a linear regression between the two variables?
$$Default = \beta_0 + \beta_1(Income) + \epsilon$$
- How would you interpret the coefficients?
- Can you see any problem(s) with this regression?

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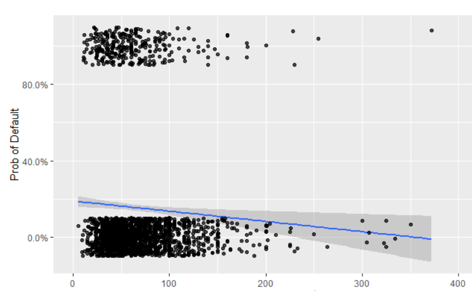
Linear Probability Model

- Running a linear regression is easy to implement… but
- It can be problematic
  - It can predict probabilities that are less than 0 or great than 1
- No micro theory behind it



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R-code to generate these figures

```r
#bar chart of defaults
def_vis1<-ggplot(data=lc_clean, aes(x=default)) +geom_bar(aes(y = (..count..)/sum(..count..))) + labs(x="Default,
1=Yes, 0=No", y="relative frequencies") +scale_y_continuous(labels=scales::percent) +geom_text(aes( label =
scales::percent((..count..)/sum(..count..) ),y=(..count..)/sum(..count..) ), stat= "count",vjust=-0.5)
def_vis1

#bar chart of defaults per loan grade
def_vis2<-ggplot(data=lc_clean, aes(x=default), group=grade) +geom_bar(aes(y = (..count..)/sum(..count..), fill =
factor(..x..)), stat="count")  + labs(title="Defaults by Grade", x="Default, 1=Yes, 0=No", y="relative frequencies")
+scale_y_continuous(labels=scales::percent) +facet_grid(~grade) + theme(legend.position = "none") +geom_text(aes( lal
= scales::percent((..count..)/sum(..count..) ),y=(..count..)/sum(..count..) ), stat= "count",vjust=-0.5)
def_vis2

#bar chart of defaults per number of Delinquencies
def_vis3<-lc_clean %>%
  filter(as.numeric(delinq_2yrs)<4) %>%
  ggplot(aes(x=default), group=delinq_2yrs) +geom_bar(aes(y = (..count..)/sum(..count..), fill = factor(..x..)),
stat="count")  + labs(title="Defaults by Number of Delinquencies", x="Default, 1=Yes, 0=No", y="relative frequencies
 +scale_y_continuous(labels=scales::percent) +facet_grid(~delinq_2yrs) + theme(legend.position = "none")
+geom_text(aes( label = scales::percent((..count..)/sum(..count..) ),y=(..count..)/sum(..count..) ), stat=
"count",vjust=-0.5)

def_vis3
#scatter plots

#We select 2000 random loans to display only to make the display less busy.
set.seed(1234)
reduced<-lc_clean[sample(0:nrow(lc_clean), 2000, replace = FALSE),]%>%
  mutate(default=as.numeric(default)-1) # also convert default to a numeric {0,1} to make it easier to plot.


# scatter plot of defaults against loan amount
def_vis4<-ggplot(data=reduced, aes(y=default,x=I(loan_amnt/1000)))  + labs(y="Default, 1=Yes, 0=No", x="Loan Amnt (1000
$)") +geom_jitter(width=0, height=0.05, alpha=0.7) #We use jitter to offset the display of defaults/non-defaults to
make the data easier to interpert. We have also changed the amount to 1000$ to reduce the number of zeros on the
horizontal axis.

def_vis4

#scatter plot of defaults against loan amount.
def_vis5<-ggplot(data=reduced, aes(y=default,x=I(annual_inc/1000)))   + labs(y="Default, 1=Yes, 0=No", x="Annual
Income(1000 $)") +geom_jitter(width=0, height=0.05, alpha=0.7) +  xlim(0,400)

def_vis5
```

Also look at the rmd file on canvas!

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The logistic “random risk” model

- Let U_P denote the “risk” of developing problem – the higher it is the more likely to develop a problem
- We assume that this U_P is linear
  - Given the features $\mathrm{X} = \{\mathrm{X}_1, \mathrm{X}_2, \ldots, X_n\}$ then $U_P|X = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_n X_n$
- U_P is **not observable**
  - We only observe if a problem has occurred or not!
- Given the “risk” U_P, the logistic regression model assumes that the probability of observing a problem is given by the logistic function

$$Prob(Default = 1) = \frac{\exp(U_P|X)}{1 + \exp(U_P|X)} = \frac{1}{1 + \exp(-U_P|X)}$$

where, $\exp(n)$ denotes the number $e = 2.71 \ldots$ raised to the power of n

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The logistic "random risk" model

- Daniel McFadden shared the 2000 Nobel prize in economics for developing these models
  - Professor at Berkeley

- Some technical notes:
  - Models such as the logistic regression are sometimes called discrete choice models
  - The risk factor U_p in discrete choice models is sometimes called Utility, i.e., it represents the utility associated with making a specific choice
  - The Logistic regression model assumes that the risk (or the utility) is random and follows a Gumbel distribution with mean U_P
  - If instead of Gumbel distribution we assumed that the risk followed a Normal distribution with mean U_P and variance 1 we would get the ***Probit*** model
    - For most applications, Logistic Regression and Probit regression give almost identical results

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The maximum likelihood principle

- We still don't know how to estimate the model coefficients $\beta = \{\beta_0, \beta_1, \dots, \beta_n\}$
- But, let's assume we know their values! We can then ask, given $\beta_0, \beta_1, \dots, \beta_n$ what is the likelihood L that the variable Y takes the values observed in my m datapoints:

$$L = \prod_{i=1}^{m} Prob(Y_i | X_i; \beta)$$

- We can then try to find the values of β that maximize this likelihood L
  - This has parallels to the least-squares estimates of linear regression (but maximizing likelihood is a harder problem than minimizing least squares)
  - We typically maximise the logarithm of the likelihood (log L) – monotone transformation that leads to smaller numbers
  - Besides logistic regression, maximum likelihood has a huge number of applications in statistics

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Back to Lending Club

No	loan_status	annual_inc	Probability of default	Prob(Y)
1	0	98	0.020	0.980
2	0	114	0.009	0.991
3	0	72	0.069	0.931
4	0	47.5	0.202	0.798
5	0	122.748	0.006	0.994
6	1	60	0.119	0.119
7	1	24	0.450	0.450
8	0	18	0.525	0.475
9	1	60	0.119	0.119
10	0	95.004	0.023	0.977

- The risk model: $U_P = \beta_0 + \beta_1 Annual\ Income$
  - Let's assume $\beta_0 = 1$ and $\beta_1 = -0.05$
- For loan 1:
  - $U_P = \beta_0 + \beta_1 \times 98 = 1 - 0.5 \times 98 = -3.9$
  - Prob of default $p_1 = \frac{1}{1+\exp(3.9)} = 0.02\%$
- Similarly, we can calculate the other loans' probabilities $(p_2, p_3, \ldots, p_{10})$
- The Likelihood of having defaults for loans (6, 7 and 9) and not in any of the other defaults

$$L = (1-p_1) \times (1-p_2) \times (1-p_3) \times (1-p_4) \times (1-p_5) \times p_6 \times p_7 \times (1-p_8) \times p_9 \times (1-p_{10})$$

- The maximum likelihood principle asks what values of β_0, β_1 maximize L?
  - Need to use a numerical solver to maximize the likelihood (or the logarithm of the likelihood)
  - Unlike linear regression it's not possible to estimate logistic regression using linear algebra

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Logistic regression

```
logistic1<-glm(default~I(annual_inc/1000), family="binomial", lc_clean)
summary(logistic1)
```

```
Call:
glm(formula = default ~ I(annual_inc/1000), family = "binomial",
    data = lc_clean)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-0.6246  -0.5786  -0.5572  -0.5115   3.6388

Coefficients:
                     Estimate Std. Error z value Pr(>|z|)
(Intercept)        -1.5191545  0.0292430  -51.95   <2e-16 ***
I(annual_inc/1000) -0.0040801  0.0003998  -10.21   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 31130  on 37868  degrees of freedom
Residual deviance: 31005  on 37867  degrees of freedom
AIC: 31009

Number of Fisher Scoring iterations: 5
```

- Using maximum likelihood we can estimate the coefficients that are most likely to have generated our data
  - I have done so here using all of the data & I have transformed annual income to be in $ 1000's
- The risk model is
- $U = -1.51915 - .00408$ * Income ($1000s)
  - What is the probability of default for a loan applicant with income 100 (000 USD)

$P = 1/(1 + \exp(1.51915 + 0.00408 \times 100)) = 12.7\%$

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Logistic regression

```
Call:
glm(formula = default ~ I(annual_inc/1000), family = "binomial",
    data = lc_clean)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-0.6246  -0.5786  -0.5572  -0.5115   3.6388

Coefficients:
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Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

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Residual deviance: 31005  on 37867  degrees of freedom
AIC: 31009

Number of Fisher Scoring iterations: 5
```

- The coefficients, std error, and p-value have a similar interpretation as the linear regression
  - To assess coefficient significance at the 5% level we look at the estimated standard error and p value the usual way
- Coefficients no longer have a ***Marginal Value*** interpretation
  - In a linear model, the coefficient β_1 had a marginal effect interpretation: An increase in variable X by 1 unit increases variable Y by 1 unit.
  - This in not the case in logistic regression because it's a non-linear model
  - Using elementary calculus, the marginal effect of variable X is equal to $\beta\, p\,(1-p)$.
    - Therefore, the marginal effect will be different for different values of *X*. Largest effect for $p = \frac{1}{2}$
    - But it has the same sign as the coefficient β

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Goodness-of-Fit

- In linear regression we assess in-sample goodness-of-fit by using the R-Squared
- The equivalent notion in logistic regression is deviance – we want deviance to be as small as possible
  - $\text{Deviance} = -2\log(L)$ where L is the maximum likelihood
- R reports the deviance of a model with only the intercept (Null Deviance) and the deviance of the model with the features
- AIC is similar to deviance but penalizes for number of coefficients (like adj-R^2in linear regression)
  - $AIC = -2\log(L) + 2k$, k is the number of estimated coefficients
  - Useful for comparing different models with different number of features estimated on the same dataset (in-sample comparison)


```
Call:
glm(formula = default ~ I(annual_inc/1000), family = "binomial",
    data = lc_clean)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-0.6246  -0.5786  -0.5572  -0.5115   3.6388

Coefficients:
                     Estimate Std. Error z value Pr(>|z|)
(Intercept)        -1.5191545  0.0292430  -51.95   <2e-16 ***
I(annual_inc/1000) -0.0040801  0.0003998  -10.21   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 31130  on 37868  degrees of freedom
Residual deviance: 31005  on 37867  degrees of freedom
AIC: 31009

Number of Fisher Scoring iterations: 5
```


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Feature Engineering

- **Same principles as linear regression**
  - Features: ***Term, income, dti, grade***, number of delinquencies, employment length, etc
    - Some of these are numerical others are factors. How do we use factor variables?
  - Interaction terms: *Perhaps the loan amount affects 36-month loans differently than 60-month loans***.** How do you model this?
    - Interactions between two factor features, a factor and a numerical feature, two numerical features
  - Non-linear terms: *Perhaps a small increase in the loan amount doesn't affect interest rate so much but a large increase does.* How would you model this?
    - ***Polynomial terms*** (powers of a feature) or any ***other non-linear transformation*** (better have a good reason for the non-linear transformation)
    - Dummy variable creation → converting a numerical variable into a factor variable (e.g., low, mid, high income, or ***deciles of income***). This is a non-parametric way of modelling non-linear relationships
- **Look for data outside your model**
- **Feature engineering is more of an art than science! Know your context (or work with people who do)!**

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Multivariate logistic regression

Logistic 1

```
Call:
glm(formula = default ~ I(annual_inc/1000), family = "binomial",
    data = lc_clean)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-0.6246  -0.5786  -0.5572  -0.5115   3.6388

Coefficients:
                     Estimate Std. Error z value Pr(>|z|)
(Intercept)        -1.5191545  0.0292430  -51.95   <2e-16 ***
I(annual_inc/1000) -0.0040801  0.0003998  -10.21   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 31130  on 37868  degrees of freedom
Residual deviance: 31005  on 37867  degrees of freedom
AIC: 31009

Number of Fisher Scoring iterations: 5
```

Logistic 2

```
Call:
glm(formula = default ~ annual_inc + term + grade + loan_amnt,
    family = "binomial", data = lc_clean)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-1.0635  -0.6079  -0.4815  -0.3455   4.0988

Coefficients:
              Estimate Std. Error z value Pr(>|z|)
(Intercept) -2.432e+00  5.056e-02 -48.093   <2e-16 ***
annual_inc  -6.019e-06  4.727e-07 -12.733   <2e-16 ***
term60       4.790e-01  3.561e-02  13.451   <2e-16 ***
gradeB       6.599e-01  5.270e-02  12.521   <2e-16 ***
gradeC       1.031e+00  5.392e-02  19.128   <2e-16 ***
gradeD       1.288e+00  5.703e-02  22.578   <2e-16 ***
gradeE       1.415e+00  6.662e-02  21.241   <2e-16 ***
gradeF       1.666e+00  8.630e-02  19.302   <2e-16 ***
gradeG       1.769e+00  1.340e-01  13.198   <2e-16 ***
loan_amnt    2.841e-06  2.347e-06   1.211    0.226
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 31130  on 37868  degrees of freedom
Residual deviance: 29286  on 37859  degrees of freedom
AIC: 29306

Number of Fisher Scoring iterations: 5
```

Which is better, logistic 1 or logistic 2?

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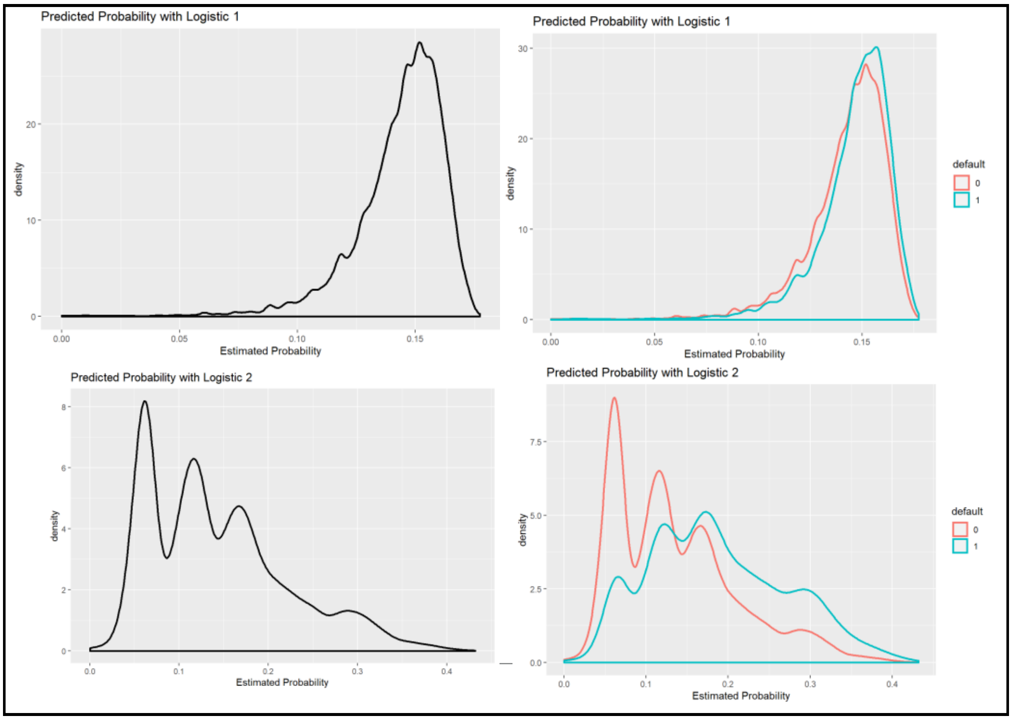

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Topics to follow

- Logistic regression is not classification. It's a probability estimate
  - How do we make use of this model to make predictions?
  - How can we assess how well the model performs?
  - How do we compare performance of different models?

  - These are more difficult questions in classification problems than linear regression. But not impossible!

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Making predictions: From probability to classification

- Logistic regression (or any other classification model for that matter) gives you a probability estimate (sometimes called score)
  - This is a number between 0 and 1
  - Intuitively, this is a number proportional to the chances of observing "success"

- But in classification problems we are interested in predicting outcomes: 0 or 1

- **We need to convert these scores (number between 0-1) to predictions ("success" of "failure")**

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Making predictions: From probability to classification

- **Converting scores to classification**
  - For each data point we can estimate the probability $Y = 1$
  - Choose a cut-off point, say 0.25
  - If the predicted probability is greater than the cutoff (>0.25) we predict that Y will take the value 1, otherwise zero

- **Some of these predictions will be right and some wrong**
  - Confusion matrix records the actual vs predicted outcomes

```
Confusion Matrix and Statistics

          Reference
Prediction     0     1
         0 29445  4131
         1  2995  1298
```

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Assessing model performance

- **Language of classification: Here is the common language we use to indicate different parts of the confusion matrix**
  - Usually we set the value of the outcome we are interested in equal to 1

	Reference				Reference	
Prediction	0	1		Prediction	0	1
0	29445	4131		0	True Negative	False Negative
1	2995	1298		1	False Positive	True Positive

- **Measure I: (Plain accuracy)**

$$Accuracy = \frac{Number\ of\ correct\ decisions\ made}{Total\ number\ of\ decisions\ made}$$

What is the accuracy of this model =(29445 + 1298)/(37869)=81.18%

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Assessing model performance

- **Consider the following scenario:**
  - 1 in 1,000,000 credit card transactions are fraud.
  - My classifier estimates all the transactions as non-fraud so we can compute its confusion matrix for a representative data as follows

	Reference	
Prediction	0	1
0	999,999	1
1	0	0

  - What is the accuracy? 99.9999%
  - Obviously, this is misleading
  - So besides accuracy, specificity and sensitivity are also important!
    - Specificity= Prob of predicting 0 if actual is 0 = 100%
    - Sensitivity=Prob of predicting 1 if actual is 1 = 0%
  - Through the choice of cut-off we can always improve sensitivity at the expense of specificity and vice versa
  - Deciding on the cut-off value will depend on the relative cost of the two types of errors

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Confusion Matrix Statistics

```
Confusion Matrix and Statistics

          Reference
Prediction     0     1
         0 29445  4131
         1  2995  1298

               Accuracy : 0.8118
                 95% CI : (0.8079, 0.8158)
    No Information Rate : 0.8566
    P-Value [Acc > NIR] : 1

                  Kappa : 0.1608

 Mcnemar's Test P-Value : <2e-16

            Sensitivity : 0.23909
            Specificity : 0.90768
         Pos Pred Value : 0.30235
         Neg Pred Value : 0.87697
             Prevalence : 0.14336
         Detection Rate : 0.03428
   Detection Prevalence : 0.11336
      Balanced Accuracy : 0.57338

       'Positive' Class : 1
```

- **Measure I:**
  - **Accuracy**: How often is the model right
  - No information rate: The accuracy of a classifier do without any information
    - Can you guess the classifier has 85.66% accuracy?
- **Measure II:**
  - **Sensitivity**: True positive rate (i.e., what proportion of bad loans did we catch?)
  - **Specificity**: True negative rate (i.e., what proportion of good loans did we identify as good loans)
    - Can you guess what would the sensitivity and specificity of the maximum accuracy no-information classifier be?

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Confusion Matrix Statistics

```
Confusion Matrix and Statistics

          Reference
Prediction     0     1
         0 29445  4131
         1  2995  1298

               Accuracy : 0.8118
                 95% CI : (0.8079, 0.8158)
    No Information Rate : 0.8566
    P-Value [Acc > NIR] : 1

                  Kappa : 0.1608

 Mcnemar's Test P-Value : <2e-16

            Sensitivity : 0.23909
            Specificity : 0.90768
         Pos Pred Value : 0.30235
         Neg Pred Value : 0.87697
             Prevalence : 0.14336
         Detection Rate : 0.03428
   Detection Prevalence : 0.11336
      Balanced Accuracy : 0.57338

       'Positive' Class : 1
```

- Is the model better than random guess?
  - Evaluating goodness of fit using the confusion matrix is a good first step
  - One problem is that it depends on the cut-off value
    - Is poor performance because of bad fit or poor choice of cut-off?
- We need sensible cutoff threshold!

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Choosing the cut-off
Expected value approach

- One way of choosing the cut-off value is by estimating the expected cost associated with decisions
- For example:
  - We will only invest in loans that are predicted not to default (i.e., prediction 0)
  - For each loan that does not default (true negative) we make a profit of $10
  - For each loan that defaults (false negative) we lose $70

Confusion Matrix				Cost Matrix		
	Reference				Reference	
Prediction	0	1		Prediction	0	1
0	True Negative	False Negative		0	10	-70
1	False Positive	True Positive		1	0	0

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Choosing the cut-off
Expected value approach

- For a given confusion matrix I can find the associated expected value

Confusion Matrix				Cost Matrix		
	Reference				Reference	
Prediction	0	1		Prediction	0	1
0	29445	4131		0	10	-70
1	2995	1298		1	0	0

Expected value = 29445 * 10 – 4131 * 70 = 5280

Or 18.95 cents per loan application

- But this is for one cut-off value and we have complete flexibility on what to choose
- How about we look at the profit as a function of the cut-off value (sensitivity analysis)

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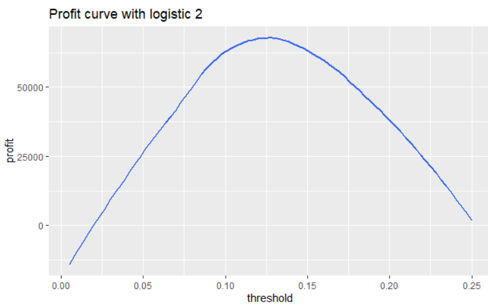
Choosing the cut-off Expected value approach



- Profit for different cut-off values is shown to the left
- In this case, a cut-off value of ~12.25% maximizes profits at just over $69,090 or $1.82 per loan application
- We can also use this curve to compare different models

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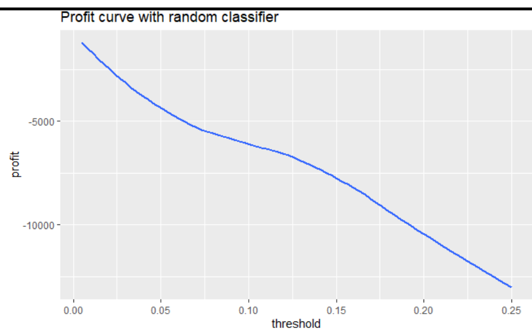
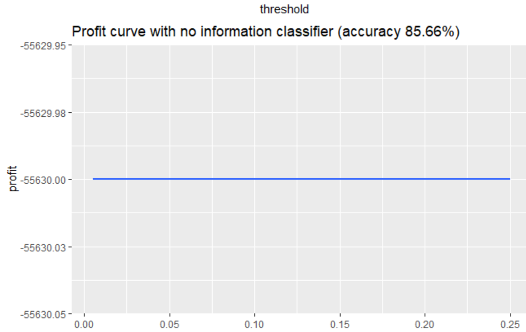
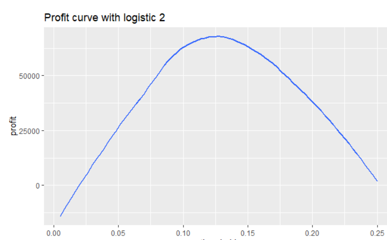
Comparing different models







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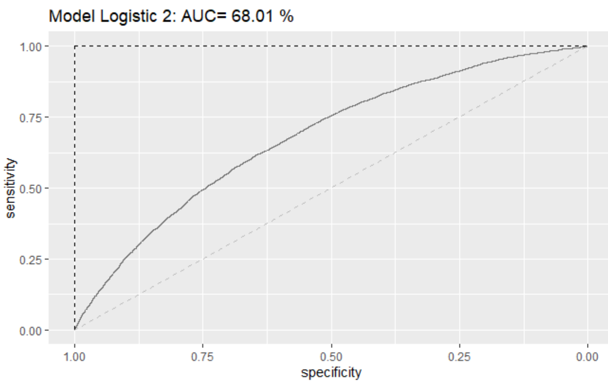
Assessing model performance



- The Receiver and Operating Characteristic (***ROC***) curve is a scatterplot of the model's sensitivity (True Positive rate) against the specificity (True Negative rate) for different cutoff values
- Typically the specificity axis (horizontal) is inverted
  - From 1->0 instead of 0->1
- The ROC curve shows for any given level of True Positives, what level of True Negatives we can expect
  - Obviously, the lower the better – so we want classifiers that are as close as possible to the top left corner (1,1)

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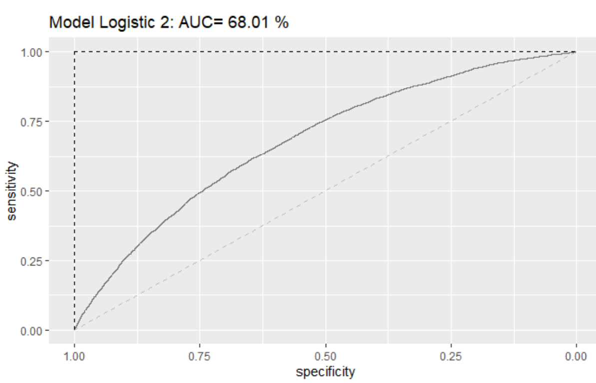
Assessing model performance



- We can use the area under the curve (AUC) to measure the predictive power of the model
  - AUC = 0.5 suggest that the model is no better than random chance (flipping a coin)
  - AUC=1 suggests that the model predicts perfectly (sensitivity=specificity=accuracy=100%)
  - The higher the AOC the better the model's predictive power

- ROC and AUC notions are not unique to logistic regression, they apply to any classification model

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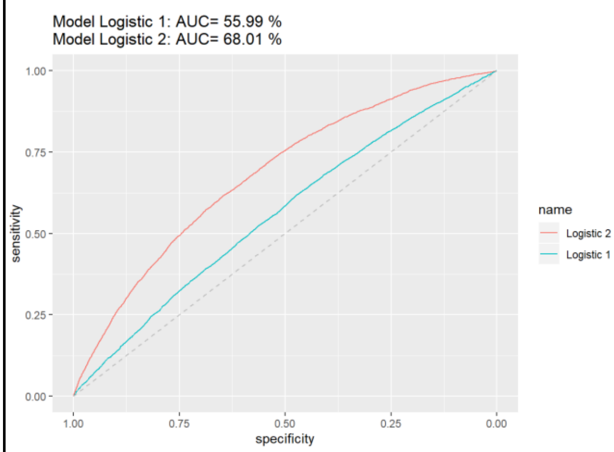
Comparing model performance



- We can use the ROC to compare models
- In this case the red model (logistic 2) ***dominates*** the blue model (logistic 1) – for any cut-off value the specificity is higher for the same sensitivity
  - This is not surprising as logistic 2 has more features and lower deviance (and AIC) than logistic 1
  - This results suggests that for any costs, the profit curve with logistic 2 will be higher than with logistic 1

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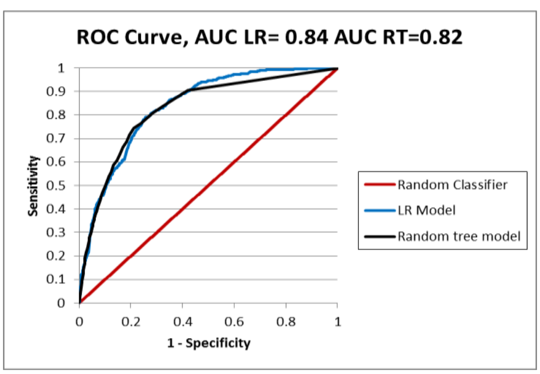
Comparing model performance



- **How can I use ROC curves to compare these two models?**
  - In this case there is no strict dominance. The “blue model” preforms better at high sensitivity, the “black model” preforms better at high specificity
  - Then we choose the one that performs better in the region we want to operate at!

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Out-of-sample testing

- So far everything was done in-sample
  - In-sample estimation of coefficients for probability model
  - In-sample estimation of cut-off point for prediction
  - In-sample estimation of errors, confusion matrix, profit, ROC and lift curves

- This is problematic as the sample may not be representative, way may be overfitting→ model may not work well on new data

- Best to do out-of-sample validation

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Out-of-sample validation

In sample (80 % of the data)

```
          Reference
Prediction     0     1
         0 23580  3308
         1  2371  1037

               Accuracy : 0.8125
                 95% CI : (0.8081, 0.8169)
    No Information Rate : 0.8566
    P-Value [Acc > NIR] : 1

                  Kappa : 0.1618

 Mcnemar's Test P-Value : <2e-16

            Sensitivity : 0.23867
            Specificity : 0.90864
```

Out of sample (20% of the data)

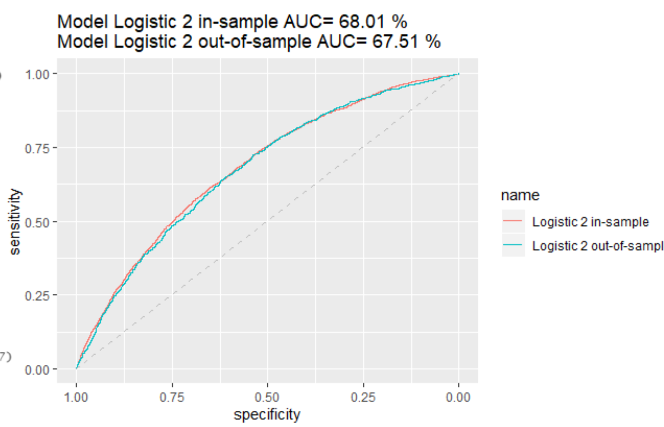
```
          Reference
Prediction    0    1
         0 5865  823
         1  624  261

               Accuracy : 0.8089
                 95% CI : (0.7999, 0.8177)
    No Information Rate : 0.8569
    P-Value [Acc > NIR] : 1

                  Kappa : 0.1566

 Mcnemar's Test P-Value : 1.939e-07

            Sensitivity : 0.24077
            Specificity : 0.90384
```



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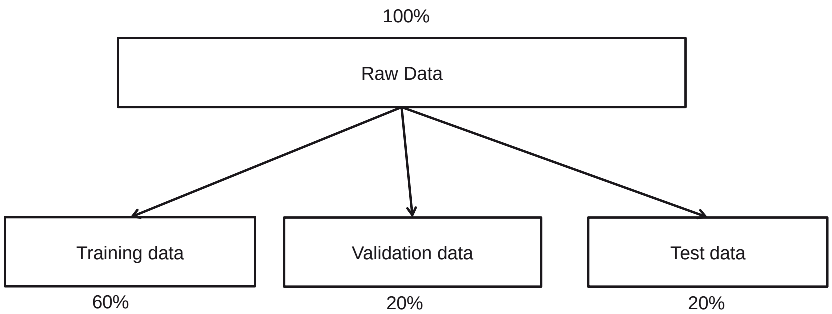
Three-way data partitioning



- **In calculating the optimal cut off we need to partition data into three samples**
  - Training → estimate the model (typically larger than the other two)
  - Validation → estimate the cutoff (or any other hyper-parameter)
  - Test → estimate overall performance

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Three-way data partitioning

- **Without splitting the data (see slide 34)**
  - Best threshold = 12.25% with a profit of $1.82 per loan
- **Using three-way partitioning method**
  - In sample: Model estimation
  - Validation set: Best threshold =11.75% with a profit of $1.81 per loan
    - Lower threshold → more conservative because now we make more errors
    - Lower profit because of the increase in error
  - Testing set: out-of-sample profit of $1.79 per loan
    - Lower than both the in-sample and the validation
- **This deterioration in performance is typical**
  - Deterioration could be worse for models that suffer from overfitting

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k-fold Cross-validation

- Splitting the data in training and validation means that
  - we reduce the data used for training which may be a problem if we have a relatively small dataset
  - we don't get to use every data point for estimation and for validation (only one of the two)

- K-fold cross-validation overcomes this problem:
  - Randomly divide data into K equal-size groups (referred to as folds)
  - Use the first fold as validation and the other K-1 for training and compute Accuracy or AUC
  - Repeat this K times
  - Use the average Accuracy or AUC to estimate model performance

- Typically set K = 5 or 10

Data

Validation	Train	Train	Train	Train
1	2	3	4	5

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Logistic Regression and Regularization

- We can also do logistic regression using the idea of regularization
  - Similar to linear regression
- Instead of maximizing log-Likelohood to estimate the model's coefficients we can maximize log-Likelihood minus λ times the sum of the absolute value of estimated coefficients
  - As in the OLS case, the penalty has the effect of shrinking the estimated coefficients towards zero
  - The penalty parameter λ is chosen to maximize performance out-of-sample (typically through k-fold cross validation)
  - Remember to standardize any features before running LASSO

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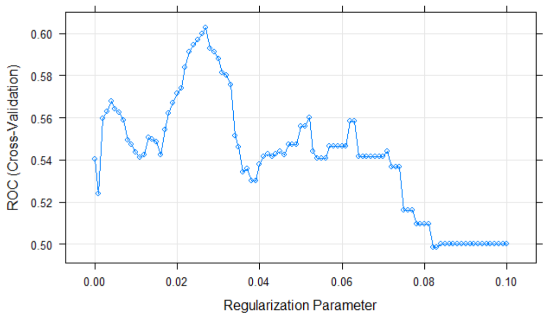
LASSO performance



- Select 1% of the data for training
- Estimate a huge model (168 coefficients) using
  - Logistic Regression
  - LASSO logistic regression with 10fold cross validation
- The chart shows AUC (based on 10-fold CV) against the parameter λ
- Highest AUC at $\lambda = 0.027$
  - This is somewhat sensitive to training sample, but the performance is not so sensitive

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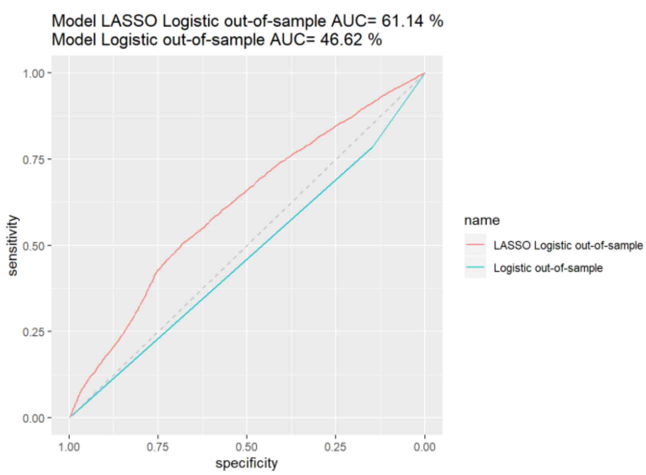
Performance of LASSO



Select 1% of the data for training (<400 loans)

Estimate a huge model (168 coefficients) using
- Logistic Regression
- LASSO logistic regression with 10fold cross validation

Use both models to construct out-of-sample ROC curves (on the 99% testing data)

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Logistic regression checklist

- Step 1. ICE the date, scatter plots & correlation tables
- Step 2. Develop model & Feature engineering. If in doubt start simple
- Step 3. Estimate the model using software (logistic / LASSO)
- Step 4. Post Estimation
  - Examine significance of individual variables: check p-values, drop out non-significant variables (e.g., p-value > 5%)
  - Check goodness of fit: deviance, ROC curve
  - Select cut-off value if needed (e.g., use profit curve)
  - Check for out of sample performance (ROC, precision, specificity, sensitivity, profit)
- Step 5: Repeat steps 2-4 to assess and compare competing models. Choose the simplest model that has good enough explanatory power and makes intuitive sense

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Rules of thumb

- The number of observations should be at least $10 \times \frac{m+2}{min(p,1-p)}$ where m is the number of explanatory variables and p is the probability of a failure
  - This is higher than the rule of thumb for linear regression
- We can construct confidence intervals for the coefficients of estimation and for odds ratios. Don't use variables that are not significant (say at the 5% level) in your model due to the problem of overfitting
- Don't rely on R-square measures for classification problems (they will tend to be low). Instead check precision, sensitivity, and specificity, ROC curves and AUC.

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Multinomial Logistic Model

- The model extends to classification in more than two categories
- Say there are three categories $Y = \{R, G, B\}$
- Choose one category as the base, say R
- Define the two risk factors
  - $U_R = 0$, $U_B = \beta_0 + \beta_1 X_1 + \cdots \beta_m X_m$, $U_G = \gamma_0 + \gamma_1 X_1 + \cdots + \gamma_m X_m$
  - The probabilities that Y takes any of the three categories given X are
  - $Prob(Y = B|X) = \frac{\exp(u_B)}{1+\exp(U_B)+\exp(U_G)}$
  - $Prob(Y = G|X) = \frac{\exp(u_G)}{1+\exp(U_B)+\exp(U_G)}$
  - $Prob(Y = R|X) = \frac{1}{1+\exp(U_B)+\exp(U_G)}$
- We can estimate the model using the maximum likelihood principle
  - Confusion matrix and ROC curve are now multidimensional

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Oversampling

- It is often the case that we are interested in rare events (<1% of the data is "success") such as fraudulent credit card transactions
- Therefore, sampling a random subset (eg training set) may yield too few events, making it difficult to train a model
- To give the model a better "chance" to train, we may want to oversample "successes" so that the dataset is more balanced
- Oversampling does not affect the estimated coefficients (except the intercept), p-values, or the ROC
- Predicted probabilities using oversampling will need to be adjusted
- Oversampling approach
  - Oversample in the training set but not in the validation set

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Next lecture

Course contents (first part of the course – Nicos)

- Session 1: The Art & Science of Regression Models For Prediction
- Session 2: More on Using Linear Regression For Prediction
  - Quiz 1, due 3 days after session 2
- Session 3: Workshop I – Engineer an algorithm that sets interest rates for new Lending Club loans
  - Group assignment 1, due 6 days after the workshop
- Session 4: Classification using Logistic Regression
  - **Quiz 2, due 3 das after session 4**
    - Instructions here
- **Session 5: Workshop – Invest in a portfolio of Lending Club loans**
  - Individual project 1, due 6 days after the end of the workshop

Course contents (second part of the course – Tolga)

- See canvas syllabus