CS147 - Lecture 04

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- Boolean values and operations
- Boolean Functions & Truth Table
- Basic Identities & Algebraic Manipulation

[Chapter 2 (2-2) of Logic & Computer Design Fundamentals, 4th Edition, M. Morris Mano, Charles R. Kime]

Binary Values & Operations ...

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Boolean Values & Operations

- A Boolean variable assumes two values TRUE or FALSE.
  - TRUE is denoted as 1 and FALSE is denoted as 0

- There are three basic Boolean algebraic operations
  - AND (.) or conjunction (∧)
  - OR (+) or disjunction (∨)
  - NOT (') or negation (¬)

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- Since dealing with 1 or 0, Boolean algebra is ideal for handling mathematics using Binary number systems.

- For the need of advance digital logic analysis, Boolean algebra is extended to handle multivalued arithmetic. For example simulator often uses value 'x' to represent unknown value and 'z' to represent electrical isolation.

- Operation 'AND' between two variables will result 1 if and only if both the variables assume value 1. Operation rules are 0.0=0; 0.1=0; 1.0=0; 1.1=1;

- Operation 'OR' between two variables will result 0 if and only if both the variables assume value 0. Operation rules are 0+0=0; 0+1=1; 1+0=1; 1+1=1;

- Operation 'NOT' is an unary operations which results in exact opposite value of the present value that a variable is holding. Operations rules are 0'=1; 1'=0;

Boolean Function & Truth Table ...

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Boolean Function & Truth Table

- A Boolean function is like any other algebraic function expressed as a function of a list of variable with a corresponding equivalent Boolean expression to evaluate the function value.

$$F(X,Y,Z) = X.Y' + Z$$

- Each part of the right hand side of the equation is called a '*term*'
  - X.Y' is a term and Z is another term.
  - X is also a term in X.Y' and thus Y'

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- Often we drop the symbol '.' from the expression. For example the equation on the slide can be written as 'F(X,Y,Z) = XY' + Z'.

- The above function is 1 if the term XY' is 1 or Z is 1. This implies that the above function is 1 if X is 1 and Y is 0 (which makes the term XY' results in 1).

Boolean Function & Truth Table

- Boolean functions are often represented in a truth table. For example the function F(X,Y,Z) = X.Y' + Z will have a truth table like the following.

X	Y	Z	F(X,Y, Z)
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

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- Truth table is a table which shows all possible combination of values of the variables in the function and corresponding evaluated value of the function if we replace the variable with its corresponding values.

- For example if X=1, Y=0 and Z=0 then F(X,Y,Z) will be evaluated as 1.0' + 0 = 1.1 = 1. This means the row in the truth table having X=1, Y=0 and Z=0 will have an entry of 1 in the function value column.

Boolean Function & Truth Table

- Accordingly, we can also create truth table for each basic Boolean operation.

Y = A.B		
A	**B**	**Y**
0	0	0
0	1	0
1	0	0
1	1	1

Y = A + B		
A	**B**	**Y**
0	0	0
0	1	1
1	0	1
1	1	1

Y = A'	
A	**Y**
0	1
1	0

Boolean Function & Truth Table

- '**Dual**' of a Boolean Function F is a function obtained by replacing '.' with '+' and '+' with '.' in the original function. Any function involving constant value 0 or 1, we interchange them too.
  - F(X,Y,Z) = XY + X'Z + YZ
  - Dual of F will be (X+Y)(X'+Z)(Y+Z)

- '**Complement**' of a Boolean function F is a function obtained by changing 1s to 0s and 0s to 1s in the truth table for the variable values of the original function F. This means like dual '.' is changed to '+' and vice-verse. However, unlike dual, variables are complemented too.
  - F(X,Y,Z) = X'YZ' + X'Y'Z
  - F'(X,Y,Z) = (X + Y' + Z)(X + Y + Z')

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- To complement a function we can change the original function to its dual expression and then we complement each literals or variables.

- For example to compute complement of F = X'YZ' + X'Y'Z we compute dual expression as (X'+Y+Z')(X'+Y'+Z) and then complement its variable as (X+Y'+Z)(X+Y+Z'), which is a complement of original function F.

- The following is the example of complementing function F(X,Y,Z) = XY' + Z, which gives F'(X,Y,Z) = (X' + Y)Z'

X	Y	Z	F(X,Y, Z)
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

X	Y	Z	F'(X,Y, Z)
1	1	1	0
1	1	0	1
1	0	1	0
1	0	0	1
0	1	1	1
0	1	0	1
0	0	1	0
0	0	0	1

Basic Identities & Algebraic Manipulation ...

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Basic Identities

1. X + 0 = X	2. X . 1 = X
3. X + 1 = 1	4. X . 0 = 0
5. X + X = X	6. X . X = X
7. X + X' = 1	8. X . X' = 0
9. (X')' = X	

10. X + Y = Y + X	11. XY = YX	Commutative
12. X+(Y+Z) = (X+Y)+Z	13. X(YZ) = (XY)Z	Associative
14. X(Y+Z) = XY + XZ	15. X + YZ = (X+Y)(X+Z)	Distributive
16. (X + Y)' = X'.Y'	17. (XY)' = X' + Y'	DeMorgan's

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- First 9 rules shows the relationship between a single variable X and its complement X' and the binary constant 0 and 1.
- The identities 10-14 are the counter part of the ordinary algebra.
- The identities 15 to 17 are only applicable for Boolean expression.
- The two columns of identities shows the dual nature of Boolean algebra. If one identity holds, its dual identity will also hold. For example if (X + 1 = 1) holds then its dual (X.0 = 0) will also hold.
- The 16 and 17 identity can be proved by using truth table equivalency. Also it can be proved as following example.
  - F = X + Y => F' = (X + Y)'
  - From definition of complement of function F' = X'.Y'
  - Hence (X+Y)' = X'.Y'
- DeMorgan's theorem is very useful to simplify complement of a function.

Boolean Algebraic Manipulation

- All the identity rules are used to simplify longer Boolean expression.

```
F = X'YZ + X'YZ' + XZ
  = X'Y(Z+Z') + XZ … by identity 14
  = X'Y.1 + XZ     … by identity 7
  = X'Y + XZ       … by identity 2
```


Boolean Algebraic Manipulation

- We can have some more commonly used theorems for Boolean expression simplification.
- Two columns shows the dual nature of Boolean algebra holds for these theorems too.

1. X + XY = X	4. X(X+Y) = X
2. XY+XY' = X	5. (X+Y)(X+Y') = X
3. X+X'Y = X + Y	6. X(X'+Y) = XY

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Boolean Algebraic Manipulation

- The 'consensus' theorem is another useful method to simplify expression.
  - $XY + X'Z + YZ = XY + X'Z$
  - It's dual also holds true
    - $(X+Y)(X'+Z)(Y+Z) = (X+Y)(X'+Z)$

$$
\begin{aligned}
XY + X'Z + YZ &= XY + X'Z + YZ(X+X') \\
&= XY + X'Z + XYZ + X'YZ \\
&= (XY + XYZ) + (X'Z + X'YZ) \\
&= XY(1 + XYZ) + X'Z(1+Y) \\
&= XY.1 + X'Z.1 \\
&= XY + X'Z
\end{aligned}
$$

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- The following example shows the usefulness of consensus theorem.

$$
\begin{aligned}
(A+B)(A'+C) &= AA' + AC + A'B + BC \\
&= AC + A'B + BC \\
&= AC + A'B
\end{aligned}
$$

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