Koç CSSM Bootcamp 2021 **Problem Set 6**

Homework X

NAME: ANSWER KEY

For the following exercises, read the problems carefully and show all your work. Attach more pages if necessary. Avoid using a calculator or the computer to solve the exercises. Please, staple your homework.

1 Continuity

Identify which of the following functions are continuous. For functions that are not continuous, identify the points of discontinuity.

1. $f(x) = x^2$

   As a polynomial, this function is continuous.

2. $f(x) = \dfrac{1}{x}$

   This function has a discontinuity at 0.

3. $f(x) = \dfrac{x-3}{x^3-27}$

   This function has a removable discontinuity at 3.

4. $f(x) = \begin{cases} x^2 & \text{for } x < 1 \\ x & \text{for } x \geq 1 \end{cases}$

   This function is continuous.

5. The function depicted below:

   (Plot omitted)

   This function has discontinuities at 1 and 2.

2 Derivatives and Slopes

For each function, find its derivative:

1. $f(x) = \frac{1}{3}x^3$

$$f'(x) = x^2$$

2. $f(x) = \dfrac{x}{e^x}$

$$\begin{aligned} f'(x) &= \frac{e^x - xe^x}{(e^x)^2} \\ &= \frac{e^x(1-x)}{(e^x)^2} \\ &= \frac{1-x}{e^x} \end{aligned}$$

3. $f(x) = \dfrac{x^2 - 1}{x - 1}$

They could use the quotient rule and simplify:

$$\begin{aligned} f'(x) &= \frac{2x(x-1) - (x^2-1)}{(x-1)^2} \\ &= \frac{2x^2 - 2x - x^2 + 1}{(x-1)^2} \\ &= \frac{x^2 - 2x + 1}{(x-1)^2} \\ &= \frac{(x-1)^2}{(x-1)^2} \\ &= 1 \end{aligned}$$

Or simplify first and see the answer quickly:

$$\begin{aligned} f(x) &= \frac{(x-1)(x+1)}{x-1} \\ &= x + 1 \\ f'(x) &= 1 \end{aligned}$$

4. $f(x) = x^2(x-1)$

$$\begin{aligned} f'(x) &= x^2 + 2x(x-1) \\ &= x^2 + 2x^2 - 2x \\ &= 3x^2 - 2x \end{aligned}$$

5. $f(y) = (1 - 1/y^2)$

$$\begin{aligned} f'(y) &= \frac{d}{dy}1 - \frac{d}{dy}\frac{1}{y^2} \\ &= -\frac{d}{dy}\frac{1}{y^2} \\ &= -\frac{(0)(y^2) - (1)(2y)}{(y^2)^2} \\ &= -\frac{-2y}{y^4} \\ &= \frac{2}{y^3} \end{aligned}$$

6. $f(y) = (y^3 - 7)(1 - 1/y^2)$

$$\begin{aligned} f(y) &= y^3 - y - 7 + \frac{7}{y^2} \\ f'(y) &= 3y^2 - 1 + \frac{-14y}{y^4} \\ &= 3y^2 - 1 - 14y^{-3} \end{aligned}$$

7. $f(x) = ln(2\pi x^2)$

$$\begin{aligned} f(x) &= \ln(2\pi) + \ln(x^2) \\ &= \ln(2\pi) + 2\ln(x) \\ f'(x) &= \frac{d}{dx}\ln(2\pi) + \frac{d}{dx}2\ln(x) \\ &= 2\frac{d}{dx}\ln(x) \end{aligned}$$

$$= \frac{2}{x}$$

8. $f(y) = (y - y^{-1})(y - y^{-2})$

$$f(y) = y^2 - y^{-1} - 1 + y^{-3}$$

$$f'(y) = 2y + y^{-2} - 3y^{-4}$$

9. $f(x) = x^6 + 5x^5 - 2x^2 + 8$

$$f'(x) = 6x^5 + 10x^4 - 4x$$

10. $m(x) = \dfrac{1}{1 + \exp(x)}$

$$m'(x) = \frac{(e^x + 1)(0) - (1)(e^x)}{(1 + e^x)^2}$$

$$= -\frac{e^x}{(e^x + 1)^2}$$

11. $y = 27x^3 + 5x^2 - x + 13$

$$y' = 81x^2 + 10x - 1$$

12. $y = 81x^2 + 10x - 1$

$$y' = 162x + 10$$

13. $y = 162x + 10$

$$y' = 162$$

For these functions, find the derivative at $x = 1$ and $x = 3$

14. $f(x) = 2x^2 + 7$

$$f'(x) = 4x$$
$$f'(1) = 4$$
$$f'(3) = 12$$

15. $f(x) = x^3 - x + 1$

$$f'(x) = 3x^2 - 1$$
$$f'(1) = 2$$
$$f'(3) = 26$$