

NAME:

For the following exercises, read the problems carefully and show all your work. Attach more pages if necessary. Avoid using a calculator or the computer to solve the exercises. Please, submit ONE pdf.

Consider the following matrices:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{B} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \mathbf{C} = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \mathbf{D} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \mathbf{E} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 5 \\ 0 & 0 & 8 \end{bmatrix}$$

1. For each of them, identify whether the matrix is: square, symmetric, triangular, idempotent, identity, \mathbf{J} , $\mathbf{0}$, or none of the above.

2. Calculate $\text{Tr}(\mathbf{A})$

3. Calculate $5(\text{Tr}(\mathbf{B}) + \text{Tr}(\mathbf{E}))$

Now consider the following matrices:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 5 \\ 1 & -2 & -1 \\ 5 & -1 & 2 \end{bmatrix} \mathbf{B} = \begin{bmatrix} 4 & 2 \\ 6 & 3 \end{bmatrix} \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{D} = \begin{bmatrix} 1 & 1 \\ 3 & -2 \end{bmatrix} \mathbf{E} = \begin{bmatrix} 0 & 1 & 2 \\ 5 & 1 & -1 \\ 2 & 4 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

4. Is $\mathbf{E}'\mathbf{E}$ square, symmetric, triangular, idempotent, identity, \mathbf{J} , $\mathbf{0}$, or none of the above?

5. For each of **A**, **B**, **C**, **D**, and **E**, find the trace.

Invert the following matrices or give a reason why you cannot:

6. $\begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}$

$$7. \begin{bmatrix} -1 & 3 \\ -2 & 6 \end{bmatrix}$$

$$8. \begin{bmatrix} 9 & 5 & 6 \\ 8 & 2 & 7 \\ 4 & 3 & 1 \end{bmatrix}$$

$$9. \begin{bmatrix} 11 & 3 & 5 \\ 3 & 2 & 19 \\ 0 & 0 & 0 \end{bmatrix}$$

$$10. \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \text{ assuming } ad \neq bc$$

11. $\begin{bmatrix} 3 & 8 & 6 \\ 0 & -3 & -5 \\ -9 & 0 & 4 \end{bmatrix}$

Provide a solution to the following systems of linear equations, or explain why you cannot:

12.

$$-3x + 5y + 5z = -43$$

$$x - 4y - 2z = 31$$

$$3x - 4z = 7$$

13.

$$x - 2y - z = -15$$

$$-x - y + z = -6$$

$$x - 6y - z = -43$$

An Applied Problem

Often political scientists use linear regression to study political phenomena. We may think about this method using matrix algebra. Essentially, the researcher has a vector \mathbf{y} of outcome observations, and a matrix \mathbf{X} of explanatory variable observations. That is, each element of \mathbf{y} is the observed outcome for the corresponding row of observations for the independent variables in \mathbf{X} . The researcher's goal is to find the vector \mathbf{b} of coefficients; that is, we assume that each explanatory variable has a linear effect on the outcome such that in expectation $\mathbf{y} = \mathbf{X}\mathbf{b}$. Given the equation $\mathbf{y} = \mathbf{X}\mathbf{b}$, find the formula for \mathbf{b} . Be sure to account for the fact that \mathbf{X} may not be a square matrix. Are there any situations where the formula you found will not work?