NAME: ANSWER KEY

For the following exercises, read the problems carefully and show all your work. Attach more pages if necessary. Avoid using a calculator or the computer to solve the exercises. Please, staple your homework.

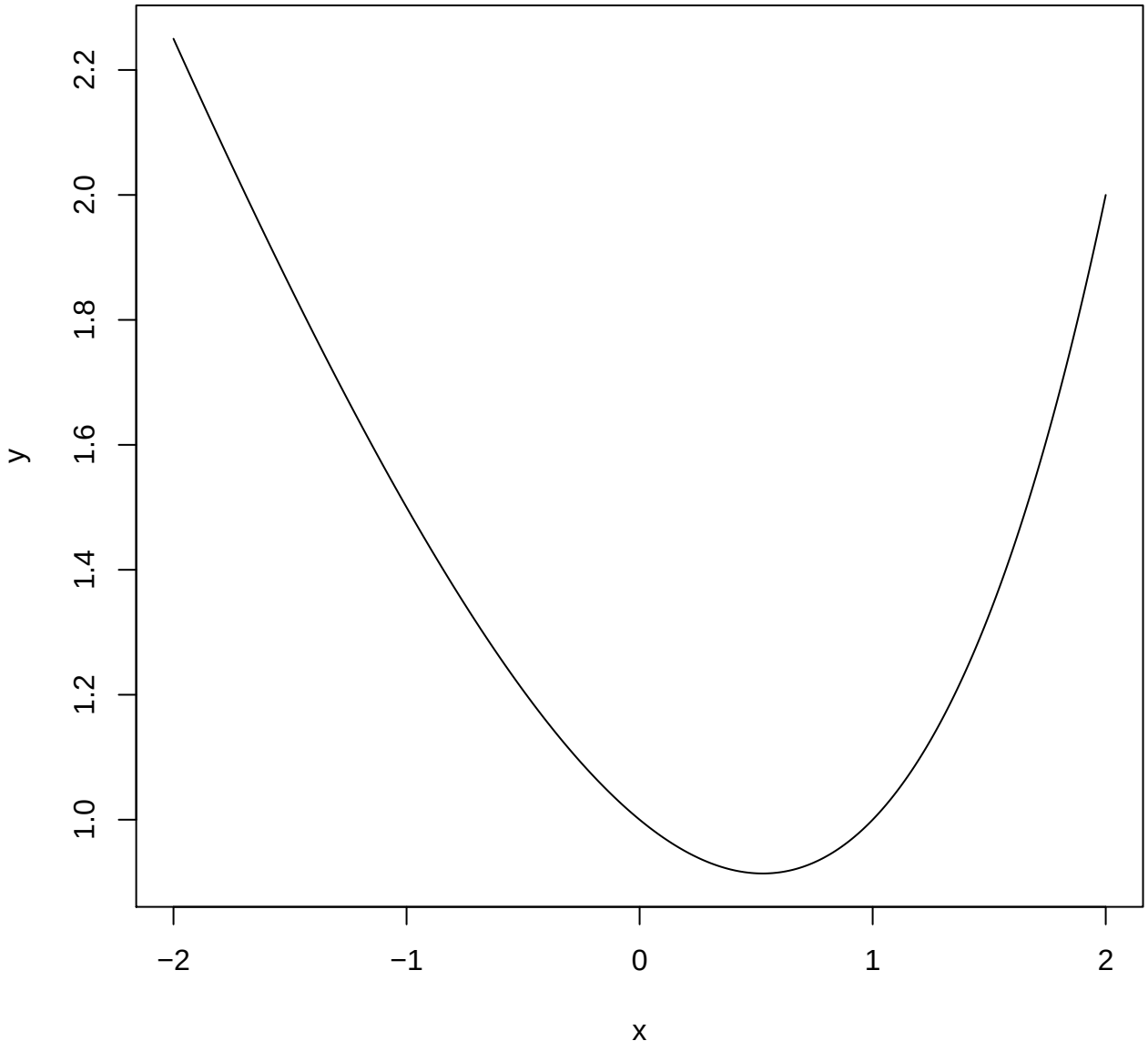
1 Cartesian Coordinates/Geometry, Lines

1. $y = 2x + 3$

2.

x	y
0 or 1	1
2	2
-1	1.5
-2	2.25

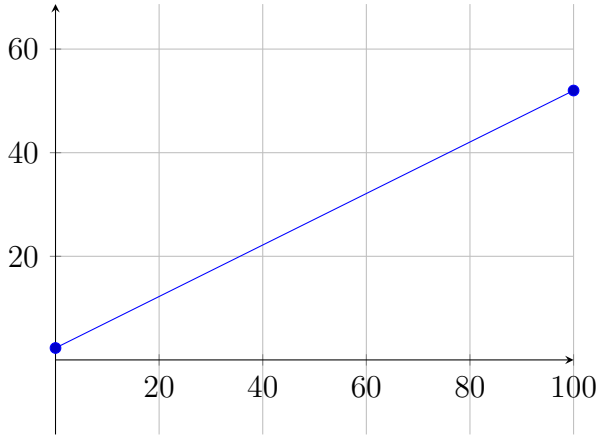
3. It should look something like this:



4. Say you were interested in the relationship between the amount of federal grant funds distributed by executive agencies in a jurisdiction and mean annual income. Suppose after collecting data and fitting a regression, you determined the relationship to be

$$Y = 2.3 + 0.5x,$$

where Y is the amount of federal grants distributed in millions and x is mean annual income in units of $1,000$. Draw a graph showing this relationship for $x \in [0, 100]$ (it may be useful to use units of ten when labeling the axes). How much federal grant money is distributed to juristidictions with a mean annual income of \$25,000? \$50,000? \$100,000? Jurisdictions with a mean annual income of \$25,000 receive \$14.8 million. Those with mean annual income of \$50,000 receive \$27.3 million. Those with mean annual income of \$100,000 receive \$52.3 million.



2 Sets

1. Let $U = \{i \in \mathbb{N} : 0 < i < 11\}$, $A = \{1, 3, 5, 7\}$, and $B = \{i \in \mathbb{N} : 1 < i < 10\}$.

   (a) Find $A \cup B$.

   $$\{i \in \mathbb{N} : 0 < i < 10\}$$

   (b) Find $A \cap B$.

   $$\{3, 5, 7\}$$

   (c) Depict these sets in a Venn diagram.

2. For any two sets A and B, what can we say about $B \setminus (B \setminus A)$?

   $B \setminus (B \setminus A) = A \cap B$.

3. For any three sets A, B, and C, what if anything can we say about $A \cup B \cap C$?

   Little or nothing since $A \cup (B \cap C)$ may differ from $(A \cup B) \cap C$. For example, let $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$, and $C = \{5, 6, 7\}$. Then

   $$A \cup (B \cap C) = \{\varnothing\}$$

   and

   $$(A \cup B) \cap C = \{5\}$$

4. Express the function $y = 2x + 6$ as a set.

   $$\{(x, y) \in \mathbb{R}^2 : y = 2x + 6\}$$

3 Functions

Solve for x:

1. $x^2 = 1$

$$x^2 - 1 = 0$$
$$(x+1)(x-1) = 0$$
$$x = \pm 1$$

2. $(x-1)(x+2) = 0$

$$x = 1 \text{ or } x = -2$$

3. $3x^2 - 1 = 6x + 8$

$$3x^2 - 6x - 9 = 0$$
$$(3x+3)(x-3) = 0$$
$$x = 3 \text{ or } x = -1$$

Expand then simplify the following expressions:

4. $(x+3)(x-4)$

$$x^2 - 4x + 3x - 12$$
$$x^2 - x - 12$$

5. $(5x+1)(2x-1)$

$$10x^2 - 5x + 2x - 1$$
$$10x^2 - 3x - 1$$

6. $(x+1)(x+y+1)$

$$x^2 + xy + x + x + y + 1$$

$$x^2 + 2x + xy + y + 1$$

Solve the following formulas:

7. $5 + 11x = -3x^2$

$$\begin{aligned} x &= \frac{-11 \pm \sqrt{11^2 - 4(3)(5)}}{2(3)} \\ &= \frac{-11 \pm \sqrt{11^2 - 4(3)(5)}}{2(3)} \\ &= \frac{-11 \pm \sqrt{121 - 60}}{6} \\ &= \frac{-11 \pm \sqrt{61}}{6} \end{aligned}$$

8. $\sqrt{4x + 13} = x + 2$

$$\begin{aligned} 4x + 13 &= (x + 2)^2 \\ 4x + 13 &= x^2 + 4x + 4 \\ 9 &= x^2 \\ x^2 - 9 &= 0 \\ (x + 3)(x - 3) &= 0 \\ x &= 3 (x = -3 doesn't hold for the original equation.) \end{aligned}$$

9. $10^{3x^2} 10^x = 100$

$$\begin{aligned} \log(10^{3x^2} 10^x) &= \log 100 \\ \log(10^{3x^2}) + \log(10^x) &= 2 \\ 3x^2 \log(10) + x \log(10) &= 2 \\ 3x^2 + x - 2 &= 0 \end{aligned}$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(3)(-2)}}{2(3)}$$
$$= \frac{-1 \pm \sqrt{1 + 24}}{6}$$
$$= \frac{-1 \pm 5}{6}$$
$$x = 2/3 \text{ or } x = -1$$

10. $6x^2 - 6x - 6 = 0$

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(6)(-6)}}{2(6)}$$
$$= \frac{6 \pm \sqrt{36 + 144}}{12}$$
$$= \frac{6 \pm \sqrt{180}}{12}$$
$$= \frac{1 \pm \sqrt{5}}{2}$$

11. $5 + 11x = -3x^2$

12. Find the inverse of $f(x) = 5x - 2$

$$\begin{aligned} y &= 5x - 2 \\ x &= 5y - 2 \\ 5y &= x + 2 \\ y &= \frac{x+2}{5} \end{aligned}$$

13. Simplify $h(x) = g(f(x))$, where $f(x) = x^2 + 2$ and $g(x) = \sqrt{x-4}$.

$$\begin{aligned} g(f(x)) &= g(x^2+2) \\ &= \sqrt{(x^2+2)-4} \\ &= \sqrt{x^2-2} \end{aligned}$$

14. Simplify $h(x) = f(g(x))$ with the same f and g. Is it the same as before?

$$\begin{aligned} f(g(x)) &= f(\sqrt{x-4}) \\ &= (\sqrt{x-4})^2 + 2 \\ &= x - 4 + 2 \\ &= x - 2 \end{aligned}$$

15. Rewrite the following by taking the log of both sides. Is the result a linear function?

$$y = \alpha \times x_1^{\beta_1} \times \beta_2 x_2 \times \beta_3 x_3$$

$$\begin{aligned}
\log(y) &= \log(\alpha x_1^{\beta_1} \beta_2 x_2 \beta_3 x_3) \\
\log(y) &= \log(\alpha) + \log(x_1^{\beta_1}) + \log(\beta_2) + \log(x_2) + \log(\beta_3) + \log(x_3) \\
&= \log(\alpha) + \beta_1 \log(x_1) + \log(\beta_2) + \log(x_2) + \log(\beta_3) + \log(x_3) \\
&= (\log(\alpha) + \log(\beta_2) + \log(\beta_3)) + \beta_1 \log(x_1) + \log(x_2) + \log(x_3)
\end{aligned}$$

This function is is linear in the variables $\log(x_1), \log(x_2)$, and $\log(x_3)$.

16. Rewrite the following by taking the log of both sides. Is the result a linear function?

$$y = \alpha \times x_1^{\beta_1} \times \frac{x_2^{\beta_2}}{x_3^{\beta_3}}$$

$$\begin{aligned}
\log(y) &= \log\left(\alpha x_1^{\beta_1} \frac{x_2^{\beta_2}}{x_3^{\beta_3}}\right) \\
\log(y) &= \log(\alpha) + \log(x_1^{\beta_1}) + \log\left(\frac{x_2^{\beta_2}}{x_3^{\beta_3}}\right) \\
&= \log(\alpha) + \beta_1 \log(x_1) + \log(x_2^{\beta_2}) - \log(x_3^{\beta_3}) \\
&= \log(\alpha) + \beta_1 \log(x_1) + \beta_2 \log(x_2) - \beta_3 \log(x_3)
\end{aligned}$$

This function is is linear in the variables $\log(x_1), \log(x_2)$, and $\log(x_3)$.