Koç CSSM Bootcamp 2021 **Problem Set 1**

NAME:

For the following exercises, read the problems carefully and show all your work. Attach more pages if necessary. Avoid using a calculator or the computer to solve the exercises. Please, turn in your homework in ONE pdf.

1 Order of Operations, Summation, and Products

Simplify the following expressions:

1. $(3 * 4)/(3 - 2) + ((4 + 3)/7)$

2. $(3 * 4)/(3 - 2) - ((4 + 3)/7)((2 + 10)/3)$

3. $\sum_{i=1}^{3} 9 + \sqrt{9^i}$

4. $\sum_{i=1}^{10} 9 + 9i$

5. $\prod_{x=1}^{5} 2x$

6. $\sum_{i=1}^{n} i$ (Even if you already know the formula, try to derive it)

7. $\frac{2g+13}{3g} + \frac{4g-5}{4g}$

8. $\frac{\frac{w^3 z^4}{(w+1)(z-3)}}{\frac{(wz)^3}{(w-2)(z-3)}}$

9. $\frac{\prod_{i=1}^{100} 2^i}{\prod_{i=2}^{100} 2^i}$

10. $\sum_{i=1}^{N} (5^i - 5^{i-1})$

2 Exponents and Logarithms

Simplify the following expressions:

1. $x^2x^5 + x^4x^3$

2. $\dfrac{x^8}{(x^4)^2}$

3. $\dfrac{x^8}{(x^8)^4}$

4. $\sqrt[3]{1000}$

5. $\sqrt[6]{1000000}$

6. $\sqrt[3]{1000000}$

7. $\log\left(2x^3 5x^8\right)$

8. $\log\left(\dfrac{x-1}{10x}\right)$

9. $5\log(x) - \log(x^4)$

10. $\log_4(16)$

Show that

$$ln\Big(\prod_{i=1}^{N}\frac{1}{\sqrt{2\pi}}e^{\frac{-(x_i-\mu)^2}{2}}\Big)$$

is equal to

$$-\frac{ln(2\pi)N}{2}-.5\sum_{i=1}^{N}(x_i-\mu)^2$$

Show that

$$\prod_{i=1}^{n} ae^{x_i}$$

is equal to

$$e^{\sum x_i + nln(a)}$$

Now consider an applied problem.

The Cobb-Douglas production function relates labor (L) and capital (K) to production (Y), such that $Y = AK^{\beta}L^{\alpha}$. (The usefulness of such functions extends beyond economics; for example, Butler (2014) utilizes a Cobb-Douglas function when studying Congressional representation). Consider that regression equations are often specified in a form such as

$$Y = \beta_0 + \beta_1 x_1 + \ldots + \beta_k x_k + \varepsilon$$

where Y is the outcome, β_0 is the intercept, $\beta_1, \ldots, \beta_k$ are coefficients, $x_1, \ldots, x_k$ are the independent variables, and ε is an error term. Without worrying about the error term, manipulate the Cobb-Douglas production function so that it is in such a form, where β and α are the coefficients.

(Hint: A variable in a regression may actually be a "transformed" variable; for example, for various reasons a researcher with one independent variable x_1 may choose to estimate an effect β_1 using $Y = \beta_0 + \beta_1\sqrt{x_1}$ rather than $Y = \beta_0 + \beta_1 x_1$, though you should note the coefficient's interpretation is changed.)