NAME:

For the following exercises, read the problems carefully and show all your work. Attach more pages if necessary. Avoid using a calculator or the computer to solve the exercises. Please, turn in ONE pdf.

1 Extrema

For each of the following functions, find any critical points and classify them as maxima, minima, or neither.

1. $f(x) = \frac{1}{3}x^3$

2. $f(x) = \dfrac{x}{e^x}$

3. $f(x) = \dfrac{x^2 - 1}{x - 1}$

4. $f(x) = x^2(x - 1)$

5. $f(x) = e^{2x} + 3e^{-4x}$

6. $f(x) = xe^{2x}$

7. $f(x) = \ln((3x-1)^2)$

8. $f(x) = \dfrac{5^x}{5}$

9. $f(x) = (1+x^2)^3$

10. $f(x) = h(g(x))$, where $h(x) = \ln(x)$ and $g(x) = x^2$

2 Concavity

For each of the functions from the previous section, identify the intervals on which the function is concave up, and the intervals on which it is concave down.

1.

2.

3.

4.

5.

6.

7.

8.

9.

10.

3 L'Hospital's Rule

Find the following limits:

1. $\lim_{x \to 9} \dfrac{x - 9}{\sqrt{x} - 3}$

2. $\lim_{x \to 0} \dfrac{8^x - 4^x}{x^3 - x^2 - x}$

3. $\lim_{x \to \infty} \frac{e^{2x}}{2x + 144}$

4. $\lim_{x \to \infty} \frac{2 + \ln(x)}{x^2 + 3}$

5. $\lim_{x \to 0} xln(x) - x$

4 An Applied Problem

Political scientists often employ rational choice theory to study politics. Political actors such as legislators are assumed to have goals, and to choose actions designed to achieve them. This is operationalized by defining an actor's *utility functions* and *feasible actions*, and determining which feasible action maximizes her utility.

For example, say a legislator i's utility function u was defined by $u_i(c) = v - c^2$, where v is the legislator's vote share in an election, and c is the portion of her wealth the legislator spent on the campaign. That is, the legislator gains utility from gaining votes, but loses utility from spending her wealth to get them. Now say vote share was determined entirely by campaign spending such that $v = c$; what level of campain spending maximizes the legislator's utility?