Koç CSSM Bootcamp 2021 **Problem Set 3**

NAME:

For the following exercises, read the problems carefully and show all your work. Attach more pages if necessary. Avoid using a calculator or the computer to solve the exercises. Please, submit ONE pdf.

Consider the following matrices:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{B} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \mathbf{C} = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \mathbf{D} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \mathbf{E} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 5 \\ 0 & 0 & 8 \end{bmatrix}$$

1. For each of them, identify whether the matrix is: square, symmetric, triangular, idempotent, identity, $\mathbf{J}$, $\mathbf{0}$, or none of the above.

2. Calculate $\text{Tr}(\mathbf{A})$

3. Calculate $5(\text{Tr}(\mathbf{B}) + \text{Tr}(\mathbf{E}))$

Now consider the following matrices:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 5 \\ 1 & -2 & -1 \\ 5 & -1 & 2 \end{bmatrix} \mathbf{B} = \begin{bmatrix} 4 & 2 \\ 6 & 3 \end{bmatrix} \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{D} = \begin{bmatrix} 1 & 1 \\ 3 & -2 \end{bmatrix} \mathbf{E} = \begin{bmatrix} 0 & 1 & 2 \\ 5 & 1 & -1 \\ 2 & 4 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

4. Is $\mathbf{E}'\mathbf{E}$ square, symmetric, triangular, idempotent, identity, $\mathbf{J}$, $\mathbf{0}$, or none of the above?

5. For each of $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$, and $\mathbf{E}$, find the trace.

Invert the following matrices or give a reason why you cannot:

6. $\begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}$

7. $\begin{bmatrix} -1 & 3 \\ -2 & 6 \end{bmatrix}$

8. $\begin{bmatrix} 9 & 5 & 6 \\ 8 & 2 & 7 \\ 4 & 3 & 1 \end{bmatrix}$

9. $\begin{bmatrix} 11 & 3 & 5 \\ 3 & 2 & 19 \\ 0 & 0 & 0 \end{bmatrix}$

10. $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, assuming $ad \neq bc$

11. $\begin{bmatrix} 3 & 8 & 6 \\ 0 & -3 & -5 \\ -9 & 0 & 4 \end{bmatrix}$

Provide a solution to the following systems of linear equations, or explain why you cannot:

12.

$$\begin{aligned} -3x + 5y + 5z &= -43 \\ x - 4y - 2z &= 31 \\ 3x - 4z &= 7 \end{aligned}$$

13.

$$\begin{aligned} x - 2y - z &= -15 \\ -x - y + z &= -6 \\ x - 6y - z &= -43 \end{aligned}$$

An Applied Problem

Often political scientists use linear regression to study political phenomena. We may think about this method using matrix algebra. Essentially, the researcher has a vector $\mathbf{y}$ of outcome observations, and a matrix $\mathbf{X}$ of explanatory variable observations. That is, each element of $\mathbf{y}$ is the observed oucome for the corresponding row of obvservations for the independent variables in $\mathbf{X}$. The researcher's goal is to find the vector $\mathbf{b}$ of coefficients; that is, we assume that each explanatory variable has a linear effect on the outcome such that in expectation $\mathbf{y} = \mathbf{Xb}$. Given the equation $\mathbf{y} = \mathbf{Xb}$, find the formula for $\mathbf{b}$. Be sure to account for the fact that $\mathbf{X}$ may not be a square matrix. Are there any situations where the formula you found will not work?