Basic notation and algebra

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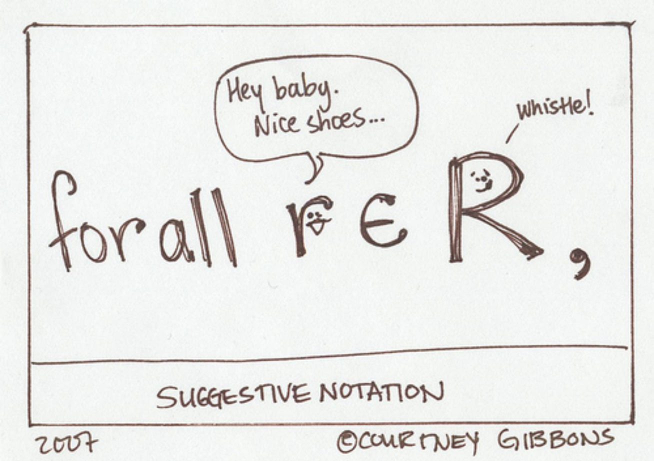
Notation and Algebra



Common math notation

- a, b, c, d: Real numbers
  - Examples: 4, $\sqrt{2}$, $\frac{2}{3}$, 3.14159265
  - The set of real numbers is denoted $\mathbf{R}$ or $\mathbf{R}^1$ and includes any number ranging from $-\infty$ to $+\infty$.
  - You will often see the expression $a \in \mathbf{R}^1$, which means that a is a real number. More correctly, it means that a is in the set of real numbers.

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- i, j, k, l: Integers (whole numbers)
  - Examples: ..., -3, -2, -1, 0, 1, 2, 3, ...
  - The set of integers is denoted $\mathcal{I}$. Positive integers are denoted $\mathcal{I}^+$. Negative integers are $\mathcal{I}^-$.

- $x, \ y, \ z$: Variables that can take on varying values.
  - $f, \ g, \ h$: Functions of some variable (e.g., $f(x)$)
  - n: Commonly denotes some non-specified positive integer. Often it represents the sample size.
- These are often used in combination:
  - Indexing: $a_1 + a_2 + \ldots + a_i$
  - Functions: $f(x) = a + bx$

- Special combinations:
  - Summation: $\sum_{i=1}^{n} x_i = x_1 + x_2 + \ldots + x_n$
  - $\displaystyle\sum_{i=1}^{n} cx_i = c \sum_{i=1}^{n} x_i$
  - $\displaystyle\sum_{i=1}^{n} (x_i + y_i) = \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} y_i$
  - $\displaystyle\sum_{i=1}^{n} c = nc$
- Product: $\prod_{i=1}^{n} x_i = x_1 \times x_2 \times \ldots \times x_n$
  - $\displaystyle\prod_{i=1}^{n} cx_i = c^n \prod_{i=1}^{n} x_i$
  - $\displaystyle\prod_{i=1}^{n} (x_i + y_i) = \text{a mess}$
  - $\displaystyle\prod_{i=1}^{n} c = c^n$

Exponents

- $a^3 = a \times a \times a$: “a to the power of 3” or”a to the 3rd.”
- $a^n = a \times a \times \ldots \times a = \prod_{i=1}^{n} a$

Some basic rules that apply at all times

- $a^1 = a$
- $a^0 = 1$
- $(a^k)^l = a^{kl}$
- $(ab)^k = a^k \times b^k$
- $(\frac{a}{b})^k = \frac{a^k}{b^k}$
- $a^{-1} = \frac{1}{a}$
- $a^{\frac{1}{2}} = \sqrt{a}$
- $\sqrt[k]{a} = a^{\frac{1}{k}}$

Rules that are true only when $a = b$ (have the same "base")

- $a^k \times b^l = a^{k+l} = b^{k+l}$
- $\frac{a^k}{b^l} = a^{k-l} = b^{k-l}$

Order of operations

- **P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally
- **P**arentheses **E**xponents **M**ultiplication **D**ivision **A**ddition **S**ubtraction
- Example:

$$((1+2)^3)^2 \times 2 + 5 = (3^3)^2 \times 2 + 5$$

$$= (27)^2 \times 2 + 5 = 729 \times 2 + 5 = 1458 + 5 = 1463$$

Drill: DROP AND GIVE ME 20!

Solve the following equations:

1. $x^1 =$ _____
2. $-a \times (-b)^2 =$ _____
3. $\sum_{i=1}^{4} i =$ _____
4. $\prod_{m=6}^{9} m =$ _____
5. $4! =$ _____
6. $9^{1/2} =$ _____
7. $27^{1/3} =$ _____
8. $(\frac{3(2-4)}{2+3})^3 =$ _____
9. $\frac{4!}{3!} =$ _____
10. $\frac{2/7}{3/8} =$ _____
11. $5 \times (3 - 4 \div 2) + 6 \div 2 - 7 \times 0 =$ ____
12. $\Omega^0 =$ ____

Simplify the following equations

1. $\sqrt[3]{x}\sqrt[5]{x}$
2. $(xy)^3x^2$
3. $\frac{(x^2/y)^3x^{-2}}{xy^2}$
4. $\frac{5+17x+4x+7}{42x}$
5. $\frac{2g+13}{3g}+\frac{4g-5}{4g}$
6. $\frac{\frac{w^3z^4}{(w+1)(z-3)}}{\frac{(wz)^3}{(w-2)(z-3)}}$
7. $\frac{\prod_{i=1}^{100}2^i}{\prod_{i=2}^{100}2^i}$
8. $\sum_{i=1}^{N}(5^i-5^{i-1})$

Logarithms

- Logarithms are the power required to raise a base to a given number:
$$y = log_a(x) \implies a^y = x$$
- "To what power should I raise a (the base) to get x?"
- It is helpful to think of the log as the inverse of exponential functions.

$$log_a(a^x) = x$$
$$a^{log_a(x)} = x$$

Common logs

- In almost all cases you will see logarithms are base 10 or base e, where e is **euler's constant**.
- Base 10: $b = log_{10}(a) \iff 10^b = a$
  - The base 10 logarithm is often simply written as "$log(x)$" with no base denoted.
- Base e: $y = log_e(x) \iff e^y = x$
  - The base e logarithm is referred to as the "natural" logarithm and is usually written as $ln(x)$.
- In statistics, you will almost always be working with $ln(x)$.

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Basic rules

- $log(1/x) = -log(x)$
- $log(x/y) = log(x) - log(y)$
- $log(x^y) = ylog(x)$
- $log(1) = 0$
- You can switch bases as necessary using the following equation:

$$log_b(x) = \frac{log_a(x)}{log_a(b)}$$

- If you see a product of an exponent, you might want to use a log to change it into sums:

$$ln(\prod_{i=1}^{n} ae^{x_i}) = \sum_{i=1}^{n}(ln(e^{x_i}) + ln(a)) = \sum_{i=1}^{n} x_i + nln(a)$$

$$\implies \prod_{i=1}^{n} ae^{x_i} = e^{\sum x_i + nln(a)}$$

Euler's constant

- Euler's constant is denoted e and is equal to $2.71828\ldots$.
- Like π, it appears in a surprising number of places in math, probability, and statistics.
- It is the only number such that the derivative of e^x is equal to itself.
- $e = \lim\limits_{x\to\infty}(1 + \frac{1}{x})^x$
- $e = \sum_{x=0}^{\infty} \frac{1}{x!}$

Drill

Simplify the following:

1. $$\log_{16}(16^8)$$
2. $$log_5 125$$
3. $$log_8 1$$
4. $$ln(e) + ln(e)$$

5.

$$ln(2) + ln(1/2)$$

6.

$$e^{ln(3)} e^{ln(2)}$$

7.

$$ln(\prod_{i=1}^{N}(2e^{a_i}))$$