NAME: ANSWER KEY

For the following exercises, read the problems carefully and show all your work. Attach more pages if necessary. Avoid using a calculator or the computer to solve the exercises. Please, staple your homework.

1 Order of Operations, Summation, Products, and Factors

Simplify the following expressions:

1. $(3 * 4)/(3 - 2) + ((4 + 3)/7)$

$$12/1 + (7/7)$$
$$12/1 + 1$$
$$12 + 1$$
$$13$$

2. $(3 * 4)/(3 - 2) - ((4 + 3)/7)((2 + 10)/3)$

$$12/1 - (7/7)(12/3)$$
$$12/1 - (1 * 4)$$
$$12 - 4$$
$$8$$

3. $\sum_{i=1}^{3} 9 + \sqrt{9^i}$

$$\sum_{i=1}^{3} 9 + \sum_{i=1}^{3} \sqrt{9^i}$$
$$9 * 3 + \sum_{i=1}^{3} \sqrt{9^i}$$
$$27 + \sqrt{9} + \sqrt{9^2} + \sqrt{9^3}$$
$$27 + 3 + 9 + \sqrt{9^2} * \sqrt{9}$$
$$27 + 3 + 9 + 27$$
$$66$$

4. $\sum_{i=1}^{10} 9 + 9i$

$$\sum_{i=1}^{10} 9 + \sum_{i=1}^{10} 9i$$
$$9 * 10 + 9 * \sum_{i=1}^{10} i$$
$$90 + 9 * (10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1)$$
$$90 + 9 * (55)$$
$$90 + 495$$
$$585$$

5. $\prod_{x=1}^{5} 2x$

$$2^5 \prod_{x=1}^{5} x$$
$$32 * 5!$$
$$32 * 120$$
$$3840$$

6. $\sum_{i=1}^{n} i$

   Since $\sum_{i=1}^{n} i = 1 + 2 + \ldots + n$, equivalently $\sum_{i=1}^{n} i = n + (n-1) + \ldots + 1$.
   Then $2\sum_{i=1}^{n} i = n * (n+1)$, so

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

2 Exponents and Logarithms

Simplify the following expressions:

1. $x^2x^5 + x^4x^3$

   $x^7 + x^7$

   $2x^7$

2. $\dfrac{x^8}{(x^4)^2}$

$$\frac{x^8}{x^8}$$

$$1$$

3. $\dfrac{x^8}{(x^8)^4}$

$$\frac{x^8}{x^{32}}$$

$$x^{-24} \text{ or } 1/x^{24}$$

4. $\sqrt[3]{1000}$

$$10$$

5. $\sqrt[6]{1000000}$

$$10/-10$$

6. $\sqrt[3]{1000000}$

$$100$$

7. $\log\left(2x^3 5x^8\right)$

$$\log\left(10x^3x^8\right)$$
$$\log(10) + \log\left(x^{11}\right)$$
$$1 + 11\log(x)$$

8. $\log\left(\dfrac{x-1}{10x}\right)$

$$\log(x-1) - \log(10x)$$
$$\log(x-1) - (\log(10) + \log(x))$$
$$\log(x-1) - \log(x) - 1$$

9. $5\log(x) - \log\left(x^4\right)$

$$\log\left(x^5\right) - \log\left(x^4\right)$$

$$\log\left(\frac{x^5}{x^4}\right)$$

$$\log\left(x\right)$$

10. $\log_4(16)$

$$\log_4(4^2)$$

$$2\log_4(4)$$

$$2$$

Show that

$$ln\Big(\prod_{i=1}^{N}\frac{1}{\sqrt{2\pi}}e^{\frac{-(x_i-\mu)^2}{2}}\Big)$$

is equal to

$$-\frac{ln(2\pi)N}{2} - .5\sum_{i=1}^{N}(x_i-\mu)^2$$

$$\begin{aligned}
ln\Big(\prod_{i=1}^{N}\frac{1}{\sqrt{2\pi}}e^{\frac{-(x_i-\mu)^2}{2}}\Big) &= \sum_{1}^{N}\ln\left(\frac{1}{\sqrt{2\pi}}e^{\frac{-(x_i-\mu)^2}{2}}\right) \\
&= \sum_{1}^{N}\ln((2\pi)^{-1/2}) + \ln\left(e^{\frac{-(x_i-\mu)^2}{2}}\right) \\
&= \sum_{1}^{N} -0.5\ln(2\pi) - 0.5(x_i-\mu)^2 \\
&= -\frac{\ln(2\pi)N}{2} - 0.5\sum_{1}^{N}(x_i-\mu)^2
\end{aligned}$$

Now consider an applied problem.

The Cobb-Douglas production function relates labor (L) and capital (K) to production (Y), such that $Y = AK^{\beta}L^{\alpha}$. (The usefulness of such functions extends beyond economics; for example, Butler (2014) utilizes a Cobb-Douglas function when studying Congressional representation). Consider that regression equations are often specified in a form such as

$$Y = \beta_0 + \beta_1 x_1 + \ldots + \beta_k x_k + \varepsilon$$

where Y is the outcome, β_0 is the intercept, $\beta_1, \ldots, \beta_k$ are coefficients, $x_1, \ldots, x_k$ are the independent variables, and ε is an error term. Without worrying about the error term, manipulate the Cobb-Douglas production function so that it is in such a form, where β and α are the coefficients.

(Hint: A variable in a regression may actually be a "transformed" variable; for example, for various reasons a researcher with one independent variable x_1 may choose to estimate an effect β_1 using $Y = \beta_0 + \beta_1\sqrt{x_1}$ rather than $Y = \beta_0 + \beta_1 x_1$, though you should note the coefficient's interpretation is changed.)

$$\begin{aligned}
\log(Y) &= \log\left(AK^{\beta}L^{\alpha}\right) \\
\log(Y) &= \log(A) + \log\left(K^{\beta}\right) + \log(L^{\alpha}) \\
\log(Y) &= \log(A) + \beta\log(K) + \alpha\log(L)
\end{aligned}$$