Sets and functions

David Carlson

2021

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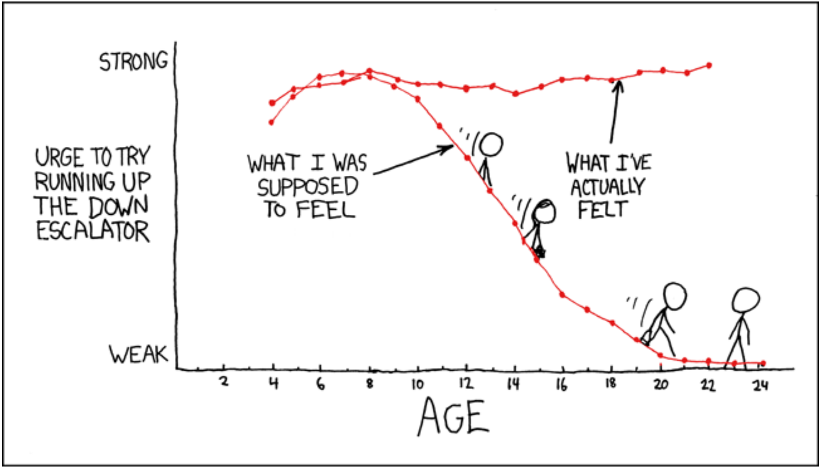

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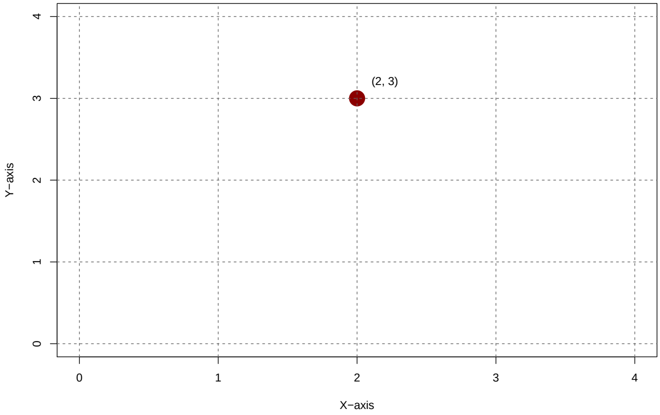
- The plot has two axes:
  - x-axis is the horizontal axis at the pottom of the plot
  - y-axis is the verticle line on the left hand of the plot
- To plot some point (x_i, y_i):
  - Move along the x-axis to the point x_i from zero
  - Move up or down to the position of y_i

Example

```r
plot(x=2, y=3, xlim=c(0, 4), ylim=c(0,4), pch=19, cex=3
     , xlab="X-axis", ylab="Y-axis", col="darkred")
abline(h=c(0,1,2,3,4),v=c(0,1,2,3,4), lty=2, col="gray40")
text(x=2.2, y=3.2, labels="(2, 3)", cex=1)
```



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- The intercept is the value of the function where $x = 0$. In other words, it is the point where the line hits the y-axis.

Example

$[2, 1], [3, 5]$

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$1 = 4(2) + b \iff b = 1 - 8 = -7$

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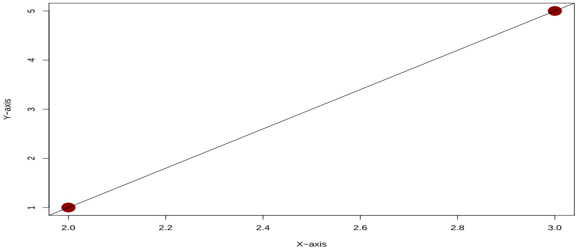
     , xlab="X-axis", ylab="Y-axis", col="darkred")
abline(a=-7, b=4)
```



Drill

1. Find the line that goes through the points $[-3, 2], [4, 20]$
2. Solve for x: $3x - 4 = \frac{2x}{2}$
3. Solve for x: $(1 - x^2) = -6$
4. What are the coordinates for the point on the line $y = 3.4x - 8$ where $x = 14$?
5. Solve for x: $5x - 7 = 0$
6. Solve for x: $-6x - 17 = 0$
7. Solve for x: $-6x - 17 < 0$

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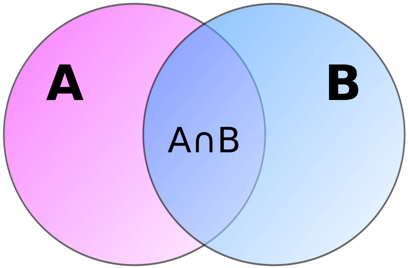
  - Empty: a set with no elements. $S = \{\}$ or $S = \emptyset$

Set operations



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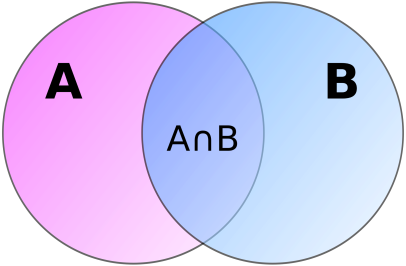

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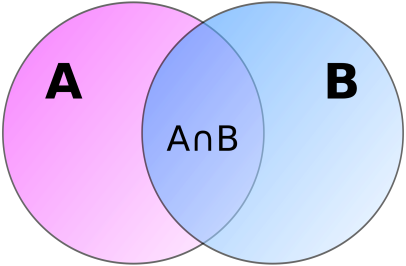

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- **Complement**: If set A is a subset of S, then the complement of A, denoted A^C, is the set containing all of the elements in S that are not in A. Sometimes notated as $\tilde{A}$.

Drill!

Draw the following:

1. $A \cap B$
2. $\tilde{A} \cap C$
3. $(A \cap B) \cup (\tilde{A} \cap C)$
4. $(A \cap \tilde{B}) \cup (\tilde{A} \cap C)$
5. $(A \cap \tilde{B}) \cap (\tilde{A} \cup C)$

Note: There are many ways to denote the complement. Another common one would be A'.

6. Give examples of $A \cap \tilde{B}$ where A is the set of all vegetables and B is the set of green foods
7. Given $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{2, 4, 6, 8\}$, $B = \{1, 3, 4, 5, 7\}, C = \{7, 8\}$, find $A \cap B$.
8. $\tilde{B}$ (in U)
9. $\tilde{A} \cup \tilde{B}$ (in U)
10. $(A \cap C) \cup (A \cap B)$

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- Points in $\mathbf{R}^n$ are ordered n-tuples, where each element of the n-tuple represents the coordinate along that dimension.

Interval notation

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- Half open, half closed:

$$(a, b] \equiv \{x \in \mathbf{R}^1 : a < x \leq b\}$$

Neighborhoods: Intervals, Disks, and Balls

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Advanced set vocabulary

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Example: The interior of the set $\{(x, y) : x^2 + y^2 \leq 4\}$ is $\{(x, y) : x^2 + y^2 < 4\}$.

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- Note: a set may be neither open nor closed.

Example: $\{(x, y) : 2 < x^2 + y^2 \leq 4\}$

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- **Compact**: A set is compact if and only if it is both closed and bounded.

DRILL!

Create a graph for the following sets:

1. $(2, 6]$
2. $[-2, 3) \cup [5, \infty)$

Express the following in plain English (note that $\mathbf{Z}$ is the set of all integers):

3. $[5, 7)$
4. $(3, \infty)$
5. $\{x \in \mathbf{Z} : x \geq 2\}$
6. $\{y \in \mathbf{R} : y \in \mathbf{Z}\}$

Express the following in math notation:

7. The set of real numbers greater than or equal to -5 and less than 4.
8. The set of real numbers greater than 12.
9. The set of real numbers that are divisible by 3 (where when I divide by 3 I will get an integer).
10. The set of numbers that solve the equation $y = 4x + 24$

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    For each x in $\mathbf{R}^1$, $f(x)$ assigns the number $x + 1$.
  - $$f(x, y) = x^2 + y^2$$
    For each ordered pair (x, y) in $\mathbf{R}^2$, $f(x, y)$ assigns the number $x^2 + y^2$.

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- Most often used when talking about a function $f : \mathbf{R}^1 \to \mathbf{R}^1$.

Example 1

$$f(x) = \begin{cases} x+1, & 1 \le x \le 2 \\ 0, & x = 0 \\ 1-x & -2 \le x \le -1 \end{cases}$$

Domain

$$X = [-2,-1] \cup \{0\} \cup [1,2]$$

Range

$$f(X) = [2,3] \cup \{0\}$$

Example 2

$$f(x) = 1/x$$

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Example 3

$$f(x, y) = x^2 + y^2$$

Domain

$$X = \mathbf{R}^2$$

Range

$$f(X, Y) = \mathbf{R}^1_+$$

DRILL!

Create a graph for the following functions (rough it out):

1. $f(x) = 3 + 2x$
2. $f(x) = 2x^2 + 3x - 4$

3. Consider the following funciton:

$$h(x) = \frac{ln(x-3)}{\sqrt{5-x}}$$

In set notation express the domain of $h(x)$.

4. For what value of x is $h(x) = 0$?
5. Does the range of $h(x)$ span all of $\mathbf{R}^1$?

Function vocabulary

- **Monomials**: $f(x) = ax^k$; a is the *coefficient*. k is the *degree*.

Examples: $y = x^2$, $y = -\frac{1}{2}x^3$

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- **Polynomials**: sum of monomials.

Examples: $y = -\frac{1}{2}x^3 + x^2$, $y = 3x + 5$
The *degree* of a polynomial is the highest degree of its monomial terms.

Function vocabulary

- **Monomials**: $f(x) = ax^k$; a is the *coefficient*. k is the *degree*.

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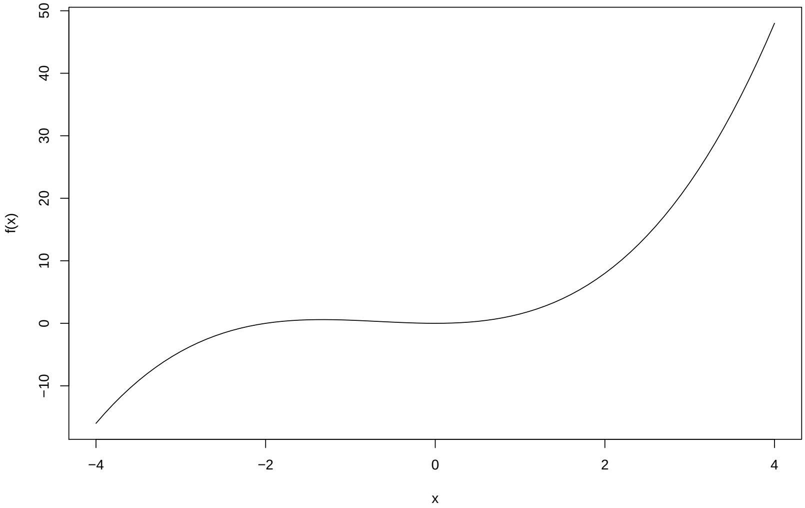
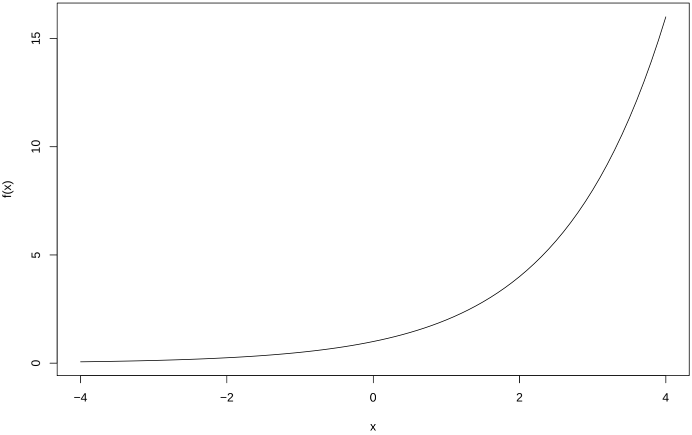
The *degree* of a polynomial is the highest degree of its monomial terms. Also, it's often a good idea to write polynomials with terms in decreasing degree.

```r
x<-seq(-4, 4, by=.1); y<-.5*x^3+x^2
plot(x, y, xlab="x", ylab="f(x)", type="l")
```



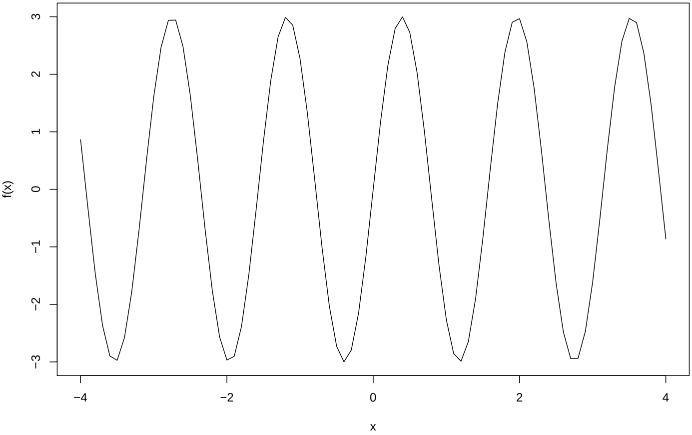
- Exponential Functions: Example: $y = 2^x$

```
x<-seq(-4, 4, by=.1); y<-2^x
plot(x, y, xlab="x", ylab="f(x)", type="l")
```



- Trigonometric Functions: Examples: $y = \cos(x)$, $y = 3\sin(4x)$

```
x<-seq(-4, 4, by=.1); y<-3*sin(4*x)
plot(x, y, xlab="x", ylab="f(x)", type="l")
```



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- Always, always, always, graph your function.

Inverse Functions

- Sometimes we're given a function $y = f(x)$ and we want to find how x varies as a function of y. That is, we are trying to "solve for" x.

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- If f is a one-to-one mapping, then it has an inverse. This will usually be donoted as $f^{-1}(x)$. That means that $f^{-1}(f(x)) = x$.
- A good way to do this is to use algebra to isolate x (your independent variable in $f(x)$) on one side of the equation.

Example 1: Find the inverse functions

$$f(x) = 3x + 2$$

Step 1: Solve for x

$$y = 3x + 2$$

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- Sometimes it's impossible to figure out what x went into the function just from the output $f(x)$.
- An undefined inverse is especially likely in non-linear functions.
- Imagine we have the function $f(x) = x^2$ (a parabola).
- Solving for x, we get $x = \sqrt{y}$ and $x = -\sqrt{y}$ — for each value of y, there are two values of x.

Roots

- You are going to be spending a lot of time finding **roots** of functions: those values where $f(x) = 0$.
  - Decision theory/game theory
  - Dynamic systems
  - Maximum likelihood

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- You are going to be spending a lot of time finding **roots** of functions: those values where $f(x) = 0$.
  - Decision theory/game theory
  - Dynamic systems
  - Maximum likelihood
- Proceedure:
  - Given $y = f(x)$
  - set $y = 0$.
  - Solve for x.

Finding the y-intercepts

- You already found the root of one function when you calculuated the **y-intercept** of a line.
- Where does the line $f(x) = a + bx$ cross the y-axis?

$$a + bx = 0$$

$$a = -bx$$

$$x = -\frac{a}{b}$$

The quadratic equation

- What is the root of $f(x) = ax^2 + bx + c$

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- What is the root of $f(x) = ax^2 + bx + c$
- The **quadratic equation** is the solution to this question that most of us have been forced to memorize at some point.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- These numbers are the two roots of $f(x)$.

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- Example

$$f(x) = x^2 + 3x - 4 = 0$$

$$(x-1)(x+4) = 0$$

$$x = \{1, -4\}$$

- **FOIL** (**F**irst **O**utside **I**nside **L**ast)
- The middle term (e.g., $3x$) is the sum of the constants. The final term is the product.

Example

$$x(x-1) = 6$$
$$x^2 - x - 6 = 0$$
$$(x-3)(x+2) = 0$$

Solution are $x = -2$ and $x = 3$.

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- **Example**

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$$(x^2 - 6x + 9) - 4 = 0$$
$$(x - 3)^2 - 4 = 0$$

$$(x - 3)^2 = 4$$

$$x - 3 = \pm 2$$

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Your turn

Solve this formula by completing the square

$$x^2 + 8x + 6 = 0$$

DRILL! (The pain means it is working)

1. Factor $-7\theta^2 + 21\theta - 14$
2. FOIL: $(2x - 3)(5x + 7)$
3. Factor: $q^2 - 10q + 9$
4. Factor and reduce: $\frac{\beta - \alpha}{\alpha^2 - \beta^2}$
5. Solve: $15\delta + 45 - 5\delta = 36$
6. Solve: $0.30\Omega + 0.05 = 0.25$
7. Solve: $-4x^2 + 64 = 8x - 32$
8. Complete the square and solve: $x^2 + 14x - 14 = 0$
9. Complete the square and solve: $\frac{1}{3}y^2 + \frac{2}{3}y - 16 = 0$
10. Solve using the quadratic formula: $2x^2 + 5x - 7$

Solve the following formulas:

11. $5 + 11x = -3x^2$
12. $\sqrt{4x + 13} = x + 2$
13. $10^{3x^2} 10^x = 100$
14. $6x^2 - 6x - 6 = 0$
15. $5 + 11x = -3x^2$

16. Find the inverse of $f(x) = 5x - 2$
17. Simplify $h(x) = g(f(x))$, where $f(x) = x^2 + 2$ and $g(x) = \sqrt{x - 4}$.
18. Simplify $h(x) = f(g(x))$ with the same f and g. Is it the same as before?
19. Rewrite the following by taking the log of both sides. Is the result a linear function?

$$y = \alpha \times x_1^{\beta_1} \times \beta_2 x_2 \times \beta_3 x_3$$

20. Rewrite the following by taking the log of both sides. Is the result a linear function?

$$y = \alpha \times x_1^{\beta_1} \times \frac{x_2^{\beta_2}}{x_3^{\beta_3}}$$