Gaussian Processes

David Carlson

May 26, 2021

Overview

- Gaussian process regression (GPR) for time-series cross sectional (TSCS) analyses

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- GPR for prediction / projection

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- GPR for prediction / projection
- Other uses of GPs

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- Variables often violate modeling assumptions
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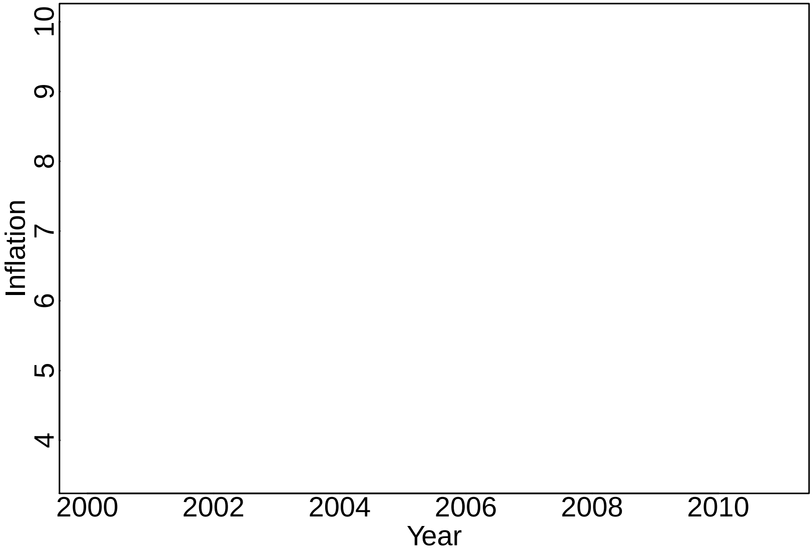
- Example: How does inflation explain anti-Americanism?

Inflation and Anti-Americanism in Mexico



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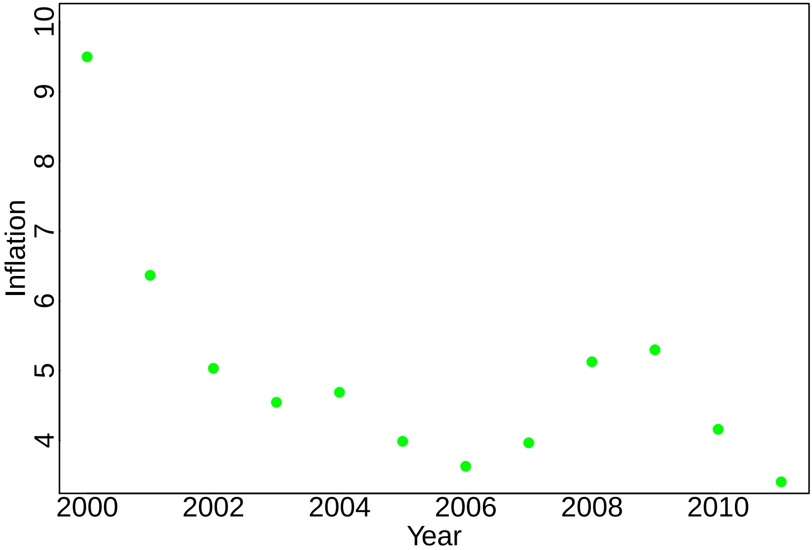

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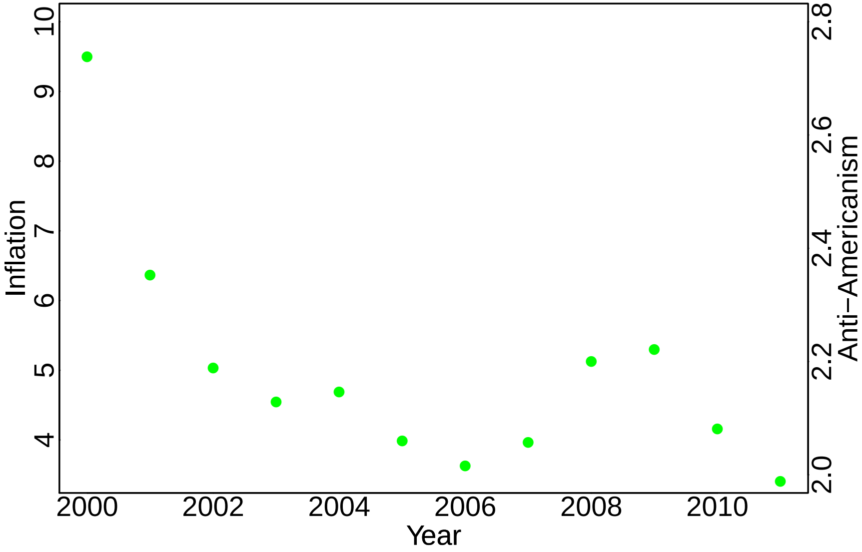

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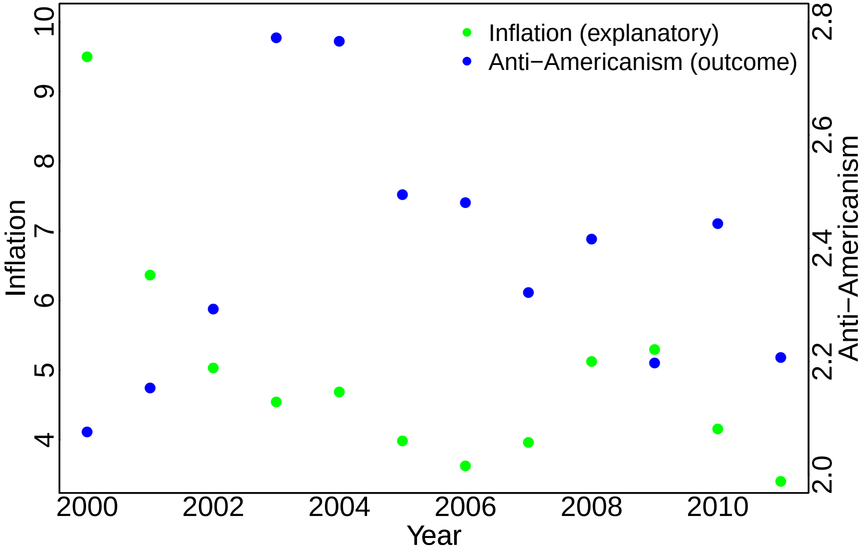

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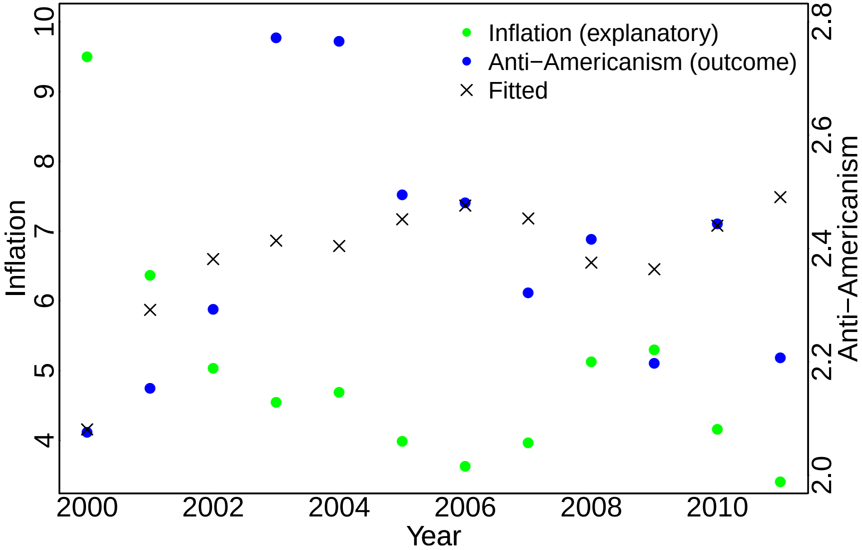

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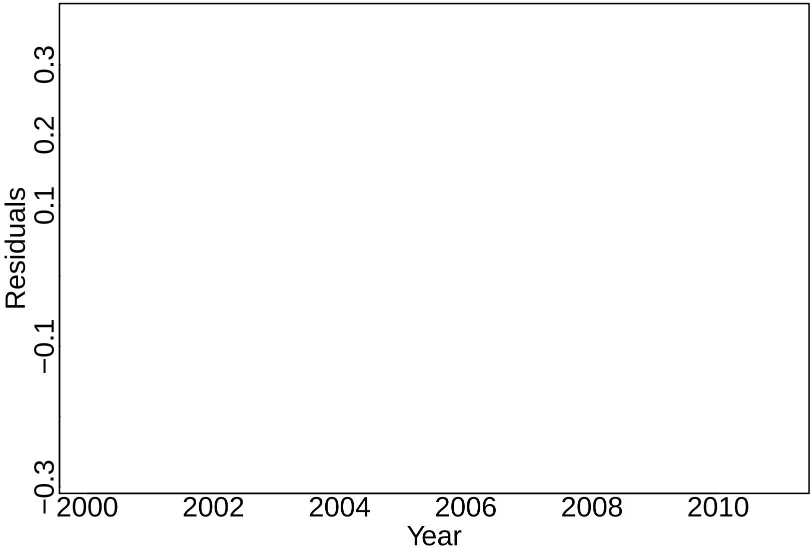

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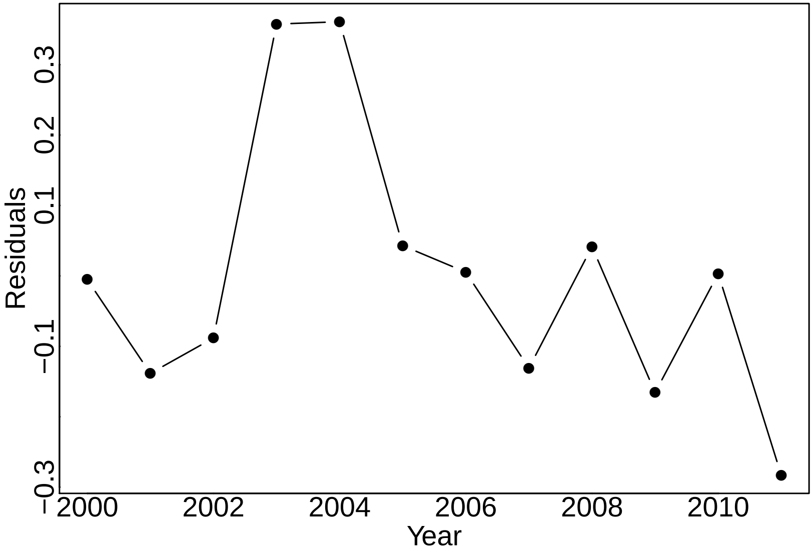

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  - Small number of units (countries)
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  - Not conditionally independent observations

Modeling TSCS Data

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- Machine learning algorithm models outcomes jointly as a process

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- Estimate variance-covariance in a learning kernel
- Very flexible, still interpretable

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- Data in mean and $\boldsymbol{\Omega}$ do not have to be the same

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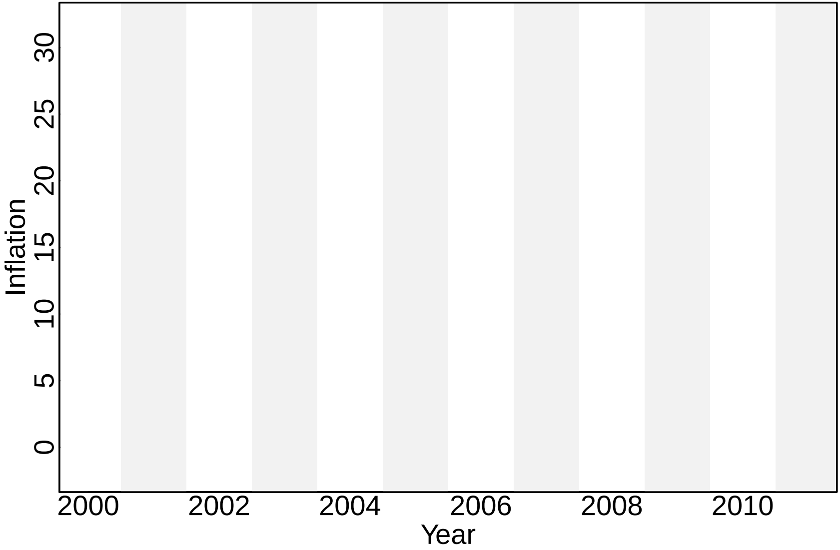
Two-way fixed effects models assume same time trends



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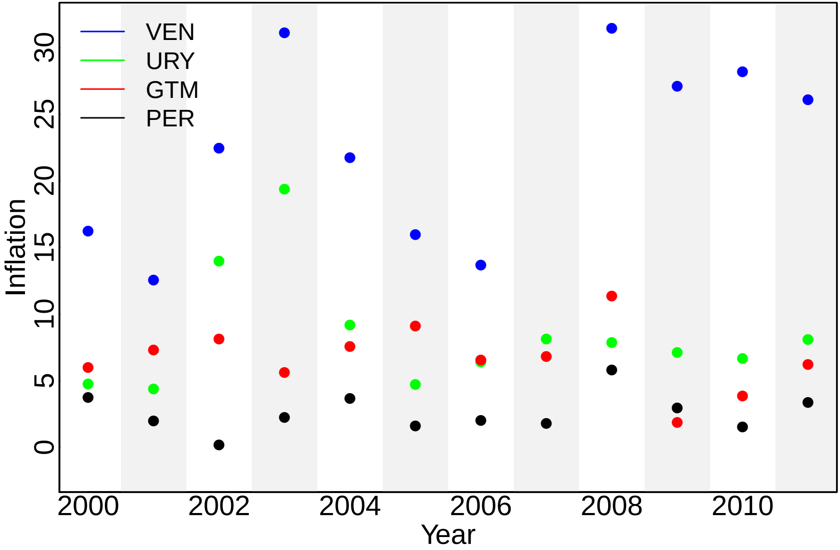

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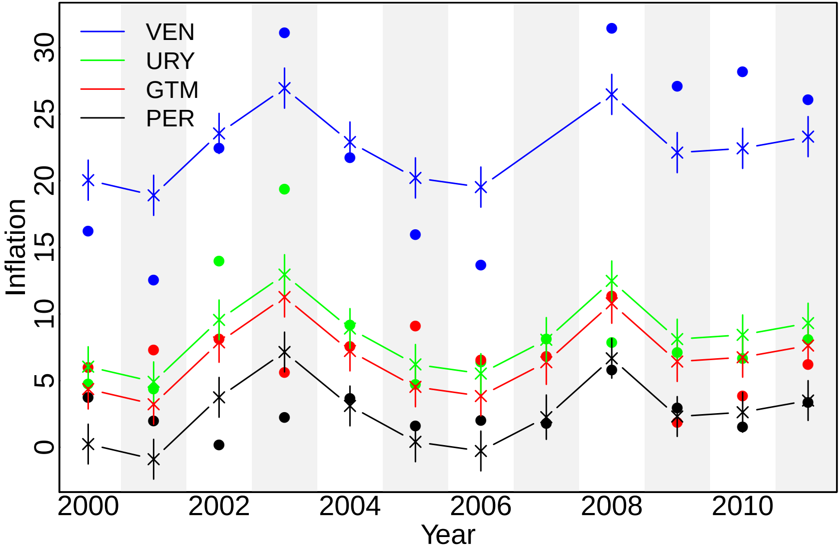

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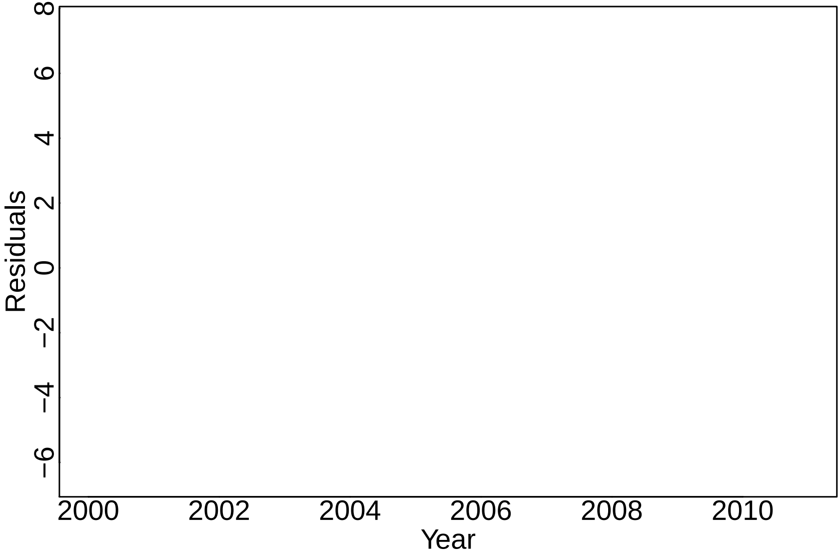

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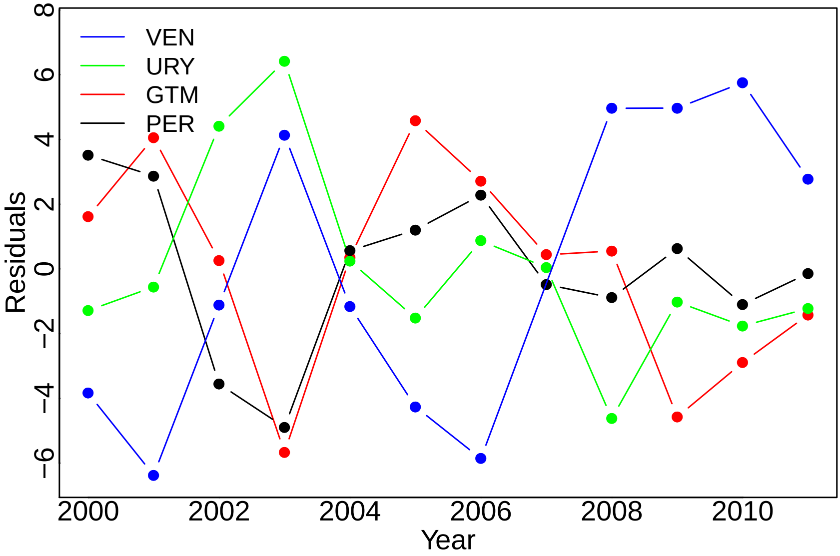

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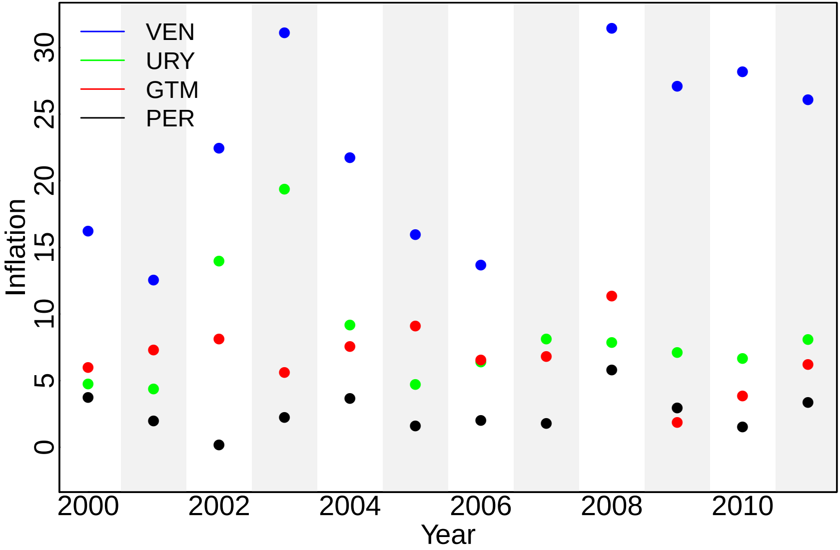

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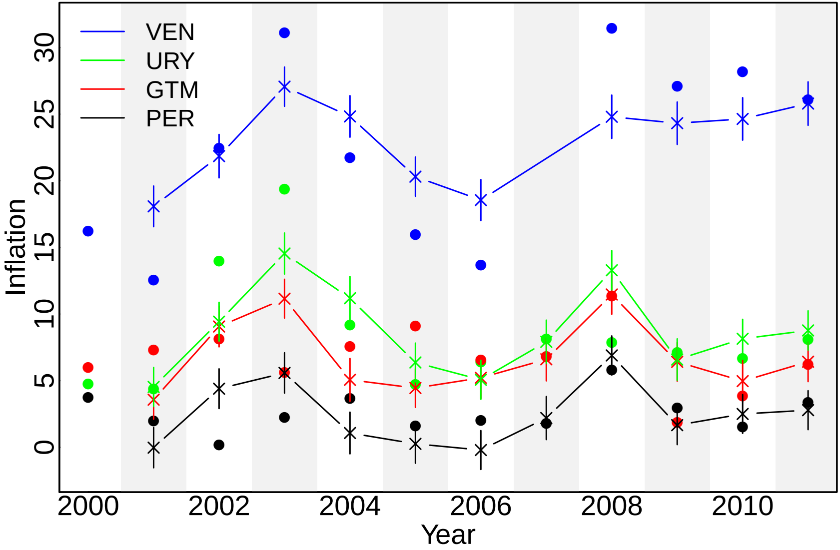

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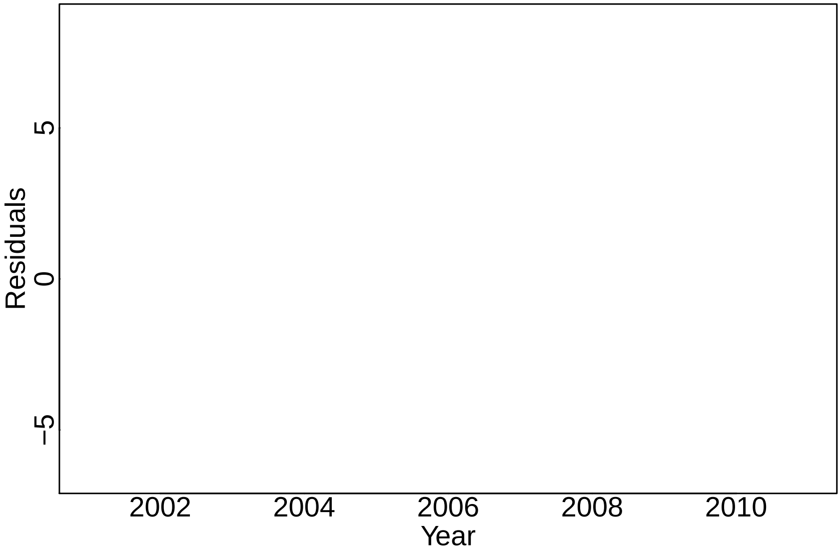

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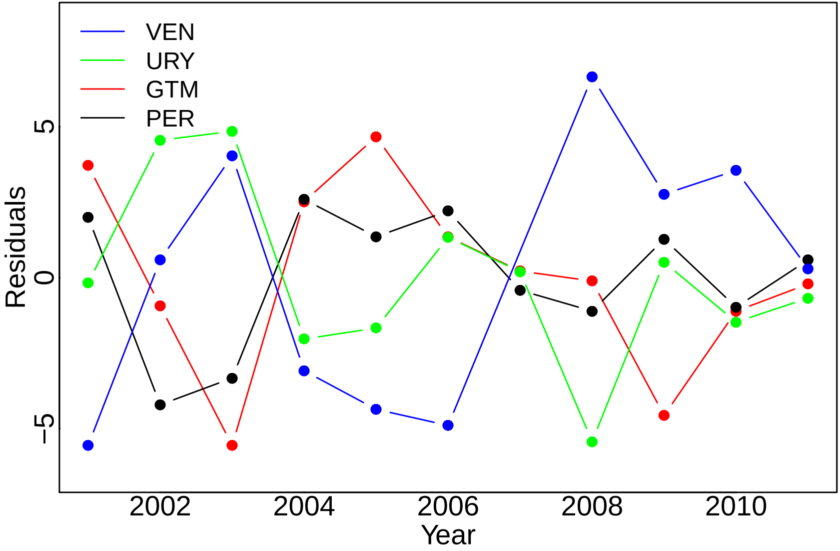

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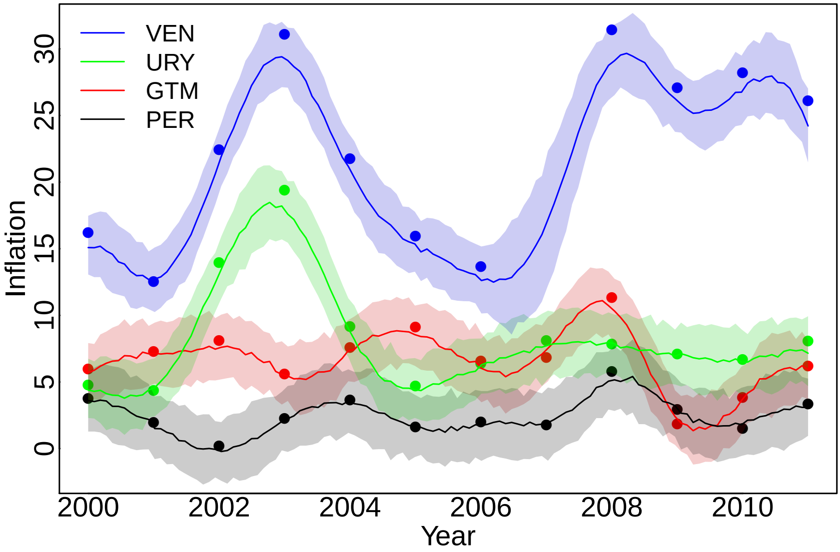
GPR allows correlated but different time trends



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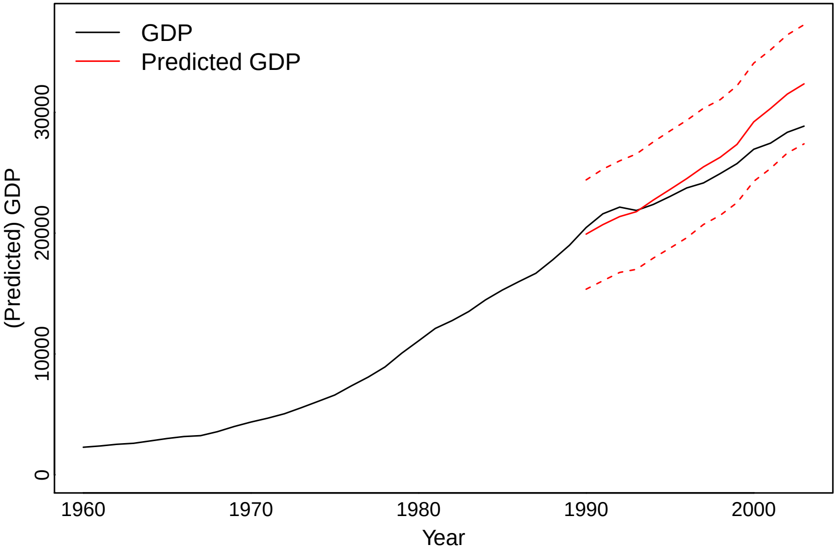
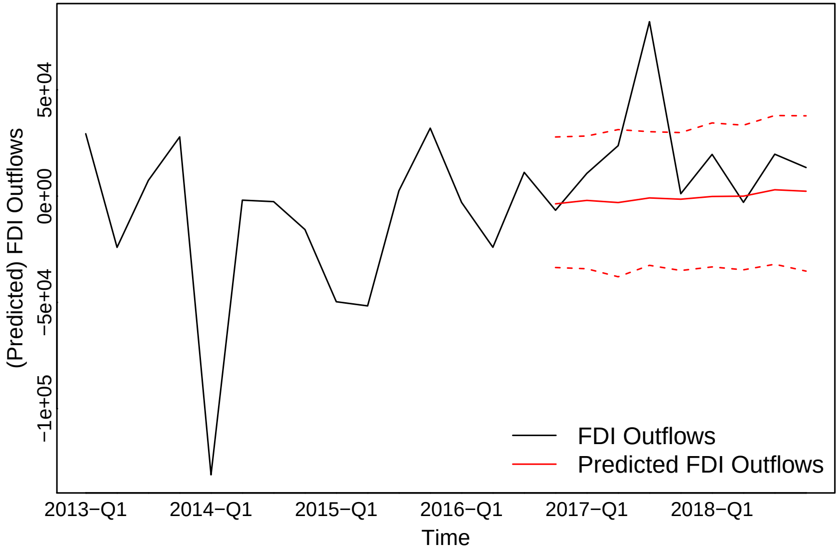
- For the applications, stationary kernel for explanatory, non-stationary kernel for outcome

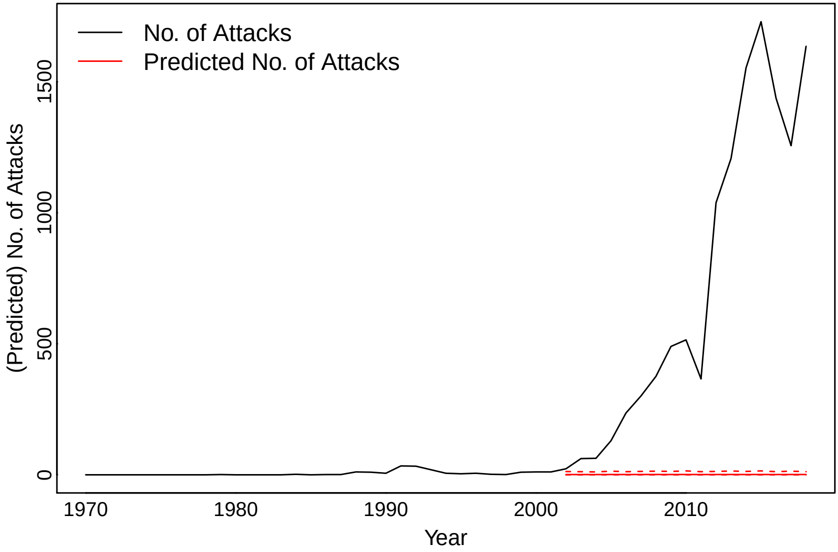
GPP Findings on Reunification



GPP Findings on UK Capital Flight



GPP Findings on Afghan Terror Attacks Following US War



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