

# Day 1 - OLS

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## Matrix Algebra

- Look at la.pdf in the folder for a simple matrix cheat sheet
- We will pretty much only deal with vectors and matrices, especially in R

```
y = c(2,4,3,2) #vector
X = matrix(c(5,3,4,5,6,5,3,2,4,5,6,3,5,5,4,2),
           nrow = 4, ncol = 4, byrow = T) #matrix (you do not need to specify both cols and rows)
y
```

```
## [1] 2 4 3 2
```

```
X
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    5    3    4    5
## [2,]    6    5    3    2
## [3,]    4    5    6    3
## [4,]    5    5    4    2
```

```
t(X) #transpose
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    5    6    4    5
## [2,]    3    5    5    5
## [3,]    4    3    6    4
## [4,]    5    2    3    2
```

```
solve(X) #inverse
```

```
##      [,1] [,2] [,3] [,4]
## [1,]   -5   30   19  -46
## [2,]    7  -42  -27   65
## [3,]   -5   29   19  -45
## [4,]    5  -28  -18   43
```

```
solve(X) %*% X #identity
```

```
##      [,1]      [,2]      [,3]      [,4]
## [1,] 1.000000e+00 -5.684342e-14 -2.842171e-14 -1.421085e-14
## [2,] 0.000000e+00 1.000000e+00 -5.684342e-14 -2.842171e-14
## [3,] 0.000000e+00 0.000000e+00 1.000000e+00 1.421085e-14
## [4,] 5.684342e-14 0.000000e+00 2.842171e-14 1.000000e+00
```

```
diag(1, 4) #identity
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    0    0    0
## [2,]    0    1    0    0
## [3,]    0    0    1    0
## [4,]    0    0    0    1
```

```

X = cbind(1, X) #add a column of 1's for the intercept
# solve(t(X) %*% X) %*% t(X) %*% y #linear model - why does it not work?
y = c(y, 6, 4.5, 5)
X = rbind(X, c(1, 2,6,4,3))
X = rbind(X, c(1, 3,4,5.5,2))
X = rbind(X, c(1, 4.6,7,3,2))
solve(t(X) %*% X) %*% t(X) %*% y #linear model

```

```

##           [,1]
## [1,] 10.2035985
## [2,] -0.9128509
## [3,]  0.1989337
## [4,] -0.6278312
## [5,] -0.3340244

```

*# lets put it in a function*

```

linMod = function(X, y){
  beta = solve(t(X) %*% X) %*% t(X) %*% y
  se = sqrt(as.vector(t(y - X %*% beta) %*% (y - X %*% beta) / as.vector(nrow(X) - ncol(X))) * diag(solve(t(X) %*% X)))
  return(cbind(beta, se))
}
linMod(X, y)

```

```

##           se
## [1,] 10.2035985 7.4799545
## [2,] -0.9128509 0.4908196
## [3,]  0.1989337 0.6054357
## [4,] -0.6278312 0.6191554
## [5,] -0.3340244 0.5838463

```

```

summary(lm(y ~ X - 1)) # -1 means do not fit an intercept (there is a column of ones)

```

```

##
## Call:
## lm(formula = y ~ X - 1)
##
## Residuals:
##      1      2      3      4      5      6      7
## -0.0547  0.8304  0.2222 -1.4546 -0.0581  0.3603  0.1545
##
## Coefficients:
##      Estimate Std. Error t value Pr(>|t|)
## X1   10.2036     7.4800   1.364   0.306
## X2   -0.9129     0.4908  -1.860   0.204
## X3    0.1989     0.6054   0.329   0.774
## X4   -0.6278     0.6192  -1.014   0.417
## X5   -0.3340     0.5838  -0.572   0.625
##
## Residual standard error: 1.228 on 2 degrees of freedom
## Multiple R-squared:  0.9736, Adjusted R-squared:  0.9076
## F-statistic: 14.76 on 5 and 2 DF, p-value: 0.06467

```

```

summary(lm(y ~ X[, -1])) # we could also drop the first column

```

```

##
## Call:

```

```
## lm(formula = y ~ X[, -1])
##
## Residuals:
##      1      2      3      4      5      6      7
## -0.0547  0.8304  0.2222 -1.4546 -0.0581  0.3603  0.1545
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  10.2036     7.4800   1.364   0.306
## X[, -1]1     -0.9129     0.4908  -1.860   0.204
## X[, -1]2      0.1989     0.6054   0.329   0.774
## X[, -1]3     -0.6278     0.6192  -1.014   0.417
## X[, -1]4     -0.3340     0.5838  -0.572   0.625
##
## Residual standard error: 1.228 on 2 degrees of freedom
## Multiple R-squared:  0.7835, Adjusted R-squared:  0.3506
## F-statistic:  1.81 on 4 and 2 DF,  p-value: 0.3861
```

## OLS Derivations

### Expectations Using Matrix Algebra

$$\begin{aligned}
 y &= X\beta + \epsilon \\
 E[y] &= E[X\beta] + E[\epsilon] \\
 E[y] &= E[X\beta] \\
 y &= XE[\beta] \\
 X'y &= X'XE[\beta] \\
 (X'X)^{-1}X'y &= (X'X)^{-1}X'XE[\beta] \\
 (X'X)^{-1}X'y &= \mathbf{I}E[\beta] \\
 E[\beta] &= (X'X)^{-1}X'y
 \end{aligned}$$

### Minimizing Loss

$$\begin{aligned}
 y &= X\beta + \epsilon \\
 \epsilon &= X\beta - y \\
 \epsilon'\epsilon &= (X\beta - y)'(X\beta - y) \\
 \frac{\partial \epsilon'\epsilon}{\partial \beta} &= \frac{\partial}{\partial \beta} ((X\beta - y)'(X\beta - y)) \\
 &= \frac{\partial}{\partial \beta} (y'y - 2\beta X'y + X'X\beta'\beta) \\
 &= -2X'y + 2X'X\beta = 0 \\
 2X'y &= 2X'X\beta \\
 X'y &= X'X\beta \\
 (X'X)^{-1}X'y &= \beta
 \end{aligned}$$

## Assumptions

- The model is linear in the parameters
- No endogeneity in the model (independent variable  $X$  and  $\epsilon$  are not correlated)
- Errors are normally distributed with constant variance
- No autocorrelation in the errors
- No multicollinearity between variable

## Reading in Data and Running a Model

```
library(readstata13)
data = readstata13::read.dta13('TamingGods.dta')
#explore
colnames(data)
```

```
## [1] "ccode"          "year"          "LND_TOTL"
## [4] "Int_maxyear"   "polity2_"      "pts"
## [7] "democracy"     "loggdp"        "logpop"
## [10] "Religion"      "Ethnic"        "lmtnest"
## [13] "pctforest"     "Meast"         "relmob_vary"
## [16] "reldemand_vary" "conflict"      "cem_strata"
## [19] "cem_matched"   "cem_weights"   "recency"
## [22] "ongoing"       "propensity"    "RASindex3_scaled"
## [25] "MX_scaled"     "SCX_scaled"    "NX_scaled"
## [28] "relconflict"   "relconflict2"  "RAS4"
## [31] "altRAS4"       "scaled_RAS42"  "relmob_vary_lessz"
## [34] "reldemand_vary_lessz" "relconflict_lessz" "SuperaltrRAS4"
```

```
head(data)
```

```
##   ccode year LND_TOTL Int_maxyear polity2_ pts democracy loggdp logpop
## 1    2 1980  9158960         0      10    1         1 10.14685 19.24145
## 2    2 1981  9158960         0      10    1         1 10.16213 19.25126
## 3    2 1982  9158960         0      10    1         1 10.13259 19.26080
## 4    2 1983  9158960         0      10    1         1 10.16762 19.26994
## 5    2 1984  9158960         0      10    1         1 10.22843 19.27860
## 6    2 1985  9158960         0      10    1         1 10.25987 19.28746
##   Religion Ethnic lmtnest pctforest Meast relmob_vary reldemand_vary
## 1 0.824078 0.4901 3.214868 33.19397    0         0         0
## 2 0.824078 0.4901 3.214868 33.19397    0         0         0
## 3 0.824078 0.4901 3.214868 33.19397    0         0         0
## 4 0.824078 0.4901 3.214868 33.19397    0         0         0
## 5 0.824078 0.4901 3.214868 33.19397    0         0         0
## 6 0.824078 0.4901 3.214868 33.19397    0         0         0
##   conflict cem_strata cem_matched cem_weights recency ongoing propensity
## 1         0         56          1   0.329019         0         1 0.2545682
## 2         0         56          1   0.329019         0         1 0.2537097
## 3         0         56          1   0.329019         0         1 0.2580184
## 4         0         56          1   0.329019         0         1 0.2547998
## 5         0         56          1   0.329019         0         1 0.2486171
## 6         0         56          1   0.329019         0         1 0.2458567
##   RASindex3_scaled MX_scaled SCX_scaled NX_scaled relconflict relconflict2
## 1                NA        NA        NA        NA            0            0
```

```

## 2      NA      NA      NA      NA      0      0
## 3      NA      NA      NA      NA      0      0
## 4      NA      NA      NA      NA      0      0
## 5      NA      NA      NA      NA      0      0
## 6      NA      NA      NA      NA      0      0
##  RAS4 altRAS4 scaled_RAS42 relmob_vary_lessz reldemand_vary_lessz
## 1      0      0      NA      NA      NA
## 2      0      0      NA      NA      NA
## 3      0      0      NA      NA      NA
## 4      0      0      NA      NA      NA
## 5      0      0      NA      NA      NA
## 6      0      0      NA      NA      NA
##  relconflict_lessz SuperaltrRAS4
## 1      NA      NA
## 2      NA      NA
## 3      NA      NA
## 4      NA      NA
## 5      NA      NA
## 6      NA      NA

```

`summary(data)`

```

##      ccode      year      LND_TOTL      Int_maxyear
## Min.   : 2.0   Min.   :1980   Min.    : 0   Min.    :0.0000
## 1st Qu.:313.8 1st Qu.:1988   1st Qu.: 25175 1st Qu.:0.0000
## Median :456.5 Median :1996   Median : 120410 Median :0.0000
## Mean   :480.2 Mean   :1996   Mean    : 602289 Mean    :0.1929
## 3rd Qu.:694.5 3rd Qu.:2005   3rd Qu.: 527970 3rd Qu.:0.0000
## Max.   :990.0 Max.   :2013   Max.    :9327489 Max.    :2.0000
##                                     NA's    :1261   NA's    :278
##      polity2_      pts      democracy      loggdp
## Min.   : -9.000   Min.   :0.000   Min.    :0.0000   Min.    : 0.000
## 1st Qu.: -6.000   1st Qu.:1.500   1st Qu.:0.0000   1st Qu.: 7.362
## Median : 4.000   Median :2.000   Median :1.0000   Median : 8.449
## Mean   : 1.991   Mean    :2.324   Mean    :0.5638   Mean    : 8.143
## 3rd Qu.: 9.000   3rd Qu.:3.000   3rd Qu.:1.0000   3rd Qu.: 9.530
## Max.   :10.000   Max.    :5.000   Max.    :5.0000   Max.    :11.723
## NA's    :2035   NA's    :1727   NA's    :1862   NA's    :1679
##      logpop      Religion      Ethnic      lmtnest
## Min.   : 5.574   Min.   :0.0023   Min.    :0.0000   Min.    :0.0000
## 1st Qu.:13.950   1st Qu.:0.2326   1st Qu.:0.2003   1st Qu.:0.7297
## Median :15.519   Median :0.4628   Median :0.4373   Median :2.3174
## Mean   :15.143   Mean    :0.4411   Mean    :0.4413   Mean    :2.1044
## 3rd Qu.:16.644   3rd Qu.:0.6469   3rd Qu.:0.6632   3rd Qu.:3.2874
## Max.   :21.024   Max.    :0.8705   Max.    :0.9302   Max.    :4.5570
## NA's    :866    NA's    :714    NA's    :782    NA's    :1904
##      pctforest      Meast      relmob_vary      reldemand_vary
## Min.   : 0.00   Min.   :0.0000   Min.    :0.0000   Min.    :0.0000
## 1st Qu.:10.83   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000
## Median :30.86   Median :0.0000   Median :0.0000   Median :0.0000
## Mean   :31.66   Mean    :0.1085   Mean    :0.0414   Mean    :0.0317
## 3rd Qu.:47.14   3rd Qu.:0.0000   3rd Qu.:0.0000   3rd Qu.:0.0000
## Max.   :94.63   Max.    :1.0000   Max.    :1.0000   Max.    :1.0000
## NA's    :816    NA's    :781    NA's    :781
##      conflict      cem_strata      cem_matched      cem_weights

```

```
## Min. :0.0000 Min. : 1.00 Min. :0.00 Min. : 0.0000
## 1st Qu.:0.0000 1st Qu.: 26.00 1st Qu.:1.00 1st Qu.: 0.3701
## Median :0.0000 Median : 89.00 Median :1.00 Median : 0.6174
## Mean :0.1851 Mean : 76.63 Mean :0.92 Mean : 0.9200
## 3rd Qu.:0.0000 3rd Qu.:124.00 3rd Qu.:1.00 3rd Qu.: 1.0000
## Max. :1.0000 Max. :144.00 Max. :1.00 Max. :45.8982
##
## recency ongoing propensity RASindex3_scaled
## Min. : 0.000 Min. :0.00 Min. :0.0031 Min. :0.000
## 1st Qu.: 0.000 1st Qu.:0.00 1st Qu.:0.0443 1st Qu.:0.000
## Median : 0.000 Median :0.00 Median :0.1220 Median :0.167
## Mean : 2.061 Mean :0.12 Mean :0.1695 Mean :0.741
## 3rd Qu.: 1.000 3rd Qu.:0.00 3rd Qu.:0.2389 3rd Qu.:1.000
## Max. :23.000 Max. :1.00 Max. :0.8225 Max. :6.500
## NA's :6018 NA's :2719 NA's :3902
## MX_scaled SCX_scaled NX_scaled relconflict
## Min. : 0.000 Min. : 0.000 Min. : 0.000 Min. :0.0000
## 1st Qu.: 0.141 1st Qu.: 2.000 1st Qu.: 0.179 1st Qu.:0.0000
## Median : 0.704 Median : 2.000 Median : 0.714 Median :0.0000
## Mean : 1.444 Mean : 3.557 Mean : 1.480 Mean :0.1044
## 3rd Qu.: 2.113 3rd Qu.: 6.000 3rd Qu.: 1.964 3rd Qu.:0.0000
## Max. :10.000 Max. :10.000 Max. :10.000 Max. :3.0000
## NA's :3960 NA's :3960 NA's :3961 NA's :848
## relconflict2 RAS4 altRAS4 scaled_RAS4
## Min. :0.00000 Min. : 0.000 Min. : 0.000 Min. :0.000
## 1st Qu.:0.00000 1st Qu.: 0.000 1st Qu.: 0.000 1st Qu.:0.200
## Median :0.00000 Median : 0.000 Median : 0.000 Median :0.589
## Mean :0.06521 Mean : 1.003 Mean : 9.154 Mean :1.073
## 3rd Qu.:0.00000 3rd Qu.: 1.121 3rd Qu.:10.000 3rd Qu.:1.544
## Max. :2.00000 Max. :10.000 Max. :107.000 Max. :5.889
## NA's : NA's :1060 NA's :3902
## relmob_vary_lessz reldemand_vary_lessz relconflict_lessz SuperaltrRAS4
## Min. :0.000 Min. :0.000 Min. :0.000 Min. : 0.000
## 1st Qu.:0.000 1st Qu.:0.000 1st Qu.:0.000 1st Qu.: 2.179
## Median :0.000 Median :0.000 Median :0.000 Median : 4.452
## Mean :0.189 Mean :0.145 Mean :0.496 Mean : 6.481
## 3rd Qu.:0.000 3rd Qu.:0.000 3rd Qu.:0.000 3rd Qu.: 9.890
## Max. :1.000 Max. :1.000 Max. :3.000 Max. :25.536
## NA's :5801 NA's :5801 NA's :5868 NA's :3961
```

- Look at the summary of the data in the pdf
- Is ethnic fractionalization correlated with religious repression?

```
mod = lm(Religion ~ Ethnic, data = data)
summary(mod)
```

```
##
## Call:
## lm(formula = Religion ~ Ethnic, data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.49617 -0.19081  0.03995  0.17651  0.42936
##
## Coefficients:
```

```

##           Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.377875    0.005692   66.39  <2e-16 ***
## Ethnic      0.149138    0.011197   13.32  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2291 on 6356 degrees of freedom
## (850 observations deleted due to missingness)
## Multiple R-squared:  0.02715,    Adjusted R-squared:  0.027
## F-statistic: 177.4 on 1 and 6356 DF,  p-value: < 2.2e-16

library(stargazer) #for making LaTeX tables

##
## Please cite as:
## Hlavac, Marek (2018). stargazer: Well-Formatted Regression and Summary Statistics Tables.
## R package version 5.2.2. https://CRAN.R-project.org/package=stargazer

stargazer(mod)

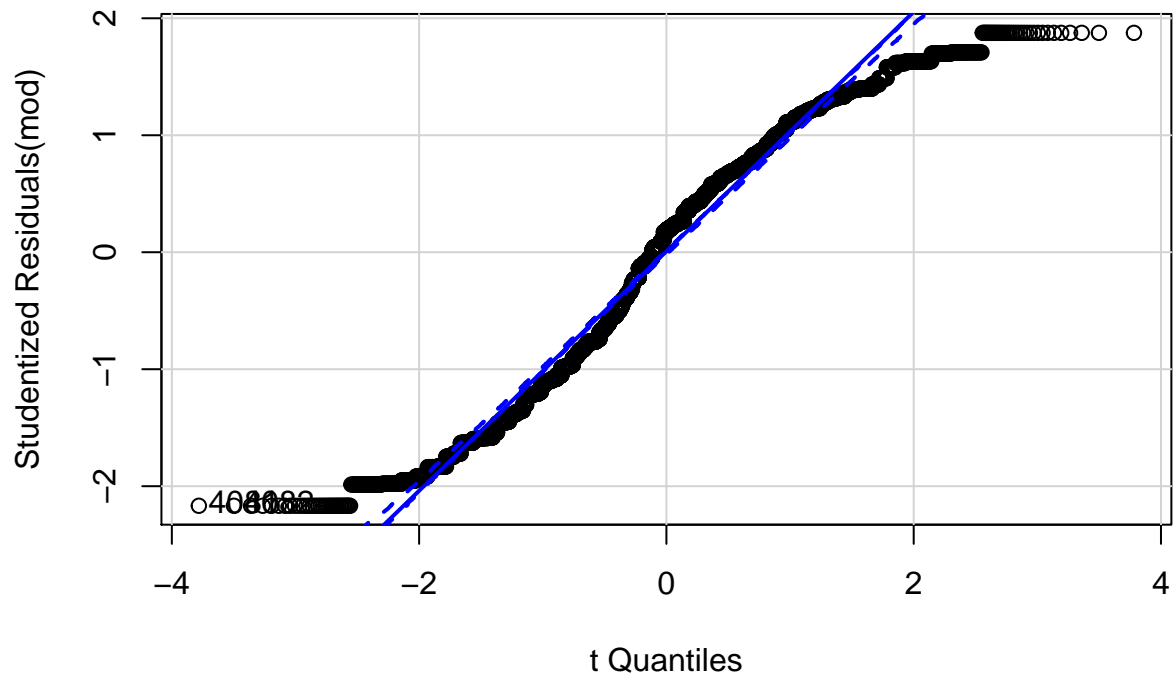
##
## % Table created by stargazer v.5.2.2 by Marek Hlavac, Harvard University. E-mail: hlavac at fas.harvard.edu
## % Date and time: Tue, Sep 15, 2020 - 02:07:16 PM
## \begin{table}[!htbp] \centering
##   \caption{}
##   \label{}
##   \begin{tabular}{@{\extracolsep{5pt}}lc}
##     \hline
##     \hline \hline
##     & \multicolumn{1}{c}{\textit{Dependent variable:}} & \\
##     \cline{2-2}
##     \hline \hline & Religion & \\
##     \hline \hline
##     Ethnic & 0.149$^{***}$ & \\
##     & (0.011) & \\
##     & & \\
##     Constant & 0.378$^{***}$ & \\
##     & (0.006) & \\
##     & & \\
##     \hline \hline
##     Observations & 6,358 & \\
##     R$^2$ & 0.027 & \\
##     Adjusted R$^2$ & 0.027 & \\
##     Residual Std. Error & 0.229 (df = 6356) & \\
##     F Statistic & 177.412$^{***}$ (df = 1; 6356) & \\
##     \hline
##     \hline \hline
##     \textit{Note:} & \multicolumn{1}{r}{\textit{$^*$}$p$<$0.1; \textit{$^{**}$}$p$<$0.05; \textit{$^{***}$}$p$<$0.01}} & \\
##     \end{tabular}
##   \end{table}

library(car)

## Loading required package: carData

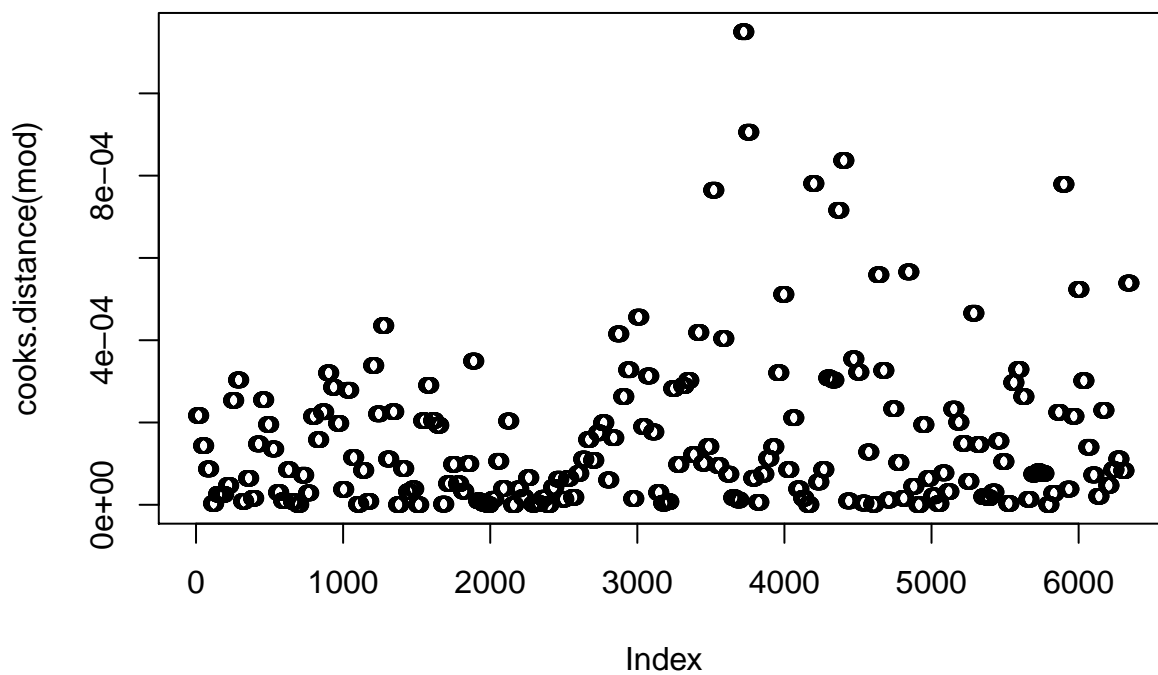
```

```
qqPlot(mod) #check for error distribution - clearly not normal
```



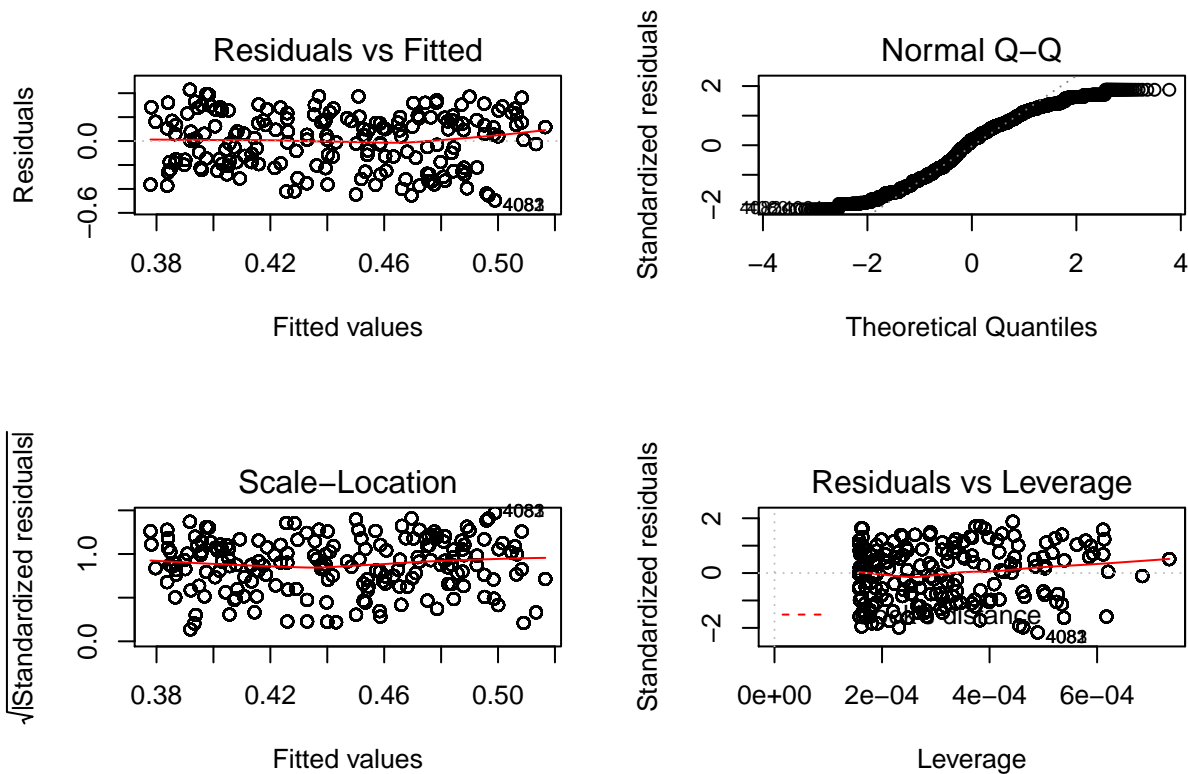
```
## [1] 4081 4082
```

```
plot(cooks.distance(mod)) #check for cook's influence
```



```
par(mfrow = c(2,2)) #set up the plot to be 2x2 rows x columns
plot(mod)
```





```
par(mfrow = c(1,1))
```

- What does  $\beta$  actually mean?
- How would you test if this relationship is conditional on democracy levels?
- Derive the conditional effect of an interaction effect

$$\hat{y} = \beta_0 + X_1\beta_1 + X_2\beta_2 + X_1X_2\beta_3$$

$$\frac{\partial y}{\partial X_1} = \beta_1 + X_2\beta_3$$

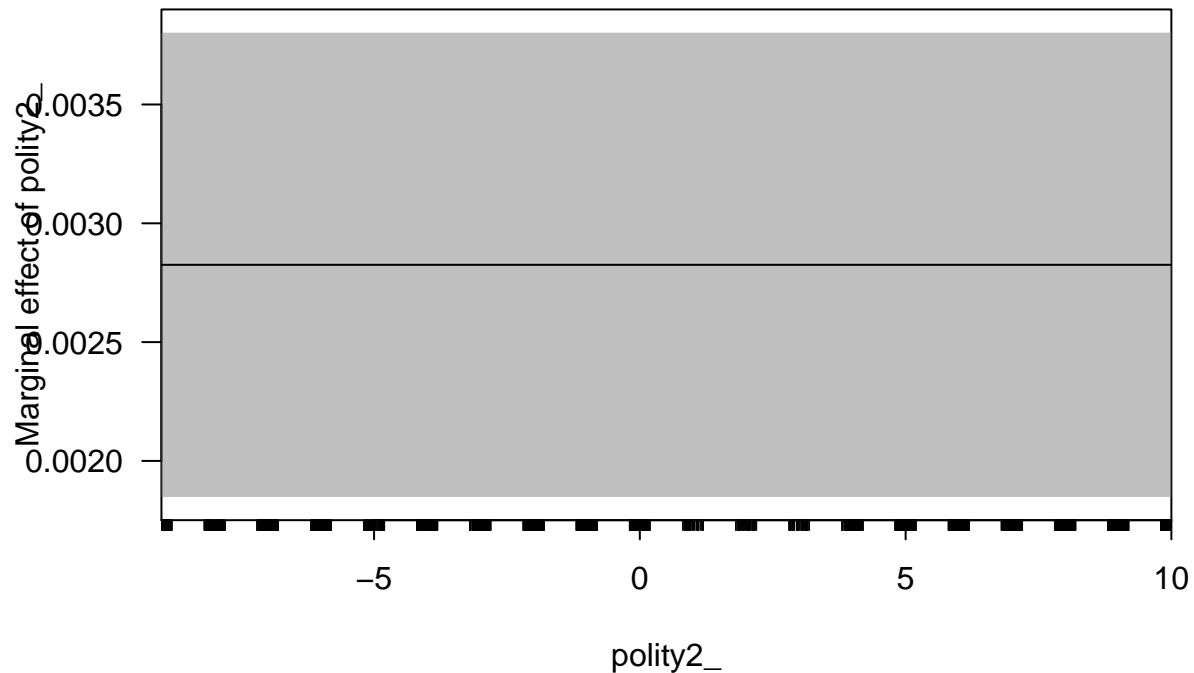
- Notice that you cannot simply look at the  $\beta$  estimates to understand an interactive effect

```
mod2 = lm(Religion ~ Ethnic*polity2_, data = data)
summary(mod2)
```

```
##
## Call:
## lm(formula = Religion ~ Ethnic * polity2_, data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.50704 -0.17763  0.02774  0.17462  0.44512
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   0.3118858   0.0077763  40.107 < 2e-16 ***
## Ethnic        0.2407606   0.0138264  17.413 < 2e-16 ***
## polity2_      0.0055995   0.0009867   5.675 1.47e-08 ***
## Ethnic:polity2_ -0.0062858   0.0019421  -3.237 0.00122 **
## ---
```

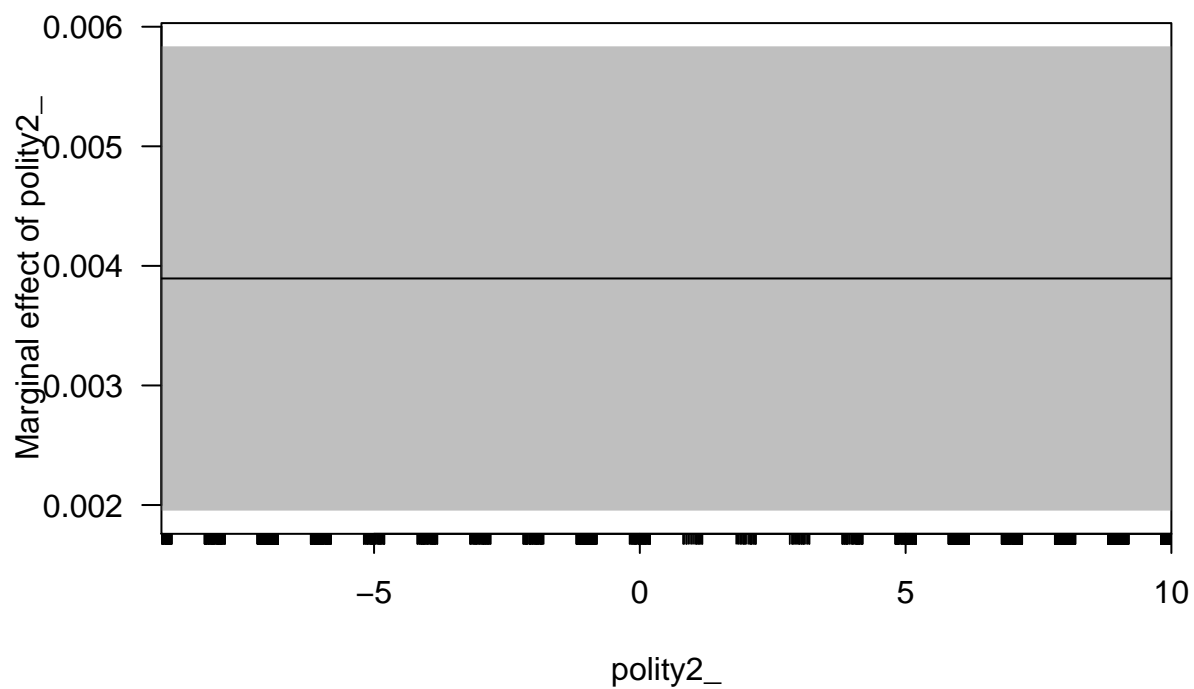
```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2321 on 4999 degrees of freedom
## (2205 observations deleted due to missingness)
## Multiple R-squared:  0.05739,    Adjusted R-squared:  0.05683
## F-statistic: 101.5 on 3 and 4999 DF,  p-value: < 2.2e-16
```

```
library(margins)
cplot(mod2, x = 'polity2_', what = 'effect', data = data)
```

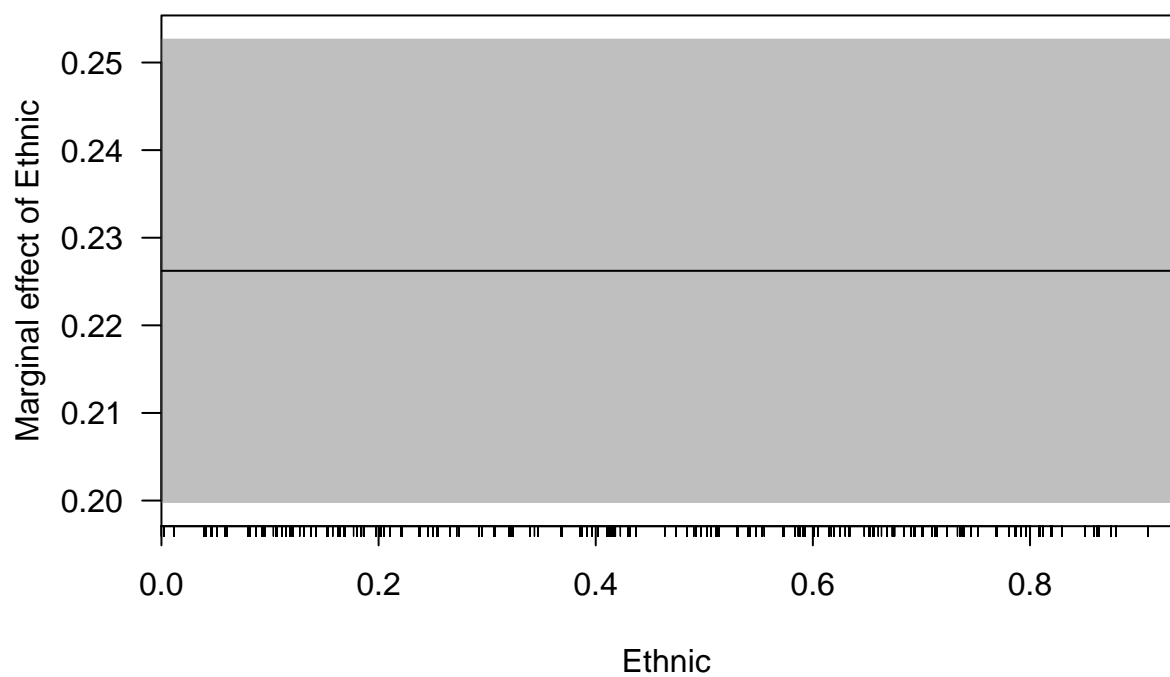


- Interpret the plot
- Let's add a factor for democracy

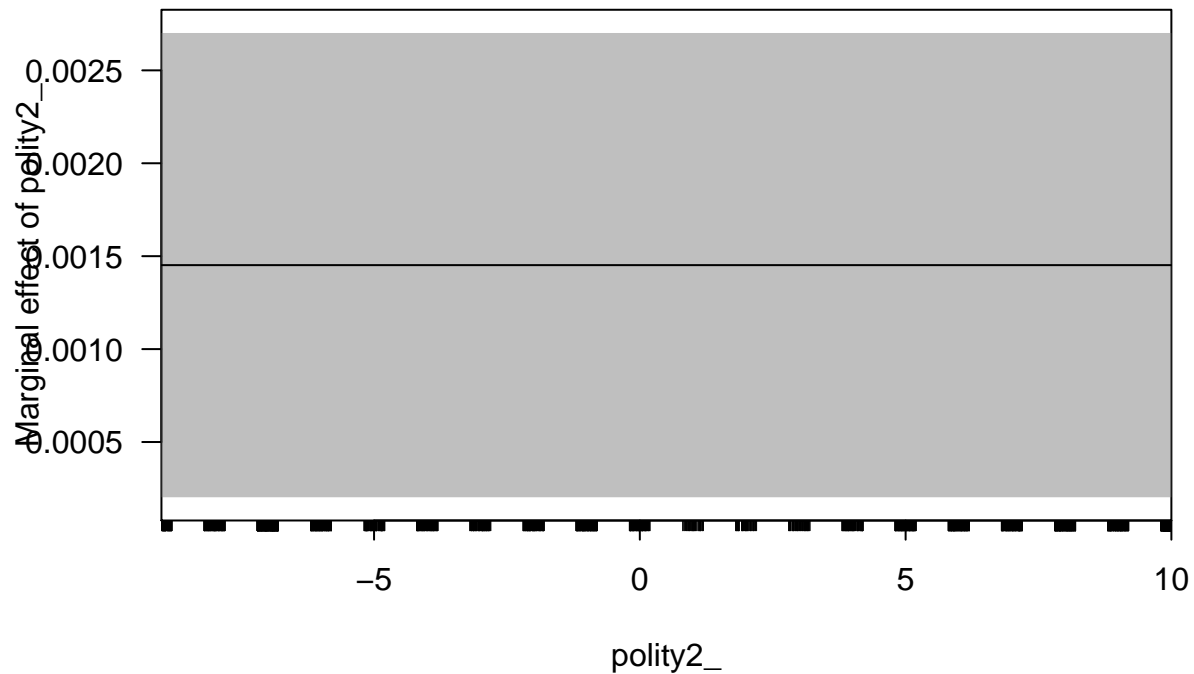
```
mod3 = lm(Religion ~ Ethnic*polity2_ + I(polity2_ > 5), data = data)
cplot(mod3, x = 'polity2_', what = 'effect', data = data)
```



```
cplot(mod3, x = 'Ethnic', what = 'effect', data = data)
```

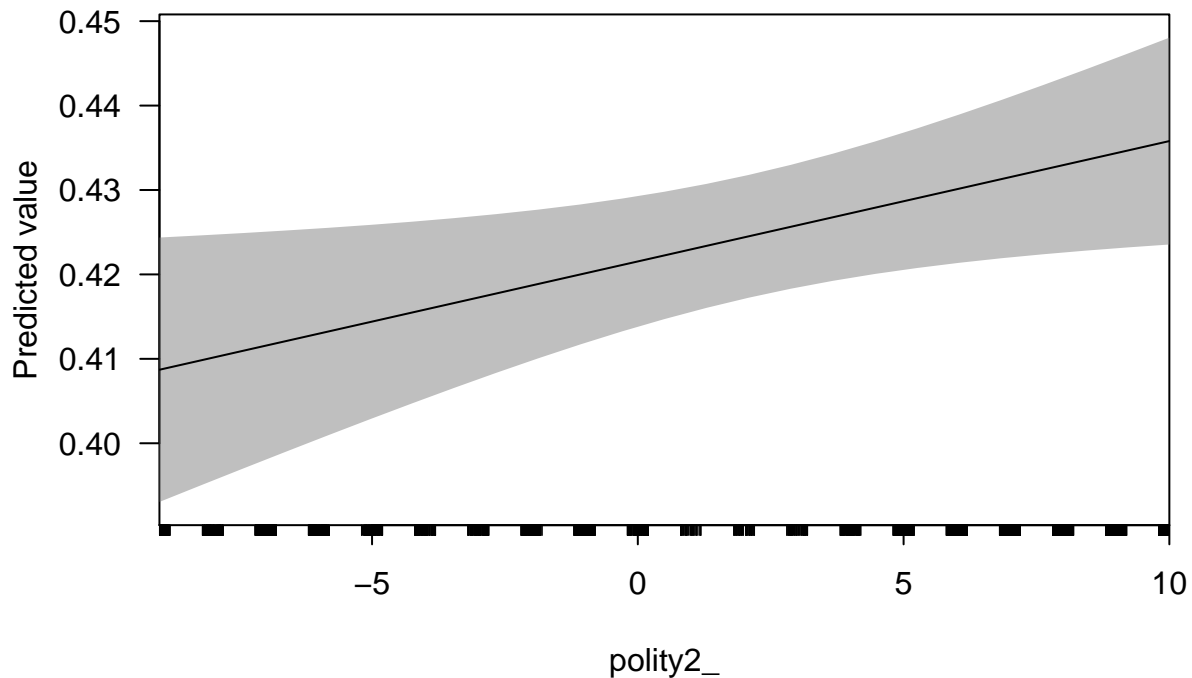


```
mod4 = lm(Religion ~ Ethnic*polity2_ + democracy, data = data)
cplot(mod4, x = 'polity2_', what = 'effect', data = data)
```



```
cplot(mod4, x = 'polity2_', what = 'prediction', data = data)
```

##	xvals	yvals	upper	lower
## 1	-9.000000	0.4087080	0.4243681	0.3930479
## 2	-8.208333	0.4098366	0.4246282	0.3950451
## 3	-7.416667	0.4109652	0.4249042	0.3970262
## 4	-6.625000	0.4120938	0.4251994	0.3989881
## 5	-5.833333	0.4132224	0.4255177	0.4009271
## 6	-5.041667	0.4143509	0.4258638	0.4028380
## 7	-4.250000	0.4154795	0.4262440	0.4047150
## 8	-3.458333	0.4166081	0.4266658	0.4065504
## 9	-2.666667	0.4177367	0.4271385	0.4083348
## 10	-1.875000	0.4188653	0.4276737	0.4100569
## 11	-1.083333	0.4199939	0.4282846	0.4117031
## 12	-0.291667	0.4211224	0.4289863	0.4132585
## 13	0.500000	0.4222510	0.4297942	0.4147078
## 14	1.291667	0.4233796	0.4307223	0.4160369
## 15	2.083333	0.4245082	0.4317804	0.4172360
## 16	2.875000	0.4256368	0.4329723	0.4183013
## 17	3.666667	0.4267654	0.4342946	0.4192361
## 18	4.458333	0.4278939	0.4357378	0.4200501
## 19	5.250000	0.4290225	0.4372879	0.4207571
## 20	6.041667	0.4301511	0.4389297	0.4213725



- Despite the reliable estimates, it is pretty clear we do not need an interaction, and the effect does not change marginally
- Let's look at a different example

```
mod5 = lm(mpg ~ wt + I(wt^2), data = mtcars)
margins(mod5)
```

```
## Average marginal effects
```

```
## lm(formula = mpg ~ wt + I(wt^2), data = mtcars)
```

```
##      wt
```

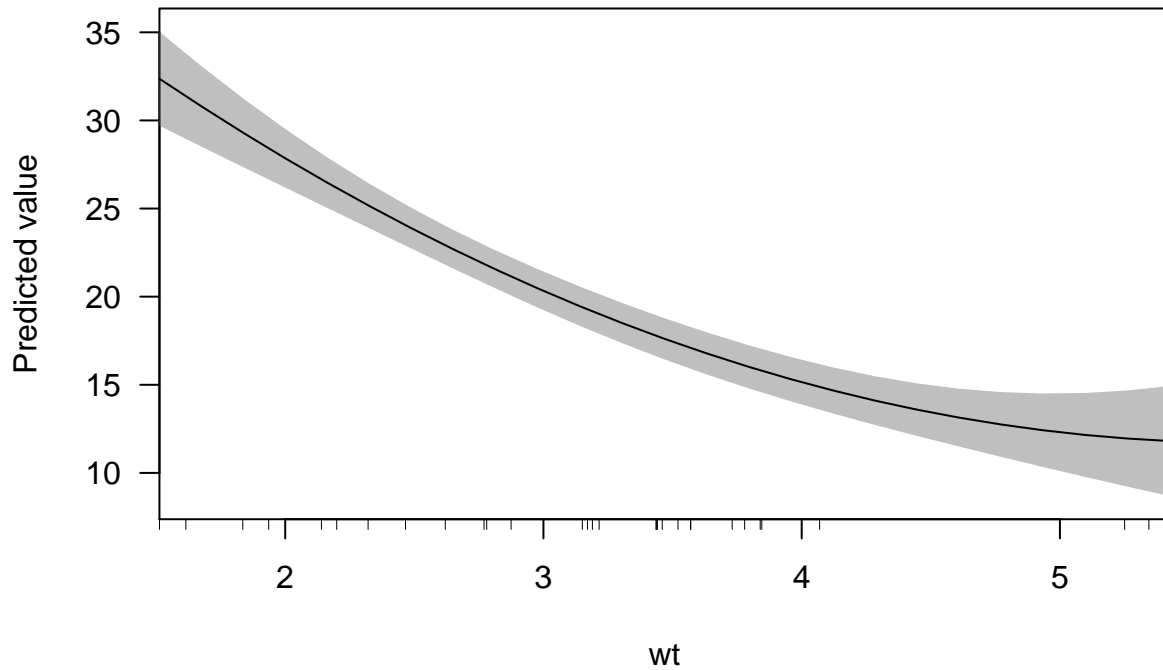
```
## -5.845
```

```
cplot(mod5, "wt", what = "prediction", main = "Predicted Fuel Economy, Given Weight")
```

```
##      xvals  yvals  upper  lower
## 1  1.513000 32.36718 35.03500 29.69936
## 2  1.675958 30.79531 33.07715 28.51348
## 3  1.838917 29.28565 31.23015 27.34115
## 4  2.001875 27.83818 29.49769 26.17868
## 5  2.164833 26.45291 27.88384 25.02199
## 6  2.327792 25.12984 26.39172 23.86796
## 7  2.490750 23.86897 25.02106 22.71687
## 8  2.653708 22.67029 23.76564 21.57494
## 9  2.816667 21.53381 22.61340 20.45422
## 10 2.979625 20.45953 21.54975 19.36930
## 11 3.142583 19.44744 20.56171 18.33318
## 12 3.305542 18.49755 19.64011 17.35500
## 13 3.468500 17.60986 18.78010 16.43963
## 14 3.631458 16.78437 17.98078 15.58796
## 15 3.794417 16.02108 17.24475 14.79740
## 16 3.957375 15.31998 16.57774 14.06221
## 17 4.120333 14.68108 15.98801 13.37414
## 18 4.283292 14.10437 15.48533 12.72342
```

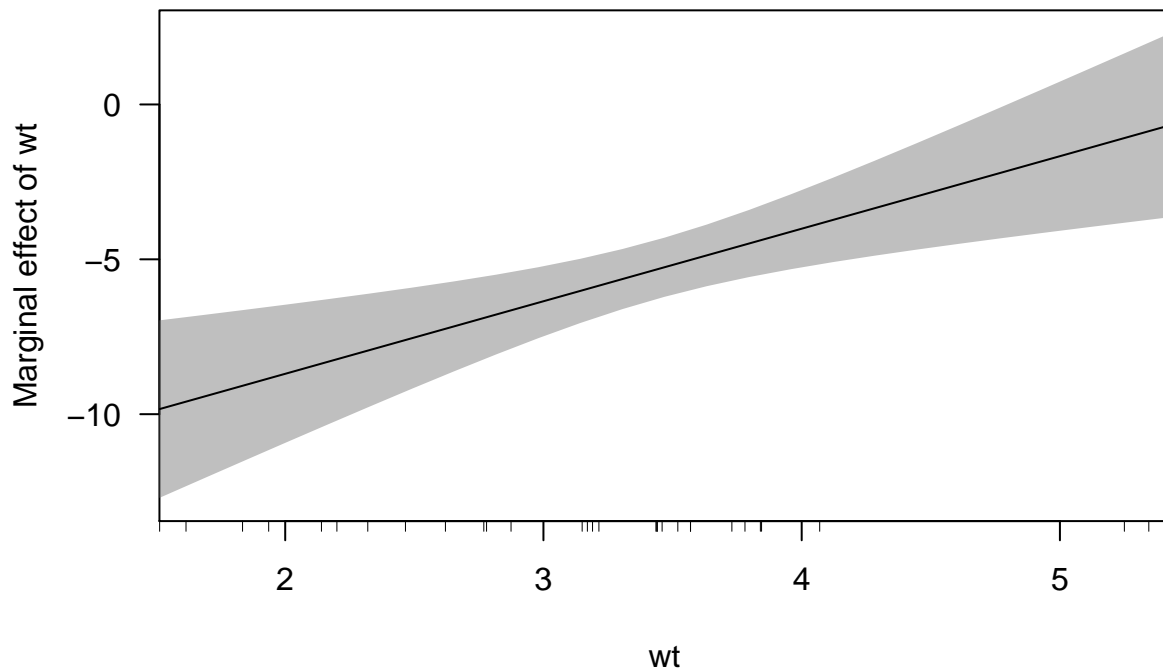
```
## 19 4.446250 13.58987 15.07937 12.10037
## 20 4.609208 13.13756 14.77796 11.49716
```

### Predicted Fuel Economy, Given Weight



```
cplot(mod5, "wt", what = "effect", main = "Average Marginal Effect of Weight")
```

### Average Marginal Effect of Weight



```
mod6 = lm(mpg ~ hp * wt, data = mtcars)
persp(mod6, "wt", "hp", theta = c(45, 135, 225, 315), what = "effect")
```

