Day 1 - OLS

David Carlson

Matrix Algebra

- Look at la.pdf in the folder for a simple matrix cheat sheet
- We will pretty much only deal with vectors and matrices, especially in R

```r
y = c(2,4,3,2) #vector
X = matrix(c(5,3,4,5,6,5,3,2,4,5,6,3,5,5,4,2),
           nrow = 4, ncol = 4, byrow = T) #matrix (you do not need to specify both cols and rows)
y
```

```
## [1] 2 4 3 2
```

```r
X
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    5    3    4    5
## [2,]    6    5    3    2
## [3,]    4    5    6    3
## [4,]    5    5    4    2
```

```r
t(X) #transpose
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    5    6    4    5
## [2,]    3    5    5    5
## [3,]    4    3    6    4
## [4,]    5    2    3    2
```

```r
solve(X) #inverse
```

```
##      [,1] [,2] [,3] [,4]
## [1,]   -5   30   19  -46
## [2,]    7  -42  -27   65
## [3,]   -5   29   19  -45
## [4,]    5  -28  -18   43
```

```r
solve(X) %*% X #identity
```

```
##              [,1]          [,2]          [,3]          [,4]
## [1,] 1.000000e+00 -5.684342e-14 -2.842171e-14 -1.421085e-14
## [2,] 0.000000e+00  1.000000e+00 -5.684342e-14 -2.842171e-14
## [3,] 0.000000e+00  0.000000e+00  1.000000e+00  1.421085e-14
## [4,] 5.684342e-14  0.000000e+00  2.842171e-14  1.000000e+00
```

```r
diag(1, 4) #identity
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    0    0    0
## [2,]    0    1    0    0
## [3,]    0    0    1    0
## [4,]    0    0    0    1
```

```r
X = cbind(1, X) #add a column of 1's for the intercept
# solve(t(X) %*% X) %*% t(X) %*% y #linear model - why does it not work?
y = c(y, 6, 4.5, 5)
X = rbind(X, c(1, 2,6,4,3))
X = rbind(X, c(1, 3,4,5.5,2))
X = rbind(X, c(1, 4.6,7,3,2))
solve(t(X) %*% X) %*% t(X) %*% y #linear model
```

```
##            [,1]
## [1,] 10.2035985
## [2,] -0.9128509
## [3,]  0.1989337
## [4,] -0.6278312
## [5,] -0.3340244
```

```r
# lets put it in a function
linMod = function(X, y){
  beta = solve(t(X) %*% X) %*% t(X) %*% y
  se = sqrt(as.vector(t(y - X %*% beta) %*% (y - X %*% beta) / as.vector(nrow(X) - ncol(X))) * diag(sol
  return(cbind(beta, se))
}
linMod(X, y)
```

```
##                       se
## [1,] 10.2035985 7.4799545
## [2,] -0.9128509 0.4908196
## [3,]  0.1989337 0.6054357
## [4,] -0.6278312 0.6191554
## [5,] -0.3340244 0.5838463
```

```r
summary(lm(y ~ X - 1)) # -1 means do not fit an intercept (there is a column of ones)
```

```
##
## Call:
## lm(formula = y ~ X - 1)
##
## Residuals:
##       1       2       3       4       5       6       7
## -0.0547  0.8304  0.2222 -1.4546 -0.0581  0.3603  0.1545
##
## Coefficients:
##    Estimate Std. Error t value Pr(>|t|)
## X1  10.2036     7.4800   1.364    0.306
## X2  -0.9129     0.4908  -1.860    0.204
## X3   0.1989     0.6054   0.329    0.774
## X4  -0.6278     0.6192  -1.014    0.417
## X5  -0.3340     0.5838  -0.572    0.625
##
## Residual standard error: 1.228 on 2 degrees of freedom
## Multiple R-squared:  0.9736, Adjusted R-squared:  0.9076
## F-statistic: 14.76 on 5 and 2 DF,  p-value: 0.06467
```

```r
summary(lm(y ~ X[, -1])) # we could also drop the first column
```

```
##
## Call:
```

```
## lm(formula = y ~ X[, -1])
##
## Residuals:
##       1       2       3       4       5       6       7
## -0.0547  0.8304  0.2222 -1.4546 -0.0581  0.3603  0.1545
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  10.2036     7.4800   1.364    0.306
## X[, -1]1     -0.9129     0.4908  -1.860    0.204
## X[, -1]2      0.1989     0.6054   0.329    0.774
## X[, -1]3     -0.6278     0.6192  -1.014    0.417
## X[, -1]4     -0.3340     0.5838  -0.572    0.625
##
## Residual standard error: 1.228 on 2 degrees of freedom
## Multiple R-squared:  0.7835, Adjusted R-squared:  0.3506
## F-statistic:  1.81 on 4 and 2 DF,  p-value: 0.3861
```

OLS Derivations

Expectations Using Matrix Algebra

$$
\begin{aligned}
y &= X\beta + \epsilon \\
E[y] &= E[X\beta] + E[\epsilon] \\
E[y] &= E[X\beta] \\
y &= XE[\beta] \\
X'y &= X'XE[\beta] \\
(X'X)^{-1}X'y &= (X'X)^{-1}X'XE[\beta] \\
(X'X)^{-1}X'y &= \mathbf{I}E[\beta] \\
E[\beta] &= (X'X)^{-1}X'y
\end{aligned}
$$

Minimizing Loss

$$
\begin{aligned}
y &= X\beta + \epsilon \\
\epsilon &= X\beta - y \\
\epsilon'\epsilon &= (X\beta - y)'(X\beta - y) \\
\frac{\partial \epsilon'\epsilon}{\partial \beta} &= \frac{\partial}{\partial \beta}\left((X\beta - y)'(X\beta - y)\right) \\
&= \frac{\partial}{\partial \beta}\left(y'y - 2\beta X'y + X'X\beta'\beta\right) \\
&= -2X'y + 2X'X\beta = 0 \\
2X'y &= 2X'X\beta \\
X'y &= X'X\beta \\
(X'X)^{-1}X'y &= \beta
\end{aligned}
$$

Assumptions

- The model is linear in the parameters
- No endogeneity in the model (independent variable X and ϵ are not correlated)
- Errors are normally distributed with constant variance
- No autocorrelation in the errors
- No multicollinearity between variable

Reading in Data and Running a Model

```r
library(readstata13)
data = readstata13::read.dta13('TamingGods.dta')
#explore
colnames(data)
```

```
##  [1] "ccode"               "year"                "LND_TOTL"
##  [4] "Int_maxyear"         "polity2_"            "pts"
##  [7] "democracy"           "loggdp"              "logpop"
## [10] "Religion"            "Ethnic"              "lmtnest"
## [13] "pctforest"           "Meast"               "relmob_vary"
## [16] "reldemand_vary"      "conflict"            "cem_strata"
## [19] "cem_matched"         "cem_weights"         "recency"
## [22] "ongoing"             "propensity"          "RASindex3_scaled"
## [25] "MX_scaled"           "SCX_scaled"          "NX_scaled"
## [28] "relconflict"         "relconflict2"        "RAS4"
## [31] "altRAS4"             "scaled_RAS42"        "relmob_vary_lessz"
## [34] "reldemand_vary_lessz" "relconflict_lessz"    "SuperaltRAS4"
```

```r
head(data)
```

```
##   ccode year LND_TOTL Int_maxyear polity2_ pts democracy   loggdp   logpop
## 1     2 1980  9158960           0       10   1         1 10.14685 19.24145
## 2     2 1981  9158960           0       10   1         1 10.16213 19.25126
## 3     2 1982  9158960           0       10   1         1 10.13259 19.26080
## 4     2 1983  9158960           0       10   1         1 10.16762 19.26994
## 5     2 1984  9158960           0       10   1         1 10.22843 19.27860
## 6     2 1985  9158960           0       10   1         1 10.25987 19.28746
##   Religion Ethnic  lmtnest pctforest Meast relmob_vary reldemand_vary
## 1 0.824078 0.4901 3.214868  33.19397     0           0              0
## 2 0.824078 0.4901 3.214868  33.19397     0           0              0
## 3 0.824078 0.4901 3.214868  33.19397     0           0              0
## 4 0.824078 0.4901 3.214868  33.19397     0           0              0
## 5 0.824078 0.4901 3.214868  33.19397     0           0              0
## 6 0.824078 0.4901 3.214868  33.19397     0           0              0
##   conflict cem_strata cem_matched cem_weights recency ongoing propensity
## 1        0         56           1    0.329019       0       1  0.2545682
## 2        0         56           1    0.329019       0       1  0.2537097
## 3        0         56           1    0.329019       0       1  0.2580184
## 4        0         56           1    0.329019       0       1  0.2547998
## 5        0         56           1    0.329019       0       1  0.2486171
## 6        0         56           1    0.329019       0       1  0.2458567
##   RASindex3_scaled MX_scaled SCX_scaled NX_scaled relconflict relconflict2
## 1               NA        NA         NA        NA           0            0
```

```
## 2               NA        NA         NA        NA            0             0
## 3               NA        NA         NA        NA            0             0
## 4               NA        NA         NA        NA            0             0
## 5               NA        NA         NA        NA            0             0
## 6               NA        NA         NA        NA            0             0
##   RAS4 altRAS4 scaled_RAS42 relmob_vary_lessz reldemand_vary_lessz
## 1    0       0           NA                NA                   NA
## 2    0       0           NA                NA                   NA
## 3    0       0           NA                NA                   NA
## 4    0       0           NA                NA                   NA
## 5    0       0           NA                NA                   NA
## 6    0       0           NA                NA                   NA
##   relconflict_lessz SuperaltRAS4
## 1                NA           NA
## 2                NA           NA
## 3                NA           NA
## 4                NA           NA
## 5                NA           NA
## 6                NA           NA
```

```
summary(data)
```

```
##      ccode            year         LND_TOTL        Int_maxyear
##  Min.   :  2.0   Min.   :1980   Min.   :      0   Min.   :0.0000
##  1st Qu.:313.8   1st Qu.:1988   1st Qu.:  25175   1st Qu.:0.0000
##  Median :456.5   Median :1996   Median : 120410   Median :0.0000
##  Mean   :480.2   Mean   :1996   Mean   : 602289   Mean   :0.1929
##  3rd Qu.:694.5   3rd Qu.:2005   3rd Qu.: 527970   3rd Qu.:0.0000
##  Max.   :990.0   Max.   :2013   Max.   :9327489   Max.   :2.0000
##                                 NA's   :1261      NA's   :278
##     polity2_          pts          democracy          loggdp
##  Min.   :-9.000   Min.   :0.000   Min.   :0.0000   Min.   : 0.000
##  1st Qu.:-6.000   1st Qu.:1.500   1st Qu.:0.0000   1st Qu.: 7.362
##  Median : 4.000   Median :2.000   Median :1.0000   Median : 8.449
##  Mean   : 1.991   Mean   :2.324   Mean   :0.5638   Mean   : 8.143
##  3rd Qu.: 9.000   3rd Qu.:3.000   3rd Qu.:1.0000   3rd Qu.: 9.530
##  Max.   :10.000   Max.   :5.000   Max.   :5.0000   Max.   :11.723
##  NA's   :2035     NA's   :1727    NA's   :1862     NA's   :1679
##      logpop          Religion          Ethnic          lmtnest
##  Min.   : 5.574   Min.   :0.0023   Min.   :0.0000   Min.   :0.0000
##  1st Qu.:13.950   1st Qu.:0.2326   1st Qu.:0.2003   1st Qu.:0.7297
##  Median :15.519   Median :0.4628   Median :0.4373   Median :2.3174
##  Mean   :15.143   Mean   :0.4411   Mean   :0.4413   Mean   :2.1044
##  3rd Qu.:16.644   3rd Qu.:0.6469   3rd Qu.:0.6632   3rd Qu.:3.2874
##  Max.   :21.024   Max.   :0.8705   Max.   :0.9302   Max.   :4.5570
##  NA's   :866      NA's   :714      NA's   :782      NA's   :1904
##    pctforest         Meast          relmob_vary     reldemand_vary
##  Min.   : 0.00   Min.   :0.0000   Min.   :0.0000   Min.   :0.0000
##  1st Qu.:10.83   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000
##  Median :30.86   Median :0.0000   Median :0.0000   Median :0.0000
##  Mean   :31.66   Mean   :0.1085   Mean   :0.0414   Mean   :0.0317
##  3rd Qu.:47.14   3rd Qu.:0.0000   3rd Qu.:0.0000   3rd Qu.:0.0000
##  Max.   :94.63   Max.   :1.0000   Max.   :1.0000   Max.   :1.0000
##  NA's   :816                      NA's   :781      NA's   :781
##     conflict        cem_strata      cem_matched    cem_weights
```

```
##  Min.   :0.0000   Min.   :  1.00   Min.   :0.00   Min.   : 0.0000
##  1st Qu.:0.0000   1st Qu.: 26.00   1st Qu.:1.00   1st Qu.: 0.3701
##  Median :0.0000   Median : 89.00   Median :1.00   Median : 0.6174
##  Mean   :0.1851   Mean   : 76.63   Mean   :0.92   Mean   : 0.9200
##  3rd Qu.:0.0000   3rd Qu.:124.00   3rd Qu.:1.00   3rd Qu.: 1.0000
##  Max.   :1.0000   Max.   :144.00   Max.   :1.00   Max.   :45.8982
##
##     recency          ongoing       propensity     RASindex3_scaled
##  Min.   : 0.000   Min.   :0.00   Min.   :0.0031   Min.   :0.000
##  1st Qu.: 0.000   1st Qu.:0.00   1st Qu.:0.0443   1st Qu.:0.000
##  Median : 0.000   Median :0.00   Median :0.1220   Median :0.167
##  Mean   : 2.061   Mean   :0.12   Mean   :0.1695   Mean   :0.741
##  3rd Qu.: 1.000   3rd Qu.:0.00   3rd Qu.:0.2389   3rd Qu.:1.000
##  Max.   :23.000   Max.   :1.00   Max.   :0.8225   Max.   :6.500
##  NA's   :6018                    NA's   :2719     NA's   :3902
##    MX_scaled        SCX_scaled       NX_scaled       relconflict
##  Min.   : 0.000   Min.   : 0.000   Min.   : 0.000   Min.   :0.0000
##  1st Qu.: 0.141   1st Qu.: 2.000   1st Qu.: 0.179   1st Qu.:0.0000
##  Median : 0.704   Median : 2.000   Median : 0.714   Median :0.0000
##  Mean   : 1.444   Mean   : 3.557   Mean   : 1.480   Mean   :0.1044
##  3rd Qu.: 2.113   3rd Qu.: 6.000   3rd Qu.: 1.964   3rd Qu.:0.0000
##  Max.   :10.000   Max.   :10.000   Max.   :10.000   Max.   :3.0000
##  NA's   :3960     NA's   :3960     NA's   :3961     NA's   :848
##   relconflict2         RAS4            altRAS4          scaled_RAS42
##  Min.   :0.00000   Min.   : 0.000   Min.   :  0.000   Min.   :0.000
##  1st Qu.:0.00000   1st Qu.: 0.000   1st Qu.:  0.000   1st Qu.:0.200
##  Median :0.00000   Median : 0.000   Median :  0.000   Median :0.589
##  Mean   :0.06521   Mean   : 1.003   Mean   :  9.154   Mean   :1.073
##  3rd Qu.:0.00000   3rd Qu.: 1.121   3rd Qu.: 10.000   3rd Qu.:1.544
##  Max.   :2.00000   Max.   :10.000   Max.   :107.000   Max.   :5.889
##                    NA's   :1060                       NA's   :3902
##  relmob_vary_lessz reldemand_vary_lessz relconflict_lessz  SuperaltRAS4
##  Min.   :0.000     Min.   :0.000        Min.   :0.000     Min.   : 0.000
##  1st Qu.:0.000     1st Qu.:0.000        1st Qu.:0.000     1st Qu.: 2.179
##  Median :0.000     Median :0.000        Median :0.000     Median : 4.452
##  Mean   :0.189     Mean   :0.145        Mean   :0.496     Mean   : 6.481
##  3rd Qu.:0.000     3rd Qu.:0.000        3rd Qu.:0.000     3rd Qu.: 9.890
##  Max.   :1.000     Max.   :1.000        Max.   :3.000     Max.   :25.536
##  NA's   :5801      NA's   :5801         NA's   :5868      NA's   :3961
```

- Look at the summary of the data in the pdf
- Is ethnic fractionalization correlated with religious repression?

```r
mod = lm(Religion ~ Ethnic, data = data)
summary(mod)
```

```
##
## Call:
## lm(formula = Religion ~ Ethnic, data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.49617 -0.19081  0.03995  0.17651  0.42936
##
## Coefficients:
```

```
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.377875   0.005692   66.39   <2e-16 ***
## Ethnic      0.149138   0.011197   13.32   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2291 on 6356 degrees of freedom
##   (850 observations deleted due to missingness)
## Multiple R-squared:  0.02715,    Adjusted R-squared:  0.027
## F-statistic: 177.4 on 1 and 6356 DF,  p-value: < 2.2e-16
```

```r
library(stargazer) #for making LaTex tables
```

```
##
## Please cite as:
```

```
##  Hlavac, Marek (2018). stargazer: Well-Formatted Regression and Summary Statistics Tables.
```

```
##  R package version 5.2.2. https://CRAN.R-project.org/package=stargazer
```

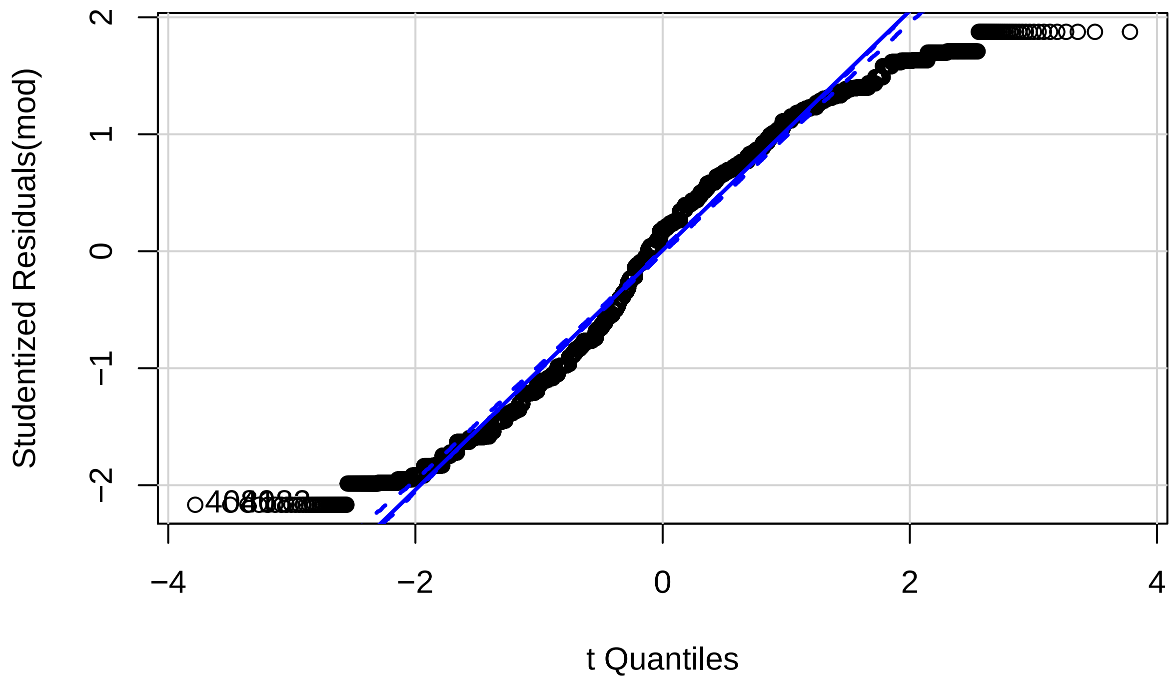
```r
stargazer(mod)
```

```
##
## % Table created by stargazer v.5.2.2 by Marek Hlavac, Harvard University. E-mail: hlavac at fas.harva
## % Date and time: Tue, Sep 15, 2020 - 02:43:47 PM
## \begin{table}[!htbp] \centering
##   \caption{}
##   \label{}
## \begin{tabular}{@{\extracolsep{5pt}}lc}
## \\[-1.8ex]\hline
## \hline \\[-1.8ex]
##  & \multicolumn{1}{c}{\textit{Dependent variable:}} \\
## \cline{2-2}
## \\[-1.8ex] & Religion \\
## \hline \\[-1.8ex]
##  Ethnic & 0.149$^{***}$ \\
##   & (0.011) \\
##   & \\
##  Constant & 0.378$^{***}$ \\
##   & (0.006) \\
##   & \\
## \hline \\[-1.8ex]
## Observations & 6,358 \\
## R$^{2}$ & 0.027 \\
## Adjusted R$^{2}$ & 0.027 \\
## Residual Std. Error & 0.229 (df = 6356) \\
## F Statistic & 177.412$^{***}$ (df = 1; 6356) \\
## \hline
## \hline \\[-1.8ex]
## \textit{Note:}  & \multicolumn{1}{r}{$^{*}$p$<$0.1; $^{**}$p$<$0.05; $^{***}$p$<$0.01} \\
## \end{tabular}
## \end{table}
```

```r
library(car)
```

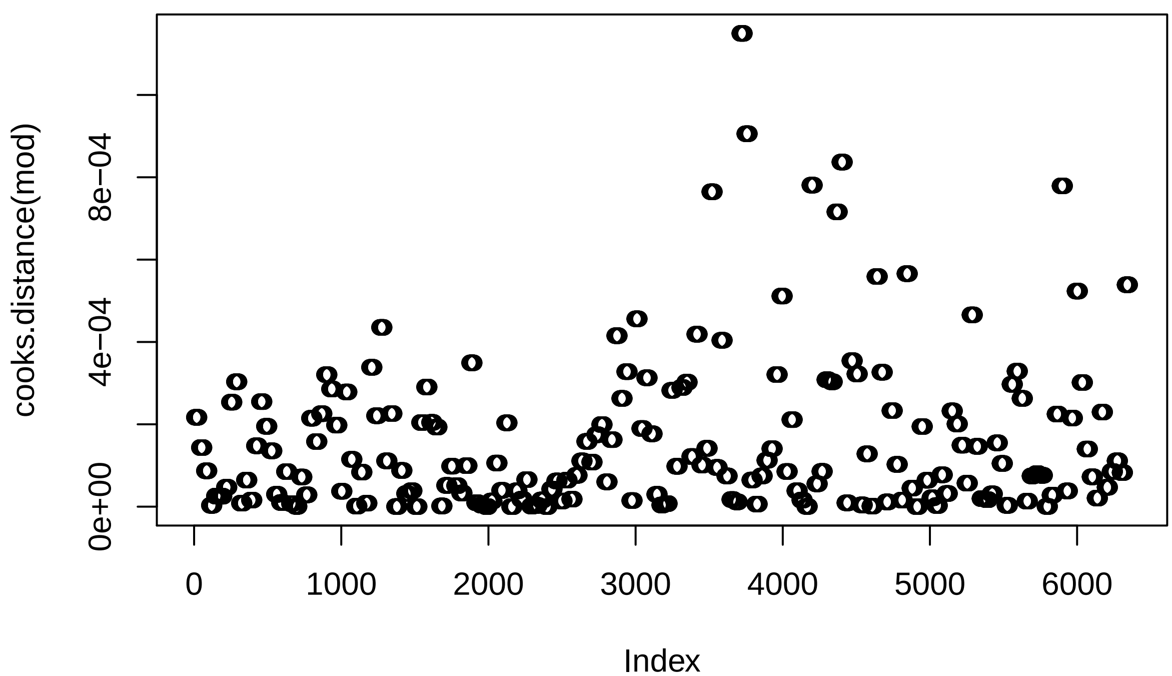
```
## Loading required package: carData
```

```r
qqPlot(mod) #check for error distribution - clearly not normal
```

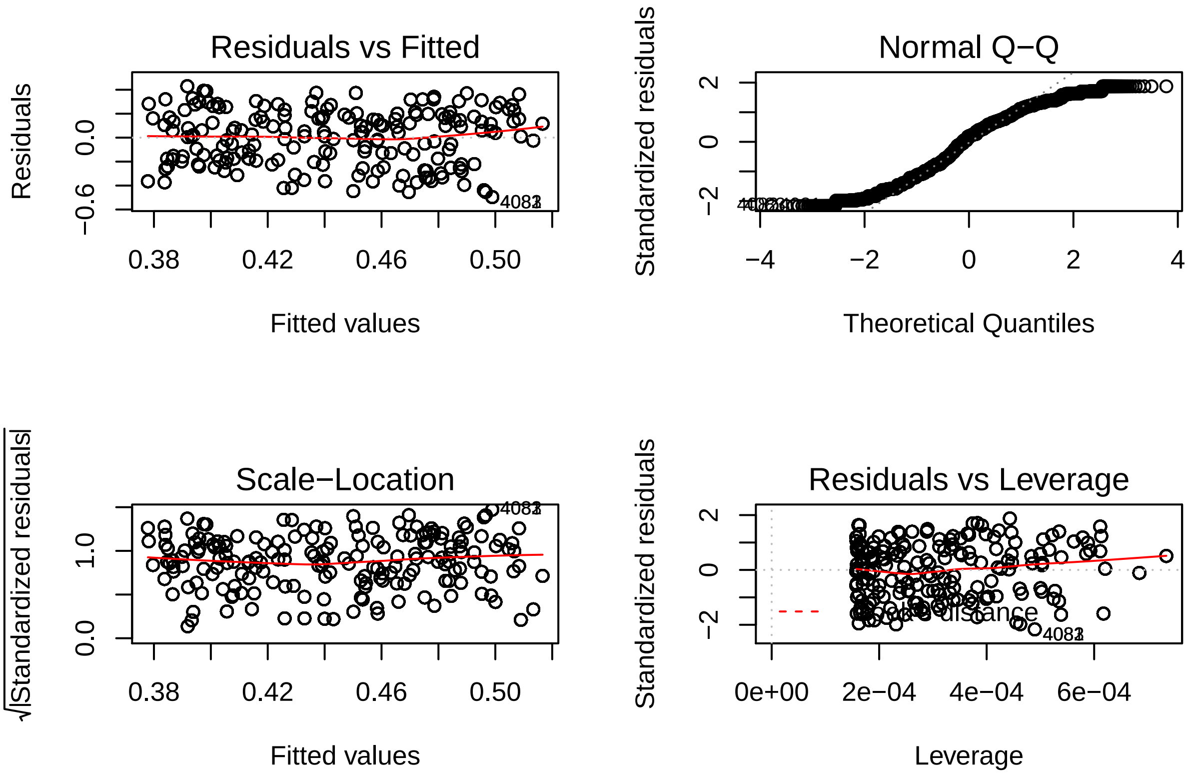



```
## [1] 4081 4082
```

```r
plot(cooks.distance(mod)) #check for cook's influence - Cook's distance shows the influence of each obs
```




```r
par(mfrow = c(2,2)) #set up the plot to be 2x2 rows x columns
plot(mod)
```




```
par(mfrow = c(1,1))
mod.null = lm(Religion ~ 1, data = data[!is.na(data$Ethnic),])
anova(mod, mod.null) #check the model against the null (typically just controls)
```

```
## Analysis of Variance Table
##
## Model 1: Religion ~ Ethnic
## Model 2: Religion ~ 1
##   Res.Df    RSS Df Sum of Sq      F    Pr(>F)
## 1   6356 333.51
## 2   6357 342.82 -1    -9.309 177.41 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- What does β actually mean?
- How would you test if this relationship is conditional on democracy levels?
- Derive the conditional effect of an interaction effect

$$\hat{y} = \beta_0 + X_1\beta_1 + X_2\beta_2 + X_1X_2\beta_3$$
$$\frac{\partial y}{\partial X_1} = \beta_1 + X_2\beta_3$$

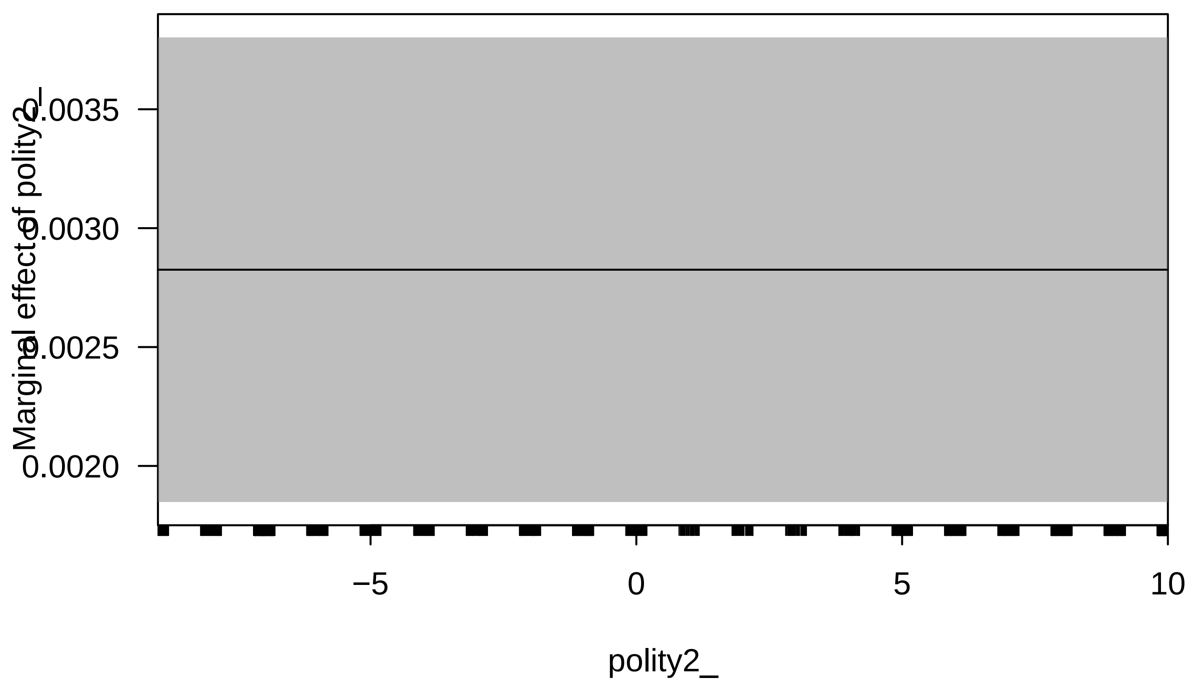
- Notice that you cannot simply look at the β estimates to understand an interactive effect

```
mod2 = lm(Religion ~ Ethnic*polity2_, data = data)
summary(mod2)
```

```
##
## Call:
## lm(formula = Religion ~ Ethnic * polity2_, data = data)
##
```

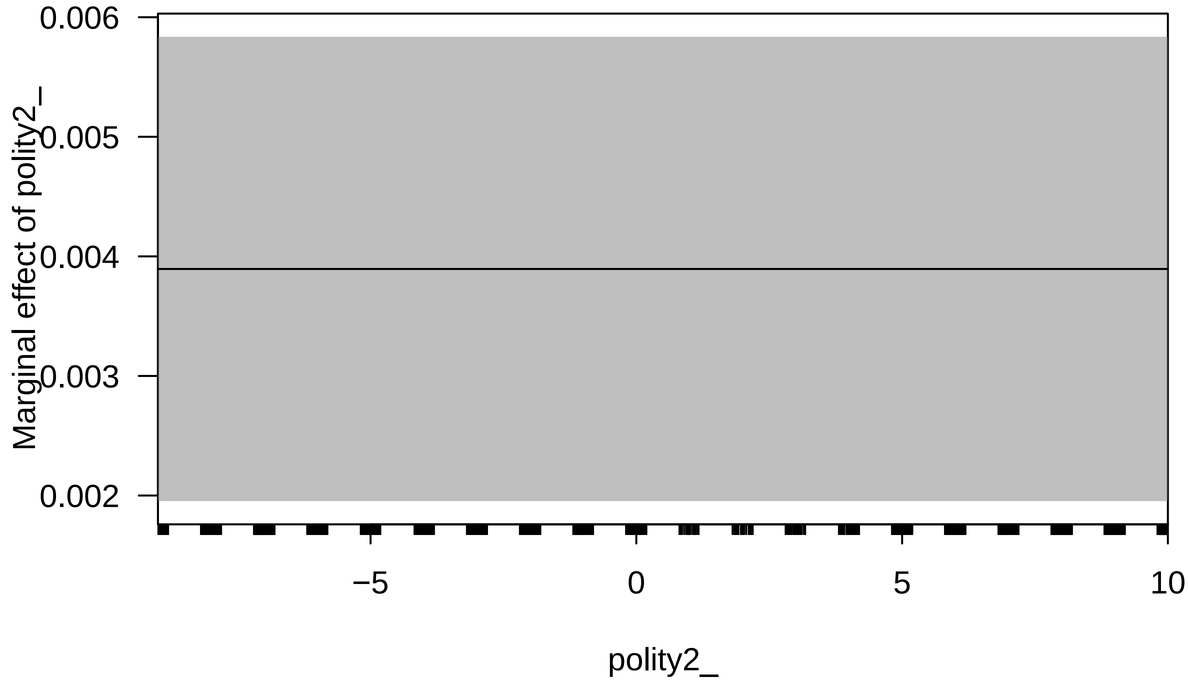
```
## Residuals:
##      Min       1Q   Median       3Q      Max 
## -0.50704 -0.17763  0.02774  0.17462  0.44512 
## 
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)    
## (Intercept)      0.3118858  0.0077763  40.107  < 2e-16 ***
## Ethnic           0.2407606  0.0138264  17.413  < 2e-16 ***
## polity2_         0.0055995  0.0009867   5.675 1.47e-08 ***
## Ethnic:polity2_ -0.0062858  0.0019421  -3.237  0.00122 ** 
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Residual standard error: 0.2321 on 4999 degrees of freedom
##   (2205 observations deleted due to missingness)
## Multiple R-squared:  0.05739,    Adjusted R-squared:  0.05683 
## F-statistic: 101.5 on 3 and 4999 DF,  p-value: < 2.2e-16
```

```r
library(margins)
cplot(mod2, x = 'polity2_', what = 'effect', data = data)
```

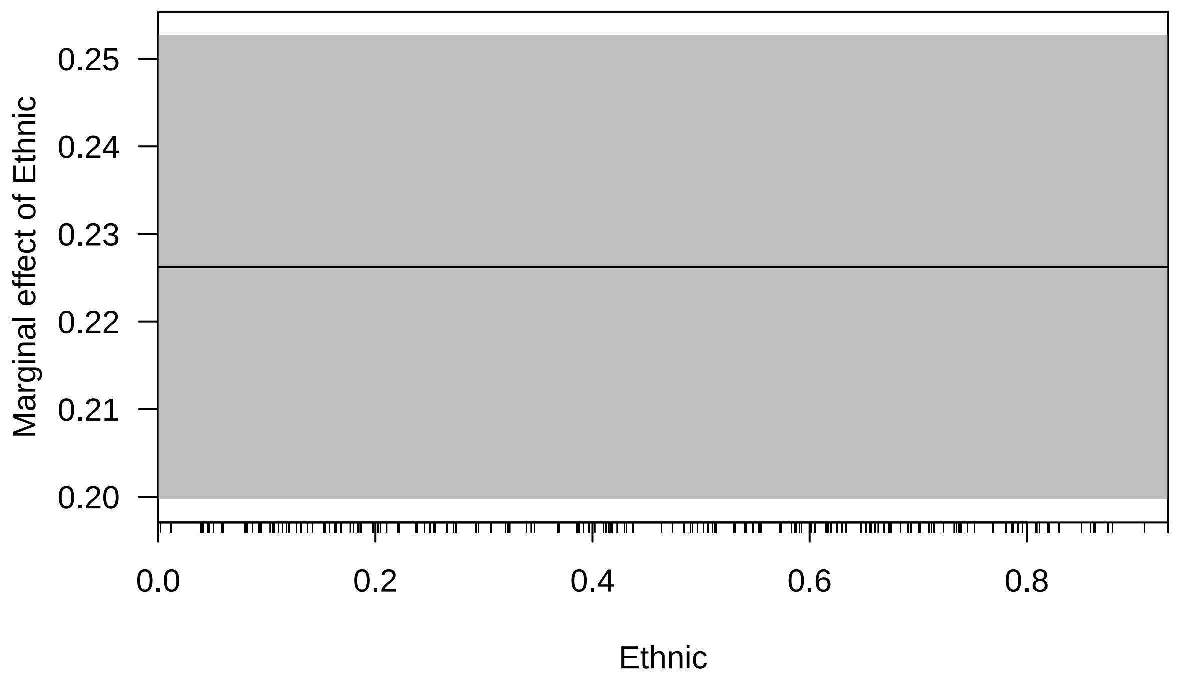


- Interpret the plot
- Let's add a factor for democracy

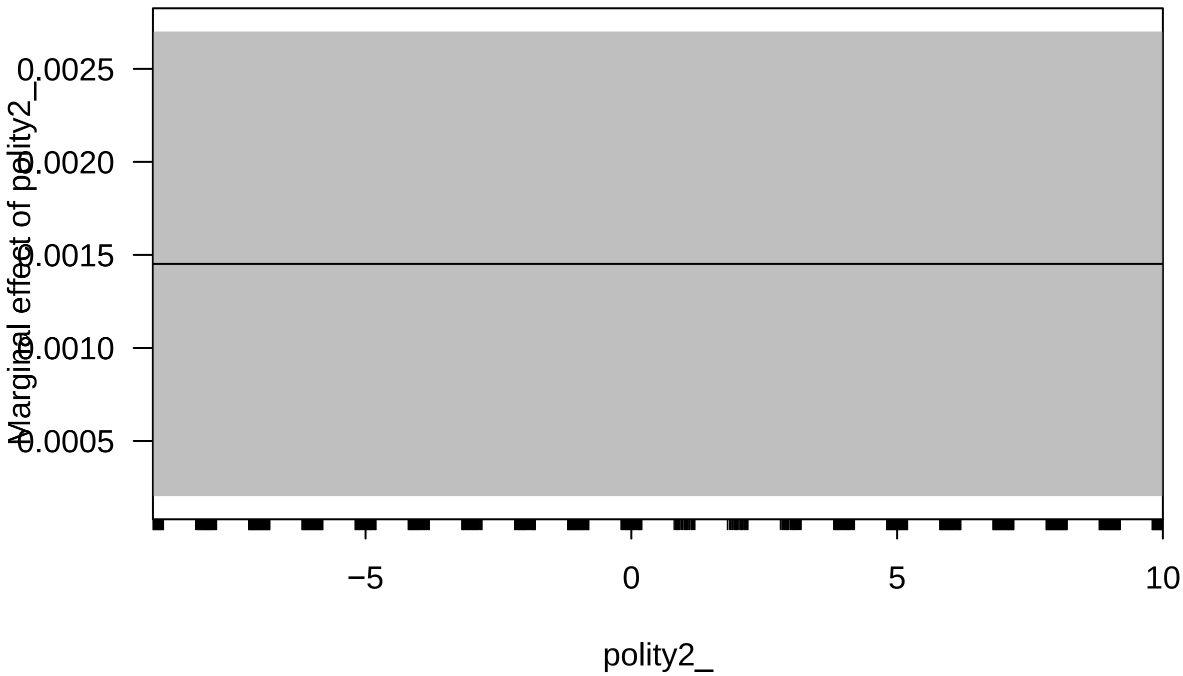
```r
mod3 = lm(Religion ~ Ethnic*polity2_ + I(polity2_ > 5), data = data)
cplot(mod3, x = 'polity2_', what = 'effect', data = data)
```




```r
cplot(mod3, x = 'Ethnic', what = 'effect', data = data)
```

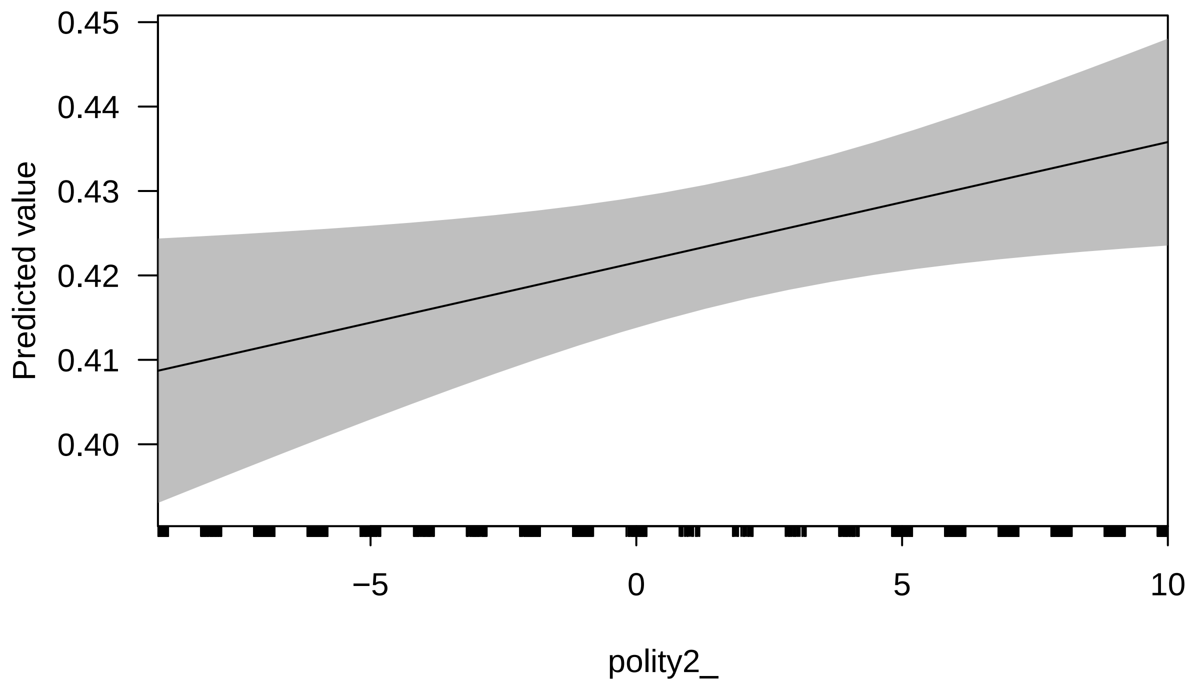



```r
mod4 = lm(Religion ~ Ethnic*polity2_ + democracy, data = data)
cplot(mod4, x = 'polity2_', what = 'effect', data = data)
```




```
cplot(mod4, x = 'polity2_', what = 'prediction', data = data)
```

```
##          xvals     yvals     upper     lower
## 1  -9.0000000 0.4087080 0.4243681 0.3930479
## 2  -8.2083333 0.4098366 0.4246282 0.3950451
## 3  -7.4166667 0.4109652 0.4249042 0.3970262
## 4  -6.6250000 0.4120938 0.4251994 0.3989881
## 5  -5.8333333 0.4132224 0.4255177 0.4009271
## 6  -5.0416667 0.4143509 0.4258638 0.4028380
## 7  -4.2500000 0.4154795 0.4262440 0.4047150
## 8  -3.4583333 0.4166081 0.4266658 0.4065504
## 9  -2.6666667 0.4177367 0.4271385 0.4083348
## 10 -1.8750000 0.4188653 0.4276737 0.4100569
## 11 -1.0833333 0.4199939 0.4282846 0.4117031
## 12 -0.2916667 0.4211224 0.4289863 0.4132585
## 13  0.5000000 0.4222510 0.4297942 0.4147078
## 14  1.2916667 0.4233796 0.4307223 0.4160369
## 15  2.0833333 0.4245082 0.4317804 0.4172360
## 16  2.8750000 0.4256368 0.4329723 0.4183013
## 17  3.6666667 0.4267654 0.4342946 0.4192361
## 18  4.4583333 0.4278939 0.4357378 0.4200501
## 19  5.2500000 0.4290225 0.4372879 0.4207571
## 20  6.0416667 0.4301511 0.4389297 0.4213725
```



- Despite the reliable estimates, it is pretty clear we do not need an interaction, and the effect does not change marginally
- Let's look at a different example

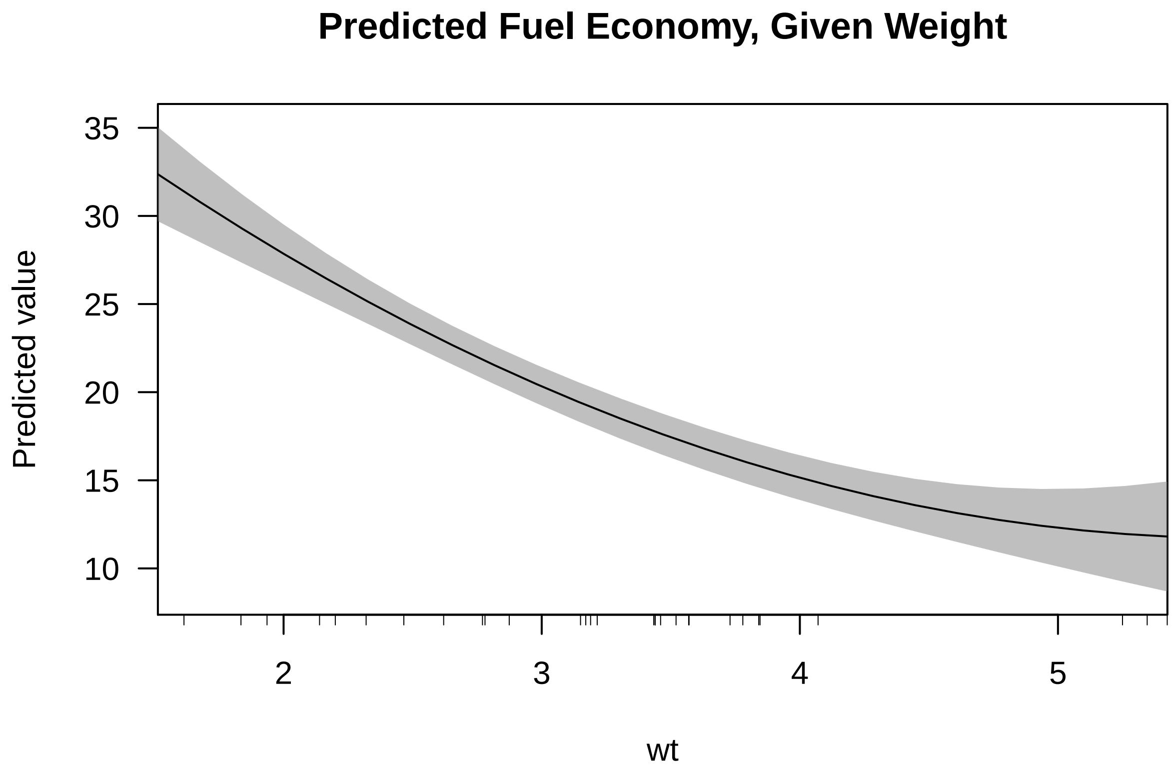
```
mod5 = lm(mpg ~ wt + I(wt^2), data = mtcars)
margins(mod5)
```

```
## Average marginal effects

## lm(formula = mpg ~ wt + I(wt^2), data = mtcars)

##      wt
##  -5.845
```

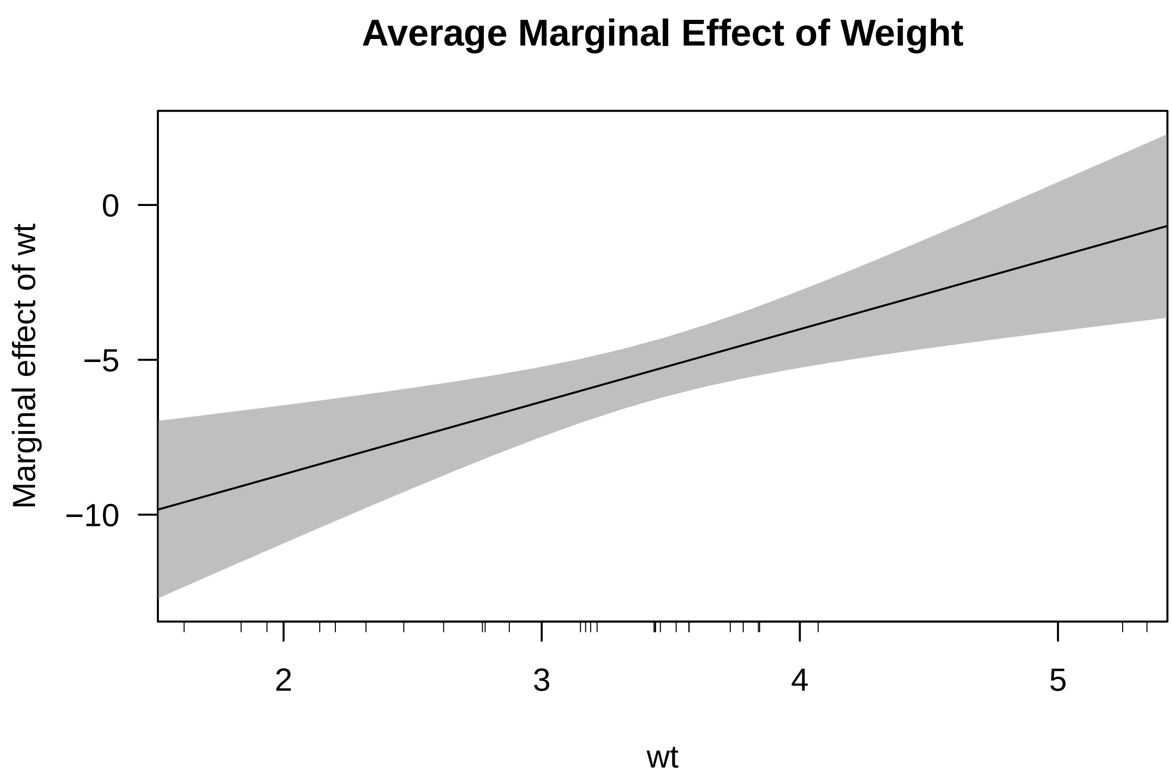
```
cplot(mod5, "wt", what = "prediction", main = "Predicted Fuel Economy, Given Weight")
```

```
##       xvals    yvals    upper    lower
## 1  1.513000 32.36718 35.03500 29.69936
## 2  1.675958 30.79531 33.07715 28.51348
## 3  1.838917 29.28565 31.23015 27.34115
## 4  2.001875 27.83818 29.49769 26.17868
## 5  2.164833 26.45291 27.88384 25.02199
## 6  2.327792 25.12984 26.39172 23.86796
## 7  2.490750 23.86897 25.02106 22.71687
## 8  2.653708 22.67029 23.76564 21.57494
## 9  2.816667 21.53381 22.61340 20.45422
## 10 2.979625 20.45953 21.54975 19.36930
## 11 3.142583 19.44744 20.56171 18.33318
## 12 3.305542 18.49755 19.64011 17.35500
## 13 3.468500 17.60986 18.78010 16.43963
## 14 3.631458 16.78437 17.98078 15.58796
## 15 3.794417 16.02108 17.24475 14.79740
## 16 3.957375 15.31998 16.57774 14.06221
## 17 4.120333 14.68108 15.98801 13.37414
## 18 4.283292 14.10437 15.48533 12.72342
```

```
## 19 4.446250 13.58987 15.07937 12.10037
## 20 4.609208 13.13756 14.77796 11.49716
```




```r
cplot(mod5, "wt", what = "effect", main = "Average Marginal Effect of Weight")
```



```r
mod6 = lm(mpg ~ hp * wt, data = mtcars)
persp(mod6, "wt", "hp", theta = c(45, 135, 225, 315), what = "effect")
```