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Bayes.pdf

1. Question:
   My neighbor has two children. Assuming that the gender of a child is like a coin flip, it is most likely, a priori, that my neighbor has one boy and one girl, with probability 1/2. The other possibilities—two boys or two girls—have probabilities 1/4 and 1/4.

   a. Suppose I ask him whether he has any boys, and he says yes. What is the probability that one child is a girl?

   b. Suppose instead that I happen to see one of his children run by, and it is a boy. What is the probability that the other child is a girl?

1. Answer:
   a. The probability that the other child is a boy is ½. The initial probability has no effect on this new situation. The new situation is: there is one child whose sex is unknown, what is the probability that it is a girl? This is answered simply by considering the probability that a child is a boy or a girl, which is given in the question as ½.
   b. ½. See above reasoning. (I don't see the distinction between these two cases. In each of them we are initially given that that man has two children, and in each we learn that at least one of his children is a boy.)

2. Question:
   Suppose a crime has been committed. Blood is found at the scene for which there is no innocent explanation. It is of a type which is present in 1% of the population.
   a. The prosecutor claims: There is a 1% chance that the defendant would have the crime blood type if he were innocent. Thus there is a 99% chance that he guilty. This is known as the prosecutor's fallacy. What is wrong with this argument?
   b. The defender claims: The crime occurred in a city of 800,000 people. The blood type would be found in approximately 8000 people. The evidence has provided a probability of just 1 in 8000 that the defendant is guilty, and thus has no relevance at all. This is known as the defender's fallacy. What is wrong with this argument?

3. Answer:
   a. The prosecutor confuses the probability of guilt with the probability of having a particular blood type. It is true that only someone with this particular blood type could be guilty of the crime, but this doesn't

imply that any person with this particular blood type has a 99% chance of guilt. In fact, as we shall see momentarily in the defenders fallacy, given a 1% probability of having this particular blood type in a city of 800,000 people, there are 8,000 total people in the city who could be guilty of the crime. As such, it is clearly not possible that each among 8,000 of them has a 99% chance of being guilty.

Knowing the blood type does help the prosecutor's case. Before hand there was a 1 in 800,000 chance that a person selected at random in the city is the criminal. By knowing the blood type the prosecutor has reduced those odds to 1 in 8,000 people who have this particular blood type. The prosecutor is still required to demonstrate that the one person selected is the guilty party.

b. The defender is again mixing up probabilities. He is confusing the probability of randomly selecting the guilty party from among 8,000 people with the probability that this particular defendant is guilty. Supposing that the prosecutor has some other evidence suggesting the defendant could be the guilty party then the defendant was not chosen at random and therefore the probability he is guilty is not 1 in 8,000.