ISLR Chapter 9 Exercises

2023-07-01

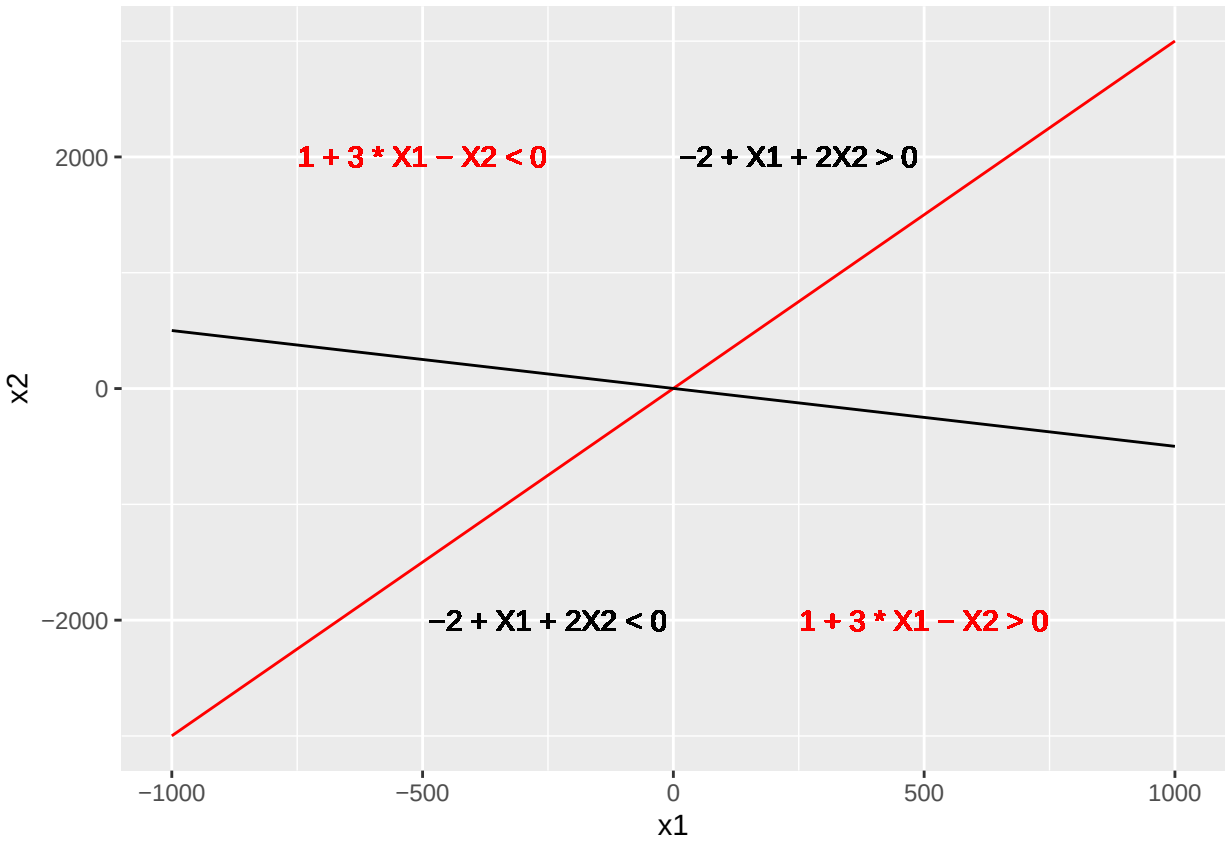
Contents


```
library(ggplot2)
```

Conceptual

Question 1

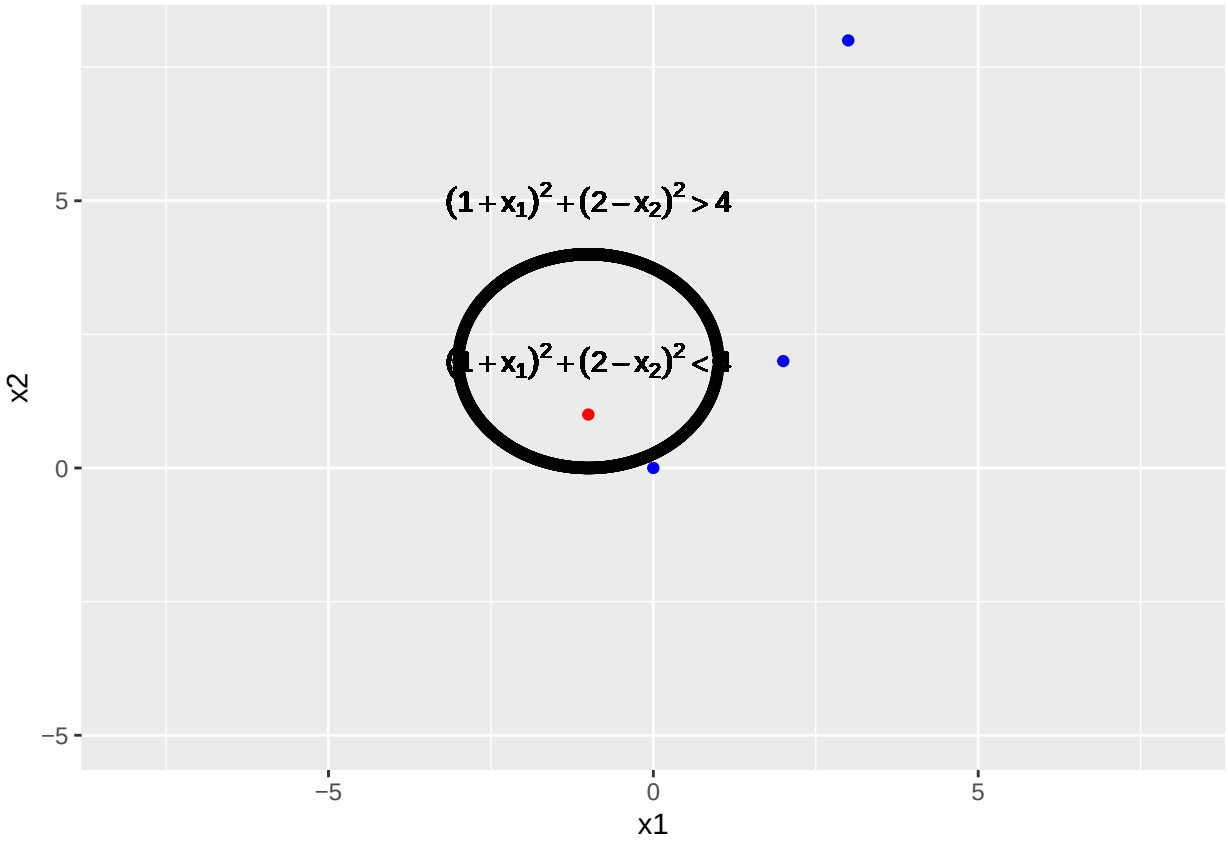
```
x1 <- seq(-1000, 1000)
x2 <- 1 + 3 * x1
x3 <- 1 - x1 / 2

df_for_plot <- data.frame(x1 = x1, x2 = x2, x3 = x3)

ggplot2::ggplot(data = df_for_plot) +
  ggplot2::geom_line(ggplot2::aes(x = x1, y = x2), color = "red") +
  ggplot2::geom_text(x = -500, y = 2000, label = "1 + 3 * X1 - X2 < 0", color = "red") +
  ggplot2::geom_text(x = 500, y = -2000, label = "1 + 3 * X1 - X2 > 0", color = "red") +
  ggplot2::geom_line(ggplot2::aes(x = x1, y = x3), color = "black") +
  ggplot2::geom_text(x = -250, y = -2000, label = "-2 + X1 + 2X2 < 0") +
  ggplot2::geom_text(x = 250, y = 2000, label = "-2 + X1 + 2X2 > 0")
```



Question 2

```r
x1 <- seq(-3, 1, length = 1000)

x2 <- c(
  2 + sqrt(4 - (1 + x1) ^ 2),
  2 - sqrt(4 - (1 + x1) ^ 2)
)
x1 <- rep(x1, 2)

df_for_plot <- data.frame(x1 = x1, x2 = x2)


df_test_points <- data.frame(
  x1 = c(0, -1, 2, 3),
  x2 = c(0, 1, 2, 8),
  predicted_class = c("blue", "red", "blue", "blue")
)

ggplot2::ggplot(data = df_for_plot) +
  ggplot2::geom_point(ggplot2::aes(x = x1, y = x2)) +
  ggplot2::geom_text(x = -1, y = 5, label = "(1 + x[1])^2 + (2 - x[2])^2 > 4", parse = TRUE) +
  ggplot2::geom_text(x = -1, y =2, label = "(1 + x[1])^2 + (2 - x[2])^2 < 4", parse = TRUE) +
  ggplot2::geom_point(data = df_test_points, ggplot2::aes(x = x1, y = x2, color = predicted_class)) +
```

```
  ggplot2::scale_color_manual(values = list(blue = "blue", red = "red"), breaks = NULL) +
  ggplot2::lims(x = c(-8, 8), y = c(-5, 8))
```



$(1 + X_1)^2 + (2 - X_2)^2 = 4$

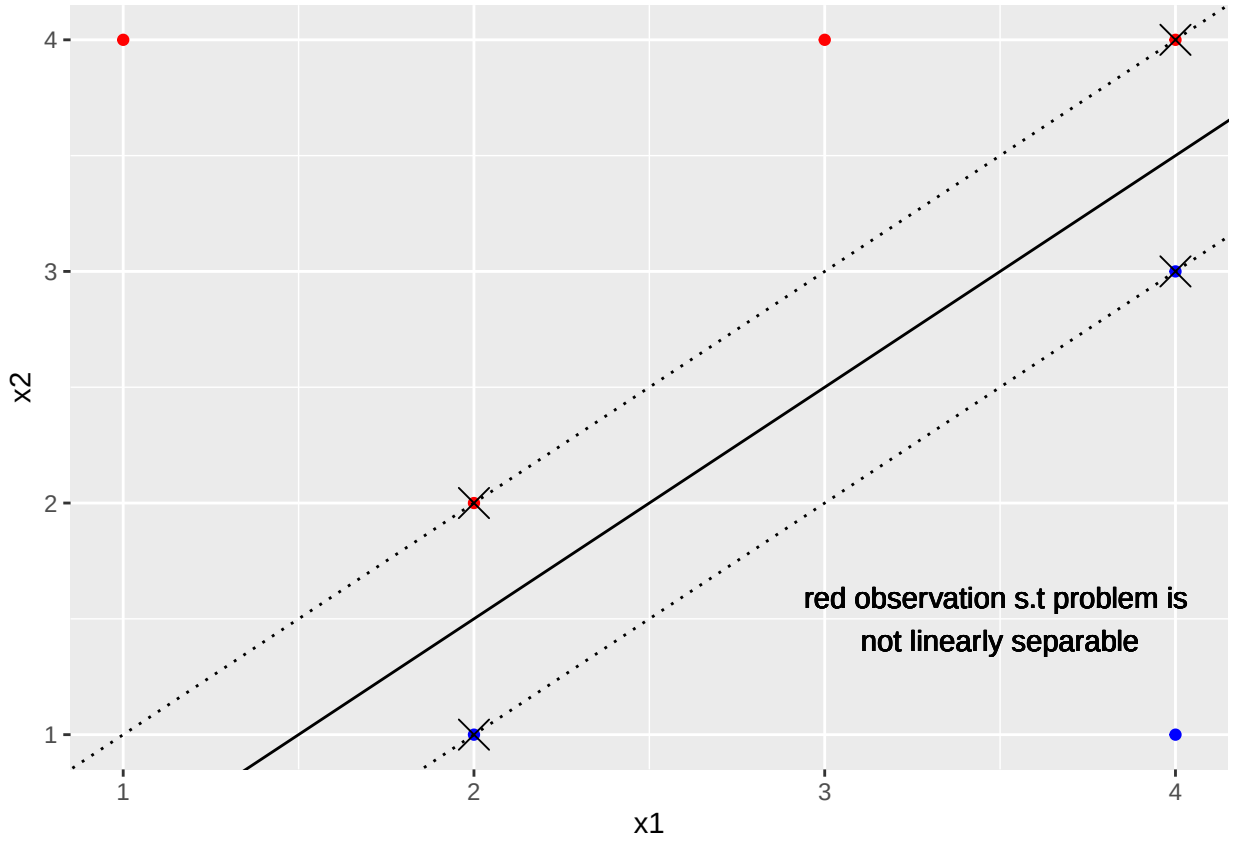
$1 + 2X_1 + X_1^2 + 4 - 4X_2 + X_2^2 = 4$

which is linear in X_1, X_1^2, X_2, X_2^2

Question 3

```
df <- data.frame(
  x1 = c(3, 2, 4, 1, 2, 4, 4),
  x2 = c(4, 2, 4, 4, 1, 3, 1),
  y = c(rep("red", 4), rep("blue", 3))
)

support_vectors <- data.frame(
  x1 = c(2, 4, 2, 4),
  x2 = c(2, 4, 1, 3)
)

slope <- 1
intercept <- -0.5

ggplot2::ggplot(data = df) +
```

```
  ggplot2::geom_point(ggplot2::aes(x = x1, y = x2, color = y)) +
  ggplot2::scale_color_manual(values = list(red = "red", blue = "blue"), breaks = NULL) +
  ggplot2::geom_abline(intercept = intercept, slope = slope) +
  ggplot2::geom_abline(intercept = intercept + 0.5, slope = slope, linetype = 3) +
  ggplot2::geom_abline(intercept = intercept - 0.5, slope = slope, linetype = 3) +
  ggplot2::geom_point(data = support_vectors, ggplot2::aes(x = x1, y = x2), shape = 4, size = 5) +
  ggplot2::geom_text(x = 3.5, y = 1.5, label = "red observation s.t problem is \nnot linearly separable
```



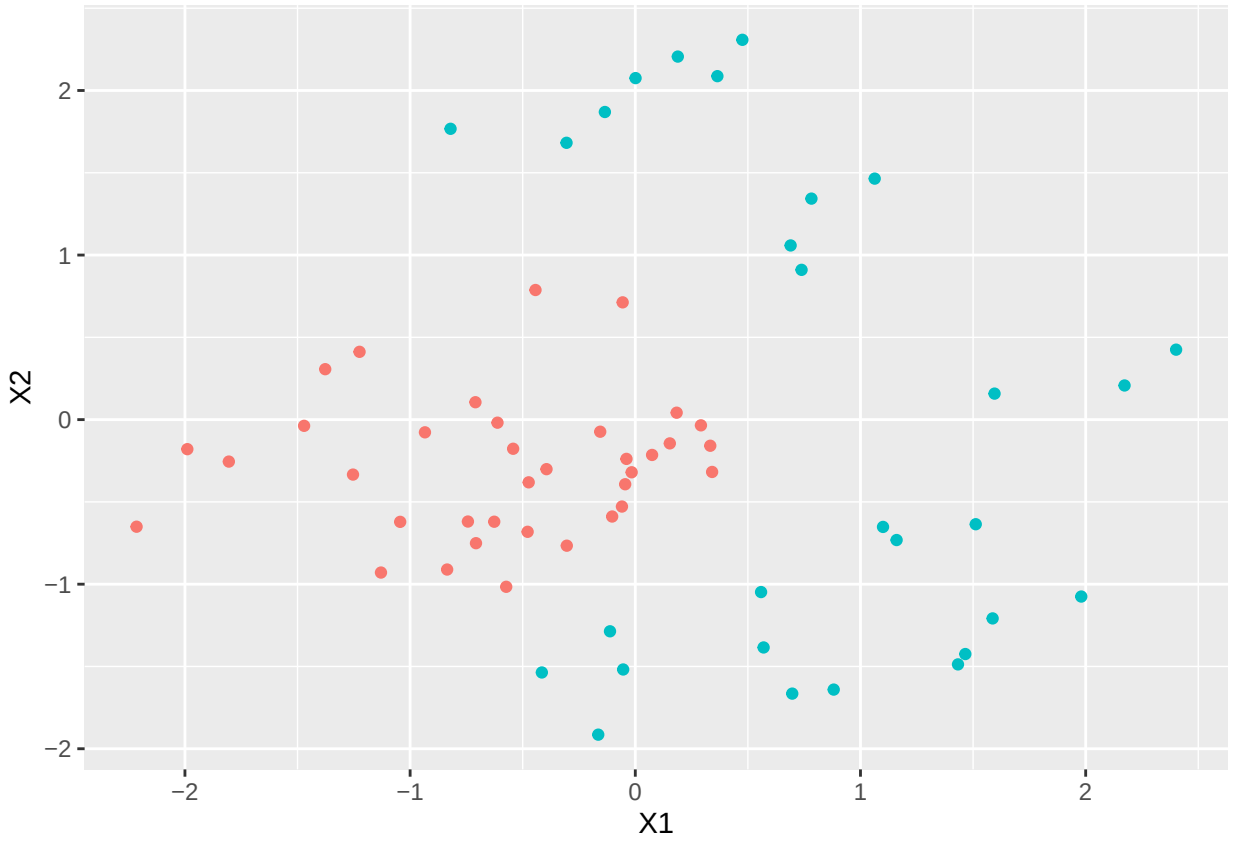
Classify to `red` if $-0.5 + \beta_1 - \beta_2 < 0$, classify to `blue` otherwise. Point 7 is not a support vector so a slight perturbation will not change the optimization problem. Either of the dotted lines would not be the optimal separating hyperplane; the equation of the top line is $\beta_1 - \beta_2 = 0$

Applied

Question 4

```
nrows <- 100
ncols <- 2
set.seed(1)
x <- matrix(rnorm(nrows * ncols), nrows, ncols)
response <- x[, 1] + x[, 2] ^ 2

y <- response > 1

# Make the classes clearly separable
```

```r
df <- as.data.frame(x) %>%
  setNames(., c("X1", "X2")) %>%
  dplyr::mutate(y = as.factor(y)) %>%
  dplyr::filter(., abs(response - 1) > 0.5)

ggplot2::ggplot(data = df) +
  ggplot2::geom_point(ggplot2::aes(x = X1, y = X2, color = y), show.legend = FALSE)
```




```r
nrows <- nrow(df)
train_idx <- sample(nrows, nrows %/% 2)
df_train <- df[train_idx, ]
df_test <- df[-train_idx, ]

linear_model <- e1071::svm(y ~ ., data = df_train, kernel = "linear")

table(linear_model$fitted, df_train$y, dnn = c("predicted", "actual"))
```

```
##           actual
## predicted FALSE TRUE
##     FALSE    18    4
##     TRUE      0   10
```

```r
mean(linear_model$fitted != df_train$y)
```

```
## [1] 0.125
```

```r
polynomial_model <- e1071::svm(y ~ ., data = df_train, kernel = "polynomial", cost = 100)
```

```r
table(polynomial_model$fitted, df_train$y, dnn = c("predicted", "actual"))
```

```
##           actual
## predicted FALSE TRUE
##     FALSE    18    1
##     TRUE      0   13
```

```r
mean(polynomial_model$fitted != df_train$y)
```

```
## [1] 0.03125
```

```r
radial_model <- e1071::svm(y ~ ., data = df_train, kernel = "radial", cost = 100)

table(radial_model$fitted, df_train$y)
```

```
##
##         FALSE TRUE
##   FALSE    18    0
##   TRUE      0   14
```

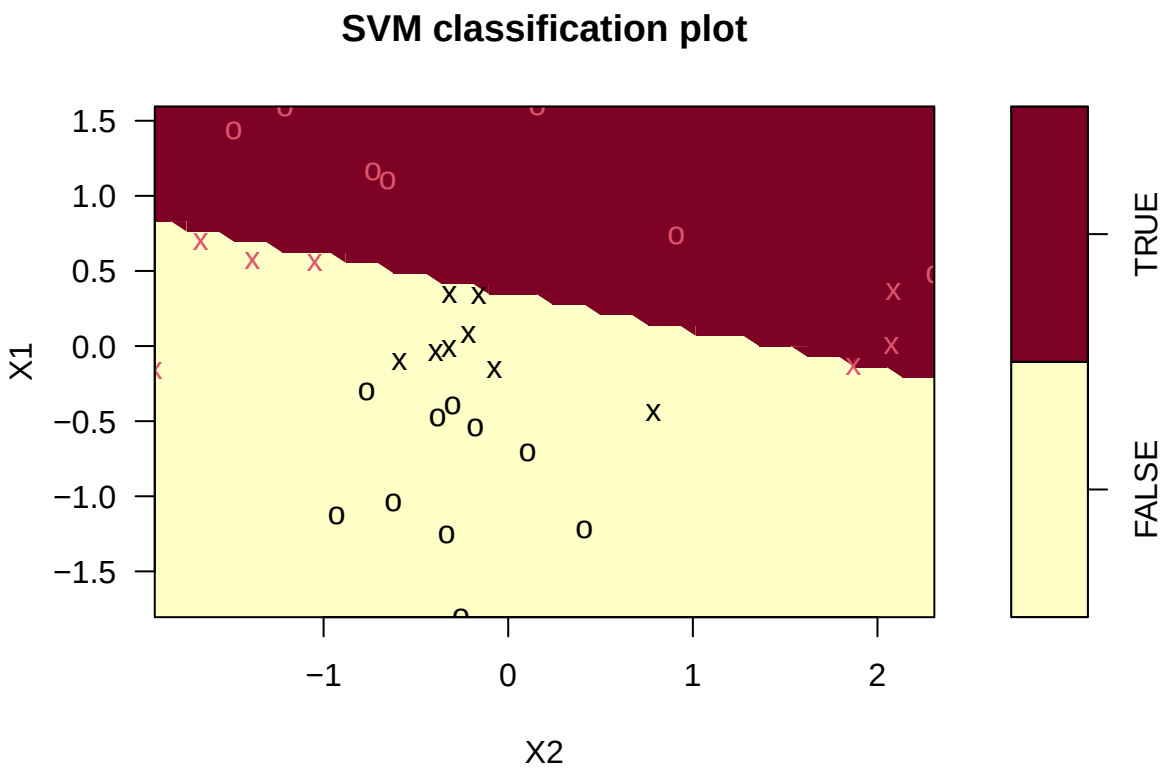
```r
mean(radial_model$fitted != df_train$y)
```

```
## [1] 0
```

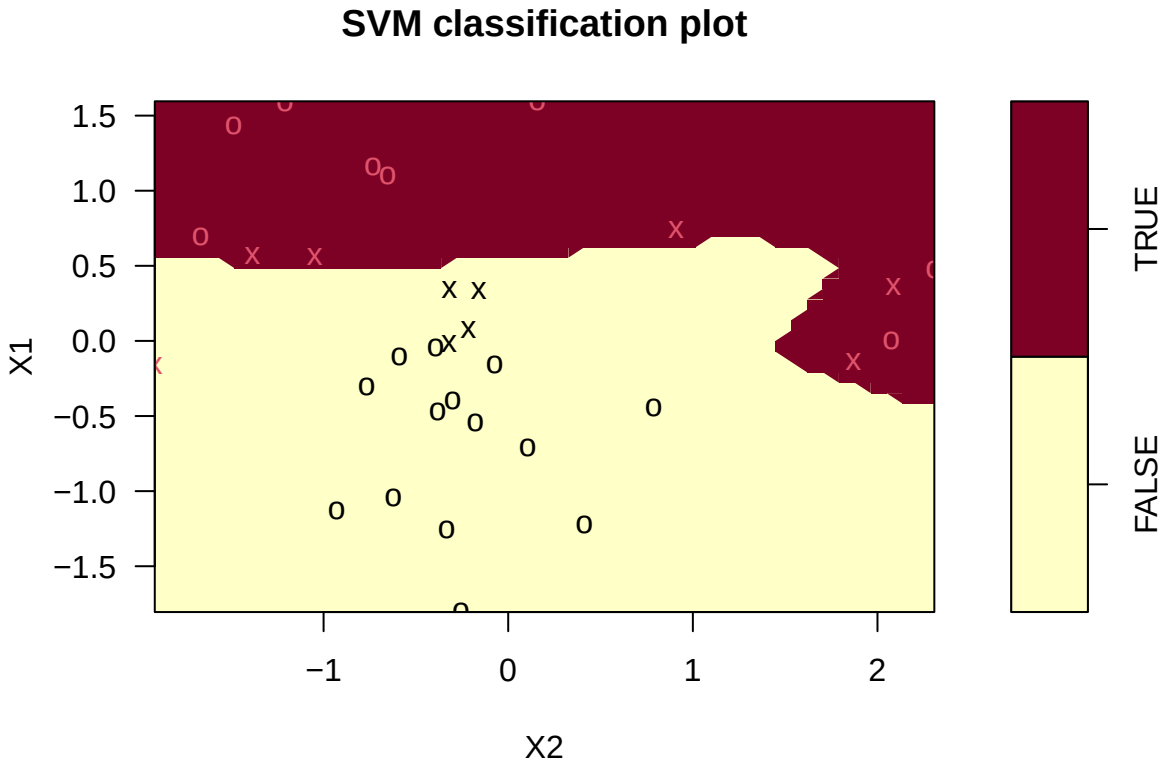
```r
models <- list(linear = linear_model, polynomial = polynomial_model, radial = radial_model)
for (model_name in names(models)) {
  preds <- predict(models[[model_name]], df_test)
  test_error_rate <- mean(preds != df_test$y)
  print(paste("Test error rate for", model_name, ":", test_error_rate))
}
```

```
## [1] "Test error rate for linear : 0.151515151515152"
## [1] "Test error rate for polynomial : 0.151515151515152"
## [1] "Test error rate for radial : 0"
```

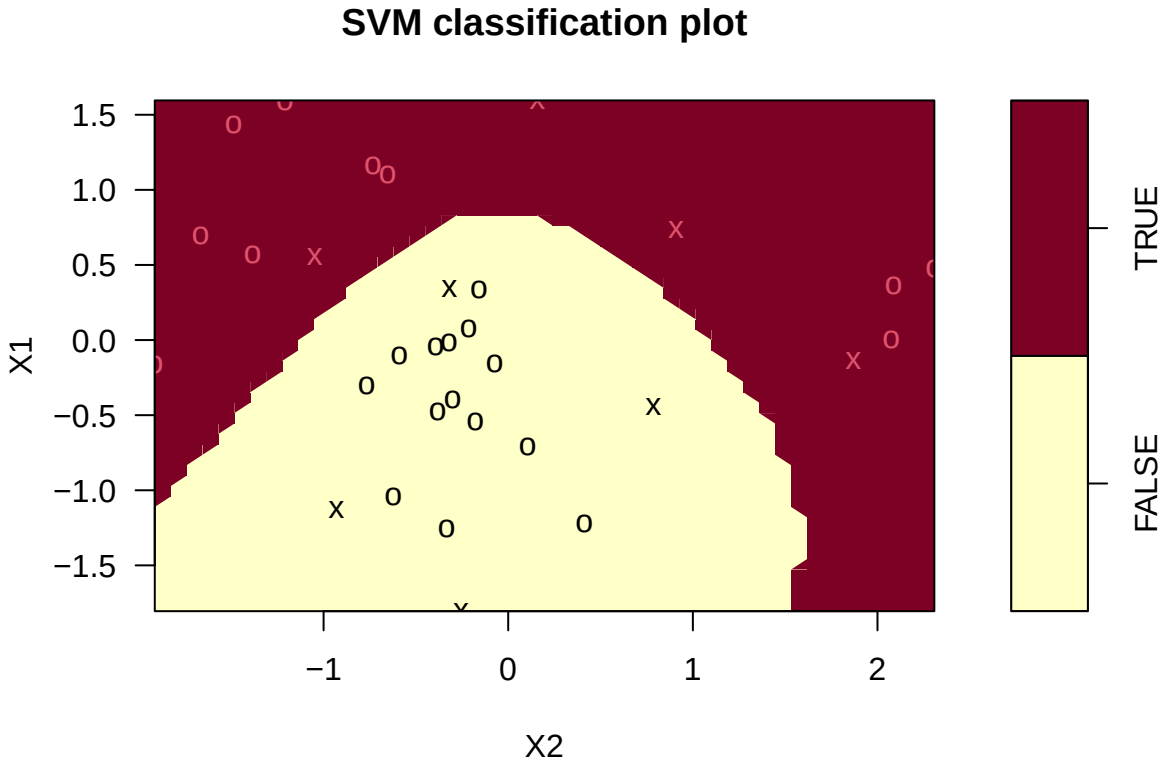
```r
plot(linear_model, data = df_train)
```




```
plot(polynomial_model, data = df_train)
```



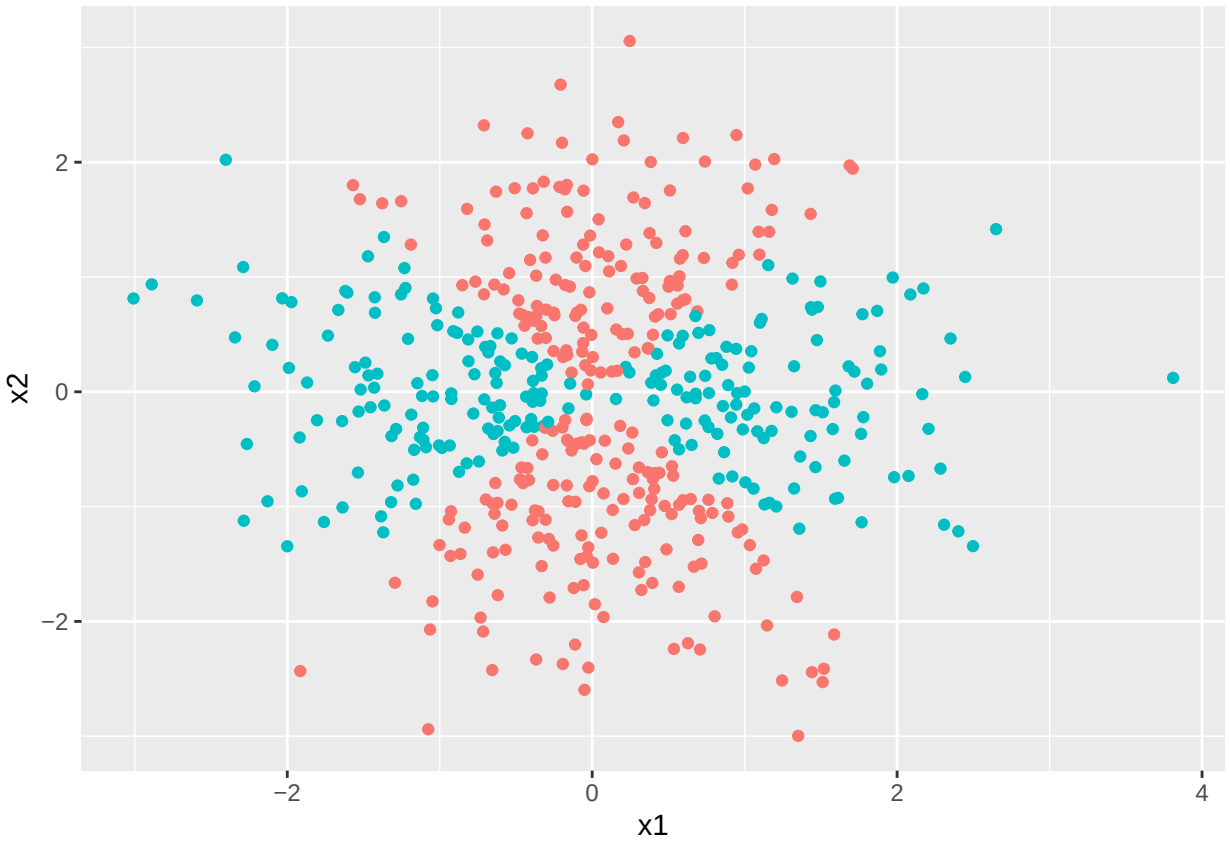

```
plot(radial_model, data = df_train)
```



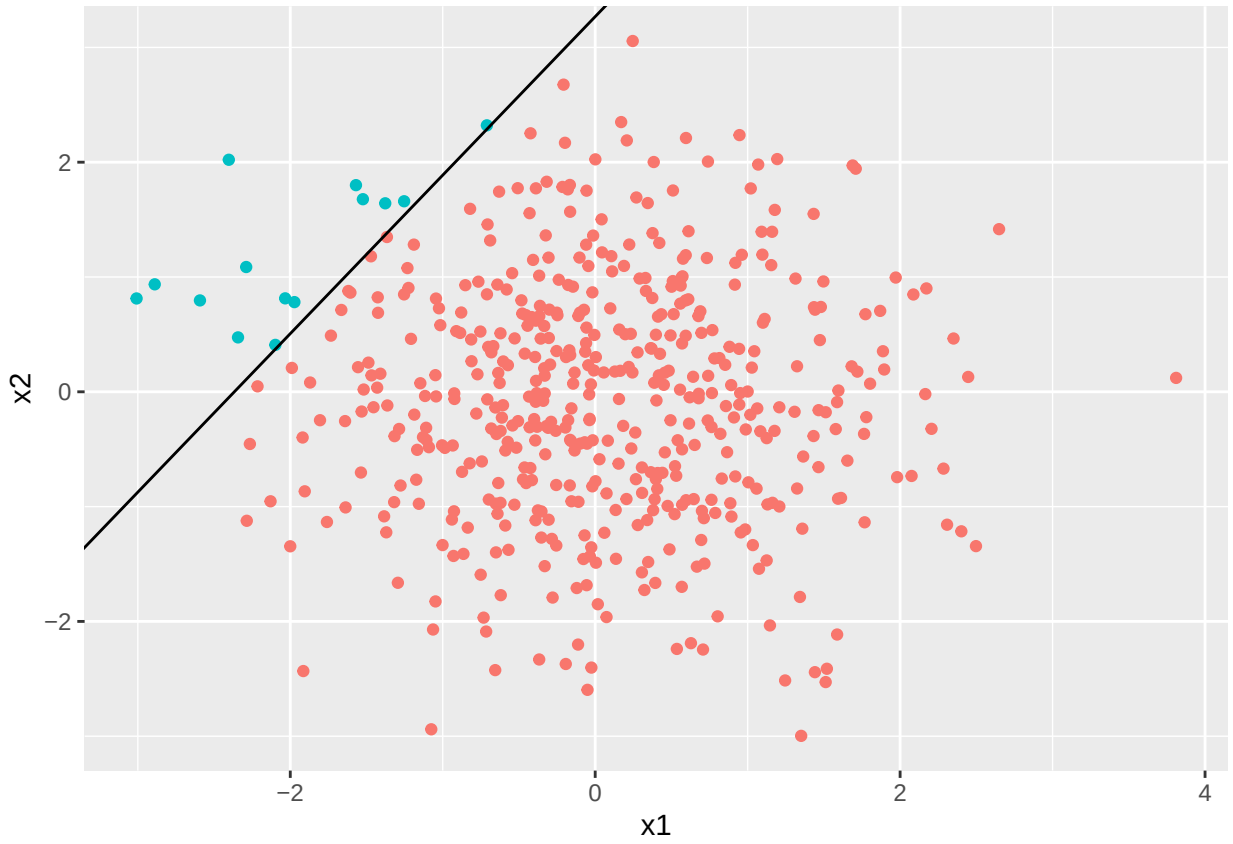
The radial model gets perfect training accuracy as well as perfect test accuracy.

Question 5

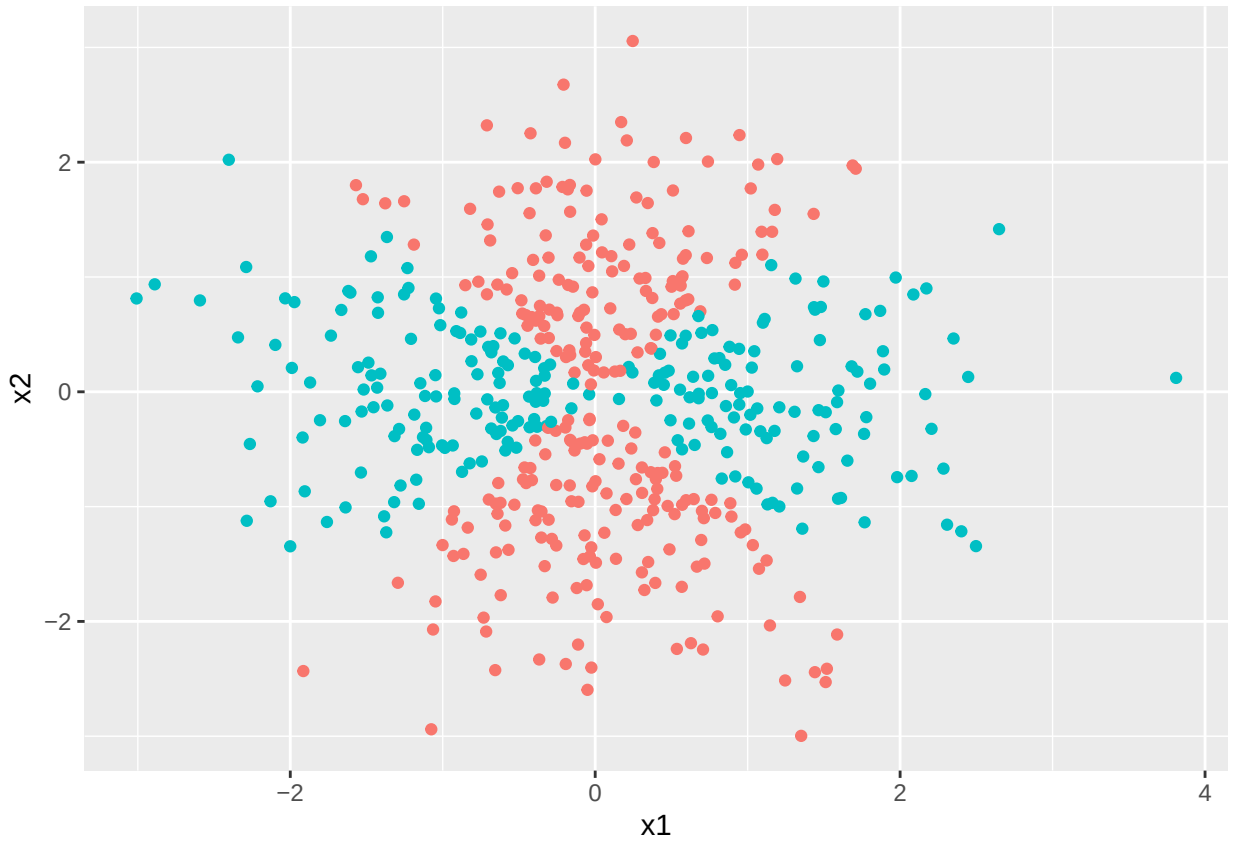
```
ncols <- 2
nrows <- 500
set.seed(1)

df <- data.frame(
  x1 = rnorm(nrows),
  x2 = rnorm(nrows)
)

df$y <- as.factor((df$x1 ^ 2 - df$x2 ^ 2) > 0)

ggplot2::ggplot(data = df) +
  ggplot2::geom_point(ggplot2::aes(x = x1, y = x2, color = y), show.legend = FALSE)
```



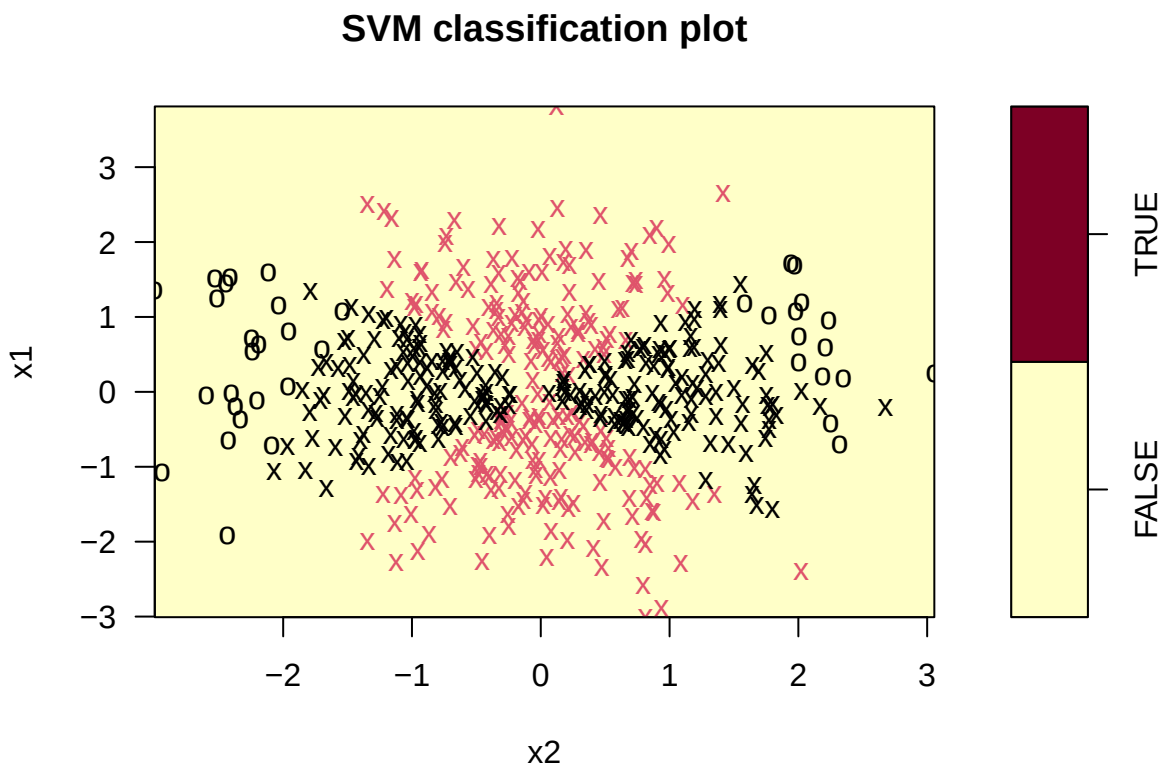

```
model <- glm(y ~ ., data = df, family = "binomial")
df$logistic_regression_pred <- as.factor(predict(model, type = "response") >= 0.5)

coefs <- coef(model)

decision_boundary_intercept <- -coefs[[1]] / coefs[[3]]
decision_boundary_slope <- -coefs[[2]] / coefs[[3]]

ggplot2::ggplot(data = df) +
  ggplot2::geom_point(ggplot2::aes(x = x1, y = x2, color = logistic_regression_pred), show.legend = FALS
  ggplot2::geom_abline(intercept = decision_boundary_intercept, slope = decision_boundary_slope)
```



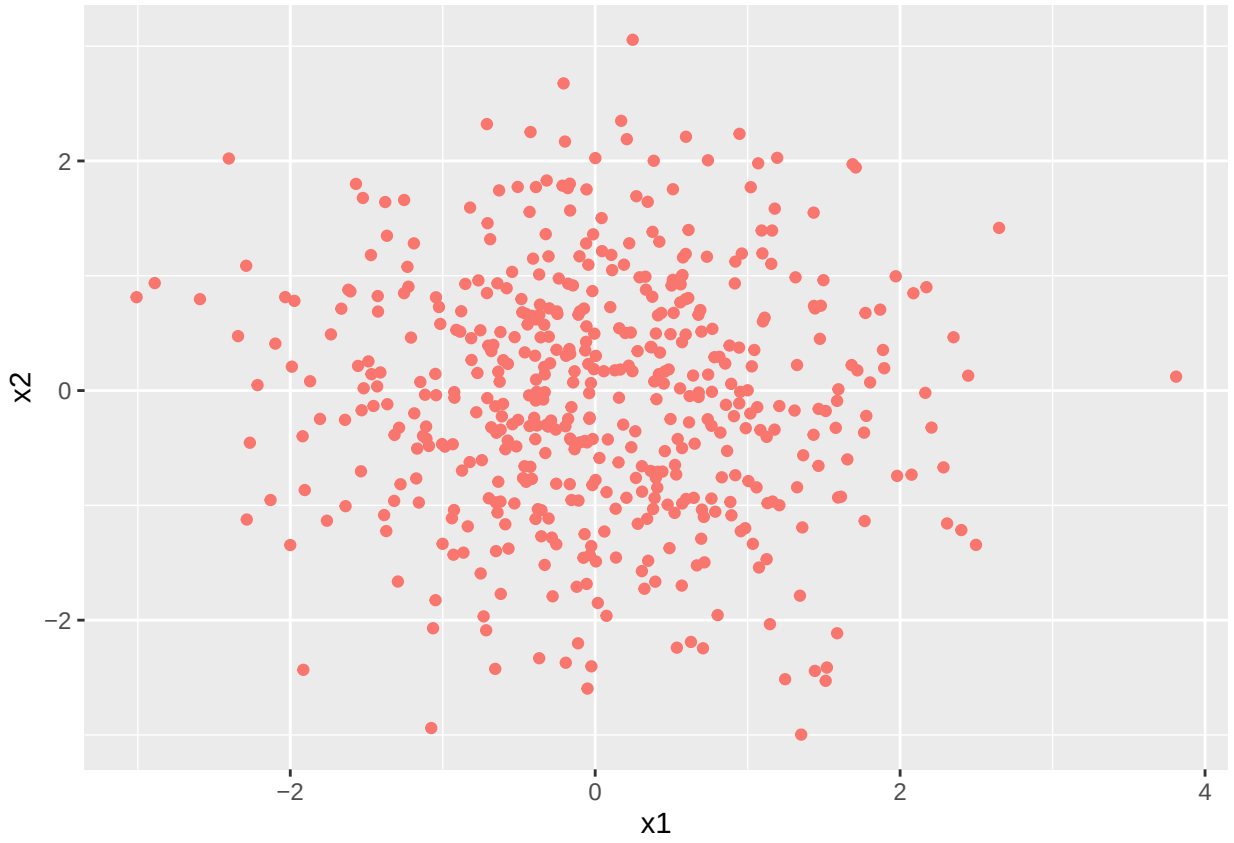

```
model <- glm(y ~ poly(x1, 2) + poly(x2, 2), data = df, family = "binomial")
df$logistic_regression_non_linear_pred <- as.factor(predict(model, type = "response") >= 0.5)

ggplot2::ggplot(data = df) +
  ggplot2::geom_point(ggplot2::aes(x = x1, y = x2, color = logistic_regression_non_linear_pred), show.le
```



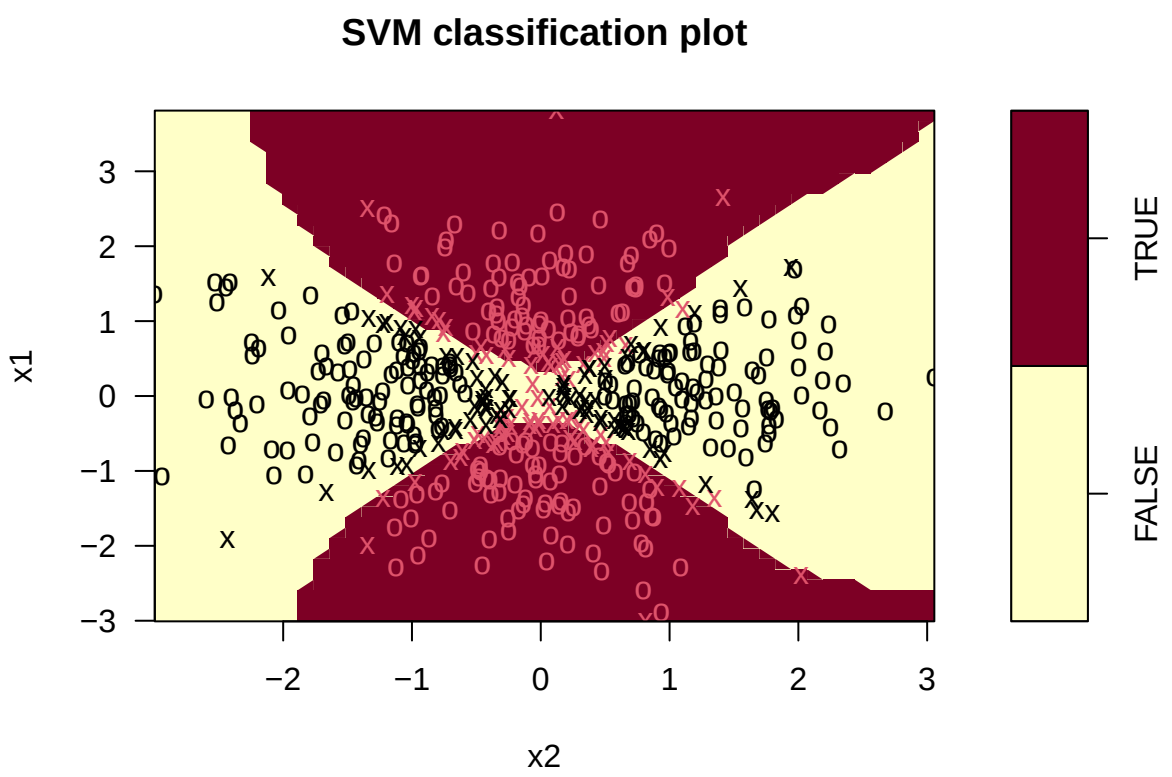

```
# Note that the classes are linearly separable so the coefficient estimates are unstable

model <- e1071::svm(y ~ x1 + x2, data = df, kernel = "linear")
df$svc_pred <- predict(model)

plot(model, df[c("y", "x1", "x2")])
```



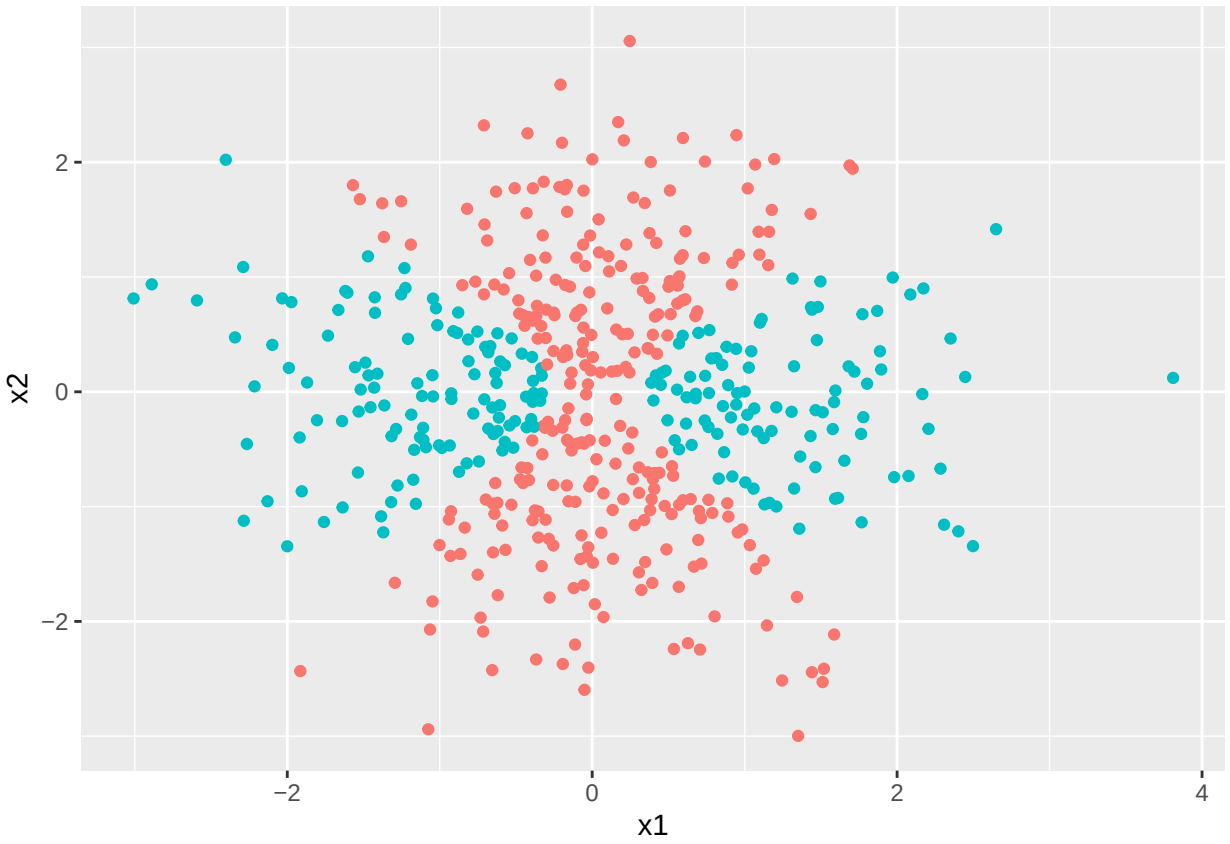

```
ggplot2::ggplot(data = df) +
  ggplot2::geom_point(ggplot2::aes(x = x1, y = x2, color = svc_pred), show.legend = FALSE)
```



```
model <- e1071::svm(y ~ x1 + x2, data = df, kernel = "radial")
df$svm_pred <- predict(model)

plot(model, df[c("y", "x1", "x2")])
```



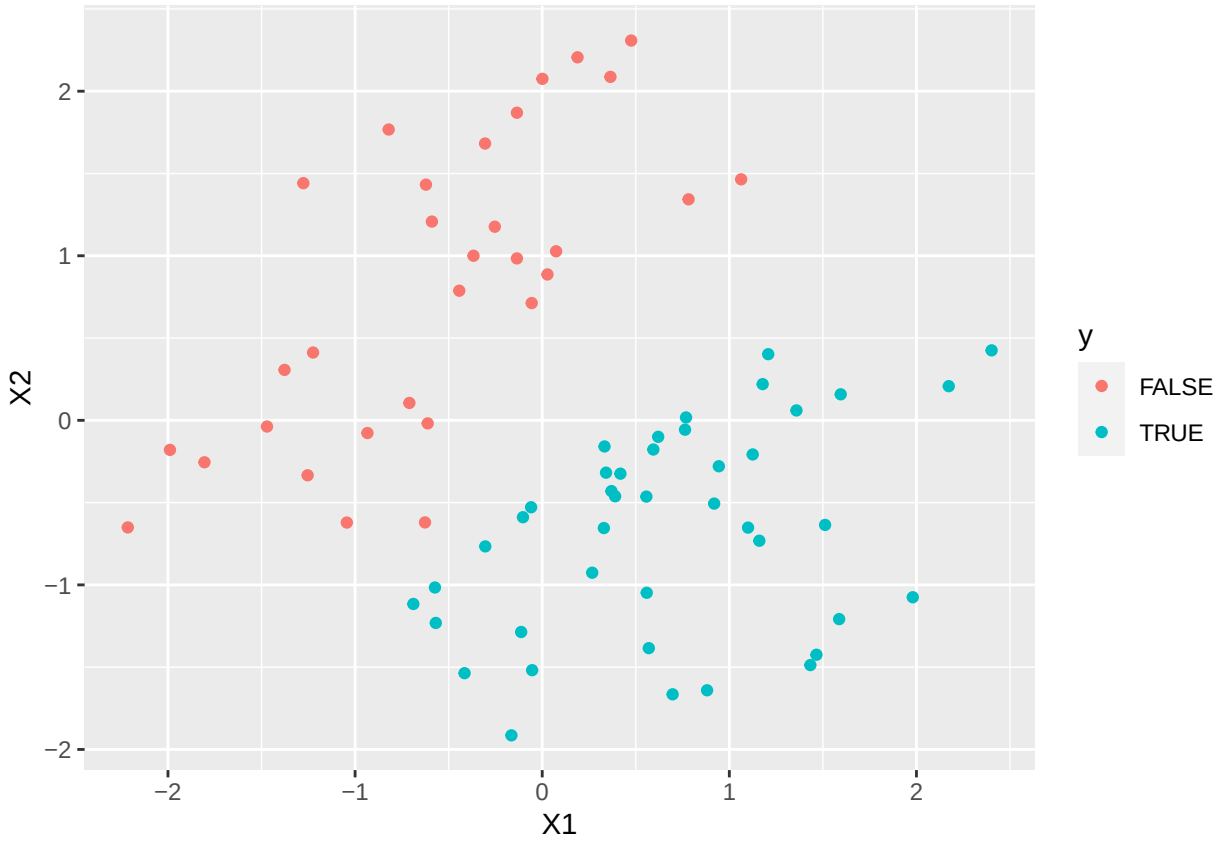

```
ggplot2::ggplot(data = df) +
  ggplot2::geom_point(ggplot2::aes(x = x1, y = x2, color = svm_pred), show.legend = FALSE)
```



Question 6

```
set.seed(1)
ncols <- 2
nrows <- 100
x <- matrix(rnorm(nrows * ncols), nrows, ncols)

y <- as.factor(x[, 1] - x[, 2] > 0)

df <- x %>%
  as.data.frame(.) %>%
  setNames(., c("X1", "X2")) %>%
  dplyr::mutate(., y = y)

df_easily_separable <- df %>%
  dplyr::filter(., abs(X1 - X2) > 0.4)

df_not_easily_separable <- df %>%
  dplyr::filter(., abs(X1 - X2) <= 0.05) %>%
  head(., 1)

df_train <- dplyr::bind_rows(df_easily_separable, df_not_easily_separable)

ggplot2::ggplot(data = df_train) +
```

```
  ggplot2::geom_point(ggplot2::aes(x = X1, y = X2, color = y), show.legend = TRUE)
```




```
x <- matrix(rnorm(nrows * ncols), nrows, ncols)

y <- as.factor(x[, 1] - x[, 2] > 0)

df_test <- x %>%
  as.data.frame(.) %>%
  setNames(., c("X1", "X2")) %>%
  dplyr::mutate(., y = y)

cv_results <- e1071::tune(
  e1071::svm, y ~ ., data = df_train,
  ranges = list(cost = c(0.01, 1, 100, 1000, 5000)), kernel = "linear"
)
best_model <- cv_results$best.model

print(summary(cv_results))
```

```
##
## Parameter tuning of 'e1071::svm':
##
## - sampling method: 10-fold cross validation
##
## - best parameters:
##  cost
##   100
```

```
##
## - best performance: 0
##
## - Detailed performance results:
##    cost     error dispersion
## 1 1e-02 0.2410714 0.25894568
## 2 1e+00 0.0125000 0.03952847
## 3 1e+02 0.0000000 0.00000000
## 4 1e+03 0.0000000 0.00000000
## 5 5e+03 0.0000000 0.00000000
```

```
train_preds <- predict(best_model)

mean(train_preds != df_train$y)
```

```
## [1] 0
```

```
models <- lapply(
  c(0.01, 1, 100, 1000, 5000) %>% setNames(., .),
  function(cost, df) {
    svm(y ~ ., data = df, kernel = "linear", cost = cost)
  },
  df = df_train
)

train_errors <- sapply(
  models,
  function(model, df) {
    mean(predict(model) != df$y)
  },
  df = df_train
)

names(train_errors)[which.min(train_errors)]
```

```
## [1] "100"
```

```
test_errors <- sapply(
  models,
  function(model, df) {
    mean(predict(model, df) != df$y)
  },
  df = df_test
)

names(test_errors)[which.min(test_errors)]
```

```
## [1] "1"
```

Question 7

```
df_auto <- ISLR::Auto %>%
  dplyr::mutate(., mpg = as.factor(ifelse(mpg >= median(mpg), 1, 0)))

cross_validation <- e1071::tune(
  e1071::svm, mpg ~ ., data = df_auto,
```

```
  ranges = list(
    cost = c(0.01, 0.1, 1, 10, 50, 100, 500),
    kernel = c("linear", "polynomial", "radial"),
    degree = c(2, 3, 4),
    gamma = c(0.01, 0.1, 0.5, 1)
  )
)

print(summary(cross_validation))
```

```
##
## Parameter tuning of 'e1071::svm':
##
## - sampling method: 10-fold cross validation
##
## - best parameters:
##  cost kernel degree gamma
##    10 radial      2   0.5
##
## - best performance: 0.08153846
##
## - Detailed performance results:
##      cost     kernel degree gamma      error dispersion
## 1   1e-02     linear      2  0.01 0.08916667 0.05395327
## 2   1e-01     linear      2  0.01 0.09173077 0.04968530
## 3   1e+00     linear      2  0.01 0.09166667 0.03974704
## 4   1e+01     linear      2  0.01 0.11192308 0.05284927
## 5   5e+01     linear      2  0.01 0.11967949 0.06065068
## 6   1e+02     linear      2  0.01 0.11724359 0.06146307
## 7   5e+02     linear      2  0.01 0.11467949 0.05906599
## 8   1e-02 polynomial      2  0.01 0.55108974 0.03316037
## 9   1e-01 polynomial      2  0.01 0.55108974 0.03316037
## 10  1e+00 polynomial      2  0.01 0.53076923 0.05746552
## 11  1e+01 polynomial      2  0.01 0.30634615 0.06755682
## 12  5e+01 polynomial      2  0.01 0.26551282 0.05595011
## 13  1e+02 polynomial      2  0.01 0.29096154 0.05073286
## 14  5e+02 polynomial      2  0.01 0.17608974 0.06102592
## 15  1e-02     radial      2  0.01 0.55108974 0.03316037
## 16  1e-01     radial      2  0.01 0.11461538 0.05038188
## 17  1e+00     radial      2  0.01 0.08916667 0.05395327
## 18  1e+01     radial      2  0.01 0.08660256 0.04658164
## 19  5e+01     radial      2  0.01 0.09153846 0.04089835
## 20  1e+02     radial      2  0.01 0.08916667 0.03203197
## 21  5e+02     radial      2  0.01 0.10955128 0.05074366
## 22  1e-02     linear      3  0.01 0.08916667 0.05395327
## 23  1e-01     linear      3  0.01 0.09173077 0.04968530
## 24  1e+00     linear      3  0.01 0.09166667 0.03974704
## 25  1e+01     linear      3  0.01 0.11192308 0.05284927
## 26  5e+01     linear      3  0.01 0.11967949 0.06065068
## 27  1e+02     linear      3  0.01 0.11724359 0.06146307
## 28  5e+02     linear      3  0.01 0.11467949 0.05906599
## 29  1e-02 polynomial      3  0.01 0.55108974 0.03316037
## 30  1e-01 polynomial      3  0.01 0.55108974 0.03316037
## 31  1e+00 polynomial      3  0.01 0.55108974 0.03316037
```

```
## 32  1e+01 polynomial      3  0.01 0.29865385 0.06957315
## 33  5e+01 polynomial      3  0.01 0.25775641 0.05052364
## 34  1e+02 polynomial      3  0.01 0.25525641 0.04726890
## 35  5e+02 polynomial      3  0.01 0.09429487 0.04645208
## 36  1e-02     radial      3  0.01 0.55108974 0.03316037
## 37  1e-01     radial      3  0.01 0.11461538 0.05038188
## 38  1e+00     radial      3  0.01 0.08916667 0.05395327
## 39  1e+01     radial      3  0.01 0.08660256 0.04658164
## 40  5e+01     radial      3  0.01 0.09153846 0.04089835
## 41  1e+02     radial      3  0.01 0.08916667 0.03203197
## 42  5e+02     radial      3  0.01 0.10955128 0.05074366
## 43  1e-02     linear      4  0.01 0.08916667 0.05395327
## 44  1e-01     linear      4  0.01 0.09173077 0.04968530
## 45  1e+00     linear      4  0.01 0.09166667 0.03974704
## 46  1e+01     linear      4  0.01 0.11192308 0.05284927
## 47  5e+01     linear      4  0.01 0.11967949 0.06065068
## 48  1e+02     linear      4  0.01 0.11724359 0.06146307
## 49  5e+02     linear      4  0.01 0.11467949 0.05906599
## 50  1e-02 polynomial      4  0.01 0.55108974 0.03316037
## 51  1e-01 polynomial      4  0.01 0.55108974 0.03316037
## 52  1e+00 polynomial      4  0.01 0.55108974 0.03316037
## 53  1e+01 polynomial      4  0.01 0.55108974 0.03316037
## 54  5e+01 polynomial      4  0.01 0.45179487 0.08673308
## 55  1e+02 polynomial      4  0.01 0.43653846 0.07638941
## 56  5e+02 polynomial      4  0.01 0.37544872 0.08139991
## 57  1e-02     radial      4  0.01 0.55108974 0.03316037
## 58  1e-01     radial      4  0.01 0.11461538 0.05038188
## 59  1e+00     radial      4  0.01 0.08916667 0.05395327
## 60  1e+01     radial      4  0.01 0.08660256 0.04658164
## 61  5e+01     radial      4  0.01 0.09153846 0.04089835
## 62  1e+02     radial      4  0.01 0.08916667 0.03203197
## 63  5e+02     radial      4  0.01 0.10955128 0.05074366
## 64  1e-02     linear      2  0.10 0.08916667 0.05395327
## 65  1e-01     linear      2  0.10 0.09173077 0.04968530
## 66  1e+00     linear      2  0.10 0.09166667 0.03974704
## 67  1e+01     linear      2  0.10 0.11192308 0.05284927
## 68  5e+01     linear      2  0.10 0.11967949 0.06065068
## 69  1e+02     linear      2  0.10 0.11724359 0.06146307
## 70  5e+02     linear      2  0.10 0.11467949 0.05906599
## 71  1e-02 polynomial      2  0.10 0.53076923 0.05746552
## 72  1e-01 polynomial      2  0.10 0.30634615 0.06755682
## 73  1e+00 polynomial      2  0.10 0.29096154 0.05073286
## 74  1e+01 polynomial      2  0.10 0.17102564 0.05017119
## 75  5e+01 polynomial      2  0.10 0.18128205 0.04489906
## 76  1e+02 polynomial      2  0.10 0.19141026 0.05466011
## 77  5e+02 polynomial      2  0.10 0.21198718 0.07114457
## 78  1e-02     radial      2  0.10 0.22467949 0.07471135
## 79  1e-01     radial      2  0.10 0.08916667 0.05117372
## 80  1e+00     radial      2  0.10 0.08660256 0.04498607
## 81  1e+01     radial      2  0.10 0.08410256 0.03401237
## 82  5e+01     radial      2  0.10 0.09929487 0.04172136
## 83  1e+02     radial      2  0.10 0.09929487 0.03993206
## 84  5e+02     radial      2  0.10 0.10448718 0.04529108
## 85  1e-02     linear      3  0.10 0.08916667 0.05395327
```

```
## 86  1e-01     linear      3  0.10 0.09173077 0.04968530
## 87  1e+00     linear      3  0.10 0.09166667 0.03974704
## 88  1e+01     linear      3  0.10 0.11192308 0.05284927
## 89  5e+01     linear      3  0.10 0.11967949 0.06065068
## 90  1e+02     linear      3  0.10 0.11724359 0.06146307
## 91  5e+02     linear      3  0.10 0.11467949 0.05906599
## 92  1e-02 polynomial      3  0.10 0.29865385 0.06957315
## 93  1e-01 polynomial      3  0.10 0.25525641 0.04726890
## 94  1e+00 polynomial      3  0.10 0.09448718 0.04548342
## 95  1e+01 polynomial      3  0.10 0.09192308 0.06079141
## 96  5e+01 polynomial      3  0.10 0.08160256 0.02896233
## 97  1e+02 polynomial      3  0.10 0.08923077 0.03222715
## 98  5e+02 polynomial      3  0.10 0.09948718 0.03703652
## 99  1e-02     radial      3  0.10 0.22467949 0.07471135
## 100 1e-01     radial      3  0.10 0.08916667 0.05117372
## 101 1e+00     radial      3  0.10 0.08660256 0.04498607
## 102 1e+01     radial      3  0.10 0.08410256 0.03401237
## 103 5e+01     radial      3  0.10 0.09929487 0.04172136
## 104 1e+02     radial      3  0.10 0.09929487 0.03993206
## 105 5e+02     radial      3  0.10 0.10448718 0.04529108
## 106 1e-02     linear      4  0.10 0.08916667 0.05395327
## 107 1e-01     linear      4  0.10 0.09173077 0.04968530
## 108 1e+00     linear      4  0.10 0.09166667 0.03974704
## 109 1e+01     linear      4  0.10 0.11192308 0.05284927
## 110 5e+01     linear      4  0.10 0.11967949 0.06065068
## 111 1e+02     linear      4  0.10 0.11724359 0.06146307
## 112 5e+02     linear      4  0.10 0.11467949 0.05906599
## 113 1e-02 polynomial      4  0.10 0.43653846 0.07638941
## 114 1e-01 polynomial      4  0.10 0.34217949 0.07026296
## 115 1e+00 polynomial      4  0.10 0.26038462 0.04709859
## 116 1e+01 polynomial      4  0.10 0.21929487 0.05086229
## 117 5e+01 polynomial      4  0.10 0.21442308 0.03300193
## 118 1e+02 polynomial      4  0.10 0.21705128 0.03963108
## 119 5e+02 polynomial      4  0.10 0.21705128 0.07327546
## 120 1e-02     radial      4  0.10 0.22467949 0.07471135
## 121 1e-01     radial      4  0.10 0.08916667 0.05117372
## 122 1e+00     radial      4  0.10 0.08660256 0.04498607
## 123 1e+01     radial      4  0.10 0.08410256 0.03401237
## 124 5e+01     radial      4  0.10 0.09929487 0.04172136
## 125 1e+02     radial      4  0.10 0.09929487 0.03993206
## 126 5e+02     radial      4  0.10 0.10448718 0.04529108
## 127 1e-02     linear      2  0.50 0.08916667 0.05395327
## 128 1e-01     linear      2  0.50 0.09173077 0.04968530
## 129 1e+00     linear      2  0.50 0.09166667 0.03974704
## 130 1e+01     linear      2  0.50 0.11192308 0.05284927
## 131 5e+01     linear      2  0.50 0.11967949 0.06065068
## 132 1e+02     linear      2  0.50 0.11724359 0.06146307
## 133 5e+02     linear      2  0.50 0.11467949 0.05906599
## 134 1e-02 polynomial      2  0.50 0.26294872 0.05053299
## 135 1e-01 polynomial      2  0.50 0.18628205 0.02456466
## 136 1e+00 polynomial      2  0.50 0.17871795 0.05030116
## 137 1e+01 polynomial      2  0.50 0.20679487 0.06146660
## 138 5e+01 polynomial      2  0.50 0.22737179 0.08736262
## 139 1e+02 polynomial      2  0.50 0.25025641 0.08229531
```

```
## 140 5e+02 polynomial      2  0.50 0.25788462 0.08095559
## 141 1e-02     radial      2  0.50 0.55108974 0.03316037
## 142 1e-01     radial      2  0.50 0.08923077 0.04828946
## 143 1e+00     radial      2  0.50 0.08660256 0.04161179
## 144 1e+01     radial      2  0.50 0.08153846 0.02592049
## 145 5e+01     radial      2  0.50 0.08410256 0.02377428
## 146 1e+02     radial      2  0.50 0.08410256 0.02377428
## 147 5e+02     radial      2  0.50 0.08410256 0.02377428
## 148 1e-02     linear      3  0.50 0.08916667 0.05395327
## 149 1e-01     linear      3  0.50 0.09173077 0.04968530
## 150 1e+00     linear      3  0.50 0.09166667 0.03974704
## 151 1e+01     linear      3  0.50 0.11192308 0.05284927
## 152 5e+01     linear      3  0.50 0.11967949 0.06065068
## 153 1e+02     linear      3  0.50 0.11724359 0.06146307
## 154 5e+02     linear      3  0.50 0.11467949 0.05906599
## 155 1e-02 polynomial      3  0.50 0.08935897 0.03884801
## 156 1e-01 polynomial      3  0.50 0.09192308 0.05833858
## 157 1e+00 polynomial      3  0.50 0.08923077 0.03222715
## 158 1e+01 polynomial      3  0.50 0.10205128 0.02691595
## 159 5e+01 polynomial      3  0.50 0.10717949 0.02910661
## 160 1e+02 polynomial      3  0.50 0.10717949 0.02910661
## 161 5e+02 polynomial      3  0.50 0.10717949 0.02910661
## 162 1e-02     radial      3  0.50 0.55108974 0.03316037
## 163 1e-01     radial      3  0.50 0.08923077 0.04828946
## 164 1e+00     radial      3  0.50 0.08660256 0.04161179
## 165 1e+01     radial      3  0.50 0.08153846 0.02592049
## 166 5e+01     radial      3  0.50 0.08410256 0.02377428
## 167 1e+02     radial      3  0.50 0.08410256 0.02377428
## 168 5e+02     radial      3  0.50 0.08410256 0.02377428
## 169 1e-02     linear      4  0.50 0.08916667 0.05395327
## 170 1e-01     linear      4  0.50 0.09173077 0.04968530
## 171 1e+00     linear      4  0.50 0.09166667 0.03974704
## 172 1e+01     linear      4  0.50 0.11192308 0.05284927
## 173 5e+01     linear      4  0.50 0.11967949 0.06065068
## 174 1e+02     linear      4  0.50 0.11724359 0.06146307
## 175 5e+02     linear      4  0.50 0.11467949 0.05906599
## 176 1e-02 polynomial      4  0.50 0.20653846 0.05133349
## 177 1e-01 polynomial      4  0.50 0.21698718 0.03319505
## 178 1e+00 polynomial      4  0.50 0.20935897 0.06858422
## 179 1e+01 polynomial      4  0.50 0.22467949 0.05171227
## 180 5e+01 polynomial      4  0.50 0.22980769 0.05743473
## 181 1e+02 polynomial      4  0.50 0.22980769 0.05743473
## 182 5e+02 polynomial      4  0.50 0.22980769 0.05743473
## 183 1e-02     radial      4  0.50 0.55108974 0.03316037
## 184 1e-01     radial      4  0.50 0.08923077 0.04828946
## 185 1e+00     radial      4  0.50 0.08660256 0.04161179
## 186 1e+01     radial      4  0.50 0.08153846 0.02592049
## 187 5e+01     radial      4  0.50 0.08410256 0.02377428
## 188 1e+02     radial      4  0.50 0.08410256 0.02377428
## 189 5e+02     radial      4  0.50 0.08410256 0.02377428
## 190 1e-02     linear      2  1.00 0.08916667 0.05395327
## 191 1e-01     linear      2  1.00 0.09173077 0.04968530
## 192 1e+00     linear      2  1.00 0.09166667 0.03974704
## 193 1e+01     linear      2  1.00 0.11192308 0.05284927
```

```
## 194 5e+01     linear      2  1.00 0.11967949 0.06065068
## 195 1e+02     linear      2  1.00 0.11724359 0.06146307
## 196 5e+02     linear      2  1.00 0.11467949 0.05906599
## 197 1e-02 polynomial      2  1.00 0.29096154 0.05073286
## 198 1e-01 polynomial      2  1.00 0.17102564 0.05017119
## 199 1e+00 polynomial      2  1.00 0.19141026 0.05466011
## 200 1e+01 polynomial      2  1.00 0.22737179 0.07376101
## 201 5e+01 polynomial      2  1.00 0.25788462 0.08095559
## 202 1e+02 polynomial      2  1.00 0.25788462 0.08095559
## 203 5e+02 polynomial      2  1.00 0.25788462 0.08095559
## 204 1e-02     radial      2  1.00 0.55108974 0.03316037
## 205 1e-01     radial      2  1.00 0.55108974 0.03316037
## 206 1e+00     radial      2  1.00 0.08416667 0.03805105
## 207 1e+01     radial      2  1.00 0.08929487 0.02741720
## 208 5e+01     radial      2  1.00 0.08929487 0.02741720
## 209 1e+02     radial      2  1.00 0.08929487 0.02741720
## 210 5e+02     radial      2  1.00 0.08929487 0.02741720
## 211 1e-02     linear      3  1.00 0.08916667 0.05395327
## 212 1e-01     linear      3  1.00 0.09173077 0.04968530
## 213 1e+00     linear      3  1.00 0.09166667 0.03974704
## 214 1e+01     linear      3  1.00 0.11192308 0.05284927
## 215 5e+01     linear      3  1.00 0.11967949 0.06065068
## 216 1e+02     linear      3  1.00 0.11724359 0.06146307
## 217 5e+02     linear      3  1.00 0.11467949 0.05906599
## 218 1e-02 polynomial      3  1.00 0.09192308 0.06079141
## 219 1e-01 polynomial      3  1.00 0.08923077 0.03222715
## 220 1e+00 polynomial      3  1.00 0.10461538 0.03294453
## 221 1e+01 polynomial      3  1.00 0.10717949 0.02910661
## 222 5e+01 polynomial      3  1.00 0.10717949 0.02910661
## 223 1e+02 polynomial      3  1.00 0.10717949 0.02910661
## 224 5e+02 polynomial      3  1.00 0.10717949 0.02910661
## 225 1e-02     radial      3  1.00 0.55108974 0.03316037
## 226 1e-01     radial      3  1.00 0.55108974 0.03316037
## 227 1e+00     radial      3  1.00 0.08416667 0.03805105
## 228 1e+01     radial      3  1.00 0.08929487 0.02741720
## 229 5e+01     radial      3  1.00 0.08929487 0.02741720
## 230 1e+02     radial      3  1.00 0.08929487 0.02741720
## 231 5e+02     radial      3  1.00 0.08929487 0.02741720
## 232 1e-02     linear      4  1.00 0.08916667 0.05395327
## 233 1e-01     linear      4  1.00 0.09173077 0.04968530
## 234 1e+00     linear      4  1.00 0.09166667 0.03974704
## 235 1e+01     linear      4  1.00 0.11192308 0.05284927
## 236 5e+01     linear      4  1.00 0.11967949 0.06065068
## 237 1e+02     linear      4  1.00 0.11724359 0.06146307
## 238 5e+02     linear      4  1.00 0.11467949 0.05906599
## 239 1e-02 polynomial      4  1.00 0.21705128 0.03963108
## 240 1e-01 polynomial      4  1.00 0.21961538 0.06591193
## 241 1e+00 polynomial      4  1.00 0.22980769 0.05743473
## 242 1e+01 polynomial      4  1.00 0.22980769 0.05743473
## 243 5e+01 polynomial      4  1.00 0.22980769 0.05743473
## 244 1e+02 polynomial      4  1.00 0.22980769 0.05743473
## 245 5e+02 polynomial      4  1.00 0.22980769 0.05743473
## 246 1e-02     radial      4  1.00 0.55108974 0.03316037
## 247 1e-01     radial      4  1.00 0.55108974 0.03316037
```

```
## 248 1e+00     radial      4  1.00 0.08416667 0.03805105
## 249 1e+01     radial      4  1.00 0.08929487 0.02741720
## 250 5e+01     radial      4  1.00 0.08929487 0.02741720
## 251 1e+02     radial      4  1.00 0.08929487 0.02741720
## 252 5e+02     radial      4  1.00 0.08929487 0.02741720
```

Question 8

```
df_oj <- ISLR::OJ

train_idx <- sample(nrow(df_oj), 800)
df_train <- df_oj[train_idx, ]
df_test <- df_oj[-train_idx, ]

linear_model <- e1071::svm(Purchase ~ ., data = df_train, kernel = "linear", cost = 0.01)
summary(linear_model)
```

```
##
## Call:
## svm(formula = Purchase ~ ., data = df_train, kernel = "linear", cost = 0.01)
##
##
## Parameters:
##    SVM-Type:  C-classification
##  SVM-Kernel:  linear
##        cost:  0.01
##
## Number of Support Vectors:  451
##
##  ( 224 227 )
##
##
## Number of Classes:  2
##
## Levels:
##  CH MM
```

```
print(paste("Train error linear default: ", mean(predict(linear_model) != df_train$Purchase)))
```

```
## [1] "Train error linear default:  0.17375"
```

```
print(paste("Test error linear default: ", mean(predict(linear_model, df_test) != df_test$Purchase)))
```

```
## [1] "Test error linear default:  0.144444444444444"
```

```
cv_model <- e1071::tune(
  e1071::svm, Purchase ~ ., data = df_train, kernel = "linear",
  ranges = list(cost = c(0.01, 0.1, 1, 100, 500, 1000))
)

print(summary(cv_model))
```

```
##
## Parameter tuning of 'e1071::svm':
##
## - sampling method: 10-fold cross validation
```

```
##
## - best parameters:
##  cost
##   0.1
##
## - best performance: 0.175
##
## - Detailed performance results:
##    cost   error dispersion
## 1 1e-02 0.18000 0.03641962
## 2 1e-01 0.17500 0.03818813
## 3 1e+00 0.17500 0.04208127
## 4 1e+02 0.17625 0.04226652
## 5 5e+02 0.17625 0.04226652
## 6 1e+03 0.17750 0.03899786
```

```
best_model <- cv_model$best.model

print(paste("Train error linear best: ", mean(predict(best_model) != df_train$Purchase)))
```

```
## [1] "Train error linear best:  0.16875"
```

```
print(paste("Test error linear best: ", mean(predict(best_model, df_test) != df_test$Purchase)))
```

```
## [1] "Test error linear best:  0.148148148148148"
```

```
polynomial_model <- e1071::svm(Purchase ~ ., data = df_train, kernel = "polynomial", cost = 0.01, degree
summary(polynomial_model)
```

```
##
## Call:
## svm(formula = Purchase ~ ., data = df_train, kernel = "polynomial",
##     cost = 0.01, degree = 2)
##
##
## Parameters:
##    SVM-Type:  C-classification
##  SVM-Kernel:  polynomial
##        cost:  0.01
##      degree:  2
##      coef.0:  0
##
## Number of Support Vectors:  652
##
##  ( 323 329 )
##
##
## Number of Classes:  2
##
## Levels:
##  CH MM
```

```
print(paste("Train error polynomial default: ", mean(predict(polynomial_model) != df_train$Purchase)))
```

```
## [1] "Train error polynomial default:  0.38"
```

```r
print(paste("Test error polynomial default: ", mean(predict(polynomial_model, df_test) != df_test$Purcha
```

```
## [1] "Test error polynomial default:  0.337037037037037"
```

```r
cv_model <- e1071::tune(
  e1071::svm, Purchase ~ ., data = df_train, kernel = "polynomial", degree = 2,
  ranges = list(cost = c(0.01, 0.1, 1, 100, 500, 1000))
)

print(summary(cv_model))
```

```
##
## Parameter tuning of 'e1071::svm':
##
## - sampling method: 10-fold cross validation
##
## - best parameters:
##  cost
##   100
##
## - best performance: 0.1775
##
## - Detailed performance results:
##    cost   error dispersion
## 1 1e-02 0.39500 0.05342440
## 2 1e-01 0.32875 0.03488573
## 3 1e+00 0.21625 0.04489571
## 4 1e+02 0.17750 0.05197489
## 5 5e+02 0.18500 0.05767485
## 6 1e+03 0.18500 0.06089609
```

```r
best_model <- cv_model$best.model

print(paste("Train error polynomial best: ", mean(predict(best_model) != df_train$Purchase)))
```

```
## [1] "Train error polynomial best:  0.14"
```

```r
print(paste("Test error polynomial best: ", mean(predict(best_model, df_test) != df_test$Purchase)))
```

```
## [1] "Test error polynomial best:  0.166666666666667"
```

```r
radial_model <- e1071::svm(Purchase ~ ., data = df_train, kernel = "radial", cost = 0.01)
summary(radial_model)
```

```
##
## Call:
## svm(formula = Purchase ~ ., data = df_train, kernel = "radial", cost = 0.01)
##
##
## Parameters:
##    SVM-Type:  C-classification
##  SVM-Kernel:  radial
##        cost:  0.01
##
## Number of Support Vectors:  648
##
##  ( 323 325 )
```

```
##
##
## Number of Classes:  2
##
## Levels:
##  CH MM
```

```
print(paste("Train error radial default: ", mean(predict(radial_model) != df_train$Purchase)))
```

```
## [1] "Train error radial default:  0.40375"
```

```
print(paste("Test error radial default: ", mean(predict(radial_model, df_test) != df_test$Purchase)))
```

```
## [1] "Test error radial default:  0.348148148148148"
```

```
cv_model <- e1071::tune(
  e1071::svm, Purchase ~ ., data = df_train, kernel = "radial",
  ranges = list(cost = c(0.01, 0.1, 1, 100, 500, 1000))
)

print(summary(cv_model))
```

```
##
## Parameter tuning of 'e1071::svm':
##
## - sampling method: 10-fold cross validation
##
## - best parameters:
##  cost
##     1
##
## - best performance: 0.175
##
## - Detailed performance results:
##    cost   error dispersion
## 1 1e-02 0.40375 0.04372023
## 2 1e-01 0.19250 0.04048319
## 3 1e+00 0.17500 0.03679900
## 4 1e+02 0.20000 0.04526159
## 5 5e+02 0.22000 0.05244044
## 6 1e+03 0.22375 0.05050096
```

```
best_model <- cv_model$best.model

print(paste("Train error radial best: ", mean(predict(best_model) != df_train$Purchase)))
```

```
## [1] "Train error radial best:  0.1525"
```

```
print(paste("Test error radial best: ", mean(predict(best_model, df_test) != df_test$Purchase)))
```

```
## [1] "Test error radial best:  0.151851851851852"
```

Linear kernel performs the best.