ISLR Chapter 6 Exercises

2023-07-01

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```r
library(ISLR)
library(tools)
library(ggplot2)
library(leaps)
library(glmnet)
library(pls)
library(dplyr)
library(R6)
library(boot)
library(splines)
library(gam)
```

Conceptual

Question 1

a For $x \le \xi$, just set $a_1 = \beta_0, b_1 = \beta_1, c_1 = \beta_2, d_1 = \beta_3$

b $f(x) = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 (x-\xi)^3 = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 (x-\xi)(x-\xi)(x-\xi) = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 (x^2 - 2x\xi + \xi^2)(x-\xi) = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 (x^3 - 2x^2\xi + x\xi^2 - x^2\xi + 2x\xi^2 - \xi^3) = (\beta_0 - \beta_4 \xi^3) + x(\beta_1 + 3\beta_4\xi^2) + x^2(\beta_2 - 3\beta_4\xi) + x^3(\beta_3 + \beta_4)$

$a_2 = \beta_0 - \beta_4 \xi^3$

$b_2 = \beta_1 + 3\beta_4\xi^2$

$c_2 = \beta_2 - 3\beta_4\xi$

$d_2 = \beta_3 + \beta_4$

c $f_1(\xi) = \beta_0 + \beta_1\xi + \beta_2\xi^2 + \beta_3\xi^3$

$f_2(\xi) = (\beta_0 - \beta_4\xi^3) + \xi(\beta_1 + 3\beta_4\xi^2) + \xi^2(\beta_2 - 3\beta_4\xi) + \xi^3(\beta_3 + \beta_4) = (\beta_0 - \beta_4\xi^3) + \xi\beta_1 + 3\beta_4\xi^3 + \xi^2\beta_2 - 3\beta_4\xi^3 + \xi^3\beta_3 + \xi^3\beta_4 = \beta_0 + \xi\beta_1 + \xi^2\beta_2 + \xi^3\beta_3$

Thus $f_1(\xi) = f_2(\xi)$

d $f\prime_1(\xi) = beta_1 + 2\beta_2\xi + 3\beta_3\xi^2$

$f\prime_2(\xi) = \beta_1+3\beta_4\xi^2+2(\beta_2-3\beta_4\xi)\xi+3(\beta_3+\beta_4)\xi^2 = \beta_1+3\beta_4\xi^2+2\beta_2\xi-6\beta_4\xi^2+3\beta_3\xi^2+3\beta_4\xi^2 = \beta_1+2\beta_2\xi+3\beta_3\xi^2$

So

$f\prime_1(\xi) = f\prime_2(\xi)$

e $f_1''(\xi) = 2\beta_2 + 6\beta_3\xi$

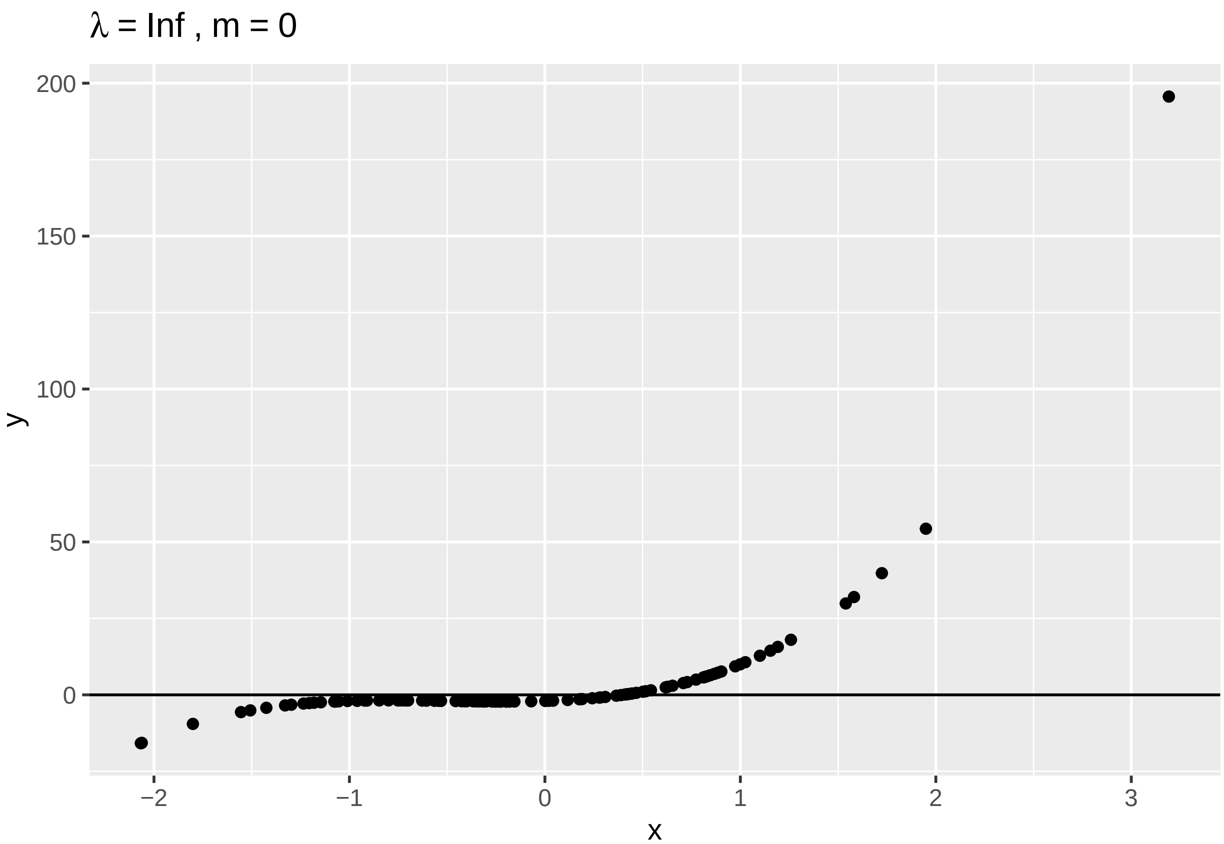
$f_2''(\xi) = 2\beta_2 - 6\beta_4\xi + 6\beta_3\xi + 6\beta_4\xi = 2\beta_2 + 6\beta_3\xi$

So

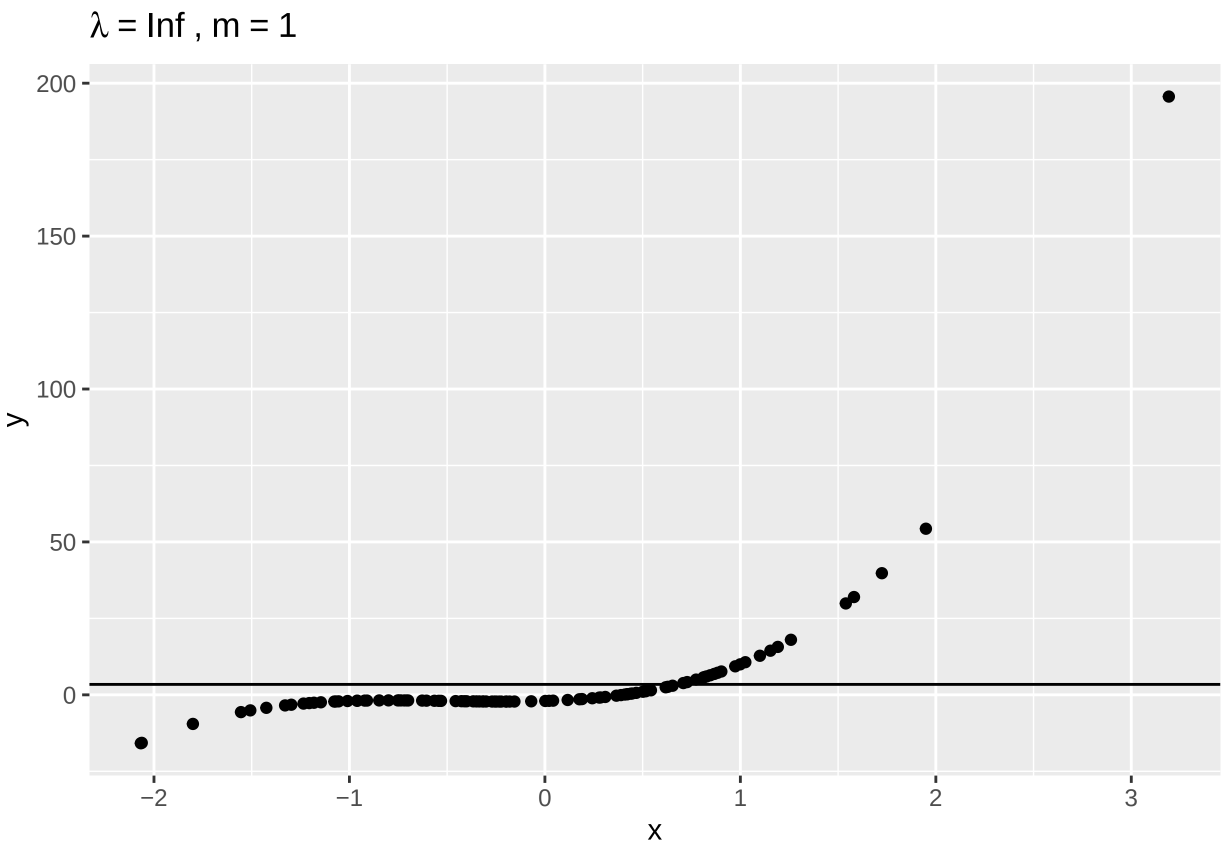
$f_1''(\xi) = f_2''(\xi)$

Question 2

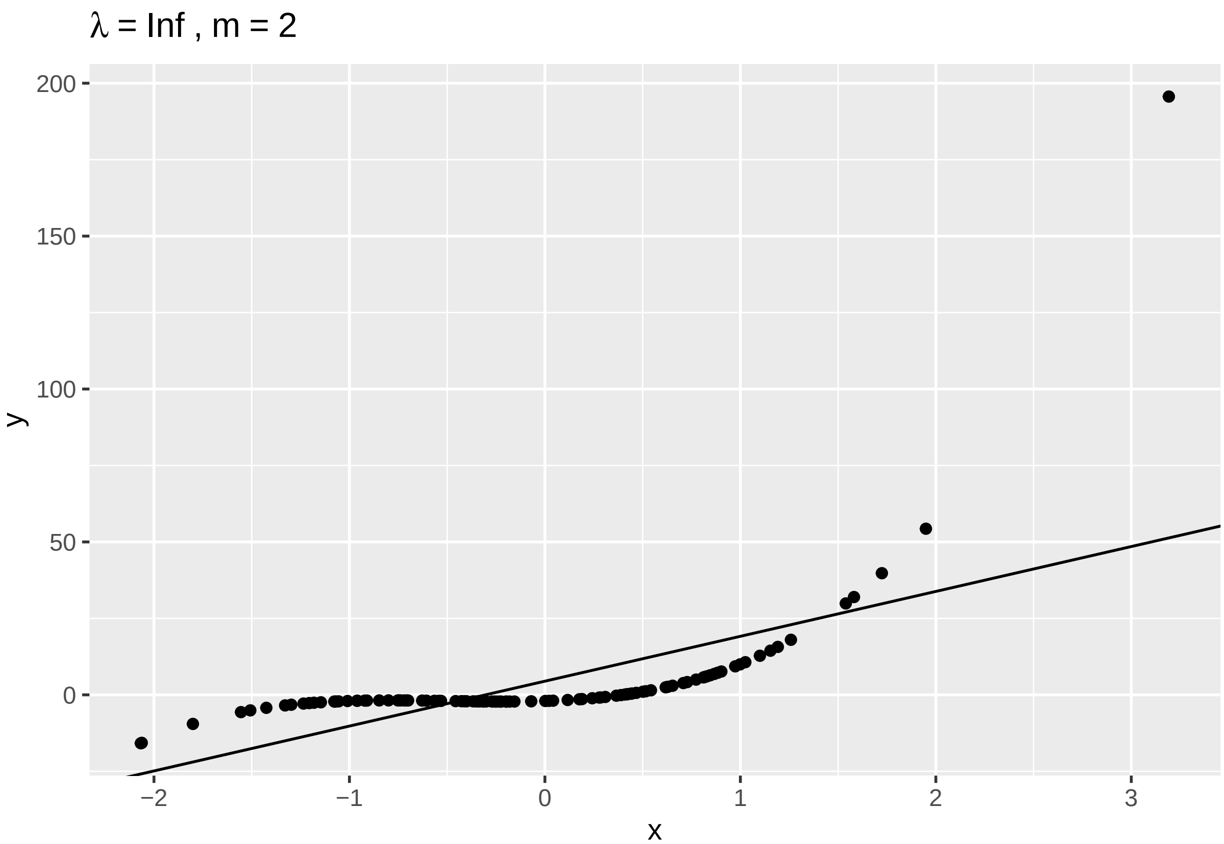
```
df <- data.frame(x = rnorm(100), eps = rnorm(100))
df$y <- -2 + 2 * df$x + 2 * 3 * df$x^2 + 4 * df$x^3

ggplot2::ggplot(data = df) +
  ggplot2::geom_point(ggplot2::aes(x = x, y = y)) +
  ggplot2::geom_hline(yintercept = 0) +
  ggplot2::ggtitle(quote(lambda~`=`~Inf~`,`~m~`=`~0))
```



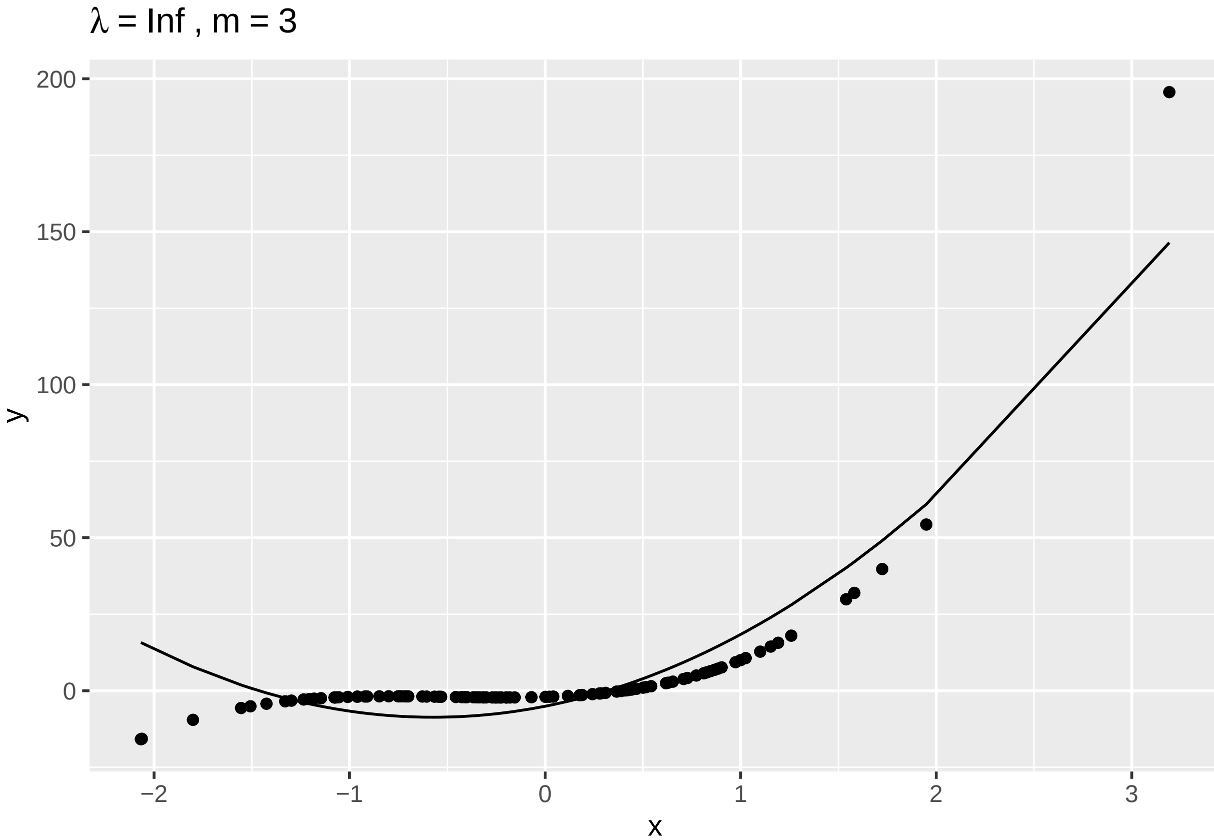

```r
ggplot2::ggplot(data = df) +
  ggplot2::geom_point(ggplot2::aes(x = x, y = y)) +
  ggplot2::geom_hline(ggplot2::aes(yintercept = mean(y))) +
  ggplot2::ggtitle(quote(lambda~`=`~Inf~`,`~m~`=`~1))
```



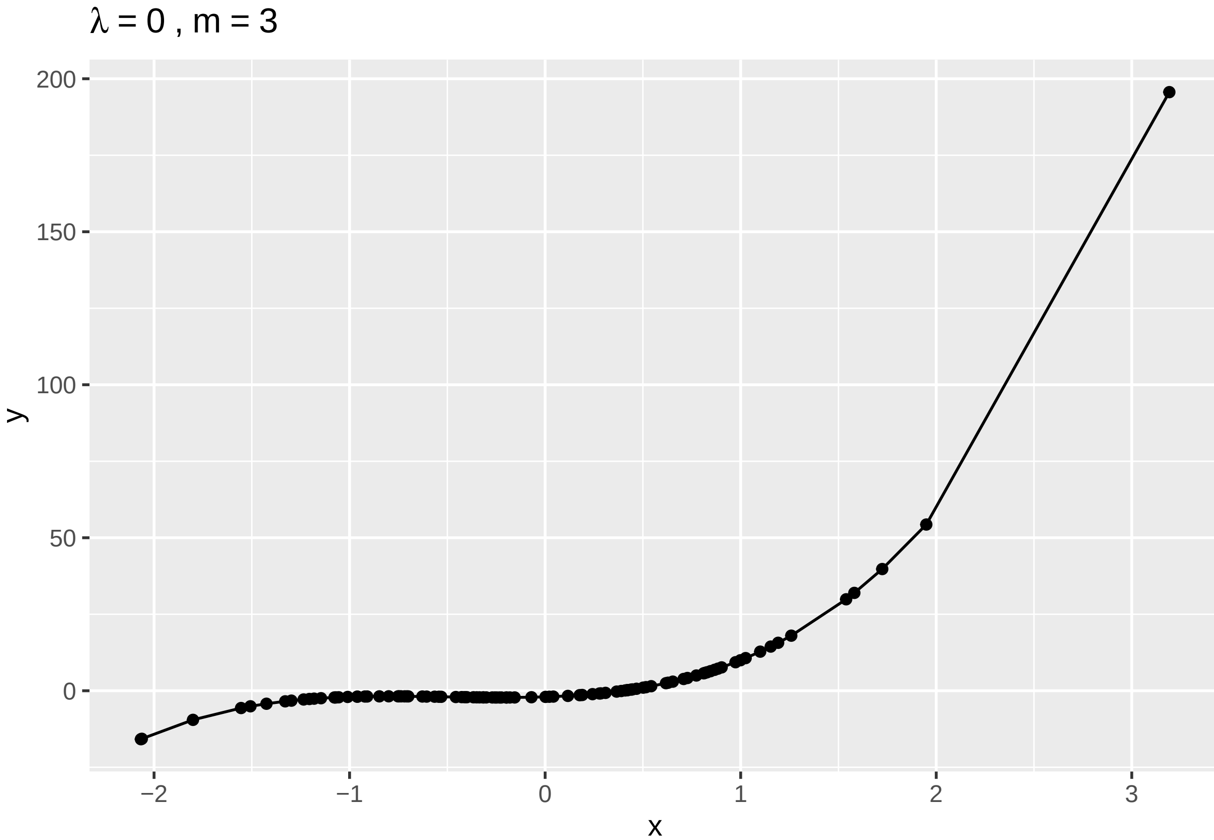

```
model <- lm(y ~ x, data = df)
coefs <- coef(model)
intercept <- coefs[[1]]
slope <- coefs[[2]]

ggplot2::ggplot(data = df) +
  ggplot2::geom_point(ggplot2::aes(x = x, y = y)) +
  ggplot2::geom_abline(intercept = intercept, slope = slope) +
  ggplot2::ggtitle(quote(lambda~`=`~Inf~`,`~m~`=`~2))
```




```
model <- lm(y ~ poly(x, 2), data = df)
preds <- predict(model)
df$preds <- preds

ggplot2::ggplot(data = df) +
  ggplot2::geom_point(ggplot2::aes(x = x, y = y)) +
  ggplot2::geom_line(aes(x = x, y = preds)) +
  ggplot2::ggtitle(quote(lambda~`=`~Inf~`,`~m~`=`~3))
```

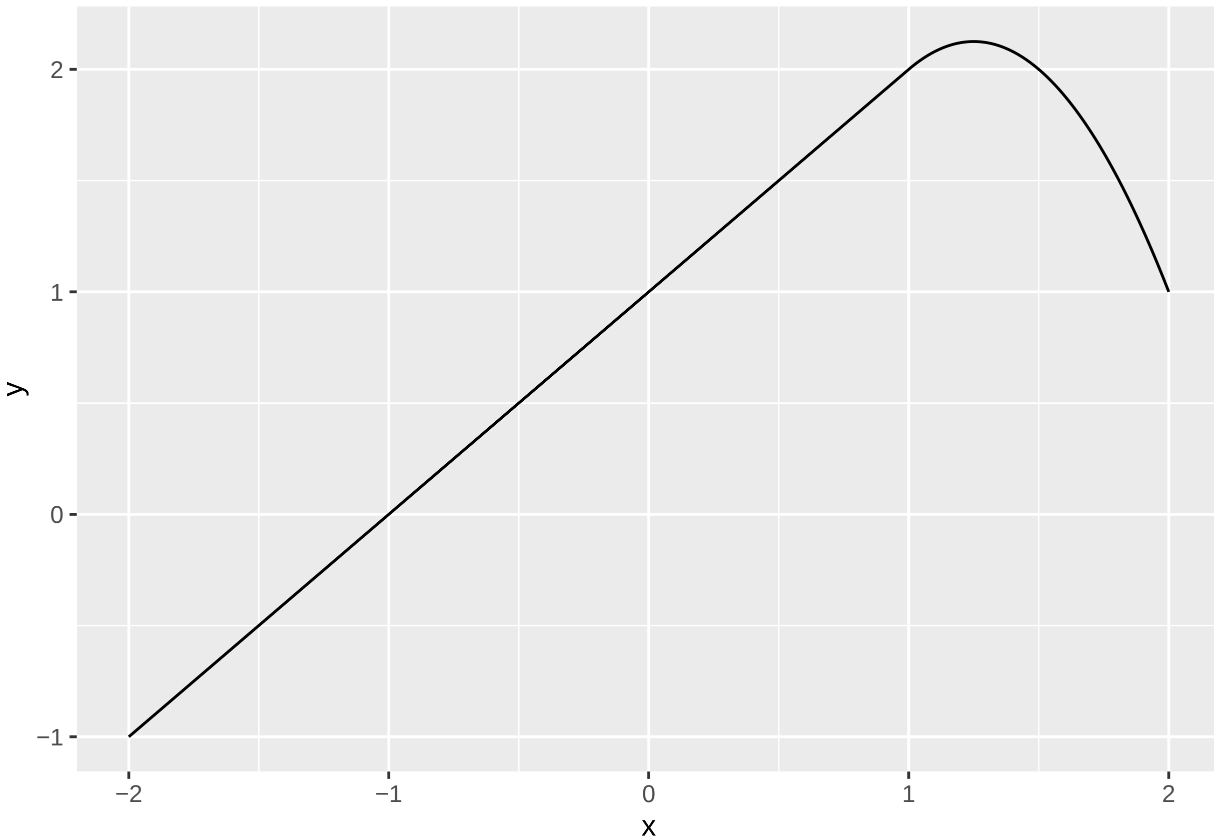



```r
ggplot2::ggplot(data = df) +
  ggplot2::geom_point(ggplot2::aes(x = x, y = y)) +
  ggplot2::geom_line(aes(x = x, y = y)) +
  ggplot2::ggtitle(quote(lambda~`=`~0~`,`~m~`=`~3))
```



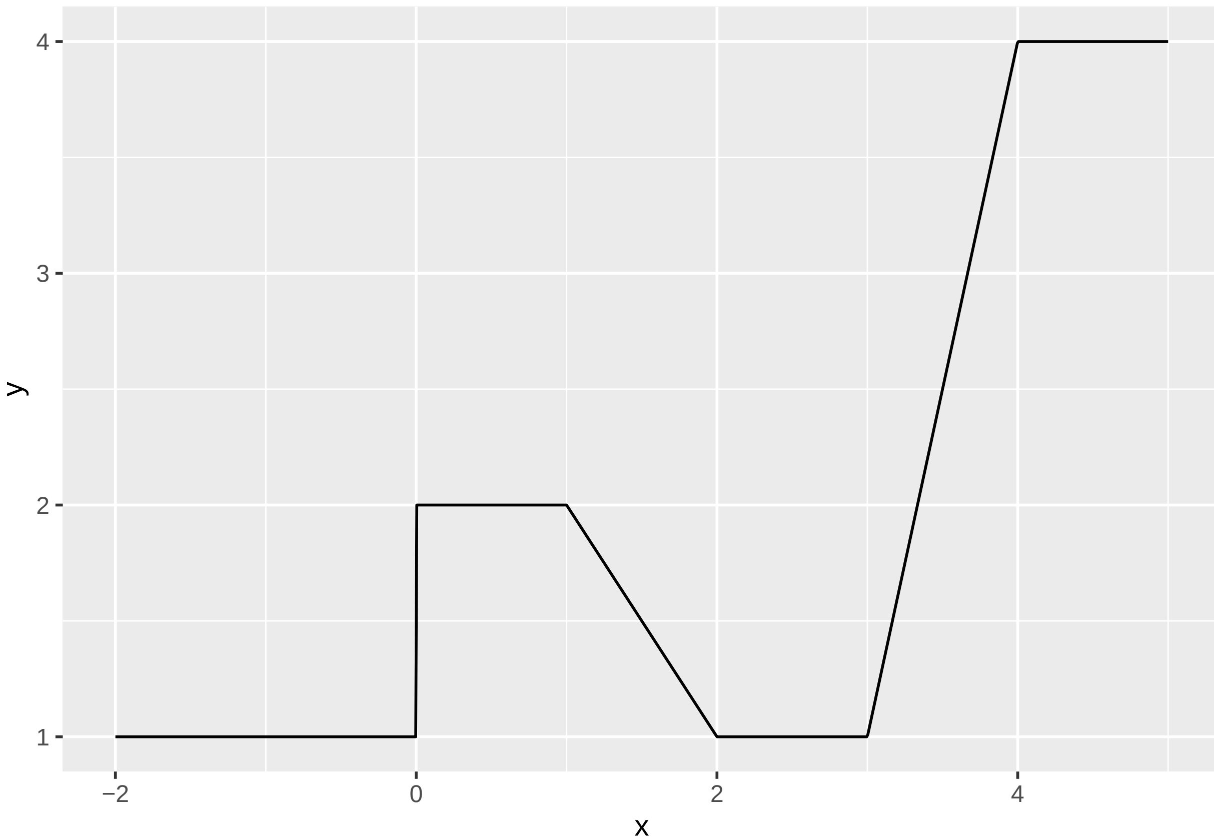
Question 3

```
x <- seq(-2, 2, length = 1000)
y <- 1 + x -2 * (x - 1)^2 * (x >= 1)

df <- data.frame(x = x, y = y)

ggplot2::ggplot(data = df) +
  ggplot2::geom_line(ggplot2::aes(x = x, y = y))
```



Question 4

```
x <- seq(-2, 5, length = 1000)
y <- 1 + (x >= 0 & x <=2) - (x - 1) * (x >= 1 & x <=2) + 3 * ((x - 3) * (x >= 3 & x <= 4) + (x > 4 & x

df <- data.frame(x = x, y = y)

ggplot2::ggplot(data = df) +
  ggplot2::geom_line(ggplot2::aes(x = x, y = y))
```



Question 5

a g2 will have smaller training RSS because the model will be more flexible.

b Impossible to tell; depends on the bias variance tradeoff.

c The model from each curve will be the same in this case; any model that interpolates the points. The training RSS will be zero no matter what, and if we choose this interpolation function to be the same for both of g1 and g2, then the test error will also be the same.

Applied

Question 6

a

```
df_wage <- ISLR::Wage

k <- 10
max_degree <- 15
degrees <- seq(max_degree)

cv_results <- rep(0, length(degrees)) %>%
  setNames(., seq_along(.))

set.seed(1)
for (degree in seq_along(degrees)) {
```

```r
  model <- glm(wage ~ poly(age, degree), data = df_wage)
  cv_results[[degree]] <- cv.glm(df_wage, model, K = 10)$delta[[1]]
}

model_full <- lm(wage ~ poly(age, max_degree), data = df_wage)

print(cv_results[cv_results == min(cv_results)])
```

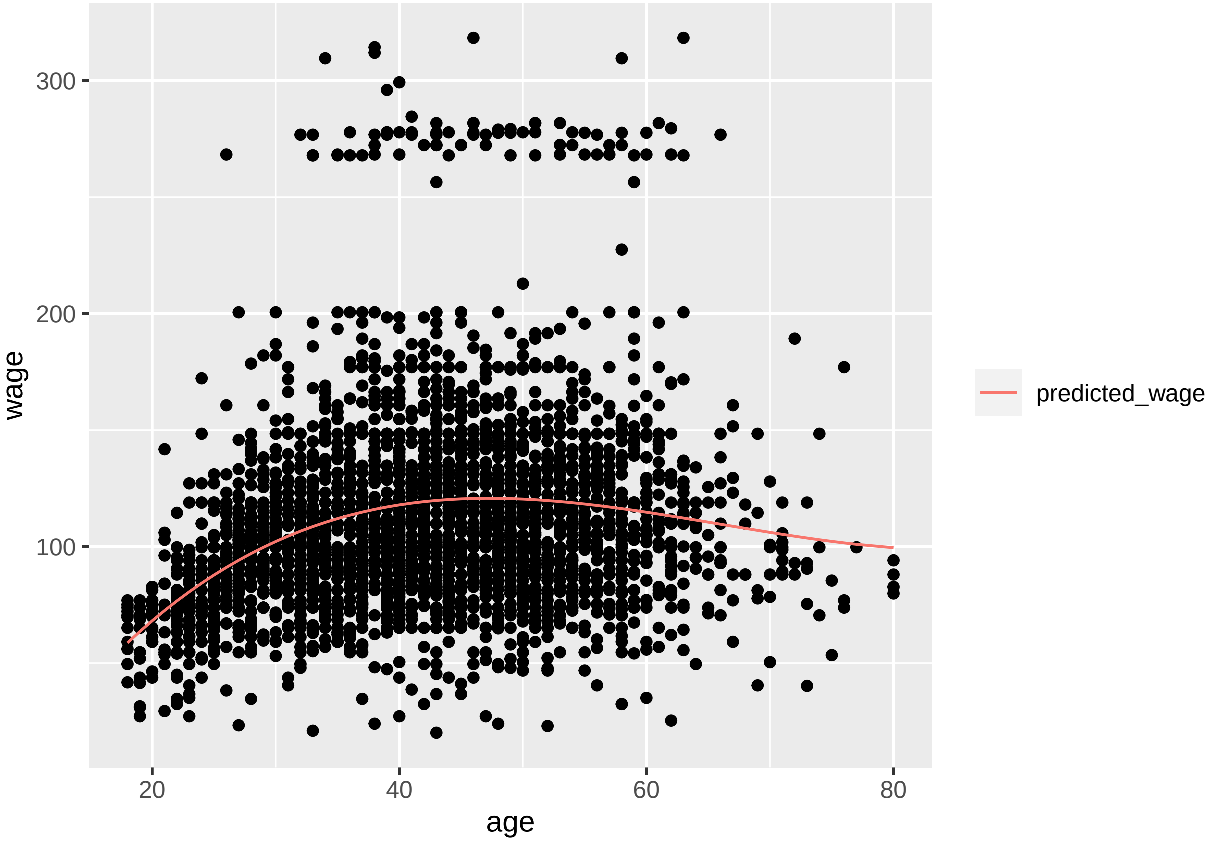
```
##        9
## 1593.913
```

```r
print(summary(model_full))
```

```
##
## Call:
## lm(formula = wage ~ poly(age, max_degree), data = df_wage)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -99.917 -24.278  -4.799  15.519 199.524
##
## Coefficients:
##                         Estimate Std. Error t value Pr(>|t|)
## (Intercept)             111.7036     0.7288 153.262  < 2e-16 ***
## poly(age, max_degree)1  447.0679    39.9202  11.199  < 2e-16 ***
## poly(age, max_degree)2 -478.3158    39.9202 -11.982  < 2e-16 ***
## poly(age, max_degree)3  125.5217    39.9202   3.144  0.00168 **
## poly(age, max_degree)4  -77.9112    39.9202  -1.952  0.05107 .
## poly(age, max_degree)5  -35.8129    39.9202  -0.897  0.36973
## poly(age, max_degree)6   62.7077    39.9202   1.571  0.11633
## poly(age, max_degree)7   50.5498    39.9202   1.266  0.20551
## poly(age, max_degree)8  -11.2547    39.9202  -0.282  0.77802
## poly(age, max_degree)9  -83.6918    39.9202  -2.096  0.03612 *
## poly(age, max_degree)10   1.6240    39.9202   0.041  0.96755
## poly(age, max_degree)11  10.1588    39.9202   0.254  0.79914
## poly(age, max_degree)12  -2.6076    39.9202  -0.065  0.94792
## poly(age, max_degree)13  13.7669    39.9202   0.345  0.73022
## poly(age, max_degree)14 -15.5730    39.9202  -0.390  0.69649
## poly(age, max_degree)15 -28.1896    39.9202  -0.706  0.48015
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 39.92 on 2984 degrees of freedom
## Multiple R-squared:  0.08937,    Adjusted R-squared:  0.0848
## F-statistic: 19.52 on 15 and 2984 DF,  p-value: < 2.2e-16
```

```r
model_poly3 <- lm(wage ~ poly(age, 3), data = df_wage)

df_for_plot <- data.frame(
  age = df_wage$age,
  predicted_wage = predict(model_poly3),
  wage = df_wage$wage,
  label = "predicted_wage"
)
```

```r
ggplot2::ggplot(data = df_for_plot) +
  ggplot2::geom_point(ggplot2::aes(x = age, y = wage)) +
  ggplot2::geom_line(ggplot2::aes(x = age, y = predicted_wage, color = label)) +
  ggplot2::labs(color = NULL)
```



Cross-validation picks a polynomial of degree 6, although the difference in MSE is trivial compared to the degree 3 model. In this case we would probably pick the degree 3 model. Since the polynomials are orthogonal, the p-value for the ANOVA between two models that differ by one degree is the same as the p-value for that degree in the larger model. So we can just look at the significance of each term in the largest model. We see that any degrees larger than 3 are insignificant.

From our plot, we can see that the model fits the bulk of the data well, but misses out on the high earners.

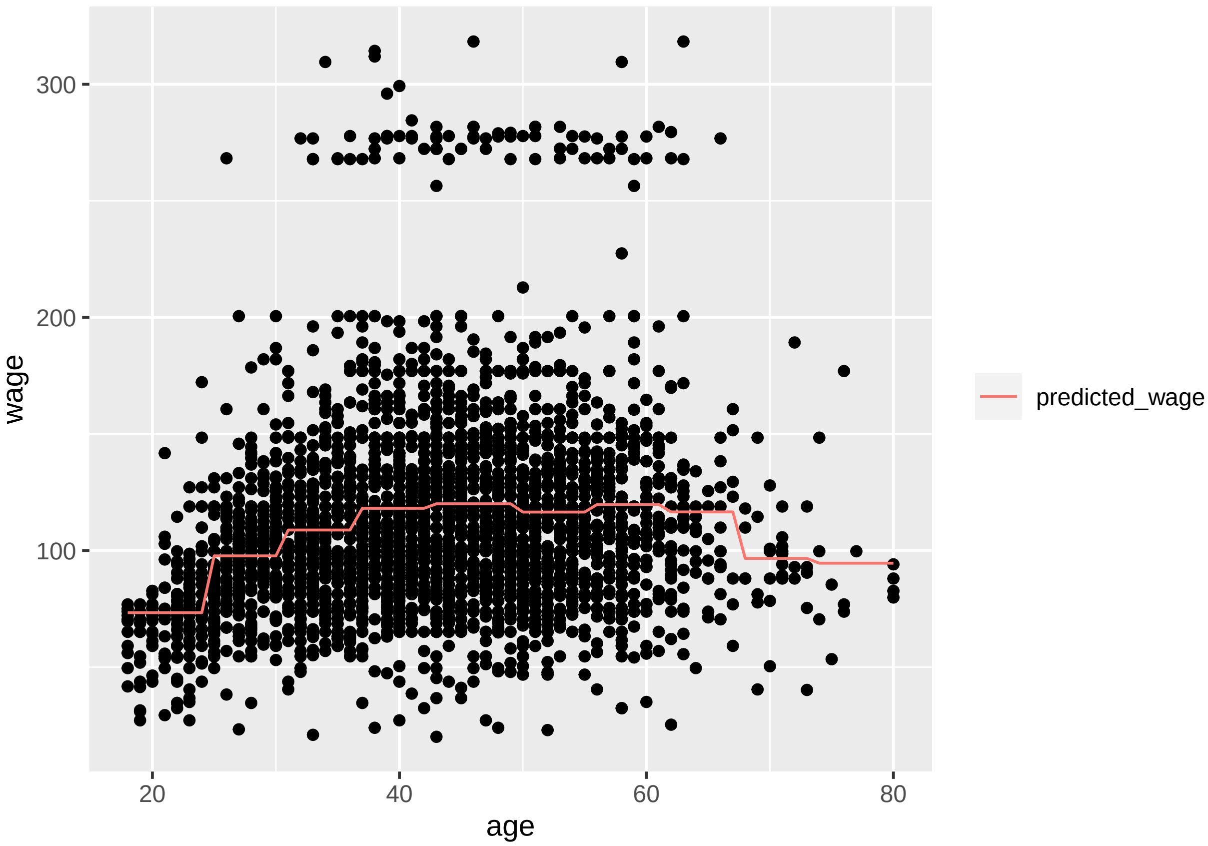
b

```r
max_cuts <- 15

# since the number of cuts is somewhat small relative to the size of the dataset,
# we don't have to worry about new factor levels being present in the test set of
# each cross-validation fold. In production we would need to handle new levels
# appropriately.

df_wage_with_cuts <- df_wage
for (n_cuts in seq(2, max_cuts)) {
  df_wage_with_cuts[[paste0("cut", n_cuts)]] <- cut(df_wage_with_cuts$age, n_cuts)
}
cut_vars <- paste0("cut", seq(2, max_cuts))
```

```r
set.seed(1)
cv_results <- rep(0, max_cuts - 1) %>%
  setNames(., seq_along(.))
for (n_cuts in seq(2, max_cuts)) {
  model_formula <- as.formula(paste("wage ~", cut_vars[[n_cuts - 1]]))
  model <- glm(model_formula, data = df_wage_with_cuts)

  cv_results[n_cuts - 1] <- cv.glm(df_wage_with_cuts, model, K = 2)$delta[[1]]
}

print(cv_results[cv_results == min(cv_results)])
```

```
##       10
## 1595.995
```

```r
final_model <- lm(wage ~ cut10, data = df_wage_with_cuts)

df_for_plot <- data.frame(
  age = df_wage_with_cuts$age,
  wage = df_wage_with_cuts$wage,
  predicted_wage = predict(final_model),
  label = "predicted_wage"
)

ggplot2::ggplot(data = df_for_plot) +
  ggplot2::geom_point(ggplot2::aes(x = age, y = wage)) +
  ggplot2::geom_line(ggplot2::aes(x = age, y = predicted_wage, color = label)) +
  ggplot2::labs(color = NULL)
```



Again we see that the model captures the relationship well for most observations, but doesn't capture the wage of the high earners.

Question 7

```
# compare polynomial regression without extra vars to polynomial regression with extra vars

model_with_extra_vars <- glm(
  wage ~ poly(age, 3) + year + maritl + race + education +
  jobclass + health +  health_ins, data = df_wage
)
cv_estimate <- cv.glm(df_wage, model_with_extra_vars, K = 10)$delta[[1]]

print(cv_estimate)
```

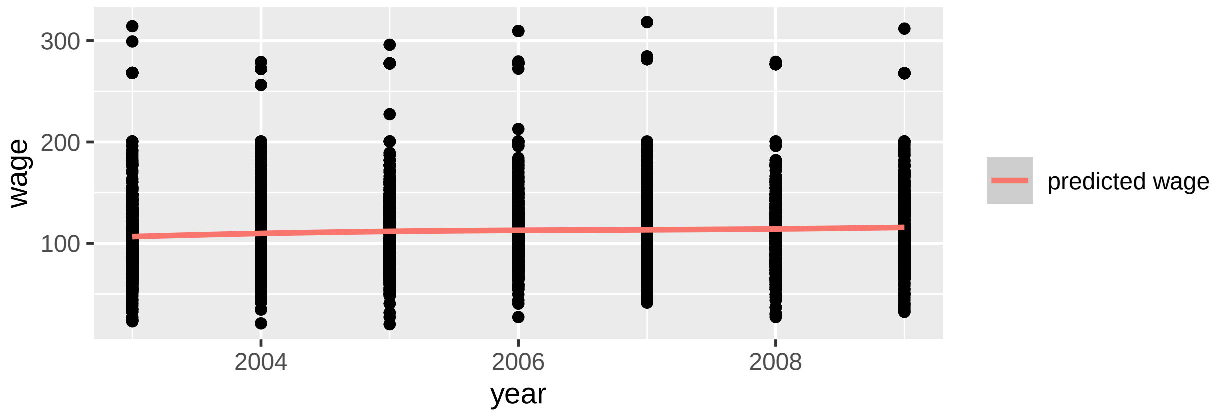
```
## [1] 1146.213
```

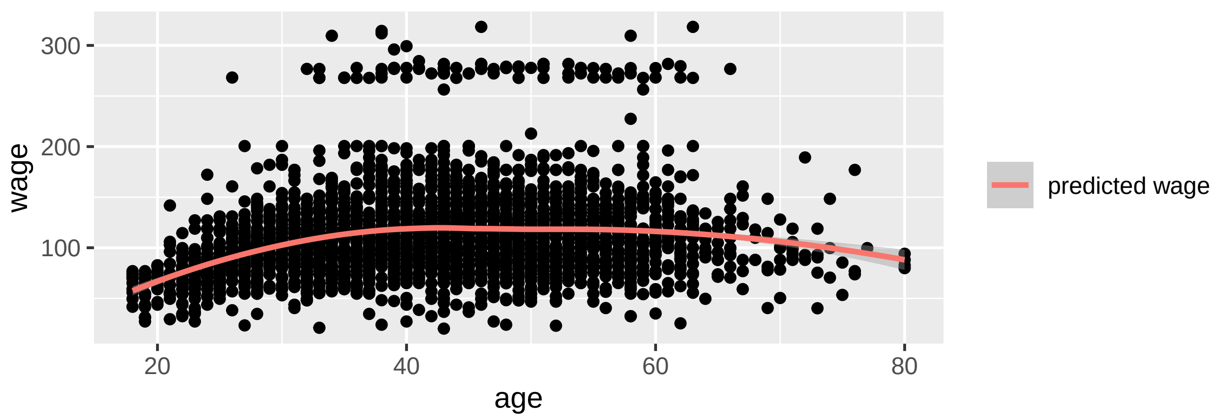
```
age_dfs <- seq(4, 6)
year_dfs <- seq(4,5)
param_grid <- expand.grid(age_dfs, year_dfs, c("ns", "bs", "s"), stringsAsFactors = FALSE) %>%
  setNames(., c("age", "year", "spline_type"))

cv_estimates <- rep(0, nrow(param_grid))
models <- vector("list", length = nrow(param_grid))

set.seed(1)
```

```r
for (i in seq(nrow(param_grid))){
  spline_method <- param_grid[i, "spline_type"]
  if (identical(spline_method, "s")) {
    model_function <- gam::gam
    spline_method <- gam::s
  } else {
    model_function <- glm
    spline_method <- get(spline_method)
  }

  df_age <- param_grid[i, "age"]
  df_year <- param_grid[i, "year"]
  model <- model_function(
    wage ~ spline_method(age, df = df_age) + spline_method(year, df = df_year) + maritl + race + educati
    jobclass + health +  health_ins, data = df_wage
  )
  models[[i]] <- model
  cv_estimates[[i]] <- boot::cv.glm(df_wage, model, K = 10)$delta[[1]]
}

param_grid$cv_estimate <- cv_estimates

best_model_idx <- which.min(param_grid$cv_estimate)

print(param_grid[best_model_idx, ])
```

```
##   age year spline_type cv_estimate
## 2   5    4          ns    1144.329
```

```r
df_for_plot <- data.frame(
  wage = df_wage$wage,
  age = df_wage$age,
  year = df_wage$year,
  predicted_wage = predict(models[[best_model_idx]]),
  label = "predicted wage"
)


plots <- lapply(
  c("year", "age"),
  function(var, df) {
    ggplot2::ggplot(data = df) +
      ggplot2::geom_point(ggplot2::aes(x = .data[[var]], y = wage)) +
      ggplot2::geom_smooth(ggplot2::aes(x = .data[[var]], y = predicted_wage, color = label), method =
      ggplot2::labs(color = NULL)
  },
  df = df_for_plot
)

do.call(gridExtra::grid.arrange, plots)
```

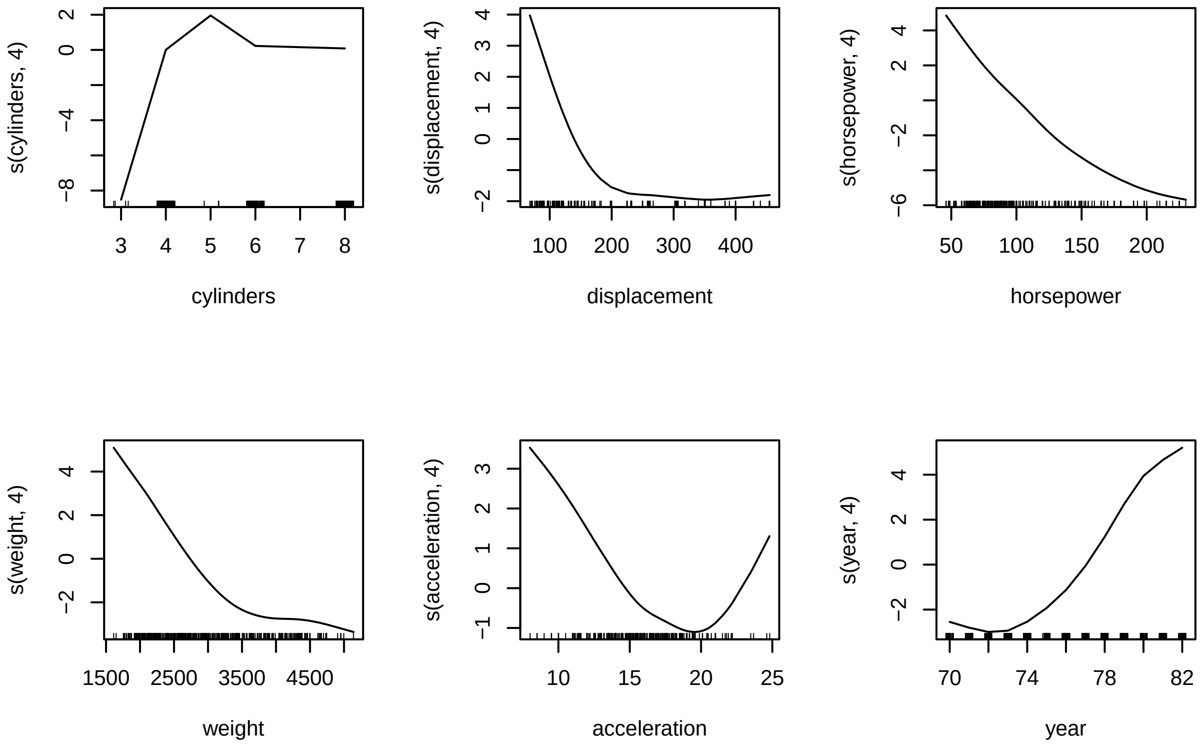




The best method is a natural spline with 5 degrees of freedom for age and 4 degrees of freedom for year. Note that the decrease in MSE is very small compared to the polynomial of degree 3 and the raw value of year, in addition to the other covariates. Looking at the plots, although we have fit a natural spline to year, the relationship looks very linear. Age is certainly non-linear.

Question 8

```r
df_auto <- ISLR::Auto

features <- setdiff(colnames(df_auto), c("mpg", "origin", "name"))

model_formula <- as.formula(
  paste(
    "mpg ~",
    paste(
      paste0("s(", features , ", 4)"),
      collapse = " + "
    )
  )
)
model <- gam::gam(model_formula, data = df_auto)

par(mfrow = c(2, 3))

plot(model)
```



There is evidence of non-linear relationships between all the variables we chose and mpg.

Question 9

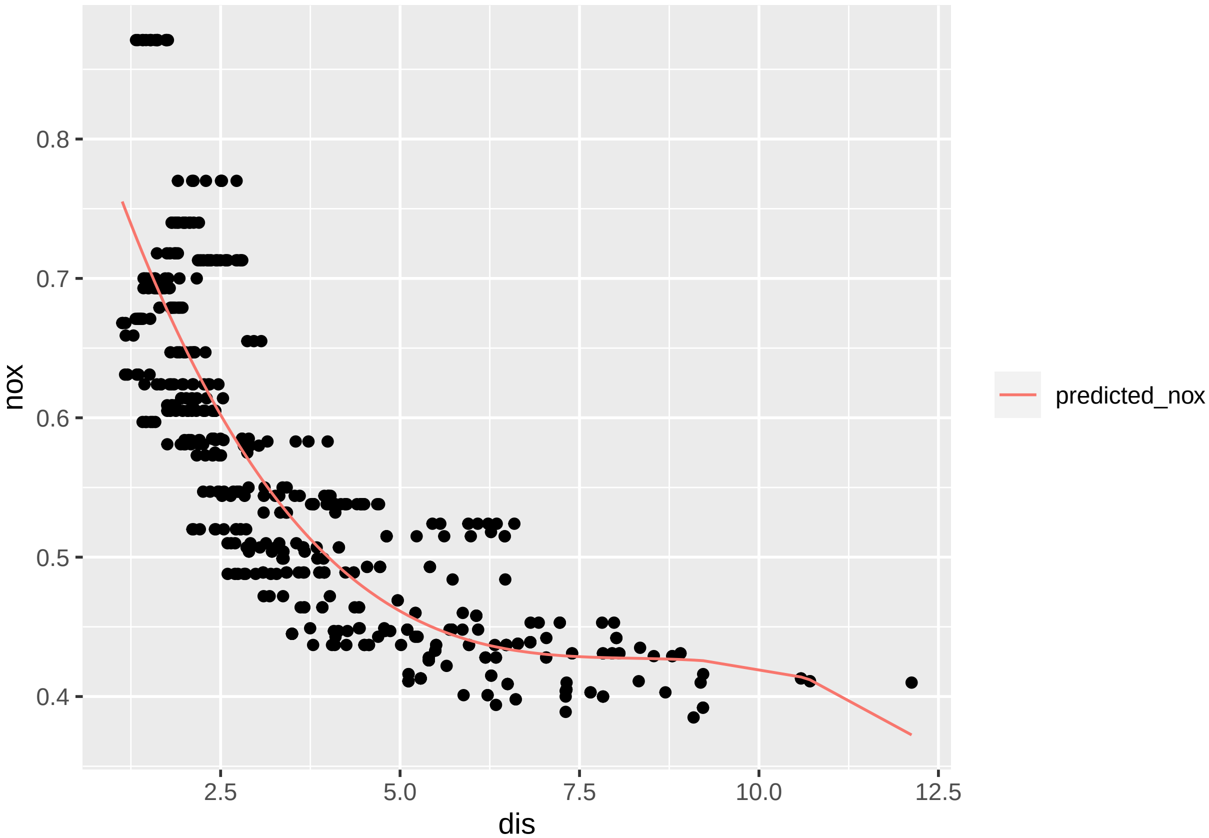
a

```r
df_boston <- MASS::Boston


model <- glm(nox ~ poly(dis, 3), data = df_boston)

print(summary(model))
```

```
##
## Call:
## glm(formula = nox ~ poly(dis, 3), data = df_boston)
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)    0.554695   0.002759 201.021  < 2e-16 ***
## poly(dis, 3)1 -2.003096   0.062071 -32.271  < 2e-16 ***
## poly(dis, 3)2  0.856330   0.062071  13.796  < 2e-16 ***
## poly(dis, 3)3 -0.318049   0.062071  -5.124 4.27e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for gaussian family taken to be 0.003852802)
##
##     Null deviance: 6.7810  on 505  degrees of freedom
## Residual deviance: 1.9341  on 502  degrees of freedom
## AIC: -1370.9
```

```
##
## Number of Fisher Scoring iterations: 2
```

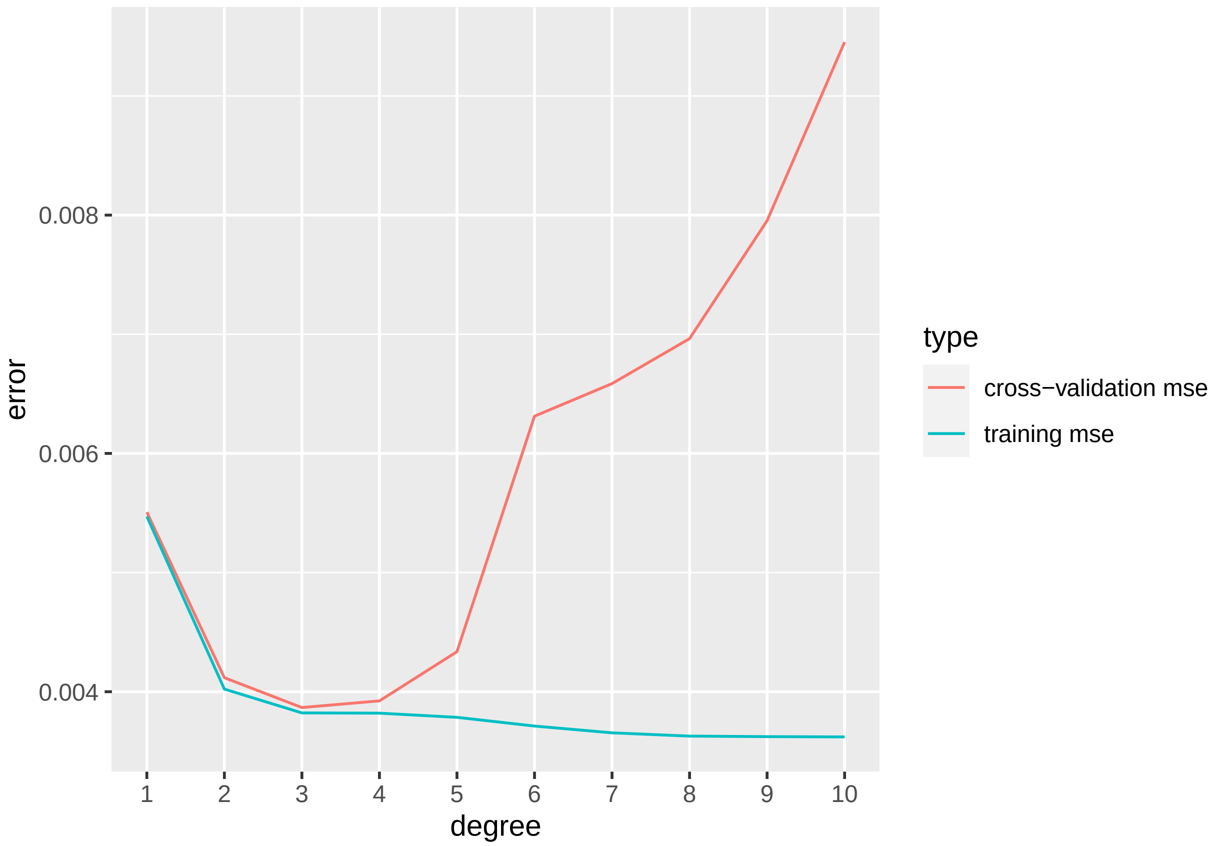
```
df_boston$preds <- predict(model)
df_boston$label <- "predicted_nox"

ggplot2::ggplot(data = df_boston) +
  ggplot2::geom_point(ggplot2::aes(x = dis, y = nox)) +
  ggplot2::geom_line(ggplot2::aes(x = dis, y = preds, color = label)) +
  ggplot2::labs(color = NULL)
```



b - c

```
max_poly_degree <- 10
cv_estimates <- rep(0, max_poly_degree)
training_rss <- cv_estimates
for (degree in seq(max_poly_degree)) {
  model <- glm(nox ~ poly(dis, degree), data = df_boston)
  training_rss[[degree]] <- model$deviance / (nrow(df_boston))
  cv_estimates[[degree]] <- cv.glm(df_boston, model, K = 10)$delta[[1]]
}

df_for_plot <- data.frame(
  degree = rep(seq_along(cv_estimates), 2),
  error = c(cv_estimates, training_rss),
  type = rep(c("cross-validation mse", "training mse"), each = max_poly_degree),
  training_rss = training_rss
```

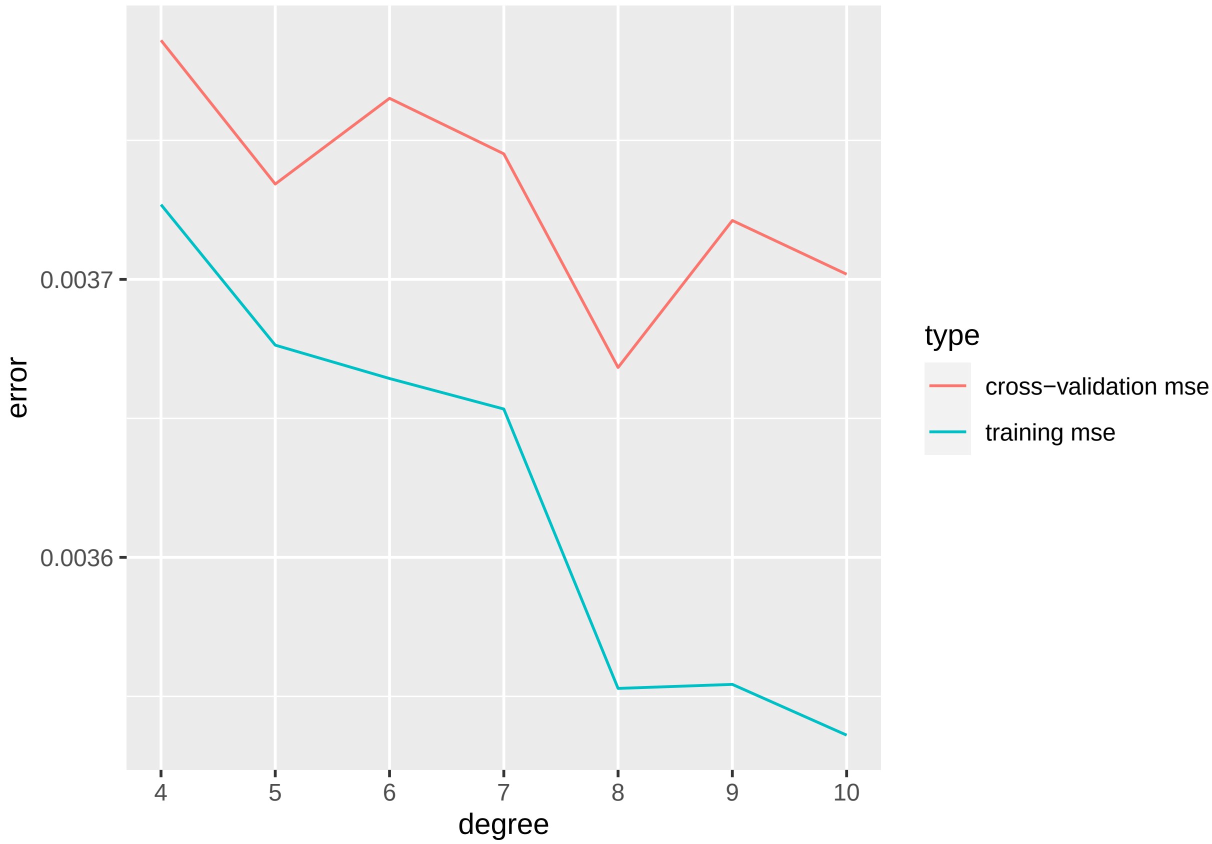
```
)

ggplot2::ggplot(data = df_for_plot) +
  ggplot2::geom_line(ggplot2::aes(x = degree, y = error, color = type)) +
  ggplot2::scale_x_continuous(breaks = seq(10), minor_breaks = NULL)
```



As expected, the MSE for the training error decreases as we add more polynomial terms, but the cross-validation error reaches a minimum at degree 3.

```
dfs <- seq(4, 10)
cv_estimates <- rep(0, length(dfs))
training_rss <- cv_estimates
for (i in seq_along(dfs)) {
  df <- dfs[[i]]
  model <- glm(nox ~ ns(dis, df = df), data = df_boston)
  training_rss[[i]] <- model$deviance / (nrow(df_boston))
  cv_estimates[[i]] <- cv.glm(df_boston, model, K = 10)$delta[[1]]
}

df_for_plot <- data.frame(
  degree = rep(dfs, 2),
  error = c(cv_estimates, training_rss),
  type = rep(c("cross-validation mse", "training mse"), each = length(dfs)),
  training_rss = training_rss
)
```

```r
ggplot2::ggplot(data = df_for_plot) +
  ggplot2::geom_line(ggplot2::aes(x = degree, y = error, color = type)) +
  ggplot2::scale_x_continuous(breaks = seq(10), minor_breaks = NULL)
```



d - f

8 degrees of freedom gives the lowest CV error.

Question 10

a

```r
df_college <- ISLR::College
nrows <- nrow(df_college)
train_idx <- sample(nrows, nrows %/% 2)

df_train <- df_college[train_idx, ]
df_test <- df_college[-train_idx, ]

n_vars <- ncol(df_train) - 1
best_subsets <- leaps::regsubsets(Outstate ~ ., data = df_train, nvmax = n_vars, method = "forward")

x_test <- model.matrix(Outstate ~ ., df_test)

mses <- rep(0, n_vars)
for (i in seq(n_vars)) {
  coefs <- coef(best_subsets, i)
  x_test_sub <- x_test[, names(coefs)]
  preds <- x_test_sub %*% coefs
```

```r
  mses[[i]] <- sum((preds - df_test$Outstate) ^ 2) / length(preds)
}

print(which.min(mses))
```

```
## [1] 17
```

```r
coefs <- names(coef(best_subsets, 13))
continuous_coefs <- setdiff(coefs, c("(Intercept)", "PrivateYes"))

model_formula <- as.formula(
  paste(
    "Outstate ~ Private +",
    paste(continuous_coefs, collapse = " + ")
  )
)

best_linear_model <- lm(model_formula, df_train)
preds <- predict(best_linear_model, df_test)
test_rmse <- sqrt(sum((preds - df_test$Outstate) ^ 2) / length(preds))

print(test_rmse)
```

```
## [1] 1947.581
```

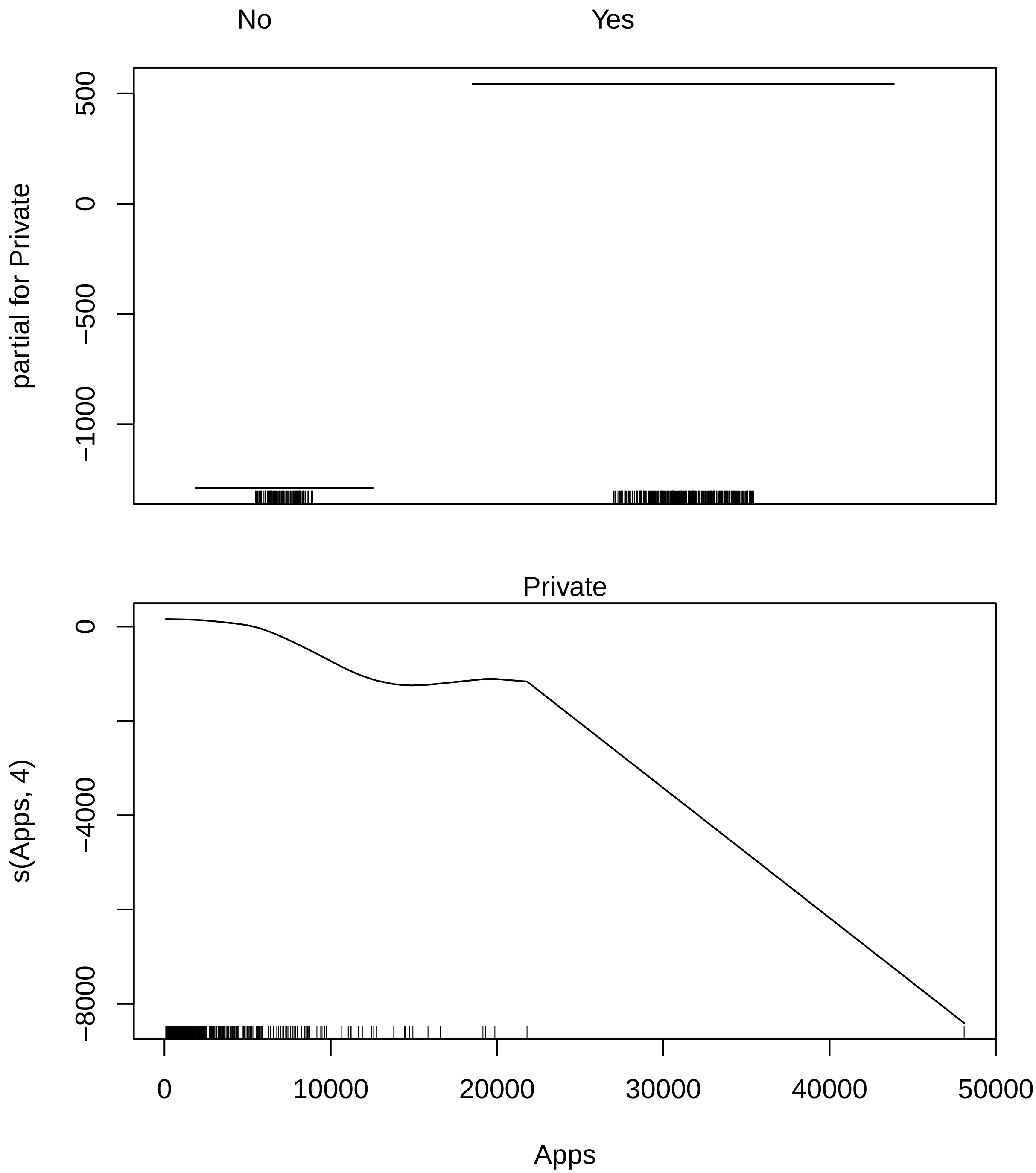
We see that the 13 variable model has the lowest test error here. So we fit a GAM using those 13 variables. Note that one of the variables is categorical so we don't apply any smoothing to that variable.

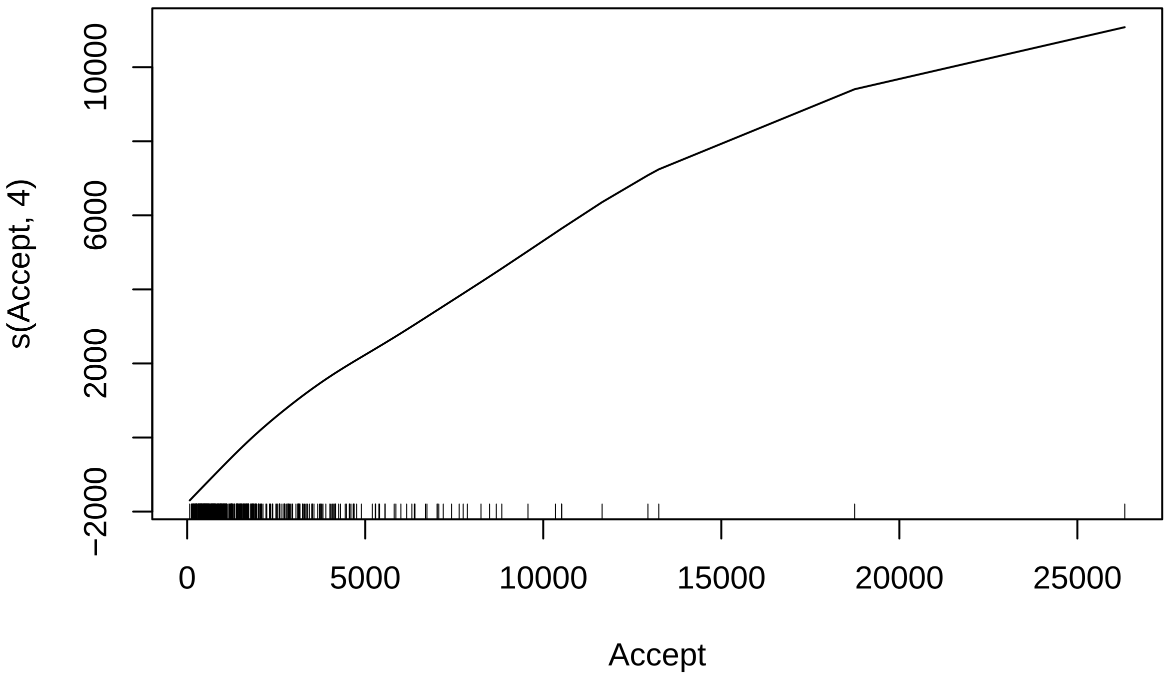
```r
model_formula <- as.formula(
  paste(
    "Outstate ~ Private +",
    paste(
        paste0("s(", continuous_coefs, ", 4)"),
        collapse = " + "
    )
  )
)

gam_model <- gam::gam(model_formula, data = df_train)

print(summary(gam_model))
```

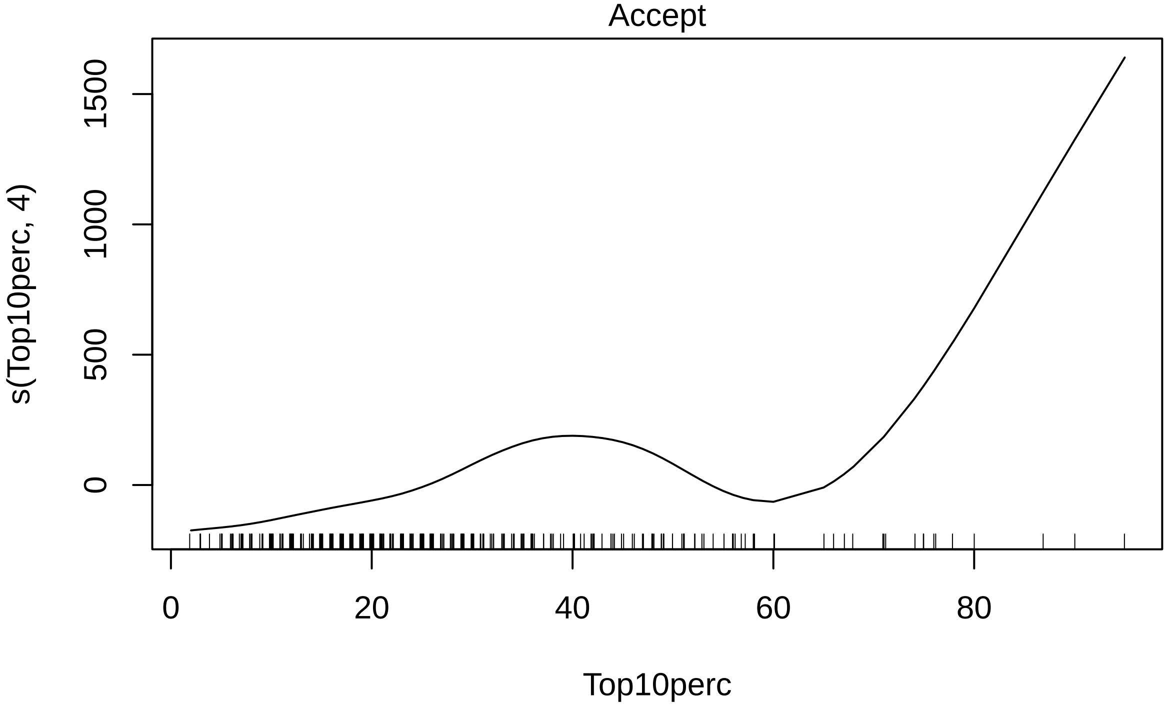
```
##
## Call: gam::gam(formula = model_formula, data = df_train)
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -6105.86 -1041.60    88.71  1096.45  7264.62
##
## (Dispersion Parameter for gaussian family taken to be 3208487)
##
##     Null Deviance: 6321773043 on 387 degrees of freedom
## Residual Deviance: 1084468156 on 337.9999 degrees of freedom
## AIC: 6962.316
##
## Number of Local Scoring Iterations: NA
##
```

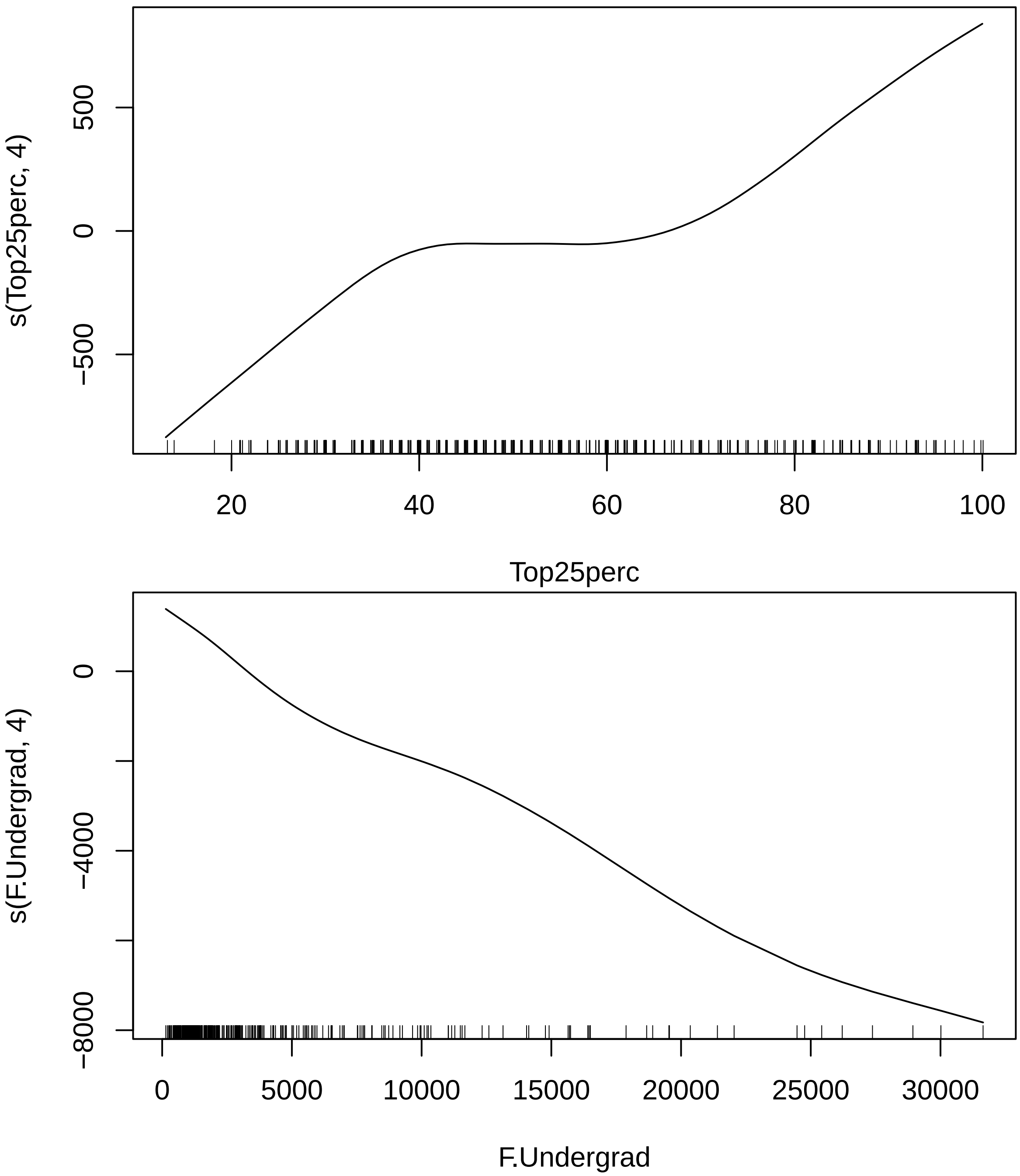
```
## Anova for Parametric Effects
##                    Df     Sum Sq    Mean Sq  F value    Pr(>F)    
## Private             1 1786659474 1786659474 556.8542 < 2.2e-16 ***
## s(Apps, 4)          1  376228460  376228460 117.2604 < 2.2e-16 ***
## s(Accept, 4)        1   43887360   43887360  13.6785 0.0002531 ***
## s(Top10perc, 4)     1  598266986  598266986 186.4639 < 2.2e-16 ***
## s(Top25perc, 4)     1      93300      93300   0.0291 0.8646988    
## s(F.Undergrad, 4)   1  362348023  362348023 112.9342 < 2.2e-16 ***
## s(Room.Board, 4)    1  410007098  410007098 127.7883 < 2.2e-16 ***
## s(Books, 4)         1   10840043   10840043   3.3786 0.0669274 .  
## s(Terminal, 4)      1  122688574  122688574  38.2388 1.805e-09 ***
## s(S.F.Ratio, 4)     1   68864020   68864020  21.4631 5.156e-06 ***
## s(perc.alumni, 4)   1  123111569  123111569  38.3706 1.699e-09 ***
## s(Expend, 4)        1  344183836  344183836 107.2729 < 2.2e-16 ***
## s(Grad.Rate, 4)     1   16384421   16384421   5.1066 0.0244713 *  
## Residuals         338 1084468156    3208487                       
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Anova for Nonparametric Effects
##                   Npar Df  Npar F     Pr(F)    
## (Intercept)                                    
## Private                                        
## s(Apps, 4)              3  2.4674  0.062006 .  
## s(Accept, 4)            3  4.0497  0.007544 ** 
## s(Top10perc, 4)         3  1.2726  0.283638    
## s(Top25perc, 4)         3  0.6673  0.572628    
## s(F.Undergrad, 4)       3  2.6677  0.047663 *  
## s(Room.Board, 4)        3  2.9149  0.034380 *  
## s(Books, 4)             3  1.2150  0.304180    
## s(Terminal, 4)          3  1.1921  0.312701    
## s(S.F.Ratio, 4)         3  2.6524  0.048631 *  
## s(perc.alumni, 4)       3  3.5466  0.014821 *  
## s(Expend, 4)            3 20.3984 3.581e-12 ***
## s(Grad.Rate, 4)         3  1.4806  0.219585    
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

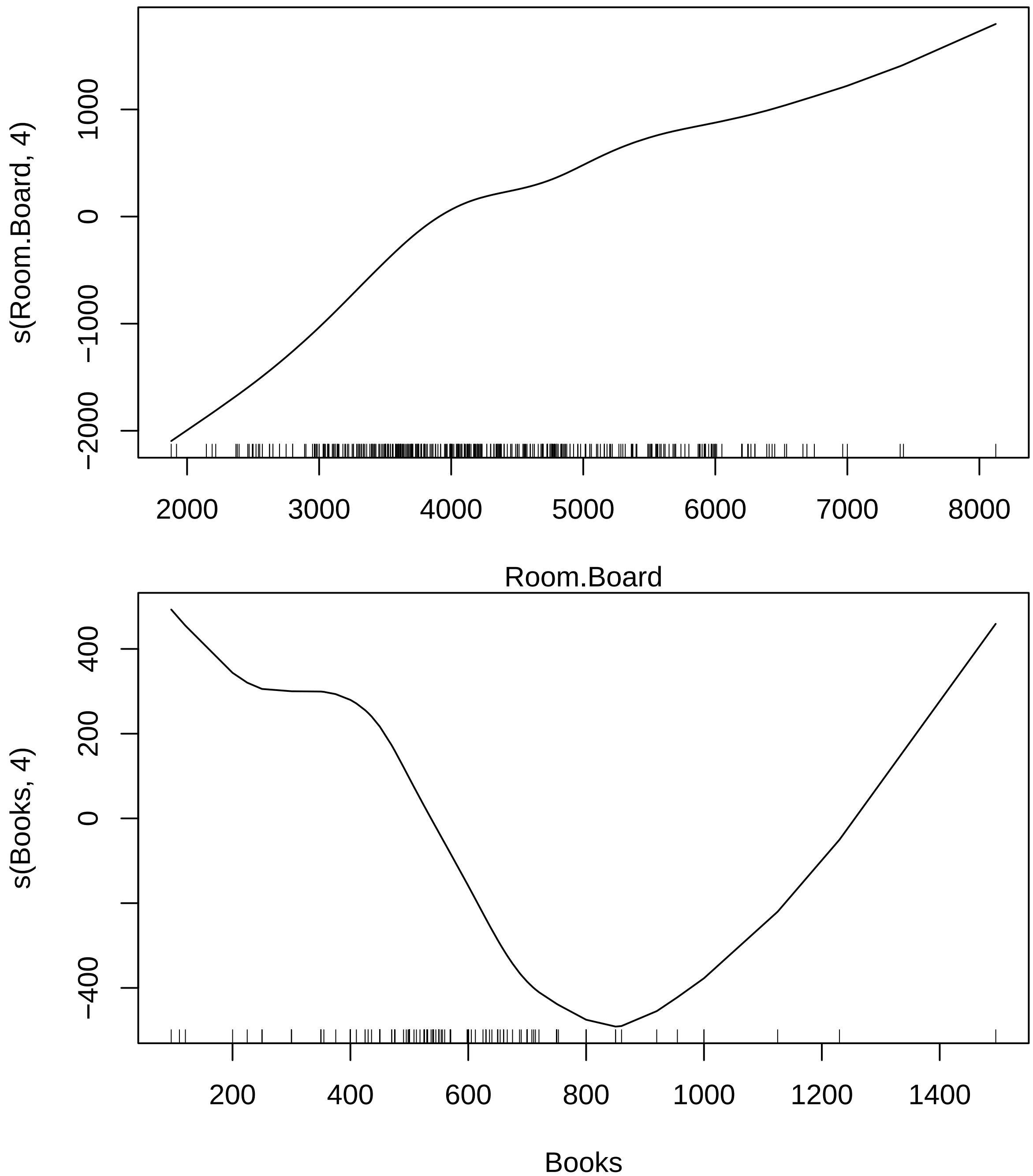
```r
plot(gam_model)
```

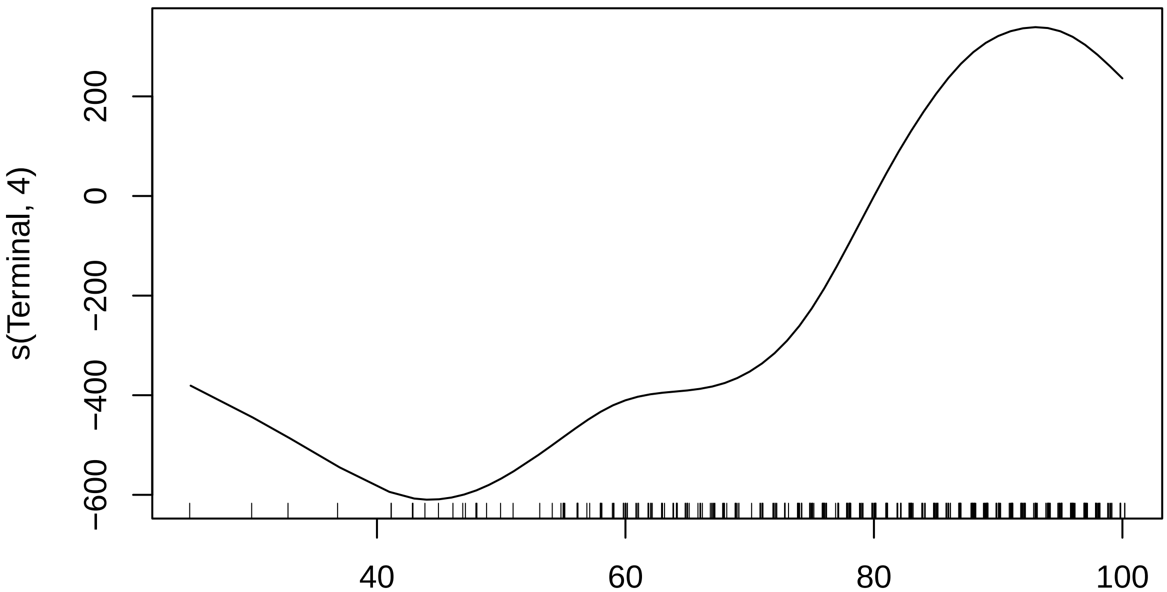

No
Yes
partial for Private
500
0
−500
−1000
Private
s(Apps, 4)
0
−4000
−8000
0
10000
20000
30000
40000
50000
Apps

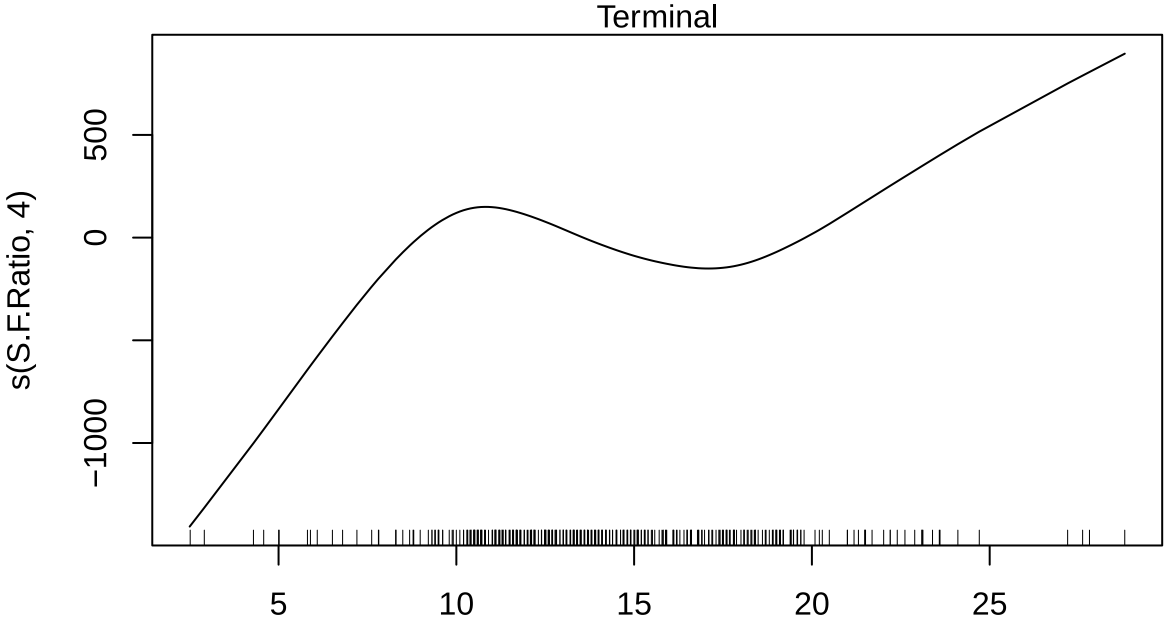

s(Accept, 4)
−2000
2000
6000
10000
0
5000
10000
15000
20000
25000
Accept

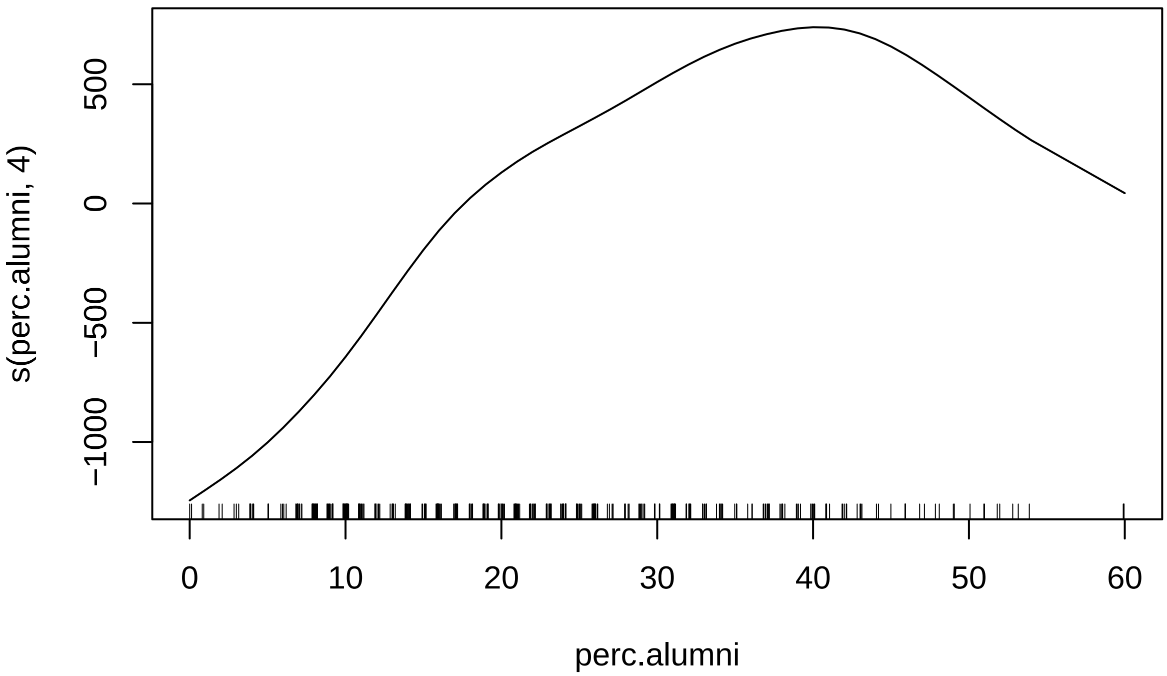

s(Top10perc, 4)
0
500
1000
1500
0
20
40
60
80
Top10perc


s(Top25perc, 4)
500
0
−500
20
40
60
80
100
Top25perc
s(F.Undergrad, 4)
0
−4000
−8000
0
5000
10000
15000
20000
25000
30000
F.Undergrad

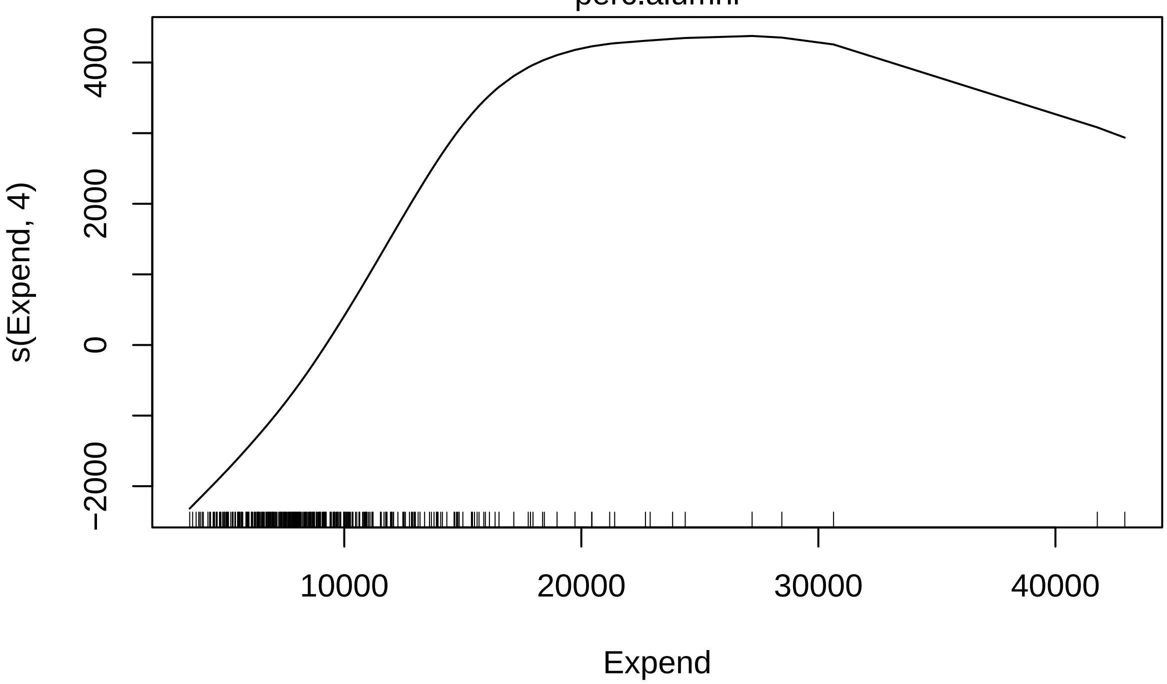

s(Room.Board, 4)
−2000
−1000
0
1000
2000
3000
4000
5000
6000
7000
8000
Room.Board
s(Books, 4)
−400
0
200
400
200
400
600
800
1000
1200
1400
Books


s(Terminal, 4)
200
0
−200
−400
−600
40
60
80
100
Terminal

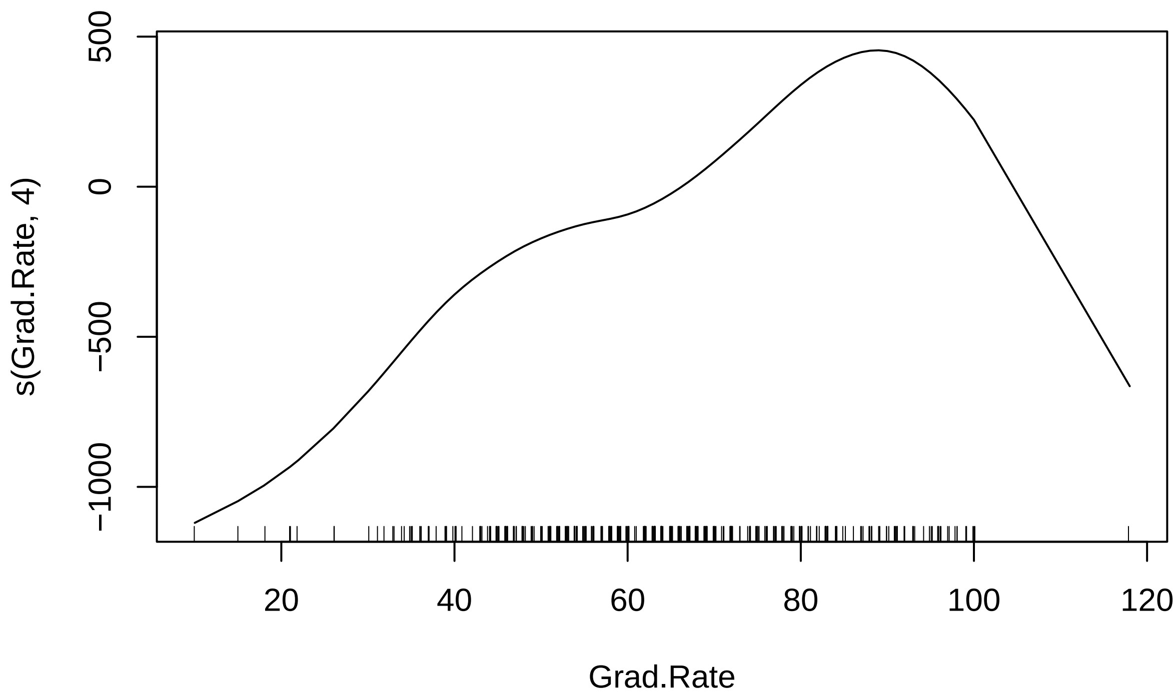

s(S.F.Ratio, 4)
500
0
−1000
5
10
15
20
25
S.F.Ratio



s(perc.alumni, 4)
500
0
−500
−1000
0
10
20
30
40
50
60
perc.alumni



s(Expend, 4)
4000
2000
0
−2000
10000
20000
30000
40000
Expend




```r
preds <- predict(gam_model, df_test)
test_rmse <- sqrt(sum((preds - df_test$Outstate) ^ 2) / length(preds))
print(test_rmse)
```

```
## [1] 1849.644
```

We see evidence of non-linearity for many but not all of the covariates, and a corresponding decrease in the test set rmse.

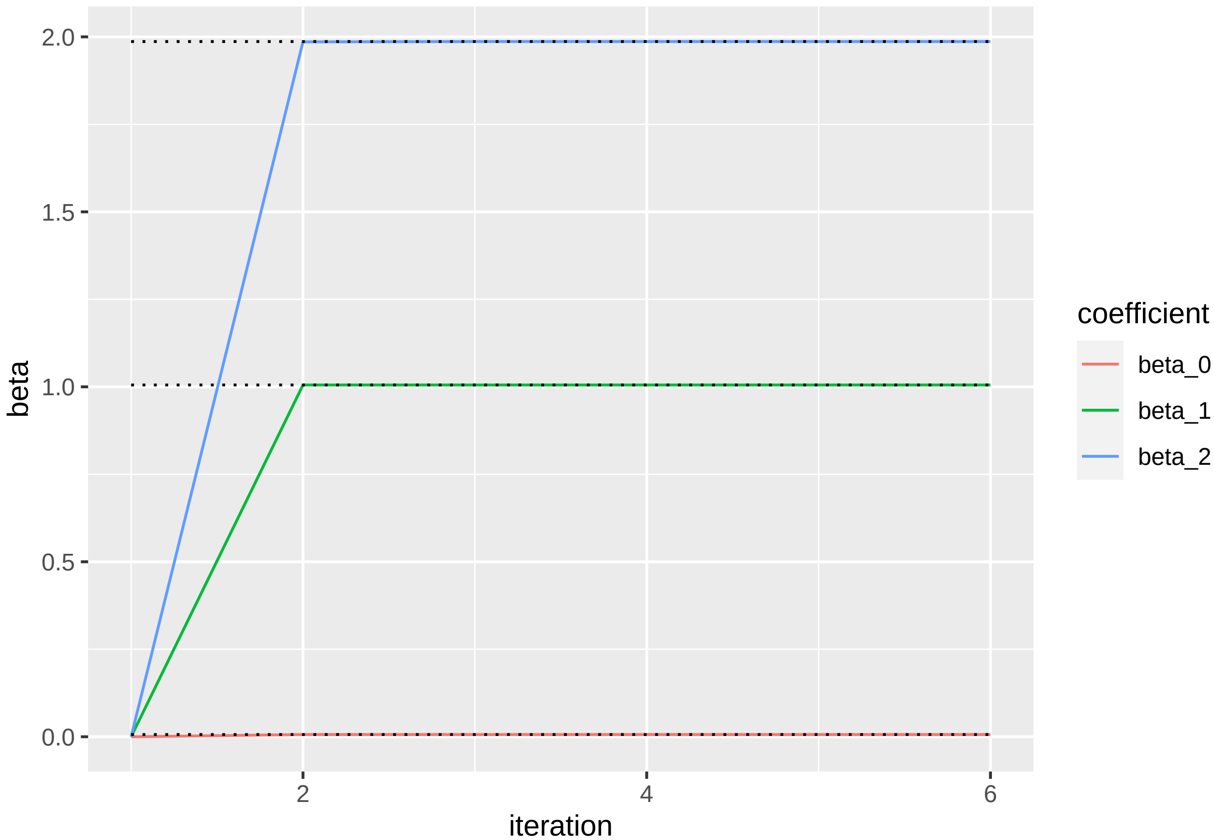
Question 11

```r
set.seed(1)
x1 <- rnorm(100)
x2 <- rnorm(100)
eps <- rnorm(100, sd = 0.25)

y <- x1 + 2 * x2 + eps

full_model <- lm(y ~ x1 + x2)
full_model_coefs <- coef(full_model)

print(full_model_coefs)
```

```
## (Intercept)          x1          x2
## 0.006338357 1.005277542 1.986633296
```

```r
n_iterations <- 5

beta_0 <- rep(0, n_iterations + 1)
beta_1 <- rep(0, n_iterations + 1)
beta_2 <- rep(0, n_iterations + 1)
```

```
current_beta_0 <- 0
current_beta_1 <- 0
current_beta_2 <- 0

beta_0[[1]] <- current_beta_0
beta_1[[1]] <- current_beta_1
beta_2[[1]] <- current_beta_2

for (i in seq(2, n_iterations + 1)) {
  current_y <- y - current_beta_1 * x1
  model <- lm(current_y ~ x2)
  current_beta_2 <- coef(model)[[2]]

  current_y <- y - current_beta_2 * x2
  model <- lm(current_y ~ x1)
  current_beta_1 <- coef(model)[[2]]

  current_beta_0 <- coef(model)[[1]]

  beta_0[[i]] <- current_beta_0
  beta_1[[i]] <- current_beta_1
  beta_2[[i]] <- current_beta_2
}

df_backfitting <- data.frame(
  beta = c(beta_0, beta_1, beta_2),
  coefficient = rep(c("beta_0", "beta_1", "beta_2"), each = n_iterations + 1),
  iteration = seq(n_iterations + 1)
)

df_ols <- data.frame(
  beta = rep(full_model_coefs, 2),
  coefficient = rep(c("beta_0_regression", "beta_1_regression", "beta_2_regression"), 2),
  iteration = rep(c(1, n_iterations + 1), each = 3)
)

ggplot2::ggplot(data = df_backfitting) +
  ggplot2::geom_line(ggplot2::aes(x = iteration, y = beta, color = coefficient)) +
  ggplot2::geom_line(data = df_ols, ggplot2::aes(x = iteration, y=beta, group = coefficient), linetype
```

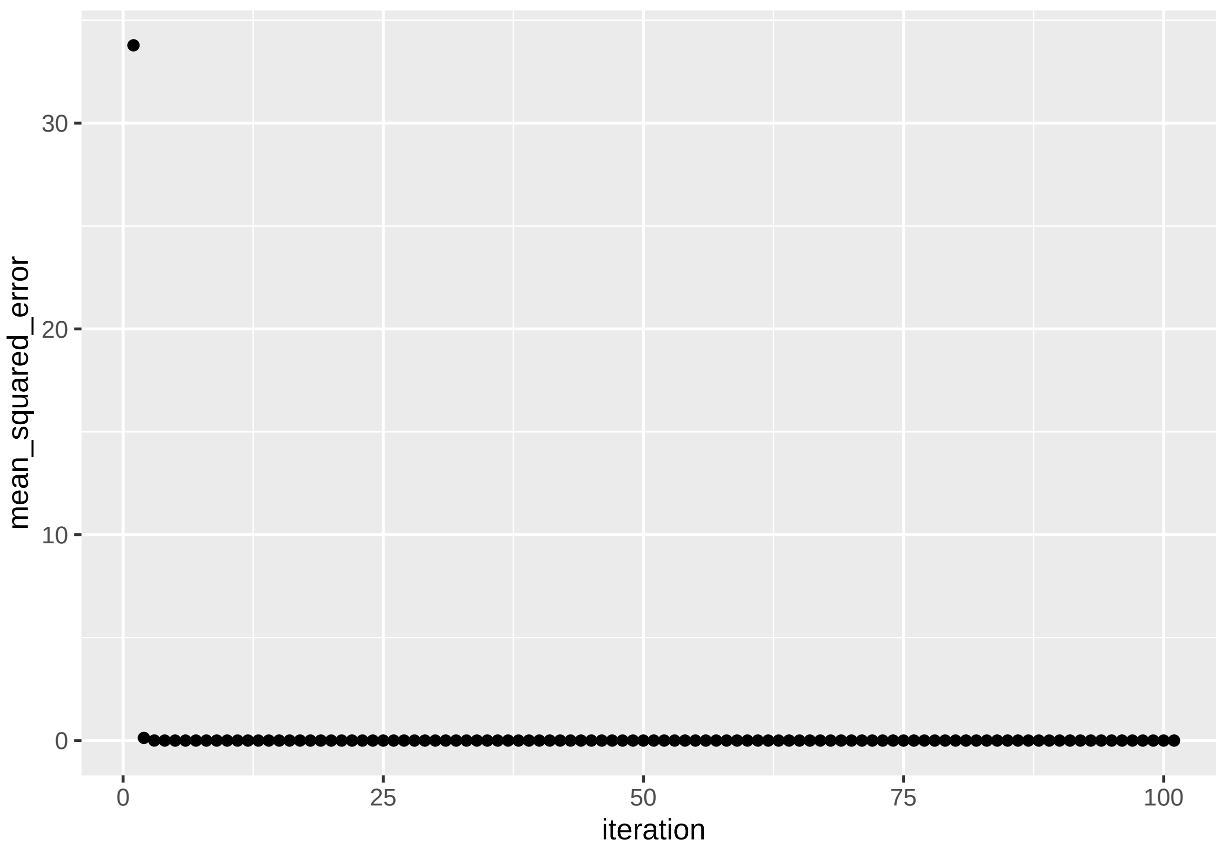


The algorithm converges in 2 iterations to the OLS coefficients, which are overlayed as dotted lines on the coefficients from backfitting.

Question 12

```
n_iterations <- 10
nrows <- 10000
ncols <- 100
X <- matrix(rnorm(nrows * ncols), nrows, ncols)
eps <- rnorm(nrows, sd = 0.25)
actual_betas <- sample(10, 100, replace = TRUE)

y <- X %*% actual_betas + eps

ols_coefs <- coef(lm(y ~ X))

backfit <- function(y, X, n_iterations, ols_coefs) {

  ncols <- ncol(X)
  current_betas <- rep(0, ncols + 1)
  mean_squared_error <- rep(0, ncols + 1)
  mean_squared_error[[1]] <- mean((ols_coefs - current_betas) ^ 2)

  for (i in seq(2, n_iterations + 1)) {
    for (j in seq(1, length(current_betas) - 1)) {
```

```r
      current_beta <- current_betas[[j + 1]]
      current_x <- X[, j]
      current_y <- y - X[, -j] %*% current_betas[2:length(current_betas)][-j]
      model <- lm(current_y ~ current_x)
      current_betas[[j + 1]] <- coef(model)[[2]]
    }
    current_betas[[1]] <- coef(model)[[1]]
    mean_squared_error[[i]] <- mean((ols_coefs - current_betas) ^ 2)

  }

  mean_squared_error
}


mean_squared_errors <- backfit(y, X, 10, ols_coefs)

df_for_plot <- data.frame(
  mean_squared_error = mean_squared_errors,
  iteration = seq_along(mean_squared_errors)
)

ggplot2::ggplot(data = df_for_plot) +
  ggplot2::geom_point(ggplot2::aes(x = iteration, y = mean_squared_error))
```



We see that after 3 iterations we have reached almost 0 error already.