ISLR Chapter 6 Exercises

2023-07-01

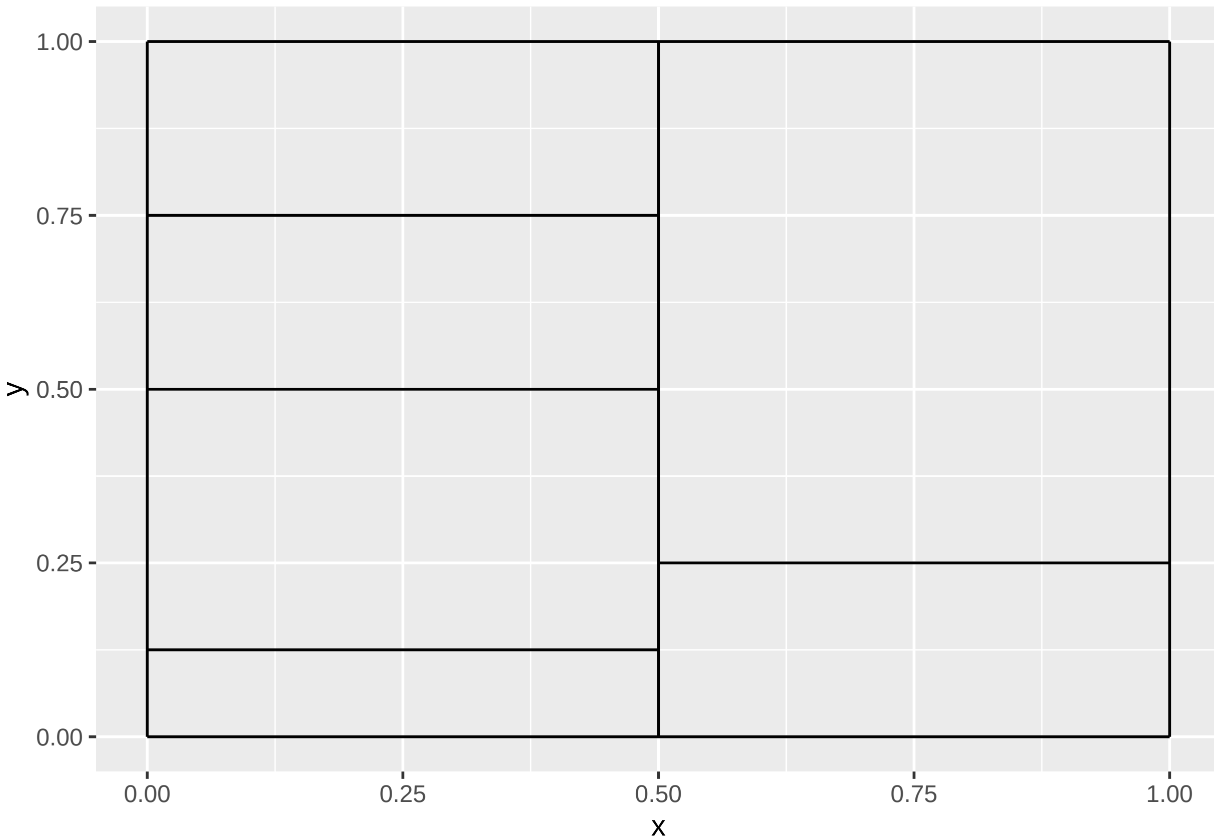
Contents


```r
library(ggplot2)
library(randomForest)
library(MASS)
library(tree)
library(ISLR)
library(ROCR)
library(gbm)
library(class)
```

Conceptual

Question 1

```r
line_df <- data.frame(
  x = c(0, 0, 0, 0.5, 0, 1, 0, 0, 0.5),
  y = c(0, 0, 0.5, 0, 1, 1, 0.75, 0.125, 0.25),
  xend = c(0, 1, 0.5, 0.5, 1, 1, 0.5, 0.5, 1),
  yend = c(1, 0, 0.5, 1, 1, 0, 0.75, 0.125, 0.25)
)
ggplot2::ggplot(data = line_df) +
  ggplot2::geom_segment(aes(x = x, y = y, xend = xend, yend = yend))
```



Question 2

Since each tree has depth 1, it follows that each tree only uses a single variable for splitting. That is each tree , indexed by `i` has the following functional form:

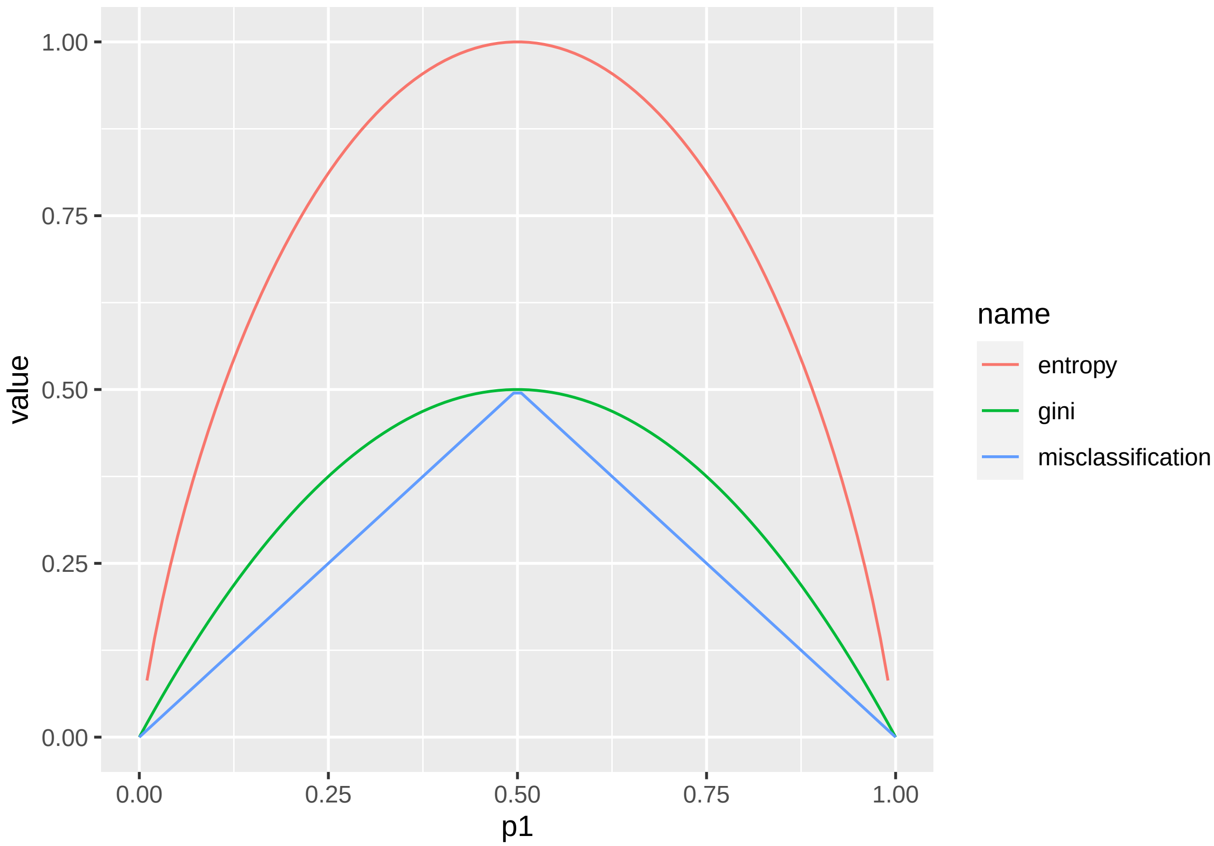
$c + d\mathbf{1}_{x_j <= t_j} = f_i(x_j)$

If multiple trees share the same variable for splitting, we can combine them into a single function $g_j = \sum_{i: f_i \, is \, a \, function \, of \, x_j} \lambda f_i(x_j)$. So the final model is of the form $\hat{f} = \sum_{j=1}^{p} g_j(x_j)$, which is an additive model of the predictors.

Question 3

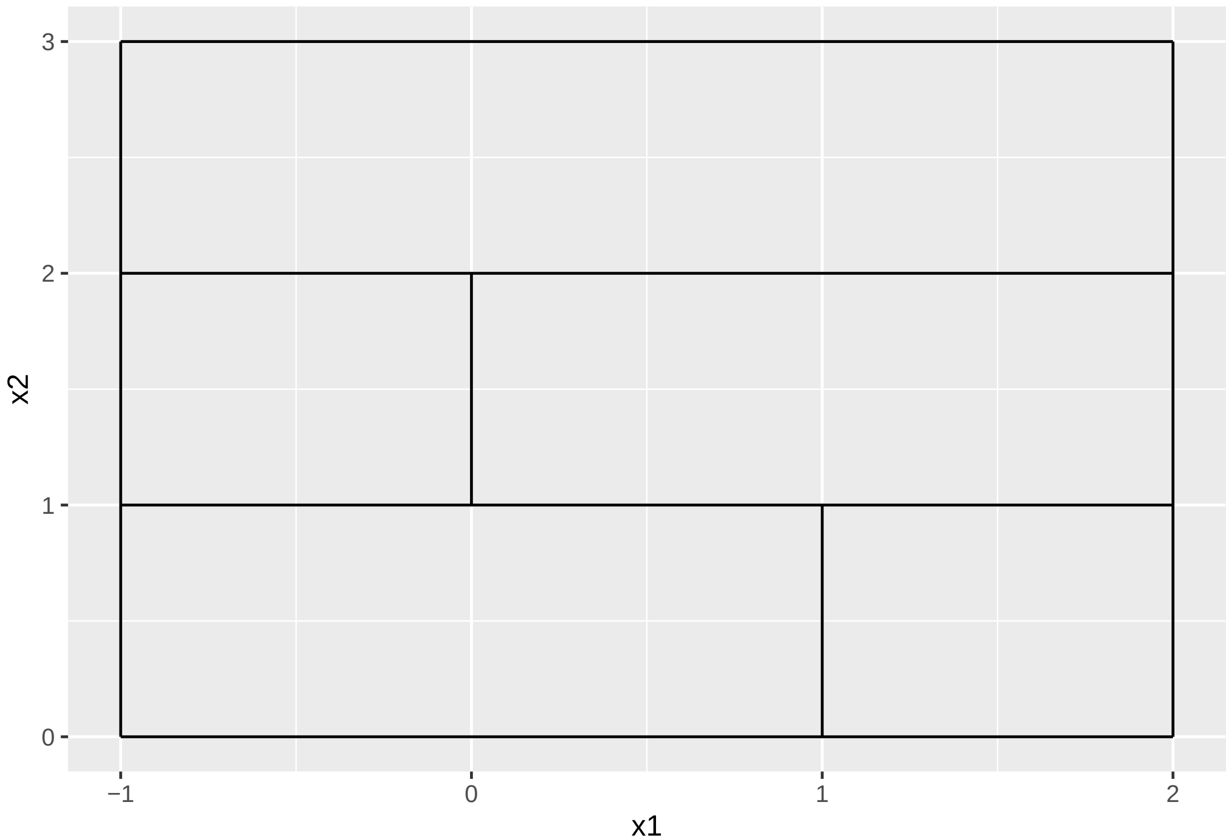
```r
p1 <- seq(0, 1, length = 100)
p2 <- 1 - p1

df <- data.frame(
  p1 = p1,
  p2 = p2,
  misclassification = 1 - pmax(p1, p2),
  gini = 2 * p1 * p2,
  entropy = -p1 * log(p1, 2) - p2 * log(p2, 2)
)

df <- tidyr::pivot_longer(df, cols = c("misclassification", "gini", "entropy"))
```

```r
ggplot2:::ggplot(data = df) +
  ggplot2::geom_line(ggplot2::aes(x = p1, y = value, color = name))
```



Question 4

```r
line_df <- data.frame(
  x1 = c(-1, -1, 1, -1, 2, -1, -1, 0),
  x2 = c(0, 1, 0, 0, 0, 2, 3, 1),
  x1end = c(2, 2, 1, -1, 2, 2, 2, 0),
  x2end = c(0, 1, 1, 3, 3, 2, 3, 2)
)
ggplot2::ggplot(data = line_df) +
  ggplot2::geom_segment(aes(x = x1, y = x2, xend = x1end, yend = x2end))
```



Question 5

```
probs <- c(0.1, 0.15, 0.2, 0.2, 0.55, 0.6, 0.6, 0.65, 0.7, 0.75)
labels <- ifelse(probs <= 0.5, "Green", "Red")
label_table <- table(labels)

print(paste("Majority Vote: ", names(label_table)[label_table == max(label_table)]))
```

```
## [1] "Majority Vote:  Red"
```

```
print(paste("Average Probability Approach:", if (mean(probs) >= 0.5) "Red" else "Green"))
```

```
## [1] "Average Probability Approach: Green"
```

Applied

Question 6

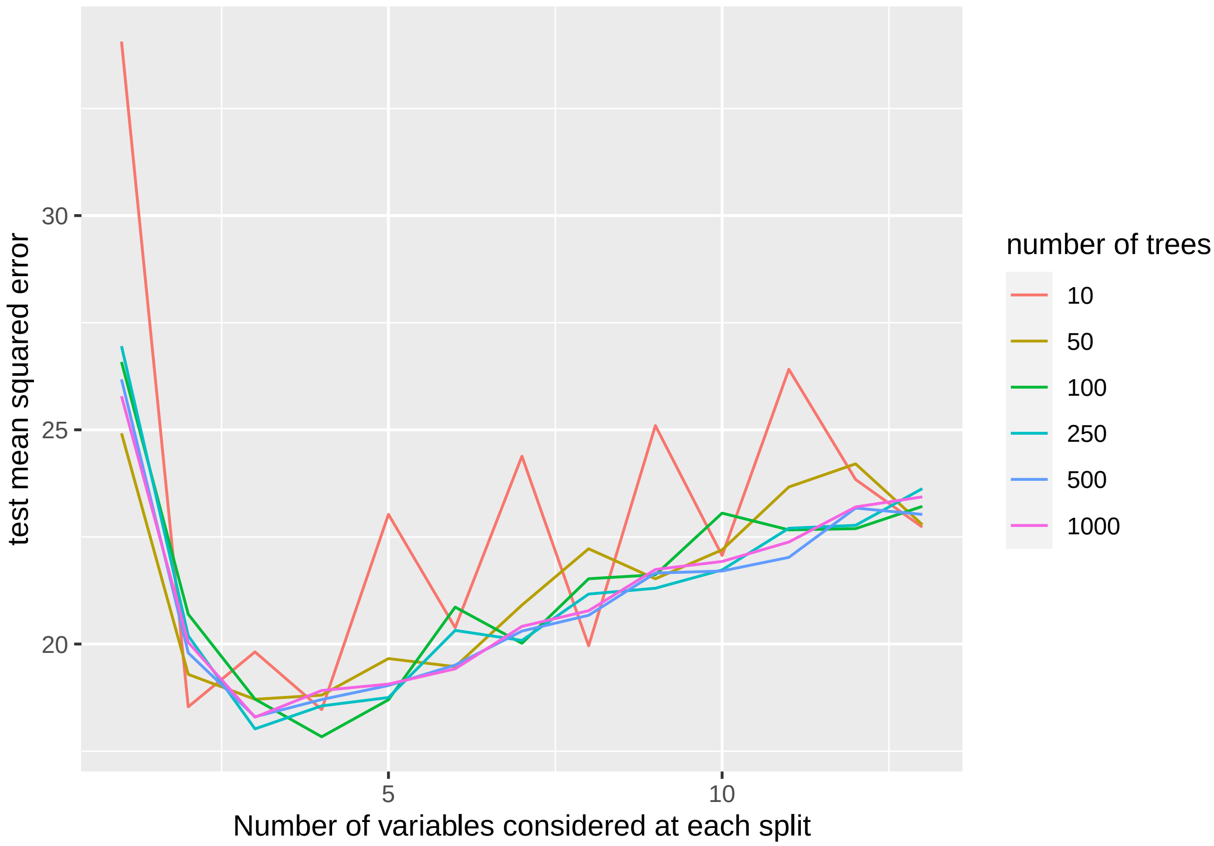
A regression tree is a recursive binary splitting algorithm. Given a set of observations in a node, for each predictor and each value of the predictor, we split the node into two sub-nodes, with observations in each node corresponding to a split of either less than or equal to the threshold, or greater than the threshold. For each split, we calculate the prediction of the children nodes as the mean of the response variable for the observations in that node. We then calculate the total sum of squared errors for each possible split using that prediction, and pick the split with the lowest total sum of squared errors. This process is repeated on the children nodes recursively until the terminal nodes have some minimum number of observations in them, or the tree has reached the maximum allowed depth.

Question 7

```r
df_boston <- MASS::Boston
nrows <- nrow(df_boston)
n_predictors <- ncol(df_boston) - 1

set.seed(1)
train_idx <- sample(nrows, nrows %/% 2)

df_train <- df_boston[train_idx, ]
df_test <- df_boston[-train_idx, ]

params <- expand.grid(mtry = seq(n_predictors), ntree = c(10, 50, 100, 250, 500, 1000))
test_error <- rep(0, nrow(params))

for (i in seq_along(test_error)) {
  model <- do.call(randomForest::randomForest, c(formula = formula(medv ~ .), data = list(df_train), pa
  preds <- predict(model, df_test)
  test_error[[i]] <- mean((preds - df_test$medv) ^ 2)
}

params$test_error <- test_error

print(params[which.min(params$test_error), ])
```

```
##    mtry ntree test_error
## 30    4   100   17.83553
```

```r
ggplot2::ggplot(data = params) +
  ggplot2::geom_line(ggplot2::aes(x = mtry, y = test_error, color = as.factor(ntree))) +
  ggplot2::labs(x = "Number of variables considered at each split", y = "test mean squared error", colo
```



Question 8

a-c

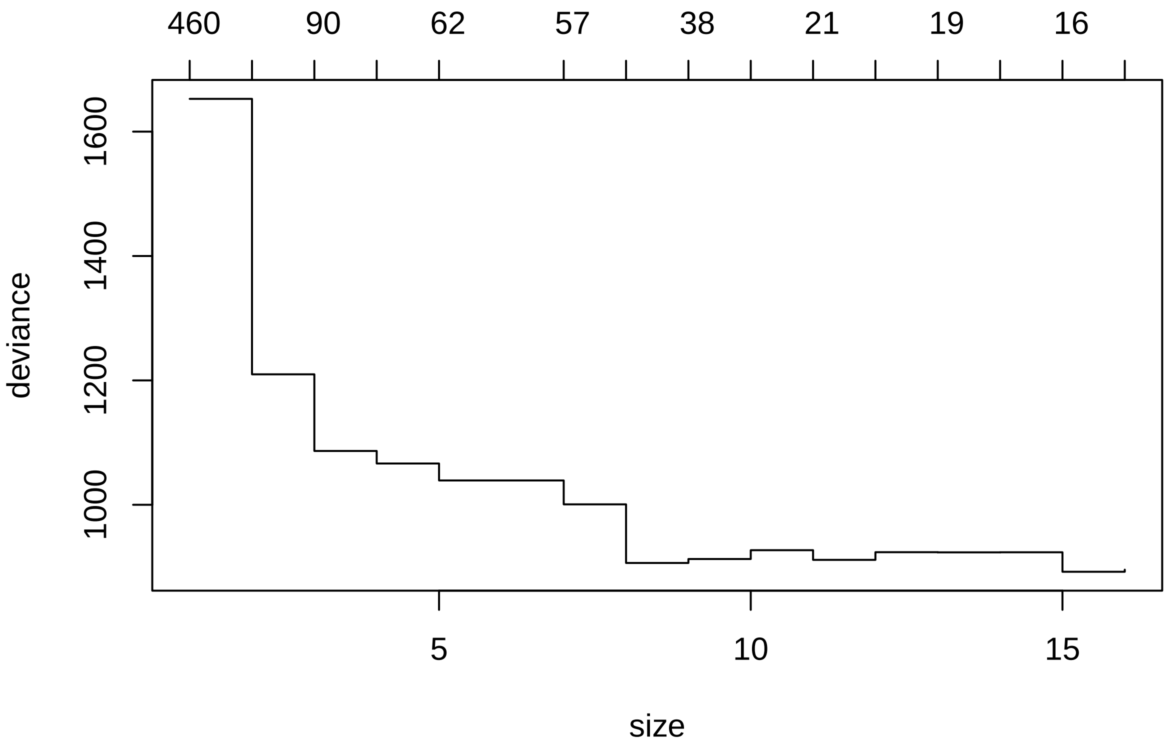
```
df_carseats <- ISLR::Carseats
nrows <- nrow(df_carseats)

train_idx <- sample(nrows, nrows %/% 2)

df_train <- df_carseats[train_idx, ]
df_test <- df_carseats[-train_idx, ]

regression_tree <- tree::tree(Sales ~ ., data = df_train)

preds <- predict(regression_tree, df_test)

test_mse <- mean((preds - df_test$Sales) ^ 2)

print(paste("test mse from unpruned regression tree: ", test_mse))
```

```
## [1] "test mse from unpruned regression tree:  5.36105674402005"
```

```
cv_regression_tree <- tree::cv.tree(regression_tree)

print(cv_regression_tree$size[which.min(cv_regression_tree$dev)])
```

```
## [1] 15
```

```r
plot(cv_regression_tree)
```



We see that the best cross-validation sum of squared errors occurs when we use the model without any pruning.

```r
bagged_model <- randomForest::randomForest(Sales ~ ., data = df_train, mtry = ncol(df_train) - 1, impor

preds <- predict(bagged_model, df_test)

test_mse <- mean((preds - df_test$Sales) ^ 2)


print(paste("test mse from bagged model: ", test_mse))
```

```
## [1] "test mse from bagged model:  2.66345847106232"
```

```r
print(importance(bagged_model))
```

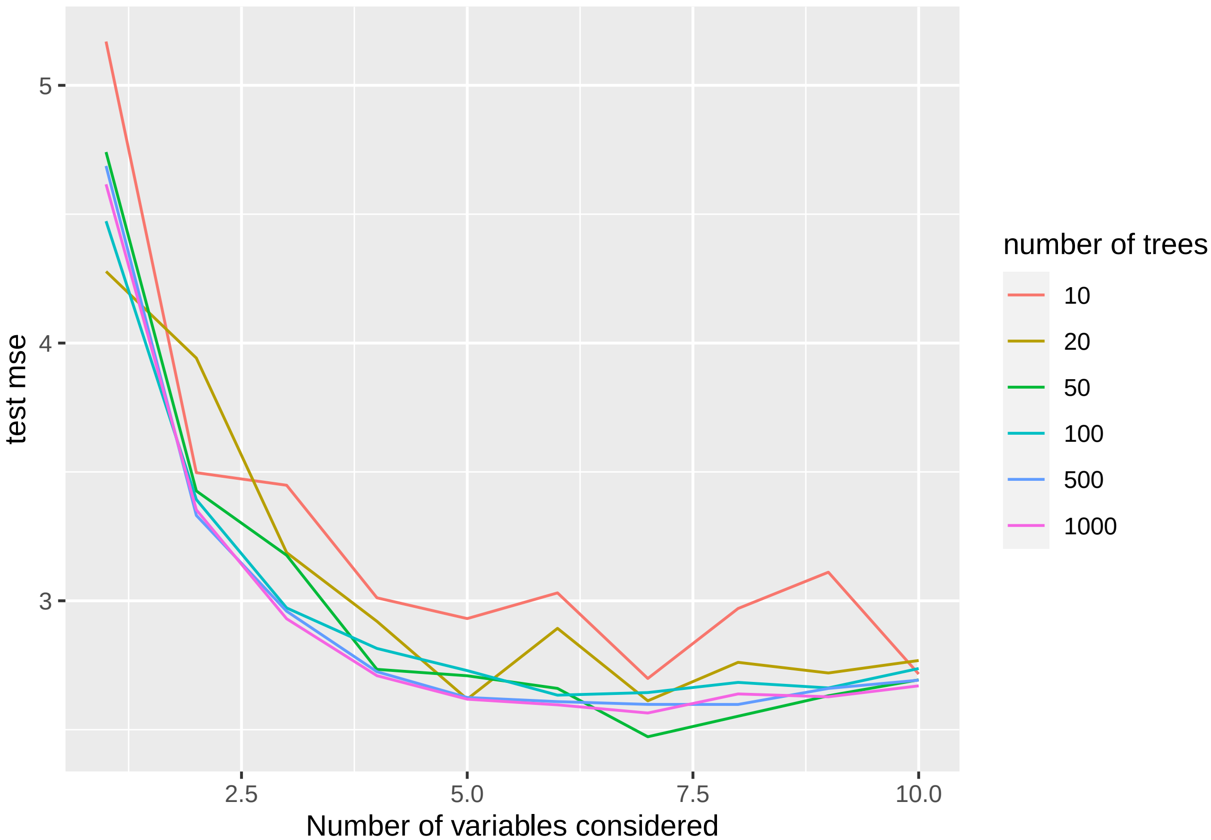
```
##                %IncMSE IncNodePurity
## CompPrice   21.1899036    149.506414
## Income       4.9379469     86.519623
## Advertising 12.6315874     93.514472
## Population   1.4900747     52.152711
## Price       58.9910503    476.387513
## ShelveLoc   60.9905744    498.372021
## Age         18.8152627    163.418586
## Education   -0.9169529     38.589570
## Urban        2.9348068      7.649432
## US           2.4007764      7.197300
```

Bagging improves the test mse considerably. The education variable actually decreases the MSE using permutation importance on the OOB samples.

```r
param_grid <- expand.grid(mtry = seq(ncol(df_train) - 1), ntree = c(10, 20, 50, 100, 500, 1000))
n_params <- nrow(param_grid)
test_mses <- rep(0, n_params)
models <- vector("list", length = n_params)

set.seed(1)
for (i in seq(n_params)) {
  model <- do.call(
    randomForest::randomForest,
    c(formula = formula(Sales ~ .), data = list(df_train), importance = TRUE, param_grid[i, ])
  )
  preds <- predict(model, df_test)
  test_mses[[i]] <- mean((preds - df_test$Sales) ^ 2)
  models[[i]] <- model
}

param_grid$test_mse <- test_mses
best_model_idx <- which.min(test_mses)
best_model_params <- param_grid[best_model_idx, ]

print(best_model_params)
```

```
##    mtry ntree test_mse
## 27    7    50 2.472395
```

```r
ggplot2::ggplot(data = param_grid) +
  ggplot2::geom_line(ggplot2::aes(x = mtry, y = test_mse, color = as.factor(ntree))) +
  ggplot2::labs(x = "Number of variables considered", y = "test mse", color = "number of trees")
```




```
print(importance(models[[best_model_idx]]))
```

```
##                %IncMSE IncNodePurity
## CompPrice    8.0966773    150.657892
## Income       2.6453460     93.208673
## Advertising  5.6666706     96.518922
## Population   2.4803834     61.938575
## Price       14.8123680    466.497913
## ShelveLoc   15.1036644    493.749884
## Age          4.5545935    163.705090
## Education   -1.5820984     46.960790
## Urban        0.5390858      9.076557
## US           1.2626993      9.027394
```

`ShelveLoc` and `Price` are the most important variables by both metrics. The random forest slightly outperforms the bagged model.

Question 9

```
df_oj <- ISLR::OJ
set.seed(1)

train_idx <- sample(nrow(df_oj), 800)

df_train <- df_oj[train_idx, ]
df_test <- df_oj[-train_idx, ]
```

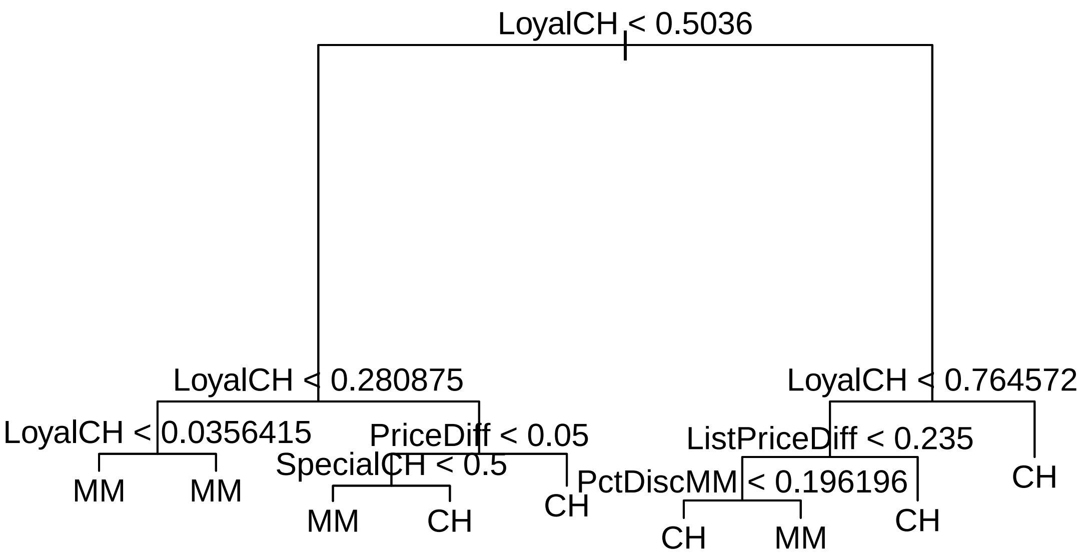
```r
models <- list(
  gini = NULL,
  deviance = NULL
)
test_aucs <- rep(0, length(models)) %>%
  setNames(., names(models))

for (method in c("gini", "deviance")) {
  model <- tree::tree(Purchase ~ ., data = df_train, split = method)
  models[[method]] <- model
  test_preds <- predict(model, df_test)[, "MM"]

  dep_var <- ifelse(df_test$Purchase == "MM", 1, 0)
  predob <- prediction(test_preds, dep_var)
  auc <- performance(predob, "auc")@y.values[[1]]
  test_aucs[[method]] <- auc
}


for (model_name in names(models)){
  print(model_name)
  print(summary(models[[model_name]]))
}
```

```
## [1] "gini"
##
## Classification tree:
## tree::tree(formula = Purchase ~ ., data = df_train, split = method)
## Variables actually used in tree construction:
##  [1] "SpecialMM"      "SpecialCH"      "DiscCH"         "DiscMM"
##  [5] "LoyalCH"        "WeekofPurchase" "STORE"          "PriceMM"
##  [9] "StoreID"        "PriceCH"        "PriceDiff"      "SalePriceMM"
## [13] "ListPriceDiff"  "Store7"
## Number of terminal nodes:  88
## Residual mean deviance:  0.6112 = 435.1 / 712
## Misclassification error rate: 0.1375 = 110 / 800
## [1] "deviance"
##
## Classification tree:
## tree::tree(formula = Purchase ~ ., data = df_train, split = method)
## Variables actually used in tree construction:
## [1] "LoyalCH"       "PriceDiff"     "SpecialCH"     "ListPriceDiff"
## [5] "PctDiscMM"
## Number of terminal nodes:  9
## Residual mean deviance:  0.7432 = 587.8 / 791
## Misclassification error rate: 0.1588 = 127 / 800
```

```r
print(test_aucs)
```

```
##      gini  deviance
## 0.8316118 0.8947829
```

```r
plot(models$deviance)
text(models$deviance)
```



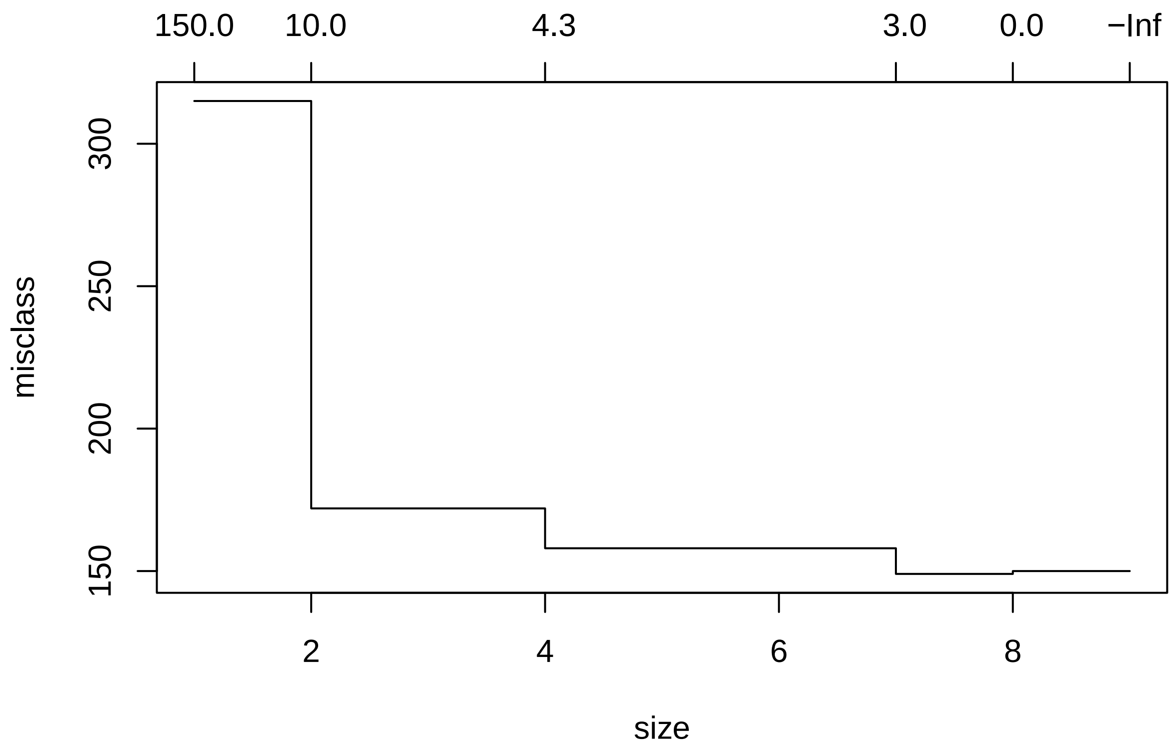

```r
test_preds <- predict(models$deviance, df_test, type = "class")

table(test_preds, df_test$Purchase, dnn = c("Predicted", "Actual"))
```

```
##          Actual
## Predicted  CH  MM
##        CH 160  38
##        MM   8  64
```

```r
mean(test_preds != df_test$Purchase)
```

```
## [1] 0.1703704
```

```r
cv_tree <- tree::cv.tree(models$deviance, FUN = prune.misclass)

plot(cv_tree)
```




```r
smaller_tree <- tree::prune.tree(models$deviance, best = 7, method = "misclass")

test_preds <- predict(models$deviance, df_test, type = "class")

table(test_preds, df_test$Purchase, dnn = c("Predicted", "Actual"))
```

```
##          Actual
## Predicted  CH  MM
##        CH 160  38
##        MM   8  64
```

```r
mean(test_preds != df_test$Purchase)
```

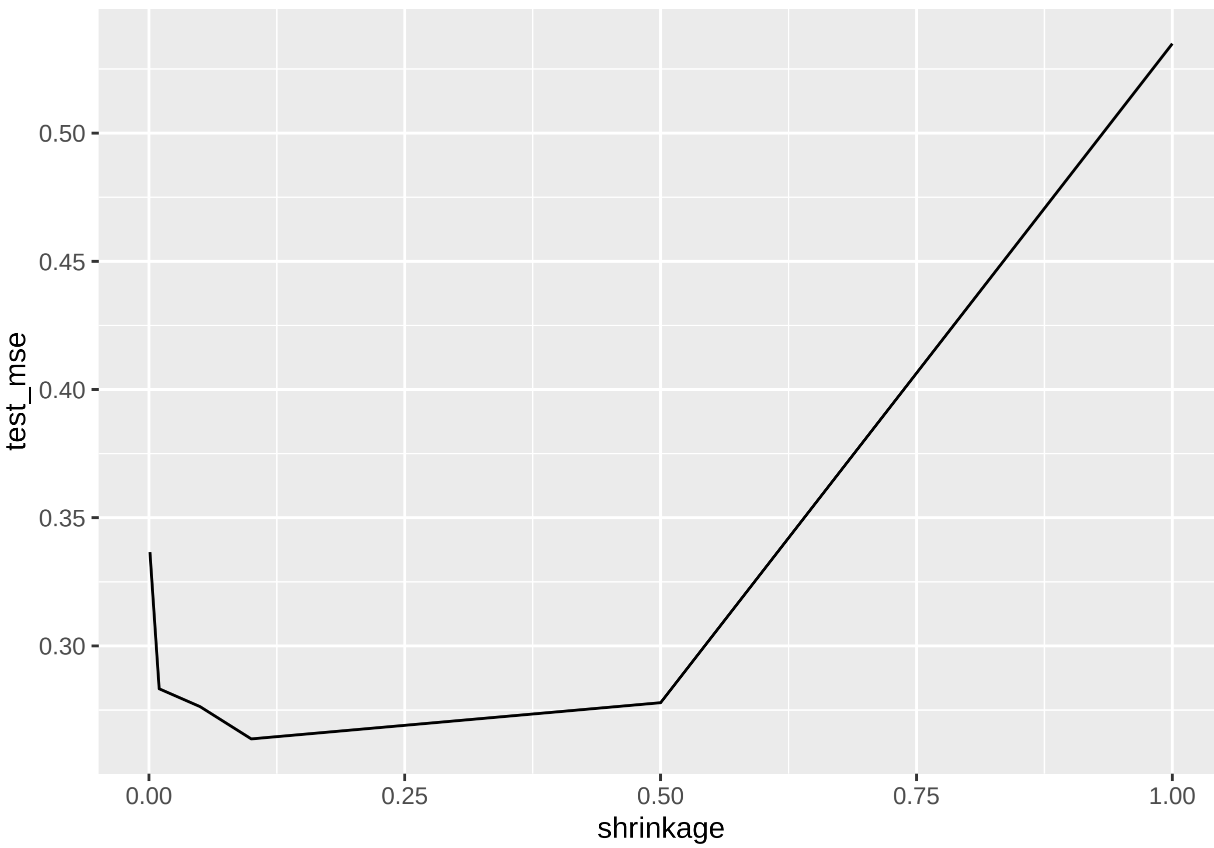
```
## [1] 0.1703704
```

We see that the model built using Gini has better training deviance but worse AUC on the test set. It is likely overfit, especially given the large number of terminal nodes. The model built using deviance has slightly higher error rate on the test set than on the training set, which is to be expected. The best tree using cross validation uses 7 terminal nodes. It has the same misclassification error as the full tree on the test set.

Question 10

```r
df_hitters <- ISLR::Hitters %>%
  dplyr::filter(., !is.na(Salary)) %>%
  dplyr::mutate(., Salary = log(Salary))

train_idx <- seq(200)
df_train <- df_hitters[train_idx, ]
df_test <- df_hitters[-train_idx, ]

etas <- c(0.001, 0.01, 0.05, 0.1, 0.5, 1)
test_mse <- rep(0, length(etas)) %>%
```

```r
  setNames(., etas)

models <- vector("list", length = length(etas)) %>%
  setNames(., etas)


for (eta in etas) {
  model <- gbm::gbm(Salary ~ ., data = df_train, n.trees = 1000, shrinkage = eta, distribution = "gauss
  models[[as.character(eta)]] <- model
  preds <- predict(model, df_test)
  test_mse[[as.character(eta)]] <- mean((preds - df_test$Salary) ^ 2)
}


df_for_plot <- data.frame(
  shrinkage = etas,
  test_mse = test_mse
)

ggplot2::ggplot(data = df_for_plot) +
  ggplot2::geom_line(ggplot2::aes(x = shrinkage, y = test_mse))
```




```r
best_idx <- which.min(test_mse)
print(min(test_mse))
```

```
## [1] 0.2637596
```

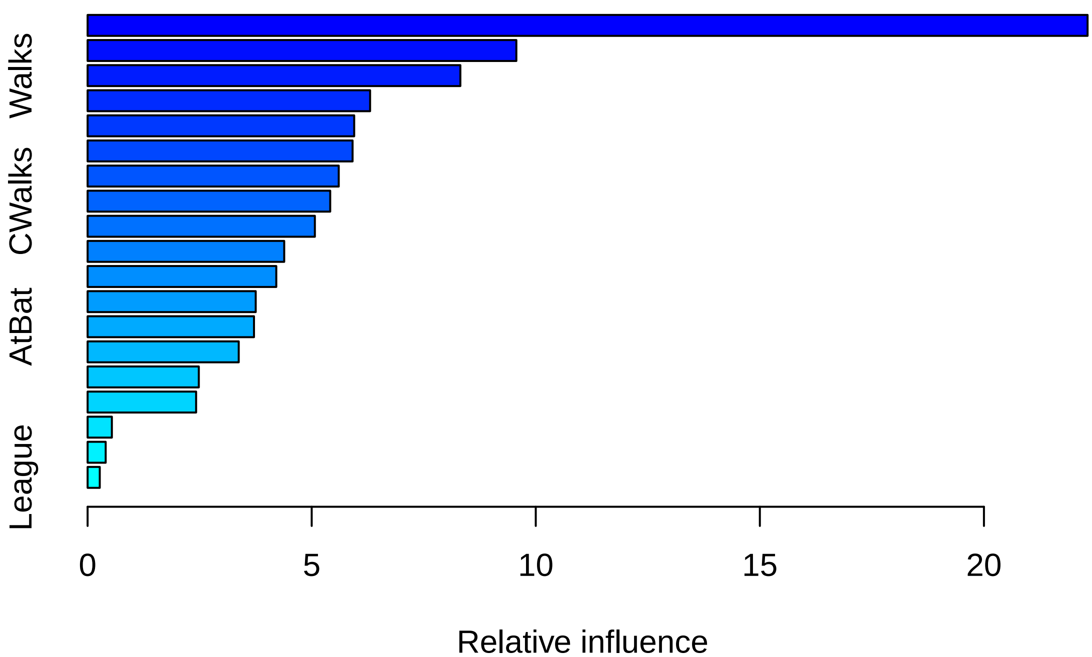
```r
x_train <- model.matrix(Salary ~ ., df_train)
y_train <- df_train$Salary

x_test <- model.matrix(Salary ~ ., df_test)

# Ridge regression
glmnet_model <- glmnet::cv.glmnet(x_train, y_train)

glmnet_preds <- predict(glmnet_model, x_test)

print(paste("Lasso test mse", mean((glmnet_preds - df_test$Salary) ^ 2)))
```

```
## [1] "Lasso test mse 0.442599198536883"
```

```r
ols_model <- lm(Salary ~ ., data = df_train)
ols_preds <- predict(ols_model, df_test)

print(paste("Ols test mse", mean((ols_preds - df_test$Salary) ^ 2)))
```

```
## [1] "Ols test mse 0.491795937545494"
```

```r
summary(models[[best_idx]])
```




```
##                   var    rel.inf
## CAtBat         CAtBat 22.3125853
## PutOuts       PutOuts  9.5668391
## Walks           Walks  8.3169907
## CRBI             CRBI  6.3034269
## Years           Years  5.9480997
## CHmRun         CHmRun  5.9125803
## CHits           CHits  5.6036941
## CWalks         CWalks  5.4129709
## Assists       Assists  5.0711934
## Hits             Hits  4.3864218
```

```
## CRuns         CRuns  4.2098295
## RBI             RBI  3.7513709
## AtBat         AtBat  3.7125735
## HmRun         HmRun  3.3722281
## Runs           Runs  2.4815950
## Errors       Errors  2.4215422
## Division   Division  0.5415515
## NewLeague NewLeague  0.4031839
## League       League  0.2713233
```

```r
bagged_model <- randomForest::randomForest(Salary ~ ., data = df_train, mtry = ncol(df_train) - 1)

bagged_preds <- predict(bagged_model, df_test)

print(paste("Bagging test mse", mean((bagged_preds - df_test$Salary) ^ 2)))
```

```
## [1] "Bagging test mse 0.232820139593086"
```

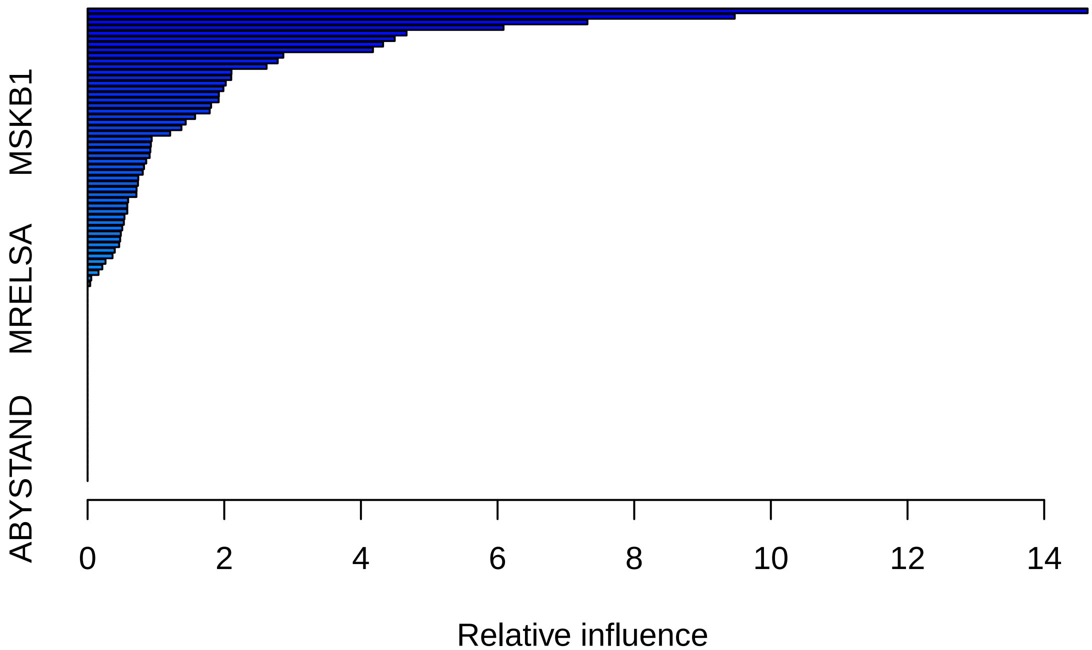
Boosting performs much better than both LASSO and OLS. `CHits` and `CAtBat` are the most important variables. Bagging performs slightly better than boosting on this problem.

Question 11

```r
set.seed(1)
df_caravan <- ISLR::Caravan
df_caravan$Purchase <- ifelse(df_caravan$Purchase == "Yes", 1, 0)

train_idx <- seq(1000)

df_train <- df_caravan[train_idx, ]
df_test <- df_caravan[-train_idx, ]

model <- gbm::gbm(Purchase ~., data = df_train, n.trees = 1000, shrinkage = 0.01)
```

```
## Distribution not specified, assuming bernoulli ...
```

```r
summary(model)
```




```
##                   var     rel.inf
## PPERSAUT PPERSAUT 14.63504779
## MKOOPKLA MKOOPKLA  9.47091649
## MOPLHOOG MOPLHOOG  7.31457416
## MBERMIDD MBERMIDD  6.08651965
## PBRAND     PBRAND  4.66766122
## MGODGE     MGODGE  4.49463264
## ABRAND     ABRAND  4.32427755
## MINK3045 MINK3045  4.17590619
## MOSTYPE   MOSTYPE  2.86402583
## PWAPART   PWAPART  2.78191075
## MAUT1       MAUT1  2.61929152
## MBERARBG MBERARBG  2.10480508
## MSKA         MSKA  2.10185152
## MAUT2       MAUT2  2.02172510
## MSKC         MSKC  1.98684345
## MINKGEM   MINKGEM  1.92122708
## MGODPR     MGODPR  1.91777542
## MBERHOOG MBERHOOG  1.80710618
## MGODOV     MGODOV  1.78693913
## PBYSTAND PBYSTAND  1.57279593
## MSKB1       MSKB1  1.43551401
## MFWEKIND MFWEKIND  1.37264255
## MRELGE     MRELGE  1.20805179
## MOPLMIDD MOPLMIDD  0.93791970
## MINK7512 MINK7512  0.92590720
## MINK4575 MINK4575  0.91745993
## MGODRK     MGODRK  0.90765539
## MFGEKIND MFGEKIND  0.85745374
## MZPART     MZPART  0.82531066
## MRELOV     MRELOV  0.80731252
## MINKM30   MINKM30  0.74126812
## MHKOOP     MHKOOP  0.73690793
```

```
## MZFONDS   MZFONDS  0.71638323
## MAUTO        MAUTO  0.71388052
## MHHUUR      MHHUUR  0.59287247
## APERSAUT APERSAUT  0.58056986
## MOSHOOFD MOSHOOFD  0.58029563
## MSKB2        MSKB2  0.53885275
## PLEVEN      PLEVEN  0.53052444
## MINK123M MINK123M  0.50660603
## MBERARBO MBERARBO  0.48596479
## MGEMOMV    MGEMOMV  0.47614792
## PMOTSCO    PMOTSCO  0.46163590
## MSKD          MSKD  0.39735297
## MBERBOER MBERBOER  0.36417546
## MGEMLEEF MGEMLEEF  0.26166240
## MFALLEEN MFALLEEN  0.21448118
## MBERZELF MBERZELF  0.15906143
## MOPLLAAG MOPLLAAG  0.05263665
## MAANTHUI MAANTHUI  0.03766014
## MRELSA      MRELSA  0.00000000
## PWABEDR    PWABEDR  0.00000000
## PWALAND    PWALAND  0.00000000
## PBESAUT    PBESAUT  0.00000000
## PVRAAUT    PVRAAUT  0.00000000
## PAANHANG PAANHANG  0.00000000
## PTRACTOR PTRACTOR  0.00000000
## PWERKT      PWERKT  0.00000000
## PBROM        PBROM  0.00000000
## PPERSONG PPERSONG  0.00000000
## PGEZONG    PGEZONG  0.00000000
## PWAOREG    PWAOREG  0.00000000
## PZEILPL    PZEILPL  0.00000000
## PPLEZIER PPLEZIER  0.00000000
## PFIETS      PFIETS  0.00000000
## PINBOED    PINBOED  0.00000000
## AWAPART    AWAPART  0.00000000
## AWABEDR    AWABEDR  0.00000000
## AWALAND    AWALAND  0.00000000
## ABESAUT    ABESAUT  0.00000000
## AMOTSCO    AMOTSCO  0.00000000
## AVRAAUT    AVRAAUT  0.00000000
## AAANHANG AAANHANG  0.00000000
## ATRACTOR ATRACTOR  0.00000000
## AWERKT      AWERKT  0.00000000
## ABROM        ABROM  0.00000000
## ALEVEN      ALEVEN  0.00000000
## APERSONG APERSONG  0.00000000
## AGEZONG    AGEZONG  0.00000000
## AWAOREG    AWAOREG  0.00000000
## AZEILPL    AZEILPL  0.00000000
## APLEZIER APLEZIER  0.00000000
## AFIETS      AFIETS  0.00000000
## AINBOED    AINBOED  0.00000000
## ABYSTAND ABYSTAND  0.00000000
```

```
preds <- predict(model, df_test, type = "response")
pred_labels <- ifelse(preds >= 0.2, 1, 0)

table(pred_labels, df_test$Purchase, dnn = c("predicted", "actual"))
```

```
##          actual
## predicted    0    1
##         0 4410  256
##         1  123   33
```

```
print(paste("Precision from gbm:", 33 / (33 + 123) ))
```

```
## [1] "Precision from gbm: 0.211538461538462"
```

```
x_train <- model.matrix(Purchase ~ ., data = df_train)
x_test <- model.matrix(Purchase ~ ., data = df_test)
y_train <- df_train$Purchase

knn_preds <- class::knn(x_train, x_test, y_train, k = 8, prob = TRUE)
probs <- attr(knn_preds, "prob")
probs <- ifelse(knn_preds == 1, probs, 1 - probs )

pred_labels <- ifelse(probs >= 0.2, 1, 0)

table(pred_labels, df_test$Purchase, dnn = c("predicted", "actual"))
```

```
##          actual
## predicted    0    1
##         0 4020  222
##         1  513   67
```

```
print(paste("Precision from knn:", 67 / (67 + 513)))
```

```
## [1] "Precision from knn: 0.11551724137931"
```

```
logistic_regression_model <- glm(Purchase ~ ., data = df_train, family = "binomial")
preds <- predict(logistic_regression_model, df_test, type = "response")

pred_labels <- ifelse(preds >= 0.2, 1, 0)

table(pred_labels, df_test$Purchase, dnn = c("predicted", "actual"))
```

```
##          actual
## predicted    0    1
##         0 4183  231
##         1  350   58
```

```
print(paste("Precision from LR:", 58 / (58 + 350) ))
```

```
## [1] "Precision from LR: 0.142156862745098"
```

Question 12

```
df_auto <- ISLR::Auto %>%
  dplyr::select(., -name) %>%
  dplyr::mutate(., mpg = ifelse(mpg >= median(mpg), 1, 0))
```

```r
nrows <- nrow(df_auto)

set.seed(1)
train_idx <- sample(nrows, nrows %/% 2)

df_train <- df_auto[train_idx, ]
df_test <- df_auto[-train_idx, ]

bagged_model <- randomForest::randomForest(as.factor(mpg) ~ ., data = df_train, mtry = ncol(df_train) -

preds_bagged <- predict(bagged_model, df_test)


print(paste("error rate from bagging trees:", mean(preds_bagged != df_test$mpg)))
```

```
## [1] "error rate from bagging trees: 0.0969387755102041"
```

```r
random_forest_model <- randomForest::randomForest(as.factor(mpg) ~ ., data = df_train)


preds_random_forest <- predict(random_forest_model, df_test)

print(paste("error rate from random forest:", mean(preds_random_forest != df_test$mpg)))
```

```
## [1] "error rate from random forest: 0.0969387755102041"
```

```r
boosted_model <- gbm::gbm(mpg ~ ., data = df_train, n.trees = 1000, shrinkage = 0.05)
```

```
## Distribution not specified, assuming bernoulli ...
```

```r
preds_boosted <- predict(boosted_model, df_test, type = "response")
preds_boosted <- preds_boosted >= 0.5


print(paste("error rate from gbm:", mean(preds_boosted != df_test$mpg)))
```

```
## [1] "error rate from gbm: 0.0816326530612245"
```

random forest and bagging have the same error rate on the test set, while boosting has slightly higher error.