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Point 2 of Homework 1 from statistics III

Data Exploration AND PCA (Point 2)

Use this data from the departments of Colombia to do the following tasks

Use the next data to calculate PCA and reduce the dimensionality from 2 dimensions to 1. For this exercise you must use the variables x_1, and x_2 and create a vector with a single dimension.

Department	GDP Millions (x_1)	Population (x_2)	GDP per capita Millions (x_3)
Amazonas	1067855.672	76589	13.94267678
Antioquia	212514957.4	6407102	33.16865524
Arauca	8548114.653	262174	32.60473828
Atlántico	63764770.77	2535517	25.1486268
Bogotá D.C.	357258620.8	7412566	48.19634938
Bolívar	51404352.37	2070110	24.83170091
Boyacá	38858162.12	1217376	31.91960588
Caldas	23953112.45	998255	23.9949837
Caquetá	5461366.78	401849	13.59059443
Casanare	23660657.37	420504	56.26737766
Cauca	25758151.71	1464488	17.58850309
Cesar	37523918.98	1200574	31.25498218
Chocó	6001844.915	534826	11.2220515
Córdoba	24991953.76	1784783	14.00279685
Cundinamarca	91945942.28	2919060	31.49847632
Guainía	497704.0127	48114	10.34426597
Guaviare	1123857.696	82767	13.57857232
Huila	24011616.06	1100386	21.8210846
La Guajira	22262575.88	880560	25.28229295
Magdalena	19738417.36	1341746	14.710994
Meta	58439500.07	1039722	56.20685151
Nariño	21775426.15	1630592	13.35430699
Norte de Santander	23056874.23	1491689	15.45689097
Putumayo	5616558.269	348182	16.13109888
Quindío	11941644.16	539904	22.11808795

Department	GDP Millions (x_1)	Population (x_2)	GDP per capita Millions (x_3)
Risaralda	23786362.42	943401	25.21341659
San Andrés, Providencia y Santa Catalina (Archipiélago)	2125410.333	61280	34.68358898
Santander	92276678.16	2184837	42.23504003
Sucre	11516270.76	904863	12.7270877
Tolima	30438180.15	1330187	22.8826324
Valle del Cauca	139863153.5	4475886	31.2481492
Vaupés	381851.6785	40797	9.359797989
Vichada	956576.6785	107808	8.872965629

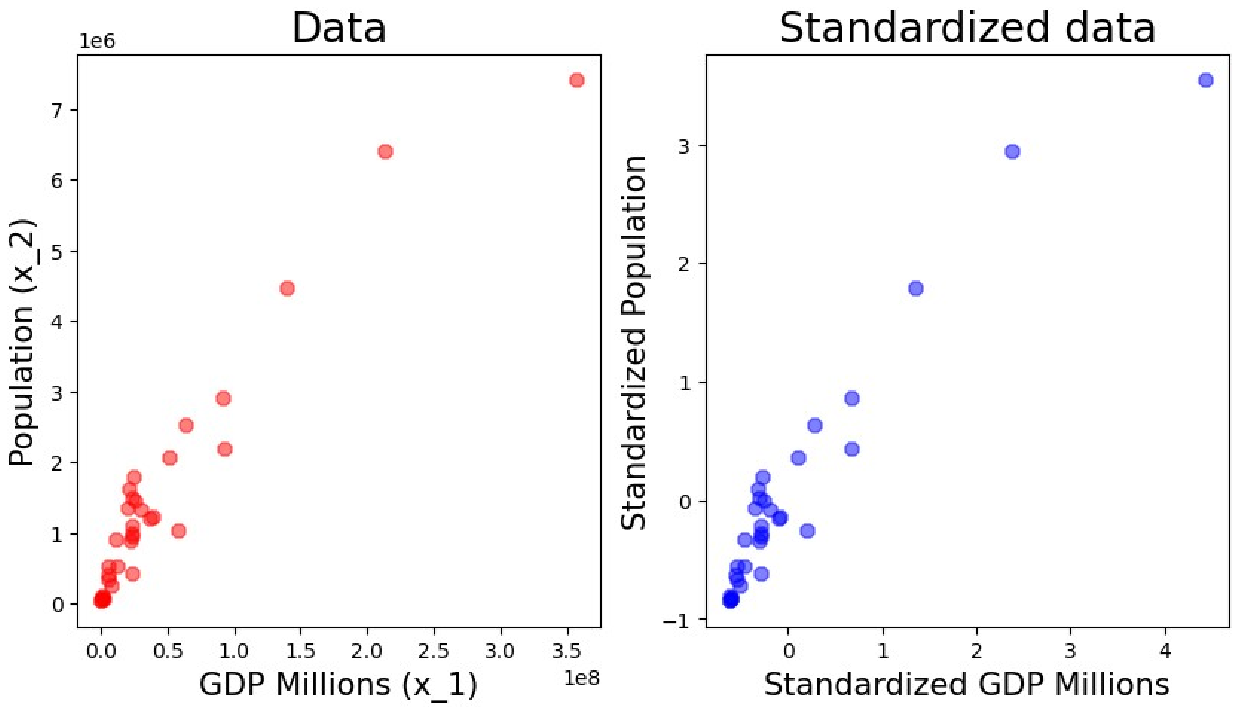
```python
import pandas as pd
import numpy as np
from sklearn.decomposition import PCA

df = pd.read_csv("../Docs/Data/DEPARTMENTS.csv")

columnx_1 = df['GDP Millions (x_1)']
columnx_2 = df['Population (x_2)']
columnx_3 = df['GDP per capita Millions (x_3)']
column_department = df['Department']

#df.describe()
```

Let's reuse some methods from point 1

```python
def calculateMean(column):
    return column.sum() / len(column)

def calculateStdDev(column, mean):
    n = len(column)
    squaredDifferences = [(x - mean) ** 2 for x in column]
    variance = sum(squaredDifferences) / n
    stdDev = variance ** 0.5
    return stdDev

def calculateCovariance(columnX, columnY):
    columnLength = len(columnX)

    if columnLength != len(columnY):
        raise ValueError("Columns must have the same length")

    meanX = calculateMean(columnX)
    meanY = calculateMean(columnY)

    return sum((columnX[i] - meanX) * (columnY[i] - meanY) for i in
range(columnLength)) / columnLength
```

```
def scaleData(data):
    scaledData = data.copy()
    for column in data.columns:
        mean = calculateMean(data[column])
        standardDeviation = calculateStdDev(data[column], mean)
        scaledData[column] = (data[column] - mean) / standardDeviation
    return scaledData

def calculateCovarianceMatrix(data):
    numFeatures = len(data.columns)
    covarianceMatrix = np.zeros((numFeatures, numFeatures))

    for i in range(numFeatures):
        for j in range(numFeatures):
            covarianceMatrix[i, j] = calculateCovariance(data.iloc[:,
i], data.iloc[:, j])

    return covarianceMatrix

def reduceDimensionalityPCA(data):
    scaledData = scaleData(data)

    # Calculate covariance matrix
    covarianceMatrix = calculateCovarianceMatrix(scaledData)

    # Manual computation of eigenvalues and eigenvectors
    eigenvalues, eigenvectors = np.linalg.eig(covarianceMatrix)

    # Manual sorting of eigenvalues and eigenvectors
    sortedIndex = np.argsort(eigenvalues)[::-1]
    sortedEigenvalues = eigenvalues[sortedIndex]
    sortedEigenvectors = eigenvectors[:, sortedIndex]

    # Manual selection of the principal component
    principalComponent = sortedEigenvectors[:, 0]

    # Adjust the sign of the principal component
    if principalComponent[0] < 0:
        principalComponent = -principalComponent

    # Manual projection of the data onto the principal component
    reducedData = np.dot(scaledData, principalComponent)

    return reducedData, covarianceMatrix, sortedEigenvalues,
sortedEigenvectors, principalComponent

# Example usage
data = pd.DataFrame({'columnx_1': columnx_1, 'columnx_2': columnx_2})
reducedData, covarianceMatrix, sortedEigenvalues, sortedEigenvectors,
principalComponent = reduceDimensionalityPCA(data)
```

```
# Calculate explained variance by each eigenvalue
explainedVariance = sortedEigenvalues / np.sum(sortedEigenvalues)

# Calculate error or difference between the projected and original
data
error = np.linalg.norm(data.values - np.outer(reducedData,
principalComponent))

point2 = {
    1: covarianceMatrix,
    2: sortedEigenvalues,
    3: explainedVariance,
    4: principalComponent,
    5: reducedData,
    6: error
}

import matplotlib.pyplot as plt
from sklearn.preprocessing import StandardScaler

fig = plt.figure(figsize=(10, 5))
ax1 = fig.add_subplot(1, 2, 1)
ax2 = fig.add_subplot(1, 2, 2)

ax1.set_title("Data", fontsize=20)
ax1.scatter(columnx_1, columnx_2, marker="8", s=50, color="red",
alpha=0.5)
ax1.set_xlabel("GDP Millions (x_1)", fontsize=15)
ax1.set_ylabel("Population (x_2)", fontsize=15)

columns = ["GDP Millions (x_1)", "Population (x_2)"]
data = df[columns]
dataScaled = pd.DataFrame(StandardScaler().fit_transform(data),
columns=columns)

ax2.set_title("Standardized data", fontsize=20)
ax2.scatter(dataScaled["GDP Millions (x_1)"], dataScaled["Population
(x_2)"], marker="8", s=50, color="blue", alpha=0.5)
ax2.set_xlabel("Standardized GDP Millions", fontsize=15)
ax2.set_ylabel("Standardized Population", fontsize=15)

plt.show()
```



1. What is the covariance matrix

```
print(pd.DataFrame(point2[1], columns=data.columns,
index=data.columns))

           columnx_1  columnx_2
columnx_1    1.00000    0.95525
columnx_2    0.95525    1.00000
```

2. What are the eigenvalues

```
print(pd.DataFrame(point2[2], columns=["Eigenvalues"]))

   Eigenvalues
0      1.95525
1      0.04475
```

3. What is the variance explained by the eigenvalue.

```
print(pd.DataFrame(point2[3], columns=["Variance explained by
eigenvalue"]))

   Variance explained by eigenvalue
0                          0.977625
1                          0.022375
```

4. What is the value of the eigenvector

```
print(pd.DataFrame(point2[4], columns=["Eigenvector"]))
```

```
   Eigenvector
0     0.707107
1     0.707107
```

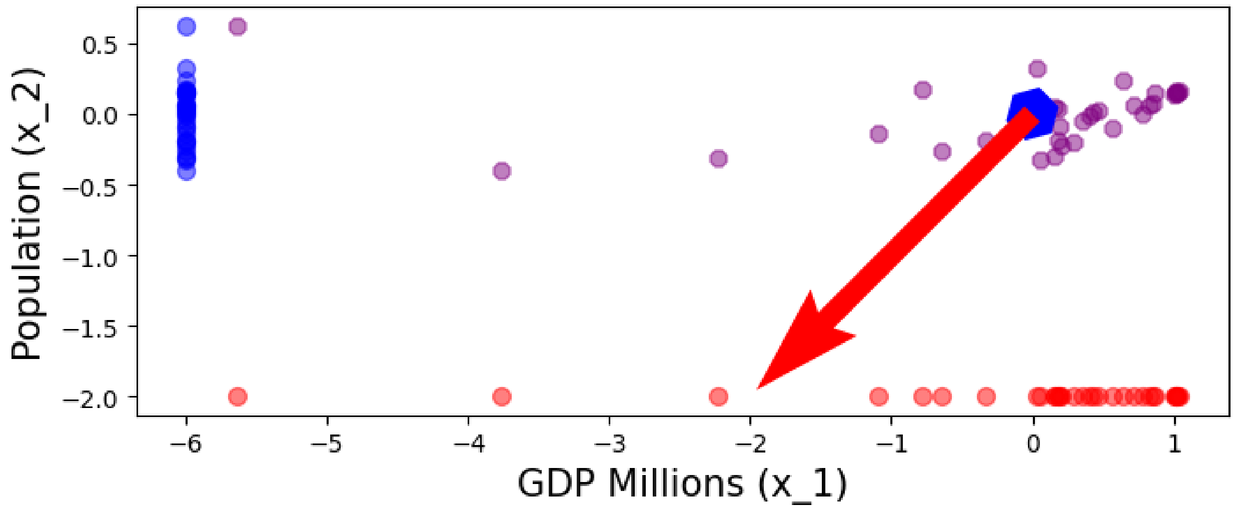
5. What is the projected vector.

```
from sklearn.decomposition import PCA

pca = PCA()
pcaResult = pca.fit_transform(scaleData(data))
print(pd.DataFrame(np.hstack([reducedData.reshape(-1, 1), pcaResult[:,
0].reshape(-1, 1)]), index=column_department, columns=["Manual",
"sklearn"]))
```

Department	Manual	sklearn
Amazonas	-1.016761	-1.016761
Antioquia	3.766923	3.766923
Arauca	-0.863684	-0.863684
Atlántico	0.646634	0.646634
Bogotá D.C.	5.640378	5.640378
Bolívar	0.326854	0.326854
Boyacá	-0.157860	-0.157860
Caldas	-0.399442	-0.399442
Caquetá	-0.835803	-0.835803
Casanare	-0.645617	-0.645617
Cauca	-0.185065	-0.185065
Cesar	-0.178301	-0.178301
Chocó	-0.774403	-0.774403
Córdoba	-0.057891	-0.057891
Cundinamarca	1.090450	1.090450
Guainía	-1.034462	-1.034462
Guaviare	-1.013599	-1.013599
Huila	-0.355857	-0.355857
La Guajira	-0.465931	-0.465931
Magdalena	-0.297051	-0.297051
Meta	-0.036477	-0.036477
Nariño	-0.155033	-0.155033
Norte de Santander	-0.200676	-0.200676
Putumayo	-0.856843	-0.856843
Quindío	-0.712756	-0.712756
Risaralda	-0.424208	-0.424208
San Andrés Providencia y Santa Catalina (Archip...	-1.012611	-1.012611
Santander	0.784641	0.784641
Sucre	-0.563362	-0.563362
Tolima	-0.194721	-0.194721

```
Valle del Cauca                                              2.225971  2.225971
Vaupés                                                      -1.038703 -1.038703
Vichada                                                     -1.004732 -1.004732
```

```
dataScaled = pd.DataFrame(StandardScaler().fit_transform(data),
columns=columns)
eigenvalues, eigenvectors = np.linalg.eig(covarianceMatrix)

vectorA = eigenvectors[:, 0]
vectorB = eigenvectors[:, 1]

projection = pd.DataFrame(dataScaled.values @ eigenvectors.T,
                          columns=["GDP Millions (x_1)", "Population
(x_2)"])

plt.figure(figsize=(8, 8))
plt.axes().set_aspect("equal")

plt.scatter(projection["GDP Millions (x_1)"], projection["Population
(x_2)"],
            marker="8", s=50, color="purple", alpha=0.5)

plt.quiver(0, 0,
           vectorA[0] / abs(vectorA[0]) * eigenvalues[0],
           vectorA[1] / abs(vectorA[1]) * eigenvalues[0],
           color="blue", angles="xy", scale_units="xy", scale=1,
width=0.05)

plt.quiver(0, 0,
           vectorB[0] / abs(vectorB[0]) * eigenvalues[1],
           vectorB[1] / abs(vectorB[1]) * eigenvalues[1],
           color="red", angles="xy", scale_units="xy", scale=1,
width=0.02)

plt.scatter(projection["GDP Millions (x_1)"], [-2] *
len(projection["GDP Millions (x_1)"]),
            s=50, color="red", alpha=0.5)

plt.scatter([-6] * len(projection["Population (x_2)"]),
projection["Population (x_2)"],
            s=50, color="blue", alpha=0.5)

plt.xlabel("GDP Millions (x_1)", fontsize=15)
plt.ylabel("Population (x_2)", fontsize=15)
plt.show()
```



6. What is the error or difference between the projected matrix

```
error =  np.linalg.norm(data.values - np.outer(reducedData, principalComponent))
print("Manual PCA error:", error)
error = np.linalg.norm(data.values - pca.inverse_transform(pcaResult), ord='fro')
print("scikit-learn PCA error:", error)
```

```
Manual PCA error: 478923461.57698685
scikit-learn PCA error: 478923461.4655835
```