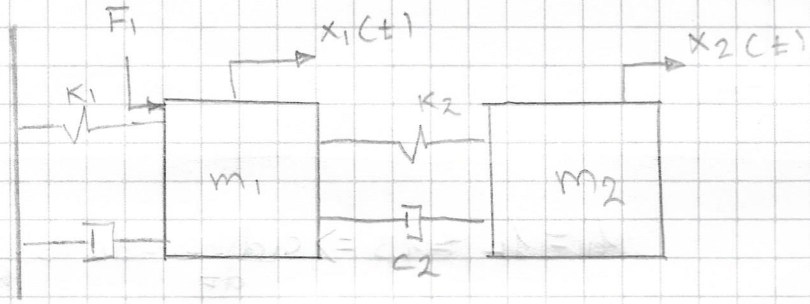
Analisis de sistemas Lineales y no Lineales
Tarea 3

1

a



$$F_1(t) - C_1\dot{x}_{11}(t) - K_1 x_{11}(t) + K_2(x_{12}(t) - x_{11}(t)) + C_2(\dot{x}_{12}(t) - \dot{x}_{11}(t)) =$$
$$m_1 \ddot{x}_{11}(t) \quad (1)$$

$$-C_2(\dot{x}_{12}(t) - \dot{x}_{11}(t)) - K_2(x_{12}(t) - x_1(t)) = m_2 \ddot{x}_{12}(t)$$

Estados

$x_1 = x_{11}$

$x_2 = \dot{x}_{11}$

$x_3 = x_{12}$

$x_4 = \dot{x}_{12}$

$\dot{x}_1 = \dot{x}_{11} = x_2 \qquad \dot{x}_3 = \dot{x}_{12} = x_4$

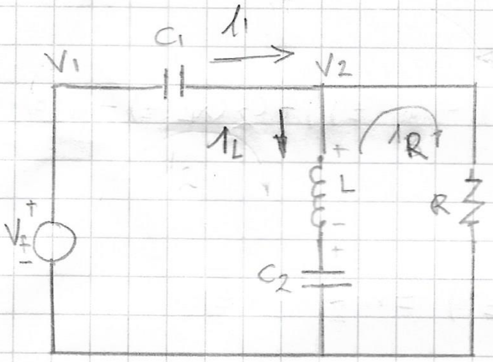
$\dot{x}_2 = \frac{U}{m_1} - \frac{C_1 x_2}{m_1} - \frac{K_1 x_1}{m_1} + \frac{K_2 x_3}{m_1} - \frac{K_2 x_1}{m_1} + \frac{C_2 x_4}{m_1} - \frac{C_2 x_2}{m_1}$

$\dot{x}_4 = \ddot{x}_{12} = -\frac{C_2 x_4}{m_2} + \frac{C_2 x_2}{m_2} - \frac{K_2 x_3}{m_2} + \frac{K_2 x_1}{m_2}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-K_1-K_2}{m_1} & \frac{-C_1-C_2}{m_1} & \frac{K_2}{m_1} & \frac{C_2}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{K_2}{m_2} & \frac{C_2}{m_2} & \frac{-K_2}{m_2} & \frac{-C_2}{m_2} \end{bmatrix}}_{4\times4} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{4\times1} + \underbrace{\begin{bmatrix} 0 \\ 1/m_1 \\ 0 \\ 0 \end{bmatrix}}_{4\times1} U$$

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{2\times 4} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}_{4\times 1}$$

b



$$i_1 = i_L + i_3 \Rightarrow C_1 \frac{dV_{C1}}{dt} = i_L + \frac{V_2}{R}$$

$$i_L = i_{C2} = C_2 \frac{dV_{C2}}{dt}$$

$$V_f = V_{C1} + V_L + V_{C2} = V_{C1} + L\frac{di_L}{dt} + V_{C2}$$

Estados

$$\boxed{x_1 = V_{C1} \qquad x_2 = V_{C2} \qquad x_3 = i_L}$$

$$\boxed{\dot{x}_3 = -\frac{1}{L}x_1 - \frac{1}{L}x_2 + U}$$

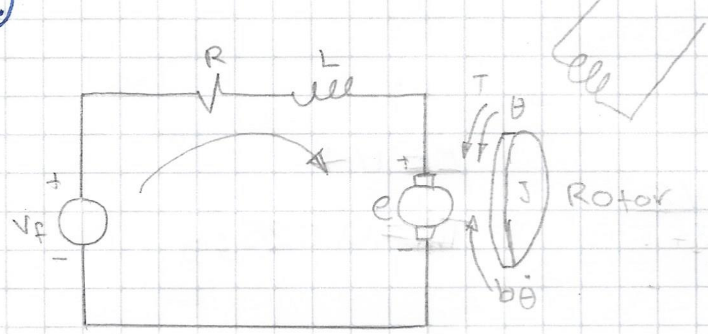
$$\boxed{\frac{dV_{C1}}{dt} = \frac{i_L}{C_1} + \frac{V_f - V_{C1}}{RC} \Rightarrow \dot{x}_1 = -\frac{x_1}{RC} + \frac{x_3}{C} + \frac{U}{RC}}$$

$$i_L = C_2\frac{dV_{C2}}{dt} \Rightarrow \boxed{\dot{x}_2 = \frac{x_3}{C_2}}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1/RC & 0 & 1/C \\ 0 & 0 & 1/C_2 \\ -1/L & -1/L & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1/RC \\ 0 \\ 1 \end{bmatrix} U$$

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

ⓒ



$$T = l(t)\, k_m$$

$$e = \frac{d\theta(t)}{dt} \cdot k_b$$

$$L\frac{dl(t)}{dt} + R\,l(t) + e = V(t)$$

$$T(t) = J\frac{d^2\theta(t)}{dt} + b\frac{d\theta(t)}{dt} = l(t)\,k_m$$

$$x_1 = l(t) \qquad \dot{x}_1 = \frac{U(t)}{L} - \frac{R x_1}{L} - \frac{x_3\, k_b}{L}$$

$$x_2 = \theta(t) \qquad \dot{x}_2 = x_3$$

$$x_3 = \dot{\theta}(t) \qquad \dot{x}_3 = \frac{x_1 k_m}{J} - \frac{b}{J}x_3$$

Estados

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -R/L & 0 & -k_b/L \\ 0 & 1 & 0 \\ \frac{k_m}{J} & 0 & -\frac{b}{J} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1/L \\ 0 \\ 0 \end{bmatrix} U$$

$$Y = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

2

a

$$G = \frac{2}{S+4} = \frac{2/4}{\frac{1}{4}S+1} = \frac{1/2}{\frac{1}{4}S+1}$$

$\tau = 1/4$

$T_m = 0.025$ seg

Polos

$$\frac{1}{4}S+1=0 \Rightarrow S=-1(4)=-4$$

$$G(z) = \frac{Kd}{z - e^{TS}} \qquad z = e^{TS} = e^{-4(0.025)} = 0.90483$$

Teorema de valor final

$$Vc = \left.\frac{2}{S+4}\right|_{S=0} = 1/2$$

$$Vd = \left.\frac{Kd}{z-0.90483}\right|_{z=1} = 1/2 \Rightarrow 1/2(1-0.90483 = Kd$$

$$Kd = 0.04758$$

$$G(z) = \frac{0.04758}{z-0.90483}$$

2)

b

$$G(s) = \frac{50}{S^2+8S+25}$$

$8 = 2\zeta w_n \quad \zeta w_n = 4 = \sigma$

$$\tau = \frac{1}{\sigma} = \frac{1}{4}$$

$$T_m = \frac{1}{40} = 0.025$$

$$Z = e^{T\sigma}(\cos(Tw) + j\,sen(Tw))$$

Polos

$$S = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-8 \pm \sqrt{8^2-4(25)}}{2(1)}$$

$S_1 = -4+3i \qquad Z_1 = e^{0{,}025(-4)}(\cos(0{,}025(3)) + J\,sen(0{,}025(3)))$

$S_2 = -4-3i \qquad Z_1 = 0.90229 + J0.06779$

$$Z_2 = e^{0.025(-4)}(\cos(0{,}025(-3)) + sen(0{,}025(-3)))$$

$Z_2 = 0.90229 - J\,0.06779$

$$G(z) = \frac{Kb}{(Z-0.90229-J0.06779)(Z-0.90229+J0.06779)}$$

$$G(z) = \frac{Kb}{Z^2 - 0.90229Z + \cancel{JZ0.06779} - 0.90229Z + 0.814127 - \cancel{J0.06116} - \cancel{JZ0.06779} + \cancel{J0.06116} + J^2\,0.00459}$$

$$G(z) = \frac{Kb}{Z^2 - 1.8045Z + 0.81871}$$

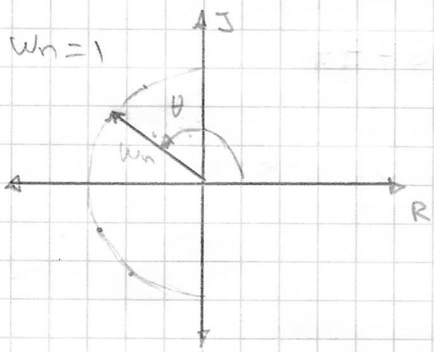
Teorema de Valor fina

$$\left.\frac{50}{S^2+8S+25}\right|_{S=0} = \frac{50}{25} = 2$$

$$\left.\frac{Kb}{Z^2-1.8045Z+0.8187}\right|_{Z=1} = 2 \qquad Kb = 2-3.609+1.6374 = 0.0284$$

$$\boxed{G(Z) = \frac{0.0284}{Z^2-1.8045+0.8187}}$$

③



$$P = -\sigma \pm J w_d$$

$$\sigma = w_n \cos(\theta)$$

$$w_d = w_n \sin(\theta)$$

④

a

$$G(s) = \frac{2}{s+4} = \frac{Y(s)}{X(s)} \cdot \frac{X(s)}{U(s)}$$

$$\frac{Y(s)}{X(s)} = 2 \qquad \frac{X(s)}{U(s)} = \frac{1}{s+4} \Rightarrow sX(s) + 4X(s) = U(s)$$

$$\Rightarrow \dot{x} + 4x = U(s)$$

Estados

$$\boxed{y = 2x_1}$$

$x_1 = x$

$\dot{x}_1 = U - 4x_1$

$$\boxed{\dot{x}_1 = -4x_1 + U}$$

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b

$$G(s) = \frac{50}{s^2+8s+25} = \frac{Y(s)}{X(s)} \cdot \frac{X(s)}{U(s)}$$

$$\frac{Y(s)}{X(s)} = 50 \Rightarrow y = 50X(s)$$

$$\frac{X(s)}{U(s)} = \frac{1}{s^2+8s+25} \Rightarrow s^2X(s) + s8X(s) + 25X(s) = U(s)$$

$$\Rightarrow \ddot{x} + 8\dot{x} + 25x = U$$

Estados

$x_1 = x$ | $\dot{x}_1 = x_2$

$x_2 = \dot{x}$ | $\dot{x}_2 = U - 8x_2 - 25x_1$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -25 & -8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} U$$

$$Y = \begin{bmatrix} 50 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

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c

$$G(s) = \frac{26}{s^3 + 8s^2 + 29s + 52} = \frac{Y(s)}{X(s)} \cdot \frac{X(s)}{U(s)}$$

$$\frac{Y(s)}{X(s)} = 26 \Rightarrow Y(s) = 26X(s)$$

$$\frac{X(s)}{U(s)} = \frac{1}{s^3 + 8s^2 + 29s + 52} \Rightarrow s^3 X(s) + 8s^2 X(s) + 29 s X(s) + 52X(s) = U(s)$$

$$\Rightarrow \dddot{x} + 8\ddot{x} + 29\dot{x} + 52x = U(s)$$

Estados

$x_1 = x$ | $\dot{x}_1 = x_2$

$x_2 = \dot{x}$ | $\dot{x}_2 = x_3$

$x_3 = \ddot{x}$ | $\dot{x}_3 = U(s) - 8x_3 - 29x_2 - 52x_1$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -52 & -29 & -8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 26 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

5

a

$$G(z) = \frac{0.06727}{(z-0.9231)} = \frac{Y(z)}{X(z)} \cdot \frac{X(z)}{U(z)}$$

$$\frac{Y(z)}{X(z)} = 0.06727 \Rightarrow Y(z) = 0.06727 X(z)$$

$$\frac{X(z)}{U(z)} = \frac{1}{z-0.9231} \Rightarrow zX(z) - 0.9231 X(z) = U(z)$$

$$X[k+1] - 0.9231 X[k] = U[k]$$

$$x_1 = x$$

$$\boxed{x_1[k+1] = U[k] + 0.9231 x_1}$$

$$\boxed{y = 0.06727 x_1}$$

b)

$$G(z) = \frac{0.01847z + 0.001847}{z^2 - 1.847z + 0.8521} = \frac{Y(z)}{X(z)} \cdot \frac{X(z)}{U(z)}$$

$$\frac{Y(z)}{X(z)} = 0.01847z + 0.001847 \Rightarrow Y(z) = 0.01847zX(z) + 0.001847X(z)$$

$$Y[k] = 0.01847x[k+1] + 0.001847x[k]$$

$$\frac{X(z)}{U(z)} = \frac{1}{z^2 - 1.847z + 0.8521} = z^2X(z) - 1.847zX(z) + 0.8521X(z) = U(z)$$

$$\Rightarrow x[k+2] - 1.847x[k+1] + 0.8521x[k] = U[k]$$

Estados

$$x_1[k] = x[k] \quad\quad x_1[k+1] = x_2[k]$$

$$x_2[k] = x[k+1] \quad\quad x_2[k+1] = U[k] + 1.847x_2[k] - 0.8521x_1[k]$$

$$\begin{bmatrix} x_1[k+1] \\ x_2[k+1] \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.8521 & 1.847 \end{bmatrix} \begin{bmatrix} x_1[k] \\ x_2[k] \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} U[k]$$

$$Y[k] = \begin{bmatrix} 0.001847 & 0.01847 \end{bmatrix} \begin{bmatrix} x_1[k] \\ x_2[k] \end{bmatrix}$$

c)

$$G(z) = \frac{0.1411z^2 + 0.2112z + 0.0224}{z^3 - 0.36z^2 + 0.168z - 0.02556} = \frac{Y(z)}{X(z)} \cdot \frac{X(z)}{U(z)}$$

$$\frac{Y(z)}{X(z)} = 0.1411z^2 + 0.2112z + 0.0224$$

$$Y(z) = 0.1411z^2X(z) + 0.2112zX(z) + 0.0224X(z)$$

$$Y[k] = 0.1411X[k+2] + 0.2112X[k+1] + 0.0224X[k]$$

$$\frac{X(z)}{U(z)} = \frac{1}{z^3 - 0.36z^2 + 0.168z - 0.02556}$$

$$X(z)z^3 - 0.36X(z)z^2 + 0.168X(z)z - 0.02556X(z) = U(z)$$

$$X[k+3] - 0.36X[k+2] + 0.168X[k+1] - 0.02556X[k] = U[k]$$

Estados

$x_1[k] = x[k]$ | $x_1[k+1] = x_2[k]$

$x_2[k] = x[k+1]$ | $x_2[k+1] = x_3[k]$

$x_3[k] = x[k+2]$ | $x_3[k+1] = U[k] + 0.36x_3[k] - 0.168x_2[k] + 0.02556x_1[k]$

$$\begin{bmatrix} x_1[k+1] \\ x_2[k+1] \\ x_3[k+1] \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.02556 & -0.168 & 0.36 \end{bmatrix} \begin{bmatrix} x_1[k] \\ x_2[k] \\ x_3[k] \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} U[k]$$

$$Y[k] = \begin{bmatrix} 0.0224 & 0.2112 & 0.1411 \end{bmatrix} \begin{bmatrix} x_1[k] \\ x_2[k] \\ x_3[k] \end{bmatrix}$$

7

a)
$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -10 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u \qquad y=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

$$G(s)=C\,(sI-A)^{-1}\times B$$

$$sI-A=\begin{bmatrix}s & 0\\ 0 & s\end{bmatrix}-\begin{bmatrix}0 & 1\\ -10 & -2\end{bmatrix}=\begin{bmatrix}s & -1\\ 10 & s+2\end{bmatrix}$$

$$(sI-A)^{-1}=\frac{\begin{bmatrix}s+2 & 1\\ -10 & s\end{bmatrix}}{s(s+2)+10}=\begin{bmatrix}\frac{s+2}{s^2+2s+10} & \frac{1}{s^2+2s+10}\\ \frac{-10}{s^2+2s+0} & \frac{s}{s^2+2s+10}\end{bmatrix}$$

$$(sI-A)^{-1}\times B=\begin{bmatrix}\frac{s+2}{s^2+2s+10} & \frac{1}{s^2+2s+10}\\ \frac{-10}{s^2+2s+10} & \frac{s}{s^2+2s+10}\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}=\begin{bmatrix}\frac{1}{s^2+2s+10}\\ \frac{s}{s^2+2s+10}\end{bmatrix}$$

$$G(s)=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}\frac{1}{s^2+2s+10}\\ \frac{s}{s^2+2s+10}\end{bmatrix}=\frac{1}{s^2+2s+10}$$

b

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -48 & -44 & -12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \qquad Y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}_{1\times3} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{3\times1}$$

$$(SI - A) = \begin{bmatrix} S & 0 & 0 \\ 0 & S & 0 \\ 0 & 0 & S \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -48 & -44 & -12 \end{bmatrix} = \begin{bmatrix} S & -1 & 0 \\ 0 & S & -1 \\ 48 & 44 & S+12 \end{bmatrix}$$

$$\mathrm{Cof}(SI - A) = \begin{bmatrix} S^2+12S+44 & -48 & -48S \\ S+12 & S^2+12S & -44S-48 \\ 1 & S & S^2 \end{bmatrix}$$

$C_{11} = (-1)^2 [S(S+12) + 44] = S^2 + 12S + 44$

$C_{12} = (-1)^3 [0 + 48] = -48$

$C_{13} = (-1)^4 [0 - 48S] = -48S$

$C_{21} = (-1)^3 [-S - 12] = S + 12$

$C_{22} = (-1)^4 [S^2 + 12S] = S^2 + 12S$

$C_{23} = (-1)^5 [44S + 48] = -44S - 48$

$C_{31} = (-1)^4 [1] = 1$

$C_{32} = (-1)^5 [-S] = S$

$C_{33} = (-1)^6 [S^2] = S^2$

$$\mathrm{Cof}(SI-A)' = \begin{bmatrix} S^2+12S+44 & S+12 & 1 \\ -48 & S^2+12S & S \\ -48S & -44S-48 & S^2 \end{bmatrix}$$

$$(SI-A)^{-1} = \begin{bmatrix} \dfrac{S^2+12S+44}{S^3+12S^2+44S+48} & \dfrac{S+12}{S^3+12S^2+44S+48} & \dfrac{1}{S^3+12S^2+44S+48} \\ \dfrac{-48}{S^3+12S^2+44S+48} & \dfrac{S^2+12S}{S^3+12S^2+44S+48} & \dfrac{S}{S^3+12S^2+44S+48} \\ \dfrac{-48S}{S^3+12S^2+44S+48} & \dfrac{-44S-48}{S^3+12S^2+44S+48} & \dfrac{S^2}{S^3+12S^2+44S+48} \end{bmatrix}$$

$$(SI-A)^{-1} \times B = \begin{bmatrix} \dfrac{1}{S^3+12S^2+44S+48} \\ \dfrac{S}{S^3+12S^2+44S+48} \\ \dfrac{S^2}{S^3+12S^2+44S+48} \end{bmatrix}$$

$$G(S) = \frac{1}{S^3+12S^2+44S+48}$$

c

$$\begin{bmatrix} X_1(K+1) \\ X_2(K+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.7408 & 1.724 \end{bmatrix} \begin{bmatrix} X_1(K) \\ X_2(K) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} U(K)$$

$$Y(K) = \begin{bmatrix} 0.02049 & 0.02264 \end{bmatrix} \begin{bmatrix} X_1(K) \\ X_2(K) \end{bmatrix}$$

$$[ZI - A] = \begin{bmatrix} Z & 0 \\ 0 & Z \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -0.7408 & 1.724 \end{bmatrix} = \begin{bmatrix} Z & -1 \\ 0.7408 & Z - 1.724 \end{bmatrix}$$

$$[ZI - A]^{-1} = \frac{\begin{bmatrix} Z - 1.724 & 1 \\ -0.7408 & Z \end{bmatrix}}{Z^2 - 1.724Z + 0.7408}$$

$$[ZI - A]^{-1} \cdot B = \begin{bmatrix} \dfrac{Z - 1.724}{Z^2 - 1.724Z + 0.7408} & \dfrac{1}{Z^2 - 1.724Z + 0.7408} \\ \dfrac{-0.7408}{Z^2 - 1.724Z + 0.7408} & \dfrac{Z}{Z^2 - 1.724Z + 0.7408} \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$[ZI - A]^{-1} B = \begin{bmatrix} \dfrac{1}{Z^2 - 1.724Z + 0.7408} \\ \dfrac{Z}{Z^2 - 1.724Z + 0.7408} \end{bmatrix}$$

$$G(Z) = \begin{bmatrix} 0.02049 & 0.02264 \end{bmatrix} \begin{bmatrix} \dfrac{1}{Z^2 - 1.724Z + 0.7408} \\ \dfrac{Z}{Z^2 - 1.724Z + 0.7408} \end{bmatrix} = \frac{0.02049 + 0.2264Z}{Z^2 - 1.724Z + 0.7408}$$

①

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5488 & -2.018 & 2.464 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} U(k)$$

$$y(k) = \begin{bmatrix} 0.00133 & 0.606189 & 0.001795 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix}$$

$$[ZI - A] = \begin{bmatrix} z & 0 & 0 \\ 0 & z & 0 \\ 0 & 0 & z \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5488 & -2.018 & 2.464 \end{bmatrix}$$

$$[ZI - A] = \begin{bmatrix} z & -1 & 0 \\ 0 & z & -1 \\ -0.5488 & 2.018 & z - 2.464 \end{bmatrix}$$

$$\mathrm{cof}[zI - A] = \begin{bmatrix} z^2 - 2.464z + 2.018 & 0.5488 & 0.5488z \\ z - 2.464 & z^2 - 2.465z & -2.018z - 0.5487 \\ 1 & z & z^2 \end{bmatrix}$$

$$\mathrm{cof}(zI - A)' = \begin{bmatrix} z^2 - 2.464z + 2.018 & z - 2.464 & 1 \\ 0.5488 & z^2 - 2.465z & z \\ 0.5488z & -2.018z - 0.5487 & z^2 \end{bmatrix}$$

$$[ZI - A]^{-1} = \frac{Cof(ZI - A)'}{Z^3 - 2{,}464Z^2 + 2.018Z - 0{,}5488}$$

$$[ZI - A]^{-1} \times B = \begin{bmatrix} \dfrac{1}{Z^3 - 2{,}464Z^2 + 2{,}018Z - 0{,}5488} \\[2ex] \dfrac{Z}{Z^3 - 2{,}464Z^2 + 2.018Z - 0{,}5488} \\[2ex] \dfrac{Z^2}{Z^3 - 2{,}464Z^2 + 2{,}018Z - 0{,}5488} \end{bmatrix}$$

$$\boxed{G(Z) = C \cdot [ZI - A]^{-1} \times B = \frac{0{,}00133 + 0{,}006189Z + 0{,}001795Z^2}{Z^3 - 2{,}464Z^2 + 2{,}018Z - 0{,}5488}}$$