①

$\dot{X}_1 = X_2$ $\quad \dot{X}_2 = -X_1 + \frac{X_1^2}{6} - X_2 \quad$ Pe (0,0)

Linealización

$$\begin{bmatrix} \delta\dot{x}_1 \\ \delta\dot{x}_2 \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} \begin{bmatrix} \delta x_1 \\ \delta x_2 \end{bmatrix} \Rightarrow \begin{bmatrix} \delta\dot{x}_1 \\ \delta\dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1+\frac{1}{3}x_{1eq} & -1 \end{bmatrix} \begin{bmatrix} \delta x_1 \\ \delta x_2 \end{bmatrix}$$

Sabiendo que el punto de equilibrio es $(\emptyset, \emptyset)$ se obtiene:

$$\begin{bmatrix} \delta\dot{x}_1 \\ \delta\dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} \delta x_1 \\ \delta x_2 \end{bmatrix}$$

Funcion cuadratica

$V(x) = x^T P X$ ① $\quad A^T P + PA = -I$ ②

Aplicando ② para encontrar la matriz P

$$\begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix} + \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -P_{12} & -P_{22} \\ P_{11}-P_{12} & P_{12}-P_{22} \end{bmatrix} + \begin{bmatrix} -P_{12} & P_{11}-P_{12} \\ -P_{22} & P_{12}-P_{22} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -2P_{12} & P_{11}-P_{12}-P_{22} \\ P_{11}-P_{12}-P_{22} & 2P_{12}-2P_{22} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$-2P_{12} = -1$

$P_{11} - P_{12} - P_{22} = 0$

$2P_{12} - 2P_{22} = -1$

$$\begin{bmatrix} 0 & -2 & 0 \\ 1 & -1 & -1 \\ 0 & 2 & -2 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}$$

$P_{12} = \frac{1}{2}$ $\quad P_{22} = 1$ $\quad P_{11} = \frac{3}{2}$

$P = \begin{bmatrix} 3/2 & 1/2 \\ 1/2 & 1 \end{bmatrix}$ // $\quad P_{11}P_{22} - P_{12}^2 > \emptyset$

$\frac{3}{2} \cdot 1 - \left(\frac{1}{2}\right)^2 > \emptyset \quad \frac{3}{2} - \frac{1}{4} = \frac{5}{4} > \emptyset$ //

Aplicando 1

$V(x) = P_{11}x_1^2 + 2P_{12}x_1x_2 + P_{22}x_2^2$

$V(x) = \frac{3}{2}x_1^2 + x_1x_2 + x_2^2 > 0$ //

$V(0) = \frac{3}{2}(\emptyset)^2 + (\emptyset)(\emptyset) + (\emptyset)^2 = 0$ //

$\dot{V}(x) = 3x_1\dot{x}_1 + x_1\dot{x}_2 + x_2\dot{x}_1 + 2x_2\dot{x}_2$

sabiendo que:

$\dot{x}_1 = x_2 \qquad \dot{x}_2 = -x_1 - x_2$

$\dot{V}(x) = 3x_1x_2 + x_1(-x_1 - x_2) + x_2(x_2) + 2x_2(-x_1 - x_2)$

$\dot{V}(x) = \cancel{3x_1x_2} - \cancel{x_1^2 - x_2} + x_2^2 - \cancel{2x_2x_1} - 2x_2^2$

$\dot{V}(x) = -x_1^2 - x_2 + x_1x_2 - x_2^2 < 0$ //

Debido a que cumple las condiciones de $V(x) > 0$; $V(0) = 0$; $\dot{V}(x) < 0$, podemos afirmar que el punto de equilibrio es asintoticamente estable

(2)

$\dot{x}_1 = x_2$

$\dot{x}_2 = -\frac{K}{m}x_1 - \frac{b}{m}x_2$

$m = 1$

$K = 1$

$b = 0,5 = 1/2$

$$V(x) = \frac{1}{2}X^T P X + \frac{1}{2}K x_1^2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1/2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Sabiendo que: $A^T P + PA = -I$ se encuentra P

$$\begin{bmatrix} 0 & -1 \\ 1 & -1/2 \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix} + \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & -1/2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -P_{12} & -P_{22} \\ P_{11} - \frac{1}{2}P_{12} & P_{12} - 1/2 P_{22} \end{bmatrix} + \begin{bmatrix} -P_{12} & P_{11} - 1/2 P_{12} \\ -P_{22} & P_{12} - 1/2 P_{22} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -2P_{12} & P_{11} - 1/2 P_{12} - P_{22} \\ P_{11} - 1/2 P_{12} - P_{22} & 2P_{12} - 1P_{22} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$-2P_{12} = -1$

$P_{11} - \frac{1}{2}P_{12} - P_{22} = 0$

$2P_{12} - P_{22} = -1$

$$\begin{bmatrix} 0 & -2 & 0 \\ 1 & -\frac{1}{2} & -1 \\ 0 & 2 & -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}$$

$P_{11} = 9/4$

$P_{12} = 1/2$

$P_{22} = 2$

$$P = \begin{bmatrix} 9/4 & 1/2 \\ 1/2 & 2 \end{bmatrix} //$$

$$P = \frac{9}{4} \cdot 2 - \left(\frac{1}{2}\right)^2 = \frac{17}{4} > 0 //$$

$$V(x) = \frac{1}{2} x^T P x + \frac{1}{2} K x_1^2$$

$$V(x) = \frac{1}{2} \left[P_{11} x_1^2 + 2 P_{12} x_1 x_2 + P_{22} x_2^2 \right] + \frac{1}{2} K x_1^2$$

$$V(x) = \frac{1}{2} \left[\frac{9}{4} x_1^2 + x_1 x_2 + 2 x_2^2 \right] + \frac{1}{2} x_1^2$$

$$V(x) = \frac{9}{8} x_1^2 + \frac{1}{2} x_1 x_2 + x_2^2 + \frac{1}{2} x_1^2$$

$$V(x) = \frac{13}{8} x_1^2 + \frac{1}{2} x_1 x_2 + x_2^2 > \emptyset //$$

$$V(\emptyset) = \frac{13}{8} (\emptyset)^2 + \frac{1}{2} (\emptyset)(\emptyset) + (\emptyset)^2 = 0 //$$

$$\dot{V}(x) = \frac{13}{4} x_1 \dot{x}_1 + \frac{1}{2} x_1 \dot{x}_2 + \frac{1}{2} x_2 \dot{x}_1 + 2 x_2 \dot{x}_2 \qquad \left\{ \begin{array}{l} \dot{x}_1 = x_2 \\ \dot{x}_2 = -x_1 - \frac{1}{2} x_2 \end{array} \right.$$

$$\dot{V}(x) = \frac{13}{4} x_1 x_2 + \frac{1}{2} x_1 \left(-x_1 - \frac{1}{2} x_2 \right) + \frac{1}{2} x_2^2 + 2 x_2 \left(-x_1 - \frac{1}{2} x_2 \right)$$

$$\dot{V}(x) = \frac{13}{4} x_1 x_2 - \frac{1}{2} x_1^2 - \cancel{\frac{1}{4} x_1 x_2} + \frac{1}{2} x_2^2 - \cancel{2 x_2 x_1} - x_2^2$$

$$\dot{V}(x) = x_1 x_2 - \frac{1}{2} x_1^2 - \frac{1}{2} x_2^2 < 0 //$$

Debido a que cumple las condiciones de $V(x) > 0$; $V(0) = 0$; $\dot{V}(x) < 0$, podemos afirmar que el sistema es asintoticamente estable en el origen.

3

Suponiendo que $x_1 = x_{1eq}$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = g - \frac{c}{m}\frac{x_3^2}{x_1}$$

$$\dot{x}_3 = -\frac{R}{L}x_3 + \frac{1}{L}U$$

$$y = x_1$$

haciendo las derivadas de los estado igual a cero para encontrar los punto de equilibrio tenemos:

$$\dot{x}_1 = x_2 = 0$$

$$\dot{x}_2 = g - \frac{c}{m}\frac{x_3^2}{x_1} = 0 \Rightarrow g = \frac{c}{m}\frac{x_3^2}{x_1} \Rightarrow \frac{gmx_1}{c} = x_3^2 \Rightarrow x_3 = \pm\sqrt{\frac{gmx_1}{c}}$$

$$\dot{x}_3 = -\frac{R}{L}x_3 + \frac{1}{L}U = 0 \Rightarrow \frac{1}{L}U = \frac{R}{L}x_3 = U = Rx_3$$

$$x_{1eq} = x_{1eq}$$

$$x_{2eq} = \varnothing$$

$x_{3eq} = \pm\sqrt{\frac{gmx_{1eq}}{c}}$ -> para el analisis se toma $\sqrt{\frac{gmx_{1eq}}{c}}$

$$U_{eq} = Rx_{3eq} = R\sqrt{\frac{gmx_{1eq}}{c}}$$

Encontrando la matriz de Linealización de forma general tenemos:

$$\begin{bmatrix} \delta \dot{x}_1 \\ \delta \dot{x}_2 \\ \delta \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{c}{m}\frac{x_{3eq}^2}{x_{1eq}^2} & 0 & -2\frac{c}{m}\frac{x_{3eq}}{x_{1eq}} \\ 0 & 0 & -R/L \end{bmatrix} \begin{bmatrix} \delta x_1 \\ \delta x_2 \\ \delta x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/L \end{bmatrix} \delta u$$

$$\delta y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta x_1 \\ \delta x_2 \\ \delta x_3 \end{bmatrix}$$

Remplazando los Puntos de equilibrio tenemo la matriz del sistema Linealizado en un Punto

$$\begin{bmatrix} \delta \dot{x}_1 \\ \delta \dot{x}_2 \\ \delta \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ g \cdot \frac{1}{x_{1eq}} & 0 & -2\frac{c}{m} \cdot \frac{\sqrt{\frac{g m x_{1eq}}{c}}}{x_{1eq}} \\ 0 & 0 & -R/L \end{bmatrix} \begin{bmatrix} \delta x_1 \\ \delta x_2 \\ \delta x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/L \end{bmatrix} \delta u$$

$$\delta y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta x_1 \\ \delta x_2 \\ \delta x_3 \end{bmatrix}$$

4

Regresor [4 4 1] debido a que el sistema es de orden 4

Entra en un rango entre 0 - 1,14 -> de lo contrario el sistema no se estabiliza