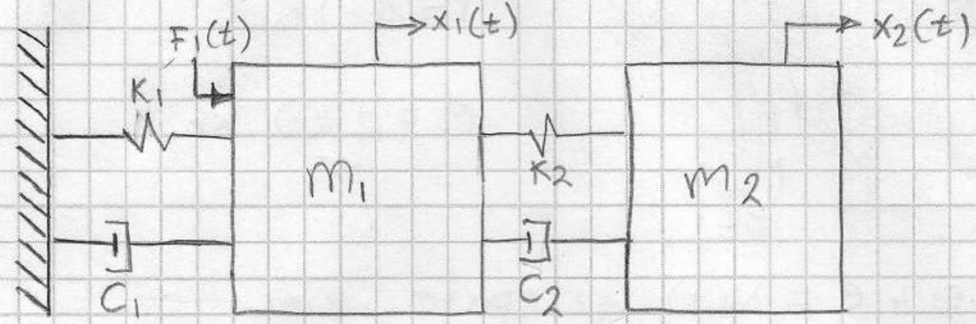
1)

a)



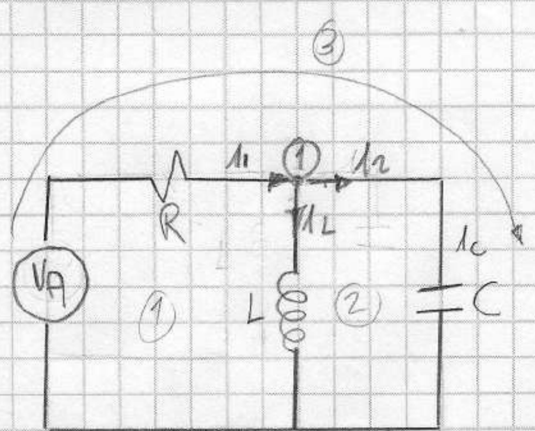
Para m1

$$F_1(t) - C_1\dot{x}_1(t) - K_1 x_1(t) + K_2\left(x_2(t) - x_1(t)\right) + C_2\left(\dot{x}_2(t) - \dot{x}_1(t)\right) = m_1\ddot{x}_1(t)$$

Para m2

$$-C_2\left(\dot{x}_2(t) - \dot{x}_1(t)\right) - K_2\left(x_2(t) - x_1(t)\right) = m_2\ddot{x}_2(t)$$

b)



$$i_c = i_2 = C\frac{dV}{dt}$$

$$V_L = L\frac{di_1}{dt}$$

Sumatoria en el nodo ①

$$i_1 - i_2 - i_L = 0$$

Malla 1

$$V_A - Ri_1 - V_L = 0 \Rightarrow V_A = R\dot{i}_1 + L\frac{di_L}{dt}$$

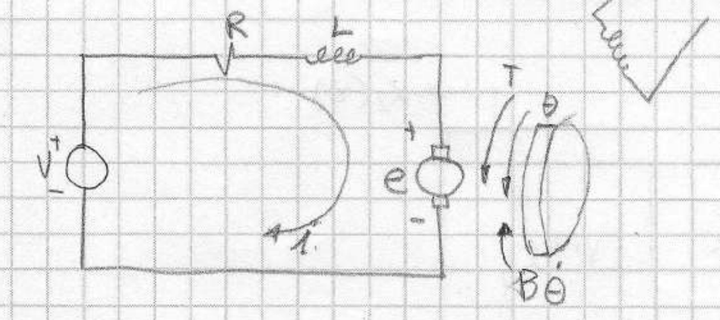
Malla 2

$$-V_C + V_L = 0 \Rightarrow L\frac{di_L}{dt} = V_C$$

Malla 3

$$VA = VR + V_C$$

c)



$$T = i(t)\, K_m$$

$$e = \frac{d\theta(t)}{dt} \cdot K_b$$

$$L\frac{di(t)}{dt} + R\,i(t) + e = V(t)$$ => parte electrica

$$T(t) = j\frac{d^2\theta(t)}{dt^2} + B\frac{d\theta(t)}{dt}$$ => parte Mecanica

2) a

a)

(1) $F_1(t) - c_1\dot{x}_1(t) - k_1 x_1(t) + k_2(x_2(t) - x_1(t)) + c_2(\dot{x}_2(t) - \dot{x}_1(t)) = m_1\ddot{x}_1(t)$

(2) $-c_2(\dot{x}_2(t) - \dot{x}_1(t)) - k_2(x_2(t) - x_1(t)) = m_2\ddot{x}_2(t)$

Aplicando Laplace

(3) $F_1(S) - Sc_1x_1(S) - k_1x_1(S) + k_2(x_2(S) - x_1(S)) + c_2S(x_2(S) - x_1(S)) = S^2m_1x_1(S)$

(4) $-c_2S(x_2(S) - x_1(S)) - k_2(x_2(S) - x_1(S)) = S^2m_2x_2(S)$

Se despeja $x_2(S)$ de (4)

$$-c_2Sx_2(S) + c_2Sx_1(S) - k_2C_2(S) + k_2x_1(S) = S^2m_2x_2(S)$$

$$+Sc_2x_1(S) + k_2x_1(S) = S^2m_2x_2(S) + c_2Sx_2(S) + k_2C_2(S)$$

$$\frac{+Sc_2x_1(S) + k_2x_1(S)}{S^2m_2 + Sc_2 + k_2} = x_2(S) \quad (5)$$

Se remplaza (5) en (3)

$$\rightarrow F_1(S) - Sc_1x_1(S) - k_1x_1(S) + k_2\left(\frac{+Sc_2x_1(S) + k_2x_1(S)}{S^2m_2 + Sc_2 + k_2} - x_1(S)\right) +$$

$$Sc_2\left(\frac{+Sc_2x_1(S) + k_2x_1(S)}{S^2m_2 + Sc_2 + k_2} - x_1(S)\right) = S^2m_1x_1(S)$$

$$F_1(S) = S^2 m_1 x_1(S) + S c_1 x_1(S) + K_1 x_1(S) + \frac{-S c_2 K_2 x_1(S) - K_2^2 x_1(S)}{S^2 m_2 + S c_2 + K_2} + K_2 x_1(S) +$$

$$\frac{-S^2 c_2^2 x_1(S) - S c_2 x_1(S) K_2}{S^2 m_2 + S c_2 + K_2} + S c_2 x_1(S)$$

$$\frac{X_1(S)}{F_1(S)} = \frac{1}{S^2 m_1 + S c_1 + K_1 + \frac{-S c_2 K_2 - K_2^2}{S^2 m_2 + S c_2 + K_2} + K_2 + \frac{-S^2 c_2^2 - S c_2 K_2}{S^2 m_2 + S c_2 + K_2} + S c_2}$$

---

2) a

b) Sabiendo que:

$$F_1(S) - S c_1 x_1(S) - K_1 x_1(S) + K_2(x_2(S) - x_1(S)) + c_2 S(x_2(S) - x_1(S)) = S^2 m_1 x_1(S) \quad (1)$$

$$-c_2 S(x_2(S) - x_1(S)) - K_2(x_2(S) - x_1(S)) = S^2 m_2 x_2(S) \quad (2)$$

Se despeja $x_1(S)$ de (2)

$$-c_2 S x_2(S) + S c_2 x_1(S) - K_2 x_2(S) + K_2 x_1(S) = S^2 m_2 x_2$$

$$x_1(S c_2 + K_2) = S^2 m_2 x_2 + S c_2 x_2(S) + K_2 x_2(S)$$

$$x_1(S) = \frac{S^2 m_2 x_2 + S c_2 x_2(S) + K_2 x_2(S)}{S c_2 + K_2} \quad (3)$$

Se remplaza (3) en (1)

$$F_1(s) - X_2(s)\left[\frac{s^3 c_1 m_2 + s^2 c_2 c_1 + s c_1 k_2}{s c_2 + k_2}\right] - X_2(s)\left[\frac{s^2 m_2 k_1 + s k_1 c_2 + k_2 k_1}{s c_2 + k_2}\right]$$

$$+ k_2 X_2(s) - X_2(s)\left[\frac{s^2 m_2 k_2 + s c_2 k_2 + k_2^2}{s c_2 + k_2}\right] + s X_2(s) c_2 - X_2(s)\left[\frac{s^3 c_2 m_2 + s^2 c_2^2 + s c_2 k_2}{s c_2 + k_2}\right]$$

$$= X_2(s)\left[\frac{s^4 m_2 m_1 + s^3 c_2 m_1 + s^2 k_2 m_1}{s c_2 + k_2}\right]$$

$$\frac{X_2(s)}{F_1(s)} = \frac{1}{\left[\frac{s^4 m_2 m_1 + s^3 c_2 m_1 + s^2 k_2 m_1}{s c_2 + k_2}\right] + \left[\frac{s^3 c_1 m_2 + s^2 c_2 c_1 + s c_1 k_2}{s c_2 + k_2}\right] + \left[\frac{s^2 m_2 k_1 + s k_1 c_2 + k_2 k_1}{s c_2 + k_2}\right] + \left[\frac{s^2 m_2 k_2 + s c_2 k_2 + k_2^2}{s c_2 + k_2}\right] + \left[\frac{s^3 c_2 m_2 + s^2 c_2^2 + s c_2 k_2}{s c_2 + k_2}\right] - s c_2 - k_2}$$

2) b

a)

$$V_A = R\,i_1 + L\frac{d i_L}{dt} \quad (1)$$

$$i_c = i_2 = C\frac{dV_L}{dt}$$

$$i_L = i_2' - i_1 \qquad V_c = V_A - V_R$$

$$-V_c + V_L = 0 \Rightarrow L\frac{d i_L}{dt} = V_c \quad (2)$$

$$V_A = V_R + L\frac{d i_L}{dt} = V_R + \frac{L\,d}{dt}[i_2 - i_1]$$

$$V_A = V_R + \frac{L\,d}{dt}\left[\frac{C\,dV_L}{dt} - V_R/R\right]$$

$$V_A = V_R + \frac{CL\,d^2}{dt^2}\left[V_A - V_R\right] - \frac{L}{R}\frac{dV_R}{dt}$$

aplicando Laplace:

$$V_A(s) = V_R(s) + CL\,s^2 V_A(s) - CL\,s^2 V_R(s) - \frac{L}{R}\,s V_R$$

$$V_A(s) - CL\,s^2 V_A(s) = V_R(s)\left[-s^2 CL - s\,1/R + 1\right]$$

$$\boxed{\frac{-s^2 CL + 1}{-s^2 CL - s\,1/R + 1} = \frac{V_R(s)}{V_A(s)}}$$

2) b

b) $V_A = V_R + V_L$ $\qquad\qquad i_2 = C\frac{dV}{dt}$

$V_A = i_1 R + V_L \Rightarrow V_A = (i_2 + i_L) R + V_L$

$V_A = CR\frac{dV_L}{dt} + Ri_L + V_L = CR\frac{dV_L}{dt} + V_L + \frac{R}{L}\int V_L \, dt$

$V_A(s) = CR\,S\,V_L(s) + V_L(s) + \frac{R}{SL} V_L(s)$

$V_A(s) = V_L(s)\left[SCR + \frac{R}{SL} + 1\right]$

$$\boxed{\frac{V_L}{V_A(s)} = \frac{1}{SCR + \frac{R}{SL} + 1}}$$

c)

$$\boxed{\frac{V_C}{V_f} = \frac{V_L}{V_f} = \frac{1}{SCR + \frac{R}{SL} + 1}}$$

2) c

a)

$$L\frac{d\imath(t)}{dt} + R\imath(t) + \omega(t)K_b = V(t) \quad ①$$

$$K_m\imath(t) = J\frac{d\omega(t)}{dt} + B\,\omega(t) \quad ②$$

Aplicando Laplace

$$SL\imath(s) + R\imath(s) + K_b\omega(s) = V(s) \quad ③$$

$$K_m\imath(s) = JS\omega(s) + B\omega(s) \quad ④$$

Se despeja $\imath(s)$ de ④ y se Remplaza en 3

$$\imath(s) = S\frac{J\omega(s)}{K_m} + \frac{B}{K_m}\omega(s)$$

$$S^2\frac{L}{K_m}\omega(s) + S\frac{LB}{K_m}\omega(s) + S\frac{R}{K_m}\omega(s) + \frac{RB}{K_m}\omega(s) + K_b\,\omega(s) = V(s)$$

$$\omega(s)\left[S^2\frac{L}{K_m} + S\left[\frac{LB}{K_m} + \frac{R}{K_m}\right] + \frac{RB}{K_m} + K_b\right] = V(s)$$

$$\boxed{\frac{\omega(s)}{V(s)} = \frac{1}{S^2\frac{L}{K_m} + S\left[\frac{LB}{K_m} + \frac{R}{K_m}\right] + \frac{RB}{K_m} + K_b}}$$

2) c

b $L\frac{di(t)}{dt} + Ri(t) + \frac{d\theta(t)}{dt} \cdot Kb = V(t)$ ①

$i(t)\,Km = J\frac{d^2\theta(t)}{dt^2} + B\frac{d\theta(t)}{dt}$ ②

Aplicando Laplace

$SL\,i(S) + R\,i(S) + S\,Kb\,\theta(S) = V(S)$ ③

$i(S)\,Km = J S^2\theta(S) + S B\,\theta(S)$ ④

Se despeja $i(s)$ de ④ y se remplaze en ③

$i(s) = S^2\frac{J}{Km}\theta(s) + S\frac{B}{Km}\theta(s)$

$\rightarrow S^3\frac{LJ}{Km}\theta(S) + S^2\frac{LB}{Km}\theta(s) + S^2\frac{RJ}{Km}\theta(S) + S\frac{RB}{Km}\theta(S) + S\,Kb\,\theta(S) = V(S)$

$$\frac{\theta(S)}{V(S)} = \frac{1}{S^3\frac{LJ}{Km} + S^2\left[\frac{Lb}{Km} + \frac{RJ}{Km}\right] + S\left[\frac{RB}{Km} + Kb\right]}$$

2) c

c

$$L\frac{di(t)}{dt} + Ri(t) + \frac{d\theta(t)}{dt}Kb = V(t)$$

$$i(t)\,Km = J\frac{d^2\theta(t)}{dt} + B\frac{d\theta(t)}{dt}$$

Aplicando Laplace

$$SLI(S) + RI(S) + SKb\,\theta(S) = V(S) \quad (1)$$

$$I(S)\,Km = S^2 J\theta(S) + SB\theta(S) \quad (2)$$

Despejando $\theta(S)$ de (2) y remplazando en (1)

$$I(S)\,Km = \theta(S)\left[S^2J + SB\right]$$

$$\frac{I(S)\,Km}{S^2J + SB} = \theta(S)$$

$$SLI(S) + RI(S) + \frac{\cancel{S}Kb\,I(S)\,Km}{S^2J + \cancel{S}B} = V(S)$$

$$I(S)\left[SL + R + \frac{Kb\,Km}{SJ + B}\right] = V(S)$$

$$\boxed{\frac{I(S)}{V(S)} = \frac{1}{SL + R + \frac{KbKm}{SJ+B}}}$$

4)

a)

$$G(s) = \frac{40}{s^2 + 13s + 40} \qquad U(s) = \frac{1}{s}$$

$$Y(s) = \frac{1}{s} \cdot \frac{40}{(s+8)(s+5)} = \frac{A}{s} + \frac{B}{s+8} + \frac{C}{s+5}$$

Para A

$$\left.\frac{40}{(s+8)(s+5)}\right|_{s=0} = \frac{40}{40} = 1$$

Para C

$$\left.\frac{40}{s(s+8)}\right|_{s=-5} = \frac{40}{-5(-5+8)} = -\frac{8}{3}$$

Para B

$$\left.\frac{40}{s(s+5)}\right|_{s=-8} = \frac{40}{-8(-8+5)} = \frac{5}{3}$$

$$Y(s) = \frac{1}{s} + \frac{5/3}{s+8} - \frac{8/3}{s+5}$$

$$\boxed{y(t) = 1 + \frac{5}{3} e^{-8t} - \frac{8}{3} e^{-5t}}$$

B)

$$G(s) = \frac{24.5}{s^2+14s+49} \qquad u(s) = \frac{1}{s}$$

$$Y(s) = \frac{1}{s} \cdot \frac{24.5}{(s+7)^2} = \frac{A}{s} + \frac{B}{(s+7)^2} + \frac{C}{s+7}$$

Para A

$$\left.\frac{24.5}{(s+7)^2}\right|_{s=0} = 1/2$$

para B

$$\left.\frac{24.5}{s}\right|_{s=-7} = -7/2$$

para C

$$\left.\frac{d}{ds}\frac{24.5}{s}\right|_{s=-7} = \left.\frac{-24.5}{s^2}\right|_{s=-7} = -1/2$$

$$Y(s) = \frac{1/2}{s} - \frac{7/2}{(s+7)^2} - \frac{1/2}{s+7}$$

$$\boxed{Y(t) = 1/2 - 1/2\, e^{-7t} - 7/2\, t\, e^{-7t}}$$

c)

$$G(s) = \frac{50}{s^2+8s+25} \qquad u(s) = \frac{1}{s}$$

$$Y(s) = \frac{50}{s(s^2+8s+25)} = \frac{A}{s} + \frac{Bs+C}{s^2+8s+25}$$

$$Y(s) = \frac{A(s^2+8s+25)+Bs^2+Cs}{s(s^2+8s+25)}$$

$$\Rightarrow As^2+8As+25A+Bs^2+Cs = 50$$

$$\Rightarrow s^2(A+B)+s(8A+C)+25A = 50$$

$$\begin{cases} A+B=0 \\ 8A+C=0 \\ 25A=50 \end{cases} \qquad A=2 \quad B=-2 \quad C=-16$$

$$Y(s) = \frac{2}{s} + \frac{-2s-16}{s^2+8s+25}$$

Se tiene que

$$\frac{-2s-16}{(s+4)^2+3^2} = -2\left(\frac{s+8}{(s+4)^2+3^2}\right) = -2\left(\frac{s+4+8-4}{(s+4)^2+3^2}\right)$$

$$= -2\left[\frac{s+4}{(s+4)^2+3^2} + \frac{4}{(s+4)^2+3^2}\right] = -2\left[\frac{s+4}{(s+4)^2+3^2} + \frac{3}{3}\frac{4}{(s+4)^2+3^2}\right]$$

$$= -2\left[\frac{s+4}{(s+4)^2+3^2} + \frac{4}{3}\frac{3}{(s+4)^2+3^2}\right] = -2.\frac{s+4}{(s+4)^2+3^2} - \frac{8}{3}\frac{3}{(s+4)^2+3^2}$$

$$Y(s) = \frac{2}{s} - 2.\frac{s+4}{(s+4)^2+3^2} - \frac{8}{3}\frac{3}{(s+4)^2+3^2}$$

$$\boxed{y(t) = 2 - 2.e^{-4t}\cos(3t) - \frac{8}{3}e^{-4t}\operatorname{sen}(3t)}$$

6)

a)

$$G(z) = \frac{0.06727}{(z-0.9231)} \qquad U(z) = \frac{z}{z-1}$$

$$Y(z) = \frac{0.06727}{z-0.9231} \cdot \frac{z}{z-1}$$

$$\frac{Y(z)}{z} = \frac{0.06727}{(z-0.9231)(z-1)} = \frac{A}{z-1} + \frac{B}{z-0.9231}$$

Para A

$$\left.\frac{0.06727}{z-0.9231}\right|_{z=1} = \frac{0.06727}{0.0769} = 0.8747$$

Para B

$$\left.\frac{0.06727}{z-1}\right|_{z=0.9231} = \frac{0.06727}{-0.0769} = -0.8747$$

$$\frac{Y(z)}{z} = \frac{0.8747}{z-1} - \frac{0.8747}{z-0.9231}$$

$$Y(z) = \left[\frac{0.8747z}{z-1} - \frac{0.8747z}{z-0.9231}\right]\frac{z^{-1}}{z^{-1}} = \frac{0.8747}{1-z^{-1}} - \frac{0.8747}{1-0.9231z^{-1}}$$

$$y[k] = 0.8747\,u[k] - 0.8747\,(0.9231)^{k}\,u[k]$$

B

$$G(z) = \frac{0.001847z + 0.001847}{(z^2 - 1.847z + 0.8521)} \qquad U[z] = \frac{z}{z-1}$$

$$Y[z] = \frac{z}{z-1} \cdot \frac{0.001847z + 0.001847}{(z-0.9509)(z-0.8960)}$$

$$\frac{Y[z]}{z} = \frac{A}{z-1} + \frac{B}{(z-0.9509)} + \frac{C}{(z-0.8960)}$$

Para A

$$\left.\frac{0.001847z + 0.001847}{(z-0.9509)(z-0.8960)}\right|_{z=1} = \frac{0.00184 + 0.001847}{(1-0.9509)(1-0.8960)} = \frac{0.003687}{0.065106} = 0.7234$$

Para B

$$\left.\frac{0.001847z + 0.001847}{(z-1)(z-0.8960)}\right|_{z=0.9509} = \frac{0.003603}{-0.002695} = -1.3367$$

Para C

$$\left.\frac{0.001847z + 0.001847}{(z-1)(z-0.9509)}\right|_{z=0.8960} = \frac{0.003501}{0.0057096} = 0.6133$$

$$\frac{Y[z]}{z} = \frac{0.7234}{z-1} - \frac{1.3367}{z-0.9509} + \frac{0.6133}{z-0.8960}$$

$$Y[z] = \left[\frac{0.7234\,z}{z-1} - \frac{1.3367z}{z-0.9509} + \frac{0.6133\,z}{z-0.8960}\right]\frac{z^{-1}}{z^{-1}}$$

$$Y[z] = \frac{0.7234}{1-z^{-1}} - \frac{1.3367}{1-0.9509z^{-1}} + \frac{0.6133}{1-0.8960z^{-1}}$$

$$\boxed{Y[k] = 0.7234 - 1.3367[0.9509]^k + 0.6133\,[0.8960]^k}$$

c)

$$G(z) = \frac{0.004528z + 0.004528}{z^2 - 1.81z + 0.8187} \qquad U[z] = \frac{z}{z-1}$$

$$\frac{Y(z)}{z} = \frac{0.004528z + 0.004528}{(z-1)(z-0.923028)(z-0.886972)}$$

$$\frac{Y(z)}{z} = \frac{A}{z-1} + \frac{B}{z-0.923028} + \frac{C}{z-0.886972}$$

para A

$$\left.\frac{0.004528z + 0.004528}{(z-0.923028)(z-0.886972)}\right|_{z=1} = 1.0409$$

Para B

$$\left.\frac{0.004528z + 0.004528}{(z-1)(z-0.886972)}\right|_{z=0.923028} = -3.1374$$

Para C

$$\left.\frac{0.004528z + 0.004528}{(z-1)(z-0.923028)}\right|_{z=0.886972} = 2.0965$$

$$\frac{Y(z)}{z} = \frac{1.0409}{z-1} - \frac{3.1374}{z-0.923028} + \frac{2.0965}{z-0.886972}$$

$$Y(z) = \left[\frac{1.0409z}{z-1} - \frac{3.1374z}{z-0.923028} + \frac{2.0965z}{z-0.886972}\right]\frac{z^{-1}}{z^{-1}}$$

$$Y(z) = \frac{1.0409}{1-z^{-1}} - \frac{3.1374}{1-0.923028z^{-1}} + \frac{2.0965}{1-0.886972z^{-1}}$$

$$Y[k] = 1.0409 - 3.1374(0.923028)^k + 2.0965(0.886972)^k$$

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D)

$$G(z) = \frac{0.02909z + 0.02721}{(z^2 - 1.753z + 0.8187)} \qquad U(z) = \frac{z}{z-1}$$

$$\frac{Y(z)}{z} = \frac{0.02909z + 0.02721}{(z-1)(z^2 - 1.753z + 0.8187)}$$

$$\frac{Y(z)}{z} = \frac{A}{z-1} + \frac{Bz + C}{z^2 - 1.753z + 0.8187}$$

$$\frac{Y(z)}{z} = \frac{Az^2 - 1.753Az + 0.8187A + Bz^2 - Bz + Cz - C}{(z-1)(z^2 - 1.753z + 0.8187)}$$

$$(A+B)z^2 + (C - B - 1.753A)z + 0.8187A - C = 0.02909z + 0.02721$$

$$A + B = 0 \qquad C - B - 1.753A = 0.02909 \qquad 0.8187A - C = 0.02721$$

$$\frac{Y(z)}{z} = \frac{0.8569}{z-1} + \frac{-0.8569z + 0.6743}{z^2 - 1.753z + 0.8187}$$

$$\frac{Y(z)}{z} = \frac{0.8569}{z-1} - 0.8569\left[\frac{z - 0.7869}{z^2 - 1.753z + 0.8187}\right]$$

Sabiendo que

$$a^k \sin(k w_0) = \frac{a \sin(w_0) z}{z^2 - 2a\cos(w_0) z + a^2}$$

$$a^k \cos(k w_0) = \frac{z^2 - a\cos(w_0) z}{z^2 - 2a\cos(w_0) z + a^2}$$

Se hallan los valores de a y w_c

$$a^2 = 0.8187 \qquad a = 0.90482$$

$$2a\cos(w_0) = 1.753 \qquad w_0 = \cos^{-1}\left(\frac{1.753}{2a}\right) = 0.2507$$

$$-0.8569\left[\frac{z-0.7869}{z^2-1.753z+0.8187}\right]=-0.8569\left[\frac{z-0.90481-0.7869+0.90481}{z^2-1.753z+0.8187}\right]$$

$$=-0.8569\left[\frac{z-0.90481}{z^2-1.753z+0.8187}+\frac{0.1179}{z^2-1.753z+0.8187}\right]$$

$$=-0.8569\left[\frac{z-0.90481}{z^2-1.753z+0.8187}+\frac{0.1179}{0.90481}\cdot\frac{0.90481}{z^2-1.753z+0.8187}\right]$$

$$Y(z)=\frac{0.8569z}{z-1}-0.8569\left[\frac{z^2-0.90481}{z^2-1.753z+0.8187}+0.1303\cdot\frac{0.90481}{z^2-1.753z+0.8187}\right]$$

$$\boxed{\begin{aligned}y[k]&=0.8569-0.8569[0.90482]^k\cos(k\,0.2507)\\&\quad-0.0875[0.90482]^k\sin(k\,0.2507)\end{aligned}}$$