Implementation for controllers PID on different methods

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Abstract—.

In this document will be implemented the mathematical modeling of a Quanser motor and the control of the variable position. For the project to be implemented a acquisition of data has been done on the Automatic Labs with the encoder board for acquisition named DAQ NI PCI-6221, which allows the usage of the tool System Identification from Matlab to determine the Transfer function that will be the key component for the build up of the controller and modeling in Simulink, this project will contain the stabilization time of the step response to different input applied.

With that the methods to obtain the information for the controller are:

- Analytic method (PID, (Discrete and continuous).
- Root Locus Method.

Key words: Quanser Motor, PID controller, Matlab.

I. INTRODUCCIÓN

The DC motor is a direct current that allows the electric power to converse to a mechanical energy as a rotative movement through a active magnetic field,

This magnetic field is generated by a winding current that allows the change for the position of the rotor motor, for that we will need a switch that will work engaging a change on the polarity of the voltage applied on the switch.

One of the key variables of interest is the speed and position, which can be modified through a voltage applied to the DC motor with a controller that acts as a regulator, by adjusting the voltage and therefore the current that circulates through the motor.

That is why in this document the experimental identification of a direct current motor can be controlled by a carried out ensemble and their respective control of the variable speed and position of a PID controller.

II. OBJECTIVES

A. General

Obtained a transfer function that will have the parameters based on the speed of the Quanser motor from experimental Data.

Implement a PID controller for the position and speed control of the Quanser motor.

B. Specs

- Configuration of the acquisition set up for speed data from the Quanser motor.
- Analyzed experimental data through System Identification Tool on Matlab.
- Understand and operate the behavior of the motor established by the PID controller in simulation.

III. Methodology

Firstly, the characteristics of the Plan is carried out by the operation from the system and how it behaves on multiple variations of step voltage inputs, there were three data recollections, these were made with sampling times of 5ms, 10ms and 15ms each.

Simulink app is used to plot the block diagram from the result of the behavior shown on the speed in real time(Fiq 1).

Sampling of the data is executed with previously mentioned sampling times, in the respective stimuli, which may vary from -5 to 5 Volts.

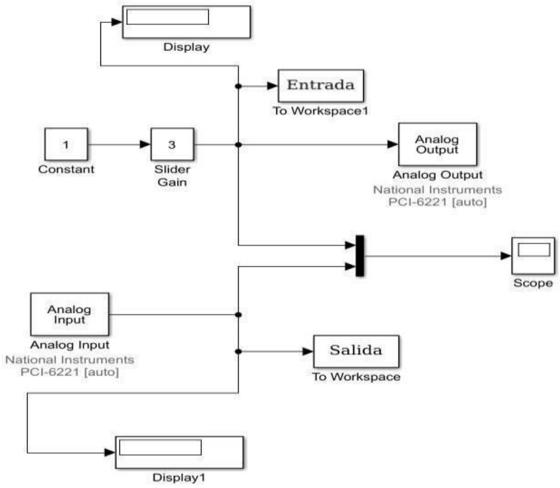


Figure 1. Block diagram for the Plant caracterización with Matlab-simulink.

The connection between the motor and the PCI-6221 board. Based on the manufacturer, the analog input from the acquisition board NI DAQ-PCI 6221 will be connected TACH motor input, and will be considered the speed of the plant, for the feedback on the output of the analog AO0 and GND of the input Command

IV. ROOT LOCUS METHOD

Teoricaly considering the transfer function regarding the speed on the Motor is considered a second order system, and with the respective combinations on the tool System Identification on Matlab, the system had 2 poles and 1 zero, for the stimuli responses on the motor from 0 to 4 Voltios.

$$\frac{0.06843\ s + 0.001901}{s^2 + 0.1241\ s + 0.002347}$$

Figure 2. Transfer function defined by the System Identification tool.

With the resulting characterization of the transfer function for the system we visualize the response versus the theoretical transfer function from the motor. and compare the information obtained versus the theoretical.

$$\frac{\Omega(s)}{V_a(s)} = \frac{K_t}{L_aJs^2 + (R_aJ + L_aB)s + (R_aB + K_tK_b)}.$$

Figure 3. Theoretical Transfer function from a DC Motor in regards to the speed.

$$\frac{0.06843\ s + 0.001901}{s^2 + 0.1241\ s + 0.002347}$$

The greater power of the denominator indicates that it is a second order system with 2 poles and a zero. and with the obtained data was processed the plotting of the Plant behavior on open loop(Fig. 3)

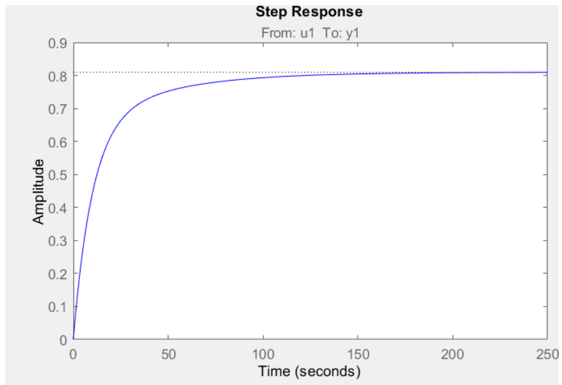


Figure 4. Step response for an open loop plant.

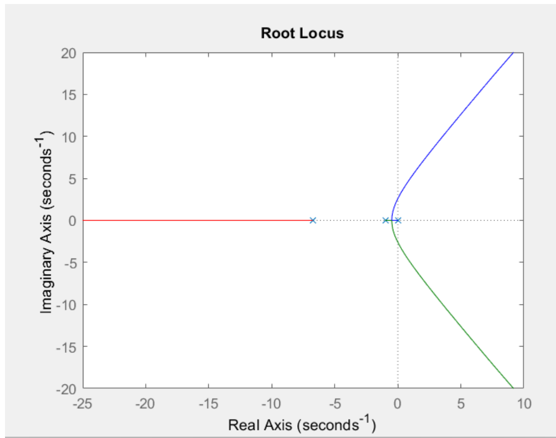


Figure 5. Root locus from poles and zeros.

To simplify the calculations the analysis will be done on a first order system , based on the canonical distribution from the first order established by the next equation:

$$\frac{C(s)}{R(s)} = \frac{1}{\tau s + 1} \qquad \text{Ecu.1}$$

The Tao parameter that indicates the speed time of the response stabilized, the behavior of the system response is really slow, more than normal indicating a issue maybe on the data acquisition of the plant, but we will continue on with the information established.

To find Tao will be calculated by the time that the response starts and when the system reaches the 63.3% of the final response. divided by 4.

For this information also we will take account on the information where the input of the system stimuli from 0 to 4 Volts, the control function is characterized;

Para este caso se aplicaron entradas desde 1V hasta 4V, realizamos tratamiento estadístico de los datos llegando a la siguiente función de transferencia:

$$G(s) = \frac{0.551}{(0.12s+1)}$$

Where the constant gain is K=0.551 and the Tao=0.12. where the stabilization time for the response is

$$Ts=10*\tau = 100 * 0.12 = 1.2sec$$

Based on the final value Theorem for the input ports the Vi=0.551when stimuli by 0 Volts.

and for when the input port is stimulated by 4 Volts the response is Vo=3.72V.

To be avail for the calculation of the controller we will have to take account of the defined education for the construction on the controller, for that the information on the stabilization time and the tao will be the next requirements:

$$T_s = 0.12seg$$
$$\xi = 0.9$$

To avoid over impulse we have the zita theoretical, for that the maximum impulse will be of :

$$MP = e^{\frac{-0.9*\pi}{\sqrt{1-0.9}}} = 0.015 * 100\% = 0.15\%$$

With this information will be replaced on the education to find the natural frequency.

$$Wn = \frac{4}{0.551*0.9} = 8.066rad/seg$$

for that we have established the second order education referenced to be the characteristic behavior:

$$M(s) = \frac{35.848}{s^2+15.58s+35.848}$$

with that equation we will compare and equal with the characteristic equation for a PID controller in a cascade dynamic..

$$Pid = \frac{Kds^2+Kps+Ki}{s}$$

$$Pid = \frac{0.551Kp + 0.551Ki}{s^2+(5.51+4.91Kp)+4.91Ki}$$

With that we found the other parameters missing Kp and Ti.

5.51+4.91Kp=15.58

Kp=2.050

4.91Ki=35.848

Ki=7.301

With all of the detail parameters found will be replaced on the characteristic equation established by

$$M(s) = \frac{35.848}{s^2+15.58s+35.848}$$

With 2 Poles = -13.655±6.09i

Angle= 57.20

With that the controller equation for the position will be established by

$$Gc = \frac{(s+14)^2}{(s+52.43)}$$

The theoretical scheme for the controller is designed.

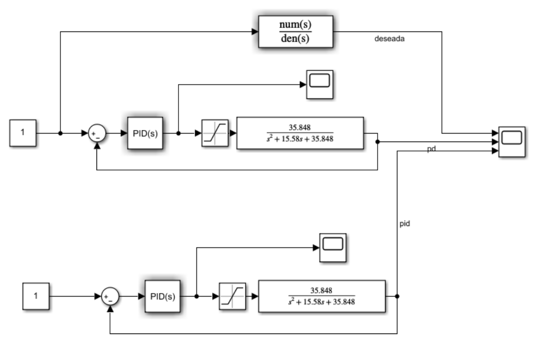


Figure 6. Simulink diagram

With this schematics the response for a stimuli on the controller has the next graph

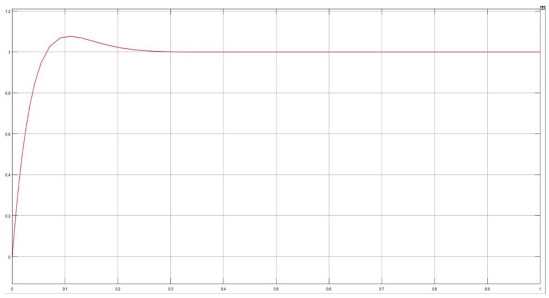


Figure 7. Theoretical response of the controller.

V. ANALYTIC METHOD (PID, DISCRETE AND CONTINUOUS)

The transfer function for the Quanser Motor is

$$Gp(s) = \frac{141300}{s^3+148.4s^2+2923s+143600}$$

Based on the two conjugate poles that domains over the dynamic of the system, there will be a transfer function of the second order using the System Identification tool on Matlab, with the combination of 2 poles and 0 zeros for all of stimuli on the 3 takes.

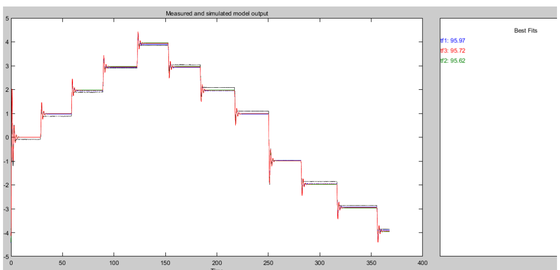


Figure 8. Controller transfer function behavior.

With the approximate data information obtained of 96.84% of accuracy the transfer function the poles of the system are determined by $S_{1,2} =- 0.689 \pm 3.02i$

$$Gp(s) = \frac{9.458}{s^2+1.378s+9.615}$$

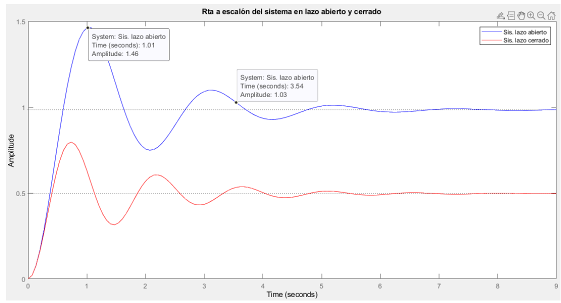


Figure 9. Controller response and stabilization.

The system has a time stabilized of 3.54s and with an over impulse 46% we will propose the new system characteristics with a Ts=1s and Mp=10%, with no stationary error state.

$$Mp = 10\% = 0.1$$

$$\zeta = \frac{-\ln ln\,(Mp)}{\sqrt{\pi^2+(\ln ln\,(Mp))^2}} = 0.5912$$

$$Ts = \frac{4}{\zeta\omega_n} \rightarrow \omega_n = \frac{4}{\zeta * Ts} = \frac{4}{0.5912*1} = 22.5547$$

To eliminate the stationary error state the plant will be multiplied by an integrator.

$$Gp(s) = \frac{9.458}{(s^2+1.378s+9.615)s}$$

And the characteristic equation is:

$$G(s) = \frac{45.78}{s^2 + 8\,s + 45.78}$$

also the poles are $S_{1,2} =- 4 \pm 5.4575i$

Algebraic controller based on the desired equation is:

$$Desired\ Eq = 2(3) - 1 = 5$$

The Elevation to the desired equation is a must for matching the equation to the characteristic equation of the system, there chosen also 3 poles in a real continuous plane of a larger distribution of 10.

$$S_{3,4} =- 50$$

$$Desired\ Eq = (s^2 + 8s + 45.78)(s + 50)(s + 51)(s + 52)$$

$$= s^5 + 161s^4 + 9071.7799s^3 + 202020.3348s^2 + 1417975.56s + 6070428$$

The controller will need to be of 2nd order

$$Gc(s) = \frac{q_2s^2+q_1s+q_0}{p_2s^2+p_1s+p_0}$$

The system will close the loop of the controller so we clear up the characteristic pole of the system.

$$\frac{C(s)}{R(s)} = \frac{(9.458)(q_2s^2+q_1s+q_0)}{(s^3+1.378s^2+9.615s)(p_2s^2+p_1s+p_0)+(9.458)(q_2s^2+q_1s+q_0)}$$

$$p_2s^5 + 1.378p_2s^4 + 9.615p_2s^3 + p_1s^4 + 1.378p_1s^3 + 9.615p_1s^2 + p_0s^3$$
$$+ 1.378p_0s^2 + 9.615p_0s + 9.458q_2s^2 + 9.458q_1s + 9.458q_0 = 0$$

The terms will be grouped and we match the equations the desired and the characteristics and resolve them

$$p_2s^5 + s^4(1.378p_2 + p_1) + s^3(9.615p_2 + 1.378p_1 + p_0) +$$
$$s^2(9.615p_1 + 1.378p_0 + 9.458q_2) + s(9.615p_0 + 9.458q_1) + 9.458q_0 = 0$$

then

$$p_2s^5 + s^4(1.378p_2 + p_1) + s^3(9.615p_2 + 1.378p_1 + p_0) + s^2(9.615p_1 +$$
$$1.378p_0 + 9.458q_2) + s(9.615p_0 + 9.458q_1) + 9.458q_0 =$$
$$s^5 + 161s^4 + 9071.7799s^3 + 202020.3348s^2.$$
$$+ 1417975.56s + 6070428$$

$$p2 = 1$$

$$1.378p_2 + p_1 = 161$$

$$9.615p_2 + 1.378p_1 + p_0 = 9071.7799$$

$$9.458q_2 + 9.615p_1 + 1.378p_0 = 202020.334$$

$$9.458q_1 + 9.615p_0 = 1417975.56$$

$$9.458q_0 = 6070428$$

The algebraic controller is shown the response taking into account the integrator added to remove the error in steady state

$$Gc(s) = \frac{q_2s^2+q_1s+q_0}{p_2s^3+p_1s^2+p_0s}$$

$$Gc(s) = \frac{19910s^2+140900s+641800}{s^3+159.6s^2+8842s}$$

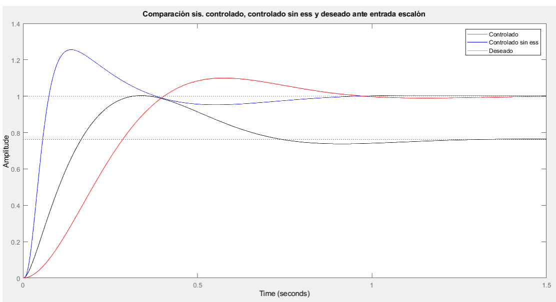


Figure 10. Controller comparison versus the desired, without the stationary error and the control.

Next step is for the controlled system that without the steady state error is discretized by the Trusting method with the sampling time chosen empirically by the rise time.

$$Tm = \frac{Tsubida}{10} = \frac{0.0746}{10} = 0.0075$$

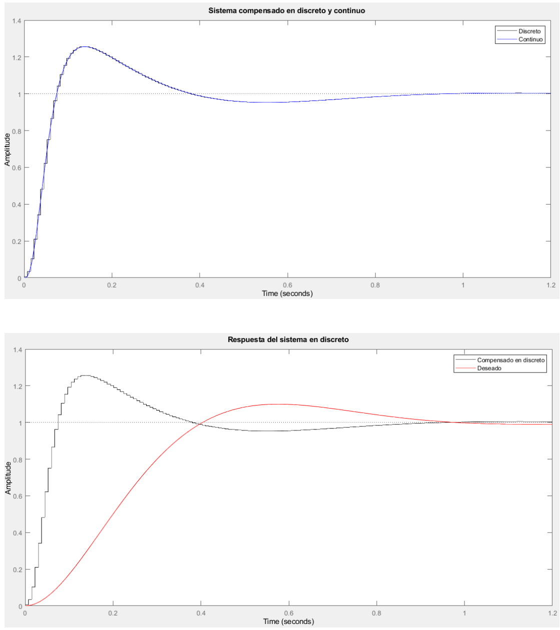


Figure 11. System compensated in discrete and continuous.

All of the information will be graph and designed the controller in continuous time and discrete in the Simulink app.

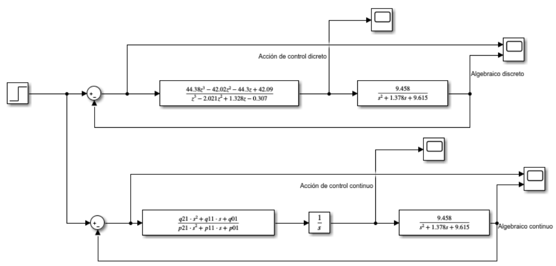
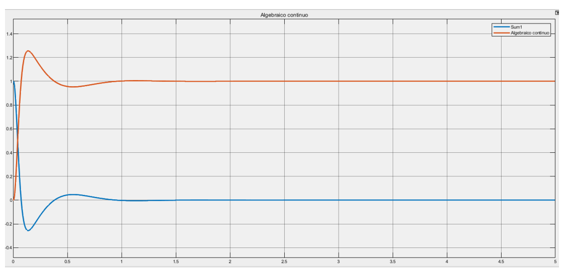


Figure 12. Simulink diagram



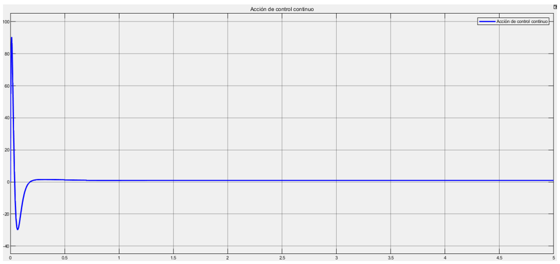


Figure 13. Continuous response of the controller

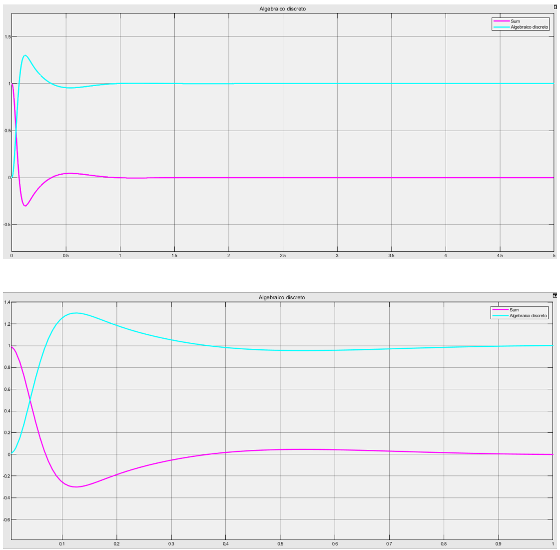


Figure 14. Discrete response of the controller

CONTROLLER PID

the characteristic desired equation will be obtained is:

$$s^2 + 8s + 45.78$$

and the controller PID is established by the equation.

$$Gc(s) = \frac{Kp}{T_is}\left(1 + T_is + T_iT_ds^2\right)$$

When closing the loop with the controller.

$$\frac{C(s)}{R(s)} = \frac{9.458*Kp\left(1+T_is+T_iT_ds^2\right)}{T_is(s^2 + 8\,s + 45.78)+9.458*Kp\left(1+T_is+T_iT_ds^2\right)}$$

$$\frac{C(s)}{R(s)} = \frac{9.458*Kp\left(1+T_is+T_iT_ds^2\right)}{T_is^3+8T_is^2+45.78T_is+9.458Kp+9.458KpT_is+9.458KpT_iT_ds^2}$$

Using the denominator we group the terms and match them.

$$(\tfrac{1}{T_i})(T_is^3 + 8T_is^2 + 45.78T_is + 9.458Kp +$$

$$9.458KpT_is + 9.458KpT_iT_ds^2) = 0(\tfrac{1}{T_i})$$

$$s^3 + 8s^2 + 45.78s + 9.458\frac{Kp}{T_i} + 9.458Kps + 9.458KpT_ds^2 = 0$$

$$s^3 + s^2\left(8 + 9.458KpT_d\right) + s(45.78 + 9.458Kp) + 9.458\frac{Kp}{T_i} = 0$$

The order of the desired characteristic equation is increased, taking into account that the poles are at a distance greater than 10 from the dominant ones (the real axis is at -4).

$$\left(s^2 + 8\,s + 45.78\right)(s + 50) = 0$$

$$s^3 + 58s^2 + 445.78s + 2289 = 0$$

Matching Polynominals

$$s^3 + s^2\left(8 + 9.458KpT_d\right) + s(45.78 + 9.458Kp)$$

$$+ 9.458\frac{Kp}{T_i} = s^3 + 58s^2 + 445.78s + 2289$$

Variables are found

$$8 + 9.458KpT_d = 58 \quad \rightarrow \quad T_d = \frac{50}{9.458*42.2922} = 0.1250$$

$$45.78 + 9.458Kp = 445.78 \quad \rightarrow \quad Kp = \frac{400}{9.458} = 42.2922$$

$$9.458\frac{Kp}{T_i} = 2289 \quad \rightarrow \quad T_i = 9.458\frac{42.2922}{2289} = 0.1747$$

The obtained PID controller enters two zeros and a pole at the origin to the system

$$Gc(s) = \frac{0.9238s^2+7.391s+42.29}{0.1747s}$$

The system controlled by the Tustin method with empirically chosen sampling time with rise time is discretized

$$Tm = \frac{Tsubida}{10} = \frac{0.047}{10} = 0.0047$$

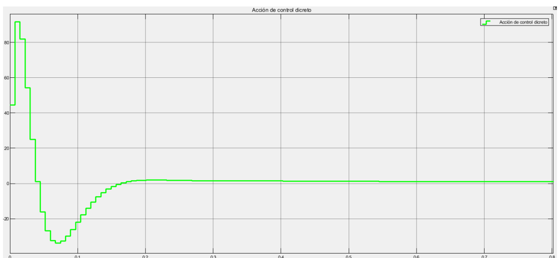


Figure 15. Discrete response

When a practice the controller schematic is built with the next structure.

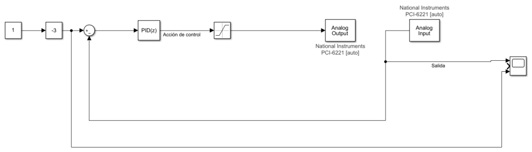
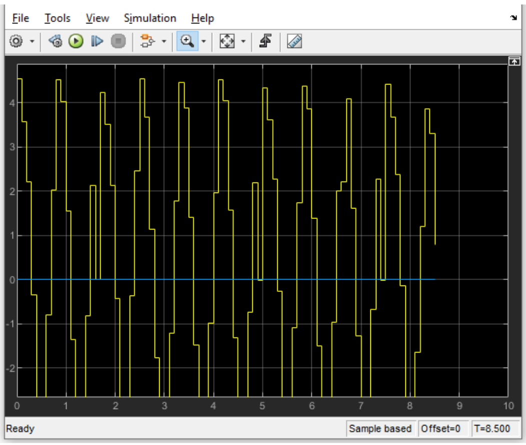


Figure 16. Simulink diagram controller to the Quanser motor.

In the results of the PID controller it was evidenced that initially the mathematical model was not adjusted to the plant since the scope showed the oscillatory behavior even with the values of Kp, Ti and Td with which it was executed, so an unstable model was observed in the system, of so that a trial and error method was carried out by which to test which conditions of the values were adjusted to what was desired, so that it came to stabilize in a moment, all this in the loop system closed.



VI. ANALYSIS AND DISCUSSION.

The selection of the stabilization time of the desired equation must be in an appropriate way since it modifies the gain of the proportional and the integrator, making our plant work in the desired way.

The closed loop generates an error in steady state, the best way to eliminate the error is by integrating the actuator input.

Theoretically the plant and controller works but when applying the variables and parameters on the Motor Quanser physically the system oscillates and is unsynchronized with the information, most probably the issue was on the data acquisition, recommended double check on the process to avoid errors and white noise.

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