Optimal Energy Control of DC Motor Speed Control: Comparative Study

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***Abstract*—In this paper, the speed control problem of DC Motor is presented. Three different control algorithms, namely PID, Fuzzy, and LQR were designed and implemented. These controller were applied to the DC motor, and their performance and energy controller output were compared each other on the basis of unit-step response, disturbance rejection, and signal tracking. The aim of this comparative is to find the most optimal energy controller in the same task. The simulation results show that LQR is the most optimal energy controller compare with PID and Fuzzy.**

Keywords— PID, Fuzzy, LQR, Energy Control, DC motor

I. INTRODUCTION

Electric motor is an actuator that is widely used in industry, for both DC motors and AC motors. However, both of them have different characteristics. DC motor has some advantages such as easy to control the speed and position [1][2][3]. DC motors are widely used as in steel rolling mills, electric trains, electric vehicles, and robotic actuators [2][4].

The research in speed control of DC motor has been done by a lot researchers based on performance viewpoint, as in [5][6][7][8], so in this paper, energy viewpoint will used. The main contribution of this paper is to determine which controller is the most energy efficient on the same task. Because energy is an important issue today, and according to [9], the electric motor is one of the appliances that consume considerable electrical energy. Some methods to be compared in this paper are: PID (state of the art of classical control), Fuzzy Logic (one of the best of intelligent control), and Linier Quadratic Regulator (LQR) that is the state of the art of optimal control.

Experiment was done for the unit step response, disturbance rejection and tracking where the performance and energy controller were observed based on MATLAB simulink platform.

This paper is organized as follows. In section II, present the mathematical model of the DC motor. In section III, Controllers are designed. In section IV, MATLAB simulation results are presented and discussed. Finally the conclusion is in section V.

II. DC MOTOR MODEL

DC motor model is shown in Fig. 1. The motor torque (*T*) is connected to the motor armature by an armature constant (K_t) whiles the back electromotive force (*EMF*) is related to the rotational velocity with motor constant(K_e) shown in Equation (1).

$$\begin{cases} T = K_t i \\ EMF = K_e \dot{\theta} \end{cases} \tag{1}$$

In SI units K_t is equal to K_e so it is assumed that $K_t = K_e = K$.

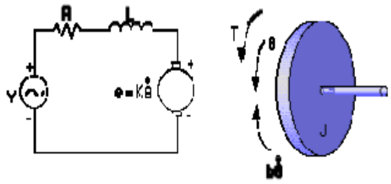

Figure 1 DC Motor Model[10]

The mathematical model of Fig. 1 is decomposed into two parts, electrical parts and mechanical parts. By using Kirchoff's law for electrical parts, and Newton's law for mechanics, we get Equation (2) and Equation (3).

$$L\frac{di}{dt} + Ri = V - K\dot{\theta} \tag{2}$$

$$J\ddot{\theta} + b\dot{\theta} = Ki \tag{3}$$

Using Laplace Transforms, Equation (2) and (3) becomes Equation (4) and (5) respectively.

$$(Ls + R)I(s) = V - Ks\theta(s) \tag{4}$$

$$(Js + b)s\theta(s) = KI(s) \tag{5}$$

By eliminating *I(s)*, the open loop transfer function with rotational speed as output and voltage as input is shown in Equation (6).

$$\frac{\theta}{V} = \frac{K}{(Js+b)(Ls+R) + K^2} \tag{6}$$

In state-space form, Equation (4) and (5) can be expressed by choosing the rotational speed and electrical current as the state variables and the voltage as an input, Equation (7). The output is chosen to be the rotational speed, Equation (8).

$$\begin{bmatrix} \ddot{\theta} \\ \dot{I} \end{bmatrix} = \begin{bmatrix} -\frac{b}{J} & \frac{K}{J} \\ -\frac{K}{L} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ I \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} V \tag{7}$$

$$\dot{\theta} = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ I \end{bmatrix} \tag{8}$$

According S.J. Chapman [11], there are three methods that are widely used in the DC motor speed control, field resistance control, armature voltage control, and armature resistant control. In this study, armature voltage control was used so that a review of energy can be done by measuring the

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applied voltage to a DC motor. The instantaneous power (p) is proportional to V^2 as

$$p = cV^2 . \tag{9}$$

Where c is constant that comes from the impedance of the motor and V is controller output or voltage input to the motor. The total energy (w) at time t, can be calculated as

$$w = \int p \ dt = c \int V^2 \ dt \tag{10}$$

Performance indices are calculated and used to evaluate the performance of the system. In this paper, ISE (integral square error), Equation (11), and energy (w) were used. Because we use the same DC motor, so in energy comparative we just compare $\int V^2 \ dt$ value.

$$ISE = \int e(t)^2 dt \tag{11}$$

Where e, error, that is the difference between set-point and system output.

For this research we refer the physical parameter from [10] as follow:

Table 1 DC Motor Physical Parameter

Symbol	Quantity	Value
J	Moment inertia of the motor	0.01 kg.m^2/s^2
b	Damping ratio of the mechanical system	0.1 Nms
K	Electromotive Force Constant	0.01 Nm/A
R	Electric resistance	1 Ω
L	Electric inductance	0.5 H
V	Source Voltage	Input
$\boldsymbol{\theta}$	Position of shaft	Output

III. Controller Design

A. Proportional-Integral-Derivative(PID) Controller

PID controller is the most extensively control algorithm used in industry control scheme [12]. PID algorithm is widely used because it is simple and easy to apply [13][14][15][16]. Performance is determined by the determination of PID parameter that is K_P, K_I, and K_D. In this paper, PID parallel structure was used as Equation (12) and the parameters were determined using MATLAB automatic tuning.

$$y(t) = (K_P * e(t)) + (K_I * \int e(t)dt) + \left(K_D * \frac{de(t)}{dt}\right) \tag{12}$$

B. Fuzzy Logic Controller

Numerous successful applications of fuzzy logic controller in industry make this branch of intelligent control so many credits and respect in control community[17]. The main advantage of this method is, it is not require precise mathematical models of plant[18][19][20]. The controller only needs the output values from sensors along with the bounds on the control signal which can be applied to the system.

The fuzzy logic controller that used is Mamdani type. This controller had two inputs, namely error (e) and change of error (ce), and one output that was controller signal. Triangular membership functions were used in the design with five membership limit (NB, NS, ZE, PS, PB) for both input and output. Fig. 2 shows the membership function of error (the first input), the second input (*ce)* had the same shape, while Fig. 3 shows the membership function of output. The rules were designed as in Table 2.

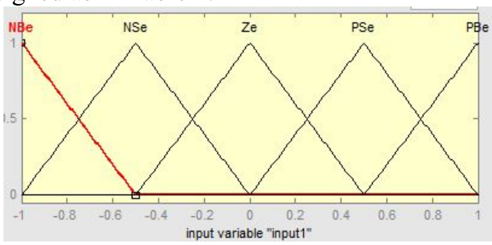

Figure 2 Input Membership

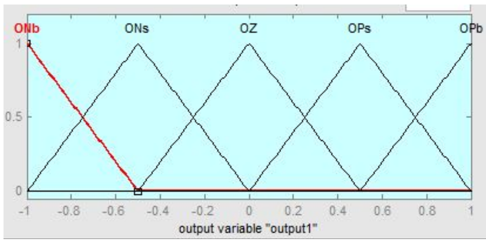

Figure 3 Output Membership

Table 2 Matrix Rules

CE\E	NB	NS	ZE	PS	PB
PB	NB	NS	ZE	PS	PB
PS	NS	ZE	ZE	ZE	PS
ZE	NB	ZE	ZE	ZE	PB
NS	NS	ZE	ZE	ZE	PS
NB	NB	NS	ZE	PS	PB

C. Linier Quadratic Regulator(LQR) Controller

LQR is one of the optimal control techniques which take into account the states of the dynamical system and control input to make optimal control decisions. This is simple as well as robust [21]. Suppose the state equation of the linier time-invariant system is:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{13}$$

$$y(t) = Cx(t) + Du(t) \tag{14}$$

The state feedback control $u(t) = -K_{LQR}\,x(t)$ leads to

$$\dot{x(t)} = \left(A - BK_{LQR}\right)x(t) \tag{15}$$

where K_{LQR} is derived from minimization of cost function as shown in (16).

$$J = \int (x^T(t)Qx(t) + u^T(t)Ru(t))dt \tag{16}$$

Q and R denote the weighting matrix of states variable and input variable where $Q = Q^T \geq 0$(positive semi-definite) and $R = R^T > 0$ (positive definite).

The LQR gain vector K_{LQR} is given by

$$K_{LQR} = R^{-1}B^T P \tag{17}$$

Where $P(t)$ is a positive definite symmetric constant matrix obtained from the solution of matrix algebraic riccatti equation(ARE) as in Equation(18). Q and R matrix were determined using experiment method, while K_{LQR} was determined using MATLAB command K_{LQR} = *lqr(A, B, Q, R).*

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{18}$$

IV. SIMULATION RESULTS

A. Unit Step Response

Fig. 4 shows the performance comparison in unit step response. LQR controller has the fastest response time, while Fuzzy has the slowest response. PID has fast response too, but it has too much overshoot. LQR doesn’t have an overshoot while Fuzzy has a small overshoot. As shown in Table 3, LQR has the lowest value of ISE. Based on the performance criteria, LQR has the best rank in unit step response.

The next step is comparison in energy controller criteria. Fig. 5 shows the energy controller comparison in unit step response. At starting time PID has the highest energy while Fuzzy has the lowest, and after 7 s (sampling time is 0.01 s), PID and Fuzzy have the same output energy. LQR has the highest energy in starting time too, but it’s still lower than PID and Fuzzy needed in normal operation time (after 7 s). As shown in Table 3, LQR has the lowest $\int V^2\,dt$ value. Based on energy controller criteria, LQR is the most efficient in unit step response and also with the best response.

Table 3 ISE and *w* in Unit-step Response

Controller	**ISE**	$\int V^2\,dt$
PID	30.635	98564.53
Fuzzy	133.471	73215.06
LQR	20.756	955.691

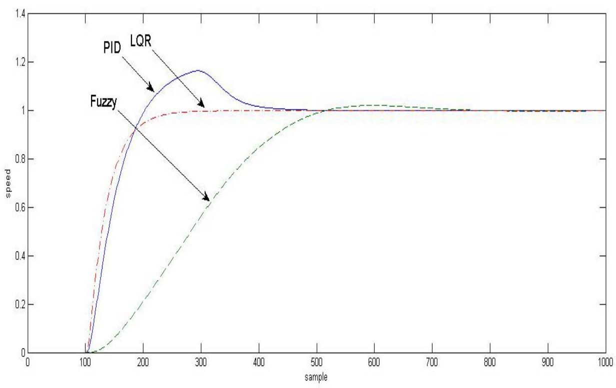

Figure 4 Performance Comparison in Unit-step Response

B. Disturbance Rejection

Fig. 6 shows the performance comparison in disturbance rejection, in which LQR controller has very small oscillation when the disturbances enter the system, while Fuzzy has the higher amplitude oscillation. PID has the oscillation but the amplitude is smaller than Fuzzy, and the settling time is faster than Fuzzy. From ISE value, Table 4, LQR has the smallest value. Concerning the disturbance rejection, LQR is the best performance because of no oscillation when disturbance happen.

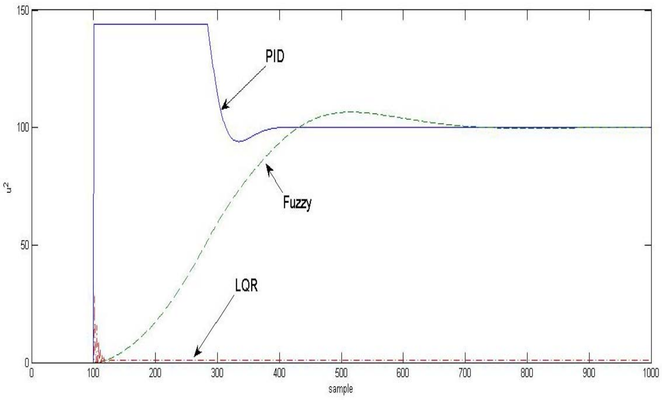

Figure 5 Energy Controller Comparison in Unit-step Response

Based on energy controller criteria, Fig. 7, at starting time PID has the highest energy while Fuzzy has the lowest. When disturbance happen, LQR need a small energy to cover it, while PID and Fuzzy need a lot. From Table 4, LQR has the smallest value of overall energy controller. PID has the highest $\int V^2\,dt$ value, because we also count the starting time energy consumption, but if we concern to the energy consumption in the disturbance rejection, Fig. 7, we can see Fuzzy consume more energy than PID in the disturbance rejection.

Table 4 ISE and *w* in Disturbance Rejection

Controller	**ISE**	$\int V^2\,dt$
PID	30.721	148708.29
Fuzzy	133.997	12346.995
LQR	20.756	1460.020

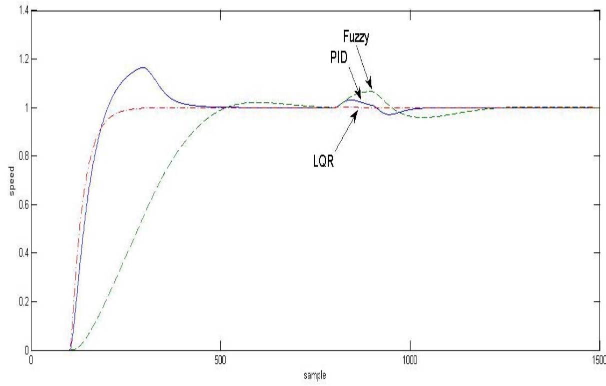

Figure 6 Performance Comparison in Disturbance Rejection

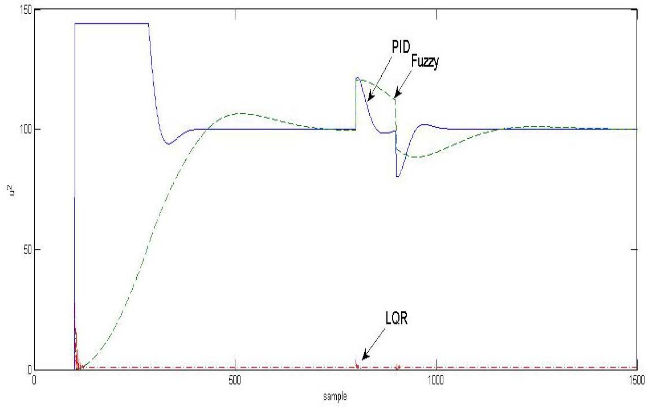

Figure 7 Energy Controller Comparison in Disturbance Rejection

C. Signal Tracking

The last experiment is tracking. Fig. 8 shows the performance comparison in tracking. It is shown that PID can track the signal although overshoot happen, Fuzzy has the worst result in tracking, and LQR can track the signal well with no overshoot. Based on Table 5, LQR has the smallest ISE so it is concluded that LQR has the best performance in signal tracking.

Comparing in energy controller criteria, Fig. 9 shows that LQR has the smallest controller energy signal. PID has higher energy when the speed increased than decreased. Fuzzy consume higher energy than LQR but lower than PID. It also has the smallest value of $\int V^2\,dt$ as in Table 5. Thus, LQR is the most efficient controller in signal tracking.

Table 5 ISE and *w* in Signal Tracking

Controller	**ISE**	$\int \boldsymbol{V^2\,dt}$
PID	49.984	70920.238
Fuzzy	234.564	39797.913
LQR	41.507	679.780

I. CONCLUSIONS

Three control algorithms for the DC motor speed control have been designed and implemented. Their performances have been compared on the basis of unit-step response, disturbance rejection and signal tracking. Performance and control energy requirement were used as the comparison criteria, but the most concern was in energy. Experiment results show that LQR is the most optimal energy controller compare with PID and Fuzzy.

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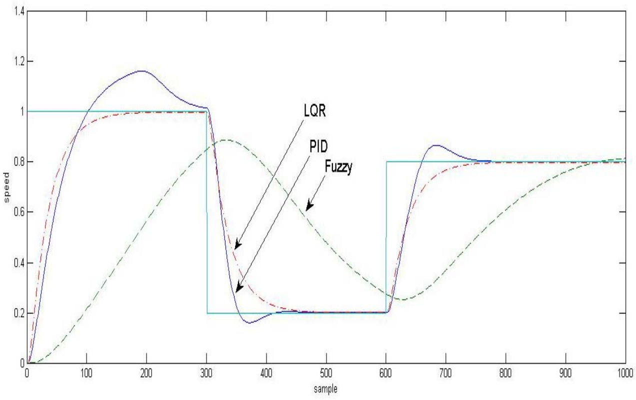

Figure 8 Performance Comparison in Signal Tracking

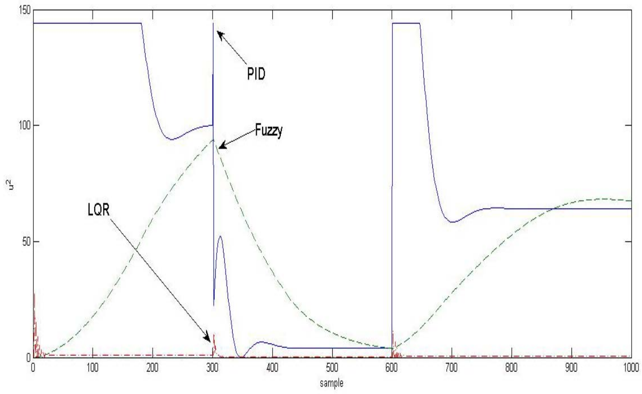

Figure 9 Energy Controller Comparison in Signal Tracking

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