Robust H_∞ Optimal Speed Control of DC Motor Using LMI Approach

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Abstract: A robust H_∞ optimal speed control scheme for a DC motor with parameters variations and disturbance torque using a linear matrix inequality (LMI) approach is presented in this paper, the stability and robust H_∞ performance index of the linear uncertain system are analyzed. The simulation results show that the closed-loop system has asymptotical stability, strong interference suppression ability and optimum performance.

Key Words: DC Motor, Speed Control, Robustness, Robust H_∞ Optimal Control, LMI

1 INTRODUCTION

DC motors and and their control drives have been widely used in various industrial processes and home appliances, such as electric wheelchairs, robotics, rolling mills, machine tools, and so on, many applications require very precise control of speed. However DC motors are unstable in their operation because system parameters may be time varying. These variations are likely due to the current sense accuracy, increase in temperature and changes in operating conditions, and to other sensor errors. In recent years, many researches have been studying the various new control techniques in order to improve speed regulation of DC motor system performance, such as digital control technique [3], adaptive variable structure control [4], optimal PID control [5], self-tuning artificial neural network control [6]. In [2], a adaptive control is presented to against machine parameter variations and maintain good performance even the DC motor at low speed. In [7], based on switched quadratic regulators, an efficient DC motor speed controller is presented and its performance robustness has been proved particularly well in spite of large parametric uncertainties.

Robust control theories have been well developed and widely applied to discuss the problems of DC motor speed control system design [8] [9] [10] [11] [12]. In this paper, we propose a robust H_∞ optimal control using LMI approach for the DC Motor Control Trainer(DCMCT) system which is based on a linear DC motor with parameters variations. Despite their complexity, robust H_∞ controllers have gained popularity due to their performance and robustness, and the robust H_∞ optimal control theory is better than conventional robust H_∞ control and PID technique, because of the low robustness of PID controller and no optimal performance of common robust H_∞ control theory.

This paper is organized as follows. In section 2, we present introduction and mathematical model of the DCMCT system. In section 3, we describe the system as a class of a linear uncertain system and give its definition. Section 4 provides the robust H_∞ optimal controller applied to our system. Simulation results of the control system are illustrated in section 5, and a conclusion is drawn in section 6.

2 MODELLING

In this section, the description and mathematical model of DCMCT are presented.

2.1 The DC Motor Control Trainer

The Quanser's DCMCT is a system which makes it possible to demonstrate motor servo control theory by practice in a variety of ways [1]. The DCMCT hardware consists of the board shown in Figure 1. The most evident feature of the DCMCT is the wheel attached to a DC motor with encoder, the wheel delivers an inertial load for experiments. The DC motor is driven by a linear power amplifier, and power to the system is delivered from a wall transformer. The interface to a PC or laptop is through a serial port connection. Control is performed using the DSP or the PC, and the controller is either hand coded or designed using commercially available design tools such as Simulink.

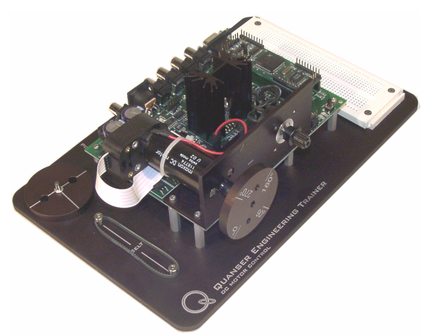

Figure 1: The DC Motor Control Trainer

Table 1: Nominal Values and Variations of the Parameters

Symbol	Nominal Value	Unit	Variation
R_m	10.6	Ω	$\pm 10\%$
k_m	0.05	$N.m/A$	$\pm 5\%$
L_m	0.82	mH	$\pm 10\%$
J_{eq}	0.00002	$kg.m^2$	-

2.2 DCMCT Model

Figure 2 represents the classic schematic of the armature circuit of a standard DC motor. In the following, the mathematical model for the DC motor system is derived through first principles.

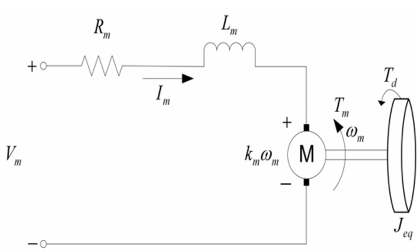

Figure 2: DC Motor Electric Circuit

From the Figure 2 ,we can determine the electrical relationship characterizing a standard DC motor. Using Kirchhoffs voltage law, we obtain the following equation

$$V_m = R_m I_m + k_m \omega_m + L_m \frac{dI_m}{dt} \tag{1}$$

where V_m denotes the voltage from the amplifier which drives the motor, R_m is motor armature resistance, I_m is motor armature current, k_m is motor torque coefficient, L_m is motor armature inductance.

Neglecting the friction in the system, the mechanics of the motor rotor with the attached inertial wheel is given by Newtons second law of motion expressing conservation of angular momentum. This is expressed below

$$J_{eq} \frac{d\omega_m}{dt} = k_m I_m + T_d \tag{2}$$

where the equivalent moment of inertia of motor rotor and the load is expressed by $J_{eq} = J_m + J_l$, J_m is the moment of inertia of motor rotor, J_l is the moment of inertia of inertial load. From formula (1) and formula (2), the state equation and the output equation for the DC motor are represented as follows

$$\begin{bmatrix} \dot{I_m} \\ \dot{\omega_m} \end{bmatrix} = \begin{bmatrix} -\frac{R_m}{L_m} & -\frac{k_m}{L_m} \\ \frac{k_m}{J_{eq}} & 0 \end{bmatrix} \begin{bmatrix} I_m \\ \omega_m \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{J_{eq}} \end{bmatrix} T_d + \begin{bmatrix} \frac{1}{L_m} \\ 0 \end{bmatrix} V_m$$
$$z = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} I_m \\ \omega_m \end{bmatrix} \tag{3}$$

The nominal values and variations of the parameters are listed in Table 1.

3 SYSTEM DESCRIPTION AND DEFINITION

In this section, we discuss the problem of robust H_∞ optimal control for the system with system matrix and input matrix uncertainties. So, we firstly describe the DCMCT system (3) as a class of form of uncertain linear system.

Let $x_1 = I_m$, $x_2 = \omega_m$, $T_d = w(t)$, $u(t) = V_m$, because k_m, R_m and L_m are time-varying parameters, then formula (3) is rewritten taking account of parameter variations as follows

$$\dot{x}(t) = (A + \Delta A(t))x(t) + B_1 w(t) + (B_2 + \Delta B_2(t))u(t)$$
$$z(t) = C_1 x(t) \tag{4}$$

where

$$A + \Delta A = \begin{bmatrix} -\frac{R_m}{L_m} & -\frac{k_m}{L_m} \\ \frac{k_m}{J_{eq}} & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ \frac{1}{J_{eq}} \end{bmatrix}$$
$$B_2 + \Delta B_2 = \begin{bmatrix} \frac{1}{L_m} \\ 0 \end{bmatrix}, C_1 = \begin{bmatrix} o & 1 \end{bmatrix} \tag{5}$$

$x(t) \in R^2$ is the state which can be denoted as $x(t) = \begin{bmatrix} x_1(t) & x_2(t) \end{bmatrix}^T$, $u(t)$ is the control input, $w(t)$ is the disturbance input, $z(t)$ is the control output, A, B_1, B_2 and C_1 are known constant matrices, $\Delta A(t)$ and $\Delta B_2(t)$ are matrices representing time-varying uncertainties, and assume they have the following forms

$$\begin{bmatrix} \Delta A(t) & \Delta B_2(t) \end{bmatrix} = DF(t) \begin{bmatrix} E_1 & E_2 \end{bmatrix} \tag{6}$$

where the matrix D, E_1, E_2 are known real constant matrices, $F(t)$ is uncertain matrix function satisfying the set Ω which defined as following

$$\Omega = \{F(t) | F^T(t)F(t) \le I, \ \forall t\} \tag{7}$$

In the following, we study the problem of robust H_∞ optimal control for system (4) when existing disturbance $w(t)$. So, we introduce the performance index of the system (4), which is defined

$$J = \int_0^\infty [x^T(t)Qx(t) + u^T(t)Ru(t)]\mathrm{d}t \tag{8}$$

where Q and R are given positive-definite symmetric weighting matrices.

Definition 1: Let $\gamma > 0$ be a given constant, for the uncertain linear system (4) and its corresponding performance index (8), if there exists a state feedback $u(t) = -Kx(t)$, for all admissible uncertainties of system such that the following three conditions hold (1) The closed-loop system is asymptotically stable;

(2) The closed-loop system is optimal in the linear quadratic(LQ) sense;

(3) When initial condition $x(0) = 0$, the H_∞ norm $\|T_{zw}(s)\|_\infty$ of the transfer function $T_{zw}(s)$ which is from external disturbance input $w(t)$ to output $z(t)$ satisfying $\|T_{zw}(s)\|_\infty < \gamma$.

Then the system (4) is robust H_∞ optimal.

Our purpose in this paper is to construct a state feedback $u(t) = -Kx(t)$, where K denotes feedback gain, for DC motor control system with uncertainties, such that the closed-loop system is robust H_∞ optimal.
Substituting $u(t) = -Kx(t)$ into (4) yields the following uncertain linear closed-loop system

$$\begin{aligned} \dot{x}(t) &= (A - B_2K + DF(E_1 - E_2K))x(t) + B_1w(t) \\ z(t) &= C_1x(t) \end{aligned} \tag{9}$$

its performance index is

$$J = \int_0^\infty [x^T(t)(Q + K^TRK)x(t)]\mathrm{d}t \tag{10}$$

4 ROBUST H_∞ OPTIMAL CONTROLLER DESCRIPTION AND DESIGN

In this section, we give the design of robust H_∞ optimal control using LMI method for uncertain linear system (4) when existing disturbance $w(t)$.
Theorem 1 : Let $\gamma > 0$ be a given constant, for uncertain linear system (4) and its performance index (8), if there exists $P = P^T > 0$, such that the following LMI holds

$$\begin{aligned} &(A + \Delta A - B_2K - \Delta B_2K)^TP + P(A + \Delta A - B_2K \\ &-\Delta B_2K) + Q + K^TRK + \gamma^{-2}PB_1B_1^TP + C_1^TC_1 < 0 \end{aligned} \tag{11}$$

then the system (4) is robust H_∞ optimal via the feedback $u(t) = -Kx(t)$.
Proof: Consider Lyapunov function $V(x(t)) = x^T(t)Px(t)$, we then have

$$\begin{aligned} \dot{V}(x(t)) &= \dot{x}^T(t)Px(t) + x(t)P\dot{x}^T(t) \\ &= x^T(t)[(A + \Delta A - B_2K - \Delta B_2K)^TP \\ &\quad + P(A + \Delta A - B_2K - \Delta B_2K)]x(t) \end{aligned}$$

From formula (11), we get

$$\begin{aligned} \dot{V}(x(t)) < -x^T(t)(Q + K^TRK + \gamma^{-2}PB_1B_1^TP \\ +C_1^TC_1)x(t) < 0 \end{aligned} \tag{12}$$

Therefore, the closed loop system (9) is asymptotically stable.
The next we proof the closed-loop system (4) has the following robust H_∞ performance index via feedback $u(t) = -Kx(t)$ when the initial condition $x(0) = 0$

$$\Gamma_{zw} = \frac{z^Tz}{w^Tw} < \gamma \tag{13}$$

Noticing that formula (8) is equivalent to

$$\int_0^\infty (z^Tz - \gamma^2w^Tw)\mathrm{d}t < 0 \tag{14}$$

So, we only proof the formula (14) holds. Considering that

$$\begin{aligned} &\int_0^\infty (z^Tz - \gamma^2w^Tw + \dot{V}(x(t)))\mathrm{d}t \\ = &\int_0^\infty (z^Tz - \gamma^2w^Tw)\mathrm{d}t + V(x(\infty)) - V(x(0)) \end{aligned}$$

and because the system is asymptotically stable, we have $V(x(\infty)) = 0$, also because the initial condition is $x(0) = 0$, so, a sufficient condition for the formula (14) is

$$z^Tz - \gamma^2w^Tw + \dot{V}(x(t) < 0 \tag{15}$$

that is also

$$\begin{aligned} z^Tz - \gamma^2w^Tw + \dot{V}(x(t) = x^T(t)[(A + \Delta A - B_2K \\ -\Delta B_2K)^TP + P(A + \Delta A - B_2K - \Delta B_2K)]x(t) \\ +w^T(t)B_1^TPx(t) + x^T(t)PB_1w(t) + x^T(t)C_1^TC_1x(t) \\ -\gamma^2w^Tw < 0 \end{aligned} \tag{16}$$

because

$$\begin{aligned} &w^T(t)B_1^TPx(t) + x^T(t)PB_1w(t) \\ < &\gamma^{-2}x^T(t)PB_1B_1^TPx(t) + \gamma^2w^T(t)w(t) \end{aligned}$$

a sufficient condition for the formula (16)holds is

$$\begin{aligned} x^T(t)[(A + \Delta A - B_2K - \Delta B_2K)^TP + P(A + \Delta A \\ -B_2K - \Delta B_2K)]x(t) + \gamma^{-2}x^T(t)PB_1B_1^TPx(t) \\ +x^T(t)C_1^TC_1x(t) < 0 \end{aligned} \tag{17}$$

According to formula (11), we know that formula (17) is tenable, from the above deduction, we know that, formula (17) is a sufficient condition for formula (13) holds, therefore, the system has robust H_∞ performance index. So, the corollary is proofed according to definition 1. So, the theorem is proofed according to definition 1.
Theorem 2 : Let $\gamma > 0$ be a given constant, for uncertain linear system (4) and its performance index (8), a sufficient condition for closed-loop system (9) robust H_∞ optimal is that exist a constant $\varepsilon > 0$, matrices $X = X^T > 0$ and Y, such that the following LMI holds

$$\begin{bmatrix} \Pi_1 & X & Y^T & B_1 & XC_1^T & \Pi_2 & D \\ * & -Q^{-1} & 0 & 0 & 0 & 0 & 0 \\ * & * & -R^{-1} & 0 & 0 & 0 & 0 \\ * & * & * & -\gamma^2I & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & -\varepsilon^{-1}I & 0 \\ * & * & * & * & * & * & -\varepsilon I \end{bmatrix} < 0 \tag{18}$$

where, $\Pi_1 = XA^T - Y^TB_2^T + AX - B_2Y$, $\Pi_2 = (E_1X - E_2Y)^T$. If formula (18) is tenable, then corresponding robust H_∞ optimal control law is

$$u(t) = -Kx(t) = -YX^{-1}x(t) \tag{19}$$

Proof: Because

$$\begin{aligned}(\Delta A-\Delta B_2K)^TP+P(\Delta A-\Delta B_2K)\\ =(E_1-E_2K)^TF^T(t)D^TP+PDF(t)(E_1-E_2K)\\ <\varepsilon(E_1-E_2K)^TF^T(t)F(t)(E_1-E_2K)+\varepsilon^{-1}PDD^TP\\ <\varepsilon(E_1-E_2K)^T(E_1-E_2K)+\varepsilon^{-1}PDD^TP\end{aligned}$$

therefore, a sufficient condition for formula (11) holds is that

$$\begin{aligned}(A-B_2K)^TP+P(A-B_2K)+Q+K^TRK\\ +\gamma^{-2}PB_1B_1^TP+C_1^TC_1+\varepsilon(E_1-E_2K)^T(E_1-E_2K)\\ +\varepsilon^{-1}PDD^TP<0\end{aligned}$$

the above LMI multiplied with P^{-1}, P^{-T} from the left and the right respectively, and noticing that $P^{-T}=P^{-1}$, we get

$$\begin{aligned}p^{-1}A^T-P^{-1}K^TB_2^T+AP^{-1}-B_2KP^{-1}+P^{-1}QP^{-1}\\ +P^{-1}K^TRKP^{-1}+\gamma^{-2}B_1B_1^T+P^{-1}C_1^TC_1P^{-1}\\ +P^{-1}\varepsilon(E_1-E_2K)^T(E_1-E_2K)P^{-1}+\varepsilon^{-1}DD^T<0\end{aligned}$$

Let $P^{-1}=X$, $KP^{-1}=Y$, then we have $K=YX^{-1}$, and we get

$$\begin{aligned}XA^T-Y^TB_2^T+AX-B_2Y+XQX+Y^TRY\\ +\gamma^{-2}B_1B_1^T+XC_1^TC_1X+\varepsilon(E_1X-E_2Y)^T\\ (E_1X-E_2Y)+\varepsilon^{-1}DD^T<0\end{aligned}$$

Using Schur complement lemma, we can get formula (18). In addition, because the formula (18) is a sufficient condition for formula (11) holds, therefore, according to theorem 1 and definition 1, we can get the above theorem. Proof is completed.

5 SIMULATION

According to the characteristic of the DCMCT system with parameters uncertainties, from system (4) and its parameters values and variations listed in table 1, we get

$$A=\begin{bmatrix}-12926.8293 & -60.9756\\ 2500 & 0\end{bmatrix}, B_1=\begin{bmatrix}0\\ 50000\end{bmatrix},$$
$$B_2=\begin{bmatrix}1219.5122\\ 0\end{bmatrix}, C_1=\begin{bmatrix}0 & 1\end{bmatrix}$$

we define the uncertain function matrix

$$F(t)=\begin{bmatrix}cos(t) & 0\\ 0 & cos(t)\end{bmatrix}$$

and select

$$D=I_2, E_1=\begin{bmatrix}2872.6287 & 10.1626\\ 125 & 0\end{bmatrix}, E_2=\begin{bmatrix}135.5014\\ 0\end{bmatrix}$$

We choose the weighting matrices $Q=diag\{400,1600\}$, $R=1$, and disturbance attenuation $\gamma=0.8$. Let $\varepsilon=0.1$, according to theorem 2, and by solving LMI (18), we obtain the robust H_∞ optimal controller

$$u(t)=-\begin{bmatrix}21.9432 & 0.0788\end{bmatrix}x(t) \quad (20)$$

Let the initial condition be $\begin{bmatrix}1 & 1\end{bmatrix}^T$, by applying the obtained controller to DCMCT system and using MATLAB/Simulink, we can get the state response curves, as shown in Figure 3, Figure 4.

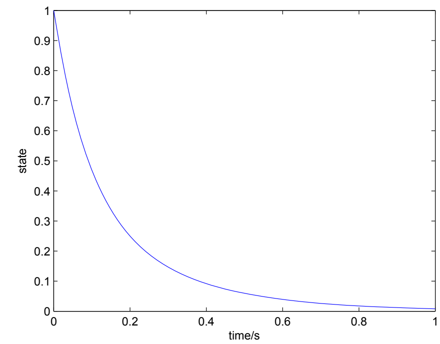

Figure 3: $x_1(t)$ State Response Curve

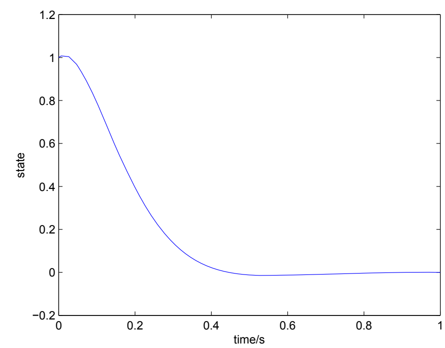

Figure 4: $x_2(t)$ State Response Curve

From Figure 3 and Figure 4, we can see that the closed-loop system is convergent, and it also illustrates fully that, the closed-loop system has strong ability of preventing interference and high stability using robust H_∞ optimal controller. This is because, in robust controller design stage, robust H_∞ had considered the parameters uncertainties and disturbance problems existing in the DCMCT system.

6 CONCLUSION

In this paper, we considered the parameters uncertainties and load torque disturbance in the DCMCT system, and gave its dynamic model. Using H_∞ control basic method, we studied robust H_∞ optimal control problem for the linear uncertain system, presented the controller design method and used LMI approach to reduce the complexity of solution. The simulation results show that the robust H_∞ optimal control system has better asymptotical stability, strong interference suppression ability and optimum performance.

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