1) Las respectivas ecuaciones de Lagrange son las siguientes:

$$(1)\,\frac{d}{dt}\left(\frac{\partial L_L}{\partial \dot{x}_1}\right)-\frac{\partial L_L}{\partial x_1}+\frac{\partial D}{\partial \dot{x}_1}=0$$

$$(2)\,\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right)-\frac{\partial L}{\partial \theta_2}+\frac{\partial D}{\partial \dot{\theta}_2}=T_m$$

**Lagrangiano:**

$$L=T-V$$

$$T=\frac{1}{2}m{\dot{x}_1}^2+\frac{1}{2}J\dot{\theta}_2^2$$

$$V=\frac{1}{2}k_1x_1^2+\frac{1}{2}k_2(\theta_2R-x_1)^2$$

$$D=\frac{1}{2}B{\dot{\theta}_2}^2+\frac{1}{2}b{\dot{x}_1}^2$$

**Empezar a derivar las respectivas ecuaciones de Lagrange**

Para $x_1$:

$$\boxed{m\ddot{x}_1+b\dot{x}_1+k_2(x_1-\theta_2R)+k_1x_1=0}\ (1)$$

Para $\theta_2$:

$$\boxed{J\ddot{\theta}_2+k_2R(\theta_2R-x_1)+B\dot{\theta}_2=T_m}\ (2)$$

Se seleccionan como estados las posiciones $x_1$ y $\theta_2$ y sus respectivas velocidades:

$$\boxed{\begin{bmatrix}\dot{\boldsymbol{x}}_1\\ \ddot{\boldsymbol{x}}_1\\ \dot{\boldsymbol{\theta}}_2\\ \ddot{\boldsymbol{\theta}}_2\end{bmatrix}=\begin{pmatrix}\mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0}\\ -\dfrac{\boldsymbol{k_1+k_2}}{\boldsymbol{m}} & -\dfrac{\boldsymbol{b}}{\boldsymbol{m}} & \dfrac{\boldsymbol{k_2R}}{\boldsymbol{m}} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1}\\ \dfrac{\boldsymbol{k_2R}}{\boldsymbol{J}} & \mathbf{0} & -\dfrac{\boldsymbol{k_2R^2}}{\boldsymbol{J}} & -\dfrac{\boldsymbol{B}}{\boldsymbol{J}}\end{pmatrix}\begin{bmatrix}\boldsymbol{x}_1\\ \dot{\boldsymbol{x}}_1\\ \boldsymbol{\theta}_2\\ \dot{\boldsymbol{\theta}}_2\end{bmatrix}+\begin{pmatrix}\mathbf{0}\\ \mathbf{0}\\ \mathbf{0}\\ \dfrac{\mathbf{1}}{\boldsymbol{J}}\end{pmatrix}\boldsymbol{T_m}}$$

**Para el G2:**

$$\boxed{\boldsymbol{y}=\begin{pmatrix}\mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0}\end{pmatrix}\begin{bmatrix}\boldsymbol{x}_1\\ \dot{\boldsymbol{x}}_1\\ \boldsymbol{\theta}_2\\ \dot{\boldsymbol{\theta}}_2\end{bmatrix}}$$

**Para el G4:** $x_2=\theta_2R$

$$\boxed{\boldsymbol{y} = \begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{R} & \mathbf{0} \end{pmatrix} \begin{bmatrix} \boldsymbol{x_1} \\ \dot{\boldsymbol{x}}_{\mathbf{1}} \\ \boldsymbol{\theta_2} \\ \dot{\boldsymbol{\theta}}_{\mathbf{2}} \end{bmatrix}}$$

Unidades de los parámetros:

$$k_1, k_2 \left[\frac{N}{m}\right]$$

$$b \left[\frac{Ns}{m}\right]$$

$$B \left[\frac{Nms}{rad}\right]$$

$$m[Kg]$$

$$J[Kgm^2]$$

$$R[m]$$

2) G2:

Unidades de los parámetros:

$$k_S \left[\frac{V}{rad}\right]$$

$$J_m, J_c[Kgm^2]$$

$$fc \left[\frac{Nms}{rad}\right]$$

$$k_b \left[\frac{Vs}{rad}\right]$$

$$k_i \left[\frac{Nm}{A}\right]$$

$$R_a[\Omega]$$

$$L_a[\mathrm{H}]$$

$$A, n_1, n_2 \; [adimensional]$$

Ecuaciones

$$v_1 = \theta_r K_s$$

$$v_2 = \theta_c K_s$$

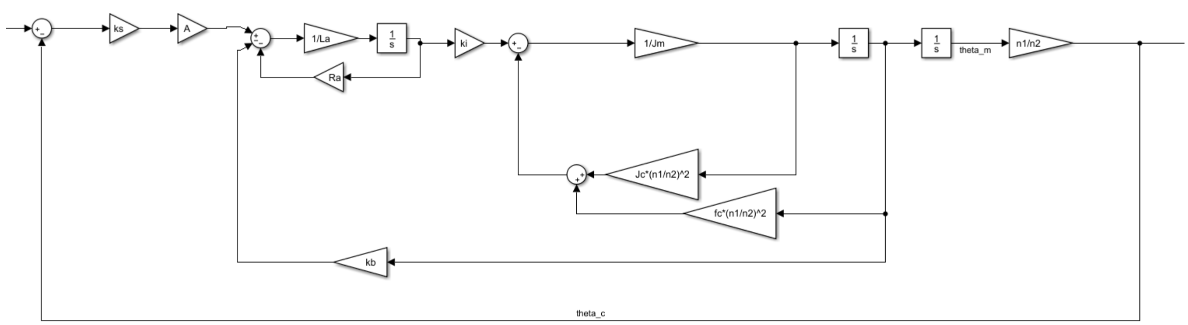
$$e_a = A(v_1 - v_2) = R_a i_a + L_a \frac{di_a}{dt} + k_b \dot{\theta}_m$$

$$J_m \ddot{\theta}_m = k_i i_a - T1$$

$$J_c\ddot{\theta}_c = T_2 - f_c\dot{\theta}_c$$

$$\frac{T_1}{T_2} = \frac{n_1}{n_2} = \frac{\theta_c}{\theta_m} = \frac{\dot{\theta}_c}{\dot{\theta}_m} = \frac{\ddot{\theta}_c}{\ddot{\theta}_m}$$

A partir de esto se plantea el siguiente diagrama de bloques:




Simplificando el diagrama de bloques, la función de transferencia es la siguiente:

$$\boxed{\boldsymbol{H(s) = \frac{\theta_c(s)}{\theta_r(s)} = \frac{\left(\frac{n_1}{n_2}\right)k_i k_s A}{J_e L_a s^3 + s^2\left(L_a f_c \left(\frac{n_1}{n_2}\right)^2 + R_a J_e\right) + s\left(R_a f_c \left(\frac{n_1}{n_2}\right)^2 + k_i k_b\right) + \left(\frac{n_1}{n_2}\right)k_i k_s A}}}$$

Donde

$$J_e = J_m + J_c\left(\frac{n_1}{n_2}\right)^2$$

**G4:**

Unidades de los parámetros:

$$J_m[Kgm^2]$$

$$b_m\left[\frac{Nms}{rad}\right]$$

$$b_G\left[\frac{Nms}{rad}\right]$$

$$k_b\left[\frac{Vs}{rad}\right]$$

$$k_i\left[\frac{Nm}{A}\right]$$

$$R_a[\Omega]$$

$$R[m]$$

$$M[kg]$$

$$n_1, n_2\ [adimensional]$$

Ecuaciones

$$e_a = R_a i_a + k_b \dot{\theta}_m$$

$$J_m \ddot{\theta}_m = k_i i_a - b_m \dot{\theta}_m - T1$$

$$T_2 - b_G\left(\dot{\theta}_2 - \dot{\theta}_G\right) = 0$$

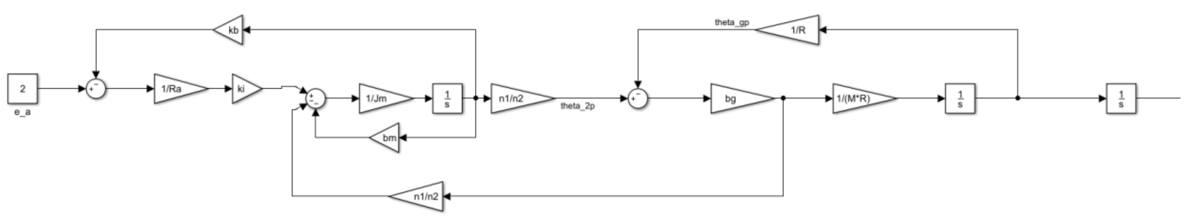
$$-b_G\left(\dot{\theta}_G - \dot{\theta}_2\right) - FR = 0$$

$$M\ddot{x} = F$$

$$\frac{T_1}{T_2} = \frac{n_1}{n_2} = \frac{\theta_2}{\theta_m} = \frac{\dot{\theta}_2}{\dot{\theta}_m} = \frac{\ddot{\theta}_2}{\ddot{\theta}_m}$$

$$R\theta_G = x, R\dot{\theta}_G = \dot{x}, R\ddot{\theta}_G = \ddot{x}$$

A partir de estas ecuaciones se plantea el siguiente diagrama de bloques




Simplificando el diagrama de bloques, la función de transferencia es la siguiente:

$$\boxed{\boldsymbol{H(s) = \frac{X(s)}{E_a(s)} = \frac{\left(\frac{n_1}{n_2}\right) k_i R b_G}{J_m M R^2 R_a s^3 + s^2\left(R_a\left(MR^2 b_m + J_m b_G + \left(\frac{n_1}{n_2}\right)^2 MR^2 b_G\right) + k_i k_b M R^2\right) + s\left(R_a b_m b_G + k_b k_i k_G\right)}}}$$

3)

```
clc
clear
close all

%Punto 1
k1=1;
k2=1;
```

```
b=1;
m=1;
B=1;
R=1;
J=1;
%matrices
A=[0 1 0 0;
  -(k1+k2)/m -b/m k2*R/m 0;
  0 0 0 1;
  k2*R/J 0 -k2*R^2/J -B/J];
Bs=[0 0 0 1/J]';
C=[1 0 0 0; 0 0 1 0]; %Grupo 2
D=[];
sisG2=ss(A,Bs,C,D);
C=[1 0 0 0; 0 0 R 0]; %Grupo 4
sisG4=ss(A,Bs,C,D);

step(sisG2)
figure
step(sisG4)

%Punto2 G2
ks=1;
ki=0.68;
kb=0.68;
A=200;
Ra=5;
La=0.1;
Jm=0.00136;
Jc=0.136;
fc=0.136;
n1=1;
n2=10;

num=ki*ks*A*(n1/n2);
den=[(Jm*La+La*Jc*(n1/n2)^2)
(La*fc*(n1/n2)^2+((n1/n2)^2*Jc+Jm)*Ra)
(Ra*fc*(n1/n2)^2+ki*kb) ki*ks*A*(n1/n2)];
figure
step(num,den)

%Punto2 G4

ki=1;
```

```
kb=1;
Ra=1;
Jm=1;
M=1;
R=2;
bg=4;
bm=1;
n2=10;
n1=1;

num=ki*bg*R*(n1/n2);
den=[(Jm*M*R^2*Ra)
(Ra*(M*R^2*bm+Jm*bg+M*R^2*bg*(n1/n2)^2)+ki*kb*M*R^2)
(Ra*bm*bg+kb*bg*ki) 0];
h=tf(num,den);
figure
step(h)
```