4

Application to a Three-tank System

4.1 Introduction

In this chapter, a hydraulic system that can be used for water treatment or storing liquids in many industrial plants is considered. During these processes, chemical reactions are supposed to occur around pre-defined operating points. Therefore, the liquid levels control in a plant is crucial in order to provide desired specifications. Using a prototype of a hydraulic system, researchers have successfully tested various methods of linear or nonlinear decoupling control and model-based fault diagnosis.

The three-tank system considered in this study is a popular laboratory-scale system designed by [4]. It is used in order to investigate linear, nonlinear multivariable feedback control as well as FDI and FTC system design. Koenig *et al*. [82] have synthesized a decoupled linear observer to detect and to isolate actuator and component faults (pipe, tank, etc.) around an operating point without fault magnitude estimation. Based on the nonlinear model, [112] have designed an observer using the bilinear model representation of the three-tank system to detect a leakage from a pipe. A diagnostic system based also on a bilinear model has been considered in [7], where time varying innovation generators combined with generalized likelihood ratio tests are designed to detect and isolate faults. The robustness of a sliding mode observer (SMO) to detect faults in the presence of noise on the measurements was tested in real-time [105]. Among model-based approaches, a differential geometric method has been successfully applied in [76]. Rather than considering a complex nonlinear model, [1] have estimated the state vector based around various operating points through a bank of decoupled observers to generate residuals for fault detection. More recently, [106] have proposed to develop a bank of decoupled observers to detect and isolate actuator/sensor faults around multiple operating points applied to the three-tank system. FDI methods based on fuzzy or neural models have also been illustrated on the three-tank system to detect and isolate faults [83, 91, 93]. In [115] the authors deal with the FDI problem of plants with unknown description.

In the FTC framework, the three-tank system has been considered as a benchmark. In the presence of sensor faults, [143] estimates the fault magnitude based on an adaptive filter with an on-line parameter estimation method developed by [142]. The sensor fault estimation is used for sensor fault masking. In [25] an effective low-order tuner for FTC of a class of unknown nonlinear stochastic sampled-data systems is proposed. The strategy is based on the modified state-space self-tuning control via the observer/Kalman filter identification method. Weighted fuzzy predictive control is used for FTC of an experimental three-tank system [95]. Furthermore, a European project "COSY" (control of complex systems) [62] has considered the three-tank system as a benchmark for all partners under the assumption: two tanks are active and the last one is used as a redundant process.

While various FDI and FTC approaches in the literature have been applied separately to the three-tank system, this chapter aims at presenting a complete approach in order to present and illustrate the application of the methods developed in Chap. 2 to this system. This is investigated in the linear case around an operating point as first presented by [121] but also on the whole operating zone using nonlinear techniques. A complete simulation platform of the three-tank system, in closed-loop, with or without actuator and sensor faults, is provided for the use of the reader via download from www.springer.com/978-1-84882-652-6 as described in the Appendix.

4.2 System Description

The considered hydraulic system is presented in Fig. 4.1.

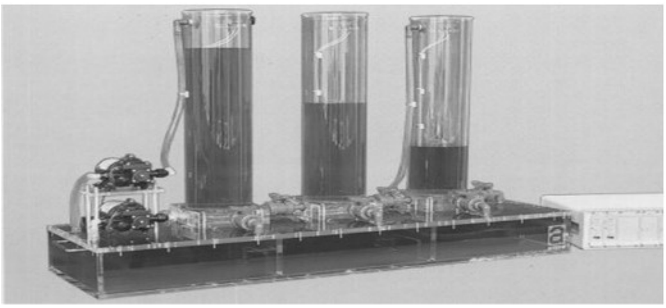

Fig. 4.1. Three-tank system

The hydraulic system consists of three identical cylindrical tanks with equal cross-sectional area S (Fig. 4.2). These three tanks are connected by two cylindrical pipes of the same cross-sectional area, denoted S_p, and have the same outflow coefficient, denoted μ_{13} and μ_{32}. The nominal outflow located at tank 2 has the same cross-sectional area as the coupling pipe between the cylinders but a different outflow coefficient, denoted μ_{20}. Two pumps driven by DC motors supply the first and last tanks. Pumps flow rates (q_1 and q_2) are defined by flow per rotation. A digital/analog converter is used to control each pump. The maximum flow rate for pump i is denoted q_{imax}. A piezo-resistive differential pressure sensor carries out the necessary level measurement. Three transducers deliver voltage signal levels. The variable ℓ_j denotes the level in tank j and ℓ_{jmax}, the associated maximum liquid level.

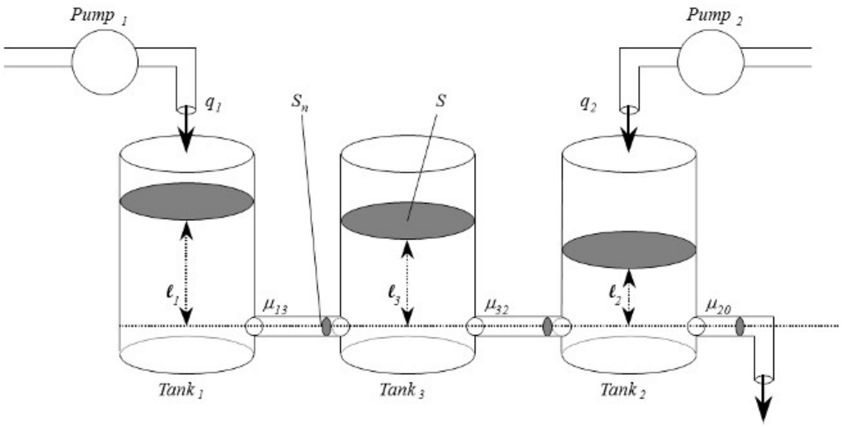

Fig. 4.2. Synoptic of the three-tank system

The system can be described by the following mass balance equations:

$$\begin{cases} S\dfrac{d\ell_1(t)}{dt} = q_1(t) - q_{13}(t) \\ S\dfrac{d\ell_2(t)}{dt} = q_2(t) + q_{32}(t) - q_{20}(t) \\ S\dfrac{d\ell_3(t)}{dt} = q_{13}(t) - q_{32}(t) \end{cases}, \tag{4.1}$$

where q_{mn} represents the flow rate from tank m to n ($m, n = 1, 2, 3\ \forall m \neq n$) which, based on the Torricelli law, is equal to

$$q_{mn}(t) = \mu_{mn} S_p sign(l_m(t) - l_n(t))\sqrt{2g \mid l_m(t) - l_n(t) \mid}, \tag{4.2}$$

and q_{20} represents the outflow rate described as follows:

$$q_{20}(t) = \mu_{20} S_p \sqrt{2gl_2(t)}. \tag{4.3}$$

The numerical values of the plant parameters are listed in Table 4.1.

Table 4.1. Parameters value of the three-tank system

Variable	Symbol	Value
Tank cross sectional area	S	$0.0154\ m^2$
Inter tank cross sectional area	S_p	$5 \times 10^{-5}\ m^2$
Outflow coefficient	$\mu_{13} = \mu_{32}$	0.5
Outflow coefficient	μ_{20}	0.675
Maximum flow rate	$q_{imax}(i \in [1\ 2])$	$1.2 \times 10^{-4}\ m^3 s^{-1}$
Maximum level	$l_{jmax}(j \in [1\ 2\ 3])$	$0.62\ m$

4.3 Linear Case

4.3.1 Linear Representation

Under the assumption $\ell_1 > \ell_3 > \ell_2$, a linear model can be established around an equilibrium point $(U_0,\ Y_0)$. The system is linearized around this operating point using Taylor expansion. The linearized system is described by a discrete LTI representation with a sampling period $T_s = 1\ s$:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ \quad y(k) = Cx(k) \end{cases}, \tag{4.4}$$

where y and u represent variations around an operating point defined by the pair (U_0, Y_0).

The purpose of this study is to control the system around the operating point (U_0, Y_0), which is fixed to

$$\begin{cases} Y_0 = \begin{bmatrix} 0.40 & 0.20 & 0.30 \end{bmatrix}^T (m) \\ U_0 = \begin{bmatrix} 0.35 \times 10^{-4} & 0.375 \times 10^{-4} \end{bmatrix}^T (m^3/s) \end{cases}. \tag{4.5}$$

In order to generate matrices A and B, the following program is written in MATLAB® code:

```
% Parameters value of three-tank system
    mu13=0.5; mu20=0.675; mu32=0.5;
    S=0.0154; Sn=5e-5; W=sqrt(2*9.81);

% Output operating Points (m)
    Y10=0.400; Y20=0.200; Y30=0.300;

% Input operating Points (m3/s)
    U10=0.350e-004; U20=0.375e-004;

% Matrix A
    A11=-(mu13*Sn*W)/(2*S*sqrt(Y10-Y30)); A12=0;
    A13=-A11;
```

```
    A21=0;A23=(mu32*Sn*W)/(2*S*sqrt(Y30-Y20));
    A22=-A23-((mu20*Sn*W)/(2*S*sqrt(Y20)));
    A31=-A11;A32=A23;A33=-A32-A31;
    A=[A11 A12 A13; A21 A22 A23; A31 A32 A33];

% Matrix B
    B11=1/S;B12=0;
    B21=0;B22=1/S;
    B31=0;B32=0;
    B=[B11 B12;B21 B22;B31 B32];

% Continuous to discret state space transformation
   [Ad, Bd] = c2d(A,B,1.0);
```

Then, matrices A, B and C are equivalent to

$$A = \begin{bmatrix} 0.9888 & 0.0001 & 0.0112 \\ 0.0001 & 0.9781 & 0.0111 \\ 0.0112 & 0.0111 & 0.9776 \end{bmatrix}; \quad B = \begin{bmatrix} 64.5687 & 0.0014 \\ 0.0014 & 64.2202 \\ 0.3650 & 0.3637 \end{bmatrix}; \quad C = I_{3\times 3}.$$

Remark 4.1. As presented in Chap. 3, which discussed the winding machine application, (4.4) may be obtained using an identification method.

4.3.2 Linear Nominal Control Law

A tracking control problem is considered in this study where the desired outputs $y_1 = [\ell_1 \ \ell_2]^T$ are required to track references y_r with

$$y(k) = \begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} x(k), \tag{4.6}$$

where $y_2 = \ell_3$.

To achieve the nominal tracking control, the solution proposed by [29] and developed in Sect. 2.4.1 has been considered for the three-tank system. Since the feedback control can only guarantee the stability and the dynamic behavior of the closed-loop system, a complementary controller is required to cause the output vector y_1 to track the reference input vector y_r such that the steady-state error is equal to zero. The technique consists of adding a vector comparator and integrator $z = [z_1 \quad z_2]^T$ that satisfies

$$z_i(k+1) = z_i(k) + T_s\left(y_{r,i}(k) - y_{1,i}(k)\right). \tag{4.7}$$

Therefore, the open-loop system can be described by an augmented state-space representation and the controllability of the system is verified off-line. Among the most popular controller design techniques for MIMO systems, a pole placement technique is considered to impose a desired behavior of the plant in closed-loop. Therefore, the feedback gain matrix K is designed