

FIGURE P3.29 Remote manipulator system.

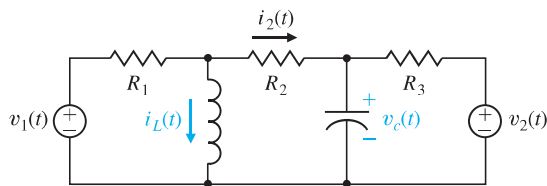


FIGURE P3.30 Two-input RLC circuit.

**P3.31** Extenders are robot manipulators that extend (that is, increase) the strength of the human arm in load-maneuvering tasks (Figure P3.31) [19, 22]. The system is represented by the transfer function

$$\frac{Y(s)}{U(s)} = G(s) = \frac{30}{s^2 + 4s + 3},$$

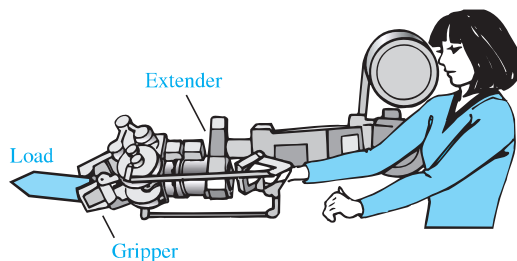


FIGURE P3.31 Extender for increasing the strength of the human arm in load maneuvering tasks.

where  $U(s)$  is the force of the human hand applied to the robot manipulator, and  $Y(s)$  is the force of the robot manipulator applied to the load. Determine a state variable model and the state transition matrix for the system.

**P3.32** A drug taken orally is ingested at a rate  $r(t)$ . The mass of the drug in the gastrointestinal tract is denoted by  $m_1(t)$  and in the bloodstream by  $m_2(t)$ . The rate of change of the mass of the drug in the gastrointestinal tract is equal to the rate at which the drug is ingested minus the rate at which the drug enters the bloodstream, a rate that is taken to be proportional to the mass present. The rate of change of the mass in the bloodstream is proportional to the amount coming from the gastrointestinal tract minus the rate at which mass is lost by metabolism, which is proportional to the mass present in the blood. Develop a state space representation of this system.

For the special case where the coefficients of  $\mathbf{A}$  are equal to 1 (with the appropriate sign), determine the response when  $m_1(0) = 1$  and  $m_2(0) = 0$ . Plot the state variables versus time and on the  $x_1 - x_2$  state plane.

**P3.33** The attitude dynamics of a rocket are represented by

$$\frac{Y(s)}{U(s)} = G(s) = \frac{1}{s^2},$$

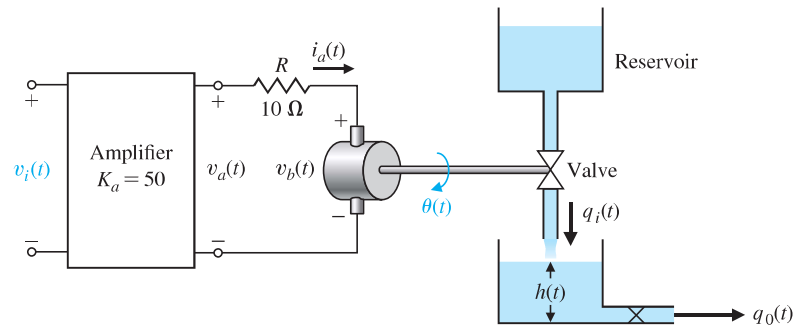
and state variable feedback is used where  $x_1(t) = y(t)$ ,  $x_2(t) = \dot{y}(t)$ , and  $u(t) = -x_2(t) - 0.5x_1(t)$ . Determine the roots of the characteristic equation of this system and the response of the system when the initial conditions are  $x_1(0) = 0$  and  $x_2(0) = 1$ . The input  $U(s)$  is the applied torques, and  $Y(s)$  is the rocket attitude.

**P3.34** A system has the transfer function

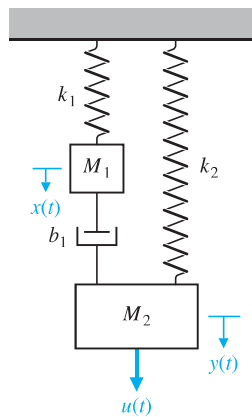
$$\frac{Y(s)}{R(s)} = T(s) = \frac{6}{s^3 + 6s^2 + 11s + 6}.$$

- Construct a state variable representation of the system.
- Determine the element  $\phi_{11}(t)$  of the state transition matrix for this system.

**P3.35** Determine a state-space representation for the system shown in Figure P3.35. The motor inductance is negligible, the motor constant is  $K_m = 10$ , the back electromagnetic force constant is  $K_b = 0.0706$ , the motor friction is negligible. The motor and valve inertia is  $J = 0.006$ , and the area of the tank is  $50 \text{ m}^2$ . Note that the motor is controlled by the armature current  $i_a(t)$ . Let  $x_1(t) = h(t)$ ,  $x_2(t) = \theta(t)$ , and  $x_3(t) = \dot{\theta}(t)$ .



**FIGURE P3.35**  
One-tank system.



**FIGURE P3.36** Two-mass system with two springs and one damper.

Assume that  $q_1(t) = 80\theta(t)$ , where  $\theta(t)$  is the shaft angle. The output flow is  $q_0(t) = 50h(t)$ .

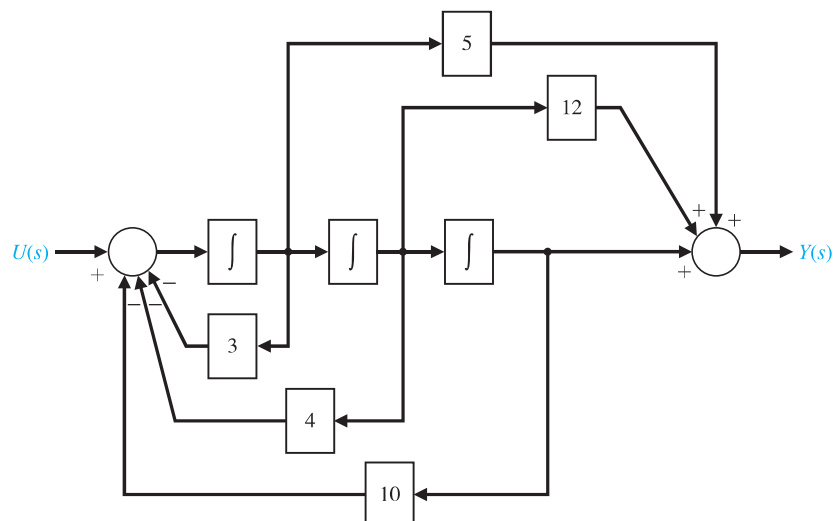
**P3.36** Consider the two-mass system in Figure P3.36. Find a state variable representation of the system. Assume the output is  $x(t)$ .

**P3.37** Consider the block diagram in Figure P3.37. Using the block diagram as a guide, obtain the state variable model of the system in the form

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t)$$

$$y(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}u(t)$$

Using the state variable model as a guide, obtain a third-order differential equation model for the system.



**FIGURE P3.37**  
A block diagram model of a third-order system.