

## Sistemas Hidráulicos:

### Variables dependientes:

$$Q, \text{ caudal } \left( \frac{m^3}{s} \right)$$

$$V, \text{ volumen } (m^3)$$

$$h, \text{ altura } (m)$$

$$p, \text{ presión } \left( \frac{N}{m^2} = Pa \right)$$

### Elementos pasivos (parámetros):

$$Q_c = C_f \frac{dP_{1r}}{dt}, Q_c = C_f \frac{dh_{1r}}{dt}, C_f \rightarrow \text{capacitancia hidráulica}$$

$$P_{12} = I \frac{dQ_I}{dt}, h_{12} = I \frac{dQ_I}{dt}, I \rightarrow \text{Inertancia hidráulica}$$

$$P_{12} = R_f Q_R, h_{12} = R_f Q_R, R_f \rightarrow \text{Resistencia hidráulica}$$

### Leyes:

#### Ley de Continuidad:

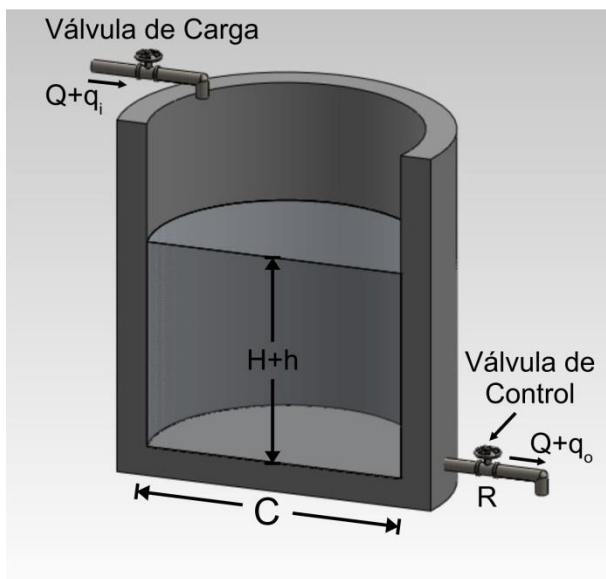
$$Q_A + Q_B + Q_C = 0$$

#### Ley de Compatibilidad:

$$P_{r1} + P_{12} + P_{2r} = 0$$

### Ejemplos

#### Ejemplo 1: Llenado de un tanque



$$H_s(s) = \frac{Q_o(s)}{Q_i(s)} = ?$$

$$q_i = C \frac{dh}{dt} + q_o$$

$$q_o = \frac{h}{R} \rightarrow h = q_o R \rightarrow \frac{dh}{dt} = R \frac{dq_o}{dt}$$

$$q_i = CR \frac{dq_o}{dt} + q_o$$

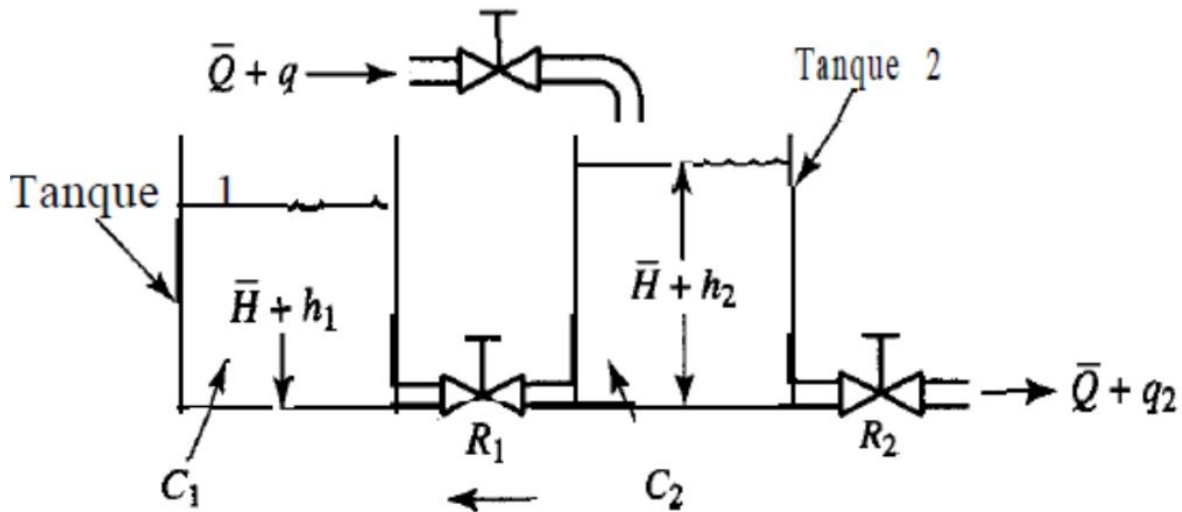
$$Q_i(s) = Q_o(s)(1 + RCs)$$

$$\frac{Q_o(s)}{Q_i(s)} = \frac{1}{RCs + 1}$$

$$\frac{H(s)}{Q_i(s)} = \frac{Q_o(s)R}{Q_i(s)} = \frac{R}{RCs + 1}$$

$$\frac{H(s)}{Q_i(s)} = \frac{R}{RCs + 1}$$

**Ejemplo 2: tanques interconectados (Ejercicio 2b del taller de sistemas hidráulicos)**



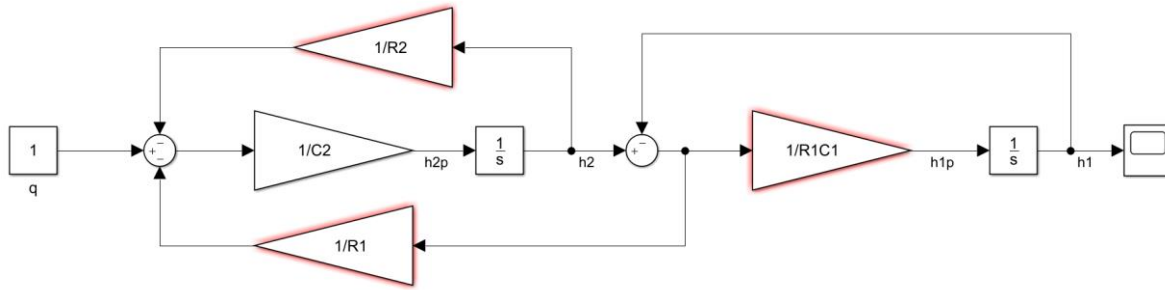
$$H(s) = \frac{H_1(s)}{Q(s)} = ?$$

$$q = C_2 \frac{dh_2}{dt} + q_2 + q_{21} \quad \text{Tanque 2}$$

$$q_{21} = C_1 \frac{dh_1}{dt} = \frac{h_2 - h_1}{R_1} \quad \text{Tanque 1}$$

$$q_2 = \frac{h_2}{R_2}$$

DB:



Simplificando el DB

$$H(s) = \frac{H_1(s)}{Q(s)} = \frac{R_2}{R_1 R_2 C_1 C_2 s^2 + (C_1 R_1 + C_1 R_2 + R_2 C_2) s + 1}$$

VE

$$q = C_2 \frac{dh_2}{dt} + \frac{h_2}{R_2} + \frac{h_2 - h_1}{R_1}$$

$$C_1 \frac{dh_1}{dt} = \frac{h_2 - h_1}{R_1}$$

$$\begin{bmatrix} \dot{h}_1 \\ \dot{h}_2 \end{bmatrix} = \begin{pmatrix} -\frac{1}{R_1 C_1} & \frac{1}{R_1 C_1} \\ \frac{1}{R_1 C_2} & -\frac{1}{C_2} \left( \frac{1}{R_2} + \frac{1}{R_1} \right) \end{pmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{pmatrix} 0 \\ \frac{1}{C_2} \end{pmatrix} [q]$$

$$y = x_1$$

$$y = (1 \quad 0) \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$

**Ejemplo 3: tanques interconectados (Ejercicio 3 del taller de sistemas hidráulicos):**

$$q_{i1} + q_1 = C_1 \frac{dh_1}{dt} \quad \text{Tanque 1}$$

$$q_1 = \frac{h_2 - h_1}{R_1}$$

$$q_{i2} = C_2 \frac{dh_2}{dt} + \frac{h_2 - h_1}{R_1} + q_o \quad \text{Tanque 2}$$

$$q_o = \frac{h_2}{R_2}$$

VE:

$$q_{i1} + \frac{h_2 - h_1}{R_1} = C_1 \frac{dh_1}{dt}$$

$$q_{i2} = C_2 \frac{dh_2}{dt} + \frac{h_2 - h_1}{R_1} + \frac{h_2}{R_2}$$

$$\begin{bmatrix} \dot{h}_1 \\ \dot{h}_2 \end{bmatrix} = \begin{pmatrix} -\frac{1}{R_1 C_1} & \frac{1}{R_1 C_1} \\ \frac{1}{R_1 C_2} & -\frac{1}{C_2} \left( \frac{1}{R_2} + \frac{1}{R_1} \right) \end{pmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{pmatrix} \frac{1}{C_1} & 0 \\ 0 & \frac{1}{C_2} \end{pmatrix} \begin{bmatrix} q_{i1} \\ q_{i2} \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$