Mathematical Modelling of Color Mixing Process and PLC Control Implementation by Using Human Machine Interface

Laurentiu Schiop*, and Marian Gaiceanu†

* Dunarea de Jos University of Galati, Galati, Romania, e-mail: *laurentiuschiop@yahoo.com*

† Dunarea de Jos University of Galati/Department of Electrical Engineering, Galati, Romania, e-mail: *Marian.Gaiceanu@ieee.org*

Abstract— **The mixing process is a multivariable and intrinsically non-linear plant. Mathematical modelling of the mixing color process has been taken into consideration in order to design an adequate control (PI controllers) to assure zero steady state, fast control and disturbance rejection. . The comparisons of the nonlinear mathematical model and linear one have been done. The control of the process has been implemented by using STEP7 Programming Software for Siemens SIMATIC Programmable Logic Controller (PLC) [1]. The entire process has been implemented in Matlab-Simulink environment. The process control has in view maintaining an appropriate level of liquid and a color as close to that required in the mixed tank, and only height control in auxiliaries tanks. Controlled process has been implemented on-line using specific language for Step7 (FBD - functional block diagram). Additionally, in order to use in industry by an operator for the implemented control has been created a friendly and intuitively HMI (human machine interface) in WinCC.**

***Index Terms*— mixing process, coloration control, height control, STEP7, HMI, WinCC**

I. DESCRIPTION OF COLOR MIXING PROCESS

COLOR mixing plant consists of a mixing tank in which colors are mixed from the two auxiliaries tanks [2]. The first auxiliary tank contains colored water, e_1, and the second tank contains clean water, e_2. The input flow to the mixing tank is controlled by two valves, which regulate the output flow from each auxiliary tank. After mixing process the resulting water has the desired coloration e. In order to find an adequate control for the mixing process, the corresponding mathematical model is developed. Color mixing plant is shown in Figure 1. The mathematical model for one tank is considered, knowing that the all three tanks have the same one.

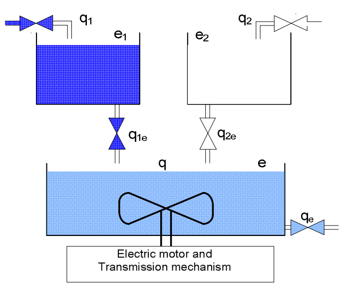


Fig.1. Color mixing plant.

II. THE MATHEMATICAL MODEL OF THE TANK

Consider the flow dynamics of a simple holding tank of which the output is determined by the pressure inside the tank [3].

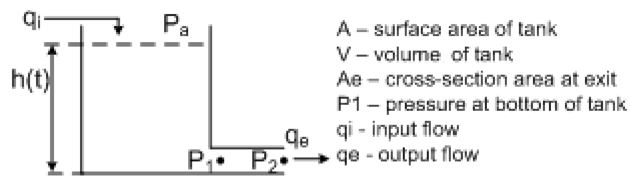


Fig.2. Tank system

The state variable is the height "*h*" of water in the tank and the desired output is the output mass flow rate, "q_e". By using the mass balance equation of the system results:

$$\frac{d}{dt}m = q_i - q_e \tag{1}$$

It is well- known that the mass is defined as a product of volume of the tank (V=Ah) and density of the liquid ρ, into a tank with vertical walls resulting as followed:

$$m = \rho A h \tag{2}$$

The output q_e can be written in terms of the output speed, v_e:

$$q_e = \rho A_e v_e \tag{3}$$

in which the output speed can be written as:

$$v_e = \sqrt{2gh} \tag{4}$$

Substituting equations (2), (3) and (4) in equation (1.0) the following eqution is obtained:

$$\rho A \frac{d}{dt}h = -\rho A_e \sqrt{2gh} + q_i \tag{5}$$

Thus, the state equation (5) for this first-order system is nonlinear. If we define the resistance, "*R*", for liquid flow in a restriction domain (between $p1$- $p2$) and the capacitance of the tank "*C*" as:

$$R(h) = \frac{h}{\rho A_e v_e} = \frac{h}{\rho A_e \sqrt{2gh}} \tag{6}$$

respectively,

$$C = \frac{\rho A h}{h} = \rho A \tag{7}$$

nonlinear equation system representing the tank model can be written as:

$$\frac{d}{dt}x(t) = -\frac{1}{RC}x(t) + \frac{1}{C}u(t) \tag{8}$$

where: $x(t) = h(t)$, $u(t) = q_i(t)$, $R(h) = \frac{\sqrt{h}}{\rho A_e \sqrt{2g}}$ and $C = \rho A$.

For process control purpose the obtained nonlinear system (8) is linearized by making a small variation $\delta(\cdot)$ around the steady state value (reference):

$$h(t) = h_0(t) + \delta h(t) \tag{9}$$

$$q_i(t) = q_{i0}(t) + \delta q_i(t) \tag{10}$$

By substituting equations (9) and (10) in equation (8) result:

$$\frac{d}{dt}h = f(h,t) + \frac{q_i}{\rho A} \tag{11}$$

$$\frac{d}{dt}h_0 + \frac{d}{dt}\delta h \approx f(h_0,t) + \left.\frac{\partial}{\partial h}f(h,t)\right|_{h_0}\delta h + \frac{q_{i0}}{\rho A} + \frac{\delta q_i}{\rho A}$$

or

$$\frac{d}{dt}\delta h = \left.\frac{\partial}{\partial h}f(h,t)\right|_{h_0}\delta h + \frac{\delta q_i}{\rho A} \tag{12}$$

where we have canceled out the reference equation evaluated at h_o and truncated the nonlinear term after first order. The Jacobian evaluated at the reference height is:

$$f(h) = -\frac{A_e}{A}\sqrt{2gh} \tag{13}$$

and

$$\left.\frac{\partial}{\partial h}f(h)\right|_{h_0} = \left.-\frac{A_e}{A}\frac{1}{2}(2gh)^{-1/2}2g\right|_{h_0} = -\frac{A_e}{A}\cdot\frac{g}{\sqrt{2gh_0}} \tag{14}$$

By substitution of equation (14) in equation (12) the following equation is obtained:

$$\frac{d}{dt}\delta h = -\frac{A_e}{A}\cdot\frac{g}{\sqrt{2gh_0}}\delta h + \frac{1}{\rho A}\delta q_i \tag{15}$$

or

$$\frac{d}{dt}x(t) = -\frac{1}{RC}x(t) + \frac{1}{C}u(t) \tag{16}$$

where: $x(t) = \delta h(t)$, $u(t) = \delta q_i(t)$, $R(h) = \frac{\sqrt{2gh_0}}{\rho g A_e}$ and $C = \rho A$.

The (16) system is a linearized representation of the original nonlinear system (8). Equation (16) it is also written in terms of variation around the reference and is given as an equation of state.

IV. Validation of the Linear Mathematical Model

To observe the effect of linearization and the behavior of tank system, a comparison between the nonlinear and linearized system results will be performed.

Analysis will be done based on the following data: q_{i0} =14,36[kg/m^3]; ρ=1000[kg/m^3]; h_0=10[m]; g=9.81[m/s^2]; A=0.46[m^2]; A_e=0.0018[m^2] gives at reference conditions (steady state for this case), v_e=14[m/s], q_e=2.52[kg/m^3], R=0.43[ms/kg], C=460[kg/m] and RC=197.8[s] (i.e.3.29[min]). These data were used in different simulation cases: for q_i increases of 10%, and of 50% respectively, for both impulse and step input changes to linearized and nonlinear systems (Fig.3). In Figure 3 the comparison responses from the linearized and nonlinear mathematical models simulations are shown. In this Figure 3 the height or level of liquid in the tank versus time is shown for the two input increased values (for an increasing impulse input value with 10% and 50%, respectively; for an increasing step input value with 10% and 50%, respectively). The results of nonlinear and linearized system are almost identical, very small error variation around the height reference. It is recalled that the impulse response of a linear system is given by:

$$\frac{d}{dt}\delta h = -\frac{1}{RC}\delta h \text{ with } \delta h(0) = \frac{1}{C}\delta q_i \text{ or } \delta h(t) = \frac{\delta q_i}{C}e^{-t/RC}$$

which is exactly the result shown in Figures 3 a-d.

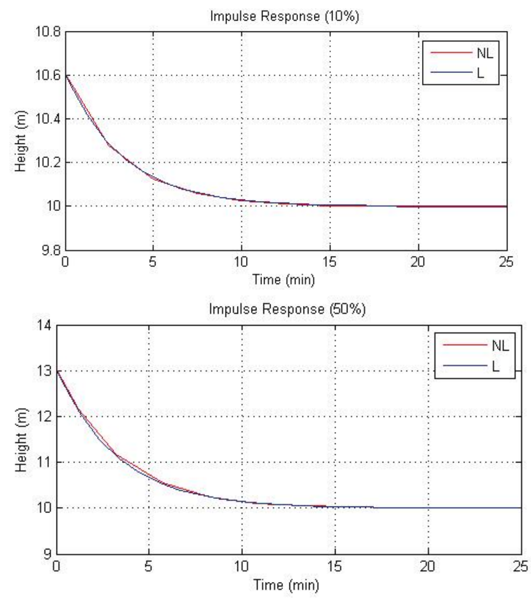
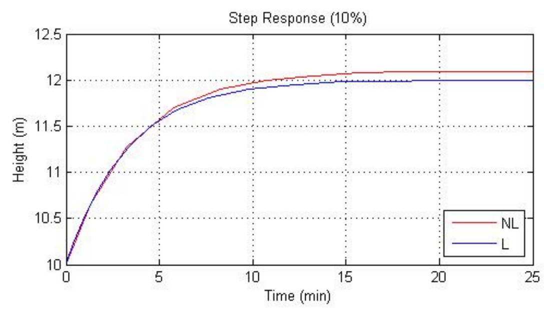


Fig.3a,b. Response comparisons of the nonlinear and linearized systems.



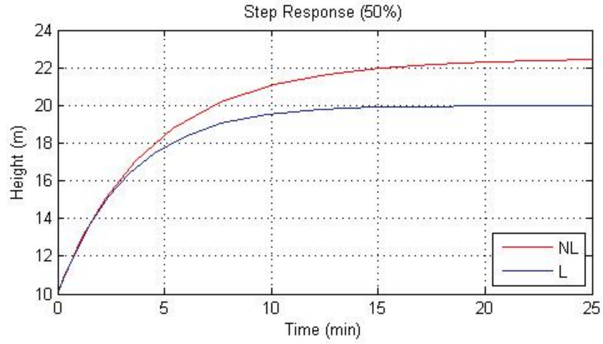


Fig.3c,d. Response comparisons of the nonlinear and linearized systems. Impulse and step responses of the system for an increased input of 10% and 50% for both types of systems.

Similarly, we can determine the step response for the linearized system by integrating the impulse response.

$$\delta h(t)=\int_0^t \frac{\partial q_i}{C} e^{-(t-\tau)} d\tau = \frac{\partial q_i}{C} e^{-t/RC}\left[RCe^{\tau/RC}\right]_0^t$$

or

$$\delta h(t)=\delta q_i R\left[1-e^{-t/RC}\right]$$

As shown, the step response given in Figure 3 also shows the expected rise in the tank height to a new equilibrium value. In this case the difference between nonlinear and linearized model is evident, and the error is considerably higher if the input flow is increased by 50%. Since the system was linearized around the reference point h_0 = 10 [m], the case of 50% increase of the input flow violate the basic criterion of linearization (i.e. that only small variations around reference are allowed). In conclusion, based on these simulations, the linearized model is adequat as long as input flow varies approximately ± 10-20% of reference, larger disturbances leading to unreliable predictions of the linearized model. Assuming that the above restrictions are not violated, further analysis of the linearized model can be perfomed. The transfer function of the linearized model is [2], [4]:

$$H(s)=\frac{R}{RCs+1}Q_i(s) \tag{17}$$

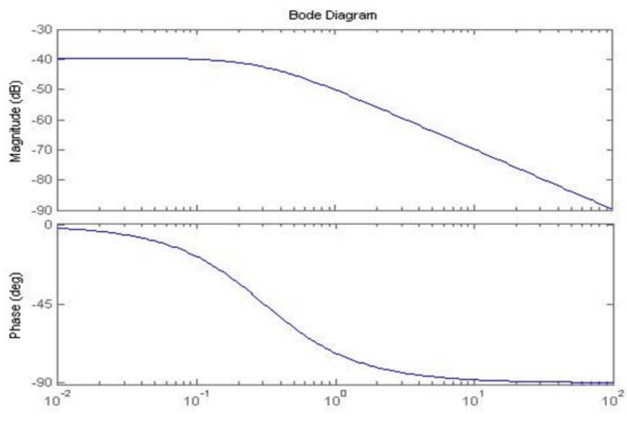


Fig.4. Bode diagram for a single linearized tank model

Bode diagram has been drawn in order to make a frequency analysis of the linearized system. From Figure 4 we can estimate the time constant of the first order system. Looking at Figure 4, the low frequency asymptote and high frequency asymptote intersect at about 0.3 radians per minute and $T_C=1/\omega_C\approx 3.3$[min], which compares closely to the value of 3.29 [min] previously calculated.

IV. COLOR MIXING PROCESS SIMULATION IN MATLAB-SIMULINK

The above deducted mathematical model of the mixing color process based on a single tank has been considerate, taking into account that all three tanks follow the same pattern. For the auxilliaries tanks there is only one height control loop, while for mixing tank there are two control loops: one for height control, and the other for coloration control. Therefore, the coloration control of the mixing plant will be designed in order to maintain a constant height in auxiliaries tanks and to result a coloration of the water in a required liquid level of tank mixing. State variables that must be controlled for the mixing tank are the heigth "h" of the tank, the coloration of the liquid "e", and the mixing tank system dynamic equations described by these variables.

Equation of heigth, h, state variable can be written as:

$$\rho A\frac{d}{dt}h=-\rho A_e\sqrt{2gh}+q$$

where q is the input flow and is defined as the sum of output flows of the auxiliary tanks: $q=q_{1e}+q_{2e}$. The above mentioned equation can be written as transfer function of first order linearized model (i.e. tank transfer function):

$$H(s)=\frac{R}{RCs+1}Q(s)$$

Color dynamic equation is obtained by combining relations:

$$\frac{d}{dt}h=\frac{1}{\rho A}\left(q_{1e}+q_{2e}-q_e\right)$$

$$e_1\cdot q_{1e}-e\cdot q_e=\frac{d}{dt}(e\rho Ah)=\rho A\left(e\frac{d}{dt}h+h\frac{d}{dt}e\right)$$

and results,

$$\frac{d}{dt}e=\frac{1}{\rho Ah}\left(e_1 q_{1e}-e\left(q_{1e}+q_{2e}\right)\right)$$

To control the color is defined proportion of the total flow of colored water entry as:

$$q_r=\frac{q_{1e}}{q_{1e}+q_{2e}}$$

The flow level in the mixing tank could be maintained at constant level if a continuous output flow of liquid with a specific coloration and the height of water is desired. In these conditions the output flow must be defined as:

$q_e = \rho A_e \sqrt{2gh}$.

Therefore, for both cases, the output flow of auxiliaries' tanks will be adjusted. But a prerequisite for the proper functioning of the mixing tank is that the auxiliaries tanks must maintain a constant level of liquid. Disturbance that occurs at the auxiliary tanks is the output flow which is controlled by the coloring process. As presented in the first part, the linearized model of a tank can be written as a transfer function: $H_{1,2}(s) = \frac{R}{RCs+1} Q_{1,2}(s)$ and tank disturbance can be written as a disturbance in the level: $h = \frac{1}{\rho A} \int_{t0}^{tf} \left(q_{1,2} - q_{1e,2e}\right)$.

With these equations the Matlab-Simulink model for the entire process can be builded (Fig.5). Has been made the simulation for the first case, where the output valve of the mixing tank is off (Manual switch).

In order to simulate the mixing process the following simplifying assumptions were made:

- the time necessary to get a uniform mixture is neglected;
- the delay time associated with the fluid flow from auxiliaries tanks in the mixing tank is neglected;
- the input valves dynamics is neglected;

The values used for the simulation of the mixing process are:

rho = 1000; % water density(kg/m^3)
g = 9.81;% gravitational, constant(m/s^2)
h0 = 3; % reference height of fluid in the mixing tank (m)
h10 = 2;% reference height of fluid in tank 1 (m)
h20 = 2; % reference height of fluid in tank 2 (m)
e1=1;% coloration of the water in tank 1
e2=0;% coloration of the water in tank 2
a = 1.13;% surface area of the mixing tank (m^2)
a1= 0.5; % surface area of tank 1 (m^2)
a2= 0.5; % surface area of tank 2 (m^2)
ae1 =0.0018; % area of exit pipe for tank 1(m^2)
ae2 =0.0018; % area of exit pipe for tank 2 (m^2)
ae =0.0018; % area of exit pipe for the mixing tank (m^2)
R=sqrt(2*g*h0)/(rho*g*ae) % resistance for liquid flow in the mixing tank (s/m^2)
R1=sqrt(2*g*h10)/(rho*g*ae1)% resistance for liquid flow in tank 1 (s/m^2)
R2=sqrt(2*g*h20)/(rho*g*ae2)% resistance for liquid flow in tank 2 (s/m^2)
C=rho*a % capacitance of the mixing tank (m^2)
C1=rho*a1 % capacitance of tank 1 (m^2)
C2=rho*a2 % capacitance of tank 2(m^2)
q, q_1, q_2, q_{1e}, q_{2e}, q_e – are the corresponding variables.

The controllers that regulate the mixing process have been calculated with Ziegler Nichols method. The mixing process has been implemented in Matlab Simulink as shown in Figure 5.

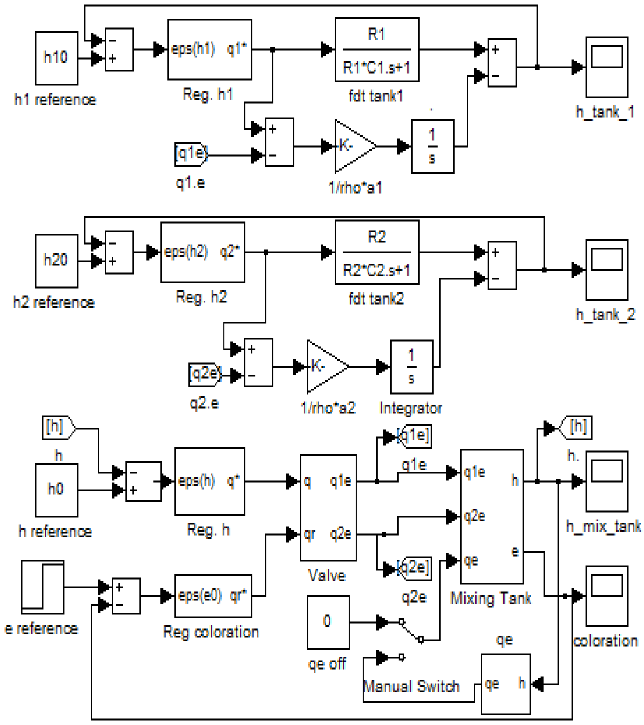


Fig.5. Mixing process in Matlab-Simulink

The control system for the fluid mixer is designed in order to maintain the coloration *e* of the output at the desired reference and the level *h* in the mixer to the desired reference value, not allowing it to be empty or overflow. The output flow, q_e, is considered as a disturbance.

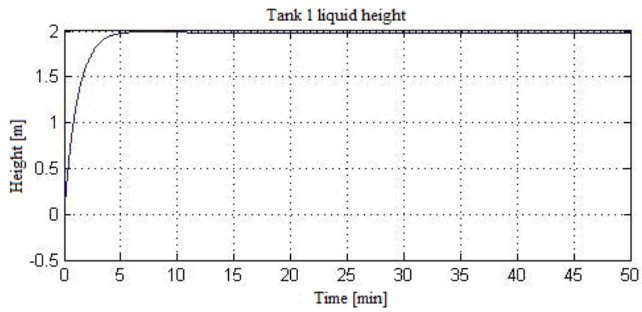


Fig. 6a. The level of liquid in tank 1.

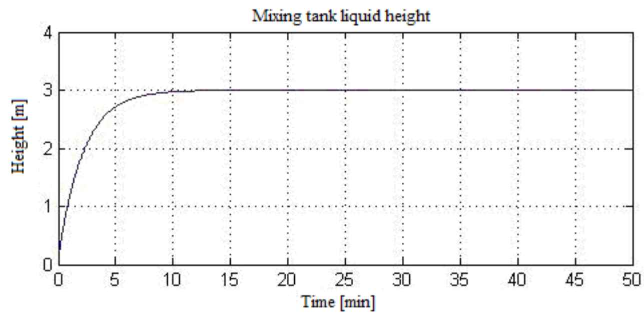


Fig. 6b. The level of liquid in mixing tank.

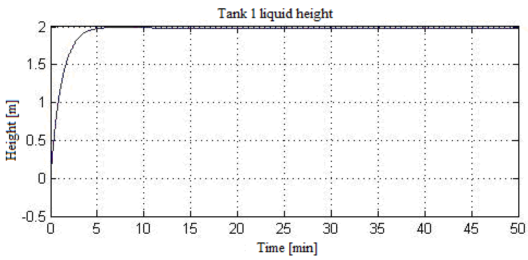


Fig. 6c. The level of liquid in tank 2

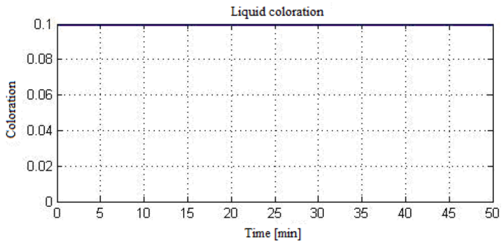


Fig. 6d. The liquid coloration for a setpoint=0.1

From the Fig. 6 we can observe that the level control in the tanks is quite good with the following time response: 5 minutes for tank 1 (Fig. 6a), 6 minutes for tank 2 (Fig. 6c) respectively 9 minutes for the mixing tank (Fig. 6d). Transient responses of a tank 1 and of the mixing tank do not have overshoot, only the transient response of tank 2 have a 5% overshoot due to a larger disturbance influence. The color reference value is set to 0.1 in the mixing tank and requires more liquid from tank 2. Level control was achieved (Fig.6b). As can be seen from the coloration graph (Fig.6d), the coloration regulator has a good ability to control the color of the liquid in the mixing tank. The coloration response does not have overshoot and the time response is fast.

IV. IMPLEMENTATION OF A PLC CONTROL INTERFACE FOR COLOR MIXING PROCESS

In order to choose the adequate power of the electric motor the following data have been considerate: D=1200 [mm], tank diameter; d = 500 [mm], mixer diameter; μ = 1.06 [cP], viscosity of water; ρ = 1000 [kg/m3], water density; n = 200[rpm], mixer rotation. The necessary motor power is a function of liquid density "ρ", liquid viscosity "μ " and number of revolutions per minute "n", P=f(ρ, μ, n). Taking into account the inertia forces, transmission efficiency of η = 0.85 and an adequate correction factor of $\varphi_1 = 1{,}3$ (has been introduced because the mixing tank has vertical metal walls) the 2,2 [kW] mechanical power of the electric motor has been choosed.

The process is on-line implemented using PLC and has been written in a specific language Step7 (FBD – funcional block diagram) of SIMATIC PLC. For understanding the control process written Step7 (FBD) language the corresponding sequence control for a tank (this sequence consists of 5 networks, Figure 7-11) is described as follows:

Network 1. Count the amount of fluid drained from the auxilliary tank in the mixing tank and decreased amount of liquid from the mixing tank when the output valve is open.

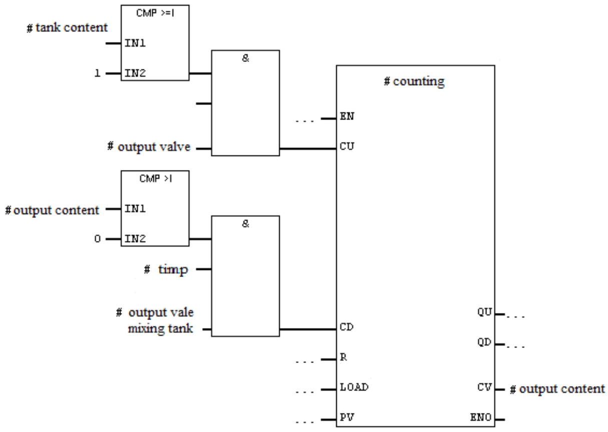


Fig.7. Network 1

Network 2. Automatic filling of the tank by imposing minimum and maximum level. If it is desired a constant level in the tank, minimum and maximum level must be set in the same point.

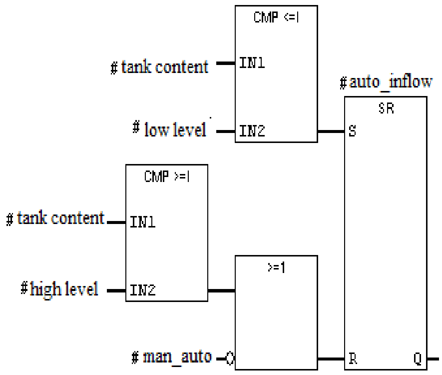


Fig.8. Network 2.

Network 3. The control of the inflow valve, manually or automatically.

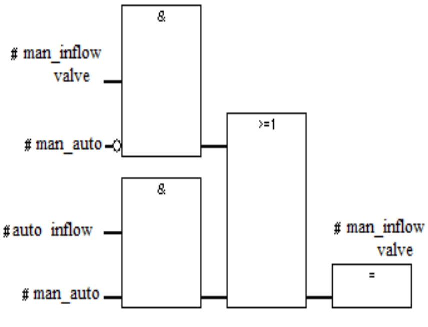


Fig.9. Network 3

Network 4. The control of the outflow valve, manually or automatically.

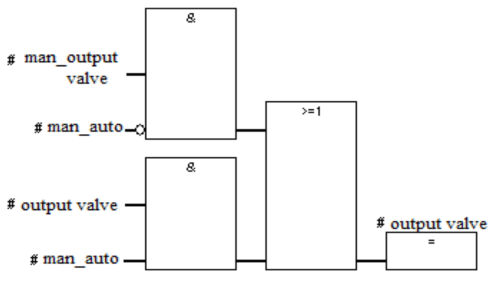


Fig.10. Network 4

Network 5. Show the amount of liquid metered in tank.

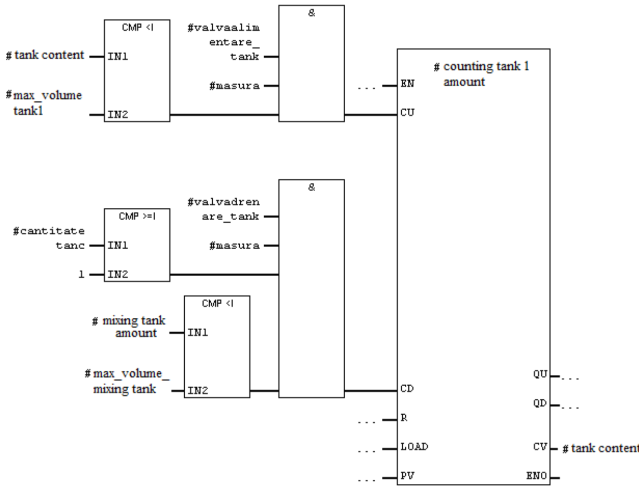


Fig.11. Network 5

The control system for the fluid mixer not allows it to be empty or overflow by imposing adequately maximum and minimum values in network 2.

Based on the designed PLC sequential control (Figs.7-11) an adequate HMI (human machine interface) has been designed and implemented in WinCC (Fig.12) for real time control purposes. The HMI comes in help for operator to not interfere in the PLC control program, being very friendly and intuitively.

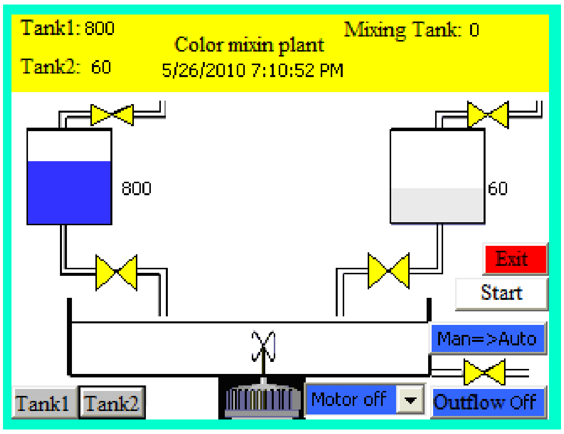


Fig.12. HMI for the coloration mixing plant.

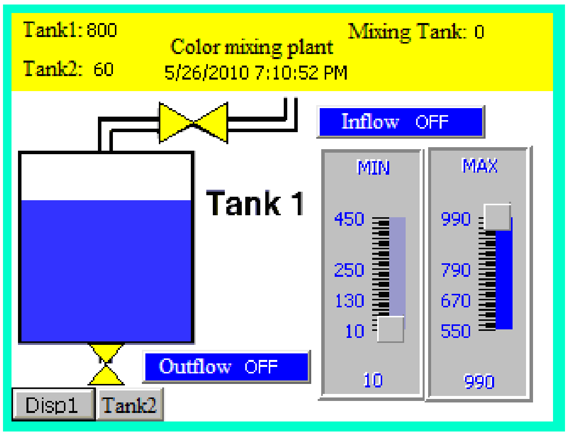


Fig. 13. Interface control of auxiliary Tank 1.

A. HMI functions:

-the entire color mixing plant operates with the designed HMI;

-the operating mode can be switched between two states: manual↔ automatic;

- the auxiliary tank levels are monitored;

- the level of the tanks are bar displayed and the fixed references are on-line displayed as numerical value;

- the motion of the mixing motor is shown graphically;

- the operator can set a maximum and a minimum level;

- a certain amount of liquid for each tanker that enters the mixing tank can be set.

By using these types of equipments:

- they provide increased productivity thanks to process automation.

- the product quality has been increasing through the use of correct amounts of liquid that occur during the process.

IV. CONCLUSION

From the comparison between nonlinear and linear model is evident that the the error is considerably higher if the input flow is increased by 50%. Since the system was linearized around the reference point $h_0 = 10$ [m], the case of 50% increase of the input flow violate the basic criterion of linearization (i.e. that only small variations around reference are allowed). In conclusion, based on these simulations, the linearized model is adequat as long as input flow varies approximately ± 10-20% of reference, larger disturbances leading to unreliable predictions of the linearized model.

As could be seen in Figures 6.a, 6.b, 6.c, 6.d control of process have been met the requirements performance . The main contribution of this paper consists of mixing process implementation by using STEP 7 FBD language into a PLC system. Additionally, the adequate human machine interface has been created. The automatized process add many advantages to mixing process: high quality control, adequate setpoints can be on-line numerically fixed, the level domain (max and min values) can be adjusted, increased productivity can be assured and the main variables of the mixing process can be on-line monitorized thanks to the adequate design of the HMI presented in this paper.

REFERENCES

[1] www.siemens.com

[2] M. Sultan, M., S. Shah, A., and C.O.David, Controllers optimization for a fluid mixing system using metamodelling approach, pp.48-60, *International Journal of Simulation Modelling*, ISSN 1726-4529, Volume 8: Number 1, March 2009

[3] M. Yianneskis, Fluid mixing: principles, practice and advancing fundamental understanding, *Chemical Engineering Science* Volume 61, Issue 9, Page 2753, Fluid Mixing VIII International Conference, May 2006

[4] M. Ali, M.S. Abdullah, S.S. Kasno, M.A., “Fuzzy logic controller optimization using Metamodeling technique for a fluid mixing system,” *Mechatronics and Automation, 2008. ICMA 2008. IEEE International Conference on*, **ISBN:** 978-1-4244-2631-7, 5-8 Aug., pp 947 – 953, 2008