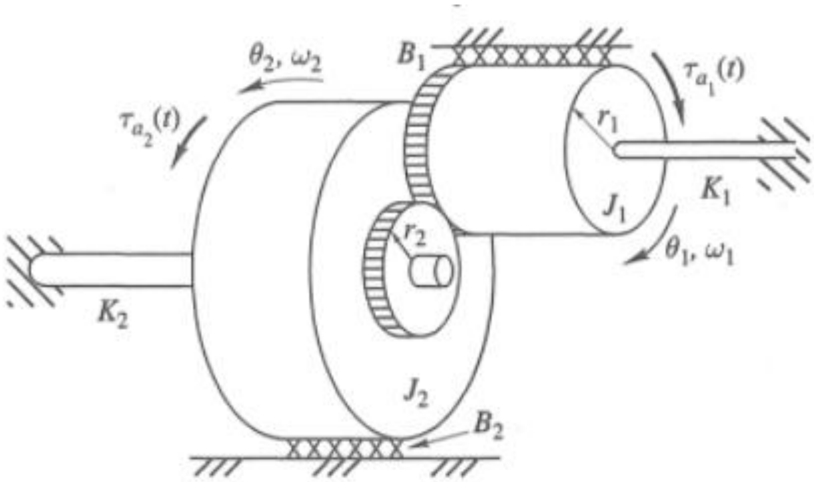
Ejercicio 3 (Taller 2 de sistemas mecánicos)



$$J_1\ddot{\theta}_1 = \tau_{a1} - T_1 - K_1\theta_1 - B_1\dot{\theta}_1$$

$$J_2\ddot{\theta}_2 = \tau_{a2} + T_2 - K_2\theta_2 - B_2\dot{\theta}_2$$

$$\frac{T_1}{T_2} = \frac{r_1}{r_2} = \frac{\theta_2}{\theta_1} = \frac{\dot{\theta}_2}{\dot{\theta}_1} = \frac{\ddot{\theta}_2}{\ddot{\theta}_1}$$

Representación en VE:

$$x_1 = \theta_2$$

$$x_2 = \dot{\theta}_2$$

$$u_1 = \tau_{a1}$$

$$u_2 = \tau_{a2}$$

$$y = \theta_2 = x_1$$

Derivamos los estados:

$$\boxed{\dot{\boldsymbol{x}}_{\boldsymbol{1}} = \boldsymbol{x}_{\boldsymbol{2}}}$$

$$\dot{x}_2 = \ddot{\theta}_2$$

$$J_2\ddot{\theta}_2 = \tau_{a2} + T_2 - K_2\theta_2 - B_2\dot{\theta}_2$$

$$T_2 = \frac{T_1 r_2}{r_1}$$

$$T_1 = \tau_{a1} - J_1\ddot{\theta}_1 - K_1\theta_1 - B_1\dot{\theta}_1$$

$$\frac{r_1}{r_2} = \frac{\theta_2}{\theta_1} = \frac{\dot{\theta}_2}{\dot{\theta}_1} = \frac{\ddot{\theta}_2}{\ddot{\theta}_1}$$

$$\theta_1 = \theta_2\frac{r_2}{r_1}, \dot{\theta}_1 = \dot{\theta}_2\frac{r_2}{r_1}, \ddot{\theta}_1 = \ddot{\theta}_2\frac{r_2}{r_1}$$

$$T_1 = \tau_{a1} - J_1\ddot{\theta}_2\frac{r_2}{r_1} - K_1\theta_2\frac{r_2}{r_1} - B_1\dot{\theta}_2\frac{r_2}{r_1}$$

$$J_2\ddot{\theta}_2 = \tau_{a2} + \frac{T_1 r_2}{r_1} - K_2\theta_2 - B_2\dot{\theta}_2$$

$$J_2\ddot{\theta}_2 = \tau_{a2} + \frac{r_2}{r_1}\left(\tau_{a1} - J_1\ddot{\theta}_2\frac{r_2}{r_1} - K_1\theta_2\frac{r_2}{r_1} - B_1\dot{\theta}_2\frac{r_2}{r_1}\right) - K_2\theta_2 - B_2\dot{\theta}_2$$

$$J_2\ddot{\theta}_2 + J_1\ddot{\theta}_2\left(\frac{r_2}{r_1}\right)^2 = \tau_{a2} + \frac{r_2}{r_1}\left(\tau_{a1} - K_1\theta_2\frac{r_2}{r_1} - B_1\dot{\theta}_2\frac{r_2}{r_1}\right) - K_2\theta_2 - B_2\dot{\theta}_2$$

$$\ddot{\theta}_2\left(J_2 + J_1\left(\frac{r_2}{r_1}\right)^2\right) = \tau_{a2} + \frac{r_2}{r_1}\left(\tau_{a1} - K_1\theta_2\frac{r_2}{r_1} - B_1\dot{\theta}_2\frac{r_2}{r_1}\right) - K_2\theta_2 - B_2\dot{\theta}_2$$

$$\ddot{\theta}_2 = \frac{\left(\tau_{a2} + \frac{r_2}{r_1}\left(\tau_{a1} - K_1\theta_2\frac{r_2}{r_1} - B_1\dot{\theta}_2\frac{r_2}{r_1}\right) - K_2\theta_2 - B_2\dot{\theta}_2\right)}{\left(J_2 + J_1\left(\frac{r_2}{r_1}\right)^2\right)}$$

$$\dot{x}_2 = \frac{\left(u_2 + \frac{r_2}{r_1}\left(u_1 - K_1x_1\frac{r_2}{r_1} - B_1x_2\frac{r_2}{r_1}\right) - K_2x_1 - B_2x_2\right)}{\left(J_2 + J_1\left(\frac{r_2}{r_1}\right)^2\right)}$$

$$\boxed{\boldsymbol{\dot{x}_2 = \frac{u_2 + \frac{r_2}{r_1}u_1 - x_1\left(K_2 + K_1\left(\frac{r_2}{r_1}\right)^2\right) - x_2\left(B_2 + B_1\left(\frac{r_2}{r_1}\right)^2\right)}{\left(J_2 + J_1\left(\frac{r_2}{r_1}\right)^2\right)}}}$$

$$\boxed{\boldsymbol{\dot{x}_1 = x_2}}$$

$$\boxed{\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{\left(K_2 + K_1\left(\frac{r_2}{r_1}\right)^2\right)}{\left(J_2 + J_1\left(\frac{r_2}{r_1}\right)^2\right)} & -\frac{\left(B_2 + B_1\left(\frac{r_2}{r_1}\right)^2\right)}{\left(J_2 + J_1\left(\frac{r_2}{r_1}\right)^2\right)}\end{pmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{pmatrix} 0 & 0 \\ \frac{\frac{r_2}{r_1}}{\left(J_2 + J_1\left(\frac{r_2}{r_1}\right)^2\right)} & \frac{1}{\left(J_2 + J_1\left(\frac{r_2}{r_1}\right)^2\right)}\end{pmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}}$$

$$\boxed{y = x_1}$$

$$\boxed{y = \begin{pmatrix}1 & 0\end{pmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}}$$

$$H_1(s) = \frac{\theta_2(s)}{\tau_{a1}(s)}, H_2(s) = \frac{\theta_2(s)}{\tau_{a2}(s)}$$

$$\theta_2(t) = L^{-1}\{H_1(s)\tau_{a1}(s) + H_2(s)\tau_{a2}(s)\}$$

Otra salida

$$J_1\ddot{\theta}_1 = \tau_{a1} - T_1 - K_1\theta_1 - B_1\dot{\theta}_1$$

$$J_2\ddot{\theta}_2 = \tau_{a2} + T_2 - K_2\theta_2 - B_2\dot{\theta}_2$$

$$\frac{T_1}{T_2} = \frac{r_1}{r_2} = \frac{\theta_2}{\theta_1} = \frac{\dot{\theta}_2}{\dot{\theta}_1} = \frac{\ddot{\theta}_2}{\ddot{\theta}_1}$$

Representación en VE:

$$\dot{x} = f(x,u) = Ax + Bu$$

$$y = g(x,u) = Cx + Du$$

$$x_1 = \theta_1$$

$$x_2 = \dot{\theta}_1$$

$$u_1 = \tau_{a1}$$

$$u_2 = \tau_{a2}$$

$$y = \theta_2, \theta_1\frac{r_1}{r_2} = \theta_2$$

$$\boldsymbol{y = \theta_1\frac{r_1}{r_2} = x_1\frac{r_1}{r_2}}$$

Derivamos los estados:

$$\boxed{\boldsymbol{\dot{x}_1 = x_2}}$$

$$\dot{x}_2 = \ddot{\theta}_1$$

$$J_1\ddot{\theta}_1 = \tau_{a1} - T_1 - K_1\theta_1 - B_1\dot{\theta}_1$$

$$T_1 = \frac{T_2 r_1}{r_2}$$

$$J_2\ddot{\theta}_2 + K_2\theta_2 + B_2\dot{\theta}_2 - \tau_{a2} = T_2$$

$$\theta_1\frac{r_1}{r_2} = \theta_2, \dot{\theta}_1\frac{r_1}{r_2} = \dot{\theta}_2, \ddot{\theta}_1\frac{r_1}{r_2} = \ddot{\theta}_2$$

$$J_2\ddot{\theta}_1\frac{r_1}{r_2} + B_2\dot{\theta}_1\frac{r_1}{r_2} + K_2\theta_1\frac{r_1}{r_2} - \tau_{a2} = T_2$$

$$J_1\ddot{\theta}_1 = \tau_{a1} - \frac{T_2 r_1}{r_2} - K_1\theta_1 - B_1\dot{\theta}_1$$

$$J_1\ddot{\theta}_1 = \tau_{a1} - \frac{r_1}{r_2}\left(J_2\ddot{\theta}_1\frac{r_1}{r_2} + B_2\dot{\theta}_1\frac{r_1}{r_2} + K_2\theta_1\frac{r_1}{r_2} - \tau_{a2}\right) - K_1\theta_1 - B_1\dot{\theta}_1$$

$$J_1\ddot{\theta}_1 + J_2\ddot{\theta}_1\left(\frac{r_1}{r_2}\right)^2 = \tau_{a1} - B_2\dot{\theta}_1\left(\frac{r_1}{r_2}\right)^2 - K_2\theta_1\left(\frac{r_1}{r_2}\right)^2 + \tau_{a2}\frac{r_1}{r_2} - K_1\theta_1 - B_1\dot{\theta}_1$$

$$\ddot{\theta}_1\left(J_1 + J_2\left(\frac{r_1}{r_2}\right)^2\right) = \tau_{a1} - B_2\dot{\theta}_1\left(\frac{r_1}{r_2}\right)^2 - K_2\theta_1\left(\frac{r_1}{r_2}\right)^2 + \tau_{a2}\frac{r_1}{r_2} - K_1\theta_1 - B_1\dot{\theta}_1$$

$$\ddot{\theta}_1 = \frac{\tau_{a1} - B_2\dot{\theta}_1\left(\frac{r_1}{r_2}\right)^2 - K_2\theta_1\left(\frac{r_1}{r_2}\right)^2 + \tau_{a2}\frac{r_1}{r_2} - K_1\theta_1 - B_1\dot{\theta}_1}{J_1 + J_2\left(\frac{r_1}{r_2}\right)^2}$$

$$\dot{x}_2 = \frac{u_1 - B_2x_2\left(\frac{r_1}{r_2}\right)^2 - K_2x_1\left(\frac{r_1}{r_2}\right)^2 + u_2\frac{r_1}{r_2} - K_1x_1 - B_1x_2}{J_1 + J_2\left(\frac{r_1}{r_2}\right)^2}$$

$$\boxed{\boldsymbol{\dot{x}_2 = \frac{u_1 - x_2\left(B_2\left(\frac{r_1}{r_2}\right)^2 + B_1\right) - x_1\left(K_2\left(\frac{r_1}{r_2}\right)^2 + K_1\right) + u_2\frac{r_1}{r_2}}{J_1 + J_2\left(\frac{r_1}{r_2}\right)^2}}}$$

$$\boxed{\boldsymbol{\dot{x}_1 = x_2}}$$

$$\boxed{\boldsymbol{y = x_1\frac{r_1}{r_2}}}$$

$$\boxed{\boldsymbol{\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{\left(K_1 + K_2\left(\frac{r_1}{r_2}\right)^2\right)}{\left(J_1 + J_2\left(\frac{r_1}{r_2}\right)^2\right)} & -\frac{\left(B_1 + B_2\left(\frac{r_1}{r_2}\right)^2\right)}{\left(J_1 + J_2\left(\frac{r_1}{r_2}\right)^2\right)}\end{pmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{pmatrix} 0 & 0 \\ \frac{1}{\left(J_1 + J_2\left(\frac{r_1}{r_2}\right)^2\right)} & \frac{\frac{r_1}{r_2}}{\left(J_1 + J_2\left(\frac{r_1}{r_2}\right)^2\right)}\end{pmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}}}$$

$$\boxed{\boldsymbol{y = \begin{pmatrix}\frac{r_1}{r_2} & 0\end{pmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}}}$$

$$H_1(s) = \frac{\theta_2(s)}{\tau_{a1}(s)}, H_2(s) = \frac{\theta_2(s)}{\tau_{a2}(s)}$$

$$\theta_2(t) = L^{-1}\{H_1(s)\tau_{a1}(s) + H_2(s)\tau_{a2}(s)\}$$