

Transformada de Fourier

$$\mathcal{F}\{x(t)\} = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt = X(j\omega)$$

$$s = \sigma + j\omega \rightarrow \sigma = 0$$

si $H(j\omega)$ es un LTI

$$Y(j\omega) = H(j\omega)X(j\omega)$$

$$|Y(j\omega)| = |H(j\omega)||X(j\omega)|$$

$$|Y(j\omega)|_{dB} = 20 \log(|Y(j\omega)|) = 20 \log(|H(j\omega)||X(j\omega)|)$$

$$|Y(j\omega)|_{dB} = 20 \log(|H(j\omega)|) + 20 \log(|X(j\omega)|)$$

$$\arg\{Y(j\omega)\} = \arg\{H(j\omega)\} + \arg\{X(j\omega)\}$$

Análisis en frecuencia de sistemas dinámicos

$$H(s) = K s^{\pm N} \prod_{i=1}^n \frac{p_i}{s + p_i} \prod_{k=1}^m \frac{s + c_k}{c_k} \prod_{o=1}^r \frac{\omega_{no}^2}{s^2 + 2\zeta_o \omega_{no} s + \omega_{no}^2} \prod_{u=1}^t \frac{s^2 + 2\zeta_u \omega_{nu} s + \omega_{nu}^2}{\omega_{nu}^2}$$

$$|H(j\omega)|_{dB} = 20 \log(|H(j\omega)|) \text{ Magnitud del sistema en dB}$$

$$\arg\{H(j\omega)\} \text{ fase del sistema}$$

Ganancia:

$$H(s) = K$$

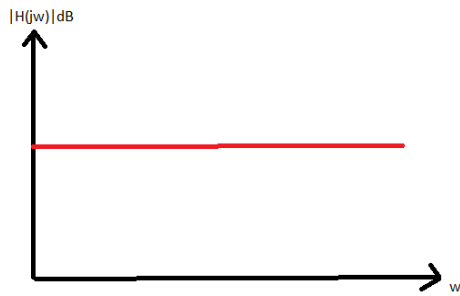
$$H(j\omega) = K$$

$$|H(j\omega)| = |K|$$

$$|H(j\omega)|_{dB} = 20 \log(|K|) > 0 \text{ if } |K| > 1$$

$$|H(j\omega)|_{dB} = 20 \log(|K|) = 0 \text{ if } |K| = 1$$

$$|H(j\omega)|_{dB} = 20 \log(|K|) < 0 \text{ if } |K| < 1$$



$$\arg\{H(j\omega)\} = 0 \text{ if } K > 0$$

$$\arg\{H(j\omega)\} = 180^\circ \text{ if } K < 0$$



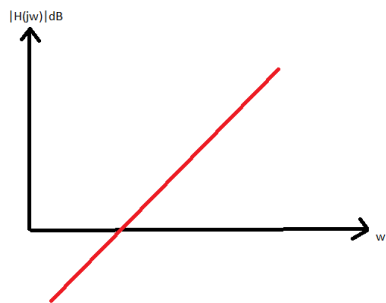
Derivador puro:

$$H(s) = s^N$$

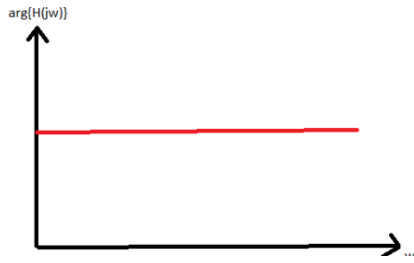
$$H(j\omega) = (j\omega)^N$$

$$|H(j\omega)| = \omega^N$$

$$|H(j\omega)|_{dB} = 20 \log(\omega^N) = 20N \log(\omega)$$



$$\arg\{H(j\omega)\} = \arg\{(j\omega)^N\} = 90^\circ N$$



Integrador puro:

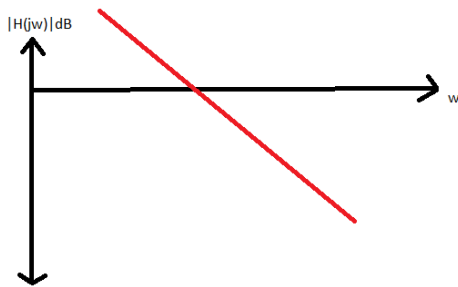
$$H(s) = s^{-N} = \frac{1}{s^N}$$

$$H(j\omega) = \frac{1}{(j\omega)^N}$$

$$|H(j\omega)| = \frac{1}{\omega^N}$$

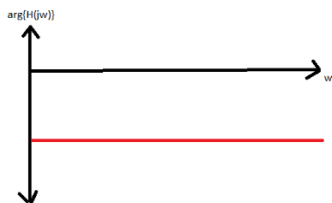
$$|H(j\omega)|_{dB} = 20\text{Log}\left(\frac{1}{\omega^N}\right) = 20\text{Log}(1) - 20\text{Log}(\omega^N)$$

$$\boxed{|H(j\omega)|_{dB} = -20N\text{Log}(\omega)}$$



$$\arg\{H(j\omega)\} = \arg\left\{\frac{1}{(j\omega)^N}\right\} = \arg\{1\} - \arg\{(j\omega)^N\}$$

$$\boxed{\arg\{H(j\omega)\} = -90^\circ N}$$



Derivador simple (ceros reales distintos de cero):

$$H(s) = \frac{s + c}{c}$$

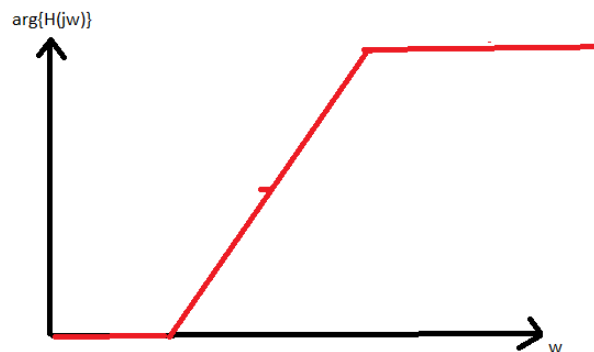
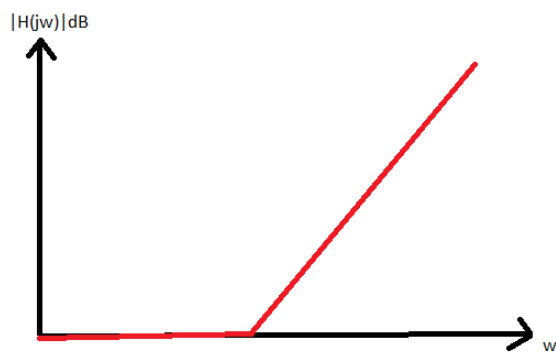
$$H(j\omega) = \frac{j\omega + c}{c}$$

$$|H(j\omega)| = \frac{|j\omega + c|}{|c|} = \frac{\sqrt{\omega^2 + c^2}}{|c|}$$

$$|H(j\omega)|_{dB} = 20 \log \left(\frac{\sqrt{\omega^2 + c^2}}{|c|} \right) = 20 \log (\sqrt{\omega^2 + c^2}) - 20 \log (|c|)$$

$$\arg \left\{ \frac{j\omega + c}{c} \right\} = \arg \{j\omega + c\} - \arg \{c\} = \operatorname{atan} \left(\frac{\omega}{c} \right)$$

ω	$ H(j\omega) _{dB}$	$\arg \left\{ \frac{j\omega + c}{c} \right\}^\circ$
(pequeña) $\omega \approx 0$	0	0
$\omega = c$	3.01	45
(grande) $\omega \approx \infty$	$20 \log (\omega)$	90



Integrador simple (polos reales distintos de cero):

$$H(s) = \frac{p}{s + p}$$

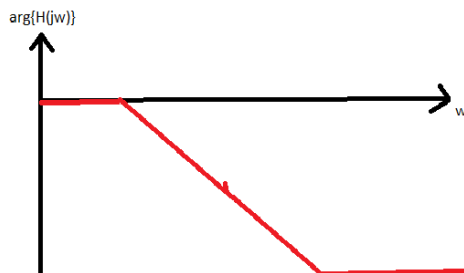
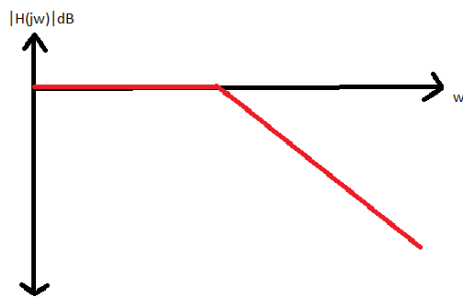
$$H(j\omega) = \frac{p}{j\omega + p}$$

$$|H(j\omega)| = \frac{|p|}{|j\omega + p|} = \frac{|p|}{\sqrt{\omega^2 + p^2}}$$

$$|H(j\omega)|_{dB} = 20\text{Log}\left(\frac{|p|}{\sqrt{\omega^2 + p^2}}\right) = 20\text{Log}(|p|) - 20\text{Log}(\sqrt{\omega^2 + p^2})$$

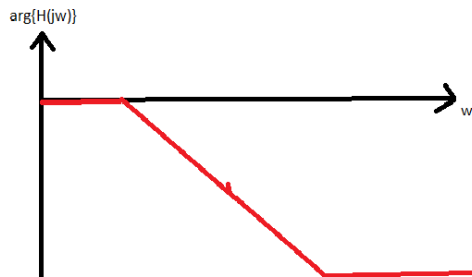
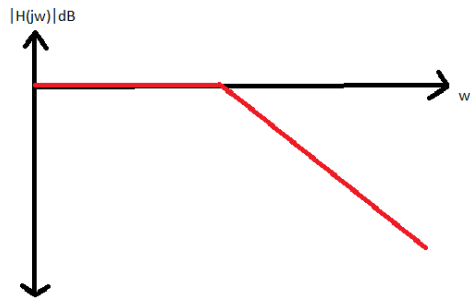
$$\arg\left\{\frac{p}{j\omega + p}\right\} = \arg\{p\} - \arg\{j\omega + p\} = -\text{atan}\left(\frac{\omega}{p}\right)$$

ω	$ H(j\omega) _{dB}$	$\arg\left\{\frac{j\omega + c}{c}\right\}^\circ$
(pequeña) $\omega \approx 0$	0	0
$\omega = p$	-3.01	-45
(grande) $\omega \approx \infty$	$-20\text{Log}(\omega)$	-90



Polos complejos conjugados:

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta_o\omega_n s + \omega_n^2}$$



Ceros complejos conjugados:

$$H(s) = \frac{s^2 + 2\zeta_u \omega_n s + \omega_n^2}{\omega_n^2}$$

