

# Paper Review - Auto-FuzzyJoin: Auto-Program Fuzzy Similarity Joins Without Labelled Examples

Peng Li, Xiang Cheng, Xu Chu, Yeye He, Surajit Chaudhuri

Carmel Gafa

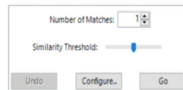
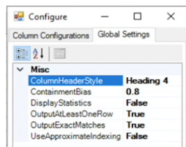
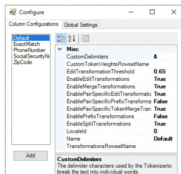
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# Fuzzy-join (or similarity join)



- Fuzzy join takes two tables as inputs and identify record pairs that refer to the same entity
- As an example, l1 and r2 refer to the same person
- The concept can be extended to rows consisting of multiple columns

# Fuzzy-join configuration



- Fuzzy-join has been integrated many commercial applications
- These systems are normally not easy to use as they have a big number of configuration parameters
- The extension in Microsoft Excel has 19 options that span across 3 dialog boxes
  - 11 are binary, thus resulting in 2048 possible configuration scenarios
  - 8 continuous, such as thresholds and biases
- In order to execute quality Fuzzy-joins, these configurations must be set up properly by the user

# Theoretical foundation: fuzzy join

Given a **reference table**  $L$  and a table  $R$  containing records that may be **imprecise** or noisy, a **fuzzy join**  $J$  establishes approximate matches between them.

- $J$  connects elements of  $R$  to similar elements in  $L$  based on a chosen **similarity measure** (e.g., Levenshtein distance, cosine similarity, Jaccard similarity).
- Each record  $r \in R$  is mapped to at most one record  $l \in L$ , or **no match at all** (denoted by  $\perp$ ).
- The join is **many-to-one** because multiple records in  $R$  can be associated with the **same** record in  $L$ , but each  $r \in R$  has only **one** possible match.

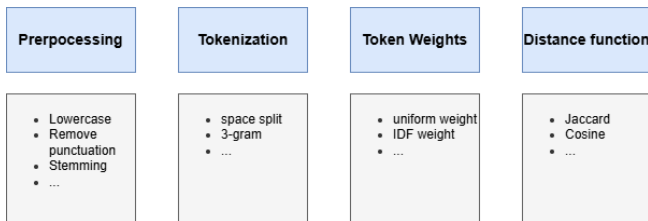
Formally:

$$J : R \rightarrow L \cup \perp$$

# Theoretical foundation: fuzzy join configuration space

A **fuzzy join**  $f$  compares two strings,  $r$  and  $l$ , by computing a distance score that reflects their similarity. The computation of this score is governed by a variety of parameters, forming a **parameter space**.

**Definition:** Each unique combination of these parameters defines a specific **join function**  $f \in \mathcal{F}$ , where  $\mathcal{F}$  is the space of all possible join functions.



# Example: Fuzzy Join Distance Score Computation

**Join Function:**  $f = (L, SP, EW, JD)$

- **L:** Lower-casing (Preprocessing)
- **SP:** Space Tokenization
- **EW:** Equal Weights
- **JD:** Jaccard Distance

**Inputs:**

- $l = \text{"2012 tigers lsu baseball team"}$
- $r = \text{"2012 lsu baseball team"}$

**Tokenization (SP):**

- $l \rightarrow \{2012, tigers, lsu, baseball, team\}$
- $r \rightarrow \{2012, lsu, baseball, team\}$

**Jaccard Distance:**

- $A \cap B = \{2012, lsu, baseball, team\} \rightarrow |A \cap B| = 4$
- $A \cup B = \{2012, tigers, lsu, baseball, team\} \rightarrow |A \cup B| = 5$
- Jaccard Similarity  $= \frac{4}{5} = 0.8$
- Jaccard Distance  $= 1 - 0.8 = 0.2$

**Result:**  $f(l, r) = 0.2$

# Theoretical foundation: threshold and join configuration

- Once the distance  $f(l, r)$  is computed:
  - It is compared to a threshold **compared to a threshold**  $\theta$  to decide whether to join the string pair  $l$  and  $r$ .
  - If  $f(l, r) \leq \theta$ , the pair is considered a **match**.
- Together, the function  $f$  and the threshold  $\theta$  define what the authors call a **join configuration**:

$$C = (f, \theta)$$

- This configuration encapsulates both:
  - How distance is computed.
  - When two strings are considered similar enough to be joined.

**Definition:** A join configuration  $C$  is a 2-tuple  $C = \langle f, \theta \rangle$ , where  $f \in \mathcal{F}$  is a join function, and  $\theta$  is a threshold.

We use  $\mathcal{S} = \{ \langle f, \theta \rangle \mid f \in \mathcal{F}, \theta \in \mathbb{R} \}$  to denote the space of join configurations.

# Theoretical foundation: fuzzy join mapping

Given two tables  $L$  and  $R$ , a join configuration  $C \in \mathcal{S}$  induces a **fuzzy join mapping**  $J_C$ , defined as:

$$J_C(r) = \arg \min_{l \in L, f(l,r) \leq \theta} f(l, r), \forall r \in R$$

That is

- For each record  $r \in R$ , find  $l \in L$  that minimizes the distance  $f(l, r)$ , **only if** that distance is less than or equal to the threshold  $\theta$ .
- If no such  $l \in L$  exists such that  $f(l, r) \leq \theta$ , then  $J_C(r)$  is **empty** — i.e., no match for that record.



# Theoretical foundation: the problem with single join configurations

Real-world data can exhibit **multiple types of variations simultaneously**, such as:

- **Typos**
- **Missing tokens**
- **Extraneous information**

As a result, relying on a **single join configuration** often fails to capture all valid matches, particularly when high **recall** is required.

To handle this diversity, the algorithm uses a **set of join configurations**:

$$U = \{C_1, C_2, \dots, C_K\}$$

Instead of relying on a single configuration, the system computes join results from each one.

This approach allows the system to:

- Accommodate diverse types of variations.
- Improve overall recall by **combining multiple perspectives** on similarity.

L-id	L-Table (Reference Table)		R-id	R-Table (Input Table)
$l_1$	2008 LSU Tigers baseball team	↔	$r_1$	2008 LSU baseball team
$l_2$	2008 LSU Tigers football team	↔	$r_2$	2008 LSU football team
$l_3$	2008 Mississippi State Bulldogs baseball team	↔	$r_3$	2008 Mississippi State <b>Bulldog</b> baseball team
$l_4$	2008 Mississippi State Bulldogs football team	↔	$r_4$	2008 Mississippi State <b>Bulldog</b> football team
$l_5$	...		$r_5$	...
$l_6$	2007 LSU Tigers football team	✗	$r_6$	2007 LSU Tigers baseball team
$l_7$	2007 Wisconsin Badgers football team	✗	$r_7$	2008 Wisconsin Badgers football team
$l_8$	2011 LSU Tigers football team	✗	$r_8$	2010 LSU Tigers football team
$l_9$	2007 LSU Tigers baseball team	✗	$r_9$	2007 LSU Tigers football team

- A **Jaccard distance** with threshold 0.2 works well for pairs like  $(l_1, r_1)$ , which differ by only one or two tokens.
- However, for pairs like  $(l_3, r_3)$  with **spelling variations**, Jaccard similarity is not enough:
  - Jaccard distance  $\approx 0.5 \rightarrow$  too high to match under the 0.2 threshold
  - A more suitable metric is **Edit Distance**, which can better align such pairs.

# Theoretical foundation: fuzzy join via multiple configurations

- To handle this diversity, the algorithm uses a **set of join configurations**:

$$U = \{C_1, C_2, \dots, C_K\}$$

- Instead of relying on a single configuration, the system computes join results from each.
- This approach allows the system to:
  - Accommodate diverse types of variations.
  - Improve overall recall by **combining multiple perspectives** on similarity.

Given  $L$  and  $R$ , a set of join configurations  $U = \{C_1, C_2, \dots, C_K\}$  induces a **fuzzy join mapping**  $J_U$ , defined as:

$$J_U(r) = \bigcup_{C_i \in U} J_{C_i}(r), \forall r \in R$$

This means that the overall result of the fuzzy join using configuration set  $U$  is the **union** of results from all individual configurations  $C_i \in U$ .

Each configuration  $C_i \in U$  is designed to capture a **specific type of string variation** (e.g., typos, missing tokens, extra tokens).

Two records are considered **joined by the set  $U$**  if and only if they are joined by **at least one** configuration  $C_i \in U$ .

- Each configuration contributes **high-quality joins** targeted at particular data challenges.
- The overall join is more **robust and comprehensive**.

## Evaluating Join Quality: Precision

Given two tables  $R$  and  $L$ , and a **space of join configurations**  $S$ , the objective is to find a subset  $U \subseteq S$  that produces **good fuzzy join results**.

Let:

- $J_U$  be the fuzzy join mapping induced by configuration set  $U$
- $J_G$  be the **ground truth** join mapping — the ideal join result

**Precision** measures how many of the predicted joins are correct:

$$\text{precision}(U) = \frac{\underbrace{|\{r \in R \mid J_U(r) \neq \emptyset, J_U(r) = J_G(r)\}|}_{\text{True Positives (TP)}}}{\underbrace{|\{r \in R \mid J_U(r) \neq \emptyset\}|}_{\text{TP + FP (all predicted joins)}}}$$

- **Numerator (TP)**: Records where a join was predicted and it matched the ground truth.
- **Denominator (TP + FP)**: All records where a join was predicted (correct or not).

# Evaluating Join Quality: Recall

**Recall** measures how many of the correct (ground truth) joins were successfully predicted:

$$\text{recall}(U) = \underbrace{|\{r \in R \mid J_U(r) \neq \emptyset, J_U(r) = J_G(r)\}|}_{\text{True Positives (TP)}}$$

- This is the **absolute count of True Positives**, i.e., records for which:
  - A join was predicted ( $J_U(r) \neq \emptyset$ ), and
  - It matches the ground truth ( $J_U(r) = J_G(r)$ )

**False Negatives (FN)** — cases where a correct join was missed — are defined as:

$$\text{FN} = |\{r \in R \mid J_G(r) \neq \emptyset, J_U(r) = \emptyset\}|$$

*Note:* The denominator  $TP + FN$  is constant across all  $U$  for a fixed dataset, so it is omitted in comparisons.

# Estimating Precision/Recall for a Single Join Configuration

Given:

- A **single join configuration**  $C = \langle f, \theta \rangle$
- Two tables:
  - $L$ : reference table
  - $R$ : query table

**Assumption: Complete Reference Table  $L$**

- $L$  is assumed to contain **all possible true matches** for records in  $R$ .
- Ensures that for each  $r \in R$ , there exists a correct match  $l \in L$ .
- Simplifies analysis by reducing the chance of missing true positives due to an incomplete reference.

**Geometric View of the Distance Function  $f$**

- Join function  $f$  embeds records into a **metric space**.
- Records are modeled as points on a **unit grid**.
- Each  $l \in L$  is surrounded by **close variants** (differing by a token, character, etc.).

**Analogy: Stars and Planets**

- Reference records  $l \in L$  are like **stars** on a grid.
- Query records  $r \in R$  are like **planets** that orbit these stars.
- Identifying the best join  $J_C(r)$  is like determining **which star a planet orbits**.

# Safe Joins and the Geometry of Fuzzy Matching

## Safe Joins with a Complete $L$

- Define the **grid width**  $w$ : typical distance between a record  $l$  and its closest neighbors in  $L$ .
- A join is considered **safe** if the distance  $d = f(l, r)$  satisfies:

$$d < \frac{w}{2}$$

- This guarantees that  $r$  lies **closer to its true match**  $l$  than to any other reference point.

## Why This Matters:

- Ensures high **precision** — avoiding false positives caused by ambiguous joins.
- Avoids joining  $r$  to an incorrect  $l'$  that lies at a similar distance.

## Analogy: Stars and Planets

- A planet that lies **equidistant** between two stars (at  $\frac{w}{2}$  each) **cannot be confidently claimed by either**.
- In fuzzy joining, such cases are inherently **ambiguous** and risky to resolve.

# Estimating Join Precision (Local Heuristic)

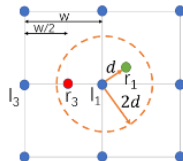
Given a query record  $r \in R$  and its closest match  $l \in L$ , with distance  $d = f(l, r)$ , we can estimate how **precise** this join is — i.e., how likely it is that  $(l, r)$  is a **correct match**.

- The **more candidate records** in  $L$  that are close to  $r$ , the **less confident** we are about any one being the true match.
- So we count how many other records  $l' \in L$  fall within the  **$2d$ -ball** centered at  $l$ :

$$\text{precision}(l, r) = \frac{1}{\underbrace{|\{l' \in L \mid f(l, l') \leq 2f(l, r)\}|}_{\text{TP} + \text{FP (local competitors)}}}$$

- A small  $2d$ -ball  $\rightarrow$  high precision (few competitors)
- A large  $2d$ -ball  $\rightarrow$  low precision (many competitors)

This provides a **data-driven estimate** of join quality **without needing ground truth**.



- To join  $r_1$ , we find the closest  $l \in L$ , say  $l_1$
- Compute the distance:  $d = f(l_1, r_1)$
- Draw a ball of radius  $2d$  centered at  $l_1$ 
  - If no other  $L$  records fall in the ball  $\rightarrow$  high confidence
- In this case, the  $2d$ -ball contains only  $l_1$ :

$$\text{precision}(l_1, r_1) = \frac{1}{1} = 1$$

- **High confidence join**

# When $L$ is Incomplete

**Problem:** When  $L$  is incomplete (i.e., some records are missing):

- Missing records in  $L$  result in **missing stars** in the grid.
- A record  $r$  may join to the wrong  $l$ , causing **false positives** and reducing **precision**.
- Example: If  $r_2$  should match with  $l_2$  (but  $l_2$  is missing), it might instead match  $l_1$  using  $d = f(r_2, l_1)$ .

**Note:**

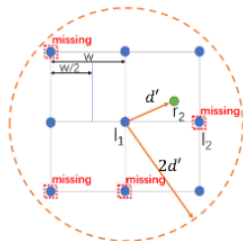
- Even if some records in  $L$  are missing, **safe decisions** can still be made.
- In 2D figure, can tolerate up to **7 out of 11** missing.

**Precision estimation:**

- $r_2$  should match  $l_2$  (missing), so  $l_1$  becomes the fallback.
- The  $2d'$ -ball around  $l_1$  contains **5 records**.
- Precision:

$$\text{precision}(l_1, r_2) = \frac{1}{5}$$

- $\Rightarrow$  **Low confidence join**



(b)  $l_1$  is the closest left record to  $r_2$ , since  $l_2$  is missing from  $L$ . We can infer that  $(l_1, r_2)$  is not a "safe" join, because we find many  $L$  records in the ball of  $2d'$ .

- $r_2$  should join with  $l_2$ , but  $l_2$  is **missing**
- $l_1$  becomes the closest available record
- Compute distance  $d' = f(l_1, r_2)$
- Draw a  $2d'$ -ball around  $l_1$ 
  - If the ball includes many other  $L$  records  $\Rightarrow d'$  is too **lax**
  - Join becomes **unreliable**



# Estimating Precision and Recall for a Configuration

A configuration  $C = \langle f, \theta \rangle$  includes:

- A join function  $f$
- A threshold  $\theta$

## 1. Local Precision for a Join

$$\text{precision}(r, C) = \frac{1}{|\{l' \in L \mid f(l, l') \leq 2f(l, r)\}|}$$

- $J_C(r) = l$ : join match for  $r \in R$
- Denominator = number of plausible alternatives

## 2. Expected True Positives

$$TP(C) = \sum_{r \in R, J_C(r) \neq \emptyset} \text{precision}(r, C)$$

## 3. Expected False Positives

$$FP(C) = \sum_{r \in R, J_C(r) \neq \emptyset} (1 - \text{precision}(r, C))$$

## 4. Overall Precision and Recall

$$\text{precision}(C) = \frac{TP(C)}{TP(C) + FP(C)} \quad \text{recall}(C) = \frac{TP(C)}{TP(C)}$$

*Note:* Recall is estimated absolutely since ground truth is unavailable.

# Precision and Recall Estimation: Example

**Setup:** Assume 3 records in  $R$ , joined to  $L$  using configuration  $C = \langle f, \theta \rangle$ .

**Join Results:**

- $J_C(r_1) = l_1, f(l_1, r_1) = 0.1$ , 5 plausible matches  $\Rightarrow \text{precision}(r_1, C) = \frac{1}{5} = 0.20$
- $J_C(r_2) = l_2, f(l_2, r_2) = 0.05$ , 2 plausible matches  $\Rightarrow \text{precision}(r_2, C) = \frac{1}{2} = 0.50$
- $J_C(r_3) = l_3, f(l_3, r_3) = 0.2$ , 4 plausible matches  $\Rightarrow \text{precision}(r_3, C) = \frac{1}{4} = 0.25$

**Estimated TP and FP:**

$$TP(C) = 0.20 + 0.50 + 0.25 = 0.95$$

$$FP(C) = (1 - 0.20) + (1 - 0.50) + (1 - 0.25) = 2.05$$

**Estimated Precision and Recall:**

$$\text{precision}(C) = \frac{0.95}{0.95 + 2.05} = \frac{0.95}{3.00} \approx 0.317$$

$$\text{recall}(C) = TP(C) = 0.95$$

*Note:* This example assumes no ground truth; hence recall is based on expected TP count.

# Precision and Recall for a Set of Configurations

Let  $U = \{C_1, C_2, \dots, C_K\}$  be a set of configurations.

## Case 1: No Conflicts in $U$

- Each record  $r \in R$  is matched by at most one configuration:

$$\forall r \in R, \quad |J_U(r)| \leq 1$$

- Then:

$$TP(U) = \sum_{C \in U} TP(C), \quad FP(U) = \sum_{C \in U} FP(C)$$

## Case 2: Conflicting Assignments in $U$

- Multiple configurations suggest different joins for the same  $r$
- Resolve conflicts by:
  - 1 Compare precision scores:  $\text{precision}(r, C_i)$  vs.  $\text{precision}(r, C_j)$
  - 2 Choose the match with higher precision
  - 3 Assign that join to  $J_U(r)$
  - 4 Recompute  $TP(U)$  and  $FP(U)$

## Final Estimates:

$$\text{precision}(U) = \frac{TP(U)}{TP(U) + FP(U)} \quad \text{recall}(U) = \frac{TP(U)}{TP(U) + FP(U)}$$

# Example: Resolving Conflicting Joins from Multiple Configurations

**Context:** Two configurations propose different joins for the same record  $r \in R$  using different string similarity methods.

## Configurations:

- $C_1 = \langle f_1, \theta_1 \rangle$ , where  $f_1$  uses Jaccard distance over space-tokenized lowercase strings with equal weights.
- $C_2 = \langle f_2, \theta_2 \rangle$ , where  $f_2$  uses Cosine similarity over character trigrams with TF-IDF weighting.

## Join Proposals for $r$ :

- $C_1$ :  $J_{C_1}(r) = l_1$  with  $\text{precision}(r, C_1) = \frac{1}{4} = 0.25$
- $C_2$ :  $J_{C_2}(r) = l_2$  with  $\text{precision}(r, C_2) = \frac{1}{2} = 0.50$

## Conflict Resolution Strategy:

- 1 Compare estimated precision:

$$\text{precision}(r, C_1) = 0.25 < \text{precision}(r, C_2) = 0.50$$

- 2 Assign  $J_U(r) = l_2$  (higher-confidence match from  $C_2$ )

## Effect:

- $TP(U)$  and  $FP(U)$  incorporate only the winning match.
- Competing matches are discarded.

# Auto-FuzzyJoin Algorithm: Single Column Case

**Problem:** Recall-Maximizing Fuzzy Join (RM-FJ) is NP-hard.

**Solution:** Use a **greedy approximation algorithm** called AutoFJ.

**Objective:**

- Maximize recall:  $TP(U)$
- Subject to: maintain precision  $(U) \geq \tau$

**Greedy Strategy:**

- Select configurations that:
  - Increase true positives (recall)
  - Minimize false positives (preserve precision)
- Guided by the **Profit Metric**:

$$\text{profit}(U) = \frac{TP(U)}{FP(U)}$$

**Blocking Heuristic:**

- Apply 3-gram-based blocking:

$LL, LR \leftarrow$  apply blocking on  $L-L, L-R$

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## Algorithm 1 AutoFJ for single column

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**Require:** Tables  $L$  and  $R$ , precision target  $\tau$ , search space  $S$

```
1:  $LL, LR \leftarrow$  apply blocking with  $L-L$  and  $L-R$ 
2:  $LR \leftarrow$  Learn negative-rules from  $LL$  and apply rules on  $LR$  (Alg. 2)
3: Compute distance with different join functions  $f \in S$ 
4: Pre-compute precision estimation for each configuration  $C \in S$ 
5:  $U \leftarrow \emptyset$ 
6: while  $S \setminus U \neq \emptyset$  do
7:    $\text{max\_profit} \leftarrow 0$ 
8:   for all  $C \in S \setminus U$  do
9:     if  $\text{profit}(U \cup \{C\}) > \text{max\_profit}$  then
10:       $C^* \leftarrow C, \text{max\_profit} \leftarrow \text{profit}(U \cup \{C\})$ 
11:   if  $\text{precision}(U \cup \{C^*\}) > \tau$  then
12:      $U \leftarrow U \cup \{C^*\}$ 
13:   else
14:     break
15: return  $U$ 
```

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# Blocking Example: 3-Grams and TF-IDF

Reference Table  $L$ :

$l_1$	"john smith"
$l_2$	"jane smythe"
$l_3$	"alice johnson"

Query Record  $r_1$ : "jon smyth"

## Step 1: Preprocessing (P)

- Lowercasing (already lowercase)
- Add padding for 3-grams: e.g., "jon"  $\rightarrow$  "##jon#"

## Step 2: Tokenization (T)

- $r_1$ : ##j, #jo, jon, on , n s, sm, smy, myt, yth, th#
- $l_1, l_2$ : similar 3-gram sequences

## Step 3: Token Weighting (W)

- Use TF-IDF to emphasize rare, meaningful trigrams (e.g., smy, yth)
- $r_1-l_2$ : **High score**,  $r_1-l_1$ : Medium,  $r_1-l_3$ : Zero

## Blocking Result:

- Only compare  $r_1$  with  $l_1, l_2 \rightarrow$  prune  $l_3$

# Optimization: Filtering with Negative Rules

*Assumption: Although 3-gram blocking may have pruned  $l_3$ , we assume here it was retained due to weak overlap, allowing us to illustrate negative-rule filtering.*

**Goal:** Use **obvious non-matches** in  $L-L$  to learn rules that help **filter unlikely  $L-R$  pairs** before costly distance computations.

**Step 1: Generate  $LL$  — Self-Join on  $L$  using 3-gram blocking**

Pair	Shared 3-grams	Interpretation
$l_1$ vs $l_2$	sm, smy, th	Possibly similar
$l_1$ vs $l_3$	jo, on	Clearly different
$l_2$ vs $l_3$	Weak overlap	Probably different

**Learn Negative Rule:**

*"If 3-gram overlap  $\leq 2$ , treat as a non-match."*

Step 2: Apply Rule on $LR$ Candidate Pairs	Pair	Overlap	Apply Rule?	Keep?
	$r_1, l_1$	$\sim 4$	No	Yes
	$r_1, l_2$	$\sim 5$	No	Yes
	$r_1, l_3$	$\sim 1$	Yes	No

**Effect:** *Filter out clearly irrelevant pairs early — no need to compute Jaccard or Edit Distance!*

# Compute Distances: Apply Join Functions

Once candidate pairs are identified (via blocking and optional negative rules), we compute the actual similarity using multiple join functions  $f \in \mathcal{S}$ .

Each join function is defined by:

- Preprocessing (e.g., lowercasing, punctuation removal)
- Tokenization (e.g., char 3-grams, word tokens)
- Token weights (e.g., TF-IDF)
- Distance function (e.g., Jaccard, Cosine, Edit)

Example Candidate Pairs (after blocking):

$r$ (query)	$l$ (reference)
"jon smyth"	"john smith"
"jon smyth"	"jane smythe"

Join Functions in $\mathcal{S}$ :	Function $f$	Tokenizer	Distance	Description
	$f_1$	char 3-grams	Jaccard	Overlap in token sets
	$f_2$	char 3-grams	Cosine (TF-IDF)	Weighted similarity
	$f_3$	raw string	Levenshtein	Edit distance

Computed Scores:	Pair	$f_1$	$f_2$	$f_3$
	jon vs john	0.4	0.5	2
	jon vs jane	0.6	0.7	3

Note: Distances may follow different scales — lower often means more similar.



# Start of Greedy Algorithm

Initialize:  $U \leftarrow \emptyset$

$U$  will hold the **selected join configurations**:

$$C = \langle f, \theta \rangle$$

Each configuration includes:

- A join function  $f \in \mathcal{F}$  (defined by  $P, T, W, D$ )
- A distance threshold  $\theta$  (max allowed distance for a match)

**Goal:**

- Select a subset  $U \subseteq \mathcal{S}$  from all candidate configurations
- Maximize recall:  $TP(U)$
- Maintain precision:  $\text{precision}(U) \geq \tau$

**Example: Precomputed Configuration Set  $\mathcal{S}$**

Config $C$	Description
$C_1 = \langle f_1, 0.37 \rangle$	Jaccard distance with $\theta = 0.37$
$C_2 = \langle f_2, 0.42 \rangle$	Cosine distance with $\theta = 0.42$
$C_3 = \langle f_3, 2 \rangle$	Edit distance with $\theta = 2$

*These  $\theta$  values were selected based on prior precision–recall evaluation for each  $f$ .*

# Main Greedy Loop

Main Loop: while  $S \setminus U \neq \emptyset$  do

We continue as long as there are still unused configurations to consider.

**Notation:**

- $S$ : full set of candidate configurations, each  $C = \langle f, \theta \rangle$
- $U$ : set of selected configurations
- $S \setminus U$ : unused configurations

**At each iteration:**

- 1 Evaluate each  $C \in S \setminus U$
- 2 Compute profit: how many true positives vs. false positives it contributes
- 3 Select the best configuration  $C^*$
- 4 If  $\text{precision}(U \cup \{C^*\}) \geq \tau$ :

$$U \leftarrow U \cup \{C^*\}$$

**Example State:**

- $S = \{\langle f_1, 0.37 \rangle, \langle f_2, 0.42 \rangle, \langle f_3, 2 \rangle\}$
- $U = \emptyset$

*Loop continues while there are remaining candidates and precision can be preserved.*