Low-energy parameterizations of X_{max} statistics

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Based on GAP-2020-058

Motivations

- Our goal is to broaden the scope of the combined fit in order to cover lower energies down to E $\gtrsim 10^{15}$ eV $\rightarrow$ Galactic-ExtraGalactic transition
- Existing parametrizations for calculating cosmic ray composition using X_{max} statistics are constrained to energies above E $\gtrsim 10^{17}$ eV (see GAP2020_058)
- It's crucial to assess whether these current models remain accurate for energies down to 10^{15} eV. Should discrepancies arise, we must consider refining our parametrization models to ensure reliable composition in the knee energy spectrum.

Conex simulations

From GAP2020_058:

- Conex version: version 4.37 for EPOS-LHC and QGSJet II-04, version 7.30 for Sibyll 2.3d
- Hadronic models used Sibyll 2.3d, EPOS-LHC and QGSJet II-04
- The log energy range 17 $\rightarrow$ 20 in 13 fixed lg E bins with $\Delta \log \mathrm{E} = 0.25$
- Number of showers 5.4k - 7.7k / bin
- The Xmax used to build the distributions is taken from the **XmxdEdX** branch of the CONEX file
- Primary nuclei: H, He, N, Si, Ca and Fe

New simulations at CNAF:

- Conex version: version 7.60
- Hadronic models used Sibyll 2.3d, EPOS-LHC, QGSJet II-04, and DPM-JET III
- The log energy range 15 $\rightarrow$ 20.5 in 23 fixed lg E bins with $\Delta \log \mathrm{E} = 0.25$
- Number of showers 10k / bin $\rightarrow$ 50k / bin
- The Xmax used to build the distributions is taken from the **XmxdEdX** branch of the CONEX file
- Primary nuclei: H, He, N, Si, and Fe

Definitions

- Mean:

$$\langle X \rangle = \frac{1}{N} \sum_{i=0}^{N} X_i \tag{1}$$

- Variance:

$$\sigma_X^2 = \frac{1}{N} \sum_{i=0}^{N} |X_i - \langle X \rangle|^2 \tag{2}$$

- Standard Deviation:

$$\sigma_X = \sqrt{\sigma_X^2} \tag{3}$$

- Error of the Mean:

$$\epsilon = \frac{\sigma_X}{\sqrt{N}} \tag{4}$$

- Error of the Standard Deviation:

$$\rho = \frac{\sigma_X}{\sqrt{2N}} \tag{5}$$

X_{max} parametrizations

- We model X_{max} as a function of $x \equiv \log(E/E_0)$ and $y \equiv \ln A$
- GAP parametrization (4 free parameters):

$$\begin{aligned} p'_0(y) &= p_0 + \alpha y \\ p'_1(y) &= p_1 + \beta y \\ f(x,y) &= p'_0(y) + p'_1(y)x \end{aligned}$$

which can be re-written as

$$f(x,y) = (p_0 + p_1 x) + (\alpha + \beta x)y \equiv f_p(x) + f_E(x)y$$

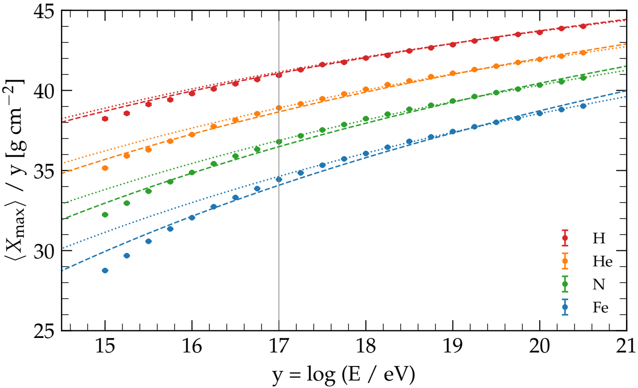
- EXT parametrization (6 free parameters):

$$\begin{aligned} p'_0(y) &= p_0 + \alpha y \\ p'_1(y) &= p_1 + \beta y \\ p'_2(y) &= p_2 + \gamma y \\ f(x,y) &= p'_0(y) + p'_1(y)x + p'_2(y)x^2 \end{aligned}$$

which can be re-written as

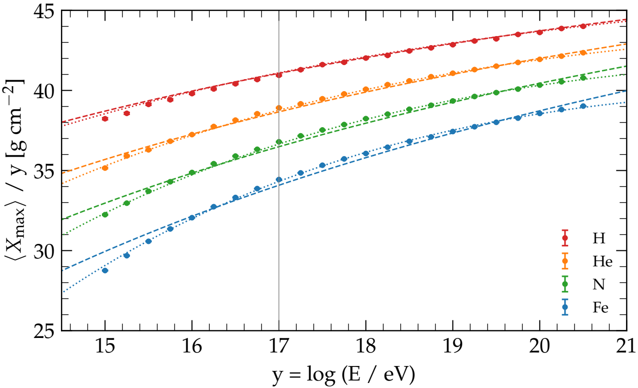
$$f(x,y) = (p_0 + p_1 x + p_2 x^2) + (\alpha + \beta x + \gamma x^2)y \equiv f_p(x) + f_E(x)y$$

X_{max} parametrizations



- Dots: CONEX simulations using Sibyll-23d. Error bars are the **std error of the mean** over N simulations
- In the y-axis we show X_{max}/y to emphasize the deviation from the linear evolution
- Dashed lines: best fit of GAP parametrization assuming $E_{min} = 10^{15}$ eV
- Dotted lines: best fit of GAP parametrization assuming $E_{min} = 10^{17}$ eV

X_{max} parametrizations



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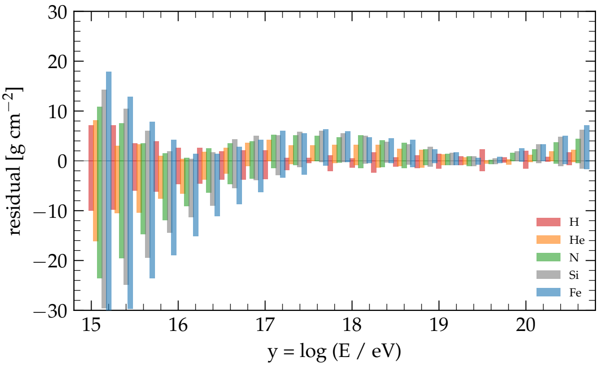
- In the y-axis we show X_{max}/y to emphasize the deviation from the linear evolution
- Dashed lines: best fit of GAP parametrization assuming $E_{min} = 10^{15}$ eV
- Dotted lines: best fit of EXT parametrization assuming $E_{min} = 10^{15}$ eV

X_{max} parametrizations

Fit parameters based on Sibyll-2.3D.

	D_0	D_1	D_2	α	β	γ
GAP-full	815.69 $\pm$ 0.11	58.76 $\pm$ 0.06	-	-26.37 $\pm$ 0.03	1.57 $\pm$ 0.02	-
GAP-hi	815.83 $\pm$ 0.11	58.09 $\pm$ 0.11	-	-26.19 $\pm$ 0.03	0.67 $\pm$ 0.03	-
EXT	816.21 $\pm$ 0.12	58.01 $\pm$ 0.11	-0.37 $\pm$ 0.04	-25.67 $\pm$ 0.04	0.56 $\pm$ 0.03	-0.46 $\pm$ 0.01

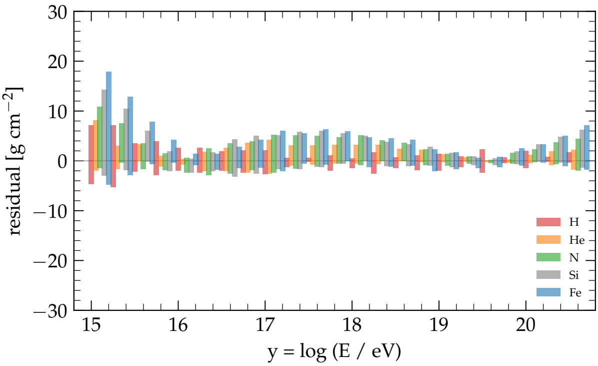
X_{max} residuals



- Positive plane: residuals of the GAP parametrization assuming $E_{min} = 10^{15}$ eV
- Negative plane: residuals of the GAP parametrization assuming $E_{min} = 10^{17}$ eV
- Mean residuals in g/cm^2. Si was not included in the fit:

	H	He	N	Si	Fe
GAP-full	2.15	2.45	3.64	4.39	4.94
GAP-hi	3.07	2.95	4.56	5.71	7.11

X_{max} residuals



- Positive plane: residuals of the GAP parametrization assuming $E_{min} = 10^{15}$ eV
- Negative plane: residuals of the EXT parametrization assuming $E_{min} = 10^{15}$ eV
- Mean residuals in g/cm^2. Si was not included in the fit:

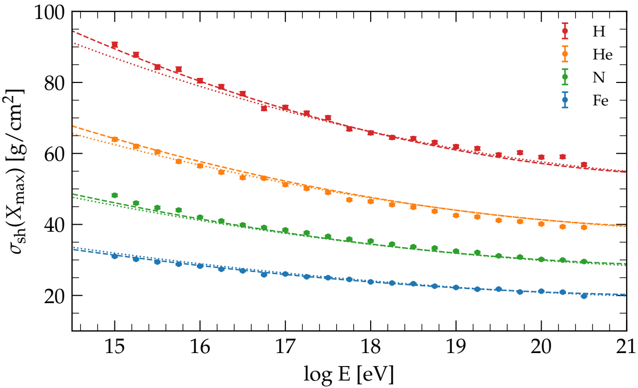
	H	He	N	Si	Fe
GAP	2.15	2.45	3.64	4.39	4.94
EXT	1.96	0.98	1.21	1.35	1.37

$\sigma(X_{max})$ parametrizations

- We model $\sigma(X_{max})$ as a function of $x \equiv \log(E/E_0)$ and $y \equiv \ln A$
- GAP parametrization (6 free parameters):

$$
\begin{aligned}
a'_0(x) &= a_0 + a_1 x \\
p(x) &= p_0 + p_1 x + p_2 x^2 \\
f(x,y) &= p(x)\left[1 + a'_0(x) y + b_0 y^2\right]
\end{aligned}
$$

$\sigma(X_{max})$ parametrizations



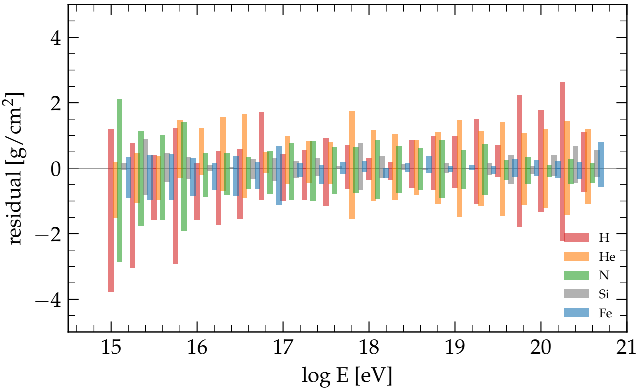
- Dots: CONEX simulations using Sibyll-23d. Error bars are the **error of the variance** over N simulations
- Dashed lines: best fit of GAP parametrization assuming $E_{min} = 10^{15}$ eV
- Dotted lines: best fit of GAP parametrization assuming $E_{min} = 10^{17}$ eV

$\sigma^2(X_{max})$ parametrizations

	p_0	p_1	p_2	a_0	a_1	b
full	61.0 ± 0.1	-4.5 ± 0.1	0.67 ± 0.02	-0.223 ± 0.001	0.0008 ± 0.0001	0.0161 ± 0.0002
hi	61.3 ± 0.1	-4.3 ± 0.1	0.53 ± 0.07	-0.228 ± 0.001	-0.0002 ± 0.0002	0.0173 ± 0.0003

Fit parameters based on Sibyll-2.3D.

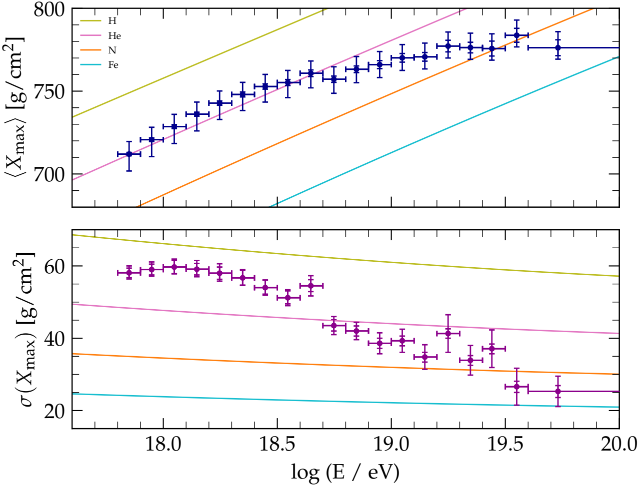
$\sigma(X_{max})$ residuals



- Positive plane: residuals of the GAP parametrization assuming $E_{min} = 10^{15}$ eV
- Negative plane: residuals of the GAP parametrization assuming $E_{min} = 10^{17}$ eV
- Mean residuals in g/cm^2. Si was not included in the fit:

	H	He	N	Si	Fe
GAP full	0.97	1.08	0.68	0.29	0.26
GAP hi	1.34	0.93	0.92	0.25	0.45

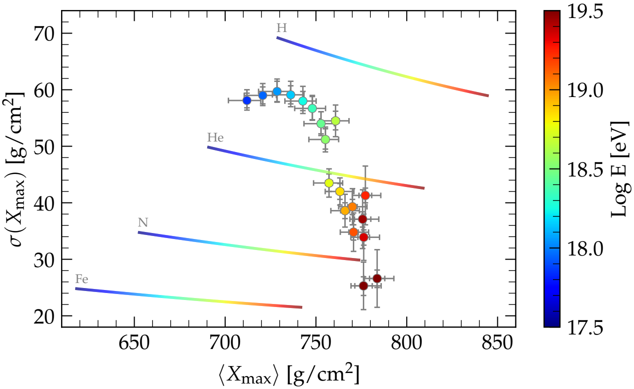
Comparison with Auger data



Based on Sibyll-2.3D.

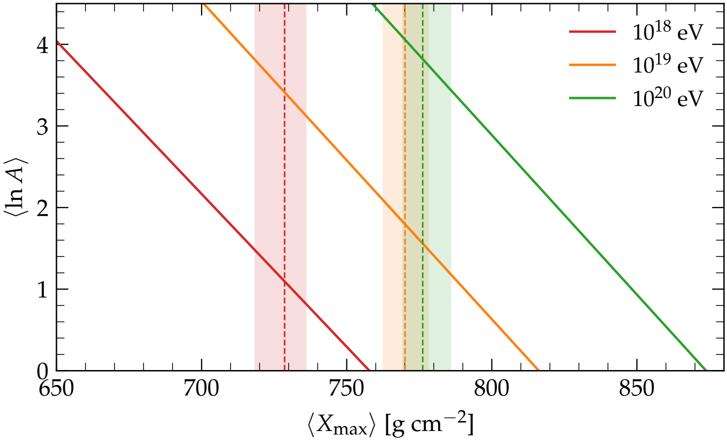
P.Auger Coll., ICRC 2019, internal use only

Comparison with Auger data



Based on Sibyll-2.3D.
P.Auger Coll., ICRC 2019, internal use only

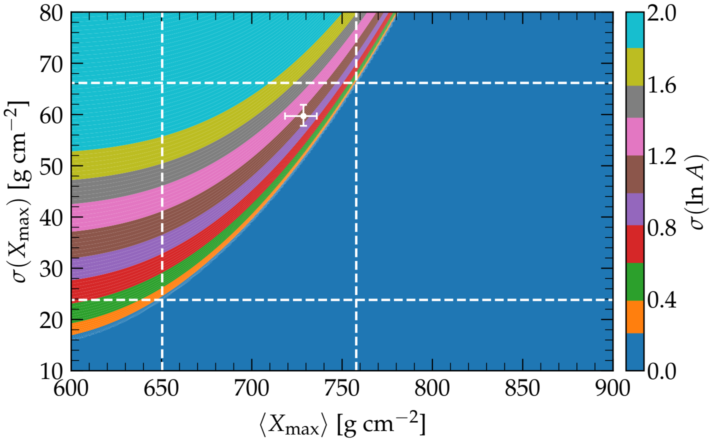
Comparison with Auger data



Based on Sibyll-2.3D.

P.Auger Coll., ICRC 2019, internal use only

Comparison with Auger data



$\mathrm{E} = 10^{18}$ eV

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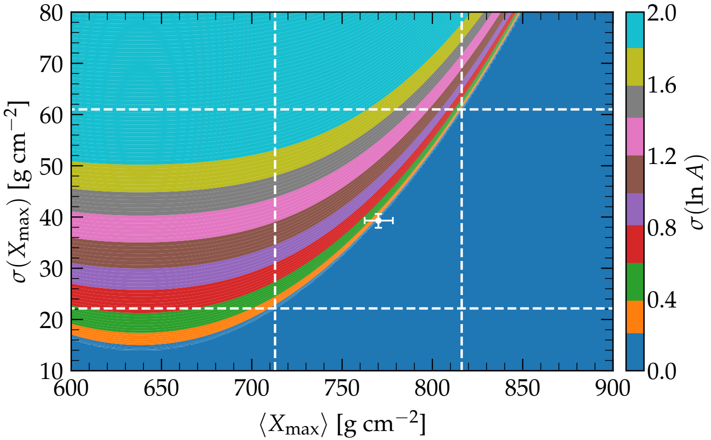
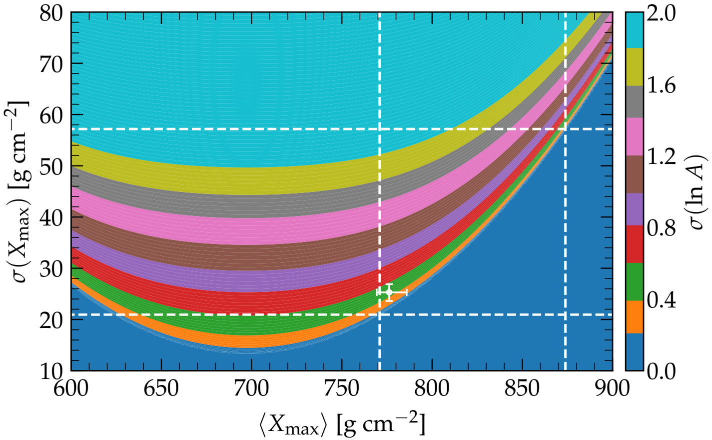
Comparison with Auger data



$\mathrm{E} = 10^{19}$ eV

P.Auger Coll., ICRC 2019, internal use only

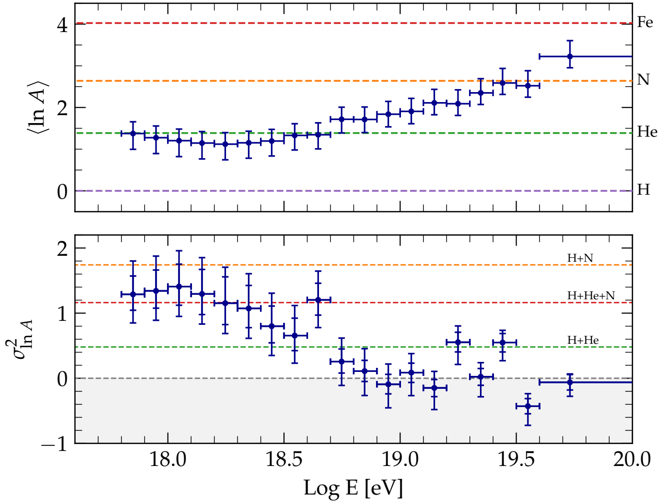
Comparison with Auger data



$E = 10^{20}$ eV

P.Auger Coll., ICRC 2019, internal use only

ln A moments in Auger data



Based on Sibyll-2.3D.

P.Auger Coll., ICRC 2019, internal use only

Variance of lnA in Auger data

- There are two independent sources of fluctuations: the intrinsic shower-to-shower fluctuations and the lnA dispersion arising from the mass distribution:

$$\sigma^2(\mathrm{X_{max}}) = \langle\sigma^2_{\mathrm{sh}}\rangle + \left(\frac{\mathrm{d}\langle \mathrm{X_{max}}\rangle}{\mathrm{d}\ln \mathrm{A}}\right)^2 \sigma^2_{\ln \mathrm{A}} = \langle\sigma^2_{\mathrm{sh}}\rangle + \mathrm{f}^2_{\mathrm{E}}\sigma^2_{\ln \mathrm{A}} \tag{6}$$

- We assume a parameterization for σ^2_{sh} as follows

$$\sigma^2_{\mathrm{sh}}(\ln \mathrm{A}) = \sigma^2_{\mathrm{p}}\left[1 + \mathrm{a}\ln \mathrm{A} + \mathrm{b}(\ln \mathrm{A})^2\right] \tag{7}$$

therefore

$$\langle\sigma^2_{\mathrm{sh}}\rangle = \sigma^2_{\mathrm{p}}\left[1 + \mathrm{a}\langle\ln \mathrm{A}\rangle + \mathrm{b}\langle(\ln \mathrm{A})^2\rangle\right] \tag{8}$$

- After substitution

$$\sigma^2(\mathrm{X_{max}}) = \sigma^2_{\mathrm{p}}\left[1 + \mathrm{a}\langle\ln \mathrm{A}\rangle + \mathrm{b}\langle(\ln \mathrm{A})^2\rangle\right] + \mathrm{f}^2_{\mathrm{E}}\sigma^2_{\ln \mathrm{A}} \tag{9}$$

- We apply the definition

$$\langle(\ln \mathrm{A})^2\rangle = \sigma^2_{\ln \mathrm{A}} + \langle\ln \mathrm{A}\rangle^2 \tag{10}$$

- As a result we arrive at

$$\sigma^2_{\ln \mathrm{A}} = \frac{\sigma^2(\mathrm{X_{max}}) - \sigma^2_{\mathrm{sh}}(\langle\ln \mathrm{A}\rangle)}{\mathrm{b}\sigma^2_{\mathrm{p}} + \mathrm{f}^2_{\mathrm{E}}} \tag{11}$$

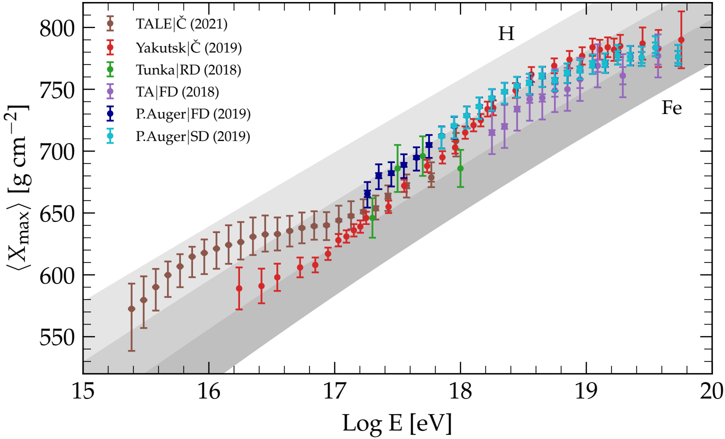
Measurements of the mean of the Xmax distribution

id	Experiment	Mode	Ref.	Table	comment
54	PAO	FD	ICRC 2019	✌	PAO public data
180	PAO	RD	UHECR 2023	✘	
193	PAO	SD	ICRC 2019	✌	PAO internal data(?)
75	TA	FD	ApJ 2018	✌	Tab. 4 but data corrected as done in Ref. 2(?)
143	TALE	Č	ApJ 2021	✌	Tab. 5 is bias-corrected(?)
178	Tunka	Č	ICRC 2021	✘	
182	Tunka	RD	PRD 2018	✌	Tab. 3
179	Yakutsk	Č	ASR 2019	✌	Tab. 2+3
183	Yakutsk	RD	ICRC 2019	✘	
181	LOFAR	radio	PRD 2021	✘	
832	Hi-Res/Mia		ApJ 2001	✘	

Datasets cited in the Snowmass paper.

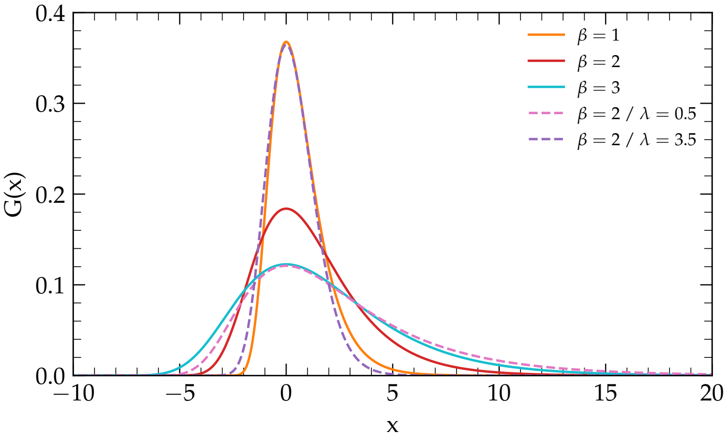
Mean lnA in other datasets

Work in progress...



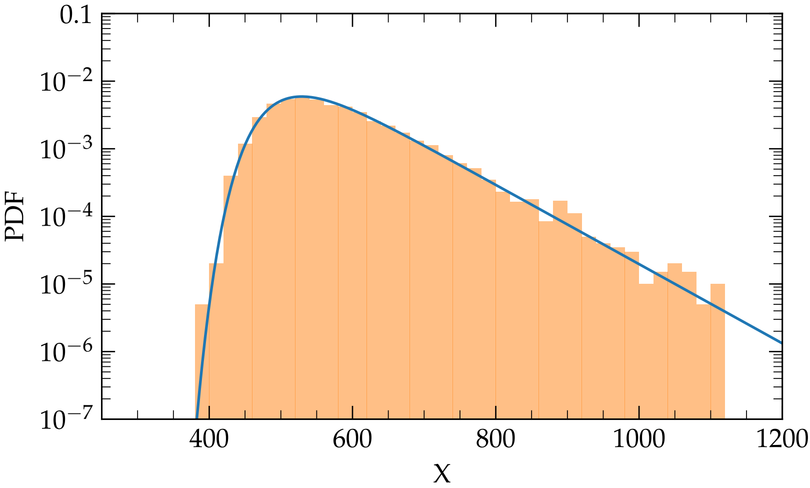
Based on Sibyll-2.3D.

Gumbel function



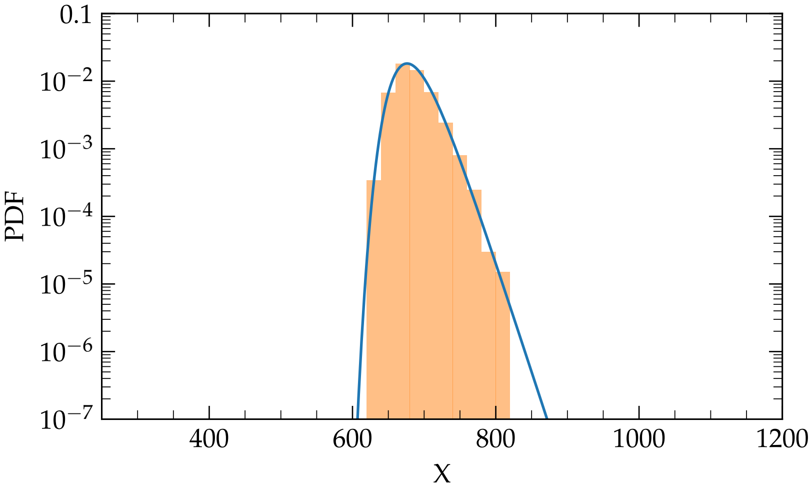
$$G(z) = \frac{1}{\beta} \frac{\lambda^\lambda}{\Gamma(\lambda)} e^{-\lambda(z + e^{-z})}$$

Gumbel function



Mass: H, Energy: 10^{16} eV

Gumbel function



Mass: H, Energy: 10^{20} eV

Parametrizations

- μ as a function of $x \equiv \log(E/E_0)$ and $y \equiv \ln A$

$$\begin{aligned} p_0^\mu(y) &= a_0^\mu + a_1^\mu y + a_2^\mu y^2 \\ p_1^\mu(y) &= b_0^\mu + b_1^\mu y + b_2^\mu y^2 \\ p_2^\mu(y) &= c_0^\mu + c_1^\mu y + c_2^\mu y^2 \\ \mu(x,y) &= p_0^\mu(y) + p_1^\mu(y)x + p_2^\mu(y)x^2 \end{aligned}$$

- σ as a function of $x \equiv \log(E/E_0)$ and $y \equiv \ln A$

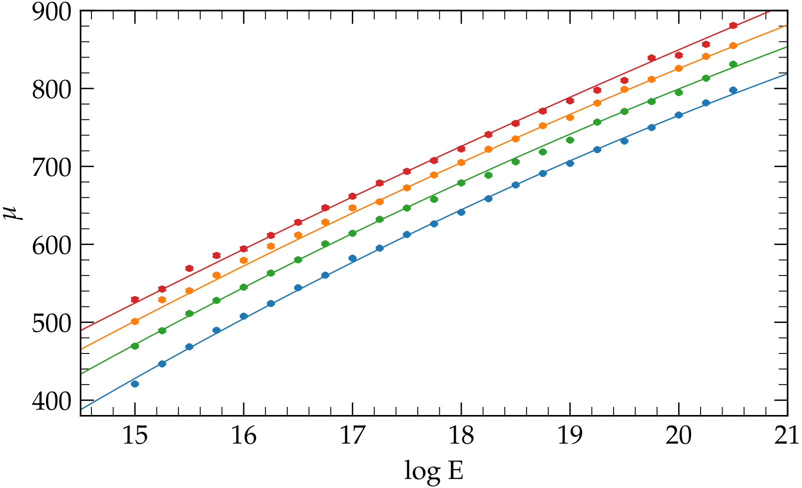
$$\begin{aligned} p_0^\sigma(y) &= a_0^\sigma + a_1^\sigma y + a_2^\sigma y^2 \\ p_1^\sigma(y) &= b_0^\sigma + b_1^\sigma y + b_2^\sigma y^2 \\ \sigma(x,y) &= p_0^\sigma(y) + p_1^\sigma(y)x + p_2^\sigma(y)x^2 \end{aligned}$$

- λ as a function of $x \equiv \log(E/E_0)$ and $y \equiv \ln A$

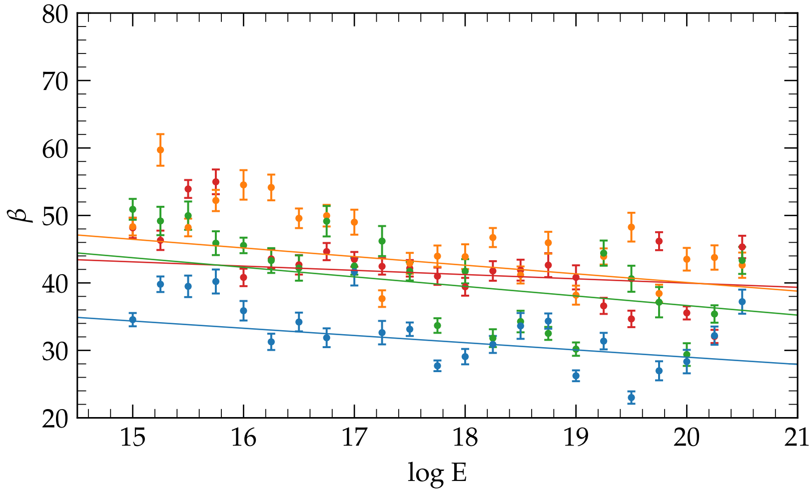
$$\begin{aligned} p_0^\lambda(y) &= a_0^\lambda + a_1^\lambda y + a_2^\lambda y^2 \\ p_1^\lambda(y) &= b_0^\lambda + b_1^\lambda y + b_2^\lambda y^2 \\ \lambda(x,y) &= p_0^\lambda(y) + p_1^\lambda(y)x + p_2^\lambda(y)x^2 \end{aligned}$$

- 21 free parameters. Cross fingers...

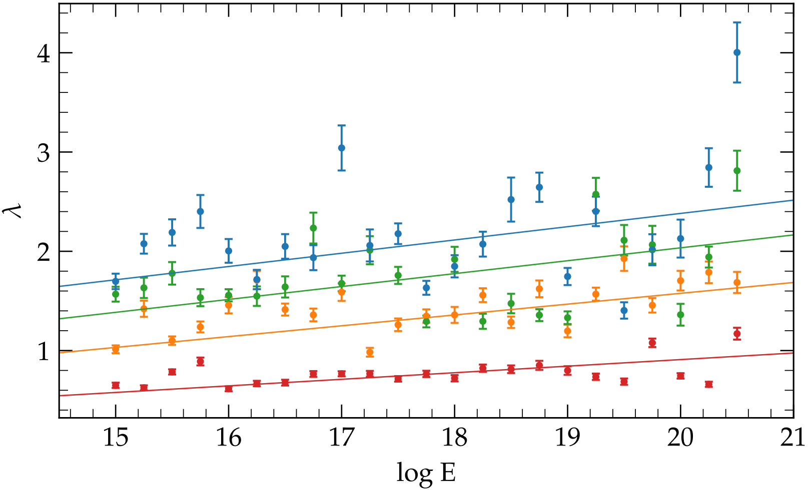
Gumbel function



Gumbel function



Gumbel function



Gumbel function

- Exponentially Modified Gaussian distribution:
- Generalized Gumbel distribution:
- Log-normal distribution:

From Luan B. Arbeletche, Vitor de Souza, Astroparticle Physics 116 (2020) 102389

Next Steps

- Calculating the parametrization parameters for 4 distinct HIMs $\longrightarrow$ assessing the variations among these models.
- We plan to augment our dataset of CONEX simulations by 10x. This increase in data volume aims to minimize the scatter in Gumbel parametrizations.
- We are actively testing additional parametrization methods rather than Gumbel functions.
- Analysis codes and simulated X_{max} databases will be public online on GitHub