Low-energy parameterizations of X_{max} statistics

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Based on GAP-2020-058

Motivations

- Our goal is to broaden the scope of the combined fit in order to cover lower energies down to E $\gtrsim 10^{15}$ eV $\rightarrow$ Galactic-ExtraGalactic transition
- Existing parametrizations for calculating cosmic ray composition using X_{max} statistics are constrained to energies above E $\gtrsim 10^{17}$ eV (see GAP2020_058)
- It's crucial to assess whether these current models remain accurate for energies down to 10^{15} eV. Should discrepancies arise, we must consider refining our parametrization models to ensure reliable composition in the knee energy spectrum.

Conex simulations

From GAP2020_058:

- Conex version: version 4.37 for EPOS-LHC and QGSJet II-04, version 7.30 for Sibyll 2.3d
- Hadronic models used Sibyll 2.3d, EPOS-LHC and QGSJet II-04
- The log energy range 17 $\rightarrow$ 20 in 13 fixed lg E bins with $\Delta \log \mathrm{E} = 0.25$
- Number of showers 5.4k - 7.7k / bin
- The Xmax used to build the distributions is taken from the **XmxdEdX** branch of the CONEX file
- Primary nuclei: H, He, N, Si, Ca and Fe

New simulations at CNAF:

- Conex version: version 7.60
- Hadronic models used Sibyll 2.3d, EPOS-LHC, QGSJet II-04, and DPM-JET III
- The log energy range 15 $\rightarrow$ 20.5 in 23 fixed lg E bins with $\Delta \log \mathrm{E} = 0.25$
- Number of showers 10k / bin $\xrightarrow{\text{goal}}$ 100k / bin
- The Xmax used to build the distributions is taken from the **XmxdEdX** branch of the CONEX file
- Primary nuclei: H, He, N, Si, and Fe

Definitions

- Mean:

$$\langle X \rangle = \frac{1}{N} \sum_{i=0}^{N} X_i \tag{1}$$

- Variance:

$$\sigma_X^2 = \frac{1}{N} \sum_{i=0}^{N} |X_i - \langle X \rangle|^2 \tag{2}$$

- Standard Deviation:

$$\sigma_X = \sqrt{\sigma_X^2} \tag{3}$$

- Error of the Mean:

$$\epsilon = \frac{\sigma_X}{\sqrt{N}} \tag{4}$$

- Error of the Standard Deviation:

$$\rho = \frac{\sigma_X}{\sqrt{2N}} \tag{5}$$

X_{max} parametrizations

- We model X_{max} as a function of $x \equiv \log(E/E_0)$ and $y \equiv \ln A$
- GAP parametrization (4 free parameters):

$$\begin{aligned} p'_0(y) &= p_0 + \alpha y \\ p'_1(y) &= p_1 + \beta y \\ f(x,y) &= p'_0(y) + p'_1(y)x \end{aligned}$$

which can be re-written as

$$f(x,y) = (p_0 + p_1 x) + (\alpha + \beta x)y \equiv f_p(x) + f_E(x)y$$

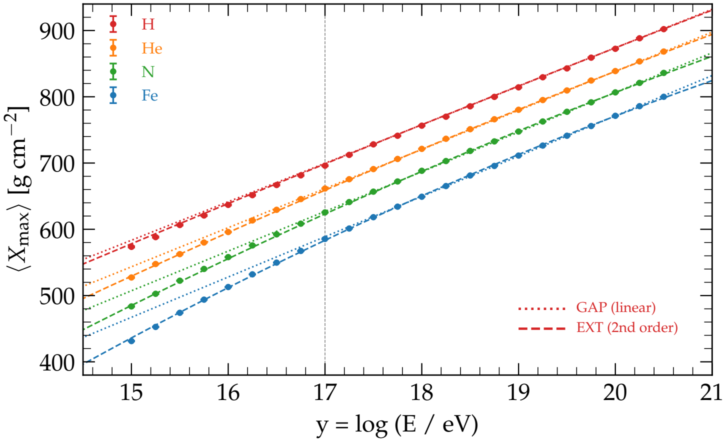
- EXT parametrization (6 free parameters):

$$\begin{aligned} p'_0(y) &= p_0 + \alpha y \\ p'_1(y) &= p_1 + \beta y \\ p'_2(y) &= p_2 + \gamma y \\ f(x,y) &= p'_0(y) + p'_1(y)x + p'_2(y)x^2 \end{aligned}$$

which can be re-written as

$$f(x,y) = (p_0 + p_1 x + p_2 x^2) + (\alpha + \beta x + \gamma x^2)y \equiv f_p(x) + f_E(x)y$$

X_{max} parametrizations



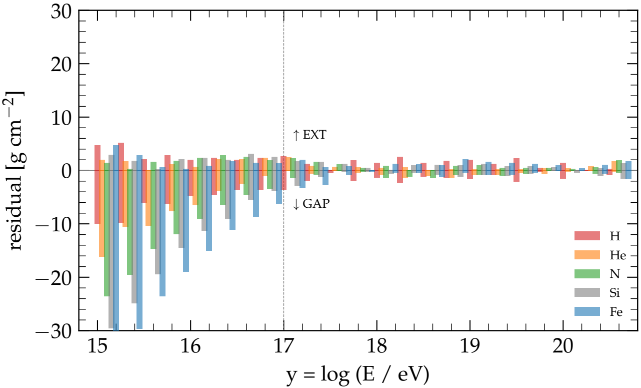
- Dots: CONEX simulations using Sibyll-23d. Error bars show the **std error of the mean** over N simulations
- Dotted lines: best fit of GAP (linear in y) parametrization assuming $E_{min} = 10^{17}$ eV
- Dashed lines: best fit of EXT (2nd order in y) parametrization assuming $E_{min} = 10^{15}$ eV

X_{max} parametrizations

Fit parameters based on Sibyll-2.3D.

	D_0	D_1	D_2	α	β	γ
GAP	815.83 ± 0.11	58.09 ± 0.11	-	-26.19 ± 0.03	0.67 ± 0.03	-
EXT	816.21 ± 0.12	58.01 ± 0.11	-0.37 ± 0.04	-25.67 ± 0.04	0.56 ± 0.03	-0.46 ± 0.01

X_{max} residuals



- Positive plane: residuals of the EXT parametrization assuming $E_{min} = 10^{15}$ eV
- Negative plane: residuals of the GAP parametrization assuming $E_{min} = 10^{17}$ eV
- Mean residuals in g/cm^2. Si was not included in the fit:

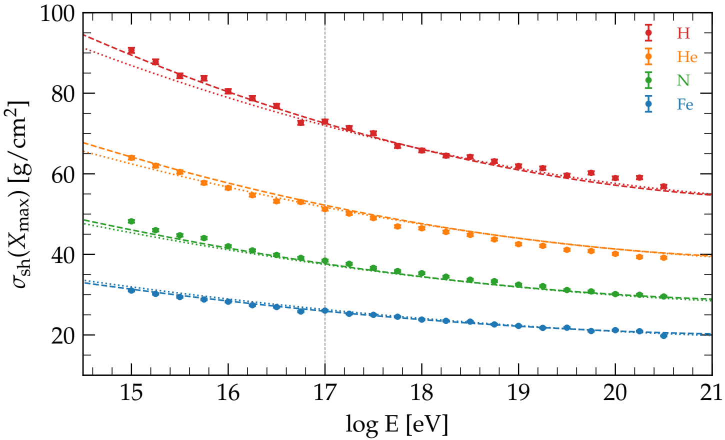
	H	He	N	Si	Fe
GAP	3.07	2.95	4.56	5.71	7.11
EXT	1.96	0.98	1.21	1.35	1.37

$\sigma(X_{max})$ parametrizations

- We model $\sigma(X_{max})$ as a function of $x \equiv \log(E/E_0)$ and $y \equiv \ln A$
- GAP parametrization (6 free parameters):

$$
\begin{aligned}
a_0'(x) &= a_0 + a_1 x \\
p(x) &= p_0 + p_1 x + p_2 x^2 \\
f(x, y) &= p(x) \left[1 + a_0'(x) y + b_0 y^2\right]
\end{aligned}
$$

$\sigma(X_{max})$ parametrizations



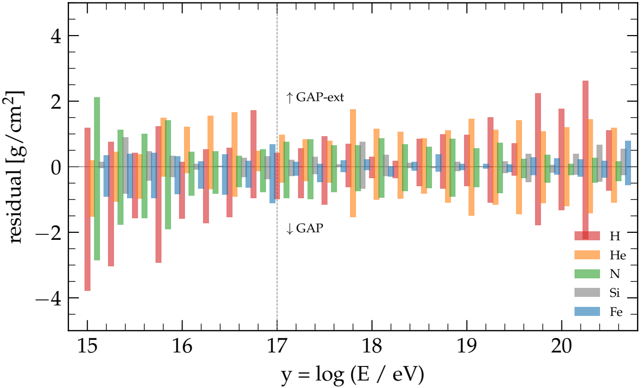
- Dots: CONEX simulations using Sibyll-23d. Error bars show the **error of the std deviation** over N simulations
- Dotted lines: best fit of GAP parametrization assuming $E_{min} = 10^{17}$ eV
- Dashed lines: best fit of GAP parametrization extended to $E_{min} = 10^{15}$ eV

$\sigma(X_{max})$ parametrizations

	p_0	p_1	p_2	a_0	a_1	b
GAP	61.3 $\pm$ 0.1	-4.3 $\pm$ 0.1	0.53 $\pm$ 0.07	-0.228 $\pm$ 0.001	-0.0002 $\pm$ 0.0002	0.0173 $\pm$ 0.0003
GAP-ext	61.0 $\pm$ 0.1	-4.5 $\pm$ 0.1	0.67 $\pm$ 0.02	-0.223 $\pm$ 0.001	0.0008 $\pm$ 0.0001	0.0161 $\pm$ 0.0002

Fit parameters based on Sibyll-2.3D.

$\sigma(X_{max})$ residuals



- Positive plane: residuals of the GAP parametrization assuming $E_{min} = 10^{15}$ eV
- Negative plane: residuals of the GAP parametrization assuming $E_{min} = 10^{17}$ eV
- Mean residuals in g/cm^2. Si was not included in the fit:

	H	He	N	Si	Fe
GAP	1.34	0.93	0.92	0.25	0.45
GAP-ext	0.97	1.08	0.68	0.29	0.26

Understanding Anomalous Shower Profiles

- Most air showers caused by high-energy cosmic rays follow a predictable pattern, showing a single, clearly defined maximum in their longitudinal development.
- A small subset exhibits deviations from this norm, with some showing significantly different profiles, including cases with two distinct maxima.
- We use simulations to analyze the frequency of these anomalous profiles as a function of the primary energy.
- We limit our investigation to primary protons (where anomalies are most pronounced) and apply a single interaction model for the moment.

Modeling the Longitudinal Shower Profile

- Gaisser–Hillas is a 3-parameter model to describe the normalized ($N_{max} = 1$) longitudinal profile of a shower. The function is given by:

$$y(X) = \left(1 + R\frac{X'}{L}\right)^{R^{-2}} \exp\left(-\frac{X'}{RL}\right) \tag{6}$$

  where

$$X' = X - X_{max} \tag{7}$$

- We evaluate the accuracy of our model using the χ^2:

$$\chi^2 = \sum_{Y_i > Y_{min}} \frac{[Y(X_i) - Y_i]^2}{V_i} \tag{8}$$

  where Y_i represents the normalized profile simulated by CONEX.

- We assume $V_i = (k/E)Y_i$ where k is a constant chosen **for each shower** such that $\sqrt{\sum V_i} / \sum Y_i = 0.01$ following

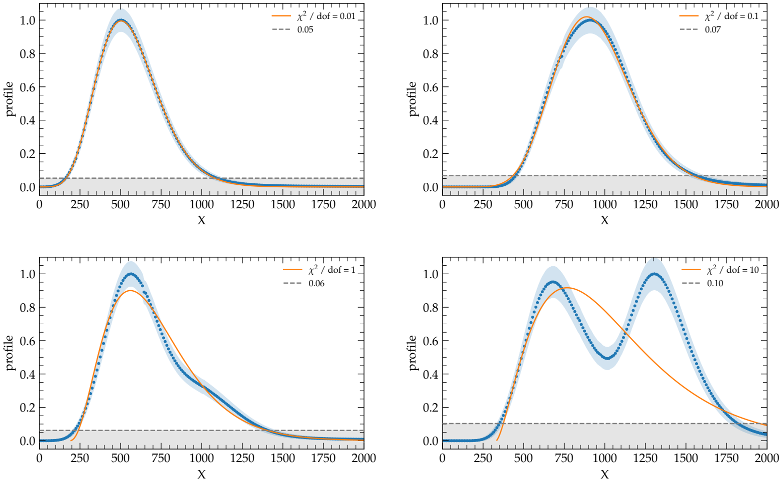
$$\frac{k}{E} = (0.01)^2 \frac{(\sum Y_i)^{\cancel{2}}}{\cancel{\sum Y_i}} = (0.01)^2 \sum Y_i \tag{9}$$

- Furthermore profile points where $\sqrt{V_i}/Y_i > 0.3$ are excluded from the fit which leads to a minimum value:

$$Y_{min} = \frac{(k/E)}{(0.3)^2} \tag{10}$$

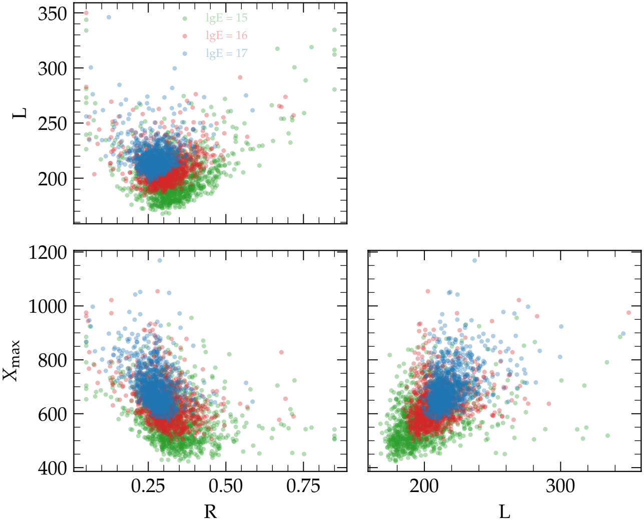
- To find the best fit parameters, (X_{max}, R, L), we perform a minimization of χ^2 starting from approximately 100 randomly selected initial points in the parameter space.

Shower examples

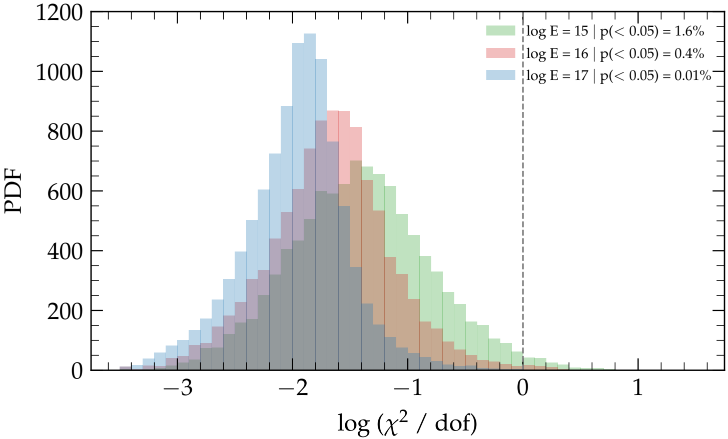


- We present four shower profiles with different best-fit values of $\chi^2 = 10^{-2}, 0.1, 1, 10$

Exploring the Parameter Space of Shower Profiles

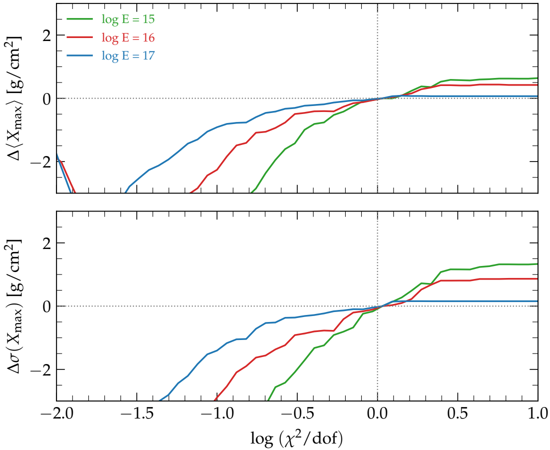


The Fraction of Anomalous Profiles



▷ For proton showers at 10^{15} eV the fraction of profiles with p-value < 0.05 is $\lesssim 2\%$

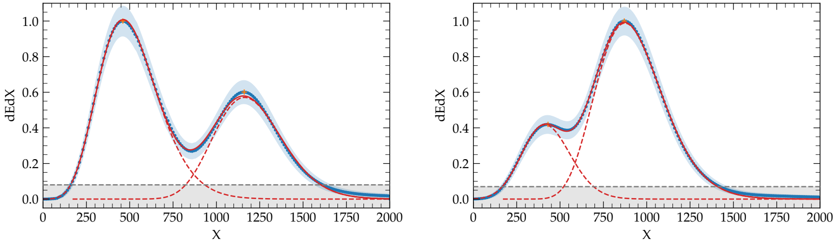
The impact on X_{max} statistics



- Upper plot: the mean X_{max} as a function of the maximum χ^2/dof allowed
- Lower plot: the X_{max} variance as a function of the maximum χ^2/dof allowed
- We conclude that the **maximum** impact of profiles with p-value < 0.05 for proton showers at E $= 10^{15}$ is $\lesssim 2$ gr/cm^2

Anomalous Profiles identification

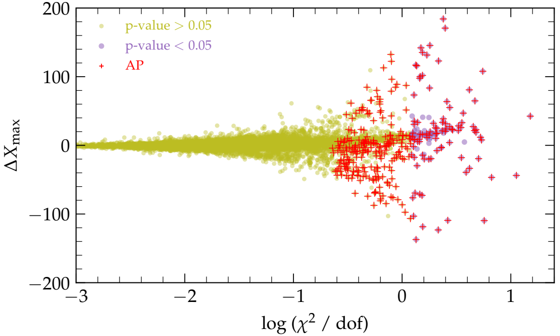
Following Baus+, 2011



Anomalous Profiles (APs) are identified when the following three conditions are satisfited:

- Two G-H functions significantly improve the goodness of the fit ($\Delta\chi^2 > 25$)
- The shower maxima are clearly separated ($|X_{\max,2} - X_{\max,1}| > 300$ g/cm^2)
- Both fitted sub-showers have more than 20% of the primary energy

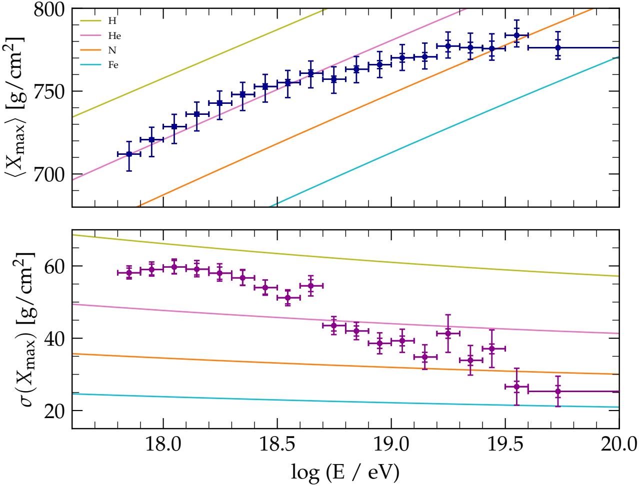
Anomalous Profiles identification



- ΔX_{max} is the difference of the best-fit X_{max} using the G-H fit and the X_{max} identified as the maximum of the shower
- ΔX_{max} increases on average for larger χ^2's
- Identified APs are about 3% for showers started by protons of 10^{15} eV, their impact is reported in the following table:

Energy	fraction of AP's	X_{max}	X_{max} without AP's	$\sigma(X_{max})$	$\sigma(X_{max})$ without AP's
15	3.25%	572.2	570.8	88.5	86.8
16	1.34%	635.8	634.8	79.1	78.0
17	0.26%	695.2	694.9	72.5	72.3

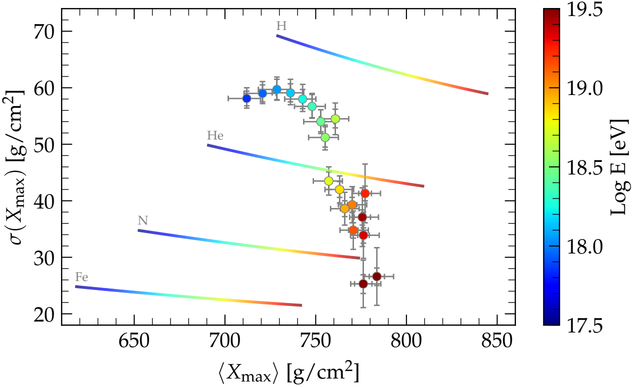
Comparison with Auger data



Based on Sibyll-2.3D.

P.Auger Coll., ICRC 2019, internal use only

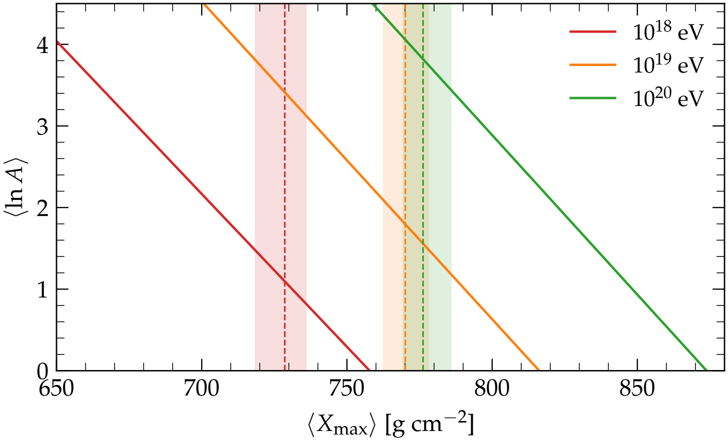
Comparison with Auger data



Based on Sibyll-2.3D.

P.Auger Coll., ICRC 2019, internal use only

Comparison with Auger data



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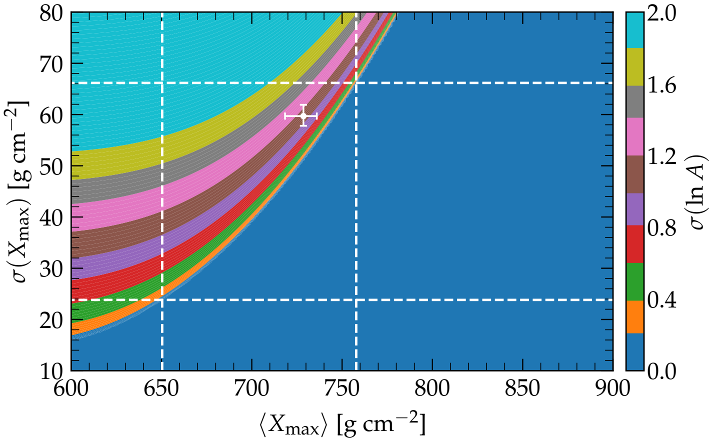
Comparison with Auger data



$E = 10^{18}$ eV

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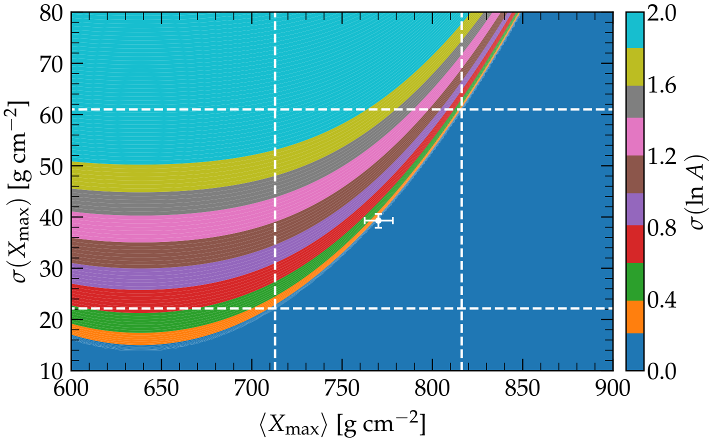
Comparison with Auger data



$E = 10^{19}$ eV

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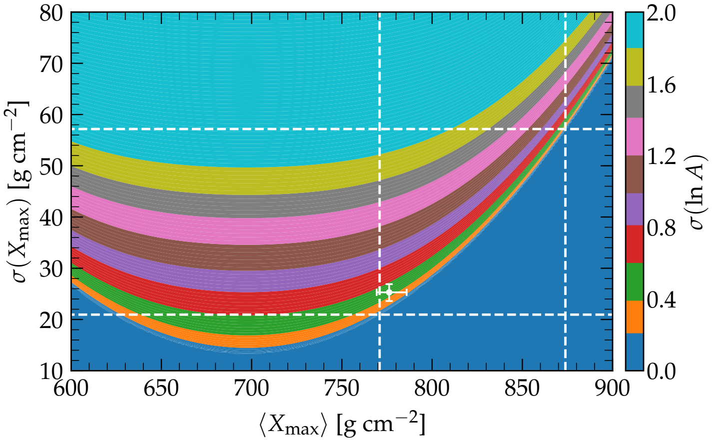
Comparison with Auger data



$E = 10^{20}$ eV

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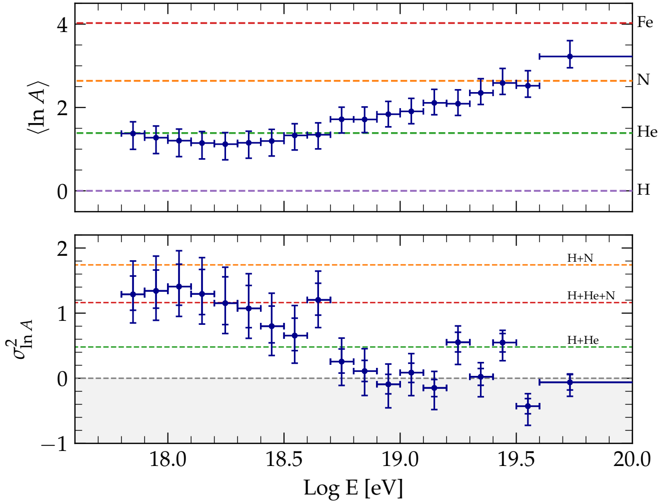
ln A **moments in Auger data**



Based on Sibyll-2.3D.

P.Auger Coll., ICRC 2019, internal use only

Variance of lnA in Auger data

- There are two independent sources of fluctuations: the intrinsic shower-to-shower fluctuations and the lnA dispersion arising from the mass distribution:

$$\sigma^2(X_{max}) = \langle \sigma_{sh}^2 \rangle + \left(\frac{d\langle X_{max} \rangle}{d \ln A} \right)^2 \sigma_{\ln A}^2 = \langle \sigma_{sh}^2 \rangle + f_E^2 \sigma_{\ln A}^2 \quad (11)$$

- We assume a parameterization for σ_{sh}^2 as follows

$$\sigma_{sh}^2(\ln A) = \sigma_p^2 \left[1 + a \ln A + b (\ln A)^2 \right] \quad (12)$$

therefore

$$\langle \sigma_{sh}^2 \rangle = \sigma_p^2 \left[1 + a \langle \ln A \rangle + b \langle (\ln A)^2 \rangle \right] \quad (13)$$

- After substitution

$$\sigma^2(X_{max}) = \sigma_p^2 \left[1 + a \langle \ln A \rangle + b \langle (\ln A)^2 \rangle \right] + f_E^2 \sigma_{\ln A}^2 \quad (14)$$

- We apply the definition

$$\langle (\ln A)^2 \rangle = \sigma_{\ln A}^2 + \langle \ln A \rangle^2 \quad (15)$$

- As a result we arrive at

$$\sigma_{\ln A}^2 = \frac{\sigma^2(X_{max}) - \sigma_{sh}^2(\langle \ln A \rangle)}{b \sigma_p^2 + f_E^2} \quad (16)$$

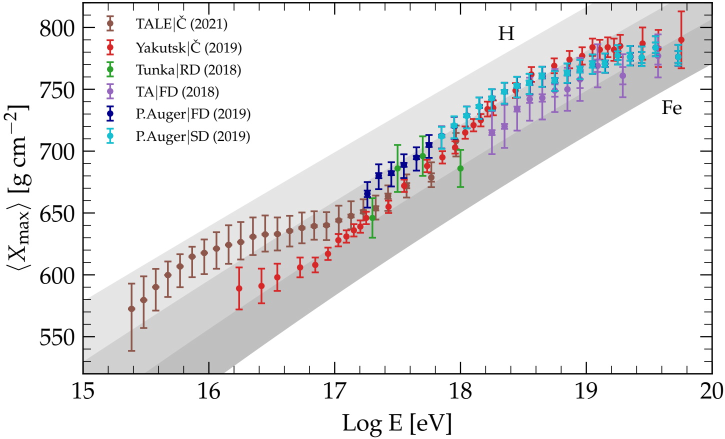
Measurements of the mean of the Xmax distribution

id	Experiment	Mode	Ref.	Table	comment
54	PAO	FD	ICRC 2019	✌	PAO public data
180	PAO	RD	UHECR 2023	✖	
193	PAO	SD	ICRC 2019	✌	PAO internal data(?)
75	TA	FD	ApJ 2018	✌	Tab. 4 but data corrected as done in Ref. 2(?)
143	TALE	Č	ApJ 2021	✌	Tab. 5 is bias-corrected(?)
178	Tunka	Č	ICRC 2021	✖	
182	Tunka	RD	PRD 2018	✌	Tab. 3
179	Yakutsk	Č	ASR 2019	✌	Tab. 2+3
183	Yakutsk	RD	ICRC 2019	✖	
181	LOFAR	radio	PRD 2021	✖	
832	Hi-Res/Mia		ApJ 2001	✖	

Datasets cited in the Snowmass paper.

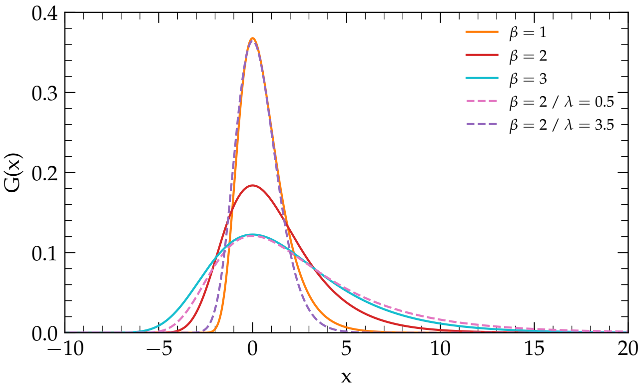
Mean lnA in other datasets

Work in progress...



Based on Sibyll-2.3D.

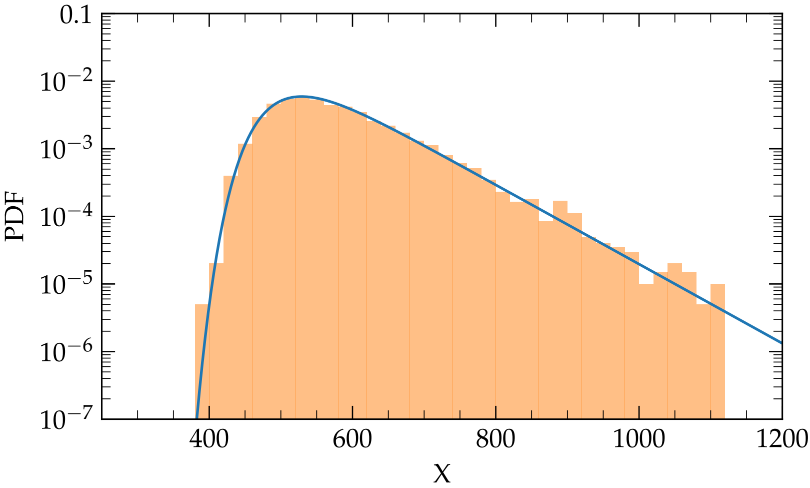
Gumbel function



$$z = \frac{x - \mu}{\beta} \quad , \quad G(z) = \frac{1}{\beta} \frac{\lambda^{\lambda}}{\Gamma(\lambda)} e^{-\lambda(z + e^{-z})}$$

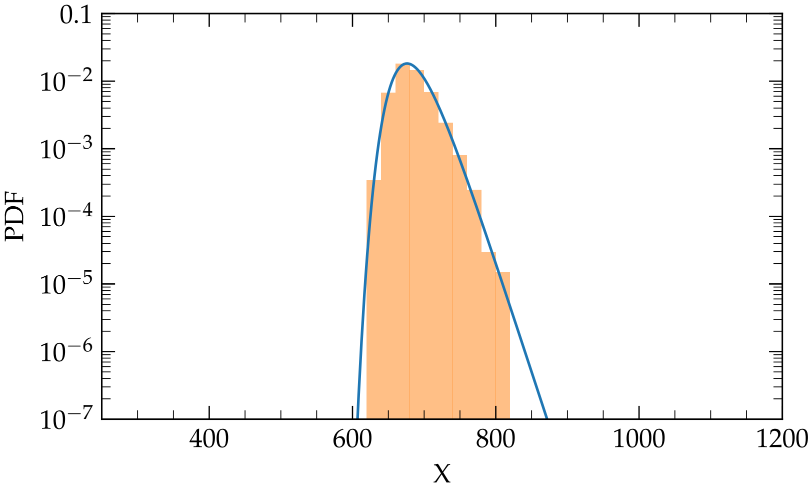
Goal: parametrize $\mu = \mu(x, y)$, $\beta = \beta(x, y)$, $\lambda = \lambda(x, y)$

Gumbel function



Mass: H, Energy: 10^{16} eV

Gumbel function



Mass: H, Energy: 10^{20} eV

Parametrizations

- μ as a function of $x \equiv \log(E/E_0)$ and $y \equiv \ln A$

$$
\begin{aligned}
p_0^\mu(y) &= a_0^\mu + a_1^\mu y + a_2^\mu y^2 \\
p_1^\mu(y) &= b_0^\mu + b_1^\mu y + b_2^\mu y^2 \\
p_2^\mu(y) &= c_0^\mu + c_1^\mu y + c_2^\mu y^2 \\
\mu(x,y) &= p_0^\mu(y) + p_1^\mu(y)x + p_2^\mu(y)x^2
\end{aligned}
$$

- β as a function of $x \equiv \log(E/E_0)$ and $y \equiv \ln A$

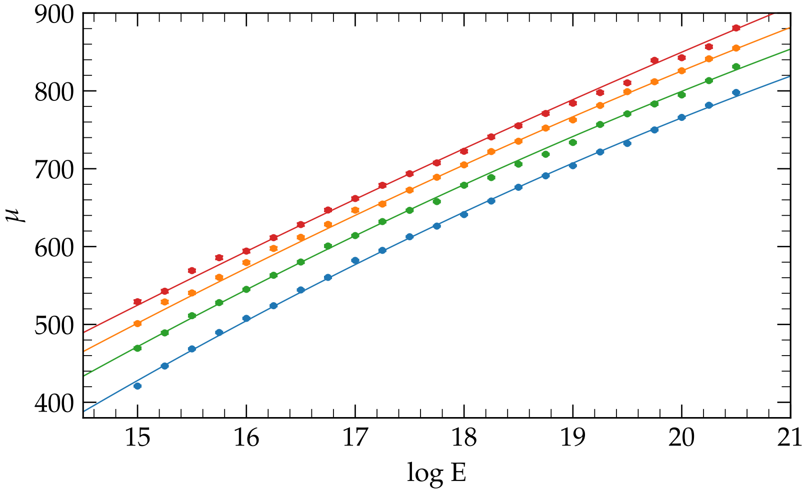
$$
\begin{aligned}
p_0^\beta(y) &= a_0^\beta + a_1^\beta y + a_2^\beta y^2 \\
p_1^\beta(y) &= b_0^\beta + b_1^\beta y + b_2^\beta y^2 \\
\beta(x,y) &= p_0^\beta(y) + p_1^\beta(y)x + p_2^\beta(y)x^2
\end{aligned}
$$

- λ as a function of $x \equiv \log(E/E_0)$ and $y \equiv \ln A$

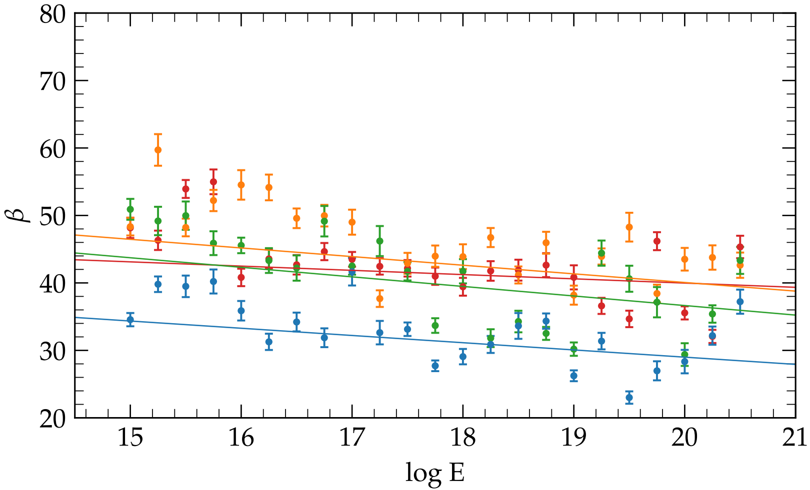
$$
\begin{aligned}
p_0^\lambda(y) &= a_0^\lambda + a_1^\lambda y + a_2^\lambda y^2 \\
p_1^\lambda(y) &= b_0^\lambda + b_1^\lambda y + b_2^\lambda y^2 \\
\lambda(x,y) &= p_0^\lambda(y) + p_1^\lambda(y)x + p_2^\lambda(y)x^2
\end{aligned}
$$

- 21 free parameters. Cross fingers...

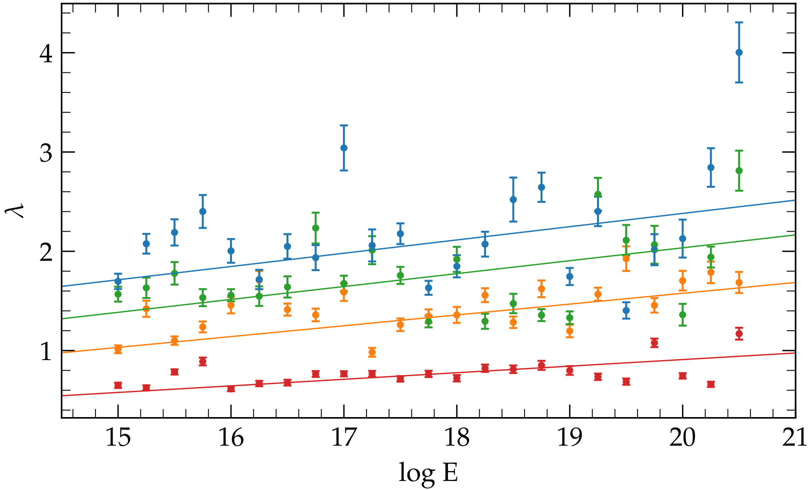
Gumbel function



Gumbel function



Gumbel function



Next Steps

- Calculating the parametrization parameters for 4 distinct HIMs $\longrightarrow$ assessing the variations among these models.
- We plan to augment our dataset of CONEX simulations by 10x. This increase in data volume aims to minimize the scatter in Gumbel parametrizations.
- We are actively testing additional parametrization methods rather than Gumbel functions (see, e.g., Luan B. Arbeletche, Vitor de Souza, Astroparticle Physics 116, 2020, 102389)
- We aim at providing parametrizations and simulated X_{max} templates public online on ZENODO