Low-energy parameterizations of $X_{\max}$ statistics

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Based on GAP-2020-058

Motivations

- Our goal is to broaden the scope of the combined fit in order to cover lower energies down to E $\gtrsim 10^{15}$ eV $\rightarrow$ Galactic-ExtraGalactic transition
- Existing parametrizations for calculating cosmic ray composition using X_{max} statistics are constrained to energies above E $\gtrsim 10^{17}$ eV (see GAP2020_058)
- It's crucial to assess whether these current models remain accurate for energies down to 10^{15} eV. Should discrepancies arise, we must consider refining our parametrization models to ensure reliable composition in the knee energy spectrum.

Conex simulations

From GAP2020_058:

- Conex version: version 4.37 for EPOS-LHC and QGSJet II-04, version 7.30 for Sibyll 2.3d
- Hadronic models used Sibyll 2.3d, EPOS-LHC and QGSJet II-04
- The log energy range 17 $\rightarrow$ 20 in 13 fixed lg E bins with $\Delta \log \mathrm{E} = 0.25$
- Number of showers 5.4k - 7.7k / bin
- The Xmax used to build the distributions is taken from the **XmxdEdX** branch of the CONEX file
- Primary nuclei: H, He, N, Si, Ca and Fe

New simulations at CNAF:

- Conex version: version 7.60
- Hadronic models used Sibyll 2.3d, EPOS-LHC, QGSJet II-04, and DPM-JET III
- The log energy range 15 $\rightarrow$ 20.5 in 23 fixed lg E bins with $\Delta \log \mathrm{E} = 0.25$
- Number of showers 10k / bin $\xrightarrow{\text{goal}}$ 100k / bin
- The Xmax used to build the distributions is taken from the **XmxdEdX** branch of the CONEX file
- Primary nuclei: H, He, N, Si, and Fe

Definitions

- Mean:

$$\langle X \rangle = \frac{1}{N} \sum_{i=0}^{N} X_i \tag{1}$$

- Variance:

$$\sigma_X^2 = \frac{1}{N} \sum_{i=0}^{N} |X_i - \langle X \rangle|^2 \tag{2}$$

- Standard Deviation:

$$\sigma_X = \sqrt{\sigma_X^2} \tag{3}$$

- Error of the Mean:

$$\epsilon = \frac{\sigma_X}{\sqrt{N}} \tag{4}$$

- Error of the Standard Deviation:

$$\rho = \frac{\sigma_X}{\sqrt{2N}} \tag{5}$$

$X_{\max}$ parametrizations

- We model $X_{\max}$ as a function of $x \equiv \log(E/E_0)$ and $y \equiv \ln A$
- GAP parametrization (4 free parameters):

$$
\begin{aligned}
p_0'(y) &= p_0 + \alpha y \\
p_1'(y) &= p_1 + \beta y \\
f(x,y) &= p_0'(y) + p_1'(y)x
\end{aligned}
$$

which can be re-written as

$$f(x,y) = (p_0 + p_1 x) + (\alpha + \beta x)y \equiv f_p(x) + f_E(x)y$$

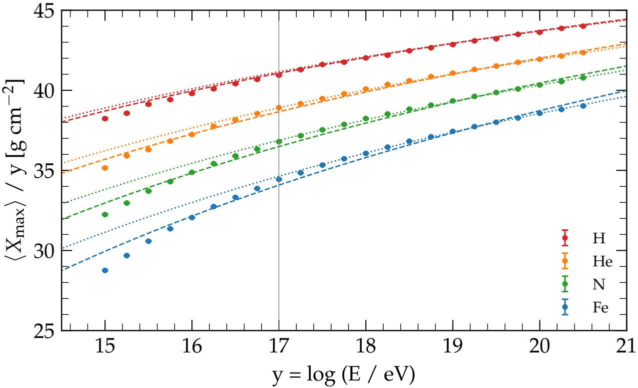
- EXT parametrization (6 free parameters):

$$
\begin{aligned}
p_0'(y) &= p_0 + \alpha y \\
p_1'(y) &= p_1 + \beta y \\
p_2'(y) &= p_2 + \gamma y \\
f(x,y) &= p_0'(y) + p_1'(y)x + p_2'(y)x^2
\end{aligned}
$$

which can be re-written as

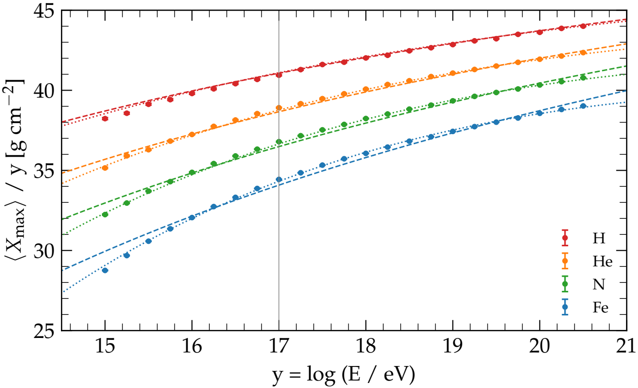
$$f(x,y) = (p_0 + p_1 x + p_2 x^2) + (\alpha + \beta x + \gamma x^2)y \equiv f_p(x) + f_E(x)y$$

X_{max} parametrizations



- Dots: CONEX simulations using Sibyll-23d. Error bars show the **std error of the mean** over N simulations
- In the y-axis we show X_{max}/y to emphasize the deviation from the linear trend
- Dashed lines: best fit of GAP (linear in y) parametrization assuming $E_{min} = 10^{15}$ eV
- Dotted lines: best fit of GAP (linear in y) parametrization assuming $E_{min} = 10^{17}$ eV

X_{max} **parametrizations**



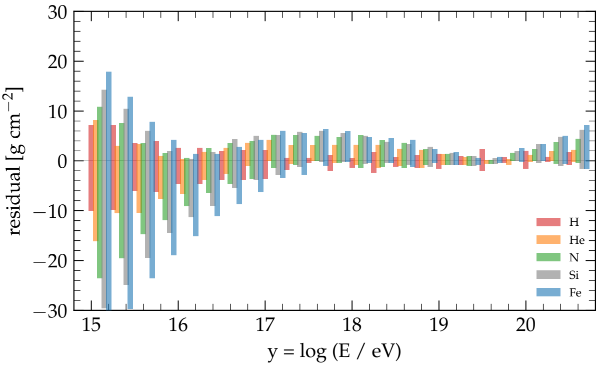
- Dots: CONEX simulations using Sibyll-23d. Error bars show the **std error of the mean** over N simulations
- In the y-axis we show X_{max}/y to emphasize the deviation from the linear trend
- Dashed lines: best fit of GAP (linear in y) parametrization assuming $E_{min} = 10^{15}$ eV
- Dotted lines: best fit of EXT (2nd order in y) parametrization assuming $E_{min} = 10^{15}$ eV

X_{max} parametrizations

Fit parameters based on Sibyll-2.3D.

	D_0	D_1	D_2	α	β	γ
GAP-full	815.69 $\pm$ 0.11	58.76 $\pm$ 0.06	-	-26.37 $\pm$ 0.03	1.57 $\pm$ 0.02	-
GAP-hi	815.83 $\pm$ 0.11	58.09 $\pm$ 0.11	-	-26.19 $\pm$ 0.03	0.67 $\pm$ 0.03	-
EXT	816.21 $\pm$ 0.12	58.01 $\pm$ 0.11	-0.37 $\pm$ 0.04	-25.67 $\pm$ 0.04	0.56 $\pm$ 0.03	-0.46 $\pm$ 0.01

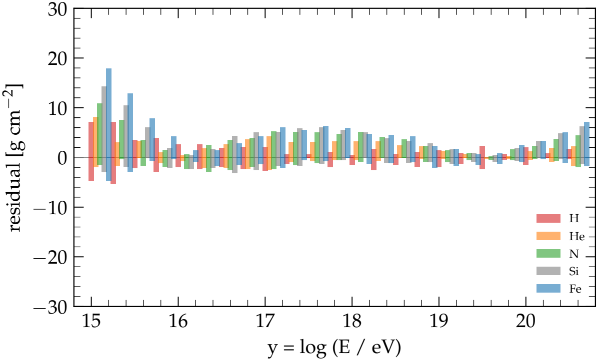
X_{max} residuals



- Positive plane: residuals of the GAP parametrization assuming $E_{min} = 10^{15}$ eV
- Negative plane: residuals of the GAP parametrization assuming $E_{min} = 10^{17}$ eV
- Mean residuals in g/cm^2. Si was not included in the fit:

	H	He	N	Si	Fe
GAP-full	2.15	2.45	3.64	4.39	4.94
GAP-hi	3.07	2.95	4.56	5.71	7.11

X_{max} **residuals**



- Positive plane: residuals of the GAP parametrization assuming $E_{min} = 10^{15}$ eV
- Negative plane: residuals of the EXT parametrization assuming $E_{min} = 10^{15}$ eV
- Mean residuals in g/cm^2. Si was not included in the fit:

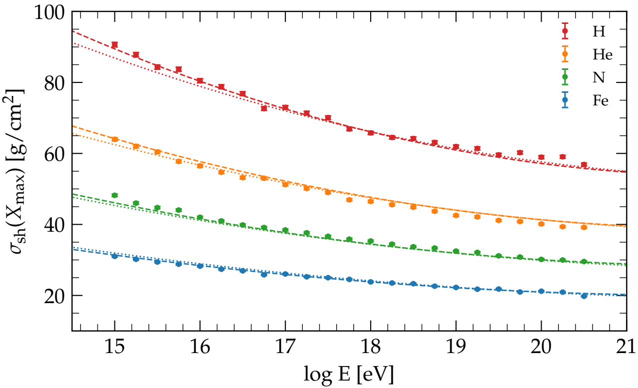
	H	He	N	Si	Fe
GAP	2.15	2.45	3.64	4.39	4.94
EXT	1.96	0.98	1.21	1.35	1.37

$\sigma(X_{max})$ parametrizations

- We model $\sigma(X_{max})$ as a function of $x \equiv \log(E/E_0)$ and $y \equiv \ln A$
- GAP parametrization (6 free parameters):

$$\begin{aligned} a_0'(x) &= a_0 + a_1 x \\ p(x) &= p_0 + p_1 x + p_2 x^2 \\ f(x,y) &= p(x)\left[1 + a_0'(x) y + b_0 y^2\right] \end{aligned}$$

$\sigma(X_{max})$ parametrizations



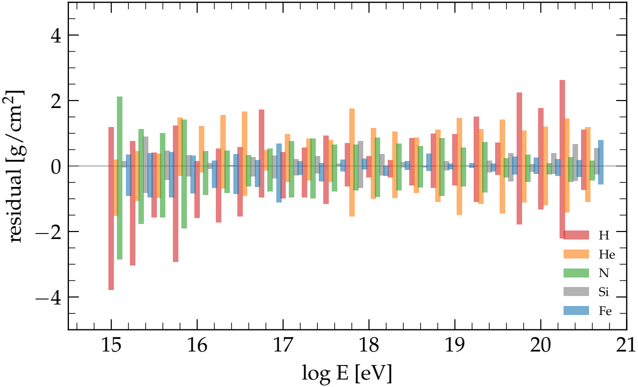
- Dots: CONEX simulations using Sibyll-23d. Error bars show the **error of the std deviation** over N simulations
- Dashed lines: best fit of GAP parametrization assuming $E_{min} = 10^{15}$ eV
- Dotted lines: best fit of GAP parametrization assuming $E_{min} = 10^{17}$ eV

$\sigma(X_{max})$ parametrizations

	p_0	p_1	p_2	a_0	a_1	b
GAP-full	61.0 ± 0.1	-4.5 ± 0.1	0.67 ± 0.02	-0.223 ± 0.001	0.0008 ± 0.0001	0.0161 ± 0.0002
GAP-hi	61.3 ± 0.1	-4.3 ± 0.1	0.53 ± 0.07	-0.228 ± 0.001	-0.0002 ± 0.0002	0.0173 ± 0.0003

Fit parameters based on Sibyll-2.3D.

$\sigma(X_{max})$ residuals



- Positive plane: residuals of the GAP parametrization assuming $E_{min} = 10^{15}$ eV
- Negative plane: residuals of the GAP parametrization assuming $E_{min} = 10^{17}$ eV
- Mean residuals in g/cm^2. Si was not included in the fit:

	H	He	N	Si	Fe
GAP full	0.97	1.08	0.68	0.29	0.26
GAP hi	1.34	0.93	0.92	0.25	0.45

Understanding Anomalous Shower Profiles

- Most air showers caused by high-energy cosmic rays follow a predictable pattern, showing a single, clearly defined maximum in their longitudinal development.
- A small subset exhibits deviations from this norm, with some showing significantly different profiles, including cases with two distinct maxima.
- We use simulations to analyze the frequency of these anomalous profiles as a function of the primary energy.
- We limit our investigation to primary protons (where anomalies are most pronounced) and apply a single interaction model for the moment.

Modeling the Longitudinal Shower Profile

- Gaisser–Hillas[1]: A foundational 3-parameter model defines the normalized ($N_{max} = 1$) longitudinal profile of a shower. The function is given by:

$$y(x) = \left(\frac{x}{m}\right)^m \exp(m - x) \tag{6}$$

where

$$y = \frac{N}{N_{max}} \tag{7}$$

$$x = \frac{X - X_0}{\lambda} \tag{8}$$

$$m = \frac{X_{max} - X_0}{\lambda} \tag{9}$$

- We evaluate the accuracy of our model using a cost function

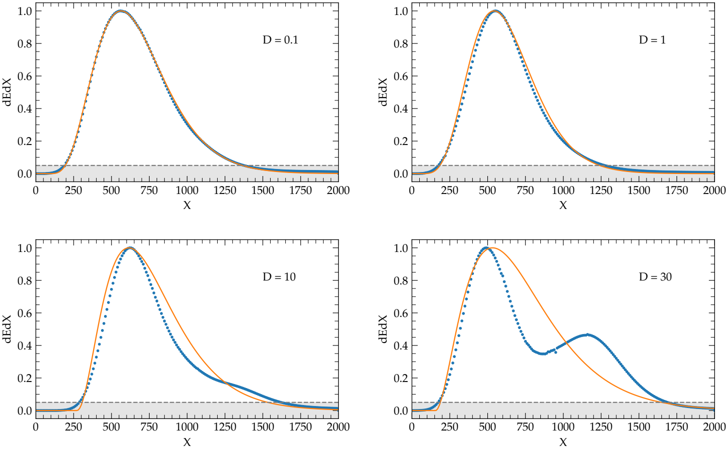
$$D = \sum_{y_i > 5\%} \frac{[y(x_i) - y_i]^2}{y_i^2} \tag{10}$$

where y_i represents the normalized profile simulated by CONEX.

- To find the best fit parameters, (X_{max}, X_0, λ), we perform a minimization of D starting from approximately 100 randomly selected initial points in the parameter space.

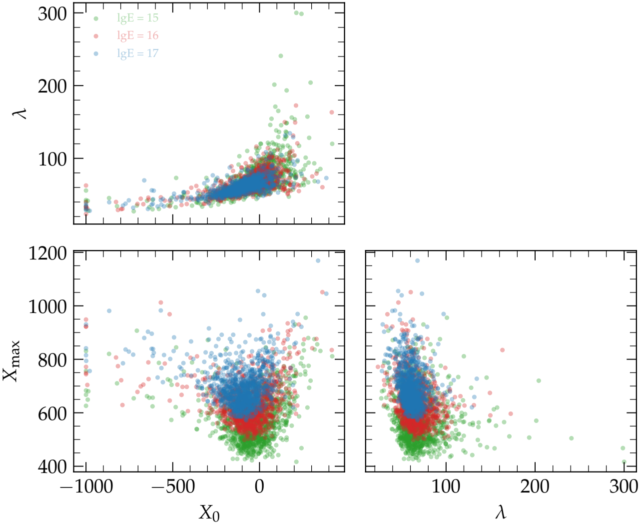
[1] https://en.wikipedia.org/wiki/Gaisser–Hillas_function

Shower examples

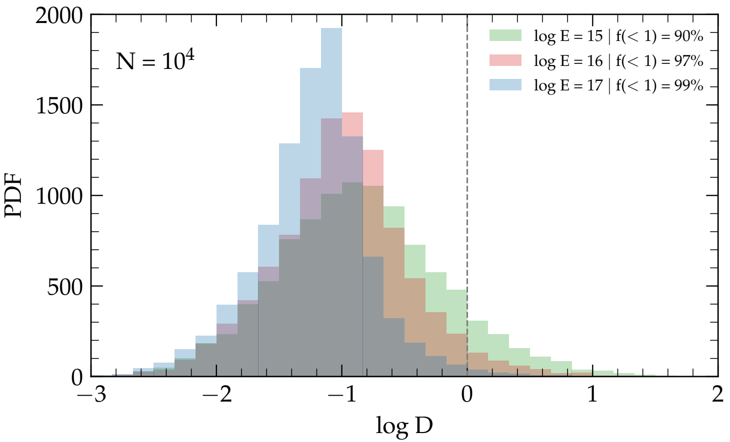


- We present four shower profiles with different best-fit values of D $= 0.1, 1, 10, 30$
- The cost function excludes the region where y < 5 %
- We concluded that setting a threshold of D < 1 to distinguish non-anomalous from anomalous profiles might be very conservative.

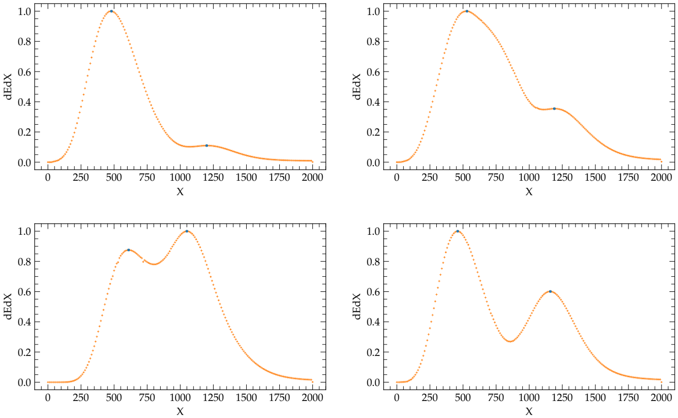
Exploring the Parameter Space of Shower Profiles



The Fraction of Anomalous Profiles



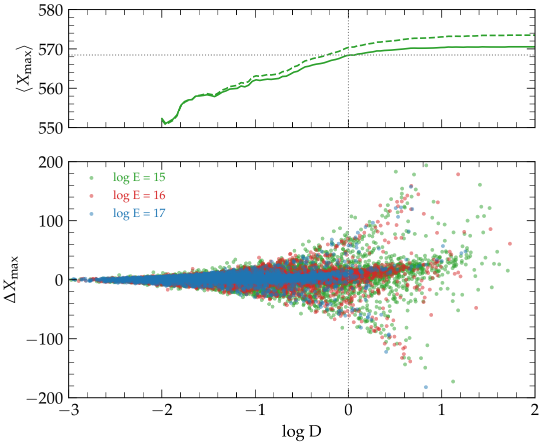
Anomalous Profiles with Double-Peak identification



- An alternative approach we tested consists in identify all local maxima by simple comparison of neighbouring values
- We tag as AP those with more than 1 peak $\longrightarrow$ we show 4 examples where 2 peaks have been identified
- With this definition (somehow similar to Colin+, ICRC 2011) the fraction of AP is much smaller:

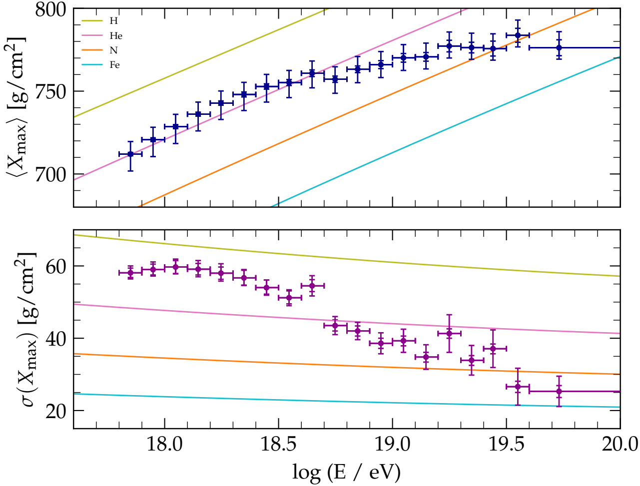
Energy	15	16	17
fraction [%]	0.65	0.17	0.03

Shower inspection



- Upper plot: the mean X_{max} as a function of the maximum D allowed
  - Solid line: the X_{max} is computed from the G-H fit
  - Dashed line: the X_{max} is taken from the CONEX file (2nd order polyfit around the max value)
- Lower plot: the ΔX_{max} is the difference between the X_{max}'s above as a function of D

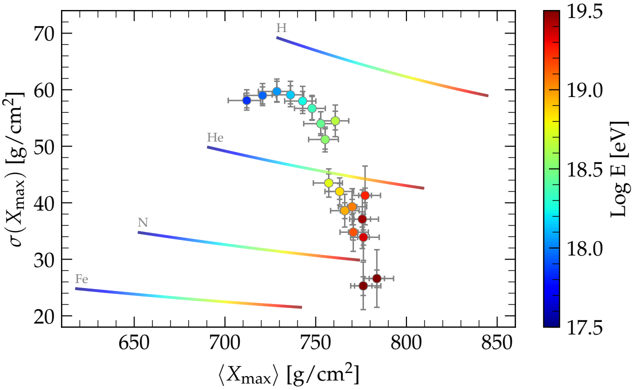
Comparison with Auger data



Based on Sibyll-2.3D.

P.Auger Coll., ICRC 2019, internal use only

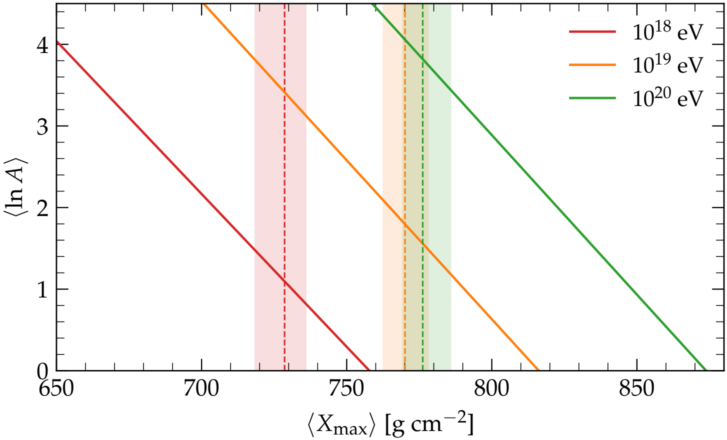
Comparison with Auger data



Based on Sibyll-2.3D.

P.Auger Coll., ICRC 2019, internal use only

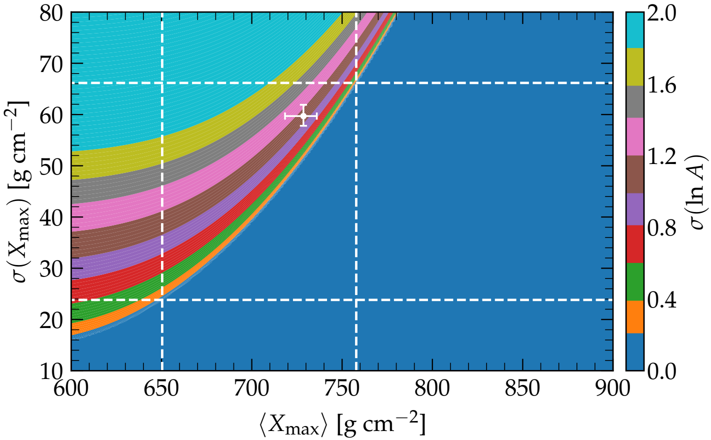
Comparison with Auger data



Based on Sibyll-2.3D.

P.Auger Coll., ICRC 2019, internal use only

Comparison with Auger data



$E = 10^{18}$ eV

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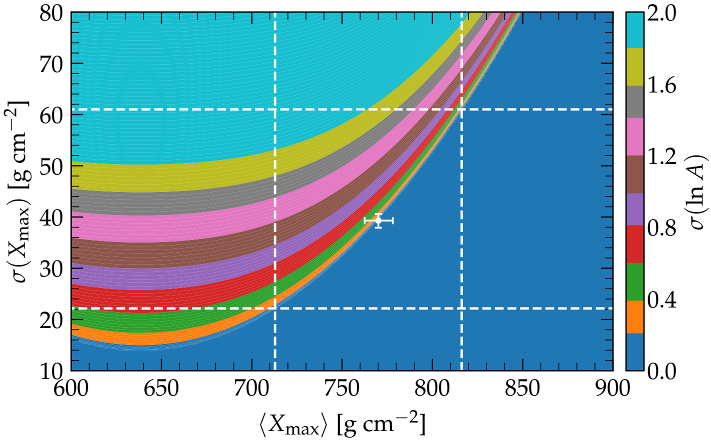
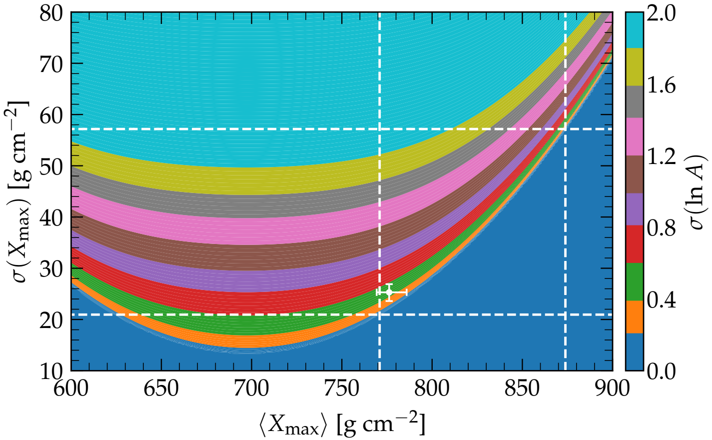
Comparison with Auger data



$E = 10^{19}$ eV

P.Auger Coll., ICRC 2019, internal use only

Comparison with Auger data



$\mathrm{E} = 10^{20}$ eV

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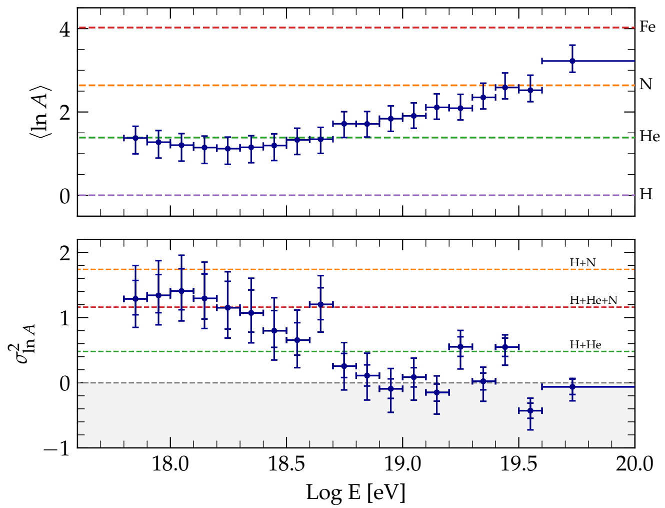
$\ln A$ moments in Auger data



Based on Sibyll-2.3D.

P.Auger Coll., ICRC 2019, internal use only

Variance of lnA in Auger data

- There are two independent sources of fluctuations: the intrinsic shower-to-shower fluctuations and the lnA dispersion arising from the mass distribution:

$$\sigma^2(X_{max}) = \langle \sigma_{sh}^2 \rangle + \left(\frac{d\langle X_{max} \rangle}{d \ln A} \right)^2 \sigma_{\ln A}^2 = \langle \sigma_{sh}^2 \rangle + f_E^2 \sigma_{\ln A}^2 \tag{11}$$

- We assume a parameterization for σ_{sh}^2 as follows

$$\sigma_{sh}^2(\ln A) = \sigma_p^2 \left[1 + a \ln A + b (\ln A)^2 \right] \tag{12}$$

therefore

$$\langle \sigma_{sh}^2 \rangle = \sigma_p^2 \left[1 + a \langle \ln A \rangle + b \langle (\ln A)^2 \rangle \right] \tag{13}$$

- After substitution

$$\sigma^2(X_{max}) = \sigma_p^2 \left[1 + a \langle \ln A \rangle + b \langle (\ln A)^2 \rangle \right] + f_E^2 \sigma_{\ln A}^2 \tag{14}$$

- We apply the definition

$$\langle (\ln A)^2 \rangle = \sigma_{\ln A}^2 + \langle \ln A \rangle^2 \tag{15}$$

- As a result we arrive at

$$\sigma_{\ln A}^2 = \frac{\sigma^2(X_{max}) - \sigma_{sh}^2(\langle \ln A \rangle)}{b \sigma_p^2 + f_E^2} \tag{16}$$

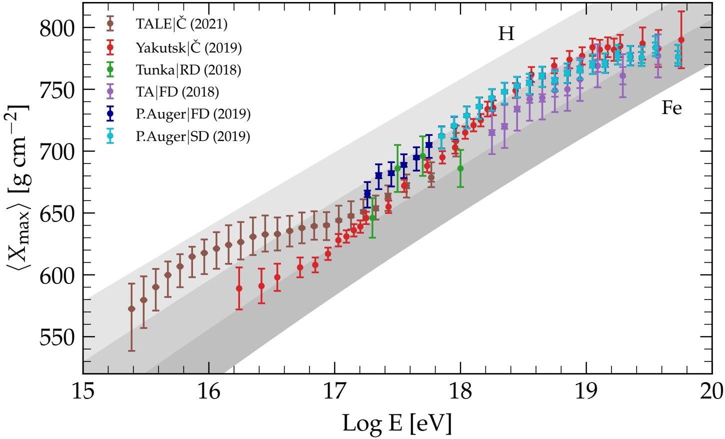
Measurements of the mean of the Xmax distribution

id	Experiment	Mode	Ref.	Table	comment
54	PAO	FD	ICRC 2019	✌	PAO public data
180	PAO	RD	UHECR 2023	✖	
193	PAO	SD	ICRC 2019	✌	PAO internal data(?)
75	TA	FD	ApJ 2018	✌	Tab. 4 but data corrected as done in Ref. 2(?)
143	TALE	Č	ApJ 2021	✌	Tab. 5 is bias-corrected(?)
178	Tunka	Č	ICRC 2021	✖	
182	Tunka	RD	PRD 2018	✌	Tab. 3
179	Yakutsk	Č	ASR 2019	✌	Tab. 2+3
183	Yakutsk	RD	ICRC 2019	✖	
181	LOFAR	radio	PRD 2021	✖	
832	Hi-Res/Mia		ApJ 2001	✖	

Datasets cited in the Snowmass paper.

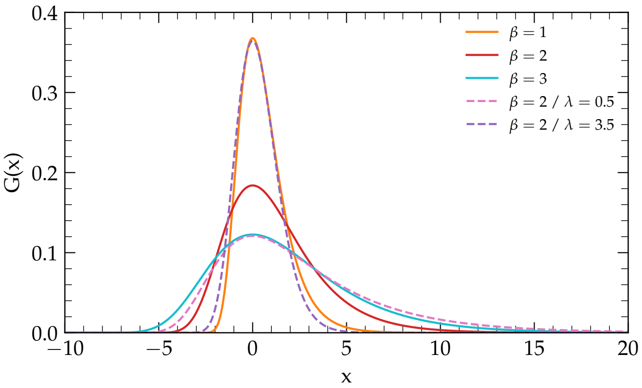
Mean lnA in other datasets

Work in progress...



Based on Sibyll-2.3D.

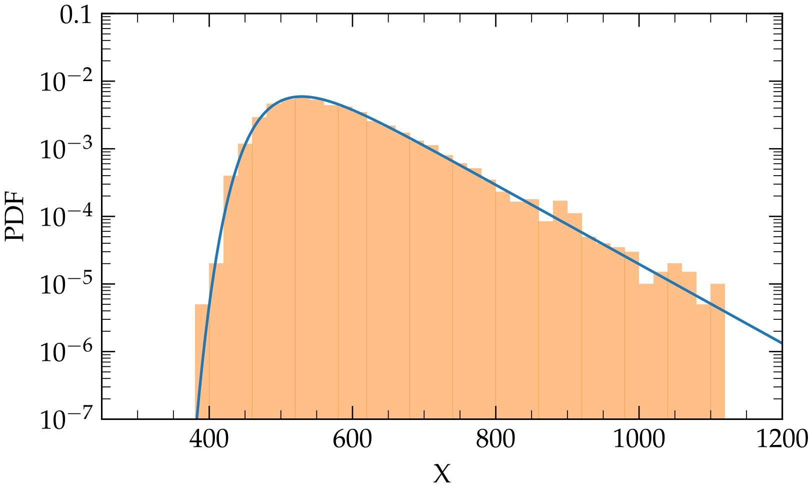
Gumbel function



$$z = \frac{x - \mu}{\beta} \quad , \quad G(z) = \frac{1}{\beta} \frac{\lambda^{\lambda}}{\Gamma(\lambda)} e^{-\lambda(z + e^{-z})}$$

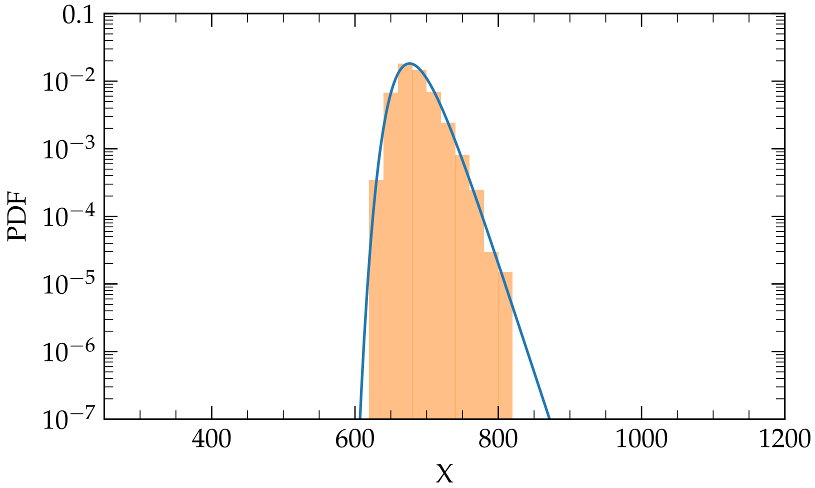
Goal: parametrize $\mu = \mu(x, y)$, $\beta = \beta(x, y)$, $\lambda = \lambda(x, y)$

Gumbel function



Mass: H, Energy: 10^{16} eV

Gumbel function



Mass: H, Energy: 10^{20} eV

Parametrizations

- μ as a function of $x \equiv \log(E/E_0)$ and $y \equiv \ln A$

$$\begin{aligned}
p_0^\mu(y) &= a_0^\mu + a_1^\mu y + a_2^\mu y^2 \\
p_1^\mu(y) &= b_0^\mu + b_1^\mu y + b_2^\mu y^2 \\
p_2^\mu(y) &= c_0^\mu + c_1^\mu y + c_2^\mu y^2 \\
\mu(x,y) &= p_0^\mu(y) + p_1^\mu(y)x + p_2^\mu(y)x^2
\end{aligned}$$

- β as a function of $x \equiv \log(E/E_0)$ and $y \equiv \ln A$

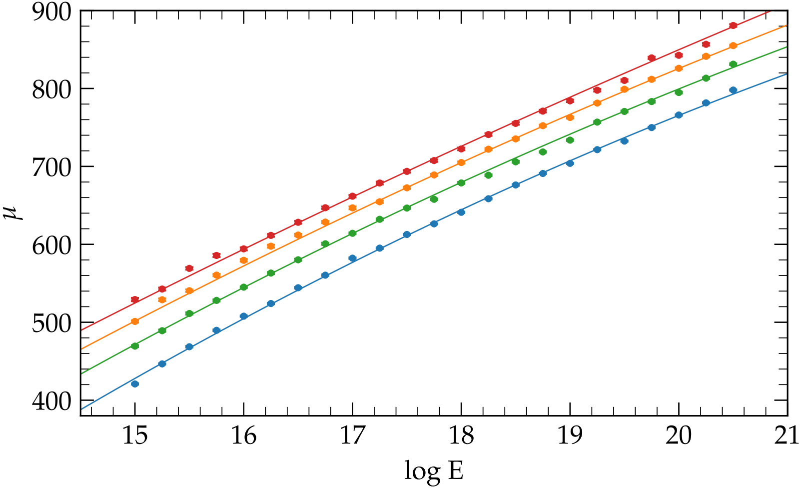
$$\begin{aligned}
p_0^\beta(y) &= a_0^\beta + a_1^\beta y + a_2^\beta y^2 \\
p_1^\beta(y) &= b_0^\beta + b_1^\beta y + b_2^\beta y^2 \\
\beta(x,y) &= p_0^\beta(y) + p_1^\beta(y)x + p_2^\beta(y)x^2
\end{aligned}$$

- λ as a function of $x \equiv \log(E/E_0)$ and $y \equiv \ln A$

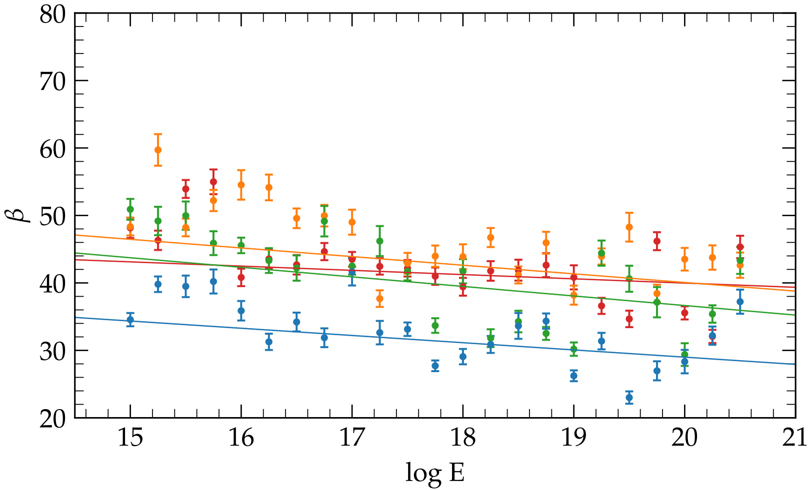
$$\begin{aligned}
p_0^\lambda(y) &= a_0^\lambda + a_1^\lambda y + a_2^\lambda y^2 \\
p_1^\lambda(y) &= b_0^\lambda + b_1^\lambda y + b_2^\lambda y^2 \\
\lambda(x,y) &= p_0^\lambda(y) + p_1^\lambda(y)x + p_2^\lambda(y)x^2
\end{aligned}$$

- 21 free parameters. Cross fingers...

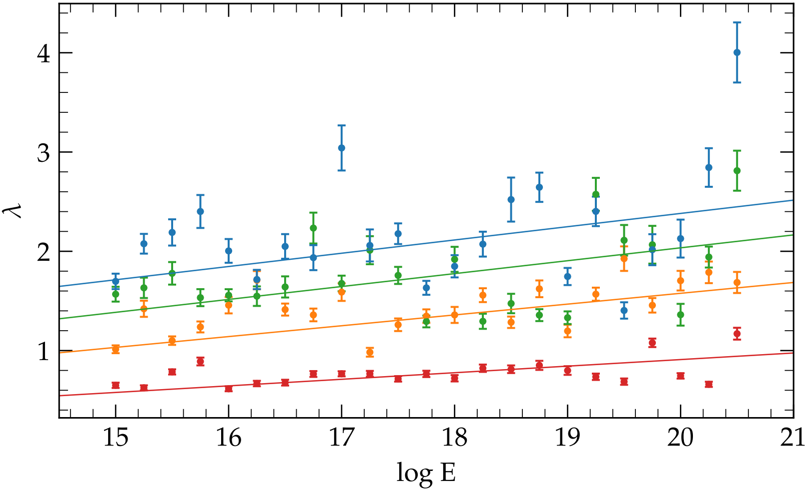
Gumbel function



Gumbel function



Gumbel function



Next Steps

- Calculating the parametrization parameters for 4 distinct HIMs $\longrightarrow$ assessing the variations among these models.
- We plan to augment our dataset of CONEX simulations by 10x. This increase in data volume aims to minimize the scatter in Gumbel parametrizations.
- We are actively testing additional parametrization methods rather than Gumbel functions (see, e.g., Luan B. Arbeletche, Vitor de Souza, Astroparticle Physics 116, 2020, 102389)
- We aim at providing parametrizations and simulated X_{max} templates public online on ZENODO