Low-energy extensions of X_{max} parameterizations

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Based on GAP-2020-058

Motivations

- Our goal is to broaden the scope of the combined fit in order to cover lower energies down to E $\gtrsim 10^{15}$ eV $\rightarrow$ Galactic-ExtraGalactic transition
- Existing parametrizations for calculating cosmic ray composition using X_{max} statistics are constrained to energies above E $\gtrsim 10^{17}$ eV (see GAP2020_058)
- It's crucial to assess whether these current models remain accurate for energies down to 10^{15} eV. Should discrepancies arise, we must consider refining our parametrization models to ensure reliable composition in the knee energy spectrum.

Conex simulations

From GAP2020_058:

- Conex version: version 4.37 for EPOS-LHC and QGSJet II-04, version 7.30 for Sibyll 2.3d
- Hadronic models used Sibyll 2.3d, EPOS-LHC and QGSJet II-04
- The log energy range 17 $\longrightarrow$ 20 in 13 fixed lg E bins with $\Delta \log \mathrm{E} = 0.25$
- Number of showers 5.4k - 7.7k / bin
- The Xmax used to build the distributions is taken from the **XmxdEdX** branch of the CONEX file
- Primary nuclei: H, He, N, Si, Ca and Fe

New simulations at CNAF:

- Conex version: version 7.60
- Hadronic models used Sibyll 2.3d, EPOS-LHC, QGSJet II-04, and DPM-JET III
- The log energy range 15 $\longrightarrow$ 20.5 in 23 fixed lg E bins with $\Delta \log \mathrm{E} = 0.25$
- Number of showers 10k / bin $\longrightarrow$ 50k / bin
- The Xmax used to build the distributions is taken from the **XmxdEdX** branch of the CONEX file
- Primary nuclei: H, He, N, Si, and Fe

Definitions

- Mean:

$$\langle X \rangle = \frac{1}{N} \sum_{i=0}^{N} X_i \tag{1}$$

- Variance:

$$\sigma_X^2 = \frac{1}{N} \sum_{i=0}^{N} |X_i - \langle X \rangle|^2 \tag{2}$$

- Standard Deviation:

$$\sigma_X = \sqrt{\sigma_X^2} \tag{3}$$

- Error of the Mean:

$$\epsilon = \frac{\sigma_X}{\sqrt{N}} \tag{4}$$

- Error of the Standard Deviation:

$$\rho = \frac{\sigma_X}{\sqrt{2N}} \tag{5}$$

X_{max} parametrizations

- We model X_{max} as a function of $x \equiv \log(E/E_0)$ and $y \equiv \ln A$
- GAP parametrization (4 free parameters):

$$
\begin{aligned}
p_0'(y) &= p_0 + \alpha y \\
p_1'(y) &= p_1 + \beta y \\
f(x, y) &= p_0'(y) + p_1'(y)x
\end{aligned}
$$

which can be re-written as

$$f(x, y) = (p_0 + p_1 x) + (\alpha + \beta x)y \equiv f_p(x) + f_E(x)y$$

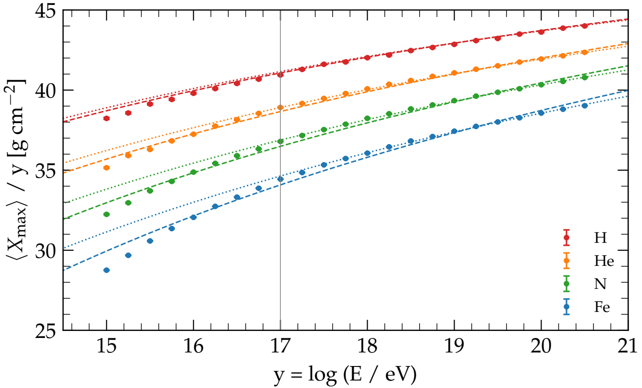
- EXT parametrization (6 free parameters):

$$
\begin{aligned}
p_0'(y) &= p_0 + \alpha y \\
p_1'(y) &= p_1 + \beta y \\
p_2'(y) &= p_2 + \gamma y \\
f(x, y) &= p_0'(y) + p_1'(y)x + p_2'(y)x^2
\end{aligned}
$$

which can be re-written as

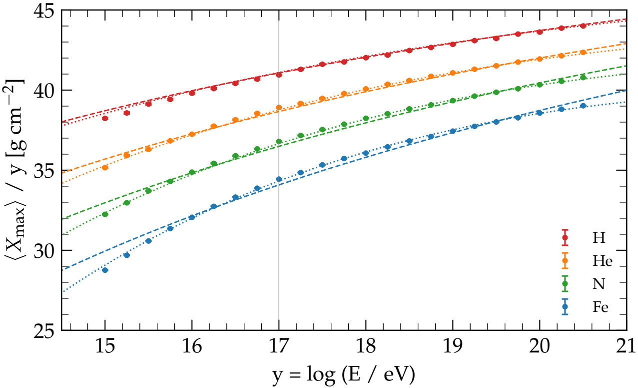
$$f(x, y) = (p_0 + p_1 x + p_2 x^2) + (\alpha + \beta x + \gamma x^2)y \equiv f_p(x) + f_E(x)y$$

X_{max} parametrizations



- Dots: CONEX simulations using Sibyll-23d. Error bars are the **std error of the mean** over N simulations
- In the y-axis we show X_{max}/y to emphasize the deviation from the linear evolution
- Dashed lines: best fit of GAP parametrization assuming $E_{min} = 10^{15}$ eV
- Dotted lines: best fit of GAP parametrization assuming $E_{min} = 10^{17}$ eV

X_{max} parametrizations



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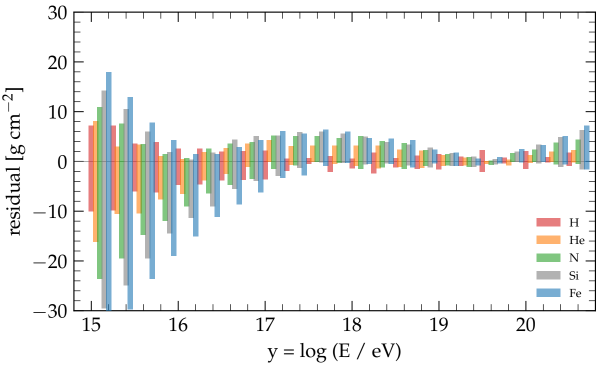
- In the y-axis we show X_{max}/y to emphasize the deviation from the linear evolution
- Dashed lines: best fit of GAP parametrization assuming $E_{min} = 10^{15}$ eV
- Dotted lines: best fit of EXT parametrization assuming $E_{min} = 10^{15}$ eV

X_{max} parametrizations

Fit parameters based on Sibyll-2.3D.

	D_0	D_1	D_2	α	β	γ
GAP-full	815.69 ± 0.11	58.76 ± 0.06	-	-26.37 ± 0.03	1.57 ± 0.02	-
GAP-hi	815.83 ± 0.11	58.09 ± 0.11	-	-26.19 ± 0.03	0.67 ± 0.03	-
EXT	816.21 ± 0.12	58.01 ± 0.11	-0.37 ± 0.04	-25.67 ± 0.04	0.56 ± 0.03	-0.46 ± 0.01

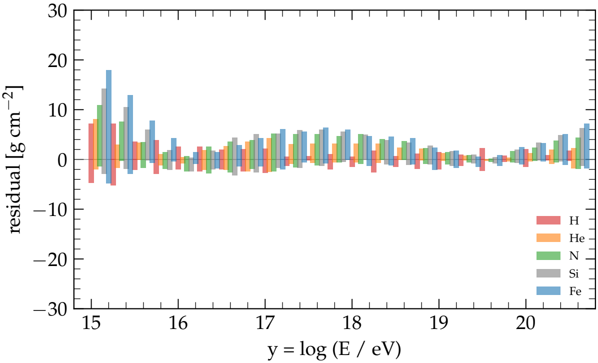
X_{max} residuals



- Positive plane: residuals of the GAP parametrization assuming $E_{min} = 10^{15}$ eV
- Negative plane: residuals of the GAP parametrization assuming $E_{min} = 10^{17}$ eV
- Mean residuals in g/cm^2. Si was not included in the fit:

	H	He	N	Si	Fe
GAP-full	2.15	2.45	3.64	4.39	4.94
GAP-hi	3.07	2.95	4.56	5.71	7.11

X_{max} residuals



- Positive plane: residuals of the GAP parametrization assuming $E_{min} = 10^{15}$ eV
- Negative plane: residuals of the EXT parametrization assuming $E_{min} = 10^{15}$ eV
- Mean residuals in g/cm^2. Si was not included in the fit:

	H	He	N	Si	Fe
GAP	2.15	2.45	3.64	4.39	4.94
EXT	1.96	0.98	1.21	1.35	1.37

$\sigma(X_{max})$ parametrizations

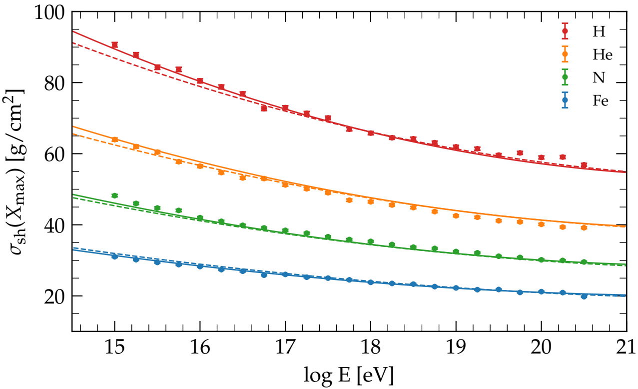
- We model $\sigma(X_{max})$ as a function of $x \equiv \log(E/E_0)$ and $y \equiv \ln A$
- Old parametrization (6 free parameters):

$$
\begin{aligned}
a_0'(x) &= a_0 + a_1 x \\
p(x) &= p_0 + p_1 x + p_2 x^2 \\
f(x,y) &= p(x)\left[1 + a_0'(x) y + b_0 y^2\right]
\end{aligned}
$$

- New parametrization (7 free parameters):

$$
\begin{aligned}
\ln p_0(x) &= a_0 + a_1 y + a_2 y^2 \\
\ln p_1(x) &= b_0 + b_1 y \\
\ln p_2(x) &= c_0 + c_1 y \\
f(x,y) &= p_0 - p_1 x + p_2 x^2
\end{aligned}
$$

$\sigma(X_{max})$ parametrizations



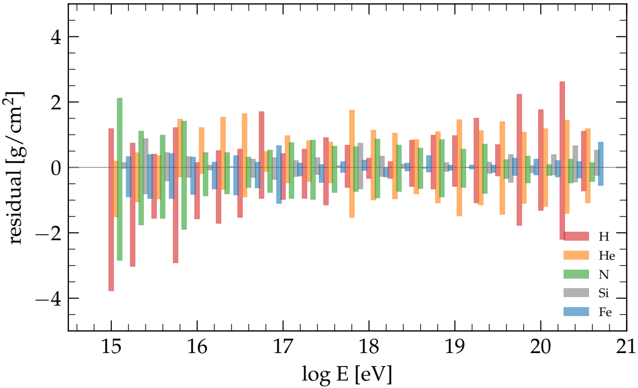
- Dots: CONEX simulations using Sibyll-23d. Error bars are the **error of the variance** over N simulations
- Dashed lines: best fit of GAP parametrization assuming $E_{min} = 10^{15}$ eV
- Dotted lines: best fit of GAP parametrization assuming $E_{min} = 10^{17}$ eV

$\sigma^2(X_{max})$ parametrizations

	p_0	p_1	p_2	a_0	a_1	b
full	61.0 ± 0.1	-4.5 ± 0.1	0.67 ± 0.02	-0.223 ± 0.001	0.0008 ± 0.0001	0.0161 ± 0.0002
hi	61.3 ± 0.1	-4.3 ± 0.1	0.53 ± 0.07	-0.228 ± 0.001	-0.0002 ± 0.0002	0.0173 ± 0.0003

Fit parameters based on Sibyll-2.3D.

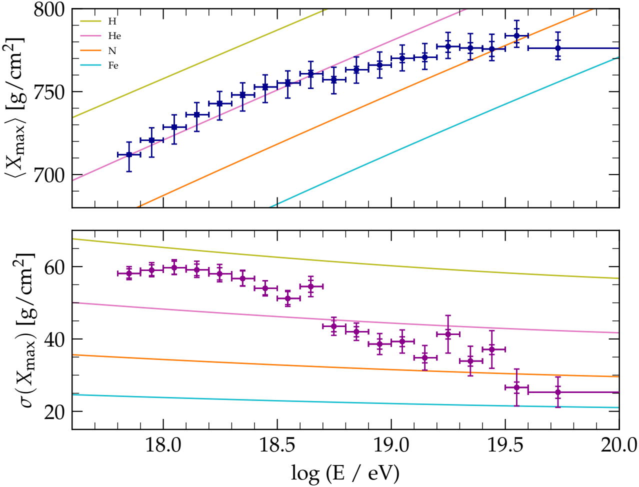
$\sigma(X_{max})$ residuals



	H	He	N	Si	Fe
GAP full	0.97	1.08	0.68	0.29	0.26
GAP hi	1.34	0.93	0.92	0.25	0.45

Mean residuals in g/cm^2. Si was not included in the fit.

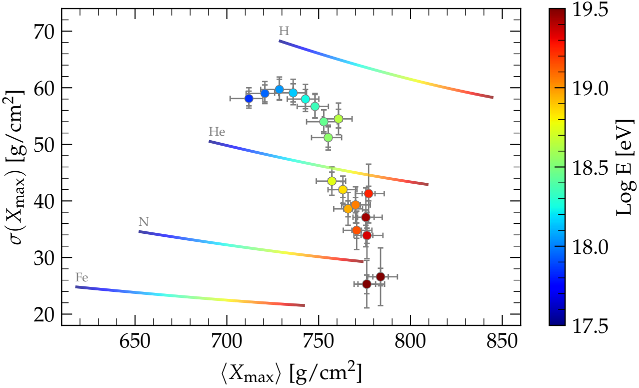
Comparison with Auger data



Based on Sibyll-2.3D.

P.Auger Coll., ICRC 2019, internal use only

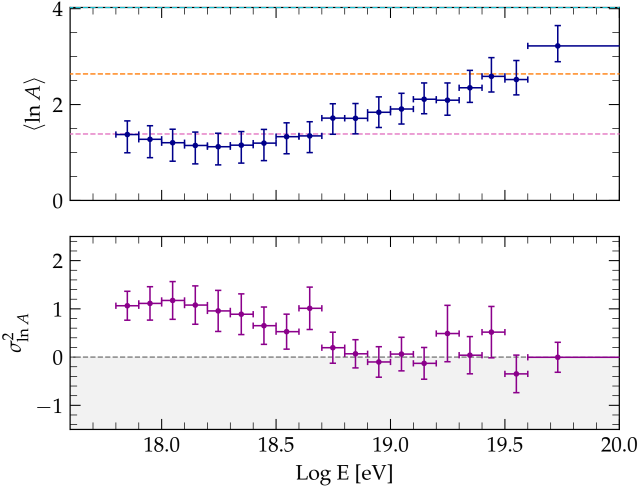
Comparison with Auger data



Based on Sibyll-2.3D.

P.Auger Coll., ICRC 2019, internal use only

Mean $\ln A$ in Auger data



Based on Sibyll-2.3D.

P.Auger Coll., ICRC 2019, internal use only

Variance of lnA in Auger data

- There are two independent sources of fluctuations: the intrinsic shower-to-shower fluctuations and the lnA dispersion arising from the mass distribution:

$$\sigma^2(X_{max}) = \langle \sigma^2_{sh} \rangle + \left(\frac{d\langle X_{max} \rangle}{d \ln A} \right)^2 \sigma^2_{\ln A} = \langle \sigma^2_{sh} \rangle + f_E^2 \sigma^2_{\ln A} \tag{6}$$

- We assume a parameterization for σ^2_{sh} as follows

$$\sigma^2_{sh}(\ln A) = \sigma^2_p \left[1 + a \ln A + b (\ln A)^2 \right] \tag{7}$$

therefore

$$\langle \sigma^2_{sh} \rangle = \sigma^2_p \left[1 + a \langle \ln A \rangle + b \langle (\ln A)^2 \rangle \right] \tag{8}$$

- After substitution

$$\sigma^2(X_{max}) = \sigma^2_p \left[1 + a \langle \ln A \rangle + b \langle (\ln A)^2 \rangle \right] + f_E^2 \sigma^2_{\ln A} \tag{9}$$

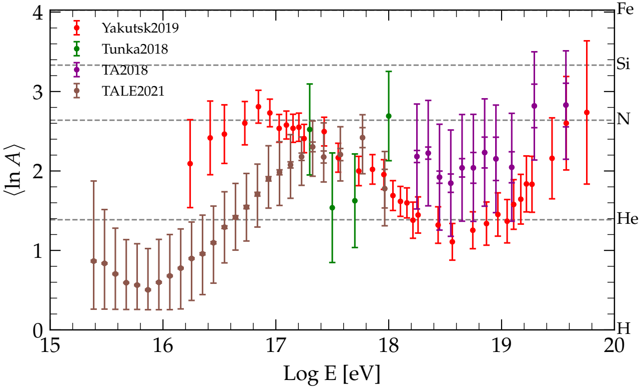
- We apply the definition

$$\langle (\ln A)^2 \rangle = \sigma^2_{\ln A} + \langle \ln A \rangle^2 \tag{10}$$

- As a result we arrive at

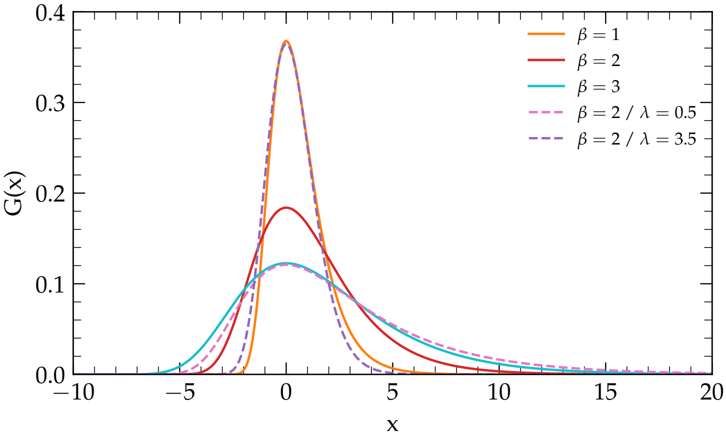
$$\sigma^2_{\ln A} = \frac{\sigma^2(X_{max}) - \sigma^2_{sh}(\langle \ln A \rangle)}{b \sigma^2_p + f_E^2} \tag{11}$$

Mean lnA in other datasets



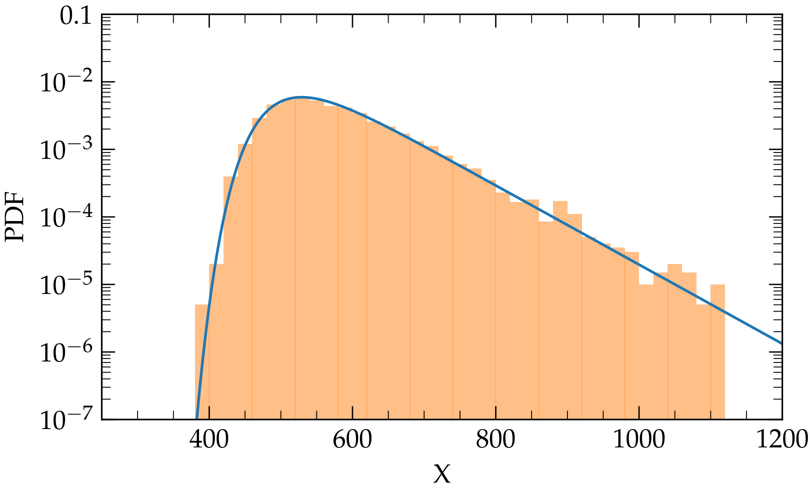
Based on Sibyll-2.3D.

Gumbel function



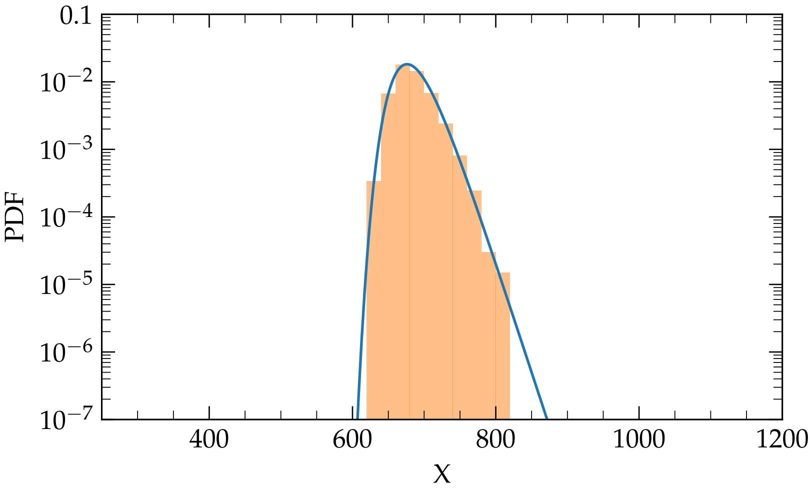
$$G(z) = \frac{1}{\beta}\frac{\lambda^\lambda}{\Gamma(\lambda)} e^{-\lambda(z+e^{-z})}$$

Gumbel function



Mass: H, Energy: 10^{16} eV

Gumbel function



Mass: H, Energy: 10^{20} eV

Parametrizations

- μ as a function of $x \equiv \log(E/E_0)$ and $y \equiv \ln A$

$$\begin{aligned}
p_0^\mu(y) &= a_0^\mu + a_1^\mu y + a_2^\mu y^2 \\
p_1^\mu(y) &= b_0^\mu + b_1^\mu y + b_2^\mu y^2 \\
p_2^\mu(y) &= c_0^\mu + c_1^\mu y + c_2^\mu y^2 \\
\mu(x,y) &= p_0^\mu(y) + p_1^\mu(y)x + p_2^\mu(y)x^2
\end{aligned}$$

- σ as a function of $x \equiv \log(E/E_0)$ and $y \equiv \ln A$

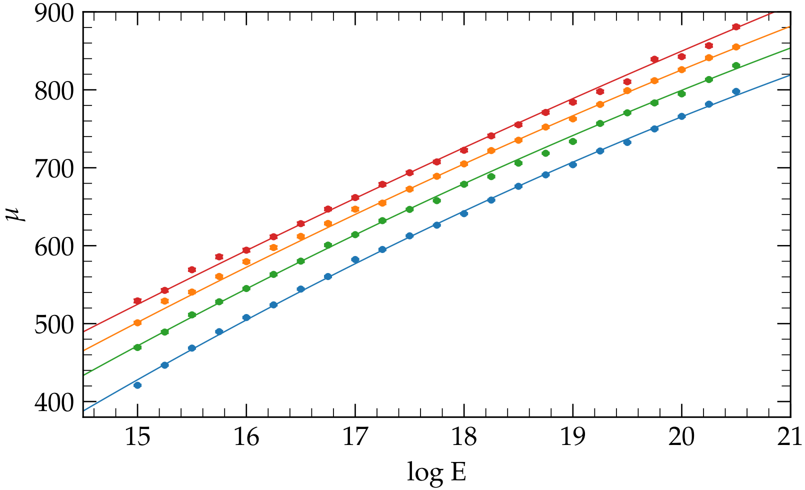
$$\begin{aligned}
p_0^\sigma(y) &= a_0^\sigma + a_1^\sigma y + a_2^\sigma y^2 \\
p_1^\sigma(y) &= b_0^\sigma + b_1^\sigma y + b_2^\sigma y^2 \\
\sigma(x,y) &= p_0^\sigma(y) + p_1^\sigma(y)x + p_2^\sigma(y)x^2
\end{aligned}$$

- λ as a function of $x \equiv \log(E/E_0)$ and $y \equiv \ln A$

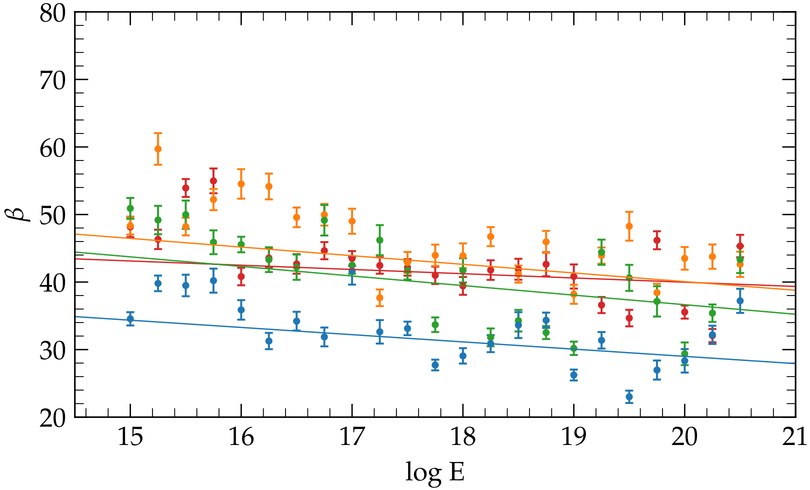
$$\begin{aligned}
p_0^\lambda(y) &= a_0^\lambda + a_1^\lambda y + a_2^\lambda y^2 \\
p_1^\lambda(y) &= b_0^\lambda + b_1^\lambda y + b_2^\lambda y^2 \\
\lambda(x,y) &= p_0^\lambda(y) + p_1^\lambda(y)x + p_2^\lambda(y)x^2
\end{aligned}$$

- 21 free parameters. Cross fingers...

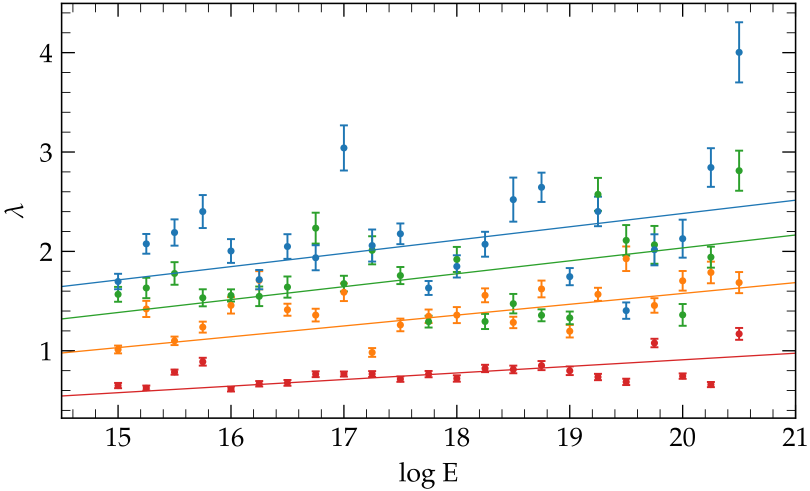
Gumbel function



Gumbel function



Gumbel function



Gumbel function

- Exponentially Modified Gaussian distribution:
- Generalized Gumbel distribution:
- Log-normal distribution:

From Luan B. Arbeletche, Vitor de Souza, Astroparticle Physics 116 (2020) 102389

Next Steps

- Calculating the parametrization parameters for 4 distinct HIMs $\longrightarrow$ assessing the variations among these models.
- We plan to augment our dataset of CONEX simulations by 10x. This increase in data volume aims to minimize the scatter in Gumbel parametrizations.
- We are actively testing additional parametrization methods rather than Gumbel functions.
- Analysis codes and simulated X_{max} databases will be public online on GitHub