Low-energy extensions of X_{max} parameterizations

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Based on GAP-2020-058

Motivations

- Our goal is to broaden the scope of the combined fit in order to cover lower energies down to E $\gtrsim 10^{15}$ eV $\rightarrow$ Galactic-ExtraGalactic transition
- Existing parametrizations for calculating cosmic ray composition using X_{max} statistics are constrained to energies above E $\gtrsim 10^{17}$ eV (see GAP2020_058)
- It's crucial to assess whether these current models remain accurate for energies down to 10^{15} eV. Should discrepancies arise, we must consider refining our parametrization models to ensure reliable composition in the knee energy spectrum.

Conex simulations

From GAP2020_058:

- Conex version: version 4.37 for EPOS-LHC and QGSJet II-04, version 7.30 for Sibyll 2.3d
- Hadronic models used Sibyll 2.3d, EPOS-LHC and QGSJet II-04
- The log energy range 17 $\longrightarrow$ 20 in 13 fixed lg E bins with $\Delta \log \mathrm{E} = 0.25$
- Number of showers 5.4k - 7.7k / bin
- The Xmax used to build the distributions is taken from the **XmxdEdX** branch of the CONEX file
- Primary nuclei: H, He, N, Si, Ca and Fe

New simulations at CNAF:

- Conex version: version 7.60
- Hadronic models used Sibyll 2.3d, EPOS-LHC, QGSJet II-04, and DPM-JET III
- The log energy range 15 $\longrightarrow$ 20.5 in 23 fixed lg E bins with $\Delta \log \mathrm{E} = 0.25$
- Number of showers 10k / bin $\longrightarrow$ 50k / bin
- The Xmax used to build the distributions is taken from the **XmxdEdX** branch of the CONEX file
- Primary nuclei: H, He, N, Si, and Fe

X_{max} parametrizations

- We model X_{max} as a function of $x \equiv \log(E/E_0)$ and $y \equiv \ln A$
- GAP parametrization (4 free parameters):

$$
\begin{aligned}
p_0'(y) &= p_0 + \alpha y \\
p_1'(y) &= p_1 + \beta y \\
f(x, y) &= p_0'(y) + p_1'(y) x
\end{aligned}
$$

which can be re-written as

$$f(x, y) = (p_0 + p_1 x) + (\alpha + \beta x) y \equiv f_p(x) + f_E(x) y$$

- EXT parametrization (6 free parameters):

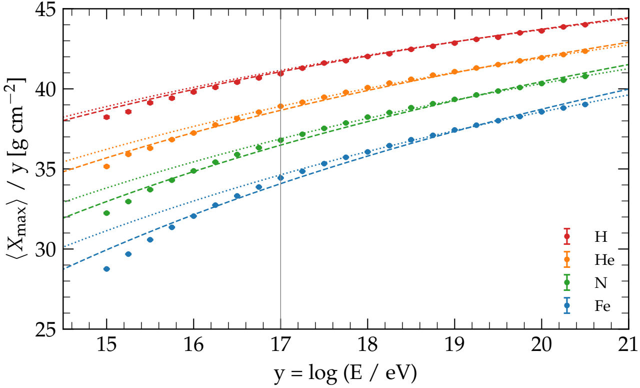
$$
\begin{aligned}
p_0'(y) &= p_0 + \alpha y \\
p_1'(y) &= p_1 + \beta y \\
p_2'(y) &= p_2 + \gamma y \\
f(x, y) &= p_0'(y) + p_1'(y) x + p_2'(y) x^2
\end{aligned}
$$

which can be re-written as

$$f(x, y) = (p_0 + p_1 x + p_2 x^2) + (\alpha + \beta x + \gamma x^2) y \equiv f_p(x) + f_E(x) y$$

X_{max} parametrizations

Shown error bars are the **mean error** over N simulations.

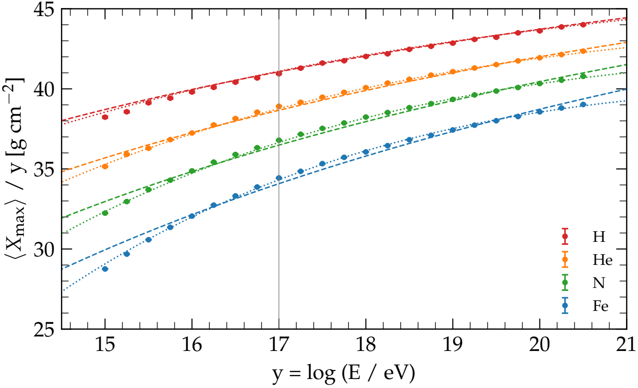


	D_0	D_1	D_2	α	β	γ
GAP-full	815.69 $\pm$ 0.11	58.76 $\pm$ 0.06	-	-26.37 $\pm$ 0.03	1.57 $\pm$ 0.02	-
GAP-hi	815.83 $\pm$ 0.11	58.09 $\pm$ 0.11	-	-26.19 $\pm$ 0.03	0.67 $\pm$ 0.03	-

Fit parameters based on Sibyll-2.3D.

X_{max} parametrizations

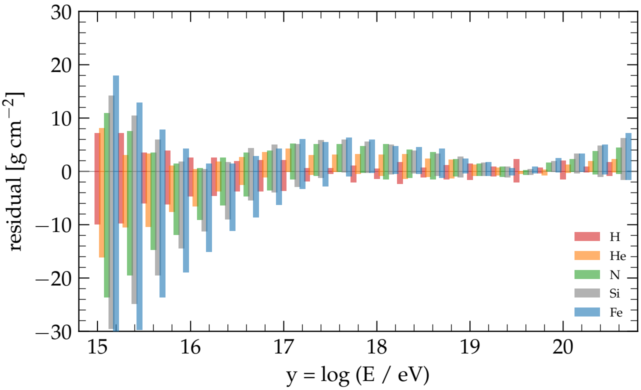
Shown error bars are the **mean error** over N simulations.



	D_0	D_1	D_2	α	β	γ
GAP	815.69 $\pm$ 0.11	58.76 $\pm$ 0.06	-	-26.37 $\pm$ 0.03	1.57 $\pm$ 0.02	-
EXT	816.21 $\pm$ 0.12	58.01 $\pm$ 0.11	-0.37 $\pm$ 0.04	-25.67 $\pm$ 0.04	0.56 $\pm$ 0.03	-0.46 $\pm$ 0.01

Fit parameters based on Sibyll-2.3D.

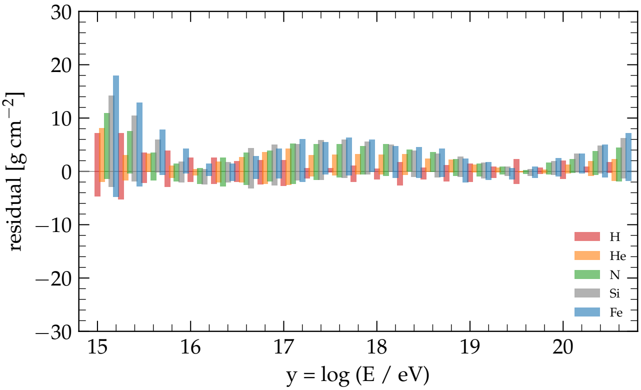
X_{max} residuals



	H	He	N	Si	Fe
GAP-full	2.15	2.45	3.64	4.39	4.94
GAP-hi	3.07	2.95	4.56	5.71	7.11

Mean residuals in g/cm^2. Si is not included in the fit.

X_{max} residuals



	H	He	N	Si	Fe
GAP	2.15	2.45	3.64	4.39	4.94
EXT	1.96	0.98	1.21	1.35	1.37

Mean residuals in g/cm^2. Si is not included in the fit.

$\sigma^2(X_{max})$ parametrizations

- We model $\sigma^2(X_{max})$ as a function of $x \equiv \log(E/E_0)$ and $y \equiv \ln A$
- Old parametrization (6 free parameters):

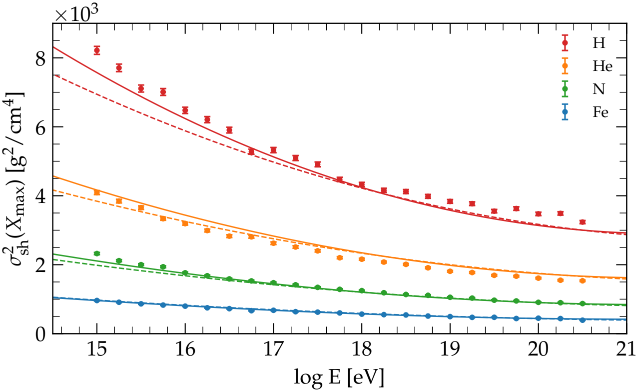
$$\begin{aligned} a_0'(x) &= a_0 + a_1 x \\ p(x) &= p_0 + p_1 x + p_2 x^2 \\ f(x,y) &= p(x)\left[1 + a_0'(x) y + b_0 y^2\right] \end{aligned}$$

- New parametrization (7 free parameters):

$$\begin{aligned} \ln p_0(x) &= a_0 + a_1 y + a_2 y^2 \\ \ln p_1(x) &= b_0 + b_1 y \\ \ln p_2(x) &= c_0 + c_1 y \\ f(x,y) &= p_0 - p_1 x + p_2 x^2 \end{aligned}$$

$\sigma^2(X_{max})$ parametrizations

Shown error bars are the **mean variance error** over N simulations.

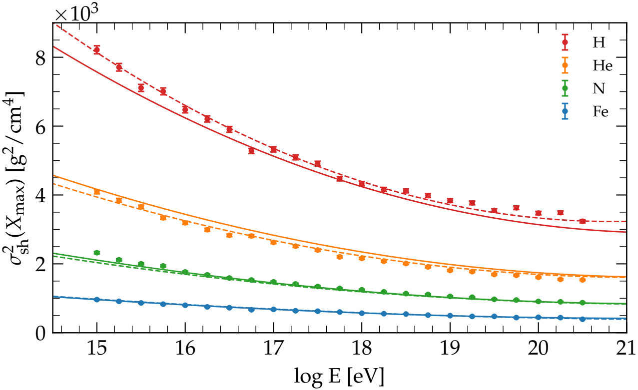


	p_0	p_1	p_2	a_0	a_1	b
full	3578 ± 11	-551.5 ± 6.4	111.7 ± 2.6	-0.378 ± 0.001	0.0007 ± 0.0001	0.0408 ± 0.0002
hi	3619 ± 14	-525.8 ± 9.5	76.3 ± 7.1	-0.380 ± 0.001	-0.0001 ± 0.0002	0.0411 ± 0.0003

Fit parameters based on Sibyll-2.3D.

$\sigma^2(X_{max})$ parametrizations

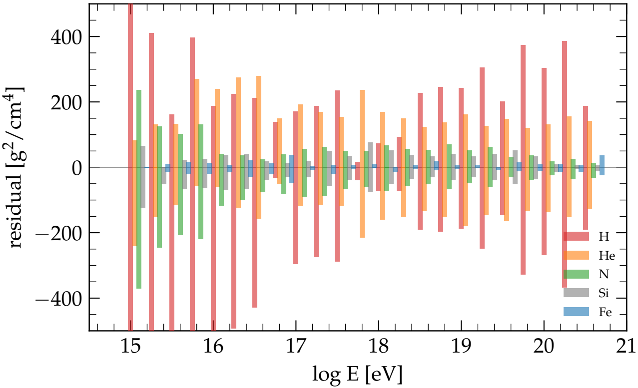
Shown error bars are the **mean variance error** over N simulations.



	a_0	a_1	a_2	b_0	b_1	c_1	c_2
EXT	8.22 ± 0.004	-0.47 ± 0.004	-0.01 ± 0.001	6.27 ± 0.02	-0.490 ± 0.007	4.97 ± 0.03	-0.63 ± 0.01

Fit parameters based on Sibyll-2.3D.

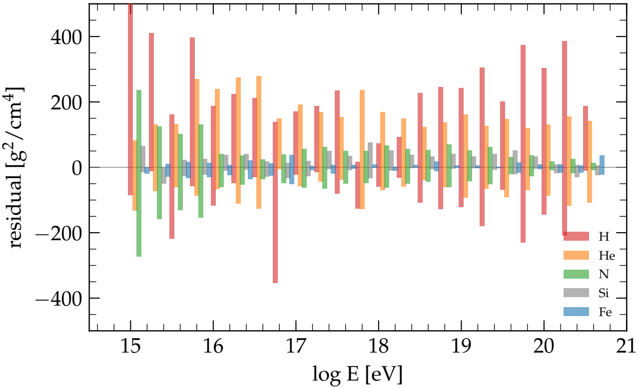
$\sigma^2(X_{max})$ residuals



	H	He	N	Si	Fe
GAP full	240.09	168.12	62.41	32.47	11.20
GAP hi	352.54	137.01	97.00	45.12	13.36

Mean residuals in g^2 cm^{-4}. Si is not included in the fit.

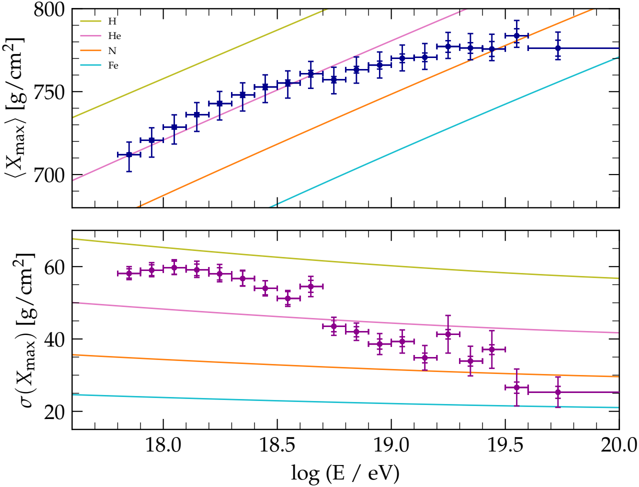
$\sigma^2(X_{max})$ residuals



	H	He	N	Si	Fe
GAP	240.09	168.12	62.41	32.47	11.20
EXP	106.36	77.57	66.40	16.97	16.52

Mean residuals in g^2 cm^{-4}. Si is not included in the fit.

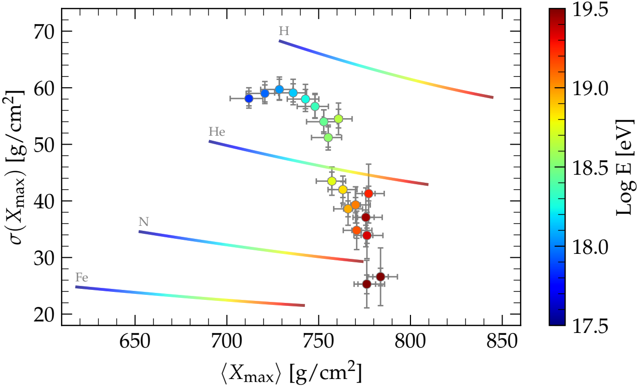
Comparison with Auger data



Based on Sibyll-2.3D.

P.Auger Coll., ICRC 2019, internal use only

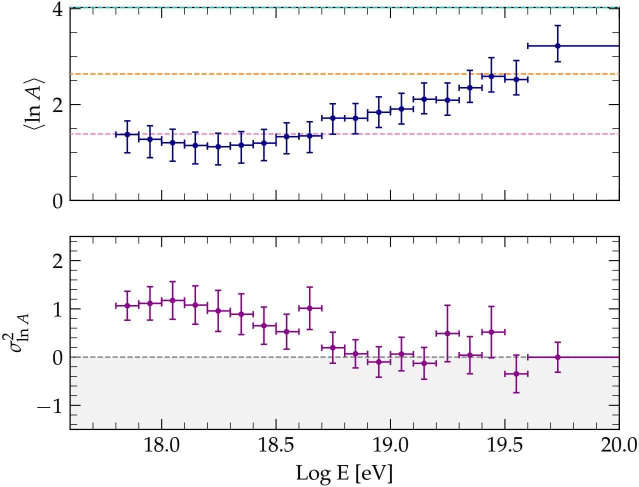
Comparison with Auger data



Based on Sibyll-2.3D.

P.Auger Coll., ICRC 2019, internal use only

Mean $\ln A$ in Auger data



Based on Sibyll-2.3D.

P.Auger Coll., ICRC 2019, internal use only

Variance of lnA in Auger data

- There are two independent sources of fluctuations: the intrinsic shower-to-shower fluctuations and the lnA dispersion arising from the mass distribution:

$$\sigma^2(\mathrm{X_{max}}) = \langle \sigma^2_{\mathrm{sh}} \rangle + \left(\frac{\mathrm{d}\langle \mathrm{X_{max}} \rangle}{\mathrm{d}\ln \mathrm{A}} \right)^2 \sigma^2_{\ln \mathrm{A}} = \langle \sigma^2_{\mathrm{sh}} \rangle + \mathrm{f}_\mathrm{E}^2 \sigma^2_{\ln \mathrm{A}}$$

- We assume a parameterization for σ^2_{sh} as follows

$$\sigma^2_{\mathrm{sh}}(\ln \mathrm{A}) = \sigma^2_\mathrm{p} \left[1 + \mathrm{a} \ln \mathrm{A} + \mathrm{b} (\ln \mathrm{A})^2 \right]$$

  therefore

$$\langle \sigma^2_{\mathrm{sh}} \rangle = \sigma^2_\mathrm{p} \left[1 + \mathrm{a} \langle \ln \mathrm{A} \rangle + \mathrm{b} \langle (\ln \mathrm{A})^2 \rangle \right]$$

- After substitution

$$\sigma^2(\mathrm{X_{max}}) = \sigma^2_\mathrm{p} \left[1 + \mathrm{a} \langle \ln \mathrm{A} \rangle + \mathrm{b} \langle (\ln \mathrm{A})^2 \rangle \right] + \mathrm{f}_\mathrm{E}^2 \sigma^2_{\ln \mathrm{A}}$$

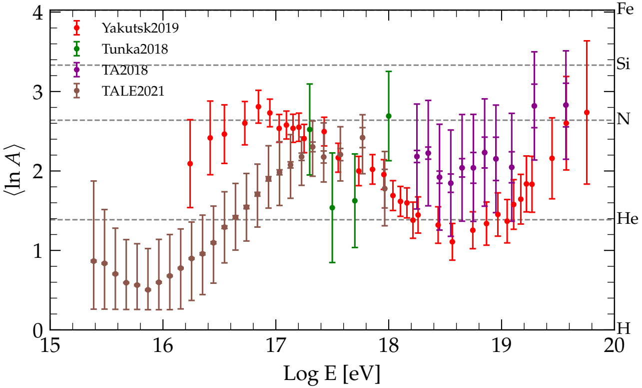
- We apply the definition

$$\langle (\ln \mathrm{A})^2 \rangle = \sigma^2_{\ln \mathrm{A}} + \langle \ln \mathrm{A} \rangle^2$$

- As a result we arrive at

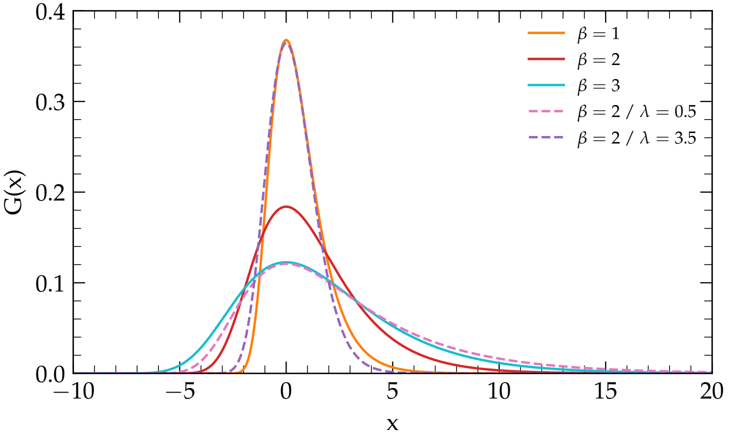
$$\sigma^2_{\ln \mathrm{A}} = \frac{\sigma^2(\mathrm{X_{max}}) - \sigma^2_{\mathrm{sh}}(\langle \ln \mathrm{A} \rangle)}{\mathrm{b}\sigma^2_\mathrm{p} + \mathrm{f}_\mathrm{E}^2}$$

Mean lnA in other datasets



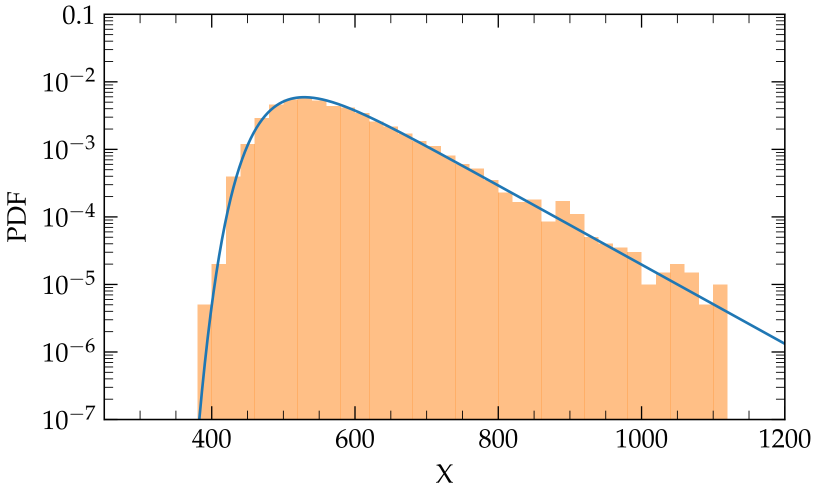
Based on Sibyll-2.3D.

Gumbel function



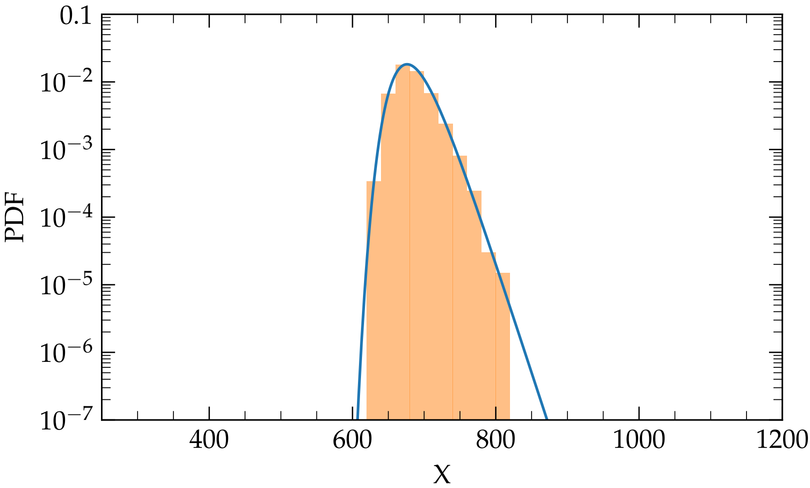
$$G(z) = \frac{1}{\beta}\frac{\lambda^{\lambda}}{\Gamma(\lambda)} e^{-\lambda(z+e^{-z})}$$

Gumbel function



Mass: H, Energy: 10^{16} eV

Gumbel function



Mass: H, Energy: 10^{20} eV

Parametrizations

- μ as a function of $x \equiv \log(E/E_0)$ and $y \equiv \ln A$

$$
\begin{aligned}
p_0^\mu(y) &= a_0^\mu + a_1^\mu y + a_2^\mu y^2 \\
p_1^\mu(y) &= b_0^\mu + b_1^\mu y + b_2^\mu y^2 \\
p_2^\mu(y) &= c_0^\mu + c_1^\mu y + c_2^\mu y^2 \\
\mu(x, y) &= p_0^\mu(y) + p_1^\mu(y)x + p_2^\mu(y)x^2
\end{aligned}
$$

- σ as a function of $x \equiv \log(E/E_0)$ and $y \equiv \ln A$

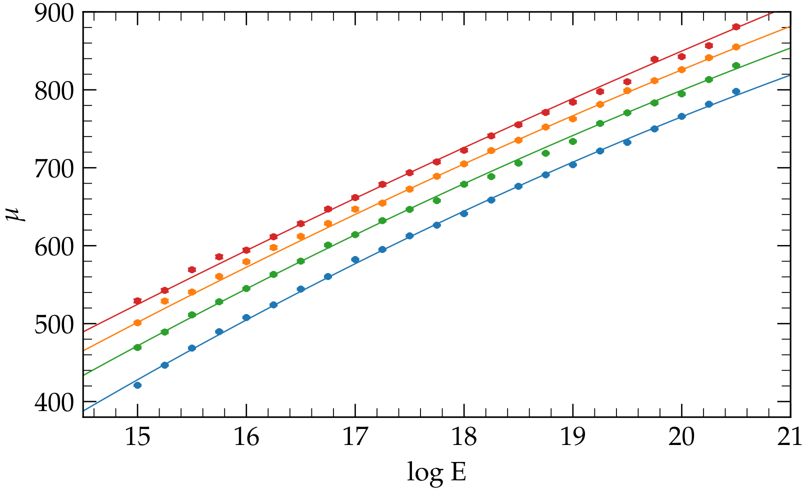
$$
\begin{aligned}
p_0^\sigma(y) &= a_0^\sigma + a_1^\sigma y + a_2^\sigma y^2 \\
p_1^\sigma(y) &= b_0^\sigma + b_1^\sigma y + b_2^\sigma y^2 \\
\sigma(x, y) &= p_0^\sigma(y) + p_1^\sigma(y)x + p_2^\sigma(y)x^2
\end{aligned}
$$

- λ as a function of $x \equiv \log(E/E_0)$ and $y \equiv \ln A$

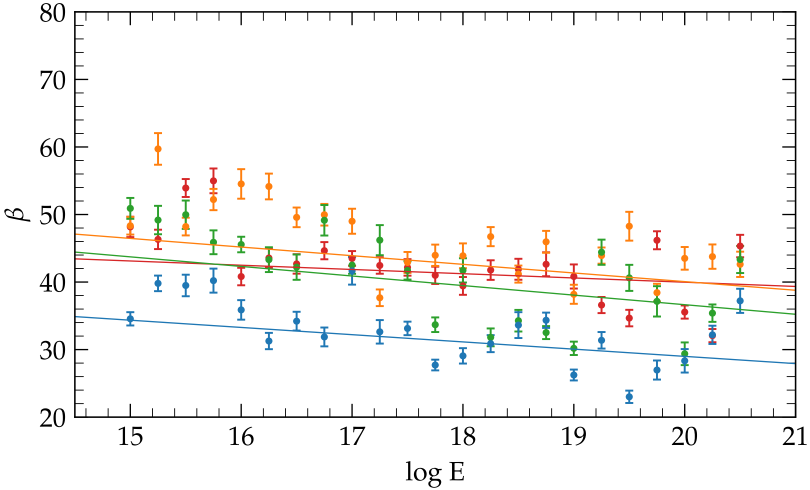
$$
\begin{aligned}
p_0^\lambda(y) &= a_0^\lambda + a_1^\lambda y + a_2^\lambda y^2 \\
p_1^\lambda(y) &= b_0^\lambda + b_1^\lambda y + b_2^\lambda y^2 \\
\lambda(x, y) &= p_0^\lambda(y) + p_1^\lambda(y)x + p_2^\lambda(y)x^2
\end{aligned}
$$

- 21 free parameters. Cross fingers...

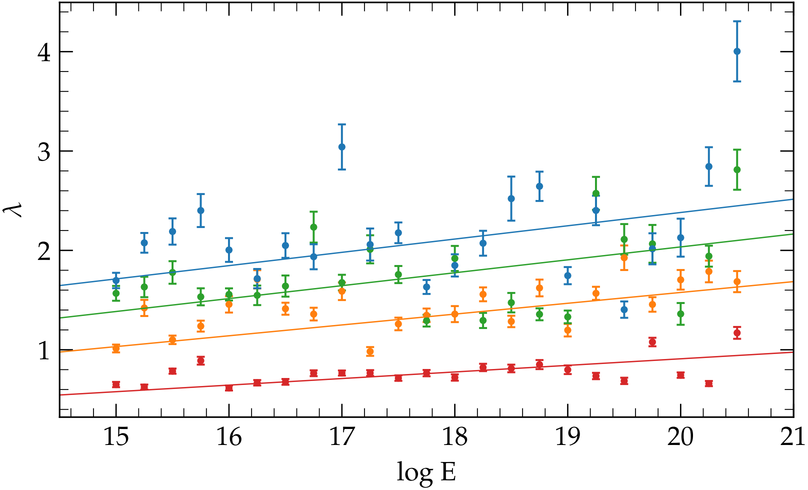
Gumbel function



Gumbel function



Gumbel function



Next Steps

- Calculating the parametrization parameters for 4 distinct HIMs $\longrightarrow$ assessing the variations among these models.
- We plan to augment our dataset of CONEX simulations by 10x. This increase in data volume aims to minimize the scatter in Gumbel parametrizations.
- We are actively testing additional parametrization methods rather than Gumbel functions.
- Analysis codes and simulated X_{max} databases will be public online on GitHub