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Primary Cosmic Radiation and Extensive Air Showers.

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Summary. — The hypothesis, that separate sources of cosmic ray particles of widely differing strength predominate in different energy regions, has been examined. In particular the following questions are discussed: Can such separate sources dominate neighbouring energy intervals of the primary particle spectrum without introducing appreciable changes in slope or discontinuities in the size-frequency distribution of air showers? Can such separate contributions, if they exist, nevertheless be distinguished experimentally with existing techniques? Do available data on extensive air showers provide evidence for or against the hypothesis? It appears that, if such separate sources existed and even if they differed in strength by factors as large as thousand, no significant departure from smoothness in the size-frequency relation of air showers would occur, provided only that all chemical components in the primary radiation have identical spectra when expressed in terms of the magnetic rigidity of the particles or energy per nucleon, and provided further that the chemical composition of the accelerated particle beam does not deviate too radically from its composition at lower energies. While under these conditions even an infinitly sharp cut-off in rigidity of the particles supplied by the stronger of the contributing sources will not produce a significant disturbance in the size frequency spectrum of air showers, it will nevertheless produce discontinuities and irregularities in the dependence of certain other shower parameters on shower size.—Irregularities which so far have found no adequate plausible explanation have been reported by various investigators at a shower size corresponding to a primary energy of about 10^{15} eV. Each of these observations seems to support the hypothesis that a rather sharp rigidity cut-off occurs in the source which supplies most cosmic ray particles below this energy. Additional measurable irregularities can be predicted which have, however, not yet been discovered.—The considerations leading to this hypothesis emphasize the important role which the complexity of the chemical composition of primary cosmic radiation must play in the interpretation of air shower phenomena. In view of the resulting complexity in the origin of showers of a given size, it is suggested that a classification of

air showers according to the maximum energy of the nuclear-active particles in the axis may be more relevant to an understanding of the various underlying nuclear and electromagnetic processes than the customary classification on which is based the total number of shower particles.

1. – Introduction.

One of the most remarkable features of cosmic radiation is presented by the apparently smooth and uniform variation of particle number with energy over a very large energy range. This fact is usually expressed by the statement that a power law with an almost constant exponent represents the energy distribution adequately over nine or even ten orders of magnitude.

This is not easy to understand. If it were strictly true, it would seem to lead to either of the following conclusions about the sources which contribute cosmic ray particles observed in our neighbourhood:

a) One single source dominates all others and gives rise to particles whose energy follows a power spectrum over the entire energy range.

b) Several or many sources contribute comparable amounts over the major portion of this energy range. In that case the sources must have precisely the same energy spectra for their contributions to be comparable both at the high and at the low energy end.

c) The bulk of the particles in different and restricted energy intervals is contributed by different sets of sources. In that case the aggregate strength of the various sets must be very accurately adjusted in order to insure that the number of cosmic ray particles decreases smoothly with a power of the energy over many orders of magnitude.

A theory of the origin of cosmic radiation like that proposed by FERMI, in which particles are accelerated throughout galactic space, corresponds essentially to case *a*) and smoothness and uniformity in the energy spectrum could be expected. However, this particular feature of Fermi's theory, namely acceleration throughout the galactic volume, has generally been abandoned after the composition of the primary radiation and the physical conditions in the interstellar space had become better known.

If one accepts instead the hypothesis that a substantial fraction of cosmic rays originate in or near some specific stellar object, as f.i. Novae or Supernovae, then one approaches case *b*) or *c*). It would be surprising if these various sources contributed nuclei with the same energy spectra, the more so since historic supernovae which have been identified, like the *Crab* and *Cassiopeia, A*, have different radio spectra and therefore are known to contain relativistic electrons whose energy distributions are quite different from each other.

In the third case, *c*), a nearly perfect correlation must exist over a very wide range between the number and strength of accelerators and the particle energy at which their main output occurs.

It is difficult to believe that either condition *b*) or *c*) could be satisfied with great accuracy over the entire range of observable cosmic ray energies. Thus, if careful examination of the intensity of the cosmic radiation as a function of energy should reveal no significant localized departures from a power law, it would seem difficult to avoid the conclusion that case *a*) prevails, namely that a single source of particles dominates the cosmic ray flux in our time and our region of space.

But before such a conclusion can be accepted, it is of great importance to establish whether or not there is any indication in the primary spectrum which points towards contributions from several different sources. It has generally been argued that the absence of marked discontinuities in the particle flux in the latitude-sensitive energy region and in the air shower region provides strong evidence against the presence of distinct, recognizable primary cosmic ray sources dominating different parts of the spectrum. This argument would be valid if cosmic radiation consisted of one kind of particles only, f.i. of protons only. If a complex radiation like that which arrives at the earth were subject to a cut-off, and another, much weaker source contributed the bulk of particles above this cut-off, the differential size-frequency spectrum of air showers f.i. would not show any gap; for the cut-off would occur at a critical *magnetic rigidity*; the *energy* at which the cut-off takes place will then be higher, the higher the atomic weight of the primary nuclei. Thus a cut-off in a complex spectrum can never be sharp as long as the quantities observed are functions of primary energy. The energy region in the primary spectrum which is affected by such a cut-off in rigidity will extend over at least a factor thirty, *i.e.* the energy ratio between iron nuclei and protons which have equal radii of curvature in interstellar magnetic fields. A factor thirty in energy corresponds to a factor of order 5000 in intensity. Another, much weaker source could therefore cut in, in such a way that the differential size-frequency spectrum for air showers does not exhibit a break, but only a discontinuity in slope, while the integral size-frequency spectrum remains almost smooth.

Nevertheless, one predicts very marked observable effects, when the primary radiation, coming from a source which dominates observations at lower energy, approaches such a critical magnetic rigidity. Such observable effects are discussed in this paper. There seems to be considerable, though not yet conclusive, experimental evidence in favour of at least one break in the primary spectrum, namely at a magnetic rigidity corresponding to that of protons with about 10^{15} eV.

2. – Observable effects produced by a primary rigidity cut-off.

In order to exhibit clearly the particular effects which a sharp cut-off in magnetic rigidity of primary particles will have on observable quantities in the air shower region, we approximate the charge composition of the primary radiation by a somewhat smoothed-out dependence of flux on a power of the atomic weight A in the range $A_{He} \leqslant A \leqslant A_{Fe}$. (A representation of the charge spectrum, which throughout varies inversely as the cube of the mass number A, gives a good approximation to the actual composition in the emulsion region (which extends up to about 10^{14} eV), as long as one is not interested in the finer details of the distribution of elements but only in the relative intensity of groups of elements.) (*)

The differential number spectrum of primary particles $n(\varepsilon, A)\,\mathrm{d}\varepsilon\,\mathrm{d}A$ can be written

$$n(\varepsilon, A) = \begin{cases} \dfrac{F_0}{A^{\sigma+1}\varepsilon^{\gamma+1}}\,, & \text{for } A_H \leqslant A \leqslant A_{Fe} \text{ and } \varepsilon < \varepsilon_c\,, \\ = 0\,, & \text{for } \varepsilon > \varepsilon_c\,, \end{cases} \tag{1}$$

where F_0 is some constant.

A more accurate representation must take into account that presumably deuterium and ^{3}He, just as Li, Be, and B are rare in the primary radiation and also that the cut-off energy per nucleon is twice as high for protons as for the other elements. With these refinements,

$$n(\varepsilon, A) = \frac{F_0}{\varepsilon^{\gamma+1}} \begin{cases} \dfrac{r_H}{A_H^\sigma}\,\delta_{A,1} + \dfrac{r_{He}}{A_{He}^\sigma}\,\delta_{A,4} + \dfrac{W}{A^{\sigma+1}}\,, & \text{for } \varepsilon < \varepsilon_c\,, \\ \dfrac{r_H^0}{A_A^\sigma}\,\delta_{A,1}\,, & \text{for } \varepsilon_c < \varepsilon < 2\varepsilon_c\,, \\ 0 & \text{for } \varepsilon > 2\varepsilon_c\,. \end{cases} \tag{1'}$$

Here δ is the Kronecker symbol, and

$$W = \begin{cases} 1\,, & \text{for } A_C \leqslant A \leqslant A_{Fe}\,, \\ 0\,, & \text{otherwise.} \end{cases}$$

r_H and r_{He} have the numerical values 0.47 and 0.45, respectively, and are related to the fraction of protons and α-particles in the primary beam at constant energy per nucleon, by

$$\sigma r_H\, A_H^{-\sigma} = A_H^{-\sigma} - A_{He}^{-\sigma} \approx \tfrac{15}{16}$$
$$\sigma r_{He}\, A_{He}^{-\sigma} = A_{He}^{-\sigma} - A_C^{-\sigma} \approx \tfrac{1}{18}\,.$$

(*) See Appendix.

In order to transform the primary spectrum into a size-frequency spectrum for air showers, one usually expresses the total number N of shower particles on the earth's surface in terms of the energy and mass of the primary by

$$N = \nu A \varepsilon^{\alpha}\,, \tag{2}$$

where ν, α are constants.

The size-frequency spectrum is obtained by inserting the value of ε from eq. (2) into eq. (1) or (1′) and integrating over the allowed range in mass numbers A. One obtains

$$n_{\sigma,\gamma}(N) = \frac{F_0}{A^{\sigma}_{\mathrm{Fe}}\,\varepsilon_c^{\gamma}}\,\frac{1}{\alpha N_M}\left(\frac{N_M}{N}\right)^{(\gamma/\alpha)+1} P_{\sigma,\gamma}\,, \tag{3}$$

where $N_M = A_{\mathrm{Fe}} N_1 = \nu A_{\mathrm{Fe}} \varepsilon_c$.

Depending on whether one uses the primary spectra of eq. (1) or (1′) one obtains (using the symbol $p = \sigma - (\gamma/\alpha)$)

$$P_{\sigma,\gamma} = \frac{1}{p}\begin{cases} [A^{p}_{\mathrm{Fe}} - 1]\,, & \text{for } N \leqslant N_1\,,\\ \left[\left(\dfrac{N_M}{N}\right)^{p} - 1\right], & \text{for } N_1 \leqslant N \leqslant N_M\,, \end{cases} \tag{3a}$$

or

$$P_{\sigma,\gamma} = \begin{cases} A^{\sigma}_{\mathrm{Fe}}\left[\dfrac{r_{\mathrm{H}}}{A^{\sigma}_{\mathrm{H}}}\left(\dfrac{A_{\mathrm{H}}}{A_{\mathrm{Fe}}}\right)^{\gamma/\alpha} + \dfrac{r_{\mathrm{He}}}{A^{\sigma}_{\mathrm{He}}}\left(\dfrac{A_{\mathrm{Fe}}}{A_{\mathrm{He}}}\right)^{\gamma/\alpha}\right] + \dfrac{1}{p}\left[\left(\dfrac{A_{\mathrm{Fe}}}{A_{\mathrm{C}}}\right)^{p} - 1\right], & \text{for } N < N_c\,,\\ A^{\sigma}_{\mathrm{Fe}}\left[\dfrac{r_{\mathrm{H}}}{A^{\sigma}_{\mathrm{He}}}\left(\dfrac{A_{\mathrm{He}}}{A_{\mathrm{Fe}}}\right)^{\gamma/\alpha}\right] + \dfrac{1}{p}\left[\left(\dfrac{A_{\mathrm{Fe}}}{A_{\mathrm{C}}}\right)^{p} - 1\right], & \text{for } N_c < N < A_{\mathrm{He}} N_1\,,\\ \dfrac{1}{p}\left[\left(\dfrac{A_{\mathrm{Fe}}}{A_{\mathrm{C}}}\right)^{p} - 1\right], & \text{for } A_{\mathrm{He}} N_1 < N < A_{\mathrm{C}} N_1\,,\\ \dfrac{1}{p}\left[\left(\dfrac{N_M}{N}\right)^{p} - 1\right], & \text{for } A_{\mathrm{C}} N_1 < N < N_M\,. \end{cases} \tag{3b}$$

Here $N_c = \nu(2\varepsilon_c)^{\alpha}$ represents the cut-off shower size for protons.

The average value $\langle F(N)\rangle$ of an observable quantity, F, which depends on the energy per nucleon ε and the number A of nucleons composing the primary particle, as

$$F = A\,f(\varepsilon)$$

is given by

$$\langle F\rangle = \frac{\int A\,f(\varepsilon)(\mathrm{d}\varepsilon/\mathrm{d}N)_A\, n(\varepsilon, A)\,\mathrm{d}A}{\int (\mathrm{d}\varepsilon/\mathrm{d}N)_{A},\, n(\varepsilon, A)\,\mathrm{d}A}\,.$$

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If one approximates F by

$$F = A\varepsilon^{\beta}, \tag{4}$$

one finds

$$\langle F\rangle = \frac{n_{\sigma-1,\gamma-\beta}}{n_{\sigma,\gamma}} = A_{\mathrm{Fe}}\varepsilon_c^{\beta}\left(\frac{N}{N_M}\right)^{\beta/\alpha}\frac{P_{\sigma-1;\gamma-\beta}}{P_{\sigma,\gamma}}\,. \tag{5}$$

Observables of the type F which, for a given energy per nucleon, depend *linearly* on the number of nucleons composing the primary particle, are for instance:

the number of secondary particles of a given type,
at a given altitude,
in a given energy interval,
at a given distance from the shower axis, or
making given angles with the shower axis.

The average value $\langle G(N)\rangle$ of an observable G, which is *independent* of the number of nucleons composing the primary particle and which can be approximated by

$$G = \varepsilon^{\beta} \tag{6}$$

is given by

$$G\rangle = \frac{n_{\sigma,\gamma-\beta}}{n_{\sigma,\gamma}} = \varepsilon_c^{\beta}\left(\frac{N}{N_M}\right)^{\beta/\alpha}\frac{P_{\sigma,\gamma-\beta}}{P_{\sigma,\gamma}}\,. \tag{7}$$

Observables of type G are f.i.:

shower age,
lateral distribution,
energy spectra of secondary particles,
intensity ratios between various types of secondary particles in different parts of the shower, etc.

The observables quantities represented by expressions of the type illustrated by eqs. (3), (5) or (7) exhibit a change in slope at the shower sizes corresponding to the cut-off value for protons ($N = N_c$) and for α-particles ($N = 4N_1$). The absence or rarity of deuterium and ^{3}He, and of Li, Be, and B, which is implicit in the assumed primary spectrum of eq. (1′), gives furthermore rise to small discontinuities at these values of shower size.

The differential shower-size spectrum based on a sharp cut-off in magnetic rigidity is shown in Fig. 1. The curve is drawn for an assumed exponent, γ, of the primary energy spectrum $\gamma = 1.65$. The exponent σ, which characterizes the chemical distributions of elements in the primary cosmic radiation, has been taken as $\sigma = 2$, *i.e.* the value appropriate for energies up to

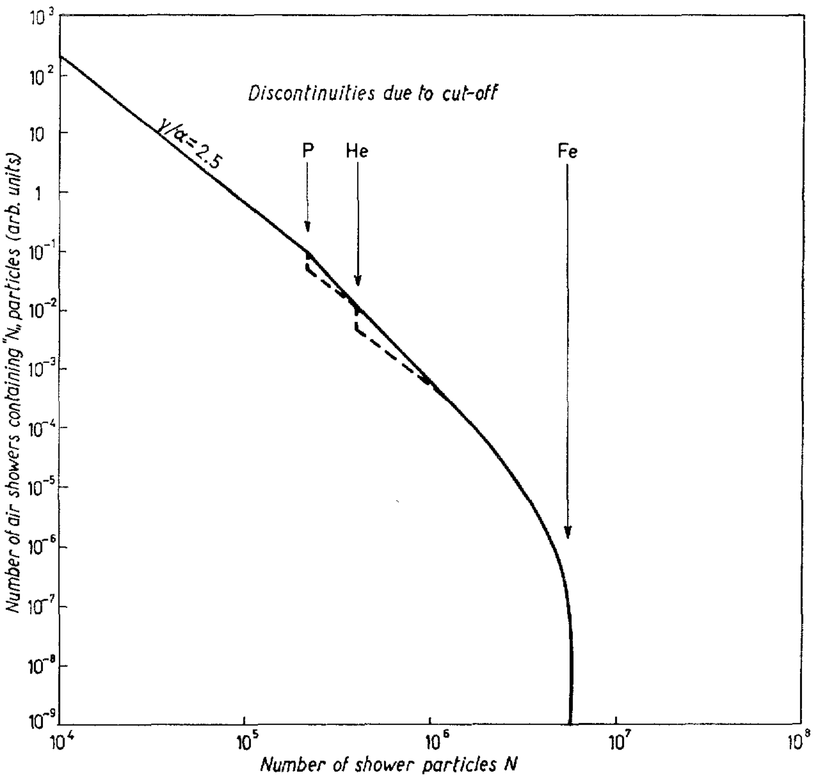

Fig. 1. – The curve represents the differential size distribution of air showers at mountain altitude in arbitrary units. This distribution results from a primary radiation which has: 1) the same chemical composition as that observed in nuclear emulsions; 2) an energy spectrum proportional to $E^{-\gamma}$ with $\gamma = 1.65$; 3) a sharp cut-off at a magnetic rigidity corresponding to protons with about 10^{15} eV. The discontinuities in the curve correspond to an assumed deficiency of ^{2}H, ^{3}He, Li, Be and B in the primary beam.

$\sim 10^{14}$ eV. The parameter α which characterizes the dependence of air-shower size on primary energy (eq. (2)) is usually assumed to lie between 1.0 and 1.2. In drawing Fig. 1, we have chosen $\alpha = 1.1$.

Fig. 2 shows the integral shower-size spectrum using the same parameters. It illustrates the fact that a *quite* smooth spectrum is obtained even in the extreme case when only two sources, differing by a factor 1000 in intensity, contribute to the particle flux and when the stronger of the two sources has an infinitely sharp cut-off at a particular magnetic rigidity.

Fig. 3 shows the behaviour of observables of type $\langle F\rangle$ and $\langle G\rangle$ as a function of shower size in the critical region. The curves are drawn for various values of the parameter β which characterizes the dependence of the observables on the energy of the incident nucleons and are based on the simpler of the two versions of the primary spectrum (eq. (1)).

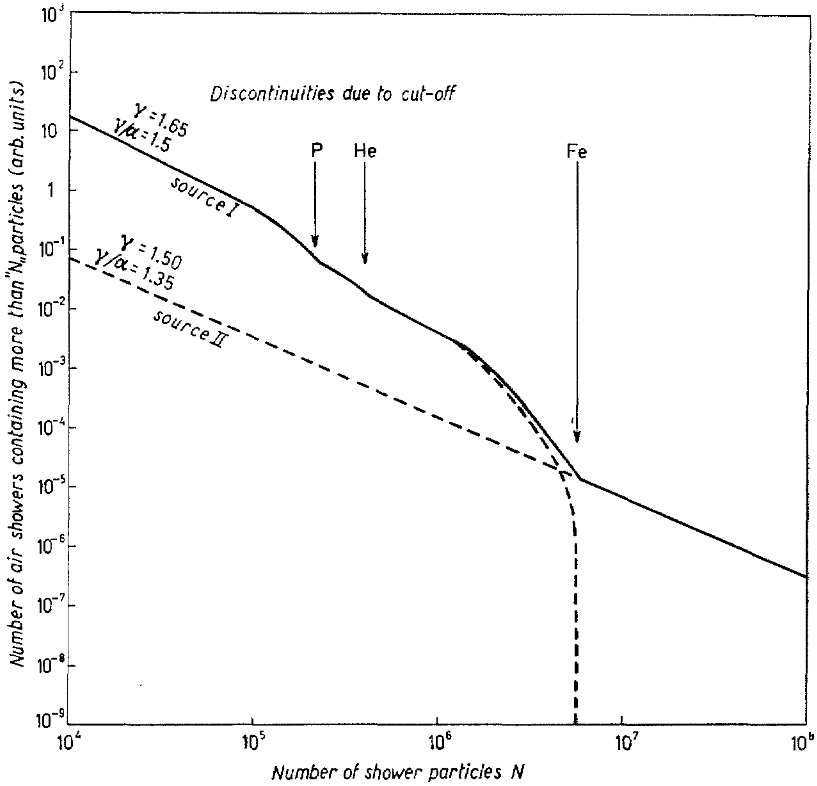

Fig. 2. – Curve I represents the integral size-frequency distribution of air showers, corresponding to the differential distribution of Fig. 1. Curve II represents the distribution contributed by a hypothetical and different source with the following properties: 1) an energy spectrum proportional to $E^{-\gamma}$ with $\gamma=1.5$; 2) an energy output which is one thousand times smaller than that of source no. 1; 3) a cut-off which lies much higher than that of source no. 1. The solid line represents the combined effect of the two sources.

The most important conclusions which can be drawn from the curves and underlying considerations are the following:

a) Discontinuities in slope occur in the frequency of air showers and in the dependence on shower size of *all* observables of types F and G at the

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same critical values of shower size. In the case of the logarithmic size-frequency shower curve the slope changes from $(\gamma/\alpha)+1 \approx 2.5$, which depends on the primary energy spectrum to $\sigma+1 \approx 3$ which is characteristic of the primary charge spectrum. The change in slope, therefore, is not very large.

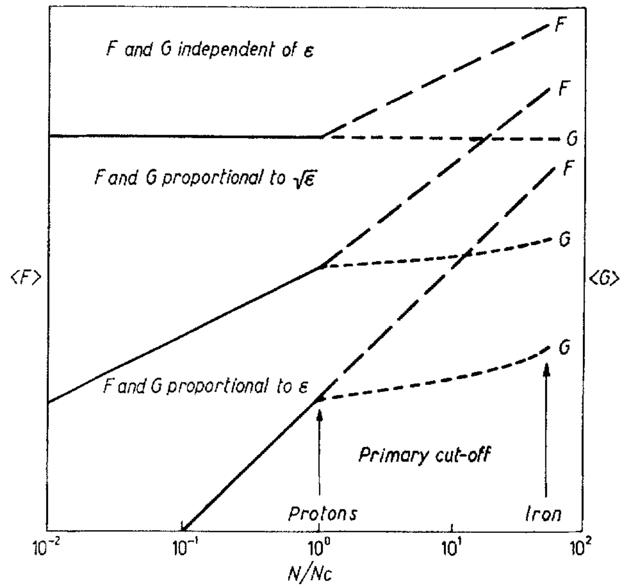

Fig. 3. – The variation of observable quantities of type F and G is shown as a function of the number of shower particles for various assumed dependences of these functions on the energy ε of primary nucleons. The curves show a break at the shower size at which a sharp magnetic rigidity cut-off in the primary spectrum begins to influence the nature of the showers. Dashed and dotted portions refer to variables of types F and G, respectively.

b) The behaviour of observables of types F and G above the critical region is well defined for a given dependence on shower size below the critical region.

c) Beyond the critical shower size, observables of types F and G behave quite differently from each other; differently from what one would predict if one assumed that both types of observables were simply functions of primary particle energy, without taking into account that they also depend in specific and distinct ways on the *mass* of the primary particle. Thus observables of type G *decrease* in their dependence on shower size and become essentially independent of shower size beyond the critical region, irrespective of whether they are slow or fast varying functions of the primary nucleon energy, while ob-

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servables of type F *increase* in their dependence on shower size and tend to increase linearly or somewhat slowlier above the critical value (*).

d) Irregularities are introduced in the size dependence of shower properties by the gaps in the primary charge spectrum (*i.e.* the gap due to the presumably low abundance of deuterium and ^{3}He and the probably equally pronounced gap due to the low abundance of elements represented by the mass numbers $5 \leqslant A \leqslant 11$). For instance, the primaries responsible for showers with a particle number $N \geqslant N_c$ should, on the average, have a *lower* energy per nucleon than those responsible for showers of size $N \leqslant N_c$; the corresponding air showers should, therefore, show ar ather abrupt change in their absorption coefficient, zenith angle dependence, and barometric coefficient.

3. – Evidence for the existence of a rigidity cut-off in the primary spectrum.

The question arises now, whether experimental data support or contradict the existence of a critical air shower size beyond which

a) observables of type F become essentially proportional to shower size,
b) observables of type G become independent of shower size,
c) the absorption coefficient of the particles in the shower core shows a sudden increase,
d) the logarithmic differential size-frequency spectrum of showers suffers a (probably slight) change of slope.

If such discontinuities are observed and if they occur at the same shower size, they constitute a strong argument in favour of a sharp drop in the primary spectrum at the corresponding magnetic rigidity.

(*) The main features of this behaviour are easily derived if one simplifies eqs. (3), (5) and (7) with the help of the approximation $A_{\mathrm{Fe}} = 56 \gg 1$. One obtains

$$\left.\begin{array}{l} n(N) \sim N^{-((\gamma/\alpha)+1)} \\ \langle F \rangle \sim N^{\beta/\alpha} \\ \langle G \rangle \sim N^{\beta/\alpha} \end{array}\right\} \quad \text{when } N \leqslant N_c$$

and

$$\text{and (if } \beta/\alpha > \tfrac{1}{2}) \qquad \left.\begin{array}{l} n(N) \sim N^{-(\sigma+1)} \\ \langle F \rangle \sim N \\ \langle G \rangle \sim \text{constant} \end{array}\right\} \quad \text{when } N > N_c \; .$$

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Detailed measurements of the variation of quantities of types F and G with shower size have been carried out only during the last few years, mostly with the large air shower array in the Pamir mountains at a residual pressure of 650 g/cm².

3'1. *Observables of type F.* – The number of μ-mesons with energies above 1 GeV was found (1) to depend on shower size N as

$$n_\mu(>1\ \text{GeV}) \sim N^{0.62\pm0.12} \qquad \text{for } N < 4\cdot10^5 \text{ electrons}$$

and

$$n_\mu(>1\ \text{GeV}) \sim N^{0.95\pm0.05} \qquad \text{for } N > 4\cdot10^5 \text{ electrons.}$$

The number of nuclear-active particles with energies above 1 GeV was found (2) to depend on shower size N as

$$n_{N.A.}(>1\ \text{GeV}) \sim N^{0.2}$$

$$n_{N.A.}(>1\ \text{GeV}) \sim N\,.$$

Here also the discontinuity was placed at $N = 4\cdot10^5$ electrons. (See Fig. 4).

Both of these quantities are of course variables of type F, *i.e.*, for a given energy per nucleon of the primary, they must vary strictly proportional to the number of nucleons which constitute the pri-

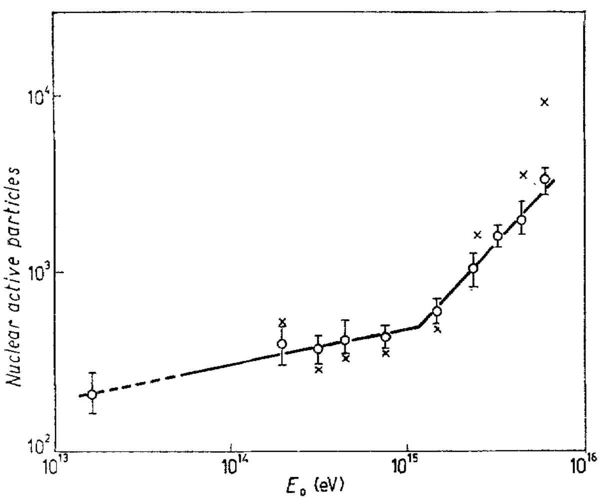

Fig. 4. – The number of nuclear-active particles, as given by NIKOLSKIJ *et al.* (4), as a function of shower size. The numbers on the abscissa represent primary energy in eV, obtained by multiplying the observed number of shower particles by $3\cdot10^9$.

(1) K. N. VAVILOV, F. ESTIGNEEV and S. I. NIKOLSKIJ: *Žurn. Èksp. Teor. Fiz.*, **32**, 1319 (1957).

(2) S. I. NIKOLSKIJ, U. N. VAVILOV and V. V. BATOV: *Dokl. Akad. Nauk SSSR*, **111**, 71 (1956).

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mary particle. They should, according to our model, become nearly proportional to N above the critical shower size N_c.

A special case of an observable of type F (when $\beta = 0$ in eq. (4)) is the atomic weight of primaries. The average atomic weight $\langle A \rangle$ of the primaries, responsible for air showers larger than N_c, should increase with N. If, as now seems likely, high energy nucleons lose only a minor fraction of their energy in meson producing collisions and always emerge as by far the most energetic product of the interaction, it follows that the number of very high energy nucleons in the shower core (at mountain altitude the energies should lie between 10^{12} and 10^{13} eV) constitute a measure of the atomic weight of the primary particle. From the transverse momenta which such nucleons transfer to π-mesons, one can estimate the scattering angles in inelastic collisions; one concludes that when the primary nucleons reach ground they should fall within a few square meters, if their energy ε_0 at the top of the atmosphere exceeded 10^{14} eV. In an apparatus of the type used by GRIGOROV *et al.* (3) and by NIKOLSKIJ and others (4), such primary nucleons shouldmani fest themselves as so-called « cores exhibiting structure ». Since, according to our hypothesis, the average atomic weight of primaries, $\langle A \rangle$, must increase proportional to N for $N > N_c$, we expect a strong increase in the probability of observing such « structure cores » when the shower size passes the critical value. Such a behaviour is for instance indicated by Table I, taken from a paper by NIKOLSKIJ *et al.* (4).

TABLE I.

Shower size N	Number of observations of at least one high energy nucleon in the shower axis	Number of cases where two or more nucleons are observed	
$1.5 \cdot 10^4$	7	2	
$2.9 \cdot 10^4$	10	3	
$5 \cdot 10^4$	8	2	$\approx 30\%$
$8 \cdot 10^4$	6	2	
$1.5 \cdot 10^5$	5	2	
$3.5 \cdot 10^5$	6	5	$\approx 85\%$
$5.8 \cdot 10^5$	1	1	

(3) N. L. GRIGOROV, V. IA. SHESTOPEROV, V. A. SOBINIAKOV and A. V. PODGURSKAIA: *Žurn. Èksp. Teor. Fiz.*, **33**, 1099 (1957).

(4) S. I. NIKOLSKIJ and A. A. POMANSKIJ: *Žurn. Èksp. Teor. Fiz.*, **35**, 618 (1958); and also O. DOVZKENKO, W. ZATSEPIN, E. MURZINA, S. NIKOLSKIJ, I. ROKOBOLSKAYA and E. TURKISH: *Dokl. Akad. Nauk SSSR*, **118**, 899 (1958).

The top curve for the observable of type F in Fig. 3 can be interpreted as representing $\langle A\rangle$, *i.e.* the average atomic weight of primaries responsible for an air shower, as a function of shower size. Below the critical region, for an air shower spectrum whose exponent $\gamma/\alpha=1.5$, it has the value

$$\langle A\rangle \approx A_{\text{Fe}}\frac{P_{\sigma-1,\gamma}}{P_{\sigma,\gamma}} = 7.5 \text{ or } 9.4$$

depending on whether one uses eq. (3*a*) or (3*b*). For larger showers, it increases towards the value $\langle A\rangle = A_{\text{Fe}} = 56$.

The contribution of various groups of elements below the critical region can be easily calculated on the basis of the primary composition given in the Appendix, valid in the emulsion region which borders on, and even slightly overlaps, the air shower region. Assuming a value $\gamma/\alpha=1.5$ the relative contributions are

$$\begin{aligned} \text{H} &= 49\,\% \\ \text{He} &= 24\,\% \\ \text{C, N, O, F,} &= 13\,\% \\ \text{Ne, Mg, Si} &= 7\,\% \\ \text{Fe} &= 7\,\% \end{aligned}$$

3·2. *Observables of type G*. – We now turn to observables of type G, which according to this model should be independent of shower size in the interval $N_c \leqslant N \leqslant 56N_1 \approx 30N_c$, whatever their behaviour below $N=N_c$. There are many well-known experimental results which have a bearing on this question. Especially, the lateral distribution function and the shower age (variation of electron number with altitude) are known to be independent of shower size. Also the energy spectrum of μ-mesons is a quantity of type G and is independent of shower size ([5]).

However, more refined measurements are expected to show a somewhat more complicated behaviour of the average value of observables beyond N_c because of the large gaps in the chemical abundances between $A=1$ and $A=4$ and between $A=4$ and $A=12$. As we go to larger showers and pass the critical size N_c, where the proton contribution vanishes and the helium contribution becomes dominant, the energy per nucleon suddenly drops by a factor four from $2\varepsilon_c$ to $\frac{1}{2}\varepsilon_c$. By the time we reach $N=4N_1$, it will have

(5) O. I. DOVZHENKO, B. A. NELOPO and S. I. NIKOLSKIJ: *Žurn. Ėksp. Teor. Fiz.*, **32**, 463 (1956).

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partially recovered and have reached $\varepsilon = \varepsilon_c$. It will then drop again to $\varepsilon_c/3$. Once more it will recover and reach a value $\varepsilon = \varepsilon_c$ at $N = 12N_1$, etc.

On this picture, we expect that the high energy nucleons in the shower core should be harder, though less numerous, below N_c than they are above N_c and that therefore the absorption coefficient of particles in a shower core, when measured with thick absorbers, should show a sudden increase at the critical shower size N_c and then recover partially at $N = 12N_1 \approx 6N_c$.

Fig. 5 shows the result of measurements by NIKOLSKIJ and POMANSKIJ (4) on the absorption of air shower cores by 230 g cm² of aluminium and carbon absorber. The data have been interpreted by the authors as showing a significant increase in the absorption coefficient at $N \approx 2 \cdot 10^5$ followed by a partial recovery.

The discontinuity in absorption of shower cores as a function of air shower size should also reveal itself as a discontinuity in the barometric coefficient and the zenith angle distribution. Until those mutually related discontinuities have been detected, the observations of NIKOLSKIJ and POMANSKIJ, which constitute perhaps the strongest evidence to date for the existence of a magnetic rigidity cut-off, cannot yet be considered conclusive.

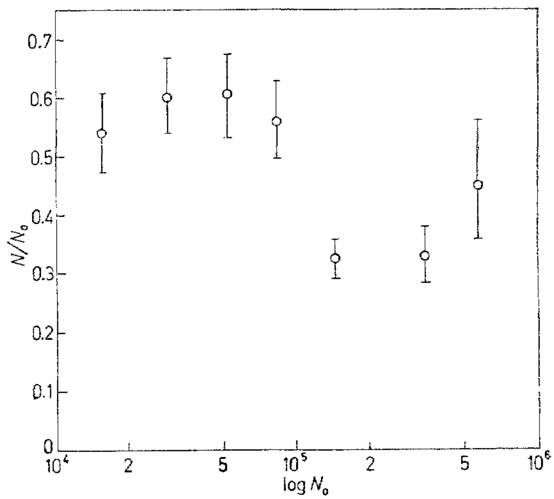

Fig. 5. – Results of NIKOLSKIJ *et al.* (4) showing the absorption of shower particles by 230 g/cm² of carbon plus aluminium as a function of shower size. The ordinate gives the ratio of particles below and above the absorber.

It seems, however, worth-while to point out that, if the interpretation given here is correct, such measurements, with only slightly better resolution, could lead to a determination of the relative abundance of elements among primaries in the energy range close to 10^{15} eV nucleon, *i.e.* an order of magnitude greater than obtainable with photographic emulsions.

One other effect which a hypothetical magnetic rigidity cut-off must produce on an observable of type G should be mentioned here.

A shower size between 2 and $4 \cdot 10^5$ at mountain altitude, where all these discontinuities seem to appear, corresponds approximately to a primary proton energy of $E_0 = 2\varepsilon_c \approx 10^{15}$ eV. Nucleons of such energy should arrive at 650 g/cm² with reduced energy E of order $E_0(1-\alpha)^{650/\lambda}$, where α is the fraction of energy transferred to mesons in each collision and λ the interaction mean free path. If one uses $\lambda = 60\ \text{g/cm}^2$ and $\alpha = 0.3$, one finds $E \approx 2 \cdot 10^{13}$ eV. One should then expect that, although the size of accompanying air showers keeps on increasing, the energy of nucleons in the shower core does not continue to increase when this value is reached. The flux of nuclei of energy larger than some value E, when plotted against the energy E, should show a decrease in the neighbourhood of $2 \cdot 10^{13}$ eV, whose sharpness will however be reduced by statistical fluctuations in the average number of inelastic atmospheric collisions.

Such a drop has been observed by MURZINA *et al.* ([6]) who measured the arrival rate of high energy nuclear-active particles (more than 80% of which occurred in air showers) as a function of energy. The drop seems to occur at $E = 1.5 \cdot 10^{13}$ eV.

3'3. *The size-frequency spectrum of air showers.* – Altogether, the experimental evidence regarding observables of types F and G seems to support the existence of a critical magnetic rigidity in the dominant part of the primary spectrum at a value which corresponds to a proton of energy 10^{15} eV and therefore to a value $\varepsilon_c = 5 \cdot 10^{14}$ eV. It is now necessary to discuss whether the observed size-frequency spectrum of air showers is in conflict with such a hypothesis.

At sea level the critical shower size corresponding to the assumed value of ε_c is about $N'_c = 7 \cdot 10^4$ electrons. The same peculiarities which have been observed at mountain altitude should have their counterpart at sea level at a somewhat smaller critical shower size. However, experimental data of sufficient accuracy are apparently not available.

If there was only one source of primary cosmic rays, then the sea level shower size spectrum should, for all practical purposes, terminate at a size

$$N_m = A_{\text{Fe}} \frac{N'_c}{2^\alpha} \approx 1.8 \cdot 10^6 \text{ electrons}\,.$$

Evidently this is not the case. (See f.i. CLARK *et al.* ([7])).

Let us, therefore, suppose that a second source exists which has the same or a slightly flatter energy spectrum than the first, characterized by an exponent γ', and which has a cut-off at a higher magnetic rigidity. Then, as illustrated in Fig. 2, no break in the differential shower size-frequency spectrum needs to occur, unless F'_0, which is a measure of the intensity of this second source, is smaller than F_0 by a factor of the order of one thousand.

([6]) E. MURZINA, S. I. NIKOLSKIJ and V. I. JAKOLEV: *Žurn. Èksp. Teor. Fiz.*, **35**, 1298 (1958).

([7]) G. CLARK, J. EARL, W. KRAUSHAAR, J. LINSLEY, B. ROSSI and F. SCHERB: *Suppl. Nuovo Cimento*, **8**, 623 (1958).

Thus the differential shower size spectrum may well be continuous and only show a change of slope when there are different primary cosmic ray sources contributing to air showers in different size intervals. At the points corresponding to a cut-off, changes in the slope of the integral shower size spectrum could be quite small. The curves in Fig. 2 illustrate such a hypothetical situation.

KULIKOV and KHRISTIANSEN (8) have drawn attention to an apparent discontinuity in the slope of the integral size-frequency spectrum at sea level, which they place at $N = 8 \cdot 10^5$ electrons. It is, however, not clear as yet how this is connected with the various discontinuities observed at mountain altitude. Apart from the changes in slope of the size frequency distribution which should occur where the proton component of the dominant source cuts off and again in the region where the proton component of a new source becomes dominant, our hypothesis suggests that there should be a general tendency for the size frequency distribution of air showers to become flatter rather than steeper as the shower size increases. For, as long as we retain the existence of power laws as a characteristic feature of cosmic ray accelerators, the source which is unimportant at low energy but becomes dominant at high energy must have a flatter particle spectrum than the source which dominates at lower primary energy. Existing measurements on the air shower frequency spectrum do not contradict this conclusion; rather they seem to support it, although, within the as yet considerable statistical uncertainty, the observed flattening could well be spurious.

4. – Concluding remarks.

The hypothesis presented in this paper emphasizes the important role which the complexity of the primary cosmic radiation plays in the interpretation of air shower data.

Observations with nuclear emulsions show that at least those primaries which give rise to the smaller air showers ($N \leqslant 10^5$ particles) must be composed of comparable numbers of nuclei whose atomic weights (and therefore whose energy per nucleon) differ from each other by large factors. Since the average atomic weight of primaries contributing to the smaller showers ought to be close to $\langle A \rangle = 9.5$, it is clear that the traditional classification of air showers according to the total number of charged particles may not be the best suited one for comprehending the underlying basic nuclear and electromagnetic processes. The « normal » shower of size N is only in about half the cases produced by a nucleon of energy ε, and in about 7 % of the cases by 50 nucleons of energy $\varepsilon/50$. Thus one should expect, especially near the

(8) G. V. KULIKOV and G. B. KHRISTIANSEN: *Žurn. Èksp. Teor. Fiz.*, **35**, 441 (1958).

core, large differences in the properties of showers of equal « size ». The interpretation of fluctuations of observables in showers of a given size in terms of fluctuations in fundamental processes seems hardly possible without taking this basic cause of fluctuations into account.

In order to arrive at a more natural classification of air showers one could start from the current picture that primary nucleons survive the meson producing collisions in the atmosphere and appear as the most energetic nuclear-active particles in the central area of the shower core. If this picture is correct, the nuclear-active particles within one meter from the shower axis could serve as a basis for classification. The events could then be classified according to the energy of the most energetic of these nuclear-active particles. This energy is presumably related to the *energy per nucleon* of the incident primary and determines all shower properties as well as the distribution of shower particles except for a scale factor. The *number* of nuclear-active particles of comparable energy close to the axis is presumably related to the atomic number of the incident primary and may furnish the scale factor.

Above energies of order 10^{14} to 10^{15} eV, nuclear emulsion data provide no guidance as to the nature of primaries. Certain measurements at mountain altitude show variations of shower properties with shower size which indicate an increasingly important role of heavy primaries in the energy region between 10^{15} and a few times 10^{16} eV. Such a preponderance of heavy nuclei could be interpreted as a magnetic rigidity cut-off in the source which contributes the bulk of primaries below 10^{15} eV. While experimental evidence in favour of this hypothesis is not yet conclusive, it appears to be fairly strong. There also seems to exist no difficulty in reconciling the hypothesis with all other observations on extensive air showers which have been reported so far.

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An abbreviated version of this coork has been published in the *Proceedings of the Moscow Cosmic Ray Conference*, Vol. VIII, 157 (July 1960). [Considerations of a similar nature can also be found in the work of UEDA, *Progr. Theor. Phys.*, **24**, 1231 (1960)]. The complete article was not sent for publication, because many of the quoted experimental results were still controversial at that time. The author wants to express his gratitude to the members of the Air Shower Group of the Tata Institute of Fundamental Research, in particular to Dr. B. V. SREEKANTAN, Dr. S. NARANAN and Mr. A. SUBRAMANIAN who have pointed out to him that the salient features of the early experiments discussed in this paper are strongly supported by more recent and partially unpublished investigations. Publication of the manuscript seems, therefore, now appropriate. The author also would like to express his gratitude to Prof. H. J. BHABHA and Prof. M. G. K. MENON for the hospitality extended to him during his recent stay in Bombay.

APPENDIX

The composition and the energy distribution of the primary cosmic radiation in the mass interval

$$1 \leqslant A \leqslant 56\,,$$

and in the energy in interval from about $1.5A$ GeV to about 4500 GeV, is represented fairly accurately by

$$n(\varepsilon, A)\,\mathrm{d}\varepsilon\,\mathrm{d}A = \frac{F_0}{A^{\sigma+1}\varepsilon^{\gamma+1}}\,\mathrm{d}\varepsilon\,\mathrm{d}A\,,$$

and the corresponding integral spectrum

$$\mathcal{N}_{A_1,A_2}(>\varepsilon) = \frac{F_0}{\gamma\sigma\varepsilon^{\gamma}}\left[A_1^{-\sigma} - A_2^{-\sigma}\right]\,, \tag{8}$$

TABLE II.

Energy per nucleon (GeV)	Group of elements	Particles/m² s sr		References
		calc.	obs.	
Texas $\varepsilon_{\mathrm{H}}=4.9$ $\lambda=41°$, $\varepsilon_A=2.6$	H	610	570 ± 50	McDONALD [9]
	He	88.5	90 ± 9	
	C, N, O, F	7.1	7.5 ± 0.65	APPA RAO *et al.* [10]
	$Z \geqslant 10$	3.45	2.2 ± 0.35	
Guam $\varepsilon_{\mathrm{H}}=16.1$ $\lambda=3°$, $\varepsilon_A=8.3$	H	116	115 ± 12	McDONALD [9]
	He	17.3	18 ± 2	
	C, N, O, F	1.38	0.93 ± .08	JAIN *et al.* [11]
	$Z \geqslant 10$	0.68	0.36 ± .06	
$\varepsilon_{\mathrm{H}}=1600$	H	0.19	0.29 ± 0.07	LAL [12]
$\varepsilon_A=1600$	He	0.01	0.035 ± 0.025	
$\varepsilon_{\mathrm{H}}=4500$	H	0.045	0.04 ± 0.015	KAPLON and RITSON [13]
$\varepsilon_A=4500$	He	0.0026	~0.004	

(9) F. B. McDONALD: *Phys. Rev.*, **109**, 1367 (1958).
(10) M. V. K. APPA RAO, S. BISWAS, R. R. DANIEL, K. A. NEELAKANTAN and B. PETERS: *Phys. Rev.*, **110**, 751 (1958).
(11) P. L. JAIN, E. LOHRMANN and M. W. TEUCHER: *Phys. Rev.* **115**, 654 (1959).
(12) D. LAL: *Proc. Ind. Acad. Sci.*, **38**, 93 (1953).
(13) M. F. KAPLON and D. M. RITSON: *Phys. Rev.*, **88**, 386 (1952).

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where F_0 is a constant, and ε is the energy per nucleon including rest mass.

This formula does not actually reproduce the detailed chemical composition but gives a smoothed-out distribution of elements. However, it does represent the various mass groups correctly, provided one uses the following values for the constants:

$$F_0 = 1.7 \cdot 10^4 ,$$
$$\sigma = 2.0 ,$$
$$\gamma = 1.40 ,$$

and expresses ε in GeV and $\mathcal{N}$ in particles/m² s sr.

In Table II, flux values calculated with the help of eq. (8), are compared with measured values. (« H » in the Table stands for all elements with mass number $1 \leqslant A \leqslant 3$; « He » stands for all elements with mass numbers $4 \leqslant A \leqslant 11$ and includes therefore the elements Li, Be, and B. But hydrogen and helium are without doubt by far the dominant elements in their respective groups.)

The formula representing the spectrum must be slightly modified in the energy range which corresponds to air showers. The size frequency spectrum of air showers has an exponent which varies slowly with size. For showers of $N \approx 10^5$ electrons the exponent is $\gamma/\alpha \approx 1.5$, which probably corresponds to an exponent for the primary spectrum, $\gamma \approx 1.65$.

A very slowly increasing exponent of the form suggested by COCCONI ([14]) could probably represent the primary spectrum over the entire range of cosmic ray energies above $\sim 1.5A$ GeV adequately within the existing errors of measurement. This small dependence of exponent on energy has been neglected in this paper, since it does not affect the validity of the arguments.

The only data on chemical composition which are available in the energy range $10^3 < E < 10^{15}$ eV come from the observation of very large meson showers in nuclear emulsions. No accurate tabulation has been made; but roughly 50 % of the observed events are due to protons, and among the largest of the observed meson showers the very heavy primaries seem to be dominant.

([14]) G. COCCONI: *Suppl. Nuovo Cimento*, **8**, 560 (1958).

RIASSUNTO (*)

Si esamina l'ipotesi che sorgenti separate di particelle dei raggi cosmici di energie fortemente differenti predominino in differenti regioni di energia. In particolare si discutono le seguenti questioni: possono tali sorgenti separate essere predominanti in intervalli di energie dello spettro primario delle particelle adiacenti senza produrre variazioni apprezzabili della pendenza o discontinuità nella distribuzione grandezza-frequenza degli sciami atmosferici? Possono, tuttavia, tali contributi separati, se esistono, essere distinti sperimentalmente con le tecniche esistenti? I dati disponibili sugli sciami

(*) *Traduzione a cura della Redazione.*

estesi dell'aria forniscono prove a favore o contro tale ipotesi? Sembra che se tali sorgenti separate esistessero ed anche se differissero in energia per fattori fino a mille, non si dovrebbero avere scostamenti significativi dalla continuità della relazione grandezza-frequenza degli sciami dell'aria, purchè tutti gli elementi chimici componenti la radiazione primaria abbiano spettri identici se espressi in termini della rigidità magnetica delle particelle o di energia per nucleone, e purchè, inoltre, la composizione chimica del fascio di particelle accelerato non si scosti troppo radicalmente dalla sua composizione ad energie inferiori. Mentre in tali condizioni anche un taglio infinitamente netto della rigidità delle particelle derivanti dalla più forte delle sorgenti agenti non produrrà un'alterazione significativa nello spettro grandezza-frequenza degli sciami atmosferici, produrrà, tuttavia, discontinuità e irregolarità nella dipendenza di alcuni altri parametri dalla grandezza dello sciame. Da vari osservatori sono state riferite irregolarità per grandezze degli sciami corrispondenti a un'energia primaria di circa 10^{15} eV, che finora non hanno trovato una spiegazione adeguata. Ognuna di tali osservazioni sembra sostenere l'ipotesi che un taglio abbastanza netto avvenga nella sorgente che fornisce il massimo numero di particelle al disotto di tale energia. Si possono predire ulteriori irregolarità misurabili che, tuttavia, non sono ancora state osservate. Le considerazioni che conducono a questa ipotesi mettono in evidenza l'importanza che la complessità della composizione chimica della radiazione cosmica primaria deve avere nell'interpretazione dei fenomeni degli sciami atmosferici. In vista della complessità risultante dell'origine degli sciami di una data grandezza si pensa che una classificazione degli sciami dell'aria secondo la massima energia delle particelle nuclearmente attive giacenti nell'asse sia più significativa per la comprensione dei vari processi nucleari ed elettromagnetici in atto, della consueta classificazione basata sul numero totale delle particelle dello sciame.

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