Lense-Thirring precession of a single vortex in an atomic superfluid

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Frame dragging and gravitomagnetism are genuine post-Newtonian effects and provide stringent tests of general relativity theory. Of those effects, the Lense-Thirring precession remains so far unobserved despite the significant effort that was put in international collaborative space missions like Gravity Probe B.

INTRODUCTION

Ultracold atomic systems have proven themselves as a versatile simulator of different classes of quantum mechanical systems (REFS), and as a platform that bespoke novel systems can be designed for (REFs). Perhaps surprisingly, ultracold atoms can also be used to simulate effects from general relativity (GR). In that analogue, the phase of the superfluid plays the role of an effective spacetime experienced by the phase fluctuations (Carusotto). This analogue gravity paradigm was used for the first ever measurement of Hawking radiation emanating from a hydrodynamic black-hole horizon (Steinhauer). The class of GR effects considered so far are mostly due to space-space or time-time terms of the effective spacetime metric, e.g., black-holes, dynamical Casimir effect (Carusotto, Galitski). Only recently it was shown how it is possible to include non-trivial space-time components in the metric (Giulio) that open up the possibility of accessing a different flavour of GR phenomena.

Novel effects arise in post-Newtonian gravitational theories due to the dynamic nature of the spacetime continuum. Here we focus on the Lense-Thirring precession (LTP) that is induced by the gravitomagnetic field of a slowly revolving mass to a satellite orbiting around it. Space based experiments have measured the LT precession of various bodies to a precision of 25%. For typical orbits it is of the order of a few marcsec/year. Gravity Probe B makes clever use of a gyroscope to null the Thomas precession and disentangle de Sitter precession from the LTP.

ANALOGUE SYSTEM

Here, we sketch the pathway from the superfluid hydrodynamics as described by the Gross-Pitaevskii equation (GPE) to GR and ultimately to a simple picture based on Maxwell's equations of electromagnetism (EM).

GPE to GR connection

To get to the effective GR representation of the hydrodynamic density-phase equations

$$\partial_t n + \nabla(n\mathbf{v}) = 0,$$
$$\hbar\partial_t\theta = -\frac{\hbar^2}{2m}(\nabla\theta)^2 - gn - V_{ext} - V_q,$$

where V_q is the quantum pressure term (and the rest of the symbols as usual) we proceed as Section 1.2.2 from Understanding Hawking radiation from simple models of atomic Bose-Einstein condensates. Doing a Bogoliubov expansion we get equations for the density and phase fluctuations n_1, θ_1. Ignoring the quantum pressure term the density fluctuations decouple and we get

$$-(\partial_t + \nabla\mathbf{v}_0)\frac{n}{mc^2}(\partial_t + \mathbf{v}_0\nabla)\theta_1 + \nabla\frac{n}{m}\nabla\theta_1 = 0$$

which can be written in a matrix form with the proper identification of matrix elements and then identified as a curved space wave equation $\Box\theta_1 = 0$ with the effective metric

$$g_{\mu\nu} = \frac{n}{mc}\begin{pmatrix} -(c^2 - \mathbf{v}_0^2) & -v_0^i \\ -v_0^j & \delta_{ij} \end{pmatrix}.$$

Most of the interesting physics on the above metric is driven by the time-time component and happens at the interface of regions where $|v_0|$ is smaller or larger than c.

We are interested in effects solely due to the space-time components v_0^i. For these to be non-trivial the space-time components are required to be space-dependent; enter artificial gauge fields. In particular using the density dependent artificial gauge field derived in Simulating an interacting gauge theory with ultracold Bose gases the authors of Curved spacetime from interacting gauge theories proved how this is possible. In brief, we consider a Raman-coupled binary mixture with Rabi frequency Ω. Including the gauge term in question the starting equation of motion is

$$i\hbar\theta_t\psi = \left(\frac{(\mathbf{p}-\mathbf{A})^2}{2m} + \mathbf{a}_1\cdot\mathbf{j} + W + gn\right)\psi,$$

where $\mathbf{A} = \mathbf{A}^{(0)} \pm \mathbf{a}_1 n_\pm(\mathbf{r})$ is the vector potential comprised of the single particle contribution $\mathbf{A}^{(0)} = \hbar/2\nabla\theta$

and a density-dependent term $\mathbf{a}_1 = \nabla\theta(g_{11} - g_{22})/8\Omega$, $\mathbf{j}$ is a (chiral) current, and $W = |\mathbf{A}^{(0)}|^2/2m$ the scalar potential. Working through the hydrodynamic density-phase representation of the above Hamiltonian and its Bogoliubov expansion we can show that the space-time components of the effective metric are $-(v_0 \pm v_\alpha)^i$, where $v_\alpha = n_0\alpha_1/m$.

In that paper they proceed by considering an effective Kaluza-Klein theory in 2+1 dimensions. We take a more pedagogical approach and relate it to a Kerr metric from which we derive the low-energy Maxwell equations for the effective EM theory.

GR to EM connection

For weak fields and non-relativistic velocities where spacetime is approximately static there is no need to deal with the full spacetime metric; we can used a linearised version of Einstein's equations akin to Maxwell's equations for electromagnetism. The electromagnetic scalar $\Phi = 1/2(g_{00} - 1)c^2$ and vector $A_i = g_{0i}c^2$ potentials can be computed from the spacetime metric tensor $g_{\mu\nu}$ and the speed of light c, and satisfy a set of equations similar to Maxwell's equations for electromagnetism albeit with a charge density $-G\rho$ where G is the gravitational constant and ρ the mass density.

Once both the electric and magnetic fields are know, the force acting on a particle in this space takes the simple form of a Lorenz force

$$\mathbf{F} = m\mathbf{E} + \frac{m}{c}\mathbf{v} \times \mathbf{H}$$

where $\mathbf{H} = \nabla \times \mathbf{A}$ is the gravitomagnetic field and is only non-zero for space-dependent g_{0i}. The newtonian term is the gravitoelectric field in this language which leads to a simple central force.

From this force, it's straightforward to get to the Larmor and Lense-Thirring precessions. The Larmor precession corresponds to $\mathbf{H} = const.$ but the gravitomagnetic field of a rotating body is that of a magnetic moment. This is what leads to LTP in addition to Larmor precession. The simplest presentation is Elementary derivation of the Lense-Thirring precession but Gravitomagnetic effects shows the connection to the Kerr metric a bit better.

Giulio only showed the Larmor precession, aka a constant magnetic field. Perhaps there is no realistic way of getting a field that looks like that of a dipole and hence this project is not going anywhere.

MEASUREMENT

What object is sensitive to phase? Perhaps a vortex core precession Vortex precession in Bose-Einstein condensates: observations with filled and empty cores? Otherwise check for phonons as in Giulio's paper. Also, plot the space-time correlator $g_2(r, r'; t)$.

NOTES

- Why is it that the time-time component of the metric is responsible for the Hawking radiation? I would have thought it's a space-space effect?
- Why do I need that density gauge field. Go through the math to see where does a naive NIST Raman-coupled gauge term drops out in the final equations.
- Although it is straightforward to derive the analogy in the non-relativistic limit the u/c term can be ≈ 1 in ultracold gases where c is the speed of sound. Will this amplify the magnitude of the LT precession and make it easier to detect?
- I suspect all of this can be derived analytically without any reference to gravity, much like Sandro has done in Diffused vorticity and moment of inertia of a spin-orbit coupled Bose-Einstein condensate.
- Tapio Simula's paper Gravitational vortex mass in a superfluid seems pretty close to some of these ideas.
- There is also a lot of useful things about precession in Sandro's book p.259-265.
- Work honestly through all the math.
- Chatting to Iacopo. Look into work of Solnyshkov and Malpuech for vortices in gauge fields and analogue gravity in polaritons. For example, Analog Kerr Black hole and Penrose effect in a Bose-Einstein Condensate seems pretty close to what I'm thinking. The project could work with rotating Rabi-coupled BECs like Luca is looking into. Rotation might be what I need to, get a $1/r$ profile.

BIBLIOGRAPHY

Analogue spacetimes from nonrelativistic Goldstone modes in spinor condensates

The influence of spacetime curvature on quantum emission in optical analogues to gravity