① If we know the plaintext and ciphertext, how can we find the key?

To decrypt a Hill-Cypher with a $n \times n$ key requires determining n^2 entries (either of the key matrix or the key inverse matrix).

If we have the mapping between plaintext and ciphertext for n-letter blocks, we can determine the key entries. The mapping allow us to establish n systems of linear congruences, each with n equations involving n unknowns.

Let's suppose we have the message of n letters and its encryption. Then, we can convert the message to a vector of integers $(v_1 \dots v_n)$ and also the encrypted message $(u_1 \dots u_n)$.

Since,

$$\begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} = \begin{pmatrix} k_{1,1} & \dots & k_{1,n} \\ \vdots & & \vdots \\ k_{n,1} & \dots & k_{n,n} \end{pmatrix} \begin{pmatrix} u_1 \\ \vdots \\ u_n \end{pmatrix}$$

we get n equations of the form: $v_i = \sum_{j=1}^{n} u_j \cdot k_{i,j} \quad \forall i = 1, \dots, n$

These are equations of congruences mod 26 that we know how to solve.

② Is this possible if we only know the ciphertext?

Without knowing the plaintext, the only way to determine the key matrix would be brute-force. This means do the product of every possible key matrix inverse and the cipher vector and checking if this produce a plaintext that looks like a valid message.

This doesn't seem possible, since the number of possible key matrices is 26^{n^2}, where n is the dimension.