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$$\boxed{y'' - y = f(x) \quad \begin{cases} y'(0) = 0 \\ y'(2) + y(2) = 0 \end{cases}} \longrightarrow \text{determine the green function.}$$

- $y'' - y = 0 \rightarrow \lambda^2 - 1 = 0 \longrightarrow y = c_1 e^x + c_2 e^{-x}$
  $\lambda = \pm 1$

From the initial conditions we have

1) $y'(0) = 0 \longrightarrow \varphi_1 = c_1 e^x + c_2 e^{-x} \longrightarrow \varphi_1' = c_1 e^x - c_2 e^{-x}$

$$\varphi_1'(0) = c_1 - c_2 = 0 \iff c_1 = c_2 \longrightarrow \boxed{\varphi_1 = e^x + e^{-x}}$$

2) $y'(2) + y(2) = 0 \longrightarrow \varphi_2 = c_1 e^x + c_2 e^{-x} \longrightarrow \varphi_2' = c_1 e^x - c_2 e^{-x}$

$$\varphi_2'(2) + \varphi_2(2) = c_1 e^2 - \cancel{c_2 e^{-2}} + c_1 e^2 + \cancel{c_2 e^{-2}} = 2c_1 e^2 = 0 \iff c_1 = 0$$

$$\boxed{\varphi_2 = e^{-x}}$$

To determine the Green function, we know that

$$G(x,s) = \begin{cases} a(s)\,\varphi_1(x) & x < s \\ b(s)\,\varphi_2(x) & x > s \end{cases}$$

Since G is continuous,

- $a(s)\,\varphi_1(s) = b(s)\,\varphi_2(s) \longrightarrow a(s)\cdot(e^s + e^{-s}) = b(s)\,e^{-s}$
- $b(s)\,\varphi_2'(s) = a(s)\,\varphi_1'(s) + 1 \longrightarrow a(s)(e^s - e^{-s}) = -b(s)e^{-s} - 1$

$$a(s)\,(2e^s) = -1 \rightarrow a(s) = -\frac{1}{2}e^{-s}$$

$$b(s) = \frac{-\frac{1}{2}\cancel{e^{-s}}(e^s + e^{-s})}{\cancel{e^{-s}}} = -\frac{1}{2}(e^s + e^{-s})$$

Then, the green function would be

$$G(x,s) = \begin{cases} -\frac{1}{2}e^{-s}(e^x + e^{-x}) & x < s \\ -\frac{1}{2}(e^s + e^{-s})e^{-x} & x > s \end{cases}$$