POLITECHNIKA GDAŃSKA
WSTEP DO MODELOWANIA STOCHASTYCZNEGO

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Computer analysis of asymptotic properties of trajectories of dynamic systems with discrete time.

by

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Abstract: Consider the properties of the trajectory of a dynamic system $\phi(x) = 4x(1-x), x \in [0, 1]$.

1. Provide a theoretical description of the issue and describe the procedures performed.
2. Based on the theoretical introduction, perform a computer analysis of the trajectory for several starting points:
   (a) draw histograms determined by the initial sections of the trajectories with increasingly larger lengths of many points simultaneously
   (b) draw (on a histogram plot) a density plot of the invariant measure
   (c) draw a graph of the empirical distribution function
3. Discuss the results.

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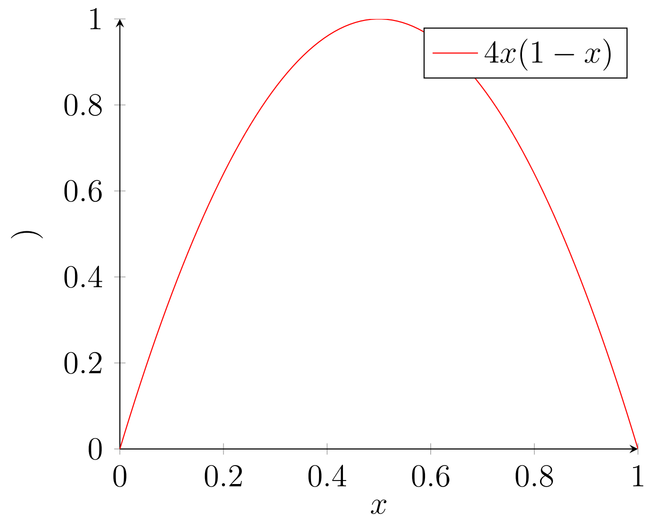
1 Description of the issue

Let $[0, 1]$ be a metric space and $\phi : [0, 1] \to [0, 1]$ a continuous mapping defined by the equation $\phi(x) = 4x(1-x)$. This function represents the evolution of the system over discrete time steps. Then the pair $([0, 1], \phi(x))$ is called dynamic system with discrete time (discrete dynamic system).



Now we are calculating fixed points, those which comply that $\phi(x) = x$.

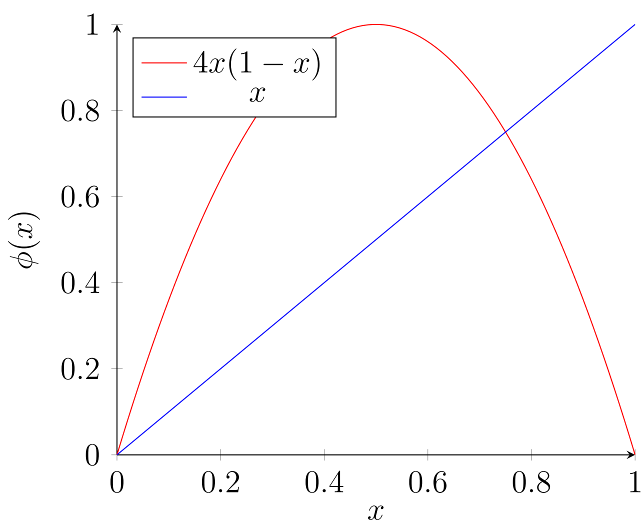
$4x(1 - x) = x$

$4x - 4x = x$

$4x - 3x = 0$

$x(4x - 3) = 0$

$x = 0 \vee x = \frac{3}{4}$



Now we are calculating the derivative of ϕ in order to analyze the stability of fixed points.

$$\phi'(x) = 4(1-x) - 4x = 4 - 8x$$

$$|\phi(0)| = 4 > 1$$

$$|\phi(\frac{3}{4})| = |-2| > 1$$

Using the Differential criterion of fixed point stability, we conclude that both fixed points are unstable. This is because a derivative greater than 1 indicates an amplification of small perturbations, leading to a rapid divergence from the fixed point.

2 Histograms

For drawing the histograms we are considering several initial values starting from 0.001 to 1 with a step of 0.001.

In the graphics below we can see the histogram of initial frequency and initial density.

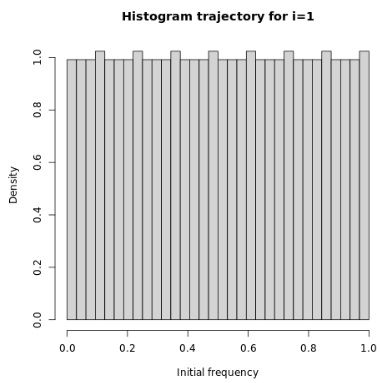


Figure 1: Frequency histogram of the trajectory for $i = 1$

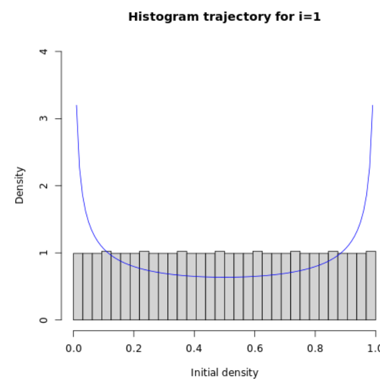


Figure 2: Density histogram of the trajectory for $i = 1$

For $i = 2$, the histograms are the number and density of values obtained on the first iteration of our system for the initial conditions introduced.

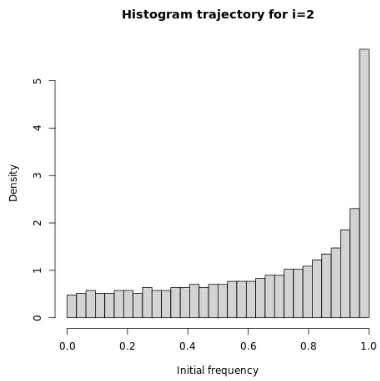


Figure 3: Frequency histogram of the trajectory for $i = 2$

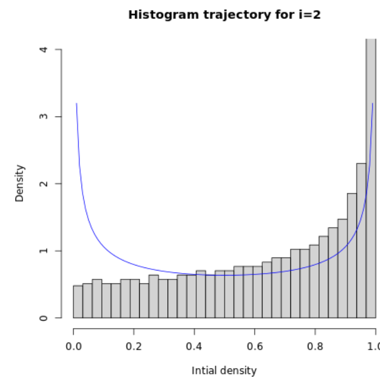


Figure 4: Density histogram of the trajectory for $i = 2$

In the graphics below we can see the frequency histogram and density histogram of trajectory for $i = 3$

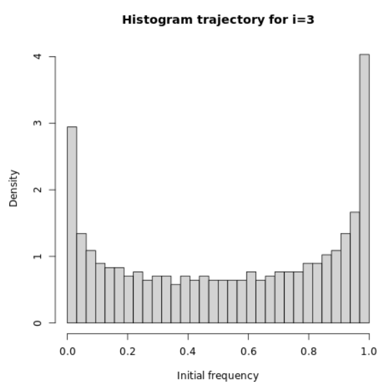


Figure 5: Frequency histogram of the trajectory for $i = 3$

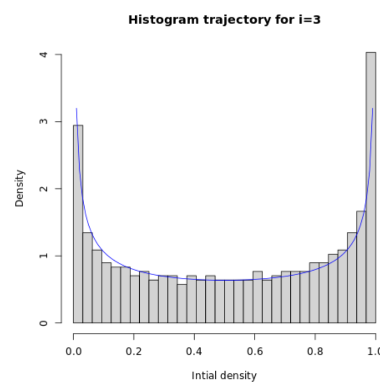


Figure 6: Density histogram of the trajectory for $i = 3$

Now we will plot the aproximations after several ranges of iterations: after 1, 10, 100 and 1000 iterations. Then the density histograms, together with the approximated boundary system seem like these:

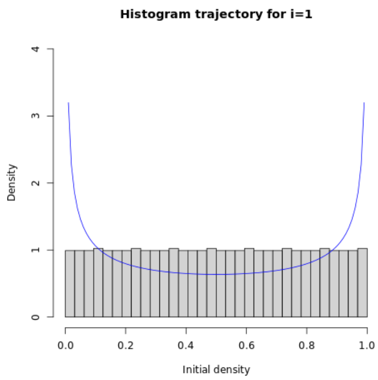


Figure 7: For $i = 1$

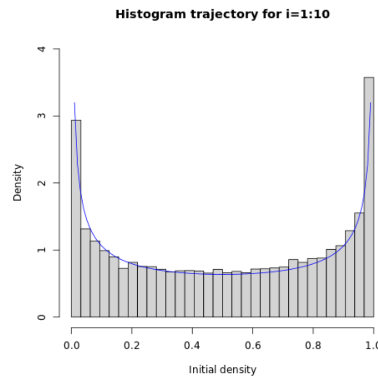


Figure 8: For $i = 1:10$

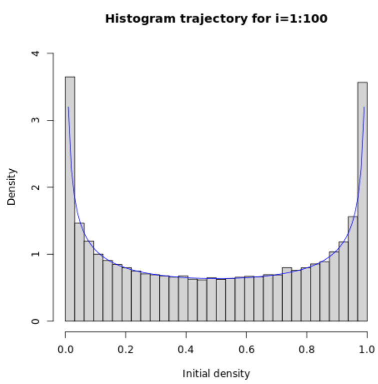


Figure 9: For $i = 1:100$

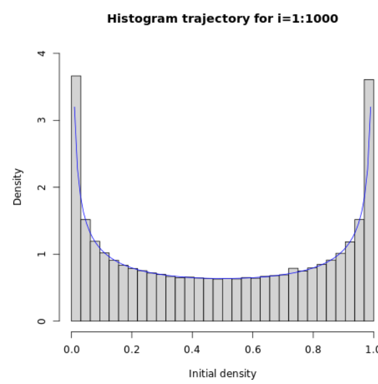


Figure 10: For $i = 1:1000$

3 Distribution graphs

In the graphic below 11 we can see the empirical distribution graph for $i = 1 : 100$. At each observed time point, the EDF indicates the cumulative proportion of trajectories that have reached a state less than or equal to the observed state.

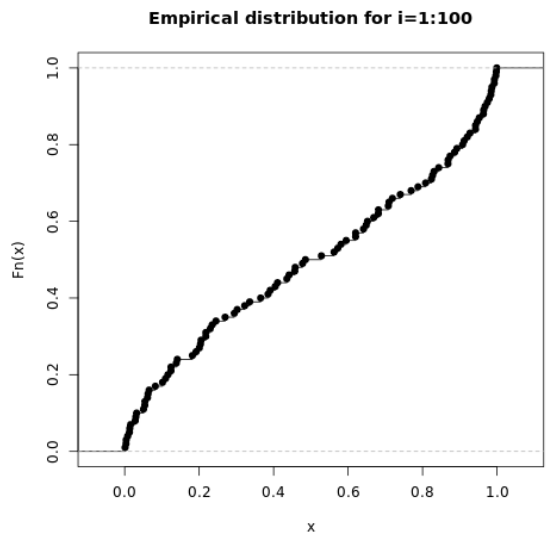


Figure 11: Empirical distribution function

4 Interpretation of the graphs

In the histograms we can clearly see that as time evolves, the trajectories tend to converge at the extremes of the function, rather than at the fixed points. This occurs because the fixed points are unstable in the dynamic system.

We can also check this with the plot of the empirical distribution. We can observe that the beginning of the function has a steep slope as well as the end, due to the accumulation of points. In contrast, the function is more linear in the middle part.