$$\frac{dx}{xz} = \frac{dy}{yz} = \frac{dz}{xy\sqrt{z^2+1}}$$

$$\frac{dx}{xz} = \frac{dy}{yz} \Longrightarrow \frac{dx}{x} = \frac{dy}{y} \Longrightarrow \ln|x| = \ln|y| + \tilde{c}_1$$

$$\Longrightarrow x = c_1 y \quad , \quad c_1 > 0$$

$$\frac{dx}{xz} = \frac{dz}{xy\sqrt{z^2+1}} \Longrightarrow y\,dx = \frac{z}{\sqrt{z^2+1}}\,dz$$

$$\Longrightarrow yx = \sqrt{z^2+1} + c_2$$

$$\begin{cases} x = c_1 y \quad , \quad c_1 > 0 \\ yx = \sqrt{z^2+1} + c_2 \quad , \quad c_2 \in \mathbb{R} \end{cases}$$