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$$\boxed{x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = z^2(x-3y)}$$

We write the so-called system of characteristics:

$$\frac{dx}{x} = -\frac{dy}{y} = \frac{dz}{z^2(x-3y)}$$

$$\frac{dx}{x} = -\frac{dy}{y} \Rightarrow \ln|x| = -\ln|y| + \tilde{c_1} \Rightarrow x\cdot y = c_1 \quad , c_1 > 0$$

$$\frac{dx+3dy}{x-3y} = \frac{dz}{z^2(x-3y)} \Rightarrow x+3y = -\frac{1}{z} + c_2 \Rightarrow c_2 = x+3y+\frac{1}{z}$$

Then $x+3y+\frac{1}{z} = f(x\cdot y)$; $z = \dfrac{1}{f(xy)-x-3y}$

We have that $x=1$ when $yz+1=0$.

If $y = -\frac{1}{z}$ $\begin{cases} c_1 = y \\ c_2 = 1+3y-y = 1+2y \end{cases}$ and $c_2 = 1+2\cdot c_1$

Substituting $xy = c_1$ and $x+3y+\frac{1}{z} = c_2$ we get

$$\boxed{x+3y+\frac{1}{z} = 1+2xy}$$